Initial coefficient bounds for a subclass of m-fold symmetric bi-univalent functions

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Abstract

Let Σ denote the class of functions

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n$$

belonging to the normalized analytic function class $\mathcal{A}$ in the open unit disk $\mathbb{U}$, which are bi-univalent in $\mathbb{U}$, that is, both the function f and its inverse f^{-1} are univalent in $\mathbb{U}$. The usual method for computation of the coefficients of the inverse function $f^{-1}(z)$ by means of the relation $f^{-1}\big(f(z)\big) = z$ is too difficult to apply in the case of m-fold symmetric analytic functions in $\mathbb{U}$. Here, in our present investigation, we aim at overcoming this difficulty by using a general formula to compute the coefficients of $f^{-1}(z)$ in conjunction with the residue calculus. As an application, we introduce two new subclasses of the bi-univalent function class Σ in which both $f(z)$ and $f^{-1}(z)$ are m-fold symmetric analytic functions with their derivatives in the class $\mathcal{P}$ of analytic functions with positive real part in $\mathbb{U}$. For functions in each of the subclasses introduced in this paper, we obtain the coefficient bounds for $|a_{m+1}|$ and $|a_{2m+1}|$.

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1 Introduction and definitions

Let $\mathcal{A}$ denote the class of functions of the following form:

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \tag{1.1}$$

which are *analytic* in the open unit disk

$$\mathbb{U} = \{z : z \in \mathbb{C} \quad \text{and} \quad |z| < 1\}.$$

We denote by $\mathcal{S}$ the class of all functions in $\mathcal{A}$ which are *univalent* in $\mathbb{U}$ (see, for details, [2, 4]).

It is well known that every function $f \in \mathcal{S}$ has an inverse f^{-1}, which is defined by

$$f^{-1}\big(f(z)\big) = z \qquad (z \in \mathbb{U}) \tag{1.2}$$

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and

$$f\big(f^{-1}(w)\big) = w \qquad \left(|w| < r_0(f);\ r_0(f) \geqq \frac{1}{4}\right). \tag{1.3}$$

In fact, the inverse function f^{-1} may be analytically continued to $\mathbb{U}$ as follows:

$$f^{-1}(w) = w - a_2w^2 + (2a_2^2 - a_3)w^3 - (5a_2^3 - 5a_2a_3 + a_4)w^4 + \cdots . \tag{1.4}$$

A function $f \in \mathcal{A}$ is said to be *bi-univalent* in $\mathbb{U}$ if both f and f^{-1} are univalent in $\mathbb{U}$. Let Σ denote the class of all functions $f \in \mathcal{A}$ which are bi-univalent in $\mathbb{U}$ and are given by the equation (1.1).

Lewin [8] investigated the class Σ of bi-univalent functions and obtained a bound given by

$$|a_2| \leqq 1.51.$$

Motivated by the work of Lewin [8], Brannan and Clunie [1] conjectured that

$$|a_2| \leqq \sqrt{2}.$$

Some examples of bi-univalent functions are given (see also the work of Srivastava *et al.* [13]):

$$\frac{z}{1-z}, \quad \frac{1}{2}\log\left(\frac{1+z}{1-z}\right) \qquad \text{and} \qquad -\log(1-z).$$

Indeed, as pointed out by Srivastava *et al.* [13], the coefficient estimate problem for each of the following Taylor-Maclaurin coefficients:

$$|a_n| \qquad (n \in \mathbb{N} \setminus \{1,2\};\ \mathbb{N} := \{1,2,3,\cdots\})$$

is still open.

In recent years, the study of bi-univalent functions has gained momentum mainly due to the work of Srivastava *et al.* [13], which has apparently revived the subject. Motivated by their work [13], many researchers (see, for example, [3, 5, 6, 11, 13, 14, 12, 15, 16, 17, 18]; see also the various closely-related papers on the subject, which are cited in some of these works) have recently investigated several interesting subclasses of the bi-univalent function class Σ and found non-sharp estimates on the first two Taylor-Maclaurin coefficients of functions belonging to these subclasses.

