INSIDE AND OUTSIDE LIQUIDITY

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OUTLINE

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The idea of writing this book on inside and outside liquidity started when we were invited to give the Wicksell lectures at the Stockholm School of Economics in 1999. We are grateful for and honored by this invitation, which gave us the impetus for thinking more deeply about the implications of the framework we developed in the mid-1990s.

As usual, our own contribution builds on the shoulders of many others, from the classic works of Wicksell, Keynes, Hicks on liquidity and government macroeconomic policy to the modern corporate finance literature, which we take as the foundation for our modeling approach. We are very grateful to the many researchers whose work is cited here, and apologize for any omission which, we are certain, will have arisen.

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Prologue: Motivation and roadmap

Why do financial institutions, industrial companies and households hold low-yielding money balances, Treasury bills and other short-term assets? The standard answer to this question, dating back at least to Keynes (1936), Hicks (1967) and Gurley and Shaw (1960), is that these assets are “liquid” as they allow their owners to better weather income shortages.*

It is unclear, though, why an economic agent’s ability to withstand shocks would not be better served by the broader concept of net wealth, including stocks and long-term bonds. While some forms of equity, such as private equity, may not be readily sold at a “fair price,” many long-term securities are traded on active organized exchanges; for example, liquidating one’s position in an open-ended S&P 500 index fund can be performed quickly and at low transaction costs. For some reason a big part of the agents’ net wealth is not liquid. It cannot be used as a substitute for liquid assets, explaining why the yield on liquid assets is lower than could be expected from standard economic models (the “risk-free rate puzzle”). The standard theory of general equilibrium offers no explanation for this phenomenon. In the Arrow-Debreu model, and its variants, economic agents are subject to a single budget constraint, implying that the consumers’ feasible consumption sets and the firms’ feasible production sets only depend on their wealth.

Similarly, financial institutions and industrial companies pay a lot of attention to risk management. They hedge against liquidity risks using short-term securities, credit facilities, currency swaps and similar instruments, adjusting their positions to meet future liquidity needs in the most efficient way. These activities cost billions of dollars. Yet, received theory is not of much help in explaining all the resources and attention spent on them. In an Arrow-Debreu world, it does not matter whether economic agents make

*The book’s title "Inside and Outside Liquidity" paraphrases Gurley and Shaw’s (1960) "inside and outside money," which distinguishes claims that private parties have on each other versus claims that private parties have on government. Our usage is consistent with this distinction. Wicksell (1898) was one of the first authors to emphasize the dual role of money as a store of value and a medium of exchange.

their consumption and production plans at the initial date or make these decisions later on provided that they can contract on transfers of numéraire from one period to the next (Arrow 1970). In particular, the Modigliani-Miller (1958) irrelevance results imply that the hoarding of liquidity or the hedging of liquidity risk (through choice of leverage, dividend payments, etc.) do not affect a firm's value.

The purpose of this monograph is to offer an explanation of the demand for and supply of liquid assets using insights from modern corporate finance and to study how such a theory can explain the pricing of assets, the role of liquidity management, real investments and also how this theory relates to some classic themes in macroeconomics and in international finance.

Macroeconomic policy rests on the presumption that the government can do things that the market cannot. Foremost among these is the provision of liquidity. The government provides liquidity in a variety of ways: through industry and banking bailouts, deposit insurance, the discount window, open-market operations, implicit insurance against major accidents or epidemics, unemployment insurance, social security, debt management and so forth. The result is often a redistribution of income from consumers to producers or from future generations to current ones. The Ricardian equivalence theorem (Barro, 1974), the macroeconomic counterpart of the Modigliani-Miller theorem, suggests that these activities are useless, since economic agents can replicate privately optimal outcomes by undoing whatever the government does.

In this monograph we will depart from the Arrow-Debreu paradigm in one single, but important way: we will assume that some part of a firm's income stream cannot be promised or pledged to investors. The implication is that the income base on which various kinds of financial claims can be built, is smaller than in the Arrow-Debreu world. For the most part, we will assume that the only imperfection in the economy is the non-pledgeability of some part of the investment income. On the pledgeable part arbitrary financial claims can be written.

There are several reasons why some of the income from investments would be non-pledgeable. One reason is that some agents may not participate in markets; indeed, they may not even be born, as in the case of future generations. This type of market incompleteness has been thoroughly investigated in the overlapping-generations literature, starting with Allais (1947), Samuelson (1958) and Diamond (1965).

A second reason is that information is imperfect and therefore the potential set of financial claims is reduced. Consumers can pledge only a small share of their future labor income, for institutional reasons (limited liability, limited slavery, priority of tax claims), verifiability problems (a liability-ridden individual may move abroad), and incentive considerations (future income is endogenous). Similarly, individual investors must share firms' proceeds with insiders (large shareholder, managers, employees) either because the latter enjoy perks, can divert resources or may exert insufficient effort, or because insiders must be given a share of the created resources to refrain from engaging in moral hazard. Adverse selection further limits the extent to which firms can market their future income to investors.

A key implication of non-pledgeability is that firms (as well as consumers) can count on liquidating only part of their wealth whenever they need funds. Instead, they must prepare themselves for adverse financial shocks by hoarding liquid assets or by contracting in other ways for the provision of liquidity. Firms are willing to pay a premium for liquidity services. We will show that in general they have to do so, because the supply of liquid assets is also constrained by the fact that the returns from productive activities cannot be fully pledged. Non-pledgeability reduces the amount of wealth in the economy, which in turn limits the ability of investors to promise, in a credible manner, future financing of firms. This observation gives rise to a demand for stores of value that can transfer wealth from today to tomorrow as well as across states of nature tomorrow. One of the basic questions we will address in this book is this: When is the corporate sector of the economy liquidity constrained in the sense that the wealth it produces is insufficient to meet its future liquidity needs? We refer to this case as a shortage of inside liquidity. This shortfall will to some extent be satisfied by two sources of outside liquidity: the government and international financial markets. In general, neither source will be sufficient and there will be a shortage of aggregate liquidity. This in turn will have interesting implications for how the government supplies and manages liquidity as well as liquidity management within firms.

The monograph is organized as follows. Part I of the book builds the foundations for the corporate demand for liquidity. Chapter 1 introduces a simple model of credit rationing with constant returns to scale. Credit rationing of some kind is essential for corporate liquidity demand. Chapter 2 introduces the workhorse model of liquidity demand that we will use throughout the book. In this model, firms with limited pledgeability must plan their liquidity in advance. Firms demand liquidity because they want to insure themselves against credit rationing. Through mechanisms such as credit lines or credit default swaps, investors commit themselves to supplying funds in states of nature in which they would not naturally have done so, i.e. in states in which liquidity needs exceed the future income that can be pledged to investors. We also examine the provision of liquidity by investors who cannot perfectly monitor the use that is made of the corresponding funds.

Chapter 2 also compares our model of corporate liquidity demand with the celebrated model of consumer liquidity demand by Bryant (1980) and Diamond-Dybvig (1983). While the two types of models differ in many respects, we show that there is a close formal relationship between them as well. Furthermore, the two can readily be merged into a single framework in which corporations and consumers compete for liquidity.

Part II of the book (chapters 3 and 4) considers the benchmark of complete markets: While there is a wedge between value and pledgeable income, the latter is traded on efficient, complete markets. The economy is then an Arrow-Debreu economy, except for the limited pledgeability. Complete markets imply that liquidity may be scarce, but is allocated efficiently.

Chapter 3 asks whether the private sector provides enough aggregate liquidity on its own. That is, do the firms in the aggregate create enough pledgeable income – *inside liquidity* – to support the financial claims necessary for implementing a second-best, state-contingent production plan. The answer is yes if the corporate sector is a net borrower and the firms' liquidity shocks are idiosyncratic. In that case the second best plan can be implemented by each firm holding a share of the market index. On the other hand, if the corporate sector is a net lender there will always be a shortage of aggregate liquidity. The same is true if all firms are hit by the same (macroeconomic) shock or more generally if macroeconomic shocks are sufficiently large relative to the idiosyncratic shocks.

A shortage of stores of values induces the private sector to try to create more, albeit at a cost. What is in short supply is not non-contingent liquidity, but liquidity in those states of nature in which the economy is doing poorly, and, as we later point out, the state has a comparative advantage in creating such contingent liquidity. Creation of more "parking space" by the private sector may involve investing in projects that deliver a safe income, more costly choices of governance that raise the corporations' pledgeable income – such as going public or resorting to monitoring structures, or costly financial innovations that enable a more efficient use of collateral. Examples of the latter include bilateral and tri-party repos (legal innovations that free posting of collateral from the vagaries of bankruptcy processes), and securitization, which transforms illiquid, low grade loans into publicly traded assets of higher quality. Our approach thus fits well with de Soto (2003)'s view that a major role of a financial system is to transform "dead capital" into "live capital". He thought of the opportunities to create collateral in developing economies. What seems to take place at the beginning of the new century was excesss savings from China and other developing countries with underdeveloped financial markets, finding their way to developed countries, and especially the U.S., which could meet the demand for parking space at a lower cost.

When there is insufficient inside liquidity, financial instruments that originate outside the private sector – *outside liquidity* – can improve productive efficiency by facilitating access to liquidity and by lowering the cost of it. Prime suppliers of outside liquidity are the government and international financial markets.

Chapter 4 shows how to compute asset prices using the analog of Arrow-Debreu state-contingent prices, but assuming that only pledgeable income can be used as the basis for contingent claims.[†] While the basic logic of our Liquidity Asset Pricing Model (LAPM) is identical to that of the Arrow-Debreu model, LAPM prices do not merely reflect the yield of the assets, but also the value that they bring by helping firms withstand liquidity shocks. The chapter also illustrates how one can use state prices to derive an optimal policy for risk management at the firm level.

Thus Part II of this monograph assumes that liquidity is dispatched efficiently through explicit or implicit state-contingent claims on all pledgeable income. The analysis of complete markets derives, in a sense, an upper bound on the available liquidity. In practice, liquidity may not be properly redispatched from economic agents who end up having an excess of it to those in need of liquidity. Put differently, efficient contracts for redispatching liquidity may not have been signed. Parts III and IV study two such situations. In the first, analyzed in Chapter 5, consumers are unable to directly pledge their future income to firms, at least in states of nature where the latter are short of liquidity; we interpret public provision of liquidity as the government filling the corresponding gaps and using its taxation power to transfer income from consumers to firms in bad macroeconomic states. In the second situation, studied in Chapters 7 and 8, the lack of coordination occurs within the corporate sector.

Chapter 5 argues that the state, through its regalian taxation power, can increase the

[†]Geanakoplos, in a series of papers, has emphasized the essential role of collateral in financial markets and studied in general equilibrium models the optimal assignment of collateral asset pricing (see Geanakoplos, 1996 and Geanakoplos et. al., 1995). The methods he uses and the questions he asks are different from ours, but the general context and perspective are related.

pledgeability of consumers' future income and thereby create liquidity for the corporate sector. Consumers and firms can be made better off by having the government act as an insurance broker, transferring funds from consumers to firms when the latter are hit by aggregate liquidity shocks. We study government policy assuming that an explicit insurance contract can be drawn up ex ante, but argue that a number of ex post policy interventions emulate the patterns of optimal government insurance. The ability of the government to provide liquidity ex post, gives it a potential advantage over privately supplied liquidity. Privately supplied liquidity often requires ex ante investments in short-term assets, incurring an opportunity cost whether liquidity is needed or not. By contrast, an ex post government policy that does not waste liquidity can be much cheaper, especially when covering liquidity shortages that occur rarely.

Chapter 6 pursues the analysis of public provision of liquidity by asking whether the presence of efficient international financial markets might eliminate all potential liquidity shortages. The answer is no in general. A country's access to international financial markets is limited by its ability to generate pledgeable income that is tradable. We study the relationship between international and domestic liquidity in an open economy with both tradable and non-tradable goods and conclude that when there is a shortage of international liquidity, the insights about the value of domestic liquidity continue to hold.

Chapters 7 and 8 depart from perfect coordination within the corporate sector and looks at situations in which each firm individually arranges its own supply of liquidity without any ex ante coordination with the other firms. The only type of coordination occurs ex post in spot markets which can reallocate liquidity. One may ask how close this kind of self-provided liquidity arrangement comes to the second-best. We show that it will in general not replicate the second-best optimum, because firms may hoard either too much or too little liquidity. Nevertheless, the government may be unable to improve on the situation in contrast to the large literature on incomplete insurance markets.

The epilogue summarizes the main lessons to be drawn from our particular approach towards the supply of inside and outside liquidity and relates it to the subprime crisis.

On terminology: Throughout this book, we will take the terms *pledgeable income, liquidity* and *collateral* to mean the same thing and use them interchangably.[‡] The financial crisis that began in 2007 we shall call *the subprime crisis* for brevity.

[‡]Clearly, collateral can be different from pledgeable income in many contexts. The value of the assets backing up debt is often higher than the value of debt (the debt is over-collateralized). This may be because the underlying assets are risky and do not protect the investor's claim in all states of nature. Or it may be, because the value of collateral is worth less to the investor than it is to the borrowing firm. Note that even if the collateral is worth very little to the investor it can provide proper incentives for repayment of debt as long as the borrower prefers to repay the debt than lose his collateral and has the means to do so.

In our complete market model, the amount of collateral is exactly equal to the amount promised in any given state. Considerations of default are already built into the notion of pledgeable income and will not happen in equilibrium. Promising more than the pledgeable income would not be credible and promising less would waste collateral.

