# Infrared observations of binary stars - I 

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Summary. JHKLM photometry is presented for the close binary systems W UMa, USge and $\beta$ Lyr, and two systems, $\eta$ Gem and RRUMi, containing late-type variable components. The results are analysed in terms of the constraints they place upon mass transfer.

## 1 Introduction

Mass transfer is an important determinant of evolution in all close binary systems. Its presence is indicated by emission lines, light curve distortion due to both absorption and scattering by gas streams, polarization, and changes in orbital period. Observations in the ultraviolet and infrared also reveal substantial continuum distortion due to free gaseous components.

Where the agents of emission reside within eclipsing binary systems, infrared excesses may vary with phase. An infrared investigation of such systems may therefore yield substantial information on the location, size, density and temperature of dust and gas components. Similarly, for stars with intrinsically high mass loss rates, close companions may modify the rate of mass loss, flow velocity, or grain formation, which may again change infrared emission characteristics, and allow a deeper understanding of the underlying mechanisms of the mass loss process itself.

In the present discussion we present new infrared observations of a wide range of binary systems. They include the prototypes of the WUMa and $\beta$ Lyr systems, and USge, a representative Algol-type binary, the prototype for which has been discussed in an earlier paper (Sanchez Magro et al. 1977). Two systems (RR UMi, $\eta \mathrm{Gem}$ ), with late-type variable components which could be expected to be losing mass in the absence of a close companion, are also included. The infrared data were obtained on the 60 -in Infrared Flux Collector on Tenerife, using an InSb detector cooled by liquid nitrogen for the $J H K L M$ results, and a
$\mathrm{Cu}: \mathrm{Ge}$ detector cooled by liquid helium for $N$ band measurements. Errors in the photometry are typically less than 0.03 mag ; particular results with errors approaching 0.1 mag are given to one decimal place.

## 2 W Ursae Majoris

W UMa is a shallow contact binary of type W, according to Binnendijk's (1970) designation (Worden \& Whelan 1973). Lucy (1968) and others (cf. Wilson 1978, and references mentioned therein) have developed a model for such systems in which the component stars have a common convective envelope, within which energy is transferred between components. Application of this model to WUMa (Worden \& Whelan 1973) leads to reasonably satisfactory results, and in particular allows an explanation of the peculiar mass-luminosity law for this system ( $L \propto M^{0.8}$ ).

W UMa systems (including the prototype) are in the great majority of cases characterized by changing period. Struve \& Horak (1950), and Worden \& Whelan (1973), have both noted Ca II $K$ line emission in W UMa, and Kuhi (1964) has observed a flare. In this latter respect W UMa is probably again not unique (Huruhata 1952; Eggen 1948). Rigterink (1972) has further interpreted distortion in the light curve as due to gas streams, and Oshchepkov

Table 1. Photometry of W UMa.

| J |  |  | H |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| JULIAN DATE 24+ | PHASE | MAGNITUDE | JULIAN DATE $24+$ | PHASE | MAGNITUDE |
| 43295.411 | . 785 | 6.57 | 43295.678 | . 585 | 6.26 |
| 43295.450 | . 902 | 6.77 | 43295.447 | . 893 | 6.40 |
| 43295.474 | . 974 | 7.08 | 43295.472 | . 968 | 6.71 |
| 43298.434 | . 846 | 6.62 | 43298.436 | . 852 | 6.34 |
| 43298.455 | . 909 | 6.77 | 43298.450 | . 894 | 6.43 |
| 43299.423 | . 810 | 6.61 | 43299.418 | . 795 | 6.28 |
| 43299.449 | . 888 | 6.71 | 43299.448 | . 885 | 6.35 |
| 43159.641 | . 846 | 6.75 | 43159.644 | 855 | 6.48 |
| 43159.710 | . 053 | 7.39 | 43159.705 | . 038 | 7.14 |
| 42850.518 | . 321 | 6.49 | 43300.388 | . 703 | 6.29 |
| 42850.540 | . 387 | 6.56 | 42850.522 | . 333 | 6.17 |
| 42850.629 | . 653 | 6.68 | 42850.545 | . 402 | 6.25 |
| 42850.636 | . 674 | 6.50 | 42850.615 | . 611 | 6.32 |
| 42850.707 | . 887 | 6.71 | 42850.642 | . 692 | 6.24 |
| 42853.597 | . 549 | 6.90 | 42850.716 | . 914 | 6.58 |
| 43300.387 | . 700 | 6.64 | 42853.605 | . 573 | 6.44 |
| 43538.397 | . 080 | 6.80 | 43538.395 | . 074 | 6.45 |
| 43538.431 | . 182 | 6.56 | 43538.435 | . 194 | 6.18 |
| 43538.482 | . 334 | 6.57 | 43538.484 | . 340 | 6.22 |
| 43538.523 | . 457 | 6.92 | 43538.525 | . 463 | 6.59 |
|  |  |  | 43538.542 | . 514 | 6.67 |
| 43538.550 | . 538 | 6.96 | 43538.552 | . 544 | 6.57 |
| 43538.372 | . 604 | 6.66 | 43538.576 | . 616 | 6.28 |
| 43538.581 | . 631 | 6.59 | 43538.584 | . 640 | 6.22 |
| 43538.590 | . 658 | 6.58 | 43538.599 | . 685 | 6.22 |
| 43539.396 | . 074 | 6.98 | 43539.393 | . 065 | 6.55 |
| 43539.400 | . 086 | 6.84 | 43539.403 | . 095 | 6.43 |
| 43539.447 | . 227 | 6.66 | 43539.449 | . 233 | 6.31 |
| 43539.468 | . 290 | 6.56 | 43539.465 | . 281 | 6.22 |
| 43539.511 | . 419 | 6.81 | 43539.497 | . 377 | 6.42 |
| 43539.548 | . 530 | 7.09 | 43539.524 | . 458 | 6.74 |
| 43539.594 | . 667 | 6.59 | 43539.565 | . 581 | 6.44 |
| 43545.419 | . 127 | 6.77 | 43539.596 | . 673 | 6.24 |
| 43545.432 | . 166 | 6.56 | 43545.416 | . 118 | 6.31 |
| 43545.507 | . 390 | 6.69 | 43545.429 | . 157 | 6.22 |
| 43545.515 | . 414 | 6.76 | 43545.506 | . 387 | 6.35 |
| 43545.524 | . 441 | 6.89 | 43545.513 | . 408 | 6.40 |
| 43545.533 | . 468 | 7.02 | 43545.522 | . 435 | 6.52 |
| 43545.542 | . 495 | 7.13 | 43545.531 | . 462 | 6.65 |
| 43545.852 | . 424 | 7.09 | 43545.541 | . 492 | 6.77 |
| 43545.561 | . 552 | 6.93 | 43545.548 | . 513 | 6.50 |
| 43545.588 | . 633 | 6.65 | 43545.585 | . 624 | 6.33 |
| 43545.603 | . 678 | 6.52 | 43545.606 | . 687 | 6.25 |

