Initialisation-Free Active Contour Segmentation

Xianghua Xie
Dept of Computer Science, University of Swansea, SA2 8PP, UK. x.xie@swansea.ac.uk

Majid Mirmehdi
Dept of Computer Science, University of Bristol, BS8 1UB, UK. majid@cs.bris.ac.uk

Abstract

We present a region based active contour model which does not require any initialisation and is capable of modelling multi-modal image regions. Its external force is based on statistically learning and grouping of image primitives in multiscale, and its numerical solution is carried out using radial basis function interpolation and time dependent expansion coefficient updating. The initialisation-free property makes it attractive to applications such as detecting unkown number of objects with unkown topologies.

1 Introduction

Initialisation of a contour to start the evolution of a snake is a nuisance for those who strive for automated active contour-based image analysis. In conventional level set methods, active contours are not able to create topological changes away from the zero level set where the deformable contours are embedded. This means, for example, that the level set would miss internal boundaries of an object if the initialised contour encircles the contour. Careful manual or automatic initialisation based on prior knowledge, such as object location, is generally required. However, in applications such as those that must detect an unknown number of objects with unknown topologies, the ability to automatically initialise and to tackle more sophisticated topological changes than splitting and merging is desirable. One not so elegant solution is to impose a mesh of small initial contours to evolve and capture the object.

In this paper, we extend our previous RBF level set method [9] to localise objects in colour images *without any initialisation*. Note this means we do not even place an initial contour at the start of the evolution of our level set function. The level set function is initialised merely as a single-value level set, which will be continuously updated according to an external force such that it develops into a multi-value level set function and creates new contours at the zero level set.

There have been only limited attempts at automating the initial contour placement stage, amongst them [11, 6], but these are either limited to specific snake types or to efficient placement on images in specific applications. For example, [11] developed a method to place an initial contour close to its object of interest by connecting the initial points of different resolutions obtained by wavelet-based multi-scale edge detection for the GVF snake, while [6] proposed a very elaborate method based on thresholding and morphological operations (to first obtain candidate regions) to find the initial contours of the epicardium and endocardium in a myocardial perfusion analysis application. The numerical method proposed by Chan and Vese in [2] can introduce new contours away from existing ones. However, due to the narrow support of the delta function, it leads to significant irregularities near the zero level set, which can hamper contour evolution [8].

Radial basis functions (RBFs) have received increasing attention in solving PDE systems in recent years. For example, Cecil *et al.* [1] used RBFs to generalise conventional FDM on a non-uniform (unstructured) computational grid to solve the high dimensional Hamilton-Jacobi PDEs with high accuracy. Very recently, Wang *et al.* Recently in [4], Morse *et al.* placed RBFs at contour landmarks to implicitly represent the active contour, thereby avoiding the manipulation of a higher dimensional function. However, the method requires dynamic insertion and deletion of landmarks which is non-trivial. Similar to the parametric representation, the resolution and position of the landmarks can affect the accuracy of contour representation.

Next, we briefly outline our RBF level set method and the associated external force based on texems [10]. We mainly focus on the manner in which, as a by-product of the methodology, we are able to offer a significant development in the field of active contour analysis, i.e. by removing altogether the need to place an initial active contour in the image. Our subjective results are then also focussed on illustrating the power of our initialisation-free approach.

2 Proposed Method

In the proposed method, the level set function is interpolated using RBFs and its shape and topology are thus determined by the coefficients of RBF interpolation. Significantly, we no longer require finite difference based numerical methods to evolve the level set function. Instead, the adaptive changes of the RBF interpolation coefficents, is an ODE problem - a much easier problem to solve. The regularisation of the level set evolution is intrinsically handled through velocity normalisation and the smoothing nature of RBF interpolation. The evolution can thus continuously take place without any periodic re-conditioning, as a result of which, perturbations away from zero level set can create new contours away from existing ones. This eliminates the awkward initialisation dependence problem and ensures our RBF-based level set to fully develop under the influence of the external forces. This also allows us to simplify the initialisation of the level set function as a single level set, i.e. a flat surface, which does not require any user interference, reduces computational costs associated with the distance transform, and simplifies RBF interpolation.