Part I

BASICS OF LEVERAGE AND LIQUIDITY

Introduction

In standard microeconomic theory, a firm that confronts financial needs can meet them as they arise by taking out loans or by issuing new securities, whose repayments and returns are secured by the cash flows that the firm generates. As long as the net present value of a re-investment is positive, investors are willing to supply the needed funds.[1] Reality seems very different. Firms keep a close watch on their current and forecasted cash positions, making sure that their essential liquidity needs can be met at all times. They do not wait until the cash register is empty. To guard against liquidity shortages, firms arrange financing in advance using both the asset and the liability sides of their balance sheets. On the asset side they may hoard liquidity by buying Treasury bills and other safe assets that can be easily sold when necessary.[2] On the liability side, they may take out credit lines or issue securities that give them flexibility in their management of cash, such as long-term debt, preferred equity and straight equity.[3] The recent subprime crisis is a vivid demonstration of how costly – and in some cases impossible – it can be to face a maturity mismatch and to seek financing in times of distress. Financial institutions have been struggling to replace the short-term, market-based financing that they had grown used to in boom times with alternative sources of funding. At the same time they have been forced to delever significantly by selling assets at distressed prices. The troubles in the financial sector in turn have shut off normal credit channels for the non-financial sector, causing bankruptcies and distress throughout the economy. A crisis of this magnitude is obviously rare, but it certainly is a stark reminder of how important

[1] In the case of a debt overhang, additional financing will require that previous contracts are renegotiated.

[2] Some non-financial firms hold surprising amounts of liquid assets at any given time. Big technology firms like Microsoft and Nokia have at times held billions of dollars in liquid instruments, mostly, if not exclusively, in the form of safe, low-yielding debt. Large accumulations of cash like this provide a readiness to make major acquisitions, but also a hedge against liquidity shocks.

[3] Brunnermeier and Pedersen (2009) talk about *funding liquidity* when the liability side is used and *market liquidity* when the asset side is used.

it is to think about one's liquidity needs in advance rather than wait until the need materializes.

By now there are several related theories in modern corporate finance that can explain why firms demand liquidity. Each theory provides a reason why a firm wants to buy insurance against higher credit costs or outright credit rationing, stemming from information problems. In von Thadden (2004), a firm that waits may be unable to get access to funding due to adverse selection problems. Adverse selection can make it very costly for a firm to obtain funding and in the worst case, asset and credit markets may dry up entirely.[4] "Signal-jamming" is an alternative rationale for advanced funding (e.g., Bolton and Scharfstein,1990 and Fudenberg and Tirole, 1985). If financiers base tomorrow's refinancing decision on a firm's current performance, competitors have an incentive to prey on each other by choosing hidden actions, such as secret price cuts, that hurt rivals and makes them look financially weak. In equilibrium, the market can see through this, but nevertheless the wasteful signaling behavior is rational. To prevent such predation, a firm has an incentive to secure its funds in advance (in a publicly observable fashion).

Throughout this book, we will employ a very simple information-based model as the driver of liquidity demand. The key assumption is that firms are unable to pledge all of the returns from their investments to the investors. Insiders, from control shareholders to managers to ordinary employees, enjoy private benefits of various kinds that create a wedge between total returns and pledgeable returns. The insiders may consume employment rents, enjoy perks, engage in empire building, or receive inducements to perform that give them an extra share of the firm's payoff. To the extent these private benefits cannot

[4] The idea that adverse selection may lead to market freezes dates back to Akerlof (1970). Some recent papers have used Akerlof's model to show how the inability to sell legacy assets may hamper reinvestment policies: see e.g., House and Masatlioglu (2010) and Kurlat (2010). Philippon and Skreta (2010) and Tirole (2010) study how the state can jumpstart asset markets when financial institutions can opt to be refinanced in a (cleaned-up) financial market instead. Daley and Green (2010) show how the thawing of an illiquid market is affected by the accrual of news about the quality of individual assets and by the waiting strategies of sellers who try to signal a high quality of their assets by conveying the message that they are not particularly eager to part with them.

be transferred or paid for up front, a part of the total surplus will not be pledgeable to outside investors.

This partial non-pledgeability of investment returns can make it more costly or impossible to finance a project. And even when the project can get off the ground, private benefits can make reinvestments difficult in the future. In general, as we will show, firms will face credit rationing both at the start as well as the future. There is however a key difference between credit rationing at the initial financing stage and at the refinancing stages: in the latter case, credit rationing can be anticipated and therefore measures can be taken to insure against it. In other words, private benefits create a demand for liquidity.

We start by showing (in chapter 1) how a simple moral hazard model with limited liability can give rise to a wedge between total and pledgeable income. Chapter 2 introduces a "liquidity shock" that may hit the firm after it has sunk its initial investment. This gives rise to a demand for liquidity. We show that if the potential liquidity shock is severe enough, the firm needs to arrange financing in advance or face costly credit rationing. Firms face both a solvency concern (the need to be adequately capitalized in order to attract financing in the first place) and a liquidity concern (the risk of facing solvency concerns in the future). The optimal design exhibits a trade-off between liquid and illiquid investments: the higher the insurance purchased by the firm in the form of a liquidity backup, the lower the investment in illiquid assets. Put differently, the firm can opt for a large scale together with an important maturity mismatch (much long-term illiquid assets and little short-term liquidity), or for a smaller, but more secure balance sheet.

Appendix 2.1 shows how private information about the magnitude of the liquidity shock limits contracting possibilities and affects the optimal solution and the demand for liquidity. Appendix 2.2 addresses an obvious question: how our model of liquidity demand by firms compares with the more extensively studied case of consumer liquidity demand.

Our main approach is to assume that parties can write fully state-contingent, enforceable contracts on all pledgeable income; that is, there is a complete market for state-contingent claims on the pledgeable part, while no contracts can be made on the private part of income. The virtue of this (unrealistic) assumption is that it makes the model very tractable and provides a rather disciplined modeling approach. The imperfection we explore is a minimal deviation from the world of complete markets. Had one assumed a more general incomplete market structure, whether exogenously or endogenously, the options would be many and harder to choose from.

Chapter 1

Leverage

1.1 A simple model of credit rationing with fixed investment scale

We will use a very simple model of credit rationing as the basic building block for our liquidity analysis. Reduced to its barest essentials, the model considers a risk-neutral entrepreneur with an investment opportunity that is worth Z_1 to him, but only $Z_0 < Z_1$ to outside investors. We assume that the initial investment I satisfies $Z_1 > I > Z_0$ (see Figure 1.1). The investment has a *positive Net Present Value*, $Z_1 > I$, but it is *not self-financing*, because the most that investors can be promised is less than the investment $Z_0 < I$. The shortfall $I - Z_0 > 0$ must be paid by the entrepreneur (or covered by claims on the market value of the firm's other existing assets).

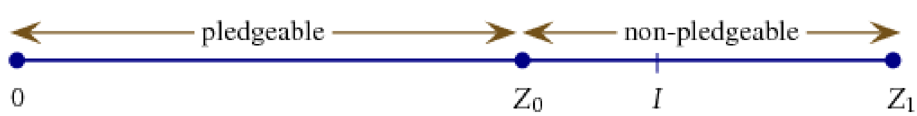


Figure 1.1: Limited pledgeability

One can give a variety of reasons why the full returns of a project cannot be paid out to the investors, that is, why there is a *positive wedge (entrepreneurial rent)* $Z_1 - Z_0 > 0$. They can be put into two general categories: explanations based on *exogenous* constraints on payouts and those based on *endogenous* constraints. The prime example of exogenous constraints are private benefits that only the entrepreneur can enjoy, such as the pleasure of working on a favorite project or the increased social status that comes with its success. A related intangible benefit arises from differences in beliefs. Entrepreneurs often have an inflated view of the chances that their project will succeed.[1] To the extent that such differences in beliefs are not based on better information, the extra utility the entrepreneur derives from overoptimism can in a one-shot setting be modeled as a private benefit that investors do not value. There are also tangible benefits that may be impossible to transfer fully, such as the increased value of human capital that comes with investment experience, or the future value that an entrepreneur may enjoy from the option to move after he has been revealed to be a good performer. [2]

In the second category, entrepreneurial rents emerge endogenously, because even though it is feasible to pay out all of the project's returns to the investors, attempts to reduce the entrepreneur's share below $Z_1 - Z_0 > 0$, will inevitably hurt the investors as well. Therefore it is optimal to let the entrepreneur enjoy a minimum rent. The simplest example is one where the entrepreneur can steal some of the output for private consumption or, equivalently, one where the entrepreneur has to be given a share of the output in order to discourage him from diverting output to private consumption (Lacker and Weinberg, 1989). Below, we will consider a standard moral hazard model with limited liability that leads to the same conclusion.

Because we assumed that the project is not self-financing, $I - Z_0 > 0$, investment will require a positive contribution from the entrepreneur. Let A be the maximum amount of capital that the entrepreneur can commit to the project either personally or through the

[1] Of course the fact that entrepreneurs often fail or that they express a high confidence in a project when asked, are as such no evidence of overconfidence and can be explained by either agency costs or confidence-maintenance strategies. However, Landier and Thesmar (2009) provide evidence of entrepreneurial overconfidence, which is consistent with Van den Steen (2004). Simsek (2010) analyzes financing of projects sponsored by optimistic entrepreneurs. He shows that heterogeneity in beliefs has a very asymmetric impact on financing, as financial market discipline operates only when entrepreneurial optimism concerns the likelihood of bad events. Entrepreneurs who are optimistic about good events can raise substantial amounts by contrast.

[2] See, e.g., Terviö (2009).

firm. The project can then go forward if and only if the pledgeable income exceeds the project's net financing need $I - A$, that is, when

$$A \geq \bar{A} \equiv I - Z_0 > 0. \tag{1.1}$$

Condition (1.1) puts a lower bound $\bar{A}$ on the amount of assets that the firm or the entrepreneur needs to have in order to be able to attract external funds. A firm with less capital than $\bar{A}$ will be *credit rationed*. It is of course possible that $A > I$, in which case no external funds are needed. This is an uninteresting case in the current model, so we will rule it out for the time being. But when we study liquidity shortages in Chapter 3, $A > I$ is a legitimate and interesting case.

It bears repeating that a positive entrepreneurial rent $Z_1 - Z_0 > 0$ is necessary for credit rationing. If $Z_1 = Z_0$, then all projects with positive net present value ($Z_1 > I$) are also self-financing ($Z_0 > I$) and hence can move forward. Another necessary condition for credit rationing is that the firm is *capital poor* in the sense that

$$A < (Z_1 - Z_0). \tag{1.2}$$

When (1.2) is violated, the firm has enough capital up front to pay for the ex post rents it earns and therefore all projects with positive net present value can go forward. One can see this formally, by rewriting (1.1) in the form

$$Z_1 - I \geq (Z_1 - Z_0) - A. \tag{1.3}$$

The left-hand side is the net present value of the project. The right-hand side is the net rent enjoyed by the entrepreneur after investing all of his net worth into the project. If the right-hand side is negative, all projects with a positive net present value can proceed. It is only when the firm is capital poor and (1.2) holds that valuable projects may be rejected. Stated more strongly condition (1.2) has the important implication that for a capital poor firm there will always be projects with a positive net present value that have to be rejected because the firm does not have enough capital.

Let us finally note that the internal cost of capital is above the market rate (0) below the point where the firm is credit rationed as can be seen by considering the entrepreneur's utility payoff U.

$$\begin{aligned} U &= A + Z_1 - I, \quad \text{if} \quad A \geq \bar{A}, \\ U &= A, \qquad\qquad\quad \text{if} \quad A < \bar{A}. \end{aligned} \tag{1.4}$$

Because utility jumps up at $A = \bar{A}$, the value of funds inside the firm is strictly higher than outside the firm below $\bar{A}$.[3] When $A < \bar{A}$ total output can be increased by transferring funds from investors to capital poor entrepreneurs, but of course such transfers will not be Pareto improving. In models with non-transferable utility, Pareto optimality does not imply total surplus maximization.

1.2 A simple moral hazard model illustrating the wedge between value and pledgeable income

The wedge as an incentive payment

Our liquidity analysis proceeds largely without reference to the particular reasons behind the non-pledgeable income wedge $Z_1 - Z_0$. But to gain a better grasp of the economic significance of this analysis, it is worth going beyond the reduced-form model. In this section, we will analyze in detail a specific model in which the wedge appears endogenously.[4] The analysis will highlight important determinants of the firm's debt capacity, illustrate the impact that credit rationing may have on the firm's choice of investments and indicate the benefits and costs of using different kinds of collateral.

We employ a standard model of investment with moral hazard.[5]. There is a single

[3] Formally, the marginal internal cost of capital is equal to 0 up to $\bar{A}$ and jumps to infinity at $\bar{A}$. With a continuous investment choice this cost varies more smoothly and exceeds the market rate: See section 1.3.

[4] We should stress that even if a number of explanations can be given for a positive wedge, it is of course not true that we can always take the wedge as a primitive in our analysis. This is one reason to provide an explicit model that justifies treating the wedge as exogenous in the analyses we will be considering.