Table 1 (continued)

| K |  |  | L |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| JULIAN DATE $24+$ | PHASE | MAGNITUDE | JULIAN DATE $24+$ | PHASE | MAGNITUDE |
| 43295.406 | . 770 | 6.15 | 43159.699 | . 020 | 7.0 |
| 43295.446 | . 890 | 6.32 | 43300.393 | . 718 | 6.20 |
| 43295.470 | . 962 | 6.62 | 42850.535 | . 372 | 6.18 |
| 43298.431 | . 837 | 6.31 | 42850.554 | . 429 | 6.35 |
| 43298.448 | . 888 | 6.38 | 42850.631 | . 659 | 6.10 |
| 43299.415 | . 786 | 6.25 | 42850.631 | . 659 | 6.10 |
| 43299.445 | . 876 | 6.34 | 42850.654 | . 728 | 6.17 |
| 43159.647 | . 864 | 6.43 | 42850.727 | . 947 | 6.48 |
| 43159.703 | . 032 | 6.07 | 42853.0621 | . 621 | 6.13 |
| 43300.390 | . 709 | 6.24 | 43538.442 | . 215 | 6.04 |
| 42850.527 | . 348 | 6.22 | 43538.490 | . 358 | 6.46 |
| 42850.549 | . 414 | 6.29 | 43538.531 | . 481 | 6.63 |
| 42850.621 | . 629 | 6.25 | 43538.546 | . 526 | 6.53 |
| 42850.649 | . 713 | 6.24 | 43538.555 | . 553 | 6.43 |
| 42850.720 | . 926 | 6.57 | 43538.579 | . 625 | 6.18 |
| 42853.613 | . 197 | 6.05 | 43538.588 | . 652 | 6.13 |
| 43538.394 | . 071 | 6.37 | 43538.607 | . 709 | 6.07 |
| 43538.437 | . 200 | 6.10 | 43539.388 | . 050 | 6.58 |
| 43538.487 | . 349 | 6.17 | 43539.408 | . 110 | 6.32 |
| 43538.528 | . 472 | 6.57 | 43539.455 | . 251 | 6.09 |
| 43538.544 | . 520 | 6.62 |  |  |  |
| 43538.553 | . 547 | 6.52 | 43539.457 | . 257 | 6.21 |
| 43538.578 | . 622 | 6.24 | 43539.487 | . 347 | 6.32 |
| 43538.586 | . 646 | 6.17 | 43539.516 | . 434 | 6.52 |
| 43538.605 | . 793 | 6.14 | 43539.571 | . 599 | 6.32 |
| 43539.390 | . 056 | 6.59 | 43539.600 | . 685 | 6.15 |
| 43539.404 | . 098 | 6.36 | 43545.499 | . 366 | 6.47 |
| 43539.452 | . 242 | 6.14 | 43545.509 | . 396 | 6.27 |
| 43539.460 | . 266 | 6.19 | 43545.517 | . 420 | 6.32 |
| 43539.492 | . 362 | 6.37 | 43545.526 | . 447 | 6.43 |
| 43539.520 | . 446 | 6.56 | 43545.535 | . 474 | 6.57 |
| 43539.567 | . 587 | 6.39 | 43545.544 | . 501 | 6.65 |
| 43539.599 | . 682 | 6.15 | 43545.554 | . 531 | 6.61 |
| 43545.426 | . 148 | 6.22 | 43545.580 | . 609 | 6.24 |
| 43545.503 | . 378 | 6.27 | 43545.611 | . 702 | 6.15 |
| 43545.511 | . 402 | 6.33 |  |  |  |
| 43545.519 | . 426 | 6.41 |  |  |  |
| 43545.528 | . 453 | 6.55 |  | M |  |
| 43545.537 | . 480 | 6.69 |  |  |  |
| 43545.546 | . 507 | 6.69 | 43159.681 | 0.966 | 6.4 |
| 43545.557 | . 540 | 6.60 |  |  |  |
| 43545.583 | . 618 | 6.29 |  |  |  |
| 43545.608 | . 693 | 6.18 |  | N |  |
|  |  |  | 42091.715 | 0.983 | >4.31 |
|  | L |  |  |  |  |
|  |  |  | 42092.591 | 0.609 | >3.44 |
| 43295.402 43295.443 | . 758 | 6.13 6.14 |  |  | $5.240{ }^{+1.72}$ |
| 43295.467 | . 953 | 6.46 | 42154.498 | 0.161 | 5.240-0.64 |
| 43298.429 | . 831 | 6.27 | 42157.421 | 0.922 | >4.720 |
| 43298.445 | . 879 | 6.32 |  |  |  |
| 43299.413 | . 780 | 6.29 | 42158.452 | 0.012 | >4.020 |
| 43299.443 | . 870 | 6.35 |  |  |  |
| 43159.651 | . 876 | 6.17 | 42160.365 | 0.746 | >3.460 |

