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and Economic Growth”**

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NIPE WP 13/ 2012

NÚCLEO DE INVESTIGAÇÃO EM POLÍTICAS ECONÓMICAS  
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# Innovation Economy, Productive Public Expenditures and Economic Growth

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## Abstract

Innovation is the main engine of growth in an increasing number of economies. Innovation economies are, according to the Quadruple Helix (QH) Innovation Theory, sustained by four pillars – Firms, Academia, Government and Consumers –, all operating in a systemic, interactive environment. We provide a model that gives analytical body to the QH theory and links formally innovation to economic growth. We aim to emphasise the equally important roles of the four helices sustaining an innovation economy and its long run growth. In particular, given the downwards pressure on Government expenditures, we analyse the effects of an increase in public expenditures on economic growth, which we find positive in the short, medium and long-run.

**Keywords:** Innovation Economy, Consumers, Quadruple Helix, Productive Public Expenditures, Economic Growth.

**JEL Classification:** O10, O31, C63.

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## **1. Introduction**

Economic growth is, in Turnovsky's (2000) words, a permanent goal in policy makers' minds. *Europe 2020 Strategy*, for instance, sets European Union's ultimate goal as the achievement of smart, sustainable and inclusive growth.

Innovation constituted the main engine of growth in countries like Austria, Finland, Sweden, the United Kingdom and the United States, between 1995 and 2006, according to the Organisation for Economic Cooperation and Development (OECD, 2010). In our days, having to tackle serious socio-economic challenges and simultaneously generate new sustainable sources of economic growth, both industrialised and emerging nations are increasingly assuming their character of innovation economies.

Meanwhile, digitalisation and globalisation have changed innovation's nature and the innovation system. Economic agents are playing different roles in the innovation process, and are guided by new functioning rules. Essential changes have occurred regarding: (i) what innovation is; (ii) who does it; and (iii) how it is done.

Innovation used to be understood as the result of research and development (R&D) activities, undertaken by researchers equipped with formal scientific and technological knowledge. Today, although science and technology are (and will continue to be) the main sources of innovation, new non-technological drivers of innovation (OECD, 2010) have come into play. Innovation is now defined as the introduction of a new product or service, a new process, or a new method (*Oslo Manual*). Formally educated researchers do not have the exclusivity of innovation

activities and incoming innovators must be equipped with new, multidisciplinary skills and competences. In this new creative environment, science and technology can be drivers and also enablers of innovation.

In a linear, top-down, inside-out philosophy, technology-driven innovation used to be performed in secrecy within companies that would subsequently use marketing techniques to reach consumers and sell their products. Nowadays, firms can no longer count on passive consumers. Being worldwide connected, informed and participative, today's citizens are empowered consumers and innovation co-creators. They interact with firms and the government giving ideas, suggestions, demanding for goods and services with specific characteristics, like smarter or greener products and services. (Arnkil et al., 2010). Such interaction forces firms and governments to acquire higher levels of social responsibility and to engage in innovation so as to meet informed and concrete demands. Indeed a new balance between technology-driven, competitive-driven and user-driven innovation has been setting in, with increasing weight given to consumers (Fora, 2009).

The ongoing structural change of societies and the dynamic nature of innovation call also for a change of culture in the public sector. Policy formulation tends to become less control-based and more influence-based, and Governments will increasingly have to innovate and work in interdependence and collaboration with private firms and organisations, universities and citizens in order to create new solutions to societal challenges, to deliver adequate public services and to design new policy instruments (Fora, 2009). Governments can also foster innovation through

infrastructure provision and maintenance, introduction of smart regulation, the exercise of intelligent demand, alleviation of bottlenecks on innovation initiatives, and through improvements in the processes of accumulation of new forms of knowledge, skills and competencies required in innovation economies.

The growing multidisciplinary, complexity and costs of innovation imply that isolation and secrecy no longer are an option for any innovative agent. Innovation results, instead, from the creative interaction and cooperation between big and small, private and public, academic and non-academic institutions, and the well informed and increasingly demanding consumers. Today's innovative agents co-create and co-produce within networks, partnerships, symbiotic relationships and collaborations.