The external force, described first next, is derived from multiscale modelling of colour images using the texem representation [10], where it was shown to be effective in novelty detection on random textures. However, instead of using patch based learning, we perform multiscale branch based primitive extraction with spatially constrained model order reduction to handle multimodal image regions.

2.1 Multiscale Learning of Colour Images

The texem representation [10] provides a generative approach to extract visual primitives from given images without resorting to image decomposition using filtering for example as in texton methods [12]. It handles the colour image in full 3D, taking into account both spatial and spectral interactions simultaneously. Let $\mathbf{m} = \{\boldsymbol{\mu}, \boldsymbol{\omega}\}$ denote a texem with mean $\boldsymbol{\mu}$ and corresponding variance $\boldsymbol{\omega}$. An image $\mathbf{I}$ is assumed to be a superposition of overlapping patches $\mathbf{Z} = \{\mathbf{Z}_i\}_{i=1}^{P}$, each of which is generated from a set of texems $\boldsymbol{\mathcal{M}} = \{\mathbf{m}_k\}_{k=1}^{K}$ ($K \ll P$). Here, we build a simple Gaussian pyramidical representation of the image and collect colour pixels across scales based on their child-parent relationship. Each branch of pixels $\mathbf{Z}_i$ is collected by tracing the parent pixel at the coarset scale to the child pixel in the finest scale. It is thus more efficient than patch-based representation in deriving the parameters associated with this mixture model:

$$p(\mathbf{Z}_i|\Theta) = \sum_{k=1}^{K} \prod_{n \in l} \mathcal{N}(\mathbf{Z}_i^{(n)}; \boldsymbol{\mu}_k^{(n)}, \boldsymbol{\omega}_k^{(n)})\beta_k, \quad (1)$$

where $\Theta = (\beta_1, ..., \beta_K, \theta_1, ..., \theta_K)$, β_k is the *priori* probability of kth texem constrained by $\sum_{k=1}^{K} \beta_k = 1$, θ_k denotes the kth texem's parameters, $\mathcal{N}(.)$ is a Gaussian distribution over $\mathbf{Z}_i^{(n)}$, l is the number of scales, $\boldsymbol{\mu}_k^{(n)}$ and $\boldsymbol{\omega}_k^{(n)}$ denote mean and variance at the nth level of the kth texem. Statistical independence is assumed for pixels within each branch.

Each pixel in image $\mathbf{I}$ is represented by a vector/branch of colour pixels, and has a measurable relationship with the texems according to the posterior probability $p(\mathbf{m}_k|\mathbf{Z}_i, \Theta)$, which can be computed using Bayes' rule. However, a single image region may contain multiple visual primitives and display complex patterns so that a single texem might not be able to fully represent such regions. Hence, multiple texems can be grouped together to jointly represent "multimodal" image regions. Here, we follow the mixture model order reduction method proposed by Manduchi [3] to group some of the texems based on their spatial coherence. This is carried out by greedily grouping two texems at a time with minimum change in descriptiveness $D = \sum_{k=1}^{K} \frac{E[p(\mathbf{m}_k|\mathbf{Z}_i)^2]}{\beta_k}$, with the grouped texem representation taking the following form:

$$\hat{p}(\mathbf{Z}_i|c) = \frac{1}{\hat{\beta}_c} \sum_{k \in G_c} \prod_{n \in l} \mathcal{N}(\mathbf{Z}_i^{(n)}; \boldsymbol{\mu}_k^{(n)}, \boldsymbol{\omega}_k^{(n)})\beta_k, \quad (2)$$

where $\hat{\beta}_c = \sum_{k \in G_c} \beta_k$, G_c is the group of texems that are combined together to form a new cluster c, and $\hat{\beta}_c$ is the *priori* for new cluster c. The posterior probability of the object or region of interest can then be computed according to Bayes' rule: $\hat{p}(c|\mathbf{Z}_i) = \frac{\hat{p}(\mathbf{Z}_i|c)\hat{\beta}_c}{\sum_{c=1}^{K} \hat{p}(\mathbf{Z}_i|c)\hat{\beta}_c}$, which can be used as the external force to drive the active contours.