( 1974,1975 ) has found large variations of polarization in W UMa and several other close binary systems, although these have not been confirmed by Piirola (1977). In short, there appears to be some evidence to suggest that WUMa systems in general, and the prototype in particular, may contain free gaseous components.

Observations at $J H K L M$ and $N$ are presented in Table 1, and (excepting $M$ and $N$ bands) plotted in Fig. 1. The primary calibration star was $\nu \mathrm{UMa}$, for which the following magnitudes were adopted:
$J=3.15 ; \quad H=2.99 ; \quad K=2.97 ; \quad L=2.95 ; \quad M=2.95$
$\mu \mathrm{UMa}(N=-0.93)$ was used to calibrate $N$ band measures. Excepting one result, these latter represent $3 \sigma$ upper limits (where $\sigma$ is 1 standard deviation).


Figure 1. Variation of magnitude with phase for W UMa at various wavelengths. The profile of the solid curves at quadrature is estimated by extrapolation from the $V$ band data using G0 intrinsic indices.

Phase was determined from (Cester et al. 1976),
MIN $=\mathrm{JD} 2441004.39769+0.33363696 \mathrm{E}$.
Comparison with more recent timings of minima at JD 2442838 (Baldinelli \& Ghedini 1976 ) indicates $\mathrm{O}-\mathrm{C}$ residuals of $\simeq 0.002 \mathrm{JD}$, corresponding to a discrepancy in phase of $\simeq 0.006$.

The profile of the comparative phase-magnitude curves (solid lines in Fig. 1) is taken from the yellow light curves from Rigterink (1972). The level of these curves at quadrature was determined by assuming both components to be type G0 (Strohmeier 1972), using the Johnson (1966) colour indices, and taking maximum visual magnitude $V=+7.5$ (Breinhorst 1971). A similar fit is obtained if spectral type F8 for primary, and F7 for secondary are assumed (Kopal \& Shapley 1956). No correction for reddening has been applied, but this is unlikely to exceed $A_{\mathrm{v}} \sim 0.1$ magnitudes at $V$ (Worden \& Whelan 1973).

Inspection of the figures indicates no excess emission at quadrature greater than $\sim 0.1$ mag at $\lambda<3.6 \mu \mathrm{~m}$, the results being well represented by the G0 extrapolation from visual wavelengths. This, and consistency in the light curve profiles, suggests that a roughly constant 'excess' of $\sim 0.4 \mathrm{mag}$ at secondary eclipse is to be attributed to the combined effects of changing reflectivity and ellipticity, and the differing effective temperatures of primary and secondary stars (Worden \& Whelan 1973).

For plasmas at temperatures comparable to the stellar components, $\mathrm{H}^{-}$free-free and bound-free emission would be expected to severely distort the optical continuum for modest infrared excesses, but this is not observed. Although shock ionization could lead to significant ionic free-free emission, the present results constrain the mass of ionized plasma to $M_{\mathrm{P}} / M_{\odot} \lesssim 3 \times 10^{-29} V^{1 / 2}$ for electron temperature $T_{\mathrm{e}} \simeq 5800 \mathrm{~K}$ (comparable to the component stars), where $V$ is plasma volume in $\mathrm{cm}^{-3}$. Identifying $V$ with the spherical volume containing the system would then imply the very low upper limit $M_{\mathrm{P}} / M_{\odot} \leqslant 4 \times 10^{-12}$.

## 3 U Sagittae

U Sge is a totally eclipsing semi-detached binary of the Algol type. The respective spectral types of primary and secondary components are B7V + G2III (Naftilan 1976; Hall 1974). The secondary shows evidence of anomalous elemental abundances (Naftilan 1975a, b; 1976), and probably represents the core of the originally more massive component, stripped of its hydrogen envelope. The rate of period increase can be attributed to a mass transfer rate of order $10^{-6} M_{\odot} \mathrm{yr}^{-1}$ (Hall \& Neff 1976).

As for many other Algol-type systems, a circumprimary disk has been detected in U Sge. Huang (1973) has suggested that grain condensation within such disks may lead to observable infrared emission. From work by Gilman (1974) on representative astrophysical condensates, we note that for silicate grains with temperature $T_{\mathrm{gr}}<T_{*}\left(\simeq 10^{4} \mathrm{~K}\right)$, then $\bar{Q}_{\mathrm{abs}}>\bar{Q}_{\mathrm{em}}$, where $\bar{Q}_{\mathrm{abs}}$ is the Planck mean absorption coefficient, and $\bar{Q}_{\mathrm{em}}$ is the corresponding parameter for emission. Since we expect $T_{\mathrm{gr}} \lesssim 10^{3} \mathrm{~K}$, it is clear that the formation zone must be substantially outside the binary for a reasonably hot primary star. Similar difficulties apply to graphite and iron grains. For these latter materials $\bar{Q}_{a b s} \propto T^{1.9}$, whence
$\frac{R}{R_{*}} \simeq \frac{1}{2}\left\{\frac{T_{*}}{T_{\mathrm{gr}}}\right\}^{2.95}$
where $T_{*}, R_{*}$ are stellar temperature and radius, $R$ is the radial distance of grain from star. Although graphite grains are more refractory than silicate (or iron) grains, it again requires an anomalously cool primary before $R / R_{*}$ becomes less than the typical dimensions of a semi-detached Algol-type system like USge. Similar arguments pertain for larger, optically thick grains. There is however a possibility that systems with gaseous rings, such as U Sge, may possess a near infrared excess due to free-free emission.