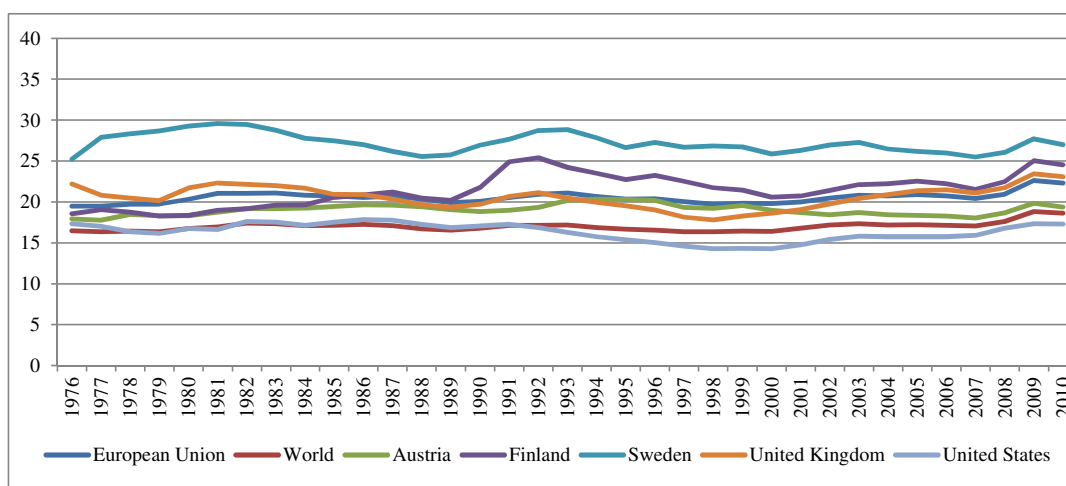
The Quadruple Helix (QH) Innovation Theory is a conceptual approach to an innovation economy. A QH innovation model is an innovation environment in which four economic pillars/helices – Consumers, Firms, Academia and the Government – cooperate and co-produce technological, social, product, service, commercial and non-commercial innovations, in an open, systemic fashion (Arnkil et al., 2010). The QH innovation concept is closely linked to the *Europe 2020 Strategy* for growth.

Current times are of downward pressure on public expenditures. However, being one of the four pillars of an innovation economy, the government's role cannot be downplayed. The OECD (2010), for instance, reminds us that the long-run growth of innovation economies relies crucially on continued baseline public investment in education, infrastructure (provision and maintenance) and research.

The existing literature regarding the impact of public expenditures on economic growth has several theoretical and empirical shortcomings, as Romp and De Haan (2007) observe. Aschauer's (1989) seminal paper and its followers find large effects of public capital on growth and productivity. However, Sturm et al. (1998) point out that this first generation of studies present substantial methodological and econometric limitations. Holz-Eakin and Lovely (1996, p.106), for instance, also note the inexistence of formal economic models predicting the effects of infrastructure on productivity. Still, recent studies, surveyed by Romp and De Haan (2007), tend to be more consensual than earlier papers in finding moderate positive effects of public expenditures on per-capita income and on economic growth.

Figure 1 illustrates the government's role in the above mentioned innovation economies and in the world, over the last four decades. It shows that the share of public expenditures on output is, in fact, stable with a slight increasing tendency.

**Figure 1. General government final consumption expenditure (% of GDP)**



Source: World Bank (available at: <http://data.worldbank.org>; accessed in April 2012)

With this paper, we propose to highlight theoretically the government's role in an innovation stylised economy. In order to formally capture the QH Innovation Theory, we build on a R&D-based growth model, which, given the new nature of innovation that we wish to capture, we rename innovation-based growth model. Introducing public expenditures in the setup, we perform one economic policy analysis. An increase in public expenditures raises the economic growth rate of an innovation economy.

In the proposed QH innovation growth model, we specify a one-sector productive structure with public productive expenditures and the presence of complementarities between intermediate inputs in the production function for all-purpose aggregate output. The one-sector structure is specified so as to capture innovation's new nature; that is, the notion that innovation is co-produced by all economic agents. Secondly, government is here assumed to provide a pure public good – expenditure on education, health, infrastructures, technological and innovation services and regulations – which increases the productivity of all inputs. Thirdly, we specify complementarities because they capture the co-creation characteristic of innovation economies and are considered essential for sustained innovation (Lundvall and Borrás, 1997), hence for sustained growth in innovation economies.

Our main finding is that an increase in the proportion of output spent on public expenditures has a positive effect on the economic growth rate in the short (initial level effect) medium (transitional dynamics) and long (steady state) run.



After this Introductory section, in Section 2 we set up the model. Section 3 shows the general equilibrium results and in Section 4 we describe the effects of an increase in public expenditures on the economic growth rate. We close the paper with some Final Remarks.