For each function $f \in \mathcal{S}$, the function

$$h(z) = \sqrt[m]{f(z^m)} \qquad (z \in \mathbb{U};\ m \in \mathbb{N})$$

is univalent and maps the unit disk $\mathbb{U}$ into a region with m-fold symmetry. A function is said to be m-fold symmetric (see [9, 10]) if it has the following normalized form:

$$f(z) = z + \sum_{k=1}^{\infty} a_{mk+1} z^{mk+1} \qquad (z \in \mathbb{U};\ m \in \mathbb{N}). \tag{1.5}$$

We denote by $\mathcal{S}_m$ the class of m-fold symmetric univalent functions in $\mathbb{U}$, which are normalized by the series expansion (1.5). In fact, the functions in the class $\mathcal{S}$ are one-fold symmetric.

Analogous to the concept of m-fold symmetric univalent functions, we here introduced the concept of m-fold symmetric bi-univalent functions. Each function $f \in \Sigma$ generates an m-fold symmetric bi-univalent function for each integer $m \in \mathbb{N}$. The normalized form of f is given as in (1.5) and the series expansion for f^{-1} is given as follows by using Theorem (1):

$$\begin{aligned} g(w) = {} & w - a_{m+1}w^{m+1} + \left[(m+1)a_{m+1}^2 - a_{2m+1}\right] w^{2m+1} \\ & - \left[\frac{1}{2}(m+1)(3m+2)a_{m+1}^3 - (3m+2)a_{m+1}a_{2m+1} + a_{3m+1}\right] w^{3m+1} + \cdots , \end{aligned} \tag{1.6}$$

where $f^{-1} = g$. We denote by Σ_m the class of m-fold symmetric bi-univalent functions in $\mathbb{U}$. For $m = 1$, the formula (1.6) coincides with the formula (1.4) of the class Σ. Some examples of m-fold symmetric bi-univalent functions are given as follows:

$$\left(\frac{z^m}{1-z^m}\right)^{\frac{1}{m}}, \quad \left[\frac{1}{2}\log\left(\frac{1+z^m}{1-z^m}\right)\right]^{\frac{1}{m}} \qquad \text{and} \qquad [-\log(1-z^m)]^{\frac{1}{m}}$$

with the corresponding inverse functions given by

$$\left(\frac{w^m}{1+w^m}\right)^{\frac{1}{m}}, \quad \left(\frac{e^{2w^m}-1}{e^{2w^m}+1}\right)^{\frac{1}{m}} \qquad \text{and} \qquad \left(\frac{e^{w^m}-1}{e^{w^m}}\right)^{\frac{1}{m}},$$

respectively.

Finally, we denote by $\mathcal{P}$ the Carathéodary class of functions $p(z)$ of the following form:

$$p(z) = 1 + p_1 z + p_2 z^2 + \cdots , \tag{1.7}$$

which are analytic in $\mathbb{U}$ such that

$$\Re\{p(z)\} > 0 \qquad (z \in \mathbb{U}). \tag{1.8}$$

The objective of the present paper is to introduce an elegant formula for computing the coefficients of the inverse functions for the class Σ_m of m-fold symmetric functions by means of residue calculus. As an application, we introduce two new subclasses of bi-univalent functions in which both f and f^{-1} are m-fold symmetric analytic functions with their derivative in the class $\mathcal{P}$ and obtain coefficient bounds for $|a_{m+1}|$ and $|a_{2m+1}|$ for functions in each of these new subclasses.

In order to derive our main results, we use the following lemma for the Carathéodary class $\mathcal{P}$.

Lemma. *If $h \in \mathcal{P}$, then*

$$|c_k| \leqq 2 \qquad (k \in \mathbb{N}),$$

where the Carathéodary class $\mathcal{P}$ is the family of all functions h, analytic in $\mathbb{U}$, for which

$$\Re\{h(z)\} > 0 \qquad (z \in \mathbb{U})$$

and

$$h(z) = 1 + c_1 z + c_2 z^2 + \cdots \qquad (z \in \mathbb{U}), \tag{1.9}$$

the extremal function being given by

$$h(z) = \frac{1+z}{1-z} \qquad (z \in \mathbb{U}).$$

2 Coefficients of the inverse functions

Coefficients of the inverse f^{-1} of a given function f are obtained generally by virtue of the relation

$$f^{-1}\big(f(z)\big) = z.$$

However, this technique is much too difficult to apply in the case of m-fold symmetric functions. To overcome this difficulty, we use a general formula [4] to compute the coefficients of f^{-1} by means of the residue calculus as follows.

