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INITIATING CHANGE IN PROSPECTIVE ELEMENTARY SCHOOL TEACHERS' ORIENTATIONS TO MATHEMATICS TEACHING BY BUILDING ON BELIEFS

ABSTRACT. Many mathematics educators have found that prospective elementary school teachers' beliefs interfere with their learning of mathematics. Often teacher educators consider these beliefs to be wrong or naïve and seek to challenge them so prospective teachers will reject them for more generative beliefs. Because of the resilience of prospective teachers' beliefs in response to these challenges, teacher educators could consider alternative ways of thinking about and addressing beliefs, particularly the potential of building on rather than tearing down pre-existing beliefs. Data from an early-field experience linked to a mathematics-for-teachers course provide evidence that when prospective teachers work intimately with children, in this case trying to teach 10-year-olds about fractions, the experience has the intensity from which beliefs can grow. Most of the prospective teachers in the study were surprised that mathematics teaching was more difficult than they had anticipated. They began to consider the importance of providing children time to think when solving mathematical problems. The change described in the study is incremental rather than monumental, suggesting that building upon prospective teachers' existing beliefs will be a gradual process.

KEY WORDS: field experience, mathematics pre-service teacher education, prospective teachers' beliefs

INTRODUCTION

Prospective elementary school teachers come to their teacher education programs with a variety of beliefs that are influenced by their experiences as students in schools. Some believe that teaching will be relatively straightforward, consisting primarily of offering clear explanations to children (Richardson, 1996). They believe that their abilities to relate to children and manage classrooms will be paramount to their success as teachers. Weinstein (1989) characterized this orientation as an *optimistic bias*, because prospective teachers enter their coursework assuming that they already know what they need to know in order to teach (see also Feiman-Nemser & Buchmann, 1986). These beliefs often lead prospective teachers to underestimate the complexity of teaching and the kind of knowledge that they will need to be successful. In particular, they often underestimate the importance of subject-matter knowledge in teaching.

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Some mathematics education researchers have argued that teachers' subject-matter knowledge is extremely important and have documented the limited content knowledge of prospective teachers (Ball, 1990; Ma, 1999). Universities often require courses designed to enhance prospective teachers' mathematical knowledge by having them make sense of mathematics and understand the principles that underlie the arithmetic they memorized as children. Despite the promising design of these courses, prospective teachers' beliefs about mathematics and teaching often diminish the outcomes of the courses. Some mathematicians have suggested that promoting changes in prospective teachers' beliefs about mathematics and mathematics teaching is critical to helping them to develop the content knowledge that they need to be effective teachers (Conference Board of Mathematical Sciences, 2001; Mathematical Sciences Education Board, 2001). But, teacher educators have been more successful in documenting the existence of beliefs that interfere with prospective teachers' learning than in promoting belief change (Wideen, Mayer-Smith & Moon, 1998).

Often, efforts to change prospective teachers' beliefs are initiated in methods courses after subject matter courses have been completed and come too late to support them in developing beliefs that will help them to develop a deep understanding of fundamental mathematics. The research reported here describes a program that attempted to initiate belief change at the beginning of prospective teachers' mathematical preparation. The Children's Mathematical Thinking Experience (CMTE), part of the Integrating Mathematics and Pedagogy (IMAP) project, helped prospective teachers begin to understand the importance of subject matter knowledge in the teaching of mathematics by having them work with children in an elementary school while they were enrolled in their first mathematics-for-teachers course. For many of the prospective teachers, the field experience was one of their first experiences of working with children in schools and, for most, it was an intense experience that caused them to reconsider their assumptions about mathematics teaching. I analyze the nature of the experience, consider the factors that contributed to its intensity and examine the effects of the experience on the prospective teachers' beliefs. Specifically, it seems that the prospective teacher's interest in relating to children proved to be a stimulus for expanding their views of teaching and affecting their beliefs about learning mathematics.

BELIEFS

Before describing the CMTE, I describe the framework I have adopted for my thinking about the process of belief change and the ways that teacher educators can promote belief change in prospective teachers. The components of this framework include several aspects of beliefs: their origins, their effects on one's interpretations of experiences, the ways separate beliefs combine to create belief systems, and the ways beliefs change within this framework. This framework provides support for the premise that providing prospective teachers opportunities to work with children can be a promising avenue to promote belief change.

Sources of Beliefs

Beliefs can be thought of as having one of two primary sources: emotion-packed experiences and cultural transmission (Pajares, 1992). The first source, emotion-packed experience, gives beliefs their "signature" quality. Many people can point to a vivid memory from which a particular belief emerged (Nespor, 1987). For example, some prospective teachers give detailed accounts of crying while they struggled to learn multiplication tables. They relate these experiences to their belief that they are incapable of learning mathematics. The emotional component of these experiences is one feature that differentiates beliefs from other forms of knowledge. In relating this feature to beliefs about teaching, Goodman (1988) suggested that these beliefs were derived from guiding images based on both positive and negative experiences that teachers had as children.

The second source of beliefs, cultural transmission, creates beliefs that may be held at a subconscious level and can be thought of as resulting from the "hidden curricula" of our everyday lives. Culturally transmitted beliefs often take the form of assumptions and stereotypes. For example, because prospective teachers' mathematics work in school consisted mostly of memorizing procedures, many assume that mathematics always requires memorization, even though they have never heard a statement to that effect. People tend to be unaware of the culturally transmitted beliefs they hold, taking them for granted because they have neither examined nor discussed them. These implicit beliefs may guide behavior in ways that could be characterized as habits, with individuals doing things in particular ways the reasons for which they are hardly cognizant.

Because of their origins, beliefs can be hard to change. It is impossible to undo intense personal experiences or wipe out 20 years of living in a culture. Teacher educators wishing to stimulate belief change in

prospective teachers might pursue the avenue of creating for them emotion-packed experiences that will leave the vivid impressions that form the basis of some beliefs. Another avenue of change is to develop a community that will instill positive implicit beliefs in prospective teachers. However, the duration of a teacher education program may be too short to achieve this kind of belief generation.

Effects of Beliefs

Beliefs have a filtering effect on one's new experiences (Pajares, 1992). This filtering effect, which can make beliefs quite durable, is evident when prospective teachers interpret experiences or information in their courses in ways different from those their instructors intended (e.g., Simon, Tzur, Heinz, Kinzel & Smith, 2000). For example, one colleague had her methods students work with kindergartners during the second week of school. Her intention was for the prospective teachers to realize that young children come to school with a great deal of informal mathematical knowledge. After this field experience, the prospective teachers returned to class impressed with how much the teacher had taught the children in the first week of school (L. Clement, personal communication, January 2001). Their beliefs that children's knowledge of mathematics comes from formal school experiences led them to interpret the experience as evidence of teaching rather than of the children's informal knowledge. This powerful filtering effect of beliefs is responsible for their important role in learning and leads to teacher educators' endeavors to affect the beliefs of prospective teachers.