2.2 RBF Active Contour

The region based approach described above has been shown as an effective alternative to the popular Mumford-Shah formulation, e.g. as in [5]. To simplify the notation, let u denote the posterior probability of the class of interest. The level set representation for the colour texem based active contour can be formulated as:

$$\frac{\partial \Phi}{\partial t} = w\kappa|\nabla\Phi| + (u - \frac{1}{m})|\nabla\Phi|, \quad (3)$$

where w is a real constant, κ denotes the curvature, m is the number of classes and $\frac{1}{m}$ is the average expectation of a class probability.

Figure 1. (1st row) Segmentation result using conventional level set; (2nd row and continuing on 3rd row) Results using proposed method.

Instead of solving the PDE problem in (3) using FDM with the upwind scheme, we interpolate the level set function $\Phi(\mathbf{x})$ using a number of RBFs and evolve it by adapting the expansion coefficients. The RBF interpolation is expressed as: $\Phi(\mathbf{x}) = p(\mathbf{x}) + \sum_{i=1}^{N} \alpha_i \psi_i(\mathbf{x})$, where ψ_i denote a RBF function, N denotes the number of RBFs, $p(\mathbf{x})$ is a first-degree polynomial $p(\mathbf{x}) = p_0 + p_1 x + p_2 y$, and α_i are the expansion coefficients. Assuming time and space are separable, the time dependence of the level set function is now due to the RBF interpolation, whose evolution is achieved by solving the following ODE:

$$\mathbf{\Psi}^T \frac{d\boldsymbol{\alpha}}{dt} + F|(\nabla \mathbf{\Psi})^T \boldsymbol{\alpha}| = 0, \qquad (4)$$

where $\mathbf{\Psi}(\mathbf{x}) = [\psi_1(\mathbf{x}) \cdots \psi_N(\mathbf{x})\ 1\ x\ y]^T$, $\boldsymbol{\alpha} = [\alpha_1 \cdots \alpha_N\ p_0\ p_1\ p_2]^T$, and $F = -w\kappa + 1/m - u$. The spatial derivative $\nabla\mathbf{\Psi}$ can be solved analytically, and the temporal derivative of the expansion coefficients is solved using the first order Euler's method. Velocity extension or speed normalisation around local maxima or minima of the level set function, where the gradient magnitude is close to zero $|(\nabla \mathbf{\Psi}^T(\mathbf{x}_i))\boldsymbol{\alpha}| \to 0$, can be used to further ensure the numerical stability. The curvature term in (4) can be ignored, since the RBF interpolation is intrinsically smoothing. Thus, the RBF active contour evolution can be formulated as:

$$\mathbf{\Psi}^T \frac{d\boldsymbol{\alpha}}{dt} + (\frac{1}{m} - u)|(\nabla \mathbf{\Psi})^T \boldsymbol{\alpha}| = 0. \qquad (5)$$

2.3 Initialisation-free segmentation

Unlike conventional level set approaches where the upwind scheme is used and re-initialisation is applied to maintain numerical stability, the RBF coefficient updating is more efficient and does not require periodic re-initialisation of the level set function. As a result, the level set surface can continuously update itself and perturbations away from evolving front are allowed so that new contours can grow out in regions away from existing ones, which is not possible for the conventional level set approach. In contrast to other approaches, such as [2], where the level set evolution largely takes place near zero level set, the interpolation and evolution is global with the proposed method. It is also more robust than heuristic driven approaches, such as [7], which insert new contours once current contours are stabilised and perform data fitting to determine whether to keep the new contours. This insertion and deletion however can be difficult for complex images, particularly when segmenting small objects, and hence can be unstable.

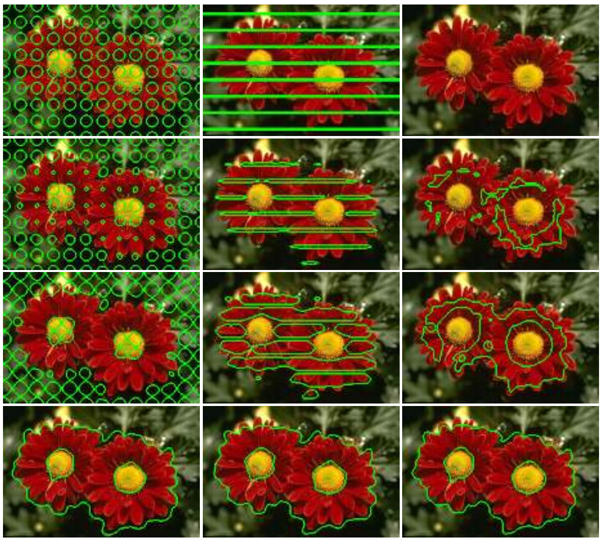

Figure 2. (1st and 2nd columns) Examples of initialisation invariancy and (final column) completely initialisation-free.