Theorem 1. Let $f(z)$ be in the class $\mathcal{S}$. Then the coefficients γ_n of the inverse function

$$f^{-1}(w) = w + \gamma_2 w^2 + \gamma_3 w^3 + \cdots$$

are given by

$$\gamma_n = \frac{1}{n!} \lim_{z\to 0} \left\{ \frac{d^{n-1}}{dz^{n-1}} \left(\frac{z}{f(z)} \right)^n \right\}. \tag{2.1}$$

Proof. Let $f(z)$, given by (1.4), be in the class $\mathcal{S}$ and suppose that

$$g(w) = f^{-1}(w)$$

is given as in the hypothesis of Theorem 1. Then, from Cauchy's integral formula and by the method described in [7], we have

$$\gamma_n = \frac{1}{2\pi i} \int_{\mathcal{C}_\varepsilon} \left(\frac{g(w)}{w^{n+1}} \right) dw = \frac{1}{2n\pi i} \int_{|z|=r} \frac{dz}{[f(z)]^n} \qquad (0 < r < 1), \tag{2.2}$$

where $\mathcal{C}_\varepsilon$ denotes a closed positively-oriented circle in the complex w-plane with centre at $w = 0$ and radius ε $(0 < \varepsilon < 1)$. Since the function $f(z)$ is univalent and $f(0) = 0$, the integrand $\frac{1}{[f(z)]^n}$ has a pole of order n at $z = 0$. Therefore, we have

$$\gamma_n = \frac{1}{n!} \lim_{z\to 0} \left\{ \frac{d^{n-1}}{dz^{n-1}} \left(\frac{z}{f(z)} \right)^n \right\}, \tag{2.3}$$

which evidently proves Theorem 1. Q.E.D.

In view of Theorem 1, the first two coefficients γ_{m+1} and γ_{2m+1} of the m-fold symmetric bi-univalent function $f(z)$ in (1.5) are given by

$$\gamma_{m+1} = \frac{1}{(m+1)!} \lim_{z\to 0} \left\{ \frac{d^m}{dz^m} \left(\frac{z}{f(z)} \right)^{m+1} \right\} \tag{2.4}$$

and

$$\gamma_{2m+1} = \frac{1}{(2m+1)!} \lim_{z\to 0} \left\{ \frac{d^{2m}}{dz^{2m}} \left(\frac{z}{f(z)} \right)^{2m+1} \right\}. \tag{2.5}$$

3 Coefficient bounds for the function class $\mathcal{H}_{\Sigma,m}^{\alpha}$

Definition 1. A function $f(z)$, given by (1.5), is said to be in the class $\mathcal{H}_{\Sigma,m}^{\alpha}$ if the following conditions are satisfied:

$$f \in \Sigma_m \qquad \text{and} \qquad |\arg\{f'(z)\}| < \frac{\alpha\pi}{2} \qquad (z \in \mathbb{U};\ 0 < \alpha \leqq 1) \tag{3.1}$$

and

$$|\arg\{g'(w)\}| < \frac{\alpha\pi}{2} \qquad (w \in \mathbb{U};\ 0 < \alpha \leqq 1)\,, \tag{3.2}$$

where the function $g(w)$ is given by (1.6).

In this section, we first state and prove the following theorem.

Theorem 2. Let the function $f(z)$, given by (1.5), be in the class $\mathcal{H}_{\Sigma,m}^{\alpha}$ $(0 < \alpha \leqq 1)$. Then

$$|a_{m+1}| \leqq \frac{2\alpha}{m\sqrt{(m+1)(\alpha m + m + 1}} \tag{3.3}$$

and

$$|a_{2m+1}| \leqq \frac{2\alpha\left[(2m+1)\alpha + m + 1\right]}{(m+1)(2m+1)}. \tag{3.4}$$