## **2. Set up of the Model**

As explained in the Introduction, we wish to frame analytically, within a growth model, an innovation economy as conceptually described by the Quadruple Helix (QH) innovation theory. In a QH innovation model, four economic helices – Consumers, Firms, Academia and the Government – cooperate and co-produce, via partnerships and simpiotic relationships, technological, social, product, service, commercial and non-commercial innovations, in a systemic fashion.

Wishing to portray an inclusive innovation economy, we assume that the whole society takes part in the innovation process. Hence, we specify a one-sector model in which innovation is undertaken with the same technology as that of the final good and inputs.

Additionally, we wish to capture the notion that in innovation economies, no single institution can innovate and work on its own. Profit seeking companies have to partner up, co-innovate and co-produce within networks. The concept of complementarities seems ideal to describe an innovation economy. Hence, we assume the presence of complementarities between all innovative intermediate companies.

Regarding the Government, Figure 1 reveals the significant constancy of the

ratio between public expenditures and GDP, over long periods of time. Therefore, we assume a behavioural version for public expenditures, specifying that in each period the flow of public expenditures is a fixed proportion of aggregate output.

## **2.1. Production Side – Technology Equation**

There is one final good,  $Y(t)$ , produced with constant labour,  $L(t)$ , public expenditure,  $G(t)$ , and the non-durable inputs,  $x_i(t)$ , of a number  $A(t)$  of Intermediate Productive Units  $i$  ( $i = 0 \dots A$ ). Each Intermediate Productive Unit (IPU) is associated with one innovation  $i$ .

### **2.1.1. Government Expenditure**

The Government's role consists in providing a pure public good – in the form of government expenditure on education, health, infrastructure, technological and innovation services and regulations –, which increases the productivity of all inputs in the same way. In our behavioural version, we assume that, in every time  $t$ , productive government expenditure,  $G(t)$ , is a constant fraction of output,  $Y(t)$ :

$$G(t) = \tau Y(t), \quad 0 < \tau < 1, \quad (1)$$

where  $\tau$  is the share of output allocated to public expenditure. Following Barro (1990), productive government expenditure is a flow variable. The government's budget is balanced in all periods. Assuming, for simplicity, zero-public-debt, and zero-consumption-taxes, the government's budget constraint is:

$$G(t) = T(t), \quad (2)$$

where  $T(t)$  represents lump-sum taxes.

### 2.1.2. Intermediate Productive Units (IPUs)

Academy&Technological Infrastructures and Firms are assumed to have an identical productive role in this economy. They constitute the intermediate productive units (IPUs)  $i$  ( $i = 0 \dots A$ ), and contribute to aggregate output production,  $Y(t)$ , by producing non-durable inputs  $x_i(t)$ . As in Afonso et al. (forthcoming), there are complementarities between the IPUs inputs in the production function for  $Y(t)$ .

### 2.1.3. Aggregate Output – Final Good

It follows that the production function for  $Y(t)$  is:

$$Y(t) = L(t)^{1-\alpha-\beta} G(t)^\beta \left( \int_0^{A(t)} x_i(t)^\gamma di \right)^\phi,$$

which, substituting  $G(t)$  by its equivalent given in equation (1), becomes:

$$Y(t) = \tau^{\frac{\beta}{1-\beta}} L(t)^{\frac{1-\alpha-\beta}{1-\beta}} \left( \int_0^{A(t)} x_i(t)^\gamma di \right)^{\frac{\phi}{1-\beta}}, \quad \gamma\phi = \alpha, \quad \frac{\phi}{1-\beta} > 1. \quad (3)$$

The parameter restriction  $\gamma\phi = \alpha$  is imposed so as to preserve homogeneity of degree one, and assumption  $\frac{\phi}{1-\beta} > 1$  is made so that the IPUs' inputs  $x_i$  are complementary to one another; i.e., so that an increase in the quantity of one input increases the marginal productivity of the other inputs.