Belief Systems

Beliefs, whatever their source, are related to one another, forming systems in which related beliefs are connected (Rokeach, 1968). Green (1971) pointed out that belief clusters might be held in isolation, unconnected to other belief clusters. He wrote, "We tend to order our beliefs in little clusters encrusted about, as it were, with a protective shield that prevents any cross fertilization among them or any confrontation between them" (p. 47). For example, some prospective teachers believe that children should have opportunities to be creative. This belief might be connected to other beliefs about art and writing and may come from childhood experiences in these domains. The belief about the importance of creativity for learning may not be connected to beliefs about mathematics because the prospective teachers have not had creative experiences in mathematics. Teacher educators could help prospective teachers connect their belief about the importance of creativity for learning with their beliefs about

learning of mathematics so that they begin to look for the creative potential in problem solving or problem posing. Green (1971) suggested that helping people to develop well-connected belief systems should be one of the primary purposes of teaching.

Rokeach (1968) argued that beliefs related to a particular situation or object, form attitudes similar to the belief clusters that Green (1971) identified. For example, prospective teachers' beliefs about teaching would together form an attitude about teaching. Rokeach pointed out that one dimension of attitudes is their degree of differentiation. Well-differentiated attitudes are those that have a large number of parts and are well articulated. Attitudes formed by culturally transmitted beliefs can be undifferentiated and fairly simplistic. These might be in the form of "teachers should be nice" or "teachers should make class interesting". These undifferentiated attitudes do not encompass the complexity of the situations to which they apply and are similar to stereotypes in that the believer assumes that they can treat all situations of this type as if they were the same, instead of taking into account the particulars of the situation.

Undifferentiated attitudes are often ones that have not been examined. Fenstermacher (1979) pointed out that the role of teacher education programs should be to support teachers in bringing tacit beliefs into the open so that these beliefs can be transformed into objectively reasonable beliefs. This process helps prospective teachers to make more principled decisions on the basis of beliefs they believe are important, instead of acting on the basis of the habits of unexamined beliefs and undifferentiated attitudes. At the beginning of their course work, prospective teachers tend to have undifferentiated attitudes about teaching because they have not had chances to refine them through the reflection that Fenstermacher discussed and may be one of the reasons that prospective teachers undervalue their subject matter preparation.

Changing Belief Systems

Four mechanisms for stimulating belief-system change in prospective teachers have been outlined above: (a) they can have emotion-packed, vivid experiences that leave an impression; (b) they can become immersed in a community such that they become enculturated into new beliefs through cultural transmission; (c) they can reflect on their beliefs so that hidden beliefs become overt; (d) they can have experiences or reflections that help them to connect beliefs to one another and, thus, to develop more elaborated attitudes. Many teacher education programs rely on reflection as a means for fostering belief change. Stofflett and Stoddard (1992) pointed out that this practice can serve to make prospective teachers more articulate

and definite about the beliefs that they held before their teacher preparation started, but may not serve to help them form new beliefs. Vivid experiences coupled with reflection may be required for new beliefs to form.

A fifth kind of belief change, the most dramatic, is the reversal of existing beliefs. A prospective teacher may change, for example, from believing that new material is best presented through a teacher's lecturing to believing that new material is best developed by students' grappling collectively with the material. This kind of conversion is the goal of many teacher educators and, all too often, they are disappointed when prospective teachers fail to reverse their existing beliefs. McDiarmid, Ball and Anderson (1989) found that the prospective teachers in their study failed to reverse their beliefs about teaching as telling. They found that the prospective teachers "expanded the range of options for teaching mathematics" (Wideen et al., 1998, p. 344) but were not converted to a whole new way of thinking about mathematics teaching.

Prospective Teachers Beliefs About the Nature of Teaching

To build on prospective teachers' beliefs, one must recognize that their beliefs about mathematics teaching and learning are part of a larger system of beliefs that also includes beliefs about teaching, generally. For example, two central beliefs of many prospective teachers are that teachers should be nice and should present instruction clearly.

Most elementary school teachers choose to enter the profession because they care about children (Hargreaves, 1994; Howes, 2002; McLaughlin, 1991) and place strong emphasis on affective and interpersonal issues (Weinstein, 1990). Gellert (2000) found that the prospective teachers in his study preferred to shelter students from challenging mathematical problems to protect them from anxiety. He attributed this attitude to the tendency of prospective teachers to see themselves in the role of nurturers. Beliefs about the importance of relationship building endure into the first years of teaching when new teachers often expend a great deal of energy trying to find ways to develop positive relationships with their students (Hollingsworth, 1992). In reviewing the research literature on learning to teach, Wideen et al. (1998) concluded, "Beginning teachers value social and peer groups, positive self-concept, and helping behaviors" (p. 142). The centrality of prospective teachers' beliefs about caring indicate that they will value experiences in which they can be intimately involved with children.

In addition to believing that nurturing is one of the primary functions of teachers, many prospective teachers believe that teaching entails presenting information that student will memorize (Richardson, 1996).

They assume that explaining is a fairly straightforward enterprise (Feiman-Nemser, McDiarmid, Melnick & Parker, 1988). "Teaching itself is seen by beginning teachers as the simple and rather mechanical transfer of information" (Wideen et al., 1998, p. 143). This is another undifferentiated attitude that eventually becomes elaborated as prospective teachers gain teaching experience. Unfortunately, as Weinstein (1989) discovered, this undifferentiated view of teaching leads prospective teachers to underestimate the importance of their subject-matter preparation. Typically, only after they have finished their subject-matter course work and have had teaching experience, do prospective teachers begin to recognize the complexity of teaching and the kind of knowledge required to do it well.

Prospective teachers' views of 'teaching as caring' and 'teaching as explaining' form the basis of their belief systems and will be likely to be retained while their belief systems change and develop. We hypothesize that prospective teachers' beliefs about children and the nature of teaching are more central to their belief systems than their beliefs about mathematics. Our goal in the IMAP project was to take advantage of prospective teachers' interest in children. Others have found that the personal relationships that emerge from individual tutoring sessions have positively affected prospective teachers' learning in the domain of reading instruction (Worthy & Patterson, 2001). We hoped that while they worked with children in the domain of doing mathematics, the prospective teachers' interest in their students would motivate them to expand their view of teaching and of mathematics when they encountered the limitations of an exclusive 'teaching as explaining' approach. We saw their interest in children as the vehicle for motivating them to care about mathematics and mathematics teaching and to begin to alter their views. We also expected that when they had experienced some teaching, their undifferentiated attitudes about teaching as telling would become elaborated.