Near infrared observations of U Sge at $J H K L$ and $M$ are presented in Table 2. The primary calibration star was $\nu$ Her, for which the following magnitudes were adopted
$I=+3.87 ; \quad J=+3.51 ; \quad H=+3.23 ; \quad K=+3.20 ; \quad L=+3.19 ; \quad M=+3.25$.
Phases were determined from:
$\operatorname{MIN}=\mathrm{JD} 2442275.4572+3.380618 \mathrm{E}$.
The period is from Kukarkin et al. (1970), the epoch of initial minimum from Pohl \& Kizilirmak (1975). The O-C residuals between the initial minimum of Kukarkin et al., and Pohl \& Kizilirmak are only JD - 0.0045.

The results are plotted in Fig. 2, together with the predicted behaviour for a B7V +G2III system (solid lines). Comparative dashed lines show the depth of minima at $V$. Intrinsic indices were taken from Johnson (1966). Although the G2III indices are estimates, the relatively slow variability with spectral type at GIII, and the dominance of the primary at visual and infrared wavelengths, preclude errors exceeding $\sim 0.1 \mathrm{mag}$. For similar reasons,

Table 2. Photometry of U Sge.

| JULIAN <br> DATE | PHASE | J | H | K | L | M |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 24+ |  |  |  |  |  |  |

the relative positions of the predicted curves are insensitive to quite severe model changes, and may be regarded with confidence. The small residuals between the extrapolated G2III + B7V fluxes, and observations, then imply a reddening which is small and, for distance $R<356 \mathrm{pc}$ (based on the observed modulus), comparable with the lowest possible value consistent with Fitzgerald's (1968) data. Consistency of the data with a G2III + B7V model also suggests that excesses are small, if present at all, and permits constraints on the physical parameters of any circumprimary disk within the system. For a typical disk, Batten (1973) gives density $N_{\mathrm{e}} \simeq 10^{13} \mathrm{~cm}^{-3}$, mass $M \simeq 10^{-9} \longrightarrow 10^{-8} M_{\odot}$, and radius $R_{\mathrm{D}} \simeq 0.6 a$, where $a$ is the binary semi-major axis. Adopting a lower limit mass, an homogeneous disk of


Figure 2. Variation of magnitude with phase for U Sge at various wavelengths. Solid lines show behaviour for an unreddened G2III + B7V model. Dashed lines show for comparison the depth of minima at $V$. The profile of the model curve is schematic; variability due to reflectivity and ellipticity has not been incorporated.
this kind has ratio of shell thickness to diameter

$$
\Gamma \simeq \frac{M}{2 \pi R_{\mathrm{D}}^{3} N_{\mathrm{e}} m_{\mathrm{H}}} \simeq 0.26
$$

where $m_{\mathrm{H}}$ is the hydrogen atomic mass, and we have taken $a \simeq 10 R_{\odot}$. Although current estimates of disk temperatures tend to be high, comparable with those of the primary components (cf. Günther 1959a, b), their validity is not unquestioned (Batten 1973). We will nevertheless assume disk and primary star temperatures to be the same.

Such a disk would be optically thick at, say, $\lambda=4.8 \mu \mathrm{~m}$, and lead to appreciable distortion of the infrared continuum, well in excess of the limits imposed by the current results. Optically thick components of the accretion disk must therefore have either very low projected surface area or, as suggested by Hall \& Neff (1976), have large optical depth in both the visible and infrared.

If $N_{\mathrm{e}}, M$ are reduced by a factor 50 , so that $\Gamma$ remains constant, then the ratio of optically thin emission from the disk ( $F_{\nu \mathrm{FF}}$ ) to stellar emission $\left(F_{\nu *}\right)$ is, assuming standard relations (Allen 1973),
$\frac{F_{\nu \mathrm{FF}}}{F_{\nu *}}=\frac{4.27 \lambda^{2} \exp \left(-1.43883 / \lambda T_{\mathrm{e}}\right) N_{\mathrm{e}} M g}{R^{2} T_{*} T_{\mathrm{e}}^{1 / 2}}$
where the stellar flux has been approximated by a Rayleigh-Jeans law. $T_{\mathrm{e}}$ is disk temperature, $g$ is the Gaunt factor. Taking $T_{*}=T_{\mathrm{e}}=10^{4} \mathrm{~K}, \lambda=4.8 \mu \mathrm{~m}$ and $R_{*}=4 R_{\odot}$, then gives
$\frac{F_{\nu \mathrm{FF}}}{F_{\nu *}} \simeq 6.2 \times 10^{-2}$
which, broadly speaking, is comparable with the limits imposed by the present results. Thus the product $N_{\mathrm{e}} M$ for the optically thin disk component is constrained to a value $\sim 2.5 \times$ $10^{3}$ times less than that of 'typical' disks as defined by Batten (1973). In reality, circumprimary disks are prone to instability (cf. Batten 1973; Baldwin 1976) and disk-emission features in U Sge are highly transient ( $c f$. McNamara 1951). The value $N_{\mathrm{e}} M$ most probably refers therefore to a period when the disk was less than fully developed. Even as such it is surprisingly low and suggests (with similar evidence for $\beta$ Persei (Sanchez Magro et al. 1977)) that Algol-type systems have, most typically, fairly low optically thin gas components. Indeed, taking a low stream electron density $N_{\mathrm{e}} \sim 4 \times 10^{11} \mathrm{~cm}^{-3}$, and upper limit plasma mass $M \sim 10^{-11} M_{\odot}$, a stream velocity $v \sim 10^{7} \mathrm{~cm} \mathrm{~s}^{-1}$ gives a binary mass transfer rate $d M / d t \sim$ $M v / 2 a \sim 10^{-9} M_{\odot} \mathrm{yr}^{-1}$; three orders of magnitude less than the (time averaged thermal) value found from changing periodicity. This estimate of $d M / d t$ may be elevated however if the stream plasma is preponderantly at a very much lower temperature than supposed above.