Assuming that it takes one unit of physical capital  $K(t)$  to produce one physical unit of any type of IPUs input,  $K(t)$  is related to inputs  $x_i(t)$  by the rule:

$$K(t) = \int_0^{A(t)} x_i(t) di. \quad (4)$$

#### 2.1.4. Innovation

Wishing to frame the idea that the whole society is involved in the innovation process, we follow Rivera-Batiz and Romer (1991) and specify a one-sector structure in that innovation is undertaken with the same technology as that of the final good  $Y(t)$  and the IPUs' inputs. We further assume that innovation  $i$  requires  $P_A i^\xi$  units of foregone output, where  $P_A$  is the fixed cost of one new innovation in units of foregone output, and  $i^\xi$  represents the additional cost of innovation  $i$  in terms of foregone output, meaning that the higher the index of one innovation, the higher its innovation cost. Like in Evans et al. (1998), this extra cost is introduced in order to obtain a balanced growth path solution. Total innovation expenditure hence mounts to  $P_A(t)\dot{A}A(t)^\xi$ .

#### 2.1.5. Total Investment

With zero depreciation for simplicity, total investment in each period,  $\dot{W}(t)$ , is equal to physical capital accumulation,  $\dot{K}(t)$ , plus innovation expenditure,  $P_A(t)\dot{A}A(t)^\xi$ .

That is:

$$\dot{W}(t) = \dot{K}(t) + P_A(t)\dot{A}A(t)^\xi. \quad (5)$$

Total capital  $W(t)$  is equal to physical capital plus innovation capital:

$$W(t) = K(t) + P_A \frac{A(t)^{\xi+1}}{\xi+1}. \quad (6)$$

Closing up the model, the economy's budget constraint is:

$$\dot{W}(t) = Y(t) - G(t) - C(t). \quad (7)$$

### 2.1.6. Technology Equation

Let us now solve for the Technology Equation, the curve that unites the pairs of constant growth rates and interest rates  $(g, r)$  for which the production side of the economy is in a Balanced Growth Path (BGP) equilibrium.

Final good producers are price takers in the market for inputs. In equilibrium they equate the rental rate on each input with its marginal productivity. The price of  $Y(t)$  is normalised to one. The demand curve faced by each IPU is, then:

$$\frac{\partial Y(t)}{\partial x_j(t)} = R_j(t) = \frac{\alpha}{1-\beta} \tau^{\frac{\beta}{1-\beta}} L(t)^{\frac{1-\alpha-\beta}{1-\beta}} x_j(t)^{\gamma-1} \left( \int_0^{A(t)} x_i(t)^\gamma di \right)^{\frac{\phi}{1-\beta}-1}. \quad (8)$$

Turning to the IPUs' production decisions. Having decided to enter the market, each IPU wishes to maximise his profits in each period of time. The physical production of each unit of the specialised input requires one unit of physical capital. Hence, in each period, the monopolistic IPU maximises profits, taking as given the demand curve for its good:

$$\max_{x_j(t)} \pi_j(t) = R_j(t)x_j(t) - rx_j(t),$$

which leads to the markup rule:

$$R_j = \frac{r}{\gamma}. \quad (9)$$

At each time  $t$ , in order to enter the market and produce the  $A$ th input, an IPU

must spend up-front an innovation cost given by  $P_A A(t)^\xi$ , where, as mentioned earlier,  $P_A$  is the fixed cost of one new innovation, in units of foregone output, and  $i^\xi$  represents an additional cost of patent  $i$  in terms of foregone output. Entering the market, each IPU will become a monopolistic producer of a differentiated input. The IPU's decision to enter the market requires comparison between the fixed innovation cost paid up-front, at time  $t$ , and the discounted value of the stream of profits obtained from  $t$  to infinity. The dynamic IPU's zero-profit condition is:

$$P_A A(t)^\xi = \int_t^\infty e^{-r(v-t)} \pi_j(v) dv,$$

which, assuming no bubbles, is equivalent to:

$$\xi g_A = r - \frac{\pi_j}{P_A A^\xi}. \quad (10)$$

The model's symmetry implies that  $R_j(t) = R(t)$ ,  $x_j(t) = x(t)$  and  $\pi_j(t) = \pi(t)$ .

Then  $R(t)$  is rewritten as:

$$R = \Omega_R A^{\frac{\phi-1+\beta}{1-\beta}} x^{\frac{\alpha-1+\beta}{1-\beta}}, \quad (11)$$

where  $\Omega_R = \frac{\alpha}{1-\beta} \tau^{\frac{\beta}{1-\beta}} L^{\frac{1-\alpha-\beta}{1-\beta}}$  is a constant. Profits,  $\pi(t) = (1-\gamma)R(t)x(t)$  are equal to:

$$\pi = (1-\gamma)\Omega_R A^{\frac{\phi-1+\beta}{1-\beta}} x^{\frac{\alpha}{1-\beta}}, \quad (12)$$

and  $x$  is rewritten as:

$$x = A^\xi \left( \frac{\Omega_R}{R} \right)^{\frac{1-\beta}{(1-\beta)-\alpha}}, \quad (13)$$

where we impose the following parameter restriction:

$$\xi = \frac{\phi - (1 - \beta)}{(1 - \beta) - \alpha}.$$

As we will see later on, in a BGP, the interest rate is constant and hence so is  $R$ . It then follows, from expression (11), that we must have:

$$\left( \frac{\phi - 1 + \beta}{1 - \beta} \right) g_A = - \left( \frac{\alpha - 1 + \beta}{1 - \beta} \right) g_x,$$

which implies:

$$g_x = \xi g_A.$$

Also because of symmetry, equation (4) simplifies to  $K = Ax$ , meaning that physical capital grows at the rate:

$$g_K = (1 + \xi) g_A.$$

Likewise, production function (3) becomes:

$$Y = \tau^{\frac{\beta}{1-\beta}} L^{\frac{1-\alpha-\beta}{1-\beta}} A^{\frac{\phi}{1-\beta}} x^{\frac{\alpha}{1-\beta}}, \quad (14)$$

whose log-time-differentiation gives the growth rate of output:

$$g_Y = \left( \frac{\phi + \alpha \xi}{1 - \beta} \right) g_A = (1 + \xi) g_A.$$

It follows that equation (10) can be presented as:

$$g_Y = \frac{1 + \xi}{\xi} \left( r - \frac{\Omega_Y}{\frac{\alpha}{R^{(1-\beta)-\alpha}}} \right), \quad \Omega_Y = \frac{(1-\gamma)}{P_A} \Omega_R^{\frac{1-\beta}{(1-\beta)-\alpha}} \quad (15)$$

Equation (15), our Technology Equation, unites the equilibrium balanced

growth path pairs  $(g, r)$  on the production side of this economy.

## 2.2. The Euler Equation

The inhabitants of this economy take part in innovation activities through co-creation, diffusion, application at work, and also through consumption. Infinitely lived, homogeneous, informed and cultivated, these citizens are Consumers who wish to consume products and services containing technological and non-technological innovation and new knowledge. They are the fourth helix of the QH innovation model.

In our model, these innovative products and services are all aggregated in the form of a final good,  $Y$ , whose production requires innovation. This means that we can use the standard specification for intertemporal consumption in order to capture the Consumers's decisions. They wish to maximise, subject to a budget constraint, the discounted value of their representative utility:

$$\max_{C(t)} \int_0^{\infty} e^{-\rho t} \frac{C(t)^{1-\sigma}}{1-\sigma} dt \quad (16)$$

$$s.t. \quad \dot{E}(t) = rE(t) + w(t) - C(t) - T(t), \quad (17)$$

where variable  $C(t)$  is consumption of  $Y(t)$  in period  $t$ ,  $\rho$  is the rate of time preference, and  $\sigma^{-1}$  is the elasticity of substitution between consumption at two periods in time. Variable  $E(t)$  stands for total assets,  $r$  is the interest rate,  $w(t)$  is the wage rate, and it is assumed that households provide one unit of labour per unit of time. The transversality condition is  $\lim_{t \rightarrow \infty} \mu(t)E(t) = 0$ , where  $\mu(t)$  is the shadow



price of assets.

Consumers's decisions are described by the familiar Euler Equation:

$$g_c = \frac{\dot{C}}{C} = \frac{1}{\sigma}(r - \rho), \quad (18)$$

according to which the interest rate,  $r$ , is constant in a BGP equilibrium.

### 3. General Equilibrium

Time-differentiation of investment equation (5) tells us that total capital  $W$  grows at the same rate as  $Y$  :

$$\frac{\dot{W}}{W} = \frac{\dot{K}}{K} \frac{K}{W} + \frac{\dot{A}}{A} \frac{A^{1+\xi}}{W} P_A,$$