[5] The model is taken from Holmström and Tirole (1998), but has many antecedents.

entrepreneur (firm) and a competitive set of outside investors. All parties are risk neutral. There is a single good used for consumption as well as investment. There are two periods. In the initial period, indexed $t = 0$, there is an opportunity to invest. The investment costs I. The gross payoff of the investment one period later ($t = 1$) is either R (a success) or 0 (a failure). The probability of success depends on an unobserved action taken by the entrepreneur. The action represents the entrepreneur's choice of where to invest the funds I. The intended purpose is to invest in an efficient technology H, which gives a probability of success p_H. The entrepreneur also has the option to invest in an inefficient technology L, which gives a lower probability of success $p_L < p_H$, but provides the entrepreneur with a private benefit B. (For instance, the inefficient technology may only cost $I - B$, leaving B for the entrepreneur's private consumption.). The choice and payoff structure is described in Figure 1.2.

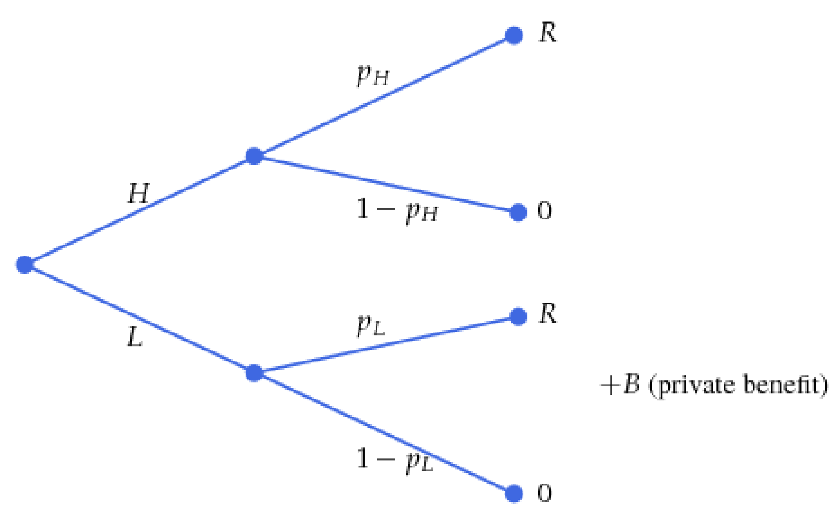


Figure 1.2: Description of moral hazard

We assume that there is no discounting between the periods and that the expected return of the investment is negative if the low action is taken and positive if the high action is taken:

$$p_H R - I > 0 > p_L R - I + B \tag{1.5}$$

Thus, it is better not to invest at all than to invest and have the firm choose the inefficient technology L.

The entrepreneur has assets worth A. These assets are liquid in the sense that they have the same value in the hands of the entrepreneur as in the hands of investors. The firm is protected by limited liability. We assume again that $A < I$ so that the firm needs to raise $I - A > 0$ from outside investors in order for the project to go forward. Investors can access an unlimited pool of funds and demand an interest rate that we normalize to 0.

Investors can be paid contingent on the outcome of the project. Let X_s (X_f) be the entrepreneur's date-1 wealth in case the project succeeds (fails). Limited liability requires that $X_i \geq 0, \;\; i = s, f$. Investors receive $Y_s = R - X_s$ if the project succeeds and $Y_f = -X_f$ if it fails.

We are interested in the conditions under which the investment can go ahead. There are two constraints that must be satisfied. First, the investors need to break even,

$$p_H(R - X_s) + (1 - p_H)(-X_f) \geq I - A. \tag{1.6}$$

Second, the entrepreneur must be induced to be diligent,

$$p_H X_s + (1 - p_H) X_f \geq p_L X_s + (1 - p_L) X_f + B \tag{1.7}$$

Simplified, this incentive compatibility constraint reads

$$X_s - X_f \geq \frac{B}{\Delta p}, \tag{1.8}$$

where

$$\Delta p \equiv p_H - p_L > 0. \tag{1.9}$$

Incentive compatibility (1.8) paired with limited liability implies that the entrepreneur earns a positive rent. This rent is minimized by setting $X_f = 0$ and $X_s = \frac{B}{\Delta p}$. The rent cuts into the amount that can be paid out to investors. The firm's *pledgeable income* is defined as the maximum expected amount that investors can be promised when the entrepreneur is paid the minimum rent. The pledgeable income is

$$Z_0 = p_H(R - \frac{B}{\Delta p}). \tag{1.10}$$

To complete the link to the reduced form discussed earlier, denote the total pie $Z_1 = p_H R$. The positive wedge is then equal to the entrepreneur's minimum rent $Z_1 - Z_0 = p_H \frac{B}{\Delta p}$.

Factors influencing pledgeable income

(a) Bias towards less risky projects. The net worth of a firm may prohibit it from investing altogether as discussed above. More generally, a firm's net worth will merely limit which projects it can invest in. Assume there is a set of projects that the firm and the investors can jointly choose from. The firm can more easily satisfy (1.1) by reducing the investment scale I or by choosing projects with a higher pledgeable income.

For example, in the incentive-payment illustration, each project is characterized by a tuple (I, R, p_H, p_L, B), where we allow the inefficient project to vary with the efficient one (the entrepreneur's alternative use of funds may depend on the project to be undertaken). Pledgeable income increases in p_H and R and decreases in p_L and B, reflecting the fact that the entrepreneur's incentive problem is less severe when the efficient project becomes more attractive relative to the inefficient one. More interestingly, consider variations in p_H and R that leave the expected payoff of the desired project Z_1 and the other parameters unaltered. Specifically, assume that p_H goes down while R goes up so that the project becomes more risky. Other things equal, the firm's pledgeable income decreases with such risk. A decrease in p_H increases the rent $\frac{p_H B}{\Delta p}$ that goes to the entrepreneur, because the entrepreneur's reward in the successful state (the only incentive instrument available) is less potent the lower is p_H. With a higher entrepreneurial rent, less can be promised to investors (Z_0 is lower), which raises the cut-off value $\bar{A}$. At the margin, therefore, capital-constrained firms will accept safer projects at the expense of lower expected returns.

(b) Diversification. As a variant on this theme, one can ask whether diversification

will help to reduce the need for own funds. Suppose a single project can be replaced by two identical, half-sized projects of the sort we have discussed. Further, assume that the projects are stochastically independent and that the entrepreneur chooses separately but simultaneously whether to be diligent in each project. One can show that in this case the optimal incentive scheme pays the entrepreneur a positive amount only when both projects succeed. In effect, the entrepreneur pledges the rewards that would accrue from a successful project as collateral for the other project (and conversely). This maximizes the pledgeable income.

For diversification to be of value, it is important that the projects be independent. If the projects were perfectly correlated (or the entrepreneur opportunistically chose them to be perfectly correlated), diversification would not raise the pledgeable income.[6]

(c) Intermediation. Another way of increasing the pledgeable income is to reduce the entrepreneur's opportunity cost of being diligent. Some projects are more conducive to misbehavior than others: for instance, those that are exceptional, that do not have tangible investments, or that involve poor accounting. A capital poor firm can sometimes increase its pledgeable income by turning to an intermediary that has monitoring expertise. A simple way to model monitoring is to assume that the intermediary can reduce B to a lower level b (and perhaps simultaneously reduce p_L), because it can place constraints on what the firm can do. Loan covenants serve this purpose: for instance, lending contracts frequently forbid the firm from paying dividends if certain financial conditions are violated. Covenants may also give the bank veto rights on the sale of strategic assets and spell out circumstances under which the bank can intervene even more aggressively by getting the right to nominate all or part of the board. Another potential interpretation of the monitoring activity is that the bank acquires information that is relevant for decision-making and uses it to convince the board not to rubberstamp (what turns

[6]For more on diversification in this type of model, see Conning (2004), Hellwig (2000), Laux (2001) and Tirole (2006, chapter 4).

out to be) the management's pet project. In the model, and apparently in reality, giving the firm less attractive outside options reduces entrepreneurial rents, increases pledgeable income and thus lowers $\bar{A}$. The carrot can be smaller if the stick is stronger.

Of course, intermediation is not free, and in order to determine whether intermediaries can really increase pledgeable income, one has to consider monitoring costs, which will move $\bar{A}$ back up. One can distinguish at least three kinds of monitoring costs from intermediation:

- First, direct costs are incurred by the intermediary as well as the firm due to the additional work involved in evaluating investments, processing loans and monitoring compliance with covenants.

- Second, the constraints imposed on a firm as part of a loan covenant do not merely cut out illegitimate opportunities; they also cut out legitimate ones. A firm that cannot sell or acquire significant assets without the approval of a bank may have to forego valuable deals. Excluding profit opportunities of this kind lowers Z_1 and reduces the project's expected return.

- Third, monitoring expertise is scarce and commands rents that depend on market conditions. In Holmström and Tirole (1997), we study a model where the monitor can itself act opportunistically and therefore has to be given a share in the firm's payoff. This increases $\bar{A}$ by an amount that gets determined by the demand for intermediation among credit-constrained firms. In equilibrium firms sort themselves into three groups as a function of their net worth: (i) those firms that have too little own capital to be able to invest; (ii) those that have enough own capital that they can go directly to the market and do not need intermediation; and (iii) those that have intermediate amounts of capital and invest with the help of intermediaries. In the last instance, funding comes both from informed investors (intermediaries) and from the uninformed investors (the general market), who invest only because the intermediary's participation has reduced the risk of opportunism.

1.3 Variable investment scale

For the upcoming liquidity analysis we need a model where the scale of investment is variable, so that we can study the important trade-off between the scale of the initial investment and the decision to save some funds to meet future liquidity shocks. A simple, tractable model is obtained by letting the investment vary in a constant-returns-to-scale fashion.

Let I be the scale of the investment (measured by cost), let ρ_1 be the expected total return, and ρ_0 the pledgeable income, both measured *per unit invested*. Thus, I results in a total payoff $\rho_1 I$ of which $\rho_0 I$ can be pledged to outside investors. The residual $(\rho_1 - \rho_0)I$ is the minimum rent going to the entrepreneur.

The moral-hazard example of section 1.2 fits this framework if we assume that a successful project returns RI and the private benefit to the entrepreneur from cheating is BI. In that case,

$$\begin{aligned} \rho_1 &= p_H R, \\ \rho_0 &= p_H(R - \tfrac{B}{\Delta p}). \end{aligned} \tag{1.11}$$

As before, we assume that projects are socially valuable, but not self-financing:

$$0 < \rho_0 < 1 < \rho_1. \tag{1.12}$$

Consequently, the entrepreneur needs own funds $A > 0$ to invest. For each unit of investment, the firm can raise ρ_0 from outside investors, leaving the *minimum equity ratio* $1 - \rho_0 > 0$ to be covered by own funds. The repayment constraint is

$$A \geq (1 - \rho_0)I,$$

implying a maximum investment scale

$$I = kA = \frac{A}{1 - \rho_0}. \tag{1.13}$$

The *equity multiplier* $k \equiv \frac{1}{1-\rho_0} > 1$, the inverse of the (minimum) equity ratio, defines the firm's maximum leverage per unit of own capital. A firm with 10 units of own capital and a required minimum equity ratio of 20% can invest a maximum of 50 units.

If the firm chooses the maximum investment scale, the entrepreneur's gross payoff is

$$U^g = \frac{(\rho_1 - \rho_0)A}{1 - \rho_0} = \mu A, \text{ where } \mu \equiv \frac{\rho_1 - \rho_0}{1 - \rho_0}. \tag{1.14}$$

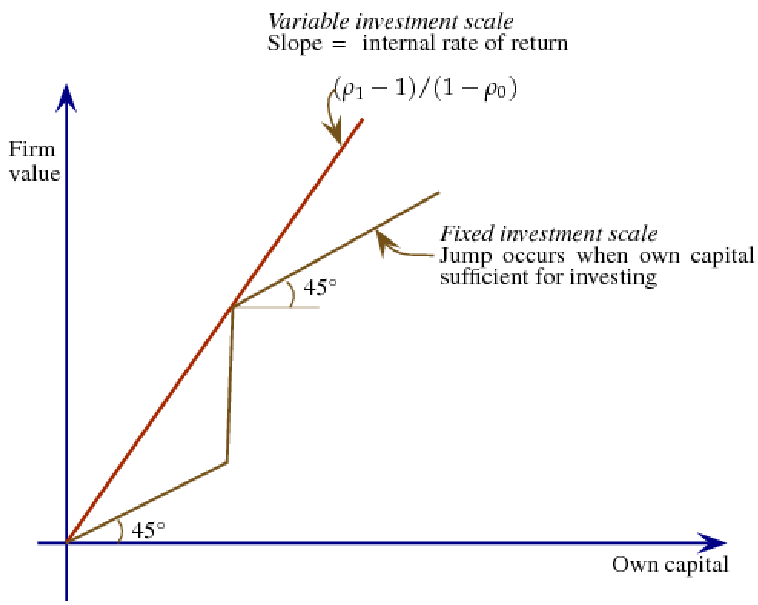


Figure 1.3: Internal rate of return

The entrepreneur's net utility is:

$$U = (\mu - 1)A = \frac{\rho_1 - 1}{1 - \rho_0} A.$$

For each unit invested, the entrepreneur enjoys a rent $\rho_1 - \rho_0$. Thanks to the equity multiplier $k > 1$, the rent gets magnified, resulting in a gross rate of return on own capital $\mu > 1$. The rate is constant, because of the constant-returns-to-scale technology. More importantly, the rate is greater than 1 because of (1.12), implying that the internal rate of return exceeds the market rate of interest. By transferring a unit of the good from investors to the entrepreneur, total social surplus $(\rho_1 I - I)$ could be increased by more than one unit. But such transfers are not Pareto improving, since the increase in total surplus cannot be arbitrarily split between the investors and entrepreneurs. In models with limited liability, total surplus maximization is not a necessary condition for Pareto optimality.