## $4 \beta$ Lyrae

$\beta \mathrm{Lyr}$ is the prototype of a relatively common set of binaries, although it is uncharacteristic of the class. The secondary component is several times more massive than the primary, although its spectrum has yet to be detected in the visible. The period is increasing rapidly (Wood \& Forbes 1963) which suggests a mass transfer rate $\sim 10^{-5} \mathrm{M}_{\odot} \mathrm{yr}^{-1}$ (Ziolkowski 1976).

An early and influential model of the system was discussed by Huang (1963), who suggested that a large accretion disk masked the secondary. Devinney (1971), Wilson (1971)

Table 3. Photometry of $\beta$ Lyrae.

| JULIAN DATE | PHASE | J | H | K | L | M |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 24+ |  |  |  |  |  |  |
| 43286.733 | 0.101 | 3.44 | 3.29 | 3.45 | 3.01 | 2.88 |
| 43294.735 | 0.721 | 3.22 | 3.00 | 2.89 | 2.59 | 2.34 |
| 43774.475 | 0.811 | 3.10 | 3.04 | 2.95 | 2.74 | - |
| 43773.424 | 0.730 | 3.12 | 3.02 | 2.93 | 2.73 | 2.47 |
| 43772.472 | 0.656 | 3.20 | 3.10 | 3.01 | 2.81 | 2.58 |
| 43772.538 | 0.662 | 3.20 | 3.13 | 3.01 | 2.81 | 2.61 |
| 43775.451 | 0.887 | 3.28 | 3.18 | 3.07 | 2.91 | 2.62 |
| 43776.359 | 0.957 | 3.57 | 3.46 | 3.34 | 3.13 | 2.89 |

and Kondo et al. (1975) have further suggested that the accretion disk surrounds a black hole, although this now seems improbable (Ziolkowski 1976; Jameson \& King 1978).

Wilson (1974) has more recently suggested that for a mass ratio $M_{2} / M_{1} \simeq 5$ to 6 , the luminosity of the secondary is twice that of the primary. Several mechanisms may account for this apparent underluminosity of the secondary, including overluminosity of the primary, and non-isotropic emission by the secondary. The former possibility arises from work by Stothers (1972), showing the primary to be the remaining helium core of a star which has lost most of its hydrogen-rich envelope to the secondary. Rotational smearing, among other factors, is invoked to explain the absence of a spectrum from the secondary.

We have obtained further $J H K L M$ results for the system and these are presented in Table 3. The primary calibration star was $\alpha$ Lyrae, for which a magnitude 0.00 was adopted at all bands. Phase was determined from
$\mathrm{MIN}=\mathrm{JD} 2439935.86+12.9327 \mathrm{E}$
taken from Rocznik Astronomiczny (1974).
These results (and in particular the earlier data) are broadly comparable with those of Jameson \& Longmore (1976). The $M$ band quadrature observations of these latter authors appear, however, to be inconsistent with their model, the trend of their results at other wavelengths, our model (to be proposed), and the present observations.

Emission at all phases is found to be in excess of the flux expected of a hot, B-type star, and the excess increases with increasing wavelength.

Table 4 gives the results of modelling the observed continuum with a mixture of optically thick and thin thermal sources, together with comparative observational data. Simulated indices are identified by the stellar continuum used ( $\mathrm{RJ}=$ Rayleigh -Jeans ), and the nature of the excess ( $\mathrm{FF}=$ Free-Free) (see preceding discussion). Data near quadrature are primarily extracted from the present, more accurate results, and correspond to two epochs separated by 479 days. The $8.6 \mu \mathrm{~m}$ results at quadrature, and the results at secondary and primary minima, are based primarily on data from Jameson \& Longmore. Their errors at 8.6 and $4.8 \mu \mathrm{~m}$ are however rather large, so that the trends are only indicative. Simulated indices are determined from a combination of optically thin free-free emission, and a stellar continuum mimicking alternately a B8Ib star, and a Rayleigh--Jeans power law (representative of a high, but unspecified temperature). The indices have been further reddened by $E_{B-V}=0.13$ using the van de Hulst curve No. 15 (van de Hulst 1949). From the accuracy of the model fit, we believe this estimate of reddening to be good to $\pm 0.02$, and may be compared to previous estimates of $E_{B-V}=0.065$ (Abt ct al. 1962), and $E_{B-V}=0.15$ (Hack et al. 1975).