Which, recalling that  $g_K = (1 + \xi)g_A$ , leads to:

$$g_W = (1 + \xi)g_A$$

Then the economy's budget constraint (7) says that, because  $G$  and  $W$  grow at the same rate as  $Y$ , a constant  $g_W$  requires that consumption  $C$  also grows at the same rate as  $W$  and  $Y$ . With labour constant, the per-capita economic growth rate is such that:

$$g_C = g_Y = g_K = g_W = (1 + \xi)g_A = g.$$

#### 3.1. The steady-state equilibrium

The BGP general equilibrium solution is obtained by solving the system of two equations, (15) and (18), in two unknowns,  $r$  and  $g$  :

$$\begin{cases} g = \frac{1}{\sigma}(r - \rho) \\ g = \frac{1 + \xi}{\xi} \left[ r - \frac{\Omega}{r^{\frac{\alpha}{(1-\beta)-\alpha}}} \right], \end{cases} \quad r > g > 0, \quad (19)$$

where  $\Omega = \Omega_y \gamma^{\frac{\alpha}{(1-\beta)-\alpha}}$ . Restriction  $r > g > 0$  is imposed so that: (i) present values will be finite; and (ii) our solution(s) have positive interest and growth rates.

**Proposition** *Existence of a unique steady-state solution.*

**Proof.**

In the space  $(g, r)$ , the linear Euler Equation (18) has inclination:  $\frac{\partial g}{\partial r} = \frac{1}{\sigma} > 0$ , and the

value it assumes on the vertical axis is:  $g = -\frac{\rho}{\sigma}$ .

The Technology Equation (15) is positively sloped and decreasing:

$$\frac{\partial g}{\partial r} = \frac{(1 + \xi)}{\xi} + \frac{(1 + \xi)}{\xi} \frac{\alpha}{1 - \beta - \alpha} r^{\frac{\beta-1}{(1-\beta)-\alpha}} \Omega > 0,$$

$$\frac{\partial^2 g}{\partial r^2} = \frac{(\beta - 1)}{1 - \beta - \alpha} \frac{(1 + \xi)}{\xi} \frac{\alpha}{1 - \beta - \alpha} r^{\frac{\alpha}{(1-\beta)-\alpha}} \Omega < 0.$$

This implies that the two curves only cross each other once in the first quadrant of the  $(r, g)$  graphic.

In order to better illustrate the unique general BGP equilibrium, and given the nonlinearity of the Technology Equation, we solve the system through a numerical exercise. The baseline chosen parameter values are:

$$\sigma = 2; \quad \rho = 0.002; \quad \alpha = 0.4; \quad \beta = 0.3; \quad \gamma = 0.1;$$

$$\phi = 4; \quad ; \xi = 11; \quad L = 1; \quad P_A = 15, \quad \tau = 0.15,$$

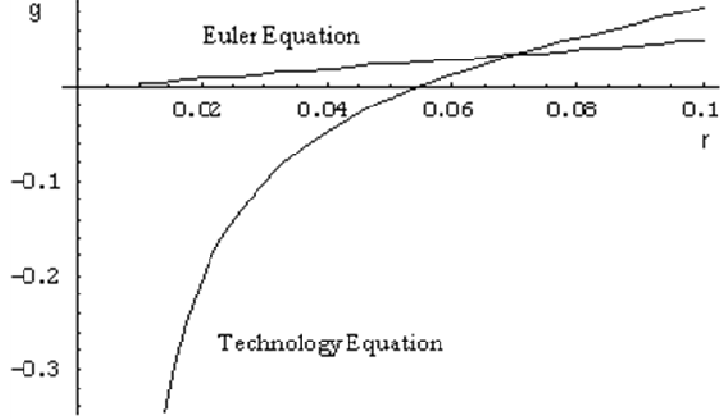
where the values for  $\alpha$ ,  $\gamma$  and consequently  $\phi = \frac{\alpha}{\gamma}$  are the same as those used by Evans et al. (1998) in their numerical example. The value for parameter  $\xi$  is, consequently,  $\xi = \frac{\phi - (1 - \beta)}{(1 - \beta) - \alpha} = 11$ . The value for the preference parameter  $\sigma$  is in agreement with those found in empirical studies such as Barro and Sala-i-Martin (2004), whereas we have chosen a small  $\rho$  in order to allow for small equilibrium interest rate values. Population is often normalised to one. The value for parameter  $\tau$  is in agreement with Irmen and Kuehnel (2009). And the value for  $P_A$  is chosen so as to give us realistic values for the equilibrium growth rate and interest rate. With the chosen parameter values, system (21) becomes:

$$\begin{cases} g = 0.5r - 0.001 \\ g = 1.091 \left[ r - \frac{0.000113}{(r)^{1.333}} \right] \end{cases} \quad r > g > 0,$$