Because the rate of return on entrepreneurial capital exceeds the market rate, it is evident that the entrepreneur maximizes his utility by choosing the maximum investment scale (1.14). He puts all his wealth in the illiquid portion of the return (the non-pledgeable return $(\rho_1 - \rho_0)I$), leaving outsiders holding the firm's liquid assets. Again, total output could be raised by transferring wealth from passive investors to active entrepreneurs, but since investors could not be compensated as they already hold all the firm's liquid claims, this would not be Pareto improving. There is nothing that the government could do to improve on private contracting.

Comparative statics and investment implications

Factors that increase ρ_0 or ρ_1 (or both) will increase the entrepreneur's utility and an increase in ρ_0 will also increase the investment scale I. Recall that in the moral-hazard model, ρ_0 increases with R and p_H and decreases with B and p_L, while ρ_1 increases with R and p_H. Investors are simply paid their market rate of return, so they remain unaffected by these changes.

If the firm could choose among investments that differed in their attributes ρ_0 and ρ_1, the firm would not want to choose the investment that maximizes the social net present value, that is, the investment with the highest ρ_1. The pledgeable income is also critical as it determines the extent to which the firm can lever its capital. From (1.14) we see that the firm's willingness to substitute ρ_0 for ρ_1 is given by

$$\frac{d\rho_1}{d\rho_0} = 1 - \mu < 0. \tag{1.15}$$

The firm will choose projects with lower ρ_1 up to the point where the reduction in ρ_1 per unit of increase in ρ_0 equals the difference between the internal rate of return and the market rate of return. Each unit of pledgeable income ρ_0 is worth μ units of ρ_1, because of scale expansion. This illustrates one of the central themes of credit constrained lending: the willingness to sacrifice net present value for an increase in pledgeable income.

Chapter 2

A simple model of liquidity demand

Firms demand liquidity in anticipation of future financing needs either because it is cheaper to get financing now or because there is a risk that financing will not be available if the firm waits until the need for funding arises. In this chapter we will analyze the demand for liquidity in a simple extension of the two-period model from section 1.3. The basic idea is easy to understand. Suppose there is an intermediate period when additional funds have to be invested in order to continue the project and realize any payoffs. We refer to this reinvestment need as a *liquidity shock* and denote it ρ (per unit of investment). If the liquidity shock ρ turns out to be larger than the pledgeable amount ρ_0, the firm cannot get outside funding to continue the project unless it has arranged for such funding in advance. This creates a demand for liquidity, as firms look to insure against shocks that have a high total return ($\rho_1 - \rho > 0$), but a negative net present value for investors ($\rho_0 - \rho < 0$). Note that the wedge $\rho_1 - \rho_0 > 0$ is crucial for the argument. If $\rho_1 = \rho_0$, the liquidity shock ρ cannot fall strictly between the total and the pledgeable return.

The ex ante demand for liquidity will depend on the size of the liquidity shock. Shocks that are high enough will not be insured (financed in advance). The second best policy trades off the scale of the initial investment against the ability to withstand higher liquidity shocks. In general, there will be credit rationing both at the initial period and in the intermediate period, because entrepreneurial capital is scarce and commands a premium relative to the market.

By definition no external claims can be issued on the private (illiquid) return $\rho_1 - \rho_0$, while arbitrary external claims can be issued on the pledgeable (liquid) return ρ_0. In particular, these claims can be made contingent on the liquidity shock ρ. In effect, we are assuming complete contracting on the liquid portion of the firm's return. This is perhaps unrealistic, but it has the attraction that it is a minimal departure from the standard Arrow-Debreu world. We will discuss how second-best contracts can be implemented using common ways such as credit lines, equity issues (involving dilution) or by holding liquid (marketable) assets in anticipation of future liquidity needs. Finally, and relaxing the assumption that the liquidity shock is observed by investors, we will also show that the implementation of the second-best policy hinges crucially on the ability of investors to keep the firm from spending funds on unauthorized projects.

2.1 The general set up

There are three dates $t = 0, 1, 2$ and a single good. At date 0 the firm chooses the scale of the project I. At date 1 the *liquidity shock* $\rho \geq 0$ takes place. The value ρ determines how much more needs to be invested per unit to continue. Continuing at a smaller scale than I is feasible. Let $i(\rho) \leq I$ denote the continuation scale when the liquidity shock is ρ. Continuing at this scale requires a date-1 investment $\rho i(\rho)$ and yields a date-2 public (pledgeable) return $\rho_0 i(\rho)$ and an illiquid (private) return $(\rho_1 - \rho_0)i(\rho)$ to the entrepreneur. There are no returns from the portion of the project that is not carried forward. If $i(\rho) = 0$ the firm is closed down and the payout, both pledgeable and private, is zero.

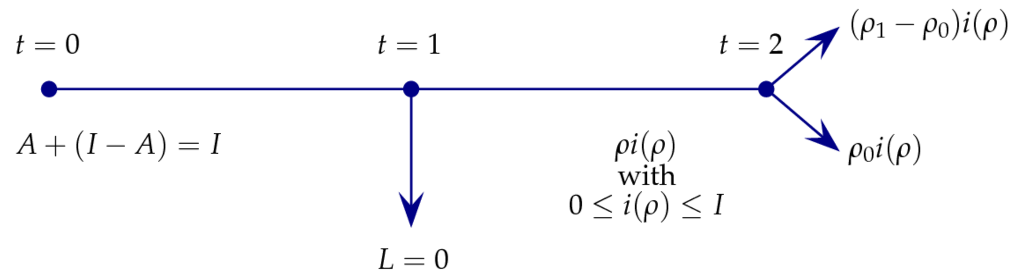


Figure 2.1. Timing

To be concrete, one can think of I as the cost of purchasing a machine. The variable cost of production – which includes payments for intermediate inputs, labor, and so on – is ρ. At date 1, after observing ρ, the firm can decide at what scale to operate the machine. Another example would be the initial purchase of land and the subsequent decision to develop all or some fraction of the land. More generally, I represents a sunk, fixed cost that caps the scale at which production can be carried out at date 1. We thus assume that it is infinitely costly to increase the scale I at date 1 (it takes time to build).

The liquidity shock is modeled this way mainly for convenience. The basic ideas we want to get across at this point are not dependent on the particular way we model the liquidity shock. In section 2.5 we will show how the analysis extends to more general cases (uncertain returns, a positive value of liquidation, an intermediate income, etc.), which are of interest for instance when we discuss implementation issues and risk management policies.

2.2 Two liquidity shocks

We start with the case in which the liquidity shock ρ can take only two values, high (ρ_H) or low (ρ_L), which are constrained to satisfy

$$0 \leq \rho_L < \rho_0 < \rho_H < \rho_1. \tag{2.1}$$

The reason for limiting the shocks in this manner is that shocks below ρ_0 do not require pre-arranged financing, while shocks above ρ_0 do. The high and low shocks in (2.1) cover these two leading cases. Let f_L and f_H denote the probabilities of a low, respectively high liquidity shock. We assume that

$$\rho_0 < \min\{1 + f_L\rho_L + f_H\rho_H, \frac{1 + \rho_L f_L}{f_L}\} < \rho_1. \tag{2.2}$$

The middle term in (2.2) is the minimum expected cost of carrying one unit of the project to completion (see below). If the project is continued in both states the expected cost is the first term in the brackets. If the project is continued only in the low state, the second term measures the expected cost per unit completed. The inequality on the right implies that the project is socially desirable, while the inequality on the left assures that the project is not self-financing (the pledgeable income does not cover the total cost of investment regardless of the optimal policy). A self-financing project could be carried out at any scale, which would lead to unbounded payoffs.

We are looking for a second-best contract. A contract specifies the level of investment I and the continuation scales $i_L \equiv i(\rho_L)$ and $i_H \equiv i(\rho_H)$ (both $\leq$ I) corresponding to the low and high liquidity shocks, respectively. A contract also specifies final payments to investors and the entrepreneur, but just as in the simpler two-period model it is easy to see that it is optimal to assign all the liquid returns $\rho_0 i(\rho)$ to the investors, leaving the entrepreneur holding only the illiquid part $(\rho_1 - \rho_0)i(\rho)$. The entrepreneur only holds illiquid claims because the return on internal liquid funds exceeds the market rate (which we take to be 0).

The second-best solution solves:

$$\max_{\{I,i_L,i_H\}}\{f_L(\rho_1 - \rho_L)i_L + f_H(\rho_1 - \rho_H)i_H - I\}, \tag{2.3}$$

subject to

$$f_L(\rho_0 - \rho_L)i_L + f_H(\rho_0 - \rho_H)i_H \geq I - A, \tag{2.4}$$

$$0 \le i_L, i_H \le I \tag{2.5}$$

The objective function is the expected social return of the investment. Evidently, the budget constraint (2.4) will bind at the optimum. By substituting the budget constraint into the objective function, eliminating I, we get an equivalent program in which the entrepreneur's expected rent rather than the expected social return is maximized. Since investors all earn the market rate of interest (0), the full social surplus goes to the entrepreneur. We will often take the entrepreneur's rent, which equals his expected *net utility*, as the objective.

The budget constraint makes clear that investors provide insurance against liquidity shocks. When the low shock occurs, the firm pays the investors $\rho_0 - \rho_L > 0$. When the high shock occurs, investors pay the firm $\rho_H - \rho_0 > 0$ per unit of continued investment.

Since $\rho_1 - \rho_L$ and $\rho_0 - \rho_L$ are both positive, it is in the interest of the investors as well as the entrepreneur to continue at full scale when a low shock occurs; hence $i_L = I$.[1] The program then boils down to choosing just two values: the initial scale I and the continuation scale i_H in the high shock state. There is a trade-off between these two investments. The bigger one chooses I the lower must i_H be, since both imply net outlays for the investors (in contrast to i_L, which relaxes the budget constraint). Let $x = i_H/I$ denote the fraction of the project that is being continued at date 1 and let

$$\overline{\rho}(x) \equiv f_L \rho_L + f_H \rho_H x \tag{2.6}$$

denote the expected unit cost of continuing. The maximal scale of the initial investment $I(x)$ as a function of the fraction x of the project that is continued in the high shock state

[1] Because the project always continues at full scale when the low shock occurs, we could have counted the low shock as part of the initial investment and adjusted both the low and the high shock correspondingly; that is, we could have chosen $\rho_L = 0$ without loss of generality. When there is a positive liquidity premium, as will be the case later on, the same nominal expenditure may have a different value in different periods; hence we refrain from this simplification.

is given by the budget constraint (2.4):

$$I(x) = \frac{A}{1 + \overline{\rho}(x) - \rho_0(f_L + x f_H)} \tag{2.7}$$

The entrepreneur's expected net utility (equal to the social surplus) is

$$\begin{aligned} U(x) &= [\rho_1(f_L + f_H x) - (1 + \overline{\rho}(x))]I(x) \\ &= \mu(x)A, \end{aligned} \tag{2.8}$$

where $\mu(x)$ is the net value of an additional unit of entrepreneurial capital and takes the form

$$\mu(x) = \frac{\rho_1(f_L + f_H x) - (1 + \overline{\rho}(x))}{(1 + \overline{\rho}(x)) - \rho_0(f_L + f_H x)}. \tag{2.9}$$

Because the Lagrangian of the program (2.3)-(2.5) is linear, we only need to evaluate the utility levels corresponding to $x = 0$ (continuing only when the shock is low) and $x = 1$ (always continuing). In either case continuation is at full scale I; partial continuation is not relevant. A direct evaluation of $U(1) - U(0)$, the difference in utility between the two cases, shows that it is optimal to cover both liquidity shocks (choose $x = 1$) if and only if

$$\frac{\rho_1 f_L - (1 + \rho_L f_L)}{(1 + \rho_L f_L) - \rho_0 f_L} = \mu(0) \leq \mu(1) = \frac{\rho_1 - (1 + \overline{\rho}(1))}{(1 + \overline{\rho}(1)) - \rho_0}. \tag{2.10}$$

When $\rho_H = \rho_L$, inequality (2.10) holds and the project will be continued in both states. As ρ_H is increased from ρ_L to ρ_1, the difference $\mu(0) - \mu(1)$ moves monotonically from being strictly negative to strictly positive. Therefore, in between the two extreme ρ_H-values there is a cut-off value c, satisfying $\rho_L < c < \rho_1$, such that the project will continue if and only if

$$\rho_H \leq c. \tag{2.11}$$

Simple manipulations of the cut-off condition $\mu(0) \leq \mu(1)$ allow us to write

$$c = \min\left\{1 + f_L\rho_L + f_H\rho_H,\ \frac{1 + f_L\rho_L}{f_L}\right\}. \tag{2.12}$$

We can interpret c as the *unit cost of effective investment*, i.e. what it costs on average to bring one unit of investment to completion. Condition (2.11) therefore has the intuitive interpretation that it is optimal to continue in the high-shock state if and only if the unit cost of effective investment is less than the cost of the shock. We will see that an analogous condition holds in the continuum case.