Surprisingly, the Rayleigh--Jeans model permits a better fit to the observed data than the B8Ib continuum, although the agreement between model and observations is within errors for both cases. A comparison of the earlier (JD 2443294.757) and later (JD 2443773.424)

Table 4. Modelling the $\beta$ Lyrae continuum.

|  | QUADRATURE <br> JD 2443294.757 |  |  | QUADRATUREJD 2443773.424 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| INDEX | OBSERVED | B8II | RJ | OBSERVED | B8II | RJ |
|  |  | +FF | +FF |  | +FF | +FF |
| H-V | -0.40 | $\bigcirc 0.42$ | -0.41 | -0.38 | -0.38 | -0.41 |
| H-J | -0.22 | -0.13 | -0.14 | -0.10 | -0.11 | -0.06 |
| $\mathrm{H}-\mathrm{H}$ | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| H-K | +0.11 | +0.08 | +0.10 | +0.09 | +0.07 | +0.09 |
| H-L | +0.41 | +0.45 | +0.45 | +0.29 | +0.34 | +0.32 |
| H-M | +0.70 | +0.62 | +0.68 | +0.55 | +0.46 | +0.55 |
| H-8.6 | $\approx+1.3$ | +1.44 | +1.52 |  |  |  |


|  | SECONDARY MINIMUM |  |  | PRIMARY MINIMUM |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| INDEX | OBSERVED | B8II | RJ | OBSERVED | B8II | RJ |
|  |  | +FF | +FF |  | +FF | +FF |
| K-V | -0.35 | -0.43 | -0.40 | -0.6 | -0.67 | -0.58 |
| K-J | -0.10 | -0.17 | -0.15 | -0.3 | -0.35 | -0.28 |
| K-K | 0.00 | 0.00 | 0.00 | 0.0 | 0.00 | 0.00 |
| K-L | +0.15 | +0.17 | +0.16 | +0.4 | +0.41 | +0.42 |
| K-M | +0.25 | +0.24 | +0.24 | +0.7 | +0.65 | +0.71 |
| K-8.6 | چ+0.75 | +0.77 | +0.71 | $\approx+1.5$ | +1.54 | +1.65 |

data at quadrature shows that the free-free component had declined by $\sim 30$ per cent over the intervening period.

A comparison of relative flux levels from the optically thin component at quadrature $\left(I_{\mathrm{Q}}\right)$, and primary $\left(I_{\mathrm{P}}\right)$ and secondary $\left(I_{\mathrm{S}}\right)$ minima yields the following ratios:

Rayleigh-Jeans model $\quad I_{\mathrm{Q}}: I_{\mathrm{P}}: I_{\mathrm{S}}=1: 0.72: 0.21$
B8Ib model
$I_{\mathrm{Q}}: I_{\mathrm{P}}: I_{\mathrm{S}}=1: 0.71: 0.26$.
Here, and in the preceding discussion, we refer to the earliest of the present data, and the data of Jameson \& Longmore. The conclusions will also apply qualitatively for more recent data.

The errors in $I_{\mathrm{P}}$ and $I_{\mathrm{S}}$ are probably of order $\pm 0.05$. The close similarity between the sum of emission at primary and secondary minima, and that at quadrature suggests that both primary and secondary components may have associated optically thin emission. The model of Flora \& Hack (1975) based on the extensive data of the $1971 \beta$ Lyrae observational campaign gives substantial support to such a proposition.

A further important point may be made. The optically thin emission component eclipsed by the primary at secondary minimum exceeds the emission due to the primary itself at longer wavelengths. The trend of emission excesses shows however that the plasma remains optically thin to $\lambda \sim 4.8 \mu \mathrm{~m}$, and possibly $\lambda \sim 8.6 \mu \mathrm{~m}$. It is evident therefore that the source of emission must almost certainly be hotter than the B8II primary. If $T_{*}$ is the temperature
of the primary, $T_{\mathrm{P}}$ the plasma temperature, and $\lambda_{\mathrm{TH}}$ the wavelength at which the plasma becomes optically thick ( $\tau=1$ ), we find

$$
\frac{T_{\mathrm{P}}}{T_{*}} \gtrsim 5.95 \times 10^{-2} \lambda_{\mathrm{TH}}^{2.1} \quad\left(\lambda_{\mathrm{TH}} \text { in } \mu \mathrm{m}\right)
$$

The inequality sign does not imply a severe lower limit to $T_{\mathrm{P}}$; although the results indicate that most if not all of the free-free emitting plasma is eclipsed at primary and secondary minima, the infrared data suggest that the eclipses are gradual, and the plasmas therefore of comparable size to the stellar components. It has already been noted that a value $\lambda_{\mathrm{TH}} \gtrsim$ $8.6 \mu \mathrm{~m}$ is probably implied by the results of Jameson \& Longmore. The latter authors also quote an upper limit magnitude +1.5 for $\lambda=18 \mu \mathrm{~m}$, and phase 0.75 , suggesting that the plasma is optically thick by this wavelength. This appears to be confirmed by data between $\lambda \lambda 8.7 \mu \mathrm{~m}$ and $19.5 \mu \mathrm{~m}$ due to Gehrz, Hackwell \& Jones (1974), suggesting $19.5 \mu \mathrm{~m}>$ $\lambda_{\mathrm{TH}}>12.6 \mu \mathrm{~m}$, for which case $T_{\mathrm{P}} \gtrsim 1.3 \times 10^{5} \mathrm{~K}$, considerably higher than the primary star ( $T_{*}=1.075 \times 10^{4} \mathrm{~K}$ (Wilson 1974)). It is pertinent to add that the radio flux (Wade \& Hjellming 1972) would be consistent with emission from such a plasma only if $\lambda_{\mathrm{TH}} \simeq$ $490 \mu \mathrm{~m}$, and therefore $T_{\mathrm{P}} \gtrsim 3 \times 10^{8} \mathrm{~K}$. Unless the $20-\mu \mathrm{m}$ results are therefore seriously in error, this indicates that an alternative plasma is necessary for the radio emission, or that the radio emission is non-thermal.