Generative Beliefs for Learning Mathematics

For our project we identified several beliefs that we hoped the prospective teachers would develop. One is the belief that mathematics is a web of interrelated concepts and procedures. Related to this view are beliefs about the relationship between concepts and procedures: that knowledge of concepts is more powerful and generative than knowledge of procedures and that one can know procedures without understanding the underlying concepts. If prospective teachers begin to appreciate the importance of concepts in developing mathematical understanding, they might try to develop their own conceptual understanding as well as think of ways to teach for conceptual understanding. Other beliefs are related to teaching

and learning mathematics and include (a) believing that children bring to school a great deal of informal mathematical knowledge that can be the basis of instruction and (b) recognizing that often the ways children think about mathematics differ from the ways of adults who have been schooled.

METHOD

Description of the Early Field Experience and the Mathematics Course

Fifteen prospective teachers volunteered to participate in the CMTE, an experimental course, which required that they enroll in both the CMTE and their mathematics course. They were compensated for their participation. At the large urban regional university where the CMTE was offered, there were twelve sections of the mathematics-for-teachers course taught by six different instructors. All instructors used the same text and the same final examination. The instructor of the CMTE also taught the mathematics course for the fifteen prospective teachers so that he could integrate the two experiences as much as possible. He was a professor with a research interest in children's mathematical thinking. The CMTE met eight times for a two hour period at a local elementary school.

The prospective teachers explored number and operations in both the mathematics course and the CMTE. The vision of the course was aligned with that described by the Conference Board of Mathematical Sciences (2001), which emphasized the importance of prospective teachers' appreciation for the intellectual richness of elementary school mathematics. The goals for the course were for prospective teachers (a) to make sense of "non-standard methods commonly created by students, the reasoning behind the procedures, and how the structure of number is used in these calculations" (Conference Board of Mathematical Sciences, 2001, Chapter 3); (b) to understand the variety of ways that the operations can be interpreted; and (c) to be able to represent mathematical concepts in a variety of ways. The domain of the course was whole and rational numbers and included analyses of children's invented approaches to problems.

In the CMTE, pairs of prospective teachers worked with individual children using specific tasks and activities designed to elicit children's thinking; the emphasis was on problem solving rather than on symbol manipulation. Each prospective teacher worked with a partner; one in each pair led the problem-solving session, and the other took notes. Each partner had a chance to perform each role several times. The partners were encouraged to help each other. They often exchanged ideas about which problem

to present next, what question to ask, and so on. The partners discussed the experience afterward and considered issues that arose during the session.

During the first weeks of the course, the prospective teachers had three sessions with 6 to 9-year-old children. The goal in this phase of the CMTE was to influence the prospective teachers' beliefs about children's informal knowledge and the children's tendencies to act out story problems. The prospective teachers provided the children with various whole number story problems that could be modeled using each of the four operations. For the last four weeks of the CMTE, each pair of prospective teachers worked with one 10-year-old child on fraction concepts. This part of the CMTE was designed to show that a concrete approach to this difficult concept can help children develop understanding. The prospective teachers worked with the same child during these sessions so that they could try to teach the child over a period of time.

Participants and Site

Of the 15 prospective teachers in the CMTE, 13 were female and two were male; five were in their first year of university course work; eight were in their third year of university course work (six of whom had transferred from 2-year colleges); and two were postgraduates completing the mathematics prerequisite for the teaching-credential program. Three of the prospective teachers spoke English as their second language; seven of the prospective teachers reported that they had enjoyed mathematics as children and felt fairly successful with it. The other eight reported that they found mathematics to be boring and difficult to learn when they were children.

The prospective teachers worked with children at a multiethnic, urban elementary school in which 46% of the students were White, 39% were Hispanic, 10% were African American, and the remaining students represented a variety of other ethnic groups. Many of the children with whom the prospective teachers worked were bilingual. The school used a drill-based mathematics curriculum, in which emphasis was on acquisition of standard procedures; independent seatwork was the predominant mode of instruction.

Data Sources and Analysis

Data for the study came from a variety of sources including surveys, interviews, prospective teachers' written work, and field notes. Each prospective teacher completed a computerized belief survey at the beginning and end of the CMTE. The survey was a pilot version of a belief survey that was later used in a large-scale study of the CMTE (see Ambrose, 2002,

for more information about the survey). The survey consisted of six open-ended items focused on whole number and rational number arithmetic. To validate the inferences we made from the belief survey, we asked each prospective teacher to discuss and elaborate on the responses given on the survey that they had just completed. Two survey items were of particular importance for this study. Both required the prospective teachers to consider alternatives to standard algorithms, one in the domain of multidigit addition and another in the domain of multidigit subtraction. These items were used to determine the degree to which the prospective teachers believed in the importance of multiple approaches.

Each prospective teacher was interviewed at the beginning and end of the semester and the interviews were transcribed. In the initial interview, interviewers followed a protocol that included questions about the prospective teachers' attitudes toward mathematics, their thoughts about teaching and learning mathematics, and follow-up questions about the belief survey. The interviews were audiotaped and later transcribed. A staff of four researchers collected field notes of the prospective teachers' problem-solving sessions with the children. Audiotapes of the problem-solving sessions were used to augment the field notes. Each prospective teacher was videotaped once during the semester while working with a child. After each problem-solving session, the prospective teachers wrote short personal-reaction papers (Quickwrites) in which they shared their initial impressions; they wrote longer reflections as homework. These reflections were collected and photocopied before being returned to the prospective teachers.

My data analysis was an emergent process similar to that used in grounded theory (Glaser, 1998) and began with intense analysis of the data of one prospective teacher (Donna) in the group. This analysis was on-going during the semester when the CMTE was held. I read and reread Donna's data, looking for emergent themes, in particular, for what aspects of the CMTE she found compelling and how these experiences affected her beliefs. I developed a set of codes and used them to analyze data from four more prospective teachers to confirm or contradict hypotheses. From this analysis, I saw that the sessions with the 10-year-olds had the greatest effect, and some factors related to that experience emerged as being critical to the prospective teachers' belief change. I then analyzed those data relating to the work with the 10-year-olds from the other 10 prospective teachers and coded it according to the nature of the problem-solving sessions, the cognitive demands the sessions placed on the prospective teachers, and the emotional aspect of the work. In charts, I organized segments of coded data from the whole group of prospective teachers to

determine the extent to which various factors affected them and to once again confirm or contradict hypotheses.

Pajares (1992) pointed out that beliefs must be inferred; they cannot be directly measured. In assessing the prospective teachers' beliefs, I looked for their statements and actions that indicated the beliefs they held. Based on the same information, others might come to different conclusions about the prospective teachers' beliefs, so I offer verbatim quotes on which others can base their conclusions about the beliefs of the prospective teachers in the study. Characterizing the beliefs of a group of individuals can be problematic because individuals in the group can hold different beliefs. In several cases, I discuss tendencies within the group and do not mean to imply that all the prospective teachers in the group developed identical belief systems. Because of the sometimes hidden nature of beliefs, even the individuals who hold them may be unaware of their presence. In this sense, any analysis of belief change will require interpretation, and I offer my interpretation with the recognition that there is a subjective component to the interpretation.