Figure 2, with  $r$  on the horizontal axis and  $g$  on the vertical axis, helps us visualise this economy's BGP general equilibrium solution, which, for the adopted parameter values, is:

$$r = 0,07; \quad g = 0,034$$

**Figure 2. BGP general equilibrium solution**



### 3.2. Transitional Dynamics

In order to examine how the economy converges towards the steady state, we proceed with transitional dynamics analysis, using numerical integration. Let us start by considering the variables marginal productivity of total capital,  $\chi_1 \equiv Y/W$ , and the consumption-total capital ratio,  $\chi_2 \equiv C/W$ , which are constant in steady state; i.e.,

$$\frac{\dot{\chi}_1}{\chi_1} = \frac{\dot{Y}}{Y} - \frac{\dot{W}}{W} \quad \text{and} \quad \frac{\dot{\chi}_2}{\chi_2} = \frac{\dot{C}}{C} - \frac{\dot{W}}{W}. \quad (20)$$

The system of autonomous differential equations in variables  $\chi_1$  and  $\chi_2$  is obtained from (2), (7), (15), (18) and (20). The explicit analytical functional expressions of the differential equations are complex and quite tedious. Hence, in a reader-friendly form, we present the system obtained for the baseline parameter values given in the previous section:

$$\begin{cases} \frac{\dot{\chi}_1}{\chi_1} = 11,9585 \chi_1 - 59,7096 \chi_2 + 0,2797 - \frac{0,0025}{(78,3020 \chi_1 - 358,7810 \chi_2 + 1,7090)^{1,3(3)}} \\ \frac{\dot{\chi}_2}{\chi_2} = 5,0227 \chi_1 - 25,9086 \chi_2 + 0,1182 \end{cases} \quad (21)$$

System (21) is solved through the fourth-order Runge-Kutta classical numerical method,<sup>1</sup> and considering the required initial values  $\chi_1(1)=0.091$  and  $\chi_2(1)=0.040$  – see Table 1 with the initial and steady-state values.

Figure 3 below depicts the decreasing paths of both  $\chi_1$  and  $\chi_2$  from their respective initial values towards steady-state values. Taking into account the paths of  $\chi_1$  (Figure 3a) and  $\chi_2$  (Figure 3b), we can easily obtain the paths of the interest rate (Figure 3c) and of the economic growth rate (Figure 3d).

### 3.3. Economic Policy Effects

In line with data in Figure 1, let us now analyse the effects of an increase in the share of output allocated to public expenditure,  $\tau$ , from 15% to 20%. Table 1 summarises the short and the long-run effects of this policy measure. Figure 3 shows the transitional dynamics from  $t=1$  towards the steady-state period,  $t=t^*$ .

An increase in  $\tau$  induces an upwards jump (short-run effect) in both  $\chi_1$  (from 0.0907 to 0.0995), and  $\chi_2$  (from 0.0400 to 0.0402). Then, both ratios decrease at decreasing rates (medium-run effect) towards their steady-state new values (long-run effect), which are higher than initially.

The increase in  $\tau$  raises both the interest rate and the economic growth rate towards their new (higher) steady-state values. Indeed, a higher  $\chi_1$ , induced by this

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<sup>1</sup> Since this classical method solves the differential equation with suitable precision, we need not consider more sophisticated methods.

policy, reflects a higher marginal productivity of total capital, thus generating a higher economic growth rate.