To summarize, the $\beta$ Lyrae system appears to contain a gaseous component providing an optically thin free-free emission continuum. More than 70 per cent of this continuum originates in the vicinity of the secondary, and the increasing contribution of this excess at longer wavelengths leads to the increasing depth of secondary minimum. The remaining $>20$ per cent appears to be associated with the primary component. Both optically thin plasmas appear to be alternately and totally eclipsed at primary and secondary minima. The plasma temperature near at least to the secondary component is extremely high. The above interpretation is also broadly consistent with data at other wavelengths.
$\mathrm{H} \alpha$ emission appears to originate from an extended volume, encompassing the binary. Apart from a particular anomalous reading, Batten \& Sahade (1973) found total $\mathrm{H} \alpha$ emission to lack phase coherence, although correlated changes of $\mathrm{H} \alpha$ and other lines may reflect variations in mass transfer (Batten 1976). In the ultraviolet, an analysis of spectra taken by the Copernicus satellite between $\sim 1100$ and $1400 \AA$ by Hack et al. $(1974,1975,1976)$ shows a rapidly rising UV excess continuum at $\lambda \lesssim 1150 \AA$, and a proliferation of displaced, and broadened, emission lines. The lines of $N V, \lambda \lambda 1238$ and 1242, indicate considerable excitation, and these and other lines show PCygni profiles, corresponding to expansion at $\sim 150 \mathrm{~km} \mathrm{~s}^{-1}$. Again, phase effects are relatively weak, but distinct, showing attenuation of line heights at secondary minimum. Line equivalent widths are less strongly affected than line heights, and this is interpreted as indicating occultation of a rapidly rotating accretion disk, the transversely moving layers (contributing to line cores) being preferentially occulated at mid-eclipse. It is clear therefore that the UV data provide qualitative support for the location of very hot gas near the secondary.

Kondo et al. (1975) and Kondo, McClusky \& Eaton (1976) have provided UV light curves from OAO II over an adjacent wavelength regime ( $\lambda 1280-\lambda 3330 \AA$ ). Secondary minimum is shown to deepen and primary minimum becomes shallower as wavelength decreases, behaviour reminiscent of the infrared light curves. Although specific criticisms have been levelled by Wilson (1971) these have been indirectly answered by Kondo et al. (1975). Whatever the final evaluation of the curves may be, it seems certain that they are at least qualitatively significant. A single anomalous curve at $\lambda 1910 \AA$, showing significantly decreased primary and secondary eclipse depths, may be attributable to particularly strong
line emission. Underhill \& Fahey (1973) have found similar behaviour in another star. The precise nature of the 'excess' ultraviolet emission found by Kondo et al. is not entirely clear, but could be due to smeared-out line emission (a very strong possibility in view of the work by Hack et al.), non-thermal continuum emission (Wilson 1971, although Wilson (1974) now maintains the primary generating source to be hot polar regions of the secondary), or even dilute black-body emission (Huang \& Brown 1976).

## $5 \eta$ Geminorum

$\eta$ Gem is a spectroscopic binary of period 2984 days (Kukarkin et al. 1958). McLaughlin \& Van Dijke (1943) have also proposed that it is an eclipsing variable, although Woolf (1973) suggests that variability arises primarily from ellipticity of the component stars. One component, an M5III star, is also an SRb variable with period 233.4 days (Hoffleit 1964).

Deutsch (1960) notes that a nearby GOIII-IV companion at a projected distance of only $\sim 1 \operatorname{arcsec}$ may show evidence for lines from the circumbinary envelope.

The present infrared data, plotted in Fig. 3, is a mean of results taken near phase 0.5 of the SRb star, and phase 0.3 of the ellipsoidal variable (ephemerides from Hoffleit 1964; Kukarkin et al. 1958). The primary calibration star was $\beta \mathrm{Gem}$, for which $H=-1.02, K=$ -1.09 and $N=-1.24$. The solid curve indicates the intrinsic continuum for an M3III star (Johnson 1956) with magnitude -1.5 at $K$.

All the observations (excepting those of Johnson et al.) agree to within a few hundredths of a magnitude at $K$. The results of Johnson et al. are fainter by $\sim 0.2$ mag at this wavelength and retain this difference with respect to the extrapolated M3III magnitudes down to the $B$ band. This appears to arise because of data acquisition within the phase range 0.003 to 0.04 of the primary minimum of ellipsoidal variability. The present $H$ data fills a previous observational gap, the general elevation in flux values at this band resulting from the $\mathrm{H}^{-}$


Figure 3. Spectrum of $\eta$ Gem. Solid curve is M3III continuum fitted to $K$ band measures. $\times$, Johnson et al. (1966); ^, Woolf (1973); +, Gehrz \& Woolf (1971); ○, Gillet et al. (1971); •, present work.


Figure 4. Spectrum of RR UMi. Solid curve is M4.4III continuum fitted to Johnson et al. (1966) $V$ band measures. $\times$, Johnson et al. (1966); $\odot \bullet$, present results. The $H K N, H K L N$ data from the present results represent sets of observations closed spaced in time.
opacity minimum. Beyond $L$ the scatter in the results is larger, although the present $10.2 \mu \mathrm{~m}$ data fit in closely with Woolf's. Data from Gehrz \& Woolf (1971), and Gillet, Merrill \& Stein (1971) have been used for the long wave extrapolation to Fig. 4, giving an excess at $11 \mu \mathrm{~m}$ of $\sim+0.4 \mathrm{mag}$, in essential agreement with Woolf (1973).