THE NATURE OF THE CHILDREN'S MATHEMATICAL THINKING EXPERIENCE

Intense Teaching Experiences

Before considering the belief change that emerged from the CMTE, I begin by establishing that the prospective teachers found the experience to be intense. In their final interviews, when asked about a CMTE episode that stood out for them, 12 of the 15 prospective teachers discussed their work with the 10-year-olds as being the most memorable. Five discussed the excitement they felt when their student told them he or she had learned something. Julie commented, "I was so impressed that he remembered. I was just excited that I actually made an impact". Five discussed their concerns that their student struggled with concepts. Holly said, "I wasn't expecting it to be such a slow process, like with fractions. I found that she was way behind what I thought she could understand". Lisa spoke about her flawed assumptions about her 10-year-old student's understanding of English, and Donna spoke about the power of a real-world context to support her 10-year-old student's thinking. At some point in their interviews, all the prospective teachers talked about their work with the 10-year-olds as being important learning experiences for them.

The focus of the four sessions with the 10-year-old children was fractions. After an assessment session, the children explored, using the pattern blocks, relative sizes of fractions and different names for fractions greater than 1. The children also solved equal-sharing problems that resulted in mixed-number answers. The tasks were designed to build conceptual understanding and to reduce the emphasis on the symbolic work that can lead children to misconceptions (Mack, 1995). The prospective teachers were encouraged to make instructional decisions, during the sessions, to adapt the work to each child's level of understanding. Some prospective teachers introduced the children to adding fractions whereas others spent more time on fractions greater than 1.

In the first fractions session, the prospective teachers found that the children were relatively unfamiliar with fractions. The children did not have a feel for the size of 12/13. They struggled to compare fractions. For example, many thought that 1 was greater than 4/4. The children were not familiar with converting improper fractions to mixed numbers. Most claimed never to have seen improper fractions and did not know how to interpret them. They could partition wholes into parts but had trouble naming the parts they drew.

The second fraction session, which involved using pattern blocks, was a high point for most of the CMTE pairs. The children enjoyed working with the pattern blocks to build representations for a variety of fractional quantities and were successful using the pattern blocks to compare simple fractions such as 1/3 and 1/2. Many of the prospective teachers conveyed this enthusiasm in their writing about the session. For example, Phan wrote, "Our child was on a roll. She would laugh out loud each time we gave her a fraction number". In reflecting on this experience, Lisa wrote,

> I was amazed by the progress he made in such a short amount of time. In the end he was able to push the blocks aside and picture them [fractional quantities] in his mind . . . . Afterwards I felt very proud because I think we honestly helped him with his understanding of fractions.

By the end of the session all the children had converted some improper fractions to mixed numbers without the aid of the blocks, and most of the prospective teachers were excited by the progress the children had made.

The third fraction session surprised most of the prospective teachers. During a group discussion prior to the session, they decided to give the children some problems identical to those they had solved in the previous session: converting improper fractions into mixed numbers. The prospective teachers elected to present the problems in symbolic form without giving the children manipulatives. They asked the children to convert, for example, 7/6 (written in symbols) into a mixed number, and

they found that the children struggled with such review problems. One child claimed that 8/5 and 5/8 were the same, and another child had difficulty drawing a representation for 3/2.

The contrast between the second and third fraction sessions impressed 13 of the 15 prospective teachers. The passages in Table I are representative of the reactions of the prospective teachers to their work in the two sessions. Each of the six prospective teachers quoted in the table worked with a different child.

TABLE I

Prospective Teacher's Reactions to Their Work with 10-Year-Olds

Prospective teacher	Comments following fraction session 2 (pattern-block session)	Comments following fraction session 3
Jane	I now have the greatest feeling because I feel that our student really progressed. I feel like we really taught him something and he understood.	He seemed to forget some of the more basic concepts of fractions. I was disappointed in some of his responses to the easier questions.
Kathy	This week was so impressive. Our student has improved so much over the past week.	Today was so shocking. Last time I walked away saying "WOW". It seems to me that he had learned so much in just a week. Then today it was like we took a backwards step and he had forgotten everything . . . . I was very humbled today.
Ana	I am amazed by the progress he made just by using the manipulatives. I really feel like he has a better understanding of fractions than he had last time when we were just focusing on the written symbols.	In an effort to make our student comfortable and relaxed, we decided to begin our interview with easy review questions. We asked him to draw one and a half. Pretty simple. He couldn't do it. I figured that writing the numbers down would help. It didn't. We assumed these tasks would be effortless for him because he seemed to understand them so easily in our last interview.
Gloria	Wow, I am really impressed. She seemed to have improved since the last time we interviewed her. My partner did a great job of teaching	It seemed like our student didn't retain anything. She made the same mistake and it took her just as log to draw the fractions and compare

TABLE I

Continued

Prospective teacher	Comments following fraction session 2 (pattern-block session)	Comments following fraction session 3
	her that 11/4 is the same as 2 3/4. That was pretty intense.	8/5 and 5/8 .... It was a frustrating session because it seemed like we went backwards .... This session was so aggravating. She did not seem to retain anything we had worked ... nothing seemed to sink in.
Phan	Overall the child improved a lot since last week .... I think she will learn from this experience and remember it forever.	This interview was shocking and yet very sad. She could not remember how to do just simple fractions. This had disappointed us because we thought she had nailed fraction problems last time we met. I learned I cannot teach a child yet.
Donna	Our session went surprisingly well. I was so stoked as we taught her how to do mixed numbers and improper fractions and she picked up on it and was able to write her own. She even was able to do 23/12 into 1 11/12 and 10/4 into 2 2/4. I was amazed.	Our student had forgotten much of what was taught her from the last time .... She was more confused that anything else today.

The terms *stoked* and *excited* used by the prospective teachers to describe the pattern-blocks session point to the intensity of the experience. The prospective teachers' initial reactions reflect the optimistic bias that Weinstein (1989) identified and might strike the reader as naïve or exaggerated. Keep in mind, however, that some of the prospective teachers were 19 years old and had limited experiences working with children. Their original assumptions that they had been successful were based on what some might argue was a relatively limited period of time with a narrow range of fractions (those that could be represented with the pattern blocks). Several of the prospective teachers' comments indicated that they thought that the teacher was responsible for the success of the session: "We really taught him something"; "My partner did a great job of teaching".

"Today was so shocking." "This had disappointed us." "So aggravating." These reactions to the follow-up session express the prospective

teachers' chagrin and surprise at their students' lack of retention. Their surprise reflects the novelty of this experience for them. Experienced teachers would be unlikely to be surprised that the children had trouble remembering a symbolic approach to converting fractions after being exposed to it for only a short amount of time, but for these novice teachers, this result was unexpected. The experience was particularly troubling to the prospective teachers because they felt that the children had been so successful during the pattern-block activity, and many felt responsible for that success. Their comments ("I was very humbled today"; "I learned I cannot teach a child yet") show how personally the prospective teachers took this experience. The emotional charge of the experience contributed to its intensity and left the kind of vivid impression from which beliefs can grow.