Proof. From (3.1) and (3.2), we get

$$f'(z) = [p(z)]^{\alpha} \tag{3.5}$$

and

$$g'(w) = [q(w)]^{\alpha}\,, \tag{3.6}$$

where the functions $p(z)$ and $q(w)$ are in the class $\mathcal{P}$ and have the following series representations:

$$p(z) = 1 + p_m z^m + p_{2m} z^{2m} + p_{3m} z^{3m} + \cdots \tag{3.7}$$

and

$$q(w) = 1 + q_m w^m + q_{2m} w^{2m} + q_{3m} w^{3m} + \cdots\,. \tag{3.8}$$

Now, equating the coefficients in (3.5) and (3.6), we find that

$$(m+1)a_{m+1} = \alpha p_m, \tag{3.9}$$

$$(2m+1)a_{2m+1} = \alpha p_{2m} + \frac{\alpha(\alpha-1)}{2} p_m^2, \tag{3.10}$$

$$-(m+1)a_{m+1} = \alpha q_m \tag{3.11}$$

and

$$(2m+1)\left[(m+1)a_{m+1}^2 - a_{2m+1}\right] = \alpha q_{2m} + \frac{\alpha(\alpha-1)}{2} q_m^2. \tag{3.12}$$

From (3.9) and (3.11), we get

$$p_m = -q_m \tag{3.13}$$

and

$$2(m+1)^2 a_{m+1}^2 = \alpha^2 (p_m^2 + q_m^2). \tag{3.14}$$

Also, from (3.10),(3.12) and (3.14), a simple computation shows that

$$\begin{aligned}(2m+1)(m+1)a_{m+1}^2 &= \alpha(p_{2m}+q_{2m}) + \frac{\alpha(\alpha-1)}{2}(p_m^2+q_m^2)\\ &= \alpha(p_{2m}+q_{2m}) + \frac{\alpha(\alpha-1)}{2}\frac{2(m+1)^2}{\alpha^2}a_{m+1}^2.\end{aligned}$$

Therefore, we have

$$a_{m+1}^2 = \frac{\alpha^2(p_{2m}+q_{2m})}{(m+1)(\alpha m + m + 1)}.$$

Applying the Lemma of the preceding section for the coefficients p_{2m} and q_{2m}, we obtain

$$|a_{m+1}| \leqq \frac{2\alpha}{\sqrt{(m+1)(\alpha m + m + 1)}}.$$

This gives the desired estimate for $|a_{m+1}|$ as asserted in (3.3).

Next, in order to find the bound on $|a_{2m+1}|$, by subtracting (3.12) from (3.10), we get

$$\begin{aligned}&2(2m+1)a_{2m+1} - (m+1)(2m+1)a_{m+1}^2\\ &\qquad = \alpha(p_{2m}-q_{2m}) + \frac{\alpha(\alpha-1)}{2}(p_m^2 - q_m^2).\end{aligned} \tag{3.15}$$

By a simple computation, and using (3.13) to (3.15), we get

$$a_{2m+1} = \frac{\alpha^2(p_m^2+q_m^2)}{4(m+1)} + \frac{\alpha(p_{2m}-q_{2m})}{2(2m+1)}.$$

Thus, by applying the Lemma of Section 1 again for the coefficients p_m,p_{2m}, q_m and q_{2m}, we find that

$$|a_{2m+1}| \leqq \frac{2\alpha\left[(2m+1)\alpha + m + 1\right]}{(m+1)(2m+1)}.$$

This completes the proof of Theorem 2. Q.E.D.

4 Coefficient bounds for the function class $\mathcal{H}_{\Sigma,m}(\beta)$

Definition 2. A function $f(z)$, given by (1.5), is said to be in the class $\mathcal{H}_{\Sigma,m}(\beta)$ if the following conditions are satisfied:

$$f \in \Sigma_m \qquad \text{and} \qquad \Re\{f'(z)\} > \beta \qquad (z \in \mathbb{U};\ 0 \leqq \beta < 1) \tag{4.1}$$

and

$$\Re\{g'(w)\} > \beta \qquad (z \in \mathbb{U};\ 0 \leqq \beta < 1)\,, \tag{4.2}$$

where the function g is defined by (1.6).

In this section, we state and prove the following theorem.