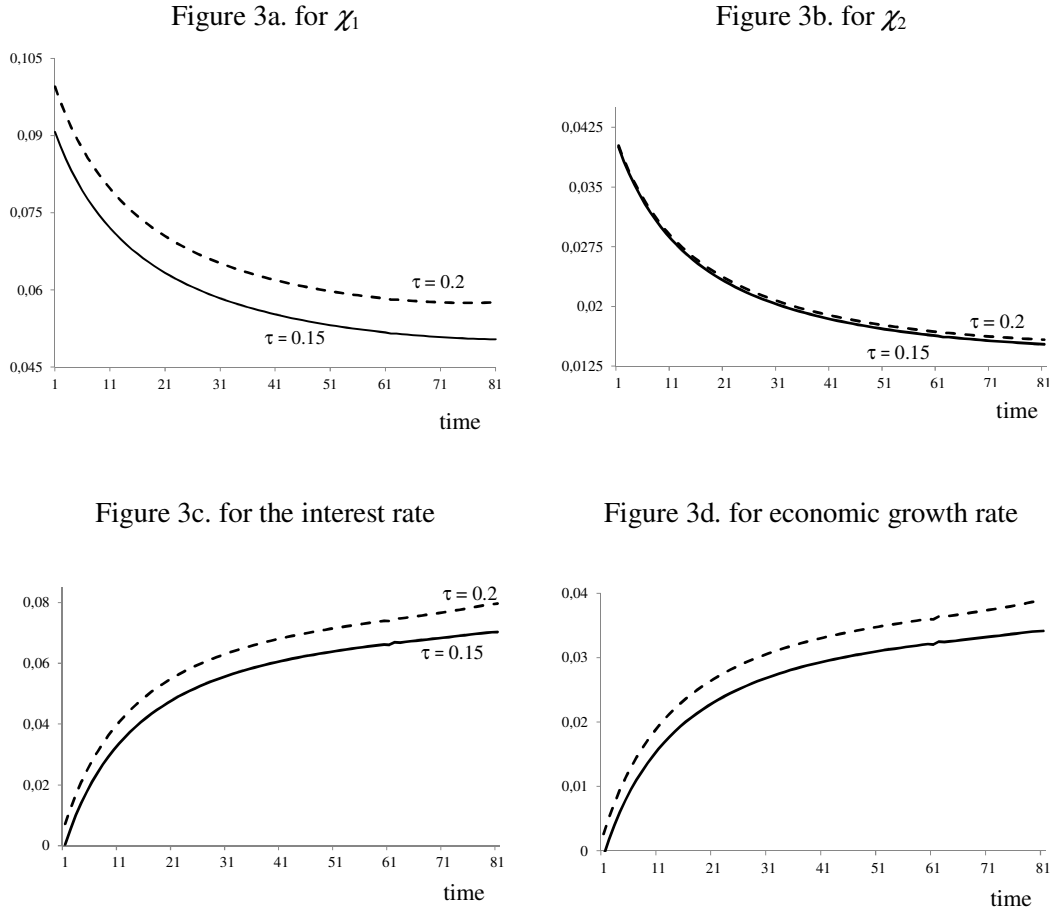
**Table 1. Initial and steady state values for relevant variables**

	With $\tau = 0.15$		With $\tau = 0.20$	
	$t = 1$	$t = t^*$	$t = 1$	$t = t^*$
$\chi_1 = Y/W$	0.0907	0.0504	0.0995	0.0576
$\chi_2 = C/W$	0.0400	0.0152	0.0402	0.0158
Interest rate, $r$	0.000443	0.070314	0.00719	0.079634
Growth rate, $g$	-0.00078	0.034157	0.002595	0.038817

Comparing with other related policy measures, Segerstrom (2000), among others, in a model where innovation results from classic R&D activities, finds that a direct subsidy to R&D activities increases the economic growth rate. As already described, in our model, innovation encompasses more than classic R&D activities, consisting in the development of a new product, service, process or method, being performed by the entire society. Thus, as we have just shown, a policy measure that increases the productivity of all economic agents constitutes an alternative policy measure to increase the economic growth rate.

Indeed, despite nowadays' downward pressure on public expenditures, Government is one of the four pillars of our stylised innovation economy. In this context, an increase in public expenditure on education, health, infrastructural provision and maintenance, technological and innovation services and regulations - which increases the productivity of all inputs - is an effective economic policy.

**Figure 3. Transitional dynamics**



## 5. Final Remarks

A growing number of developed and emerging economies are assuming the character of innovation economies, in which innovation is the main source of economic growth.

At the same time, innovation's nature has been extending beyond classic R&D activities. *The Oslo Manual* defines innovation as the introduction of a new product or service, a new process, or a new method. This type of innovation is growingly multidisciplinary and extremely competitive. Thus, innovative agents are compelled

to co-create and co-produce within networks, partnerships, symbiotic relationships and collaborations. Indeed, in these economic environments, innovation results from the creative interaction and cooperation between all private and public institutions and increasingly demanding Consumers.

Wishing to provide an analytical frame for an innovation economy, we have followed the Quadruple Helix Innovation Theory, according to which four pillars - Firms, Academia, Government and Consumers - sustain the economy. Intending to stress the equally important role of these four helices, in our model, innovation is the engine of growth and it is performed by the entire Society, in a one-sector productive structure. We have also introduced the assumption of complementarities between intermediate inputs in the production function, so as to capture analytically the need for co-creation and partnerships between all innovative agents.

In this innovation-based growth model, public expenditure has an important economic role. Having formally linked innovation to economic growth, we have found an economic policy with positive effects on growth. An increase in Government expenditures increases economic growth, not only in the short-run, but also in the medium and the long-run.

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