We can also restate the inequality (2.10) as the following necessary and sufficient condition for continuing in both the low and the high state[2]

$$f_L(\rho_H - \rho_L) \leq 1. \tag{2.13}$$

The effects of ρ_H and ρ_L in (2.13) are intuitive. They reflect the fact that both an increase in ρ_H and a decrease in ρ_L will work in favor of larger ex-ante scale at the expense of less ex-post liquidity; a lower ρ_L increases the return to initial scale, while a higher ρ_H makes it more costly to continue ex post. The role played by f_L in (2.13) is less obvious, since both the benefits and costs of continuing in the high state go up as f_L decreases. The issue then is how the firm should divide an extra unit between the initial investment I and liquidity provision at date 1. As f_L goes to zero, the net return from investing only in the low-shock state goes to zero, while the net return from continuing in both states is bounded below by a strictly positive number. Hence, if f_L is small enough it is better to continue also in the high state.

Remark (repeated liquidity shocks). A limitation of our analysis is that it does not do justice to the rich dynamics of liquidity management. Indeed, after the initial contracting stage, date 0, there is only one period, date 1, at which the firm will possibly need new cash. There is accordingly no point hoarding liquidity at date 1. Relatedly, all liquidity hoarded at date 0 is usable,[3] with the caveat, studied in Appendix 2.1, that available liquidity may be abused in the presence of alternative uses of this liquidity. Recent

[2] That neither ρ_0 or ρ_1 show up in (2.13) is a consequence of the constant returns to scale technology; however, they enter implicitly through the parameter restrictions (2.1) and (2.2).

[3] See Goodhart (2008) for a discussion of usable liquidity.

work has studied optimal liquidity management in related, but infinite-horizon models of repeated moral hazard. Biais et al. (2007, 2008) and DeMarzo and Fishman (2007 a,b) shed light on how over time investment and available liquidity adjust to profit realizations in an optimal contract. For example Biais et al. show that liquidity is not meant to be fully depleted even though it is indeed reduced after an adverse shock. Discipline is ensured by downsizing when things go wrong, not by a complete exposure to liquidity risk. This policy is in the spirit of proportionality for compulsory reserves as well as for capital requirements in banking regulation.

2.3 Implementation of the second best contract

Implementation of the second-best contract presents two kinds of problems. The first problem is that the entrepreneur may use the funds in a different way than the contract specifies (either at date 0 or at date 1). The second problem is that investors may not be able to deliver on the promise to provide funds at date 1. Such promises must be backed up by claims on real assets that the investor owns at date 1 and can use as collateral. We will discuss collateral problems at length in the coming chapters. The purpose of this section is to illustrate some of the problems that may arise on the firm's side.

One potential problem is that, when the second best optimum specifies that the firm should not continue when facing the high shock, the entrepreneur may nonetheless want to invest less than the agreed-upon amount I at date 0 in order to keep extra funds in store to meet the high liquidity shock. Alternatively, when the second best optimum recommends to withstand even the high liquidity shock, the entrepreneur may not want to save funds for reinvestment in the high state and instead spend the funds on a higher initial investment I. This can be a tempting possibility, since the entrepreneur knows that investors will always finance a low shock, even if the scale is higher than initially intended. The entrepreneur can rely on a soft-budget constraint for the low shock, because investors face a fait accompli. The downside to the entrepreneur of such a policy is that there will not be enough funds to finance the high shock, and no investor will be ready to make up the short-fall at date 1.

In order to implement the second-best policy, there must be a way to enforce the right levels and kinds of investment at date 0 as well as date 1. Intermediaries, venture capitalists, large block holders and others monitor in varying degrees and in different ways a firm's use of funds. A rich literature in corporate finance has investigated these issues in depth. We will not study intermediation explicitly, even though it constitutes an integral part of the financing of firms.[4] Instead, we will assume that investors can directly monitor the firm's liquidity position but not the firm's liquidity shock and consider two illustrative cases: in the first, the firm has no alternative uses of funds and will therefore behave; in the second case, the entrepreneur can divert funds for some personal benefit.

Costless implementation: firm has no alternative use of funds

Assume condition (2.10) holds so that reinvestment is desirable both in the low and the high shock state. Consider first the rather hypothetical scenario in which the entrepreneur has no alternative use of date-1 funds; date-1 liquidity is of value only because it allows the firm to meet liquidity shocks. Under this scenario, there are many alternative ways to implement the second-best solution.

One option is to give the firm a line of credit up to $\rho_H I$, which it can freely use at date 1. This allows the firm to meet both kinds of shocks. If the low shock occurs, the firm will leave $(\rho_H - \rho_L)I$ unspent, since we assumed that there is no alternative use for the excess funds. If the high shock occurs, the firm will draw down the full line of credit, which has a negative net present value, because the firm's pledgeable date-2 income $\rho_0 I$ is less than the credit $\rho_H I$ used. A minor variant would be to reduce the credit line to $(\rho_H - \rho_0)I$ and giving the entrepreneur the right to dilute the initial investors' up to their maximum stake $\rho_0 I$. Either way, the entrepreneur has to pay for the expected use of liquidity up

[4] See Holmström and Tirole (1997) for an analysis of intermediation using the same basic model as here and Tirole (2006) for a more extensive treatment.

front.

One may wonder to what extent banks actually honor credit lines in states of nature where they would prefer not to lend. Empirically, it is not an easy question to distinguish involuntary lending from voluntary lending or lending that is done to preserve a reputation in the credit market as has been suggested by Boot, Thakor and Udell (1987).[5] However, studies of bank lending during the subprime crisis as well as the 1998 crisis associated with the collapse of Long Term Capital Management, indicate that drawdowns on existing credit lines can be substantial. Ivanisha and Scharfstein (2009), who compared patterns of bank lending in the period August-November 2008 note that while new commercial and industrial loans fall dramatically (37%) in this period compared with the same period in 2007, firms that secured credit lines likely made extensive use of them, since the level of loans on bank balance sheets increased somewhat during that time. They identify $16 billion worth of drawdowns from press releases alone, but considering the large stock of credit lines (about $3,500 billion) the total drawdowns must be much larger. The paper further documents that banks that were liquidity constrained had to reduce their other lending in order to handle drawdowns – an opportunity cost argument that points to the involuntary nature of honoring credit lines. Anecdotal evidence indicates that a large number of firms also drew down credit lines in anticipation of future liquidity problems caused by the crisis, suggesting that firms have been concerned about either the credit worthiness of banks or changes in the terms of the credit facility. Strahan, Gater and Schuermann's (2006) study of the LTCM crisis likewise concludes that banks and other credit institutions had to accept costly drawdowns, though the overall effect on the banking system was moderated by the fact that the funds came back in the form of deposits. On balance, the evidence indicates that credit lines do serve an insurance role

[5] Banks' implicit liabilities are common and create serious issues for prudential regulators as they are not really covered by any capital charge. For example, in the summer of 2007, Bear Stearns bailed out two funds it had sponsored even though it had no legal obligation to do so.

of the sort envisioned in our model.[6]

Another common arrangement to guarantee that the firm will have enough resources in adverse circumstances is for the firm to buy protection. Credit default swaps (CDSs), which amounted to $62 trillions at the onset of the recent crisis, allow firms to buy protection against the default of other firms.[7] For example, if an amount exceeding $(\rho_H - \rho_0)I$ constitutes a shortfall of income due to the default of trading partners, the shortfall can be offset through the use of CDSs.

Finally, firms routinely hoard liquid funds, sometimes very large ones, both to cushion smaller liquidity shocks as well as in anticipation of future spending needs such as acquisitions and other kinds of investments.[8]. This can be represented in our model by investors paying the firm $I + (\rho_H - \rho_0)I - A$ at date 0 and making sure the firm hoards liquid assets, for instance treasury bonds, at date 0 in the amount $(\rho_H - \rho_0)I$. At date 1, the firm can raise up to $\rho_H I$ in fresh funds by selling the bonds and by diluting the stake of the initial investors through additional equity. If the liquidity shock is low, the firm simply leaves the excess liquidity unused – does not issue new equity and/or does not use all of the bonds. Note that this way of implementing the second-best outcome, while equivalent to a credit line, does not require the firm to raise fresh funds at date 1 for a project that has a negative net present value. The funds covering negative net present value actions at date 1 have been paid already at date 0. Naturally, one can mix hoarding with credit lines in a lot of ways as long as the firm can access the right amount

[6] Credit lines are often contingent on satisfying loan covenants, which could be interpreted as an effort to deal with states where firms, even in the second-best, should not be extended credit. In the recent crisis and especially in the final years before the collapse, many banks appear to have abandoned usual precautions and extended "covenant-light" credit lines, tempted by the generous up-front fees for contingent credit.

[7] The $62 trillions figure is the gross value of CDSs and greatly overstates the actual insurance coverage for this reason. Also, many participated in these markets for reasons other than insuring themselves. The CDS market was in part a place where bets could be placed on the future of the economy.

[8] For instance, Microsoft had at one point over $40 billion in liquid assets. Very little is typically invested in the stock of other companies, unless there is some strategic purpose to do so or the firm is in the investment business as such. Our focus is on liquidity shocks, but as the analysis in section 2.5 shows, we could include other motives. However, given our "complete market" assumption, we cannot explain the heavy use of debt and near-debt assets.

of liquidity at date 1.

Costly implementation: firm can divert funds at date 1

The assumption that the firm has no alternative use for excess liquidity is extreme. Firms always have alternative investments to consider, especially if the funds come at no extra charge as above. In Appendix 2.1 we illustrate with a simple example how the optimal contract responds to the presence of an alternative investment option.

Consider again the two-shock model in which (2.10) holds, so that the second-best contract prescribes continuation for both the low (ρ_L) and the high (ρ_H) liquidity shock. Suppose that the firm can divert funds at date 1 to a project that only generates private benefits. Each unit diverted into the alternative project generates a benefit to the entrepreneur that is equal to ξ units of consumption. We assume that the value of a diverted unit satisfies

$$\xi \leq 1, \tag{2.14}$$

so that it is more efficient for the entrepreneur to receive one unit from the pledgeable income at date 2 than to divert one unit at date 1.

Investors can observe how much funds the firm uses in total at date 1 as well as the scale at which the project is continued, but they cannot see how much is used for reinvestments and how much for diversion, nor can they observe the realization of the date-1 liquidity shock. The presence of the alternative project makes the design of liquidity provision more challenging. In general, the second-best solution from section 2.2 can no longer be implemented, because the optimal contract must ensure that the entrepreneur does not want to divert funds for private benefit.

Appendix 2.1 determines the optimal second-best contract with potential diversion, using a standard mechanism-design approach. To summarize our findings briefly, we find that diversion can be avoided most efficiently in one of two ways. Either the entrepreneur is asked to hold more than the minimum stake at the beginning, which he forgoes if he claims that the high shock has occurred, or investors scale down the investment in response to a reported high shock. Which of the two options are used, depends on actual parameter values. Both schemes add to the cost of financing and therefore reduce the social surplus. The idea of discouraging false reporting of high costs by lowering the continuation scale of investment corresponds to standard distortions in adverse selection models. The other solution, that forces the entrepreneur to hold some of the pledgeable income in addition to the non-pledgeable one, is more interesting. The entrepreneur's incentives are made to align with the investors by having his extra shares diluted along with the initial investors' shares when a high liquidity shock occurs. Both solutions resonate with practice.

Finally, we note that when $\xi = 1$, i.e., when there is no dead-weight loss for the entrepreneur to consume unneeded funds, the option to divert destroys all outside insurance opportunities. In that case, the firm is left to take care of itself using market instruments that are not contingent on idiosyncratic shocks.[9]

2.4 Continuum of liquidity shocks

For some applications (e.g., risk management, see section 4.3) it is analytically more convenient to deal with a continuum of liquidity shocks. We will provide only a brief treatment here. For a more detailed analysis see Holmström and Tirole (1998) and Tirole (2006).

Let $F(\rho)$ be the distribution function and $f(\rho)$ the density function of the liquidity shock ρ. A contract specifies the initial investment level I and the continuation levels $i(\rho) \leq I$ for each contingency ρ. The contract also specifies payments to each party. As in the two-shock case, it is optimal to let the investors retain all of the pledgeable income $\rho_0 i(\rho)$ and have the entrepreneur hold only the illiquid part $(\rho_1 - \rho_0)i(\rho)$. This maximizes

[9] There are other technologies that result more readily in self-insurance. For instance, in Caballero and Krishnamurthy's (2003b) closely related model of liquidity demand, reinvestments only create non-pledgable income. Since the reinvestment cannot be observed/controlled by the investors, self-insurance is the only option in their model.

the return on the entrepreneur's initial assets A.

The second-best solution can therefore be found by solving the following program:

$$\max_{\{I,i(\rho)\}} \int (\rho_1 - \rho_0) i(\rho) f(\rho) d\rho, \tag{2.15}$$

$$\text{subject to}$$

$$\int (\rho_0 - \rho) i(\rho) f(\rho) d\rho \geq I - A, \tag{2.16}$$

$$0 \leq i(\rho) \leq I, \quad \text{for every } \rho.$$

Note that we have taken the entrepreneur's rent as the objective function, because the budget constraint binds. The left-hand side of the budget constraint is the expected pledgeable income given the reinvestment policy. This has to cover the investors' date-0 contribution $I - A$.