Two of the prospective teachers were less affected than the others by these two sessions and were cautious in their evaluations of what the children understood from the pattern-block session. Nina noted that her child had been successful, but she was not convinced that the child's understanding was completely developed. She wrote, "I was impressed by my student's advancement from the week before. She came a long way. However, I don't feel like she really understands what she is doing fully". Holly was concerned by her child's belief that there are no fractions in which the numerator is greater than the denominator. She wrote, "I was frustrated. I could see Stephanie's confusion, but I don't know if I parted the clouds for her". These two prospective teachers seemed not to have felt the elation or let down as intensely as their peers, and the sessions may not have engendered new beliefs for them. Their caution in reacting to the pattern block session indicates the reflective stance that these two prospective teachers had throughout the CMTE. For these particular teachers, the belief-system change that they exhibited may have been due more to their ongoing reflection than to an intense experience.

Several features were in place that created the conditions necessary for these teaching experiences to have the intensity that they did for the majority of the prospective teachers. These features will be explored in the next section.

Focus on Mathematics Learning Rather Than on Classroom Management

The fact that prospective teachers worked with individual children contributed to the intensity of the experience. Because they were working one-on-one with the child, the prospective teachers did not face the cognitive overload that can accompany teaching. When prospective teachers are in student-teaching situations, they must attend to all the issues of class

management *and* content, and their cognitive capacity is overwhelmed by all the stimuli they are trying to assimilate (Hollingsworth, 1989). Through their work with an individual child, the prospective teachers had clear evidence that the child was struggling and that their teaching had not been entirely successful. They could concentrate their thinking on this issue because they were not distracted by the host of management issues that typically preoccupy student teachers. They could not attribute the children's difficulties to their behavior, attention span, or attendance. Nor did they have the option to turn to a different child to get the answers they sought; no other children could "bail out" the children or the prospective teachers. They had to face the fact that this was difficult material to teach and to learn.

High Cognitive Demand

Although the CMTE was stripped of some factors that occupy teachers' minds, the prospective teachers had to think about several things while they worked with their children. They experienced what some call "knowledge in use" (e.g., Ball, 2000), that is, the knowledge that teachers must use while teaching. They had to consider the mathematical concept at hand, attend to what the child was doing and consider what understanding the child had. They had to decide what question to ask or what problem to provide in order to extend the child's understanding and what representation might help the child better understand the concept. Lisa talked about the challenge of this cognitive demand, observing "trying to work with them and think on my toes and figure out what questions to ask really fast was hard".

Donna talked about the difficulty of finding an appropriate vocabulary that would make sense to the child:

Donna:	Sometimes you need to change your vocabulary or whatever so that it fits their world. Sometimes you don't know what to say. You feel like you understand – like you're explaining the right thing.
Interviewer:	It makes perfect sense to you.
Donna:	Yeah. but not to the child. Sometimes you can't explain things.

Many prospective teachers talked about developing explanations as one of the most challenging aspects of their work with the children. Julie said,

I had no clue how to explain this one problem. He was just looking at me like, "Explain it". I was just like, "I don't know!" I didn't even know how to explain it . . . . I know how

to find the common denominator for an addition problem, but I didn't know how to teach it. So that was what was hard.

Kathy said, "I was trying to explain it to him, and I confused myself. It was horrible". When they struggled to find the words to generate clear explanations, the prospective teachers experienced the high cognitive demand of teaching. This was a novel experience for most of them, because few had ever been in the position of having to think on their feet in this way. The cognitive demands of teaching coupled with the novelty of this kind of thinking contributed to the intensity of the experience.

Connecting with Children through Mathematics

One critical feature of the work in the CMTE was the interpersonal component. Working with the children on fraction concepts provided the prospective teachers opportunities to connect with children and to develop relationships, a feature evident in some prospective teachers' comments about their children. Julie wrote, after her last session, "I'm so sad that I won't be seeing him anymore. He had to have been the most polite child I have ever seen. He was so sweet . . . . I miss him already". Although this comment may seem overly sentimental, the prospective teacher writing it was one of the 19-year-old freshmen who had limited experiences working with children. These were the first words she wrote when reflecting on her final problem session, showing that this personal relationship was the most salient aspect of the work for her. Joe brought a present for his child to the last interview and mentioned how much he had enjoyed getting to know her. Tom and Alison commented that they were touched that their child cared so much about their work with her that she held on to a mathematics paper for two weeks and brought it back to one of their sessions.

The one-on-one teaching situation was intimate in that the child was asked to share his or her thinking, and the prospective teachers were committed to listening. Goldstein (1999) wrote about this kind of interaction as involving both an intellectual component and an emotional component, requiring engagement and receptivity on the part of the teacher. As Tom noted,

Going and dealing with the student really kept me on my toes, because we had that added responsibility. I wasn't just responsible for my own time and knowledge, but I was actually going to be meddling with somebody else's knowledge and time. I think that made me focus more.

Kathy noted how important working with a child was for her: "Where else do you get an opportunity to sit with a child and have the child share what she is thinking? It was almost an honor to have the child do that. I felt

very privileged". The prospective teachers developed emotional ties that contributed to their learning because of their personal investment, which, along with the focus on the mathematics learning of one child and the high cognitive demands of this work, made the CMTE the kind of engrossing experience with the emotional charge that leaves an impression which can give rise to beliefs.

FRACTIONS WORK'S EFFECTS ON PROSPECTIVE TEACHERS' BELIEFS

Beliefs about Teaching Mathematics

The fractions-teaching experience affected the ways that many of the prospective teachers viewed teaching mathematics. They began to recognize that teaching requires more than simply presenting information to students. At the end of the semester, Donna commented about what she would tell other prospective teachers: "Well, like I said before, not to expect that a child knows what you've taught 'em, because just because you've taught 'em doesn't mean that they understand it". Although Donna may not be using the term *understanding* in a conceptual sense and may instead be talking about a student's ability to remember a procedure, her comments reflect her recognition that teaching does not equate with student learning. We inferred, from the way she phrased her response, that before the CMTE this prospective teacher had expected that "a child would know what you've taught 'em", reflecting an initial stereotypical attitude that teaching simply entailed presenting information to students.

Tom commented about how his views of teaching had changed: "You think, 'Oh well, I'll just tell them this and they'll understand it'. And then when you work with kids, you realize that it doesn't work that way". Tom's transmission view of teaching had been expanded when he realized that children did not readily learn the material that he thought he had transmitted to them. Kathy stated,

> I went into class that day thinking, "I'm so excited. I'm going to teach him this. By the end of the hour, he's going to know it and he'll be able to do it forever". And it didn't happen that way, so I guess to just keep that in mind and to know that it's not going to only take an hour for a child to understand a concept.