Although logically straight forward, it is significant to realise that the ability to develop new contours in fact indicates there is actually *no need to place the initial contour* in the first place. This greatly simplifies the initialisation process as it means the initial level set function can be simply single-value, i.e. a flat surface. This single-value level set function will then gradually develop into a multi-value function which contains zero crossings, colocating with object boundaries. In other words, the proposed active contour based segmentation method is in fact completely initialisation-free.

3 Experimental Results

The proposed method requires very little parameter tunning. The only active contour parameter, ω, which controls the smoothness of the contour is fixed for all experiments. Since the level set evolution is solved as

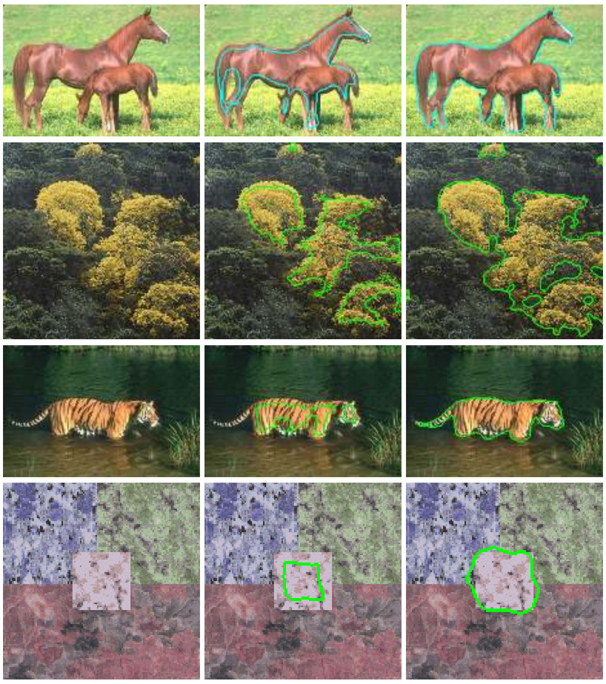

Figure 3. Segmentation without initialisation.

an ODE problem, it is much more efficient than solving PDEs. However, the computational bottle neck in our current implementation is the direct RBF interpolation, which however can be significantly improved using fast interpolation algorithms.

Fig. 1 provides an example of the proposed method in dealing with more complex topological changes than split-and-merge. The initial snake was placed outside of the object of interest and was forced to *shrink* according to the same external force derived from colour texem modelling. The conventional level set method (top row) failed to localise the object due to its inability to develop new contours. However, the proposed method (second row and continuing on the third row, from left to right) succeeded by growing out new contours inside the object. In this case, the conventional method requires the initial snake to be specifically placed overlapping or inside the object. This assumes that prior knowledge of spatial position of the object of interest, as well as its topology, is available for initialisation, which is not always the case in real world applications. The proposed method does not require any such assumptions and thus is particularly useful in detecting unknown numbers of objects with complex topologies.

Due to its ability to create new contours without any dedicated numerical or heuristic driven remedies, the proposed method can carry out segmentation without an initial snake. The final column in Fig. 2 gives such an example. The first and second columns of Fig. 2 also provide two completely different initialisations with the first one commonly seen in region based methods as an approach to initialisation invariancy.

Fig. 3 provides several more initialisation-free examples. In each case, the proposed method carried out the segmentation without placing an initial snake, successfully developing new contours and localising the objects. Also note that the texem based region force handled regional colour and feature variations very well.

4 Conclusion

We have presented an RBF level set based active contour model for colour image segmentation. Its external force is derived from multiscale texem modelling and mixture model order reduction. The initial PDE problem is transformed into an ODE problem based on RBF interpolation and expansion coefficient updating. The proposed method can tackle multi-modal colour images without decomposition and the active contour evolves without any initialisation. We believe this is a major step forward in the area of active contours.

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