Theorem 3. Let the function $f(z)$, given by (1.5), be in the class $\mathcal{H}_{\Sigma,m}(\beta)$ $(0 \leqq \beta < 1)$. Then

$$|a_{m+1}| \leqq 2\sqrt{\frac{(1-\beta)}{(m+1)(2m+1)}} \tag{4.3}$$

and

$$|a_{2m+1}| \leqq 2(1-\beta)\left(\frac{(1-\beta)(2m+1)+m+1}{(m+1)(2m+1)}\right). \tag{4.4}$$

Proof. First of all, the argument inequalities in (4.1) and (4.2) can be written in the following forms:

$$f'(z) = \beta + (1-\beta)p(z) \tag{4.5}$$

and

$$g'(w) = \beta + (1-\beta)q(w), \tag{4.6}$$

where the functions $p(z)$ and $q(w)$ are in the class $\mathcal{P}$ and have the forms given by (3.7) and (3.8), respectively. Now, as in the proof of Theorem 2, by equating the coefficients in (4.5) and (4.6), we get

$$(m+1)a_{m+1} = (1-\beta)p_m, \tag{4.7}$$

$$(2m+1)a_{2m+1} = (1-\beta)p_{2m}, \tag{4.8}$$

$$-(m+1)a_{m+1} = (1-\beta)q_m \tag{4.9}$$

and

$$(2m+1)\left[(m+1)a_{m+1}^2 - a_{2m+1}\right] = (1-\beta)q_{2m}. \tag{4.10}$$

From (4.7) and (4.9), we get

$$p_m = -q_m \tag{4.11}$$

and

$$2(m+1)^2 a_{m+1}^2 = (1-\beta)^2(p_m^2 + q_m^2). \tag{4.12}$$

Also, from (4.8) and (4.10), we obtain

$$(m+1)(2m+1)a_{m+1}^2 = (1-\beta)(p_{2m}+q_{2m}).$$

Thus, clearly, we have

$$|a_{m+1}|^2 \leqq \frac{(1-\beta)\left(|p_{2m}|+|q_{2m}|\right)}{(m+1)(2m+1)} \leqq \frac{4(1-\beta)}{(m+1)(2m+1)}.$$

This gives the bound on $|a_{m+1}|$ as asserted in (4.3).

Next, in order to find the bound on $|a_{2m+1}|$, by subtracting (4.10) from (4.8), we get

$$2(2m+1)a_{2m+1} - (m+1)(2m+1)a_{m+1}^2 = (1-\beta)(p_{2m}-q_{2m})$$

or, equivalently,

$$a_{2m+1} = \frac{(m+1)}{2}a_{m+1}^2 + \frac{(1-\beta)(p_{2m}-q_{2m})}{2(2m+1)}. \tag{4.13}$$

Upon substituting the value of a_{m+1}^2 from (4.12), we get

$$a_{2m+1} = \frac{(1-\beta)^2(p_m^2+q_m^2)}{4(m+1)} + \frac{(1-\beta)(p_{2m}-q_{2m})}{2(2m+1)}. \tag{4.14}$$

Applying the Lemma of Section 1 for the coefficients p_m, p_{2m}, q_m and q_{2m}, we find that

$$|a_{2m+1}| \leqq 2(1-\beta)\left(\frac{(1-\beta)(2m+1)+m+1}{(m+1)(2m+1)}\right) \tag{4.15}$$

which is the bound on $|a_3|$ as asserted in (4.4). Q.E.D.

5 Corollaries and consequences

For one-fold symmetric bi-univalent functions, Theorem 2 and Theorem 3 reduce to Corollary 1 and Corollary 2, respectively, which were proven earlier by Srivastava *et al.* [13].

Corollary 1. (see [13]) Let the function $f(z)$, given by (1.1), be in the class $\mathcal{H}_\Sigma^\alpha$ $(0 < \alpha \leqq 1)$. Then

$$|a_2| \leqq \alpha\sqrt{\frac{2}{2+\alpha}} \tag{5.1}$$

and

$$|a_3| \leqq \frac{\alpha(3\alpha+2)}{3}. \tag{5.2}$$

Corollary 2. (see [13]) Let the function $f(z)$, given by (1.1), be in the class $\mathcal{H}_\Sigma(\beta)$ $(0 \leqq \beta < 1)$. Then

$$|a_2| \leqq \sqrt{\frac{2(1-\beta)}{3}} \tag{5.3}$$

and

$$|a_3| \leqq \frac{(1-\beta)(5-3\beta)}{3}. \tag{5.4}$$

Numerous other (presumably new) corollaries and consequences of our main results can also be deduced.