Kathy originally assumed that her student would absorb and retain the information she presented, and, through the CMTE, she learned that teaching was not as straight forward as she had initially thought.

After their experiences in the CMTE, most of the prospective teachers talked about the importance of providing children time to think, both

during individual problem-solving sessions and over long periods of time. Donna replied, in responding to the question "What did you learn from the CMTE?"

> . . . not drill it into their heads. When children learn, they need their own space and time to learn on their own. Let them have a chance first, and then see what they need help with.

We inferred from Donna's suggestion, "Let them have a chance first and then see what they need help with", that she was beginning to develop a more student-centered perspective toward teaching that might include a teaching-as-telling orientation but also included a concern for carefully timing her lectures and assessing what the children knew first. Her more student-centered perspective was reflected when she later said,

> I wanted to help her, but I also wanted her to do it on her own. I didn't want to step in too much . . . . I have learned to give my child enough time on her own to try and figure out a problem before I jump in and help her.

These comments illustrate how Donna's attitude toward teaching became differentiated. She began with a stereotypical and simplistic attitude, consisting of the belief that teaching entails presenting information. She continued to hold this belief, as was evident in her comments about wanting to help the student. She added to that belief another belief about teaching as stepping back to allow students time to think. Her attitude toward teaching grew to include two beliefs that were connected, and in this way the attitude became differentiated.

The issue of providing children with time was a recurring theme in many of the prospective teachers' suggestions for people who might participate in the CMTE. Kathy mentioned, "Give them time – don't bombard them with questions". Jane stated,

> They don't really learn anything when you just give them the answer. You just have to give them time. You can't just push them and keep asking questions, because when I did that, they were just frustrated.

Julie said, "Give them time and don't show them everything. You have to let them discover it for themselves, but you can help them along".

Jane and Julie equated providing children time to think with allowing the children to figure things out for themselves. They cautioned against "giving answers" and "showing them everything", indicating that they had expanded their attitude toward teaching beyond merely presenting information to include facilitating children's thinking. Nina explained her expanded view:

> Teaching is not me giving the information, and then them absorbing it, but rather giving them the tools that they need to learn on their own. I think that's probably the most important thing that I learned.

The prospective teachers characterized the idea of giving children time to think and letting them discover things for themselves as insights that they had gained from the CMTE. Apparently they had not started the semester with these ideas but developed them while working with the children. We took this as evidence that their attitude about teaching expanded as a result of their experience.

All the prospective teachers recognized that teaching is not as simple as they had expected it to be. Holly summarized her learning:

> It was a lot more complex than I expected. It was also good talking to all my classmates, seeing that it's not as clean-cut as we thought it would be.

Most came to believe that providing children with "think time" was an important element to good teaching. Several came to believe that children should have opportunities to figure things out for themselves, and a few developed faith that children could learn on their own when given appropriate tools.

Beliefs about Multiple Solution Strategies

The prospective teachers grew to appreciate the importance of multiple solution strategies in mathematics. Their appreciation was apparent in their comments in interviews as well as in their belief-survey responses but was less apparent in their work in the problem-solving sessions.

In their interviews, the prospective teachers focused on the importance of knowing different mathematical approaches for successful teaching. In her final interview, when asked what she had learned from the CMTE, Donna stated that she needed "to be able to know how to do things more than just your way – that your way doesn't work for everybody. Kids learn in different ways". Nina noted the importance of being flexible: "You have to be able to be flexible and approach math problems from different perspectives. Each child's learning is going to be different". For these prospective teachers, being familiar with different approaches would allow them to assist different children.

Tom noted that being familiar with different approaches to problem solving helped him in his teaching. He mentioned "getting away from formulas. Using manipulatives and drawing pictures seemed to really help them make sense of what was going on". These comments indicate an interest in having all children use manipulatives and pictures as a way to deepen their understanding of the concepts. He was not advocating multiple strategies to meet multiple needs; instead, he was arguing in favor of individuals knowing multiple strategies as a means for making sense of the mathematics. Responses to the belief survey provided further

evidence for changes in the prospective teachers' beliefs about multiple solution strategies. In considering the addition segment (see Figure 1), all the prospective teachers wanted more strategies shared at the end of the semester than they had at the beginning of the semester.

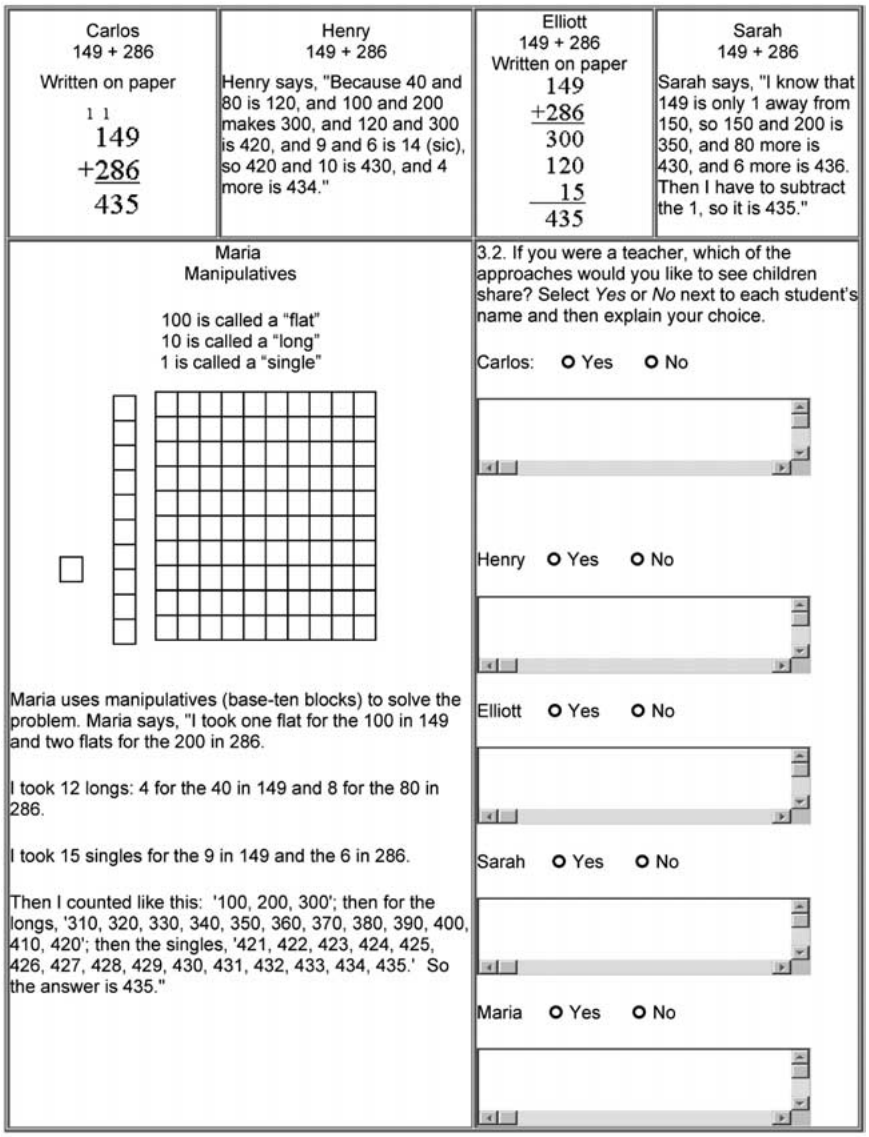

Figure 1.