It is intuitive, and also easy to show, that the solution to the second-best program takes the form of a cut-off value $\widehat{\rho}$ such that the project continues at full scale $(i(\rho) = I)$ if $\rho \leq \widehat{\rho}$ and it is discontinued $(i(\rho) = 0)$ if $\rho > \widehat{\rho}$. We will show that the optimal cut-off level, denoted ρ^*, falls strictly between the pledgeable and the total income:

$$\rho_0 < \rho^* < \rho_1, \tag{2.17}$$

To this end, consider the maximal investment level consistent with an arbitrary cut-off level $\widehat{\rho}$. This level, obtained from the budget constraint (2.16), can be written:

$$I = k(\widehat{\rho})A, \tag{2.18}$$

where the *investment multiplier* $k(\widehat{\rho})$ is

$$k(\widehat{\rho}) = \frac{1}{1 + \int_0^{\widehat{\rho}} \rho f(\rho) d\rho - F(\widehat{\rho})\rho_0} \tag{2.19}$$

The denominator of the investment multiplier gives the amount of entrepreneurial funding that is required per unit of investment. Substituting the maximal investment level into the objective function (2.15), the entrepreneur's net expected utility can, after some algebraic manipulations, be written

$$U(\hat{\rho}) \equiv \mu(\hat{\rho})A = m(\hat{\rho})k(\hat{\rho})A = \frac{\rho_1 - c(\hat{\rho})}{c(\hat{\rho}) - \rho_0}A, \tag{2.20}$$

where the *total expected return per unit of investment* $m(\hat{\rho})$ is

$$m(\hat{\rho}) = F(\hat{\rho})\rho_1 - 1 - \int_0^{\hat{\rho}} \rho f(\rho) d\rho \tag{2.21}$$

and the *expected unit cost of effective investment* $c(\hat{\rho})$ is

$$c(\hat{\rho}) \equiv \frac{1 + \int_0^{\hat{\rho}} \rho f(\rho) d\rho}{F(\hat{\rho})}. \tag{2.22}$$

The interpretation of $c(\hat{\rho})$ parallels that of c in the two-shock case; see (2.12). It is intuitive that the entrepreneur's expected utility is the product of the total expected return per unit invested $m(\hat{\rho})$ times the maximal investment scale $I(\hat{\rho}) = k(\hat{\rho})A$ for the cut-off $\hat{\rho}$. To see why the optimal cut-off ρ^* falls strictly between ρ_0 and ρ_1 as posited in (2.17), note first that the investment scale $k(\hat{\rho})$ is largest at $\hat{\rho} = \rho_0$, because the investor's date-1 expected net income is maximized by continuing at full scale whenever the financial return is non-negative ($\rho_0 - \rho \geq 0$). On the other hand, the total expected return per unit of investment $m(\hat{\rho})$ is maximized by continuing at full scale whenever the total return from continuing is positive ($\rho_1 - \rho \geq 0$), that is, by setting $\hat{\rho} = \rho_1$. The second-best cut-off ρ^* is set strictly above ρ_0, because $m(\hat{\rho})$ is strictly increasing at ρ_0; it is set strictly below ρ_1 because $k(\hat{\rho})$ is strictly decreasing at ρ_1.

The model with a continuum of shocks highlights the fundamental trade-off facing the firm: that between investing in initial scale versus saving funds to meet liquidity shocks. The argument above shows that the *the second-best solution is a compromise featuring credit-rationing both at the initial investment date 0 and at the re-investment date 1.*[10]

[10] With our constant-returns-to-scale technology, the cut-off ρ^* does not depend on the entrepreneur's endowment. Rampini and Viswanathan (2010), in a richer model, show that if the intial investment

From the entrepreneur's expected utility (2.20) follows immediately that the optimal cut-off value $\widehat{\rho} = \rho^*$ is determined by minimizing the expected unit cost of effective investment $c(\widehat{\rho})$. The first-order condition for minimizing $c(\widehat{\rho})$ can after some manipulation be expressed as

$$\int_0^{\rho^*} F(\rho)d\rho = 1. \tag{2.23}$$

Using (2.21) the condition for the optimal cut-off ρ^* can also be written

$$c(\rho^*) = \rho^*, \tag{2.24}$$

which is the natural counter-part to the cut-off condition for the two-shock case.

Equation (2.23) (or, equivalently, (2.24)) determines implicitly the second-best cut-off value ρ^*. Note that the optimal cut-off ρ^*does not depend on the parameters ρ_0 or ρ_1 (as long as it is desirable to invest in the project at all). Equation (2.23) implies that a mean-preserving spread of the distribution F lowers ρ^*.[11] Substituting (2.24) into (2.22) it follows that a mean-preserving spread also reduces $c(\rho^*)$, implying that more uncertainty raises the value of the project. The simple intuition behind these conclusions is that higher uncertainty increases the option value of terminating the project when the liquidity shock exceeds ρ^*. Indeed, it is essential that ρ be stochastic. If the liquidity shock were a constant, the firm could raise external funds only if this constant were less than ρ_0. To finance shocks above ρ_0 there must also be shocks below ρ_0.

Finally, it is worth noting that the implementation options described in Section 2.4

exhibits decreasing returns to scale, then the trade-off will depend on the entrepreneur's endowment. Most importantly, entrepreneurs with little wealth will buy no insurance at all, since the returns from the initial investment trumps the value of liquidity.

[11]Consider two cumulative distribution functions $F(\rho)$ and $G(\rho)$ on $[0, \overline{\rho}]$. Distribution G is a mean-preserving spread of distribution F if for all ρ:

$$\int_0^{\rho} G(\rho)d\rho \geq \int_0^{\rho} F(\rho)d\rho$$

and

$$\int_0^{\overline{\rho}} G(\rho)d\rho = \int_0^{\overline{\rho}} F(\rho)d\rho$$

(they have the same mean).

would not work with a continuum of liquidity shocks. In the continuum model, both a credit line and the purchase of bonds would permit the entrepreneur to continue at a scaled-down level even when the shock exceeds ρ^*, the second-best cut-off. Unless prevented, the entrepreneur would take advantage of such an opportunity.

2.5 General shocks.

It is easy to generalize the model to include income at date-1 as well as a payoff from liquidation. We can also let all the variables, including the date-2 payoffs, be uncertain until date 1. The purpose of looking into such an extension is two-fold. It gives a richer interpretation of the liquidity shock by pointing to a variety of sources that give rise to a demand for liquidity. Our original focus on shocks to the cost of reinvestment ρ was mainly for convenience. The general model will also be the basis for discussing risk management in section 4.3

Let ω describe the state of nature at date 1 and $F(\omega)$ be the probability distribution as seen by the parties at date 0. Contracts at date 0 can be made contingent on ω as before. Let $\rho(\omega)$, $\rho_0(\omega)$, $\rho_1(\omega)$ be the various per-unit costs and payoffs in state ω and $i(\omega)$ the corresponding reinvestment level at date 1. Liquidation is also possible at date 1: the value of liquidation is $(I - i(\omega))w(\omega)$, where $I - i(\omega)$ is the amount liquidated and $w(\omega)$ represents the per-unit value of liquidation. We also allow for uncertain income at date 1. It takes the form $r(\omega)I + y(\omega)$, where $r(\omega)$ is the per-unit income from the initial investment and $y(\omega)$ is income from old assets in place. At this stage of the analysis, where liquidity has no price premium, a random income $y(\omega)$ is equivalent to making the endowment A have an uncertain value that gets revealed at date 1.

The problem is the same as before: to decide the level of the date-0 investment I and the date-1 continuation scale $i(\omega)$. This problem is solved by the following program, which is a straightforward extension of (2.15):

$$\max_{\{I,i(\cdot)\}} \int b(\omega)i(\omega)dF(\omega), \tag{2.25}$$

subject to

$$\int \{(\rho_0(\omega) - \rho(\omega))i(\omega) + w(\omega)(I - i(\omega))\}dF(\omega) + \int [r(\omega)I + y(\omega)]dF(\omega) \geq I - A,$$

$$0 \leq i(\omega) \leq I, \quad \text{for every } \omega. \tag{2.26}$$

We have chosen to replace $\rho_1(\omega)$ with the entrepreneur's private benefit $b(\omega) = \rho_1(\omega) - \rho_0(\omega)$ (per unit of continuation investment) to separate the effect that $\rho_0(\omega)$ has on the objective function from the effect it has on the budget constraint. The budget constraint includes the value of liquidation as well as the date-1 incomes from investment and from legacy assets. As before, assuming that $I > 0$ and that the budget constraint binds, the maximization of social surplus is equivalent to the maximization of the entrepreneur's rent $b(\omega)$.

Let $1 + \lambda \geq 1$ be the shadow price of the budget constraint. Then the optimal choice of $i(\omega)$ is determined by the sign of

$$\Psi(\omega) = b(\omega) - (1 + \lambda)[\rho(\omega) + w(\omega) - \rho_0(\omega)] \tag{2.27}$$

as follows:

$$i(\omega) = 0, \text{ if } \Psi(\omega) < 0$$

$$i(\omega) = I, \text{ if } \Psi(\omega) > 0$$

$$i(\omega) \in [0, I], \text{ if } \Psi(\omega) = 0.$$

We see that variations in date-1 income $r(\omega)I + y(\omega)$ do not affect the decision rule directly. However, the expected value of $r(\omega)$ does influence the return on the initial investment I and that way the cost of capital $1+\lambda$. The expected value of $y(\omega)$ merely changes the value of entrepreneurial capital as noted earlier and therefore the scale of investment, but it does not influence the balance between date 0 and date 1 investments. We return to these observations when we discuss risk management.

The decision to continue in any given state ω depends on three terms: (i) the entrepreneur's private value of continuing, $b(\omega) = \rho_1(\omega) - \rho_0(\omega)$; (ii) the net financial cost $\rho(\omega) + w(\omega) - \rho_0(\omega)$ of continuing, including the cost of forgoing the opportunity to liquidate the project and (iii) the entrepreneur's cost of capital $1 + \lambda$. Thus, $\Psi(\omega)$ is the marginal social as well as entrepreneurial value of continuing in state ω. The higher the private benefit or the pledgeable income in a state, the more likely it is that the project is continued in that state. The reverse is true the higher is the total cost of continuing, where total cost is the sum of the direct reinvestment cost $\rho(\omega)$ and the liquidation benefit $w(\omega)$.

For empirical work, it is important to note that these "comparative statics" results pertain to variations across realized states of nature. Changes in the (random) parameters of the model would in general affect λ and that way investments as well as the cut-off.[12] Nevertheless, the qualitative features of the model would not change. In particular, instead of taking ρ to be random, we could have let the liquidation value w or the pledgeable income ρ_0 be random (keeping $b(\omega)$ fixed) without changing our main conclusions (e.g., the optimality of the cut-off and the incomplete insurance result). In this sense, the liquidity shock can be given a broader interpretation (also for many of the qualitative results that follow up to part IV). This should be contrasted with comparative statics performed on the parameters of the basic model. In that model, changes in the parameters ρ_0 and ρ_1 had no effect on continuation decisions (the cut-off rule implicit in (2.23) does not include these parameters) as long as the parameter values are such that I is strictly positive but finite.

[12] This need not be the case. In the basic model, changes in the parameters ρ_0 and ρ_1 had no effect on continuation decisions (the cut-off rule implicit in (2.23) does not include these parameters) as long as the parameter values are such that I is strictly positive but finite.

2.6 Summary and concluding comments

In this chapter we have introduced our basic model of liquidity demand. Firms demand liquidity, because they want to insure themselves against credit rationing. Credit rationing can occur, due to a wedge between the pledgeable income to investors and the total income to the firm. When the cost of continuing a project falls between the pledgeable income and the total income, the project can continue only if its funding has been arranged in advance. This is the sense in which firms demand liquidity. In standard financial models, firms never demand liquidity.[13]

As in standard corporate finance models, investments have, at least at the margin, a negative net present value based on pledgeable income, so the entrepreneur has to put in some of his own capital to get the project off the ground. Because entrepreneurial capital has a higher rate of return than investor capital, it is optimal for the entrepreneur to commit all of the firm's pledgeable income to the investors and only hold illiquid claims for himself. The amount of entrepreneurial capital determines the scale of operations.

Adding a liquidity demand to this standard solvency-requirement story, we have shown how the firm must also plan for its liquidity. The model is one of asset-liability-management. Not hoarding liquidity is akin to accepting a maturity mismatch. Liquidity hoarding is akin to purchasing insurance. In general (and always, with a variable investment scale and a continuum of liquidity shocks), the firm is credit rationed both ex ante (initial investment) and ex post (continuation investment). All strictly positive net present value investments will not be undertaken.

The notion that investors knowingly fund projects with a negative continuation value may be surprising at first, yet it is an essential ingredient of a theory of liquidity shortages. If all liquidity shocks were associated with a positive continuation value for the investors

[13] This statement is just a variant on the indeterminacy of the firm's capital structure in traditional finance.

there would be no need to seek funding in advance.[14] The demand for insurance is always associated with one side winning and the other side losing ex post. In that sense, our model is conceptually no different than Diamond and Dybvig's (1983) seminal model of consumer liquidity demand. Indeed, the two models are in some respects formally quite similar, though liquidity shocks to the productive sector have quite different implications for the economy as we will see later.

The extent to which investors can commit additional funds to projects that have a negative continuation value, depends on their ability to monitor the contingencies. If investors cannot observe the firm's liquidity shock, a contingent contract has to rely on information reported by the firm. This is not a problem if the firm has no alternative use for funds. However, if the firm can divert excess funds for private benefit, the contract needs to be adjusted as shown in Appendix 2.1. Diversion can be avoided in one of two ways. Either the entrepreneur is asked to hold more than the minimum stake at the beginning, which he forgoes if he claims that the high shock has occurred, or investors scale down the investment in response to a reported high shock. In both cases, the solution is less efficient than when the shock is contractible.