## 6 RR Ursae Minoris

RR UMi is a system similar to $\eta \mathrm{Gem}$. It is a spectroscopic binary of period 750 days, and a photovisual variation of $+6.2 \rightarrow+6.5$ (Kukarkin et al. 1958) arises from semi-regular variability of the M5III component with period of $\sim 40$ days (this period is not well

Table 5. Photometry of RR UMi.

| JULIAN <br> DATE <br> $24+$ | PHASE | H | K | L |
| :--- | :---: | :---: | :---: | :---: |
| 42086.797 | 0.066 | -0.96 | -1.22 | - |
| 42087.781 | 0.064 | -1.06 | -1.10 | - |
| 42089.804 | 0.062 | -0.66 | -0.79 | - |
| 42153.482 | 0.023 | -1.20 | -1.24 | -1.35 |
| 42159.552 | 0.031 | -0.60 | -0.80 | - |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  | JULIAN | PHASE |  |  |
|  | DATE |  |  |  |
|  |  |  |  |  |
|  | $424+$ |  |  |  |
|  | 42156.494 | 0.059 | -1.3 |  |
|  | 42157.588 | 0.029 | -1.35 |  |
|  | 42158.490 | 0.030 | -0.46 |  |
|  | 42161.603 | 0.030 | -1.17 |  |
|  | 42163.615 | 0.037 | -1.39 |  |

established). Joy \& Wilson (1949) note Ca II emission to the red and violet side of the absorption lines (red emission being about twice as strong as the violet).

The observations are presented in Table 5; $\mu \mathrm{UMa}$ was used for calibration, for which magnitudes $H=-0.68, K=-0.82, L=-0.95, N=-0.93$ were adopted. There is a variability of $\sim 0.7 \mathrm{mag}$ at all wavelengths. This is similarly reflected in the three sets of $I J K$, and two sets of JKL observations recorded by Johnson et al. (1966). Plots of the present data, and that of Johnson et al. with respect to phase, assuming 40-day and 750-day variability, show no systematic trends.

The spectrum of RR UMi is given in Fig. 4.
Near infrared and visible data are well represented by an M4III continuum (solid curve in Fig. 4, based on Johnson's (1966) indices, and fitted to the Johnson et al. (1966) magnitude at $V$ ). From the modulus and extinction rate (Fitzgerald 1968) reddening is probably small and of order $E_{B-V} \simeq 0.01$.

The extension of the intrinsic curve to the $N$ band again uses photometry by Gehrz \& Woolf (1971) and Gillet et al. (1971). Allowing for a small depression in the data of Johnson et al. (to which the intrinsic curve is fitted) relative to our results, gives an $N$ band excess of 0.05 mag ; less, that is, than the combined error of observed and intrinsic indices.

## 7 Restrospective and conclusions

Results for U Sge and $\beta$ Persei (Sanchez Magro et al. 1977; Nadeau et al. 1978) suggest that mass transfer within Algol-type systems will not, under normal circumstances, be sufficient to lead to appreciable distortion of the infrared continuum.

Similar reasoning appears to apply to the contact binary W UMa. Although there is ample evidence of considerable mass transfer between the respective stellar components, and for weak optical emission from tenuous, optically thin gas, the present results show that the system is remarkably clean. The ability to detect small masses of plasma in the infrared indicates however that transient emission from flares, for instance, might well lead to measurable infrared excesses.

A rather more interesting case is that of $\beta$ Lyrae. Here, both secondary and primary eclipse depths vary strongly with wavelength. Using old and new data, we have shown that this is almost certainly due to eclipse of regions of free-free emission associated with primary and secondary components of the system. It seems likely that the plasmas become optically thick at $\lambda \approx 10 \mu \mathrm{~m}$, and in consequence have temperatures at least several times that of the B8II primary. A careful evaluation of the infrared variation of intensity with phase, and wavelength, should provide interesting data on the temperature, density, location and dimensions of these non-stellar plasma components. $\beta$ Lyrae is in many respects spectroscopically unusual, although systems with similar characteristics are HD 30353 and $\nu \mathrm{Sgr}$ (Woolf 1965). $\nu \mathrm{Sgr}$ is already known to have a broad infrared excess continuum reminiscent of the kind to be expected for a cool, optically thin, free-free emitting plasma, upon which is superimposed a silicate emission feature (Woolf 1973; Humphreys \& Ney 1974). HD 30353 has also been observed at a single phase by Woolf (1973), and may have an optically thin free-free excess at $\lambda \gtrsim 5 \mu \mathrm{~m}$.

In contrast, $\eta$ Geminorum and RR UMi present no substantial surprises. Woolf (1973) found an excess of $\sim 0.3 \mathrm{mag}$ for $\eta \mathrm{Gem}$ at $\lambda=11.3 \mu \mathrm{~m}$, and present results substantially agree with this. Similarly, a negligible excess is found for RR UMi. Dust formation does not therefore appear to be unusually prolific compared with behaviour to be expected of such late-type variables. This could be due to a complex set of factors; although a close companion may for instance induce greater mass flow in a late-type giant it may, at the same
time, suppress (through heating and consequent sublimation) the formation of grains. The verdict of Woolf (1973) that the close companion of $\eta$ Gem (and, from the present paper, that of RRUMi) does not significantly influence mass loss seems however the most likely interpretation of the results.

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