Six of the prospective teachers who originally wanted one or two strategies shared wanted four or five strategies shared at the end of the

semester. At the beginning of the semester, two prospective teachers noted the connection between the addition strategies and place-value understanding whereas at the end of the semester, eight commented on the importance of helping children make this connection. By the end of the semester all the prospective teachers believed in the importance of multiple solution strategies. For some this belief was evident at the beginning of the semester but seems to have been strengthened; for others it had developed over the course of the semester. All the prospective teachers saw value in multiple solution strategies, and some saw multiple solution strategies as a vehicle for promoting conceptual understanding.

The prospective teachers were asked, in another segment on the belief survey, which of two strategies they would prefer their students use (see Figure 2). At the beginning of the semester, nine preferred that students in their classrooms exclusively use the standard algorithm for multidigit subtraction. In supporting this choice, many wrote that it is less prone to error, faster or "more of a sure thing". We inferred from these responses that these prospective teachers were focused on the production of answers. At the end of the semester, 13 wanted their students to have access to both the standard algorithm and an alternative approach. (Of the two remaining, one wanted her students to use the standard algorithm, and the other wanted her students to use the alternative approach.) In explaining why they wanted children to have access to both strategies, five mentioned that children learn in different ways and so should have a choice of strategies. We inferred from their responses that although these prospective teachers were open to multiple solution strategies, they also believed that computing correct answers is a focal point for instruction. In contrast, six mentioned that children should learn both ways so that they would better understand the concepts associated with the procedures. We inferred that these prospective teachers had become interested in having their students develop conceptual understanding. Julie wrote, "I would want them to understand the concepts behind the traditional way". This response was in contrast to Alison's: "I think both are important because some kids may be able to solve the problem easier with the other way"; her focus is on generating correct answers rather than on understanding concepts. As was evident in the interview data, some prospective teachers were interested in multiple strategies to accommodate different learning styles, whereas others were interested in multiple strategies as a means to promote understanding.

Although the prospective teachers talked about valuing multiple solution strategies, this belief was not always evident in their work with the 10-year-olds. When they had opportunities to make instructional decisions,

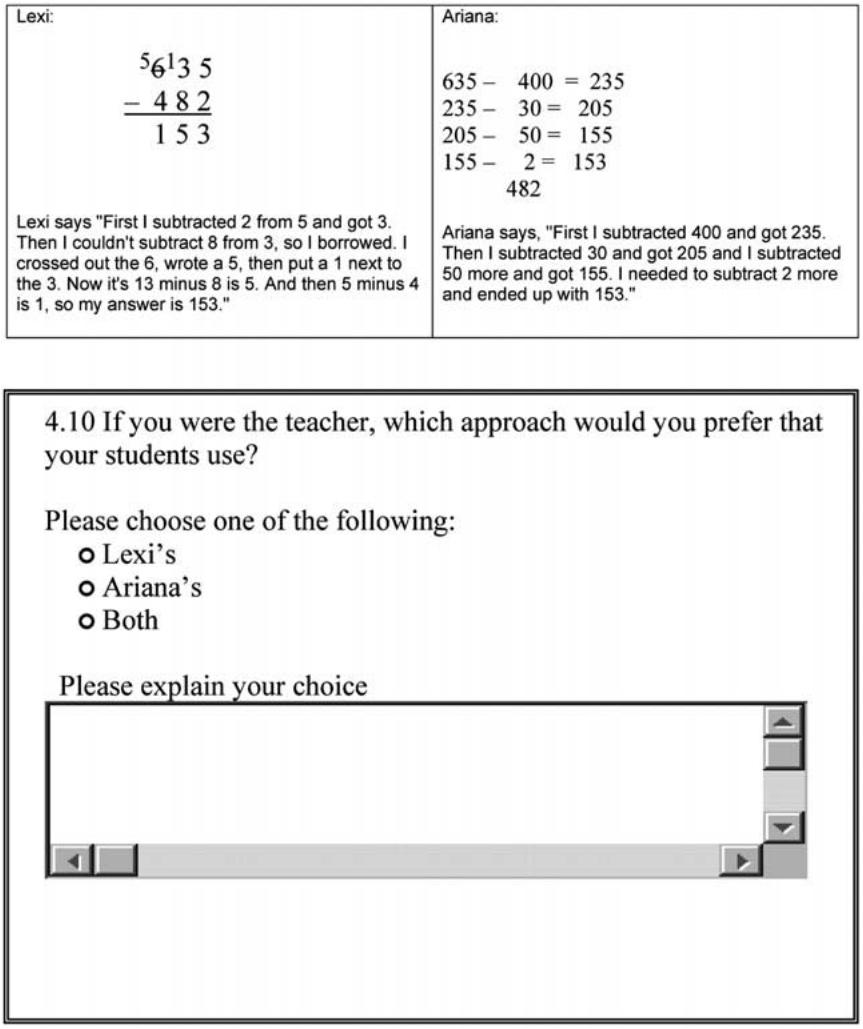

Figure 2.

three of the seven groups chose to teach their students the symbolic procedure for adding fractions by finding common denominators. When they did so, they neither supported their students in developing their own approaches for solving these problems nor presented multiple strategies. They attempted to explain the procedure for finding common denominators and in so doing ran into difficulties. Margie explained her experience: "I totally confused her so much . . . . I didn't even think of showing her with the blocks. I was just showing her the procedure way. Then afterwards I thought, 'Oh God, I should have showed her this way' ". We were some-

what disappointed when we saw the prospective teachers focusing on the standard symbolic procedure after they had discussed the value of multiple approaches. We were relieved that at least Margie recognized that she might have approached her instruction differently.

Beliefs about the Importance of Mathematics Understanding for Teaching

The prospective teachers' experiences in the CMTE helped them to realize that their understanding of mathematical concepts was essential to their success as teachers. Cindy stated,

> I want to teach young children, so I didn't think I needed to know a whole lot of actual mathematical skills and I really disagree with that now. In order to come up with a creative way to teach it, you need to understand what you're talking about and you need to have the math skills to do that.

This prospective teacher came to the realization that even mathematics for young children was complicated to teach and required conceptual understanding.

Many of the prospective teachers talked about how the CMTE helped them to appreciate the importance of the material that they were learning in their mathematics course. Margie noted, "When we were like, 'Why do we need to know the meaning of this?' and it's so you can explain it, because you can't teach something to a student if you don't know what it means".