References

[1] D. A. Brannan and J. G. Clunie (Editors), *Aspects of contemporary complex analysis* (Proceedings of the NATO Advanced Study Institute held at the University of Durham, Durham; July 1–20, 1979), Academic Press, New York and London, 1980.

[2] P. L. Duren, *Univalent functions*, Grundlehren der Mathematischen Wissenschaften, Band **259**, Springer-Verlag, New York, Berlin, Heidelberg and Tokyo, 1983.

[3] B. A. Frasin and M. K. Aouf, *New subclasses of bi-univalent functions*, Appl. Math. Lett. **24** (2011), 1569–1573.

[4] A. W. Goodman, *Univalent functions*, Vols. 1 and 2, Mariner Publishing Company, Tampa, Florida, 1983.

[5] S. P. Goyal and P. Goswami, *Estimate for initial Maclaurin coefficients of bi-univalent functions for a class defined by fractional derivatives*, J. Egyptian Math. Soc. **20** (2012), 179–182.

[6] T. Hayami and S. Owa, *Coefficient bounds for bi-univalent functions*, Pan Amer. Math. J. **22** (4) (2012), 15–26.

[7] J. G. Krzyż, R. J. Libera and E. J. Złotkiewicz, *Coefficients of inverses of regular starlike functions*, Ann. Univ. Mariae Curie-Skłodowska Sect. A **33** (1979), 103–110.

[8] M. Lewin, *On a coefficient problem for bi-univalent functions*, Proc. Amer. Math. Soc. **18** (1967), 63–68.

[9] W. Koepf, *Coefficients of symmetric functions of bounded boundary rotations*, Proc. Amer. Math. Soc. **105** (1989), 324–329.

[10] C. Pommerenke, *On the coefficients of close-to-convex functions*, Michigan Math. J. **9** (1962), 259–269.

[11] H. M. Srivastava, *Some inequalities and other results associated with certain subclasses of univalent and bi-univalent analytic functions*, in Nonlinear Analysis: Stability, Approximation, and Inequalities (Panos M. Pardalos, Pando G. Georgiev and Hari M. Srivastava, Editors.), Springer Series on Optimization and Its Applications, Vol. **68**, Springer-Verlag, Berlin, Heidelberg and New York, 2012, pp. 607–630.

[12] H. M. Srivastava, S. Bulut, M. Çağlar and N. Yağmur, *Coefficient estimates for a general subclass of analytic and bi-univalent functions*, Filomat **27** (2013), 831–842.

[13] H. M. Srivastava, A. K. Mishra and P. Gochhayat, *Certain subclasses of analytic and bi-univalent functions*, Appl. Math. Lett. **23** (2010), 1188–1192.

[14] H. M. Srivastava, G. Murugusundaramoorthy and N. Magesh, *Certain subclasses of bi-univalent functions associated with the Hohlov operator*, Global J. Math. Anal. **1** (2) (2013), 67–73.

[15] Q.-H. Xu, H. M. Srivastava and Z. Li, *A certain subclass of analytic and close-to-convex functions*, Appl. Math. Lett. **24** (2011), 396-401.

[16] Q.-H. Xu, Y.-C. Gui and H. M. Srivastava, *Coefficient estimates for a certain subclass of analytic and bi-univalent functions*, Appl. Math. Lett. **25** (2012), 990–994.

[17] Q.-H. Xu, H.-G. Xiao, and H. M. Srivastava, *A certain general subclass of analytic and bi-univalent functions and associated coefficient estimate problems*, Appl. Math. Comput. **218** (2012), 11461–11465.

[18] Q.-H. Xu, C.-B. Lv and H. M. Srivastava, *Coefficient estimates for the inverses of a certain general class of spirallike functions*, Appl. Math. Comput. **219** (2013), 7000-7011.