In Appendix 2.2 we develop in more detail the formal relationship between consumer liquidity demand and producer liquidity demand by reinterpreting our firms as Diamond-Dybvig consumers. Here is a brief summary of the key points. The basic model of consumer liquidity demand has a three-period structure like the model with producer liquidity demand. Consumers hoard liquidity to smooth consumption and meet unexpected liquidity needs. Modeling this has traditionally assumed that consumers save at date 0 because they face uncertainty about the timing of their consumption needs, that is, they

[14] With a strictly concave production function, it is possible of course to have a demand for liquidity even if continuation yields a positive net present value. The ex ante commitment required from investors occurs at the margin: they may prefer a smaller scale than is optimal. More formally, if the pledgeable income $\rho_0(I)$ is concave, one can have $\rho_0(I) \geq \rho I$ while $\rho_0'(I) < \rho$. There is no need to hoard stores of value or to obtain a credit line, but the entrepreneur must still plan liquidity management by securing from investors the right to dilute their stake.

are unsure about their marginal rate of substitution between consumption at date 1 and date 2. If one is to compare the models of producer and consumer liquidity demand, this suggests considering firms that have net savings at date 0 ($I < A$) and, as in this chapter, a random liquidity need that accrues either at date 1 or at date 2 (and income accruing at date 3).

With firms as net lenders, we are able to show the precise sense in which the two models are isomorphic. The central optimal insurance result in Diamond and Dybvig's analysis, whereby patient consumers end up subsidizing impatient ones (provided relative risk aversion is greater than 1), has an exact counterpart in the producer liquidity context as long as one interprets the entrepreneur's "consumption" as the "average liquidity withdrawals" We also discuss Jacklin's (1987) critique of a breakdown in insurance (cross-subsidization) between patient and impatient consumers when consumers have access to financial markets. Jacklin's critique in our context comes down to whether firms have alternative uses for excess funds withdrawn at date 1.

Finally, it is worth noting that our model can support some degree of cross-subsidization within a firm. Conglomerates are often criticized for subsidizing divisional investments, though the empirical evidence by now is rather mixed (for a survey of the evidence, see Stein, 2003). In light of the model here, subsidies can be viewed as optimal insurance policies for the human capital invested in the firm or alternatively as efficient ways to motivate the management and employees of divisions. Conglomerates have specific information about divisional circumstances as well as the right to reallocate capital based on this information. Reputation and culture can provide the necessary commitment power (see Shleifer and Summers, 1988, for a related argument).

Appendix 2.1 Mechanism design under fund diversion

In this appendix we provide a full treatment of the second-best optimum and its implementation when the entrepreneur can divert funds at date 1 as reviewed in section 2.3. By the revelation principle the optimal mechanism or contract will have to induce the firm to reveal its type (the date-1 liquidity shock ρ) truthfully. A generic contract specifies: (i) the initial investment level I; and for each state $j = L, H$, (ii) the level of reinvestment i_j; and (iii) payments t_{1j} and t_{2j} made to the firm at dates 1 and date 2, respectively. The date-1 payment t_{1j} will at the optimum be used to cover the firm's liquidity shock as it is not optimal to induce the entrepreneur to divert funds at date 1. We assume that the payment t_{2j} does not vest until date 2 and cannot be used to secure additional funding for the firm at date 1 (this feature is common in executive incentive schemes, for instance, restricted stock). The vesting constraint is important, since it provides an additional instrument for screening. If the firm could use t_{2j} as collateral for borrowing at date 1 there would be no distinction between t_{1j} and t_{2j}.

It is clear that we can restrict attention to cases where the entrepreneur at date 1 is given no more than he claims to need; that is, $t_{1j} = \rho_j i_j$. Indeed, keeping liquidity t_{1L} at its minimum level makes it infeasible for an H-firm to claim that it is an L-firm; since $t_{1L} = \rho_L i_L < \rho_H i_L$, there will not be enough date-1 funds for the H-firm to invest i_L. Any additional compensation for the entrepreneur is most efficiently paid by increasing t_{2j}.

The diversion option creates the following incentive problem: an L-firm can claim a high liquidity shock ρ_H and then divert the excess funds $(\rho_H - \rho_L)i_H$ for private consumption; the entrepreneur cannot turn more of the date-1 funds into private consumption, because the scale of the continuation investment is verifiable. Note that diversion is an issue only if (2.10) holds, which we will assume in this Appendix. Otherwise $i_H = 0$ without diversion in which case diversion would not feasible in the first place.

With these preliminary observations, the second-best program can be written:

$$\max_{\{I, i_j, t_{1j}, t_{2j}\}} \{\sum_j f_j\,(t_{2j} + (\rho_1 - \rho_0)\,i_j)\},$$

subject to

$$\sum_j f_j\,(\rho_0 i_j - t_{1j} - t_{2j}) \geq I - A, \tag{2.28}$$

$$t_{2L} + (\rho_1 - \rho_0)\,i_L \geq t_{2H} + (\rho_1 - \rho_0)\,i_H + \xi(\rho_H - \rho_L)i_H, \tag{2.29}$$

$$t_{1j} = \rho_j i_j \ , \quad j = L, H, \tag{2.30}$$

$$0 \leq i(\omega) \leq I \ \ , \quad \omega = L, H, \tag{2.31}$$

$$t_{2j} \geq 0 \ \ , \quad j = L, H. \tag{2.32}$$

Constraint (2.28) is the budget constraint and constraint (2.29) the (remaining) incentive constraint. We argued that the liquidity constraint (2.30) is binding. This fact has been used to eliminate the t_{1j} terms from the objective function and the incentive constraint. The last term on the right-hand side of the incentive constraint is the L-type's private benefit when claiming to be an H-type and diverting the excess funds for private consumption.

The budget constraint (2.28) will obviously bind. If the incentive constraint (2.29) did not bind, the solution would be the same as without diversion. That solution sets $i_L = i_H = I$, since we assumed that (2.10) holds, and $t_{2L} = 0$, because the entrepreneur's utility (and the initial investment scale) are maximized by distributing all the pledgeable income to investors. But these values violate the incentive constraint, so (2.29) must hold as an equality.

Suppose $t_{2H} > 0$. By reducing t_{2H} to 0 and increasing t_{2L} by the amount $f_H t_{2H}/f_L$, we can relax the incentive constraint (2.29) while keeping the objective function and the budget constraint intact. This cannot be optimal, hence $t_{2H} = 0$. Put differently, setting $t_{2H} = 0$ maximizes the leverage of A.

It is easy to see that $i_L \geq i_H$. It is true already without the diversion option and the incentive constraint (2.29) pushes in the same direction. It follows that $i_L = I$, else we could reduce the cost of investment by lowering the initial scale I.

Eliminating t_{1j} using (2.30) and setting $t_{2H} = 0$ and $i_L = I$, we can rewrite program (2.28)–(2.31) as:

$$\max_{\{I, i_H, t_{2L}\}} \{f_L t_{2L} + f_L (\rho_1 - \rho_0) I + f_H (\rho_1 - \rho_0) i_H\}, \tag{2.33}$$

subject to

$$f_L(\rho_0 - \rho_L)I - f_L t_{2L} - f_H(\rho_H - \rho_0) i_H = I - A \tag{2.34}$$

$$t_{2L} + (\rho_1 - \rho_0)I = (\rho_1 - \rho_0) i_H + \xi(\rho_H - \rho_L) i_H \tag{2.35}$$

$$t_{2L} \geq 0, \quad 0 \leq i_H \leq I. \tag{2.36}$$

This is a linear program and therefore one of the extreme points must be optimal. There are three candidate solutions:

Case I $\quad t_{2L} = 0$ and $i_H = \delta I < I$, where

$\delta \equiv \frac{(\rho_1 - \rho_0)}{(\rho_1 - \rho_0) + \xi(\rho_H - \rho_L)} < 1,$ from (2.35)

Case II $\quad i_H = I$ and

$t_{2L} = \xi (\rho_H - \rho_L) I \; > 0,$ from (2.35)

Case III $\quad t_{2L} = 0$ and $i_H = 0$.

We will shortly argue that Case I always dominates Case III. Looking at the incentive constraint (2.35), we see that Cases I and II represent two extreme ways to prevent an L-firm from claiming that it has been hit by a high liquidity shock and diverting $(\rho_H - \rho_L) i_H$ for private consumption. In Case I, the incentive is established by reducing the scale of operations by a factor $1-\delta$ when the firm reports a high liquidity shock. This corresponds to the investment distortion in a standard screening model. In Case II, the firm is allowed to continue at full scale even if it claims a high shock, but a bribe $t_{2L} > 0$ is used to induce truth-telling. The bribe corresponds to an information rent in a standard screening model. Normally, we would both distort the investment and pay an information rent, but in our linear model one or the other is optimal. Note that $\xi \leq 1$ implies that a bribe is a more efficient way of paying the information rent than letting the firm consume the excess liquidity.

Case II can be interpreted as follows. Initially, the entrepreneur keeps a fraction $\beta = t_{2L}/(\rho_0 I)$ of the "cash flow rights" of the firm. If the entrepreneur reports a high shock ρ_H, all initial shareholders as well as the entrepreneur lose their pledgeable shares through dilution. The missing amount $(\rho_H - \rho_0) I$ is paid by the investors according to the date-0 financial agreement (e.g., using a credit line). If the entrepreneur reports a low shock ρ_L, he retains his share β of the firm's pledgeable income, which pays t_{2L} at date 2. and receives $t_{1L} = \rho_L$ to cover the liquidity shock. This incentive scheme wastes collateral relative to the case without diversion. The entrepreneur would rather hold just illiquid assets, but to prevent diversion he also has to hold a stake in the pledgeable portion of the firm, which he can give up if he claims a high shock.

Let us compare cases I and II. Using the budget and incentive constraints we can calculate the maximum investment scale I for the two cases. It is easy to see that $I_I > I^* > I_{II}$ where I^* is the second-best solution without diversion. Plugging these investment levels into the objective function the maximum objective value in Case I is

$$\text{Max}_I = \frac{A(\rho_1 - \rho_0)(f_L + f_H\delta)}{1 + f_H(\rho_H - \rho_0)\delta - f_L(\rho_0 - \rho_L)}. \tag{2.37}$$

In Case II the maximum objective value is

$$\text{Max}_{\text{II}} = \frac{A[(\rho_1 - \rho_0) + f_L \xi(\rho_H - \rho_L)]}{1 - f_L(\rho_0 - \rho_L) + f_L \xi(\rho_H - \rho_L) + f_H(\rho_H - \rho_0)}. \tag{2.38}$$

At this point we can explain why Case I always dominates Case III. The value function Max_{I} is the same as the value function without diversion, except that δ corresponds to a smaller scale in the H-state. Assumption (2.10), which assures that it is optimal to continue in both states in the model without diversion, implies that Max_{I} is increasing in δ and hence that we should raise i_H as high as constraint (2.29) permits.

The comparison between Case I and Case II is ambiguous. For $\xi = 0$, there is no temptation to divert funds and therefore no need to pay a bribe or distort the reinvestment scale. In this case both solutions coincide with the solution without diversion. For any value $0 < \xi \leq 1$ simulations show that either Case I or Case II can be optimal depending on the choice of the other parameters. For instance, when the probability of a high shock is large (f_H is large) it is optimal to use a bribe (Case II) rather than scale down the operations (Case I), while the reverse is true for small values of f_H. This is intuitive, because the expected cost of reducing the scale of operations is higher when the probability of the H-state is higher, while the expected cost of forcing the entrepreneur to hold a piece of the pledgeable income (i.e. have him pay for the continuation ex post) is lower. Both effects work in the direction of making a bribe more desirable. In general, however, the difference $\text{Max}_{\text{I}} - \text{Max}_{\text{II}}$ may be non-monotone as parameters are changed.

It is of interest to consider $\xi = 1$ when Case II is optimal (which happens for small enough f_L). We have

$$t_{2L} = \xi(\rho_H - \rho_L) I = (\rho_H - \rho_L) I. \tag{2.39}$$

Substituting this expression into the budget constraint (2.28) we see that the investors payoff in the low state will now be

$$\rho_0 i_L - t_{1L} - t_{2L} = (\rho_0 - \rho_H) I < 0.$$

Consequently, investors end up paying the same amount $(\rho_H - \rho_0)\,I$ in both the high and the low state.

There is no scope for insurance at all when $\xi = 1$. The firm is merely using investors to transfer some of its initial funds A to date 1 as if they were investing in a bond or some other storage technology. In effect, the best the firm can do is self-insure. We will study self-insurance in more detail in chapters 7 and 8.

To sum up, whether diversion is avoided by asking the entrepreneur to hold more than the minimum stake at the beginning (which he forgoes if he claims that the high shock has occurred) or by a scaling down of investment in response to a reported high shock, the solution is less efficient than when the shock is contractible. Sometimes the option to divert destroys all outside insurance opportunities and the firm is left to take care of itself using market instruments that are not contingent on idiosyncratic shocks.

Appendix 2.2 Relationship with consumer liquidity demand

(a) The Diamond-Dybvig model

To focus on corporate liquidity demand, our analysis has assumed that consumers are indifferent about the date of consumption (see, however, section 5.3.1 below). In reality, of course, consumers hoard substantial amounts of liquid assets in order to insure themselves against liquidity shocks. They are willing to sacrifice returns to ensure that they will have enough money for various expenditures such as buying a house or a car when the opportunity arises, to send their children to college, or to protect themselves against illness or unemployment. Thus, consumers compete with corporations for the available stock of liquidity.