Many noted that the experience of the CMTE made the contents of the mathematics class more compelling. Phan noted, "We understand that knowing math is easy but knowing how to teach math to children is hard work". Ana expanded on this observation:

> I think that as students, we tend to think when we are learning these math concepts that it's so obvious and easy and that it will be easy to teach them to students. But it's not as easy as we think it is, and the students tend not to know as much as we think they know. We're not just going to go in and knock them dead. It's going to take work and thought.

The prospective teachers began to distinguish *doing mathematics*, which they equated with using memorized procedures and considered easy, from *teaching mathematics*, which they equated with understanding and considered more difficult.

Many shared Ana's observation that the children did not know as much as the prospective teachers had expected them to and remarked that without the CMTE they would have doubted their instructor when he told them that children have trouble with particular concepts or tend to think about problems in specific ways. Gloria observed,

> The instructor would have said, "This is how the kids are learning", but that wouldn't have meant anything to me. I would have been like, "So . . .? Okay. I'll just concentrate on doing

the math problem". But during the CMTE, it's like you're right there and applying what you're learning and seeing whether or not it works.

When the prospective teachers had first-hand experiences teaching mathematics, they realized that they needed to understand it well and began to appreciate the value of their mathematics class. As Gloria pointed out, without the practical experience, most would have focused only on mastering techniques for solving problems instead of on the mathematical concepts related to those techniques.

Beliefs about Children's Informal Knowledge

The first three sessions of the CMTE, those with the 6–9-year-old children, were intended to acquaint the prospective teachers with the informal knowledge that children bring to school. By having the prospective teachers pose story problems to young children, we had hoped that the prospective teachers would see that the children could model the action of the problem using manipulatives and solve a variety of story problems without much guidance. Most of the children that were interviewed had difficulty with several of the story problems, perhaps because they had not had any exposure to using manipulatives to model story problems in their classrooms. Although many children can readily solve such problems (Carpenter, Fennema, Franke, Levi & Empson, 1999), the children in the CMTE could not. They tended simply to add the two numbers in the problems posed, regardless of the action in the problem. Although many of the children could solve multiplication and division problems set in story contexts, they were so unsuccessful with some of the other problem types (comparison situations, join change unknown, etc.) that the prospective teachers tended to focus on the children's difficulties rather than on their successes.

The prospective teachers found the interviews with the primary-grade children to be awkward because they saw each child only once and did not have opportunities to develop rapport with the children. They also had difficulty observing while their child was working instead of showing the child how to solve problems. Holly noted, "It was challenging for me to hold back and not help him by hinting at ways to solve the problems he was having difficulty with". The children struggled to explain their thinking, probably because explanations were not asked of them in their classrooms. Ana noted, "The primary interviews didn't make much of an impression on me, because I didn't expect much out of the kids, to be perfectly honest, and we didn't get much". Unfortunately, the work with the primary children left little impression on most of the prospective teachers, and it was unclear how their work with the primary children affected their beliefs.

BUILDING ON EXISTING BELIEF SYSTEMS

The belief-system change that I have attempted to illustrate focused on the elaboration of attitudes through the development of new beliefs and the formation of new connections among beliefs due to an intense experience coupled with reflection. We speculate that the prospective teachers' original attitudes about teaching were undifferentiated, consisting of few beliefs, because of their limited experiences. Inasmuch as they had not had opportunities to reflect on their beliefs about teaching, the beliefs remained at the subconscious level. When they struggled to teach fractions to the 10-year-olds, their beliefs about teaching became more salient to them. They added to their beliefs the notions that teachers should listen to children to determine at what point to begin instruction, provide children with time to think, and be prepared to explain concepts in a variety of ways. This is an example of the elaboration of an attitude when new beliefs emerge and become connected to existing beliefs. The basis of this belief change was an intense experience.

The struggles that many of the prospective teachers had, when they attempted to help their children understand fractions better, could be characterized as failed teaching experiences. Weinstein (1990) suggested that failed teaching experiences were critical in helping prospective teachers to overcome their optimistic bias about their abilities as teachers. Weinstein speculated that prospective teachers need to see that teaching is not as easy as they had believed and that facing the challenge of teaching students, particularly those who struggle, would affect their beliefs. This seems to be the case for the prospective teachers in the CMTE. As Lisa wrote, "I am beginning to find out that teaching is not as easy as is looks. It takes a lot more to be a teacher than enjoying working with children". The prospective teachers' CMTE experiences helped them to recognize that mathematics teaching is much more complicated than they expected it to be and helped them to appreciate the value of the material in the mathematics course.

Note that most of the prospective teachers continued to hold onto their beliefs that teaching involves explaining things to children, even though they spoke of the importance of giving children time to think for themselves. This was evident in the prospective teachers' actions in the problem solving sessions when they presented children with the standard algorithm for fraction addition. We concluded from their actions that, for at least the six prospective teachers in these three partnerships, their actions indicated a "teaching as telling" belief along with a belief about mathematics learning as the acquisition of standard symbolic procedures. We did not conclude that our efforts to help them change their beliefs were wasted. Instead we interpreted these examples as evidence that prospective

teachers do not let go of old beliefs while they are forming new ones. Their new beliefs about the importance of multiple solution strategies, or their knowledge of these approaches, or both of these, were not strong enough to compel them to introduce multiple approaches for adding fractions. We were encouraged that the prospective teachers could be critical of their own actions, and we concluded that while their beliefs grew, several new beliefs which might compel them to make different teaching decisions in the future, would be added to their belief systems. Their lack of success in teaching the addition of fractions may accelerate their acquisition of new beliefs while they experience the limitation of their existing belief system. The mismatch between the prospective teachers' comments in their interviews and surveys and their decisions with their students may be construed, not as evidence of conflicting beliefs, but as evidence of evolving beliefs. Although belief change had been initiated, the IMAP team recognized that the prospective teachers had not developed all the beliefs we would like them eventually to develop. Over time, the prospective teachers' belief systems may continue to change when they have more experiences upon which they have opportunities to reflect. More beliefs may be added to their belief systems, and their beliefs about teaching-as-telling may still exist but may be weaker or less central.

Providing prospective teachers with intense experiences that involve them intimately with children poses a promising avenue for belief change. Coupling these experiences with reflection allows the beliefs that arise from these experiences to be examined and refined. The CMTE came early in the teacher preparation program. Given the incremental nature of belief change, teacher educators might consider creating several such experiences throughout the teacher preparation program, especially while prospective teachers are doing their subject matter preparation, to ensure that the beliefs become well connected and that attitudes become differentiated. One intense experience is a starting point but seems insufficient to catalyze all the belief change that teacher educators might desire. Various experiences are required to help prospective teachers develop new beliefs about mathematics, and teacher educators would be wise to recognize that these beliefs will probably coexist with, rather than replace, the beliefs that preceded them.

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