Consumer liquidity demand has been the focus of a large and interesting literature, starting with the seminal papers of Bryant (1980) and Diamond and Dybvig (1983). The purpose of this section is to compare the implications of the Diamond-Dybvig approach to consumer demand for liquidity (CDL) with our approach to producer demand for liquidity (PDL). To this end, we will first go over the Diamond-Dybvig model with our notation.

Timing. Like our model, the Diamond-Dybvig model, has three periods, $t = 0, 1, 2$.

Consumer preferences. Consumers are ex-ante identical. For notational simplicity, let us assume that they have no demand for consumption at date 0, and therefore invest their entire date-0 endowment a (per consumer). They receive no additional endowments at dates 1 and 2. At date-1 consumers are revealed to be one of two types. A fraction $1-\alpha$ are *impatient*: they only care about date-1 consumption. The remaining fraction α of consumers are *patient*: they are indifferent about date-1 and date-2 consumption. The consumer's state-contingent preferences over consumption c_1 and c_2 is

$$\begin{cases} u(c_1) & \text{if impatient (probability } 1-\alpha) \\ u(c_1+c_2) & \text{if patient (probability } \alpha), \end{cases} \tag{2.40}$$

where the function u is increasing and *strictly* concave, and $u'(0) = \infty$. Consumers do not know at date 0 whether they will be impatient ("face a liquidity shock") or not. In the simplest version of this model (covered here), there is no aggregate uncertainty and exactly a fraction $(1 - \alpha)$ of consumers will want to consume at date 1.[15] In the CDL model risk aversion creates a consumer demand for liquidity. In the PDL model, as we discussed, future credit rationing generates a form of risk aversion that creates demand for liquidity by firms.

Technology. At date 0, consumers allocate their endowment a between (liquid) short-term projects and (illiquid) long-term projects. Investment in liquidity carries an opportunity cost, since long-term projects have a higher yield: Short-term projects yield 1 at date 1 (date 2) per unit of date-0 (date-1) investment, while long-term projects yield nothing at date 1 and r_2 at date 2 per unit of date-0 investment, where

$$r_2 > 1. \tag{2.41}$$

Let ℓ and i denote a consumer's investments in long- and short-term projects, respectively. The consumer budget constraint is

$$\ell + i = a. \tag{2.42}$$

There are two ways for the consumer to insure against liquidity risk, self-insurance and risk pooling:

- In *autarky*, each consumer is on her own. Self-insurance solves the problem:

$$\max_{\{\ell,i|\ell+i=a\}} \left\{(1-\alpha)u(\ell) + \alpha u(\ell + r_2 i)\right\}. \tag{2.43}$$

[15] As in the case of PDL, aggregate uncertainty raises the issue of how much of it can be insured by foreigners (ideally, all of it for a small economy). On this, see Allen and Gale (2000) and Castiglionesi et.al. (2010). In the absence of possibilities for international insurance, impatient and patient consumers must share the aggregate risk, and thus (safe) deposit contracts are not optimal; see Hellwig (1994). In chapter 6 we will discuss international insurance.

The optimal solution is

$$\begin{aligned} \ell &= a \quad \text{if} \quad 1-\alpha \geq \alpha(r_2 - 1), \\ (1-\alpha)u'(\ell) &= \alpha(r_2-1)u'\left(\ell + r_2(a-\ell)\right) \quad \text{otherwise.} \end{aligned} \tag{2.44}$$

• In *pooling*, the consumer's use of liquidity is coordinated through some sort of intermediary, for instance a bank. Consumers are better off with pooling. To see why, note that a consumer who turns out to be patient receives income ℓ from the low-yielding, short-term investment, which is wasteful as a patient consumer would rather enjoy the returns from the higher-yielding long-term investment. In the autarky case patient (impatient) consumers "overconsume" ("underconsume") liquidity. In contrast, because there is no aggregate risk, it is optimal in a pooling solution to let the impatient consumers consume all the proceeds of the short-term assets and the patient consumers consume all the proceeds from the long-term assets. The optimal pooling solution maximizes the representative consumer's utility

$$(1-\alpha)u(c_1) + \alpha u(c_2), \tag{2.45}$$

subject to the constraints

$$\begin{aligned} (1-\alpha)c_1 &= \ell, \\ \alpha c_2 &= r_2 i, \\ \ell + i &= a. \end{aligned} \tag{2.46}$$

The optimal insurance-consumption policy is therefore

$$u'(c_1) = r_2 u'(c_2). \tag{2.47}$$

If the coefficient of relative risk aversion exceeds 1 (i.e., $u'(c)c$ decreasing), condition (2.47) implies that the ratio of the two levels of consumption is smaller than the marginal rate of transformation:

$$\frac{c_2}{c_1} < r_2$$

The optimal policy does not call for equalization of the marginal utilities because of the difference in returns. However it brings the marginal utilities in the two states of nature closer to each other in comparison with the marginal rate of transformation, and thus involves insurance between consumers. The optimal insurance policy can be implemented by a deposit contract in which the consumers can withdraw c_1 at date 1, and if they have not done so, c_2 at date 2.

As in the PDL model to be studied in Chapter 3, the CDL model stresses the superiority of liquidity pooling due to insurance across idiosyncratic consumer risk. Also, long-term investments have higher returns than short-term ones, liquidity ought to be hoarded sparingly and dispatched properly. The absence of coordination/planning generally results in overprovision of liquidity.

There are two well-known issues raised by the Diamond-Dybvig analysis worth highlighting.

The first concerns bank runs and their prevention. When the consumers' type cannot be verified at date 1, patient consumers may pretend that they are impatient and withdraw early. They have no incentive to do so as long as the other patient consumers do not withdraw early, since $c_2 > c_1$. However, there is also a bad equilibrium where patient consumers withdraw their money at date 1. Suppose that a fraction of consumers exceeding $(1 - \alpha)$ come to the liquidity pool to withdraw their money. Provided that liquidating long-term assets at date 1 provides a bit of income, the financial intermediary may liquidate these assets early in order to (partly) serve the withdrawing customers. But this induces other patient consumers to run since their claim to date-2 income is depreciated. Indeed, if all run, then it is in the interest of each to run.[16]

Diamond and Dybvig have offered several ways out of such bad equilibria. The financial intermediary (perhaps with the assent of a prudential supervisor) can suspend con-

[16] Recent theoretical contributions (Goldstein and Pauzner, 2005, Morris and Shin, 1998, and Rochet and Vives, 2004) have looked at asymmetric information environments in which the prediction is unique (either a panic or no panic).

vertibility of demand deposits when the fraction of withdrawers reaches $(1-\alpha)$. Knowing that their long-term claim will not be jeopardized by early liquidation, patient consumers are better off waiting until date 2. Similarly, the presence of a Central Bank acting as a lender of last resort or international markets extending sufficient credit lines to intermediaries, can destroy the incentive for a run (we will have more to say on the limits on international credit in chapter 6).

The second well-known point is that trading in financial markets can have an adverse impact on efficient liquidity provision. Jacklin (1987) showed that it is necessary to prevent consumer trading at date 1 in order for financial intermediaries to be able to provide efficient insurance. To see this, suppose that an entrepreneur uses his date-0 endowment a to set up a mutual fund that invests only in long-term assets at date 0. The entrepreneur does not issue shares in the mutual fund until date 1. Patient consumers are better off withdrawing their deposits c_1 and reinvesting the money in the mutual fund as long as the price of a share in the mutual fund does not exceed $(c_1/c_2)r_2$. Let the entrepreneur keep the shares of the mutual fund for himself if he turns out to be patient and sell them at price c_1r_2/c_2 if he turns out to be impatient. That way, the entrepreneur secures $r_2a > c_2$ when patient and $(c_1r_2/c_2)a > c_1$ when impatient.

In summary, consumer insurance against liquidity shocks involves a cross-subsidy that can be arbitraged by financial markets. The arbitrageurs free-ride on the liquidity (the investments in short-term assets) provided by the financial intermediaries. Trading must be prevented to maintain the optimal subsidy. More generally, Diamond (1997) shows that, when some consumers do not have access to financial markets, there is still scope for insurance, but the social cost of financial markets increases as the number of consumers who have access to them increases.

Farhi *et al.* (2009), considering general preferences $u(c_1, c_2, \theta)$ where θ is a one-dimensional preference parameter learned by the consumer at date 1, provide a general treatment of the Jacklin critique. They show that the inefficiency created by self-provision of liquidity paired with trading at date 1 can be entirely attributed to a failure to control the interest rate between dates 1 and 2; indeed the latter is equal to the technologically driven interest rate and in general does not coincide with the optimal one, which involves cross-subsidies between patient and impatient consumers.[17]

More formally, let p denote the date-1 price of one unit of income at date 2. In equilibrium $p < 1$. The representative consumer solves:

$$\max_{\{i,\ell\}} \left\{ (1-\alpha)u(\ell + p(r_2 i)) + \alpha u(r_2 i + \frac{\ell}{p}) \right\}$$

subject to

$$\ell + i \leq a.$$

The market clearing condition in the date-1 asset market is:

$$(1-\alpha)p(r_2 i) = \alpha\ell.$$

The first-order conditions yield:

$$(1-\alpha)u'(c_1) + \frac{\alpha}{p}u'(c_2) = (1-\alpha)pr_2u'(c_1) + \alpha r_2 u'(c_2).$$

This equation is satisfied for the Jacklin price:

$$p = \frac{1}{r_2}.$$

The term structure is driven by technology alone.[18]

Farhi *et al* demonstrate that the state can implement the efficient allocation by imposing on all financial intermediaries a liquidity requirement: a liquidity floor if the optimal

[17] Their paper contains a useful discussion, for the separable case, of the difference between liquidity shocks à la Diamond-Dybvig ($\theta u(c_1) + u(c_2)$) and discount shocks of the type ($u(c_1) + \theta u(c_2)$).

[18] If the utility of impatient consumers were:

$$u(c_1 + \underline{\rho} c_2) \text{ with } \frac{1}{r_2} < \underline{\rho} < 1$$

instead of $u(c_1)$, however, the equilibrium allocation would involve an inefficient reallocation of liquidity ex post. We will discuss such inefficient reallocations in more detail in chapters 7 and 8.

More generally, the analysis of self-provision of liquidity in these chapters will provide the PDL analog of Jacklin's CDL analysis.

cross-subsidy runs from patient to impatient consumers (the case of a coefficient of relative risk aversion exceeding 1) and a liquidity ceiling if the cross-subsidy runs the other way.

(b) Reinterpreting firms as Diamond-Dybvig consumers

We could introduce consumer liquidity demand into our model simply by treating consumers as firms that are net lenders ($I > A$) with no pledgeable income ($\rho_0 = 0$). Here, we instead compare our approach to the extensive literature on consumer liquidity demand by making our firms similar to Diamond-Dybvig consumers.

To this end, consider an economy with ex ante identical firms of the following sort. The representative firm at date 0 has assets A and invests a variable amount $I < A$ in an illiquid project. Note that the initial investment is less than the firm's endowment in contrast to our earlier assumption. This assumption is essential since it makes firms save just as consumers do in the CDL model.

The representative firm faces a shock ρ per unit of investment

- either at date 1, with probability $1 - \alpha$, drawn from distribution $F^{ST}(\rho)$,
- or at date 2, with probability α, drawn from distribution $F^{LT}(\rho)$.

Provided that it meets the liquidity shock, the firm produces $\rho_1 I$ at date 2, all non-pledgeable ($\rho_0 = 0$). To draw the parallel with the Diamond-Dybvig analysis, we assume that there is no aggregate uncertainty; and we posit the existence of technologies that will help firms face liquidity needs. At date 0 they can invest both in short-term projects that yield one per unit of investment at date 1 and in long-term projects that yield nothing at date 1 and r_2 at date 2 per unit of investment, where

$$r_2 > 1.$$

As before, we assume that investors are risk neutral and willing to invest at a zero rate of return, and that state-contingent policies can be used (there is monitoring of the use of liquidity). The representative firm maximizes the date-2 expected proceed subject to the date-0 constraint on the allocation of savings. Letting $\overline{\rho}^{ST}$ and $\overline{\rho}^{LT}$ denote the cut-off levels, that is, the maximum shocks that the firm will meet, we solve the second-best program:

$$\max_{\{I,\ \overline{\rho}^{ST}, \overline{\rho}^{LT}\}} \left\{[(1-\alpha)F^{ST}(\overline{\rho}^{ST}) + \alpha F^{LT}(\overline{\rho}^{LT})]\rho_1 I\right\}$$

subject to the budget constraint

$$I + \left[(1-\alpha)\int_0^{\overline{\rho}^{ST}} \rho f^{ST}(\rho)d\rho + \frac{\alpha}{r_2}\int_0^{\overline{\rho}^{LT}} \rho f^{LT}(\rho)d\rho\right] I \leq A. \tag{2.48}$$

Paralleling our earlier analyses, the solution to this program minimizes the expected cost of bringing one unit of investment to completion:

$$\min_{\{\overline{\rho}^{ST}, \overline{\rho}^{LT}\}} \left\{ c = \frac{1 + (1-\alpha)\int_0^{\overline{\rho}^{ST}} \rho f^{ST}(\rho)d\rho + \frac{\alpha}{r_2}\int_0^{\overline{\rho}^{LT}} \rho f^{LT}(\rho)d\rho}{(1-\alpha)F^{ST}(\overline{\rho}^{ST}) + \alpha F^{LT}(\overline{\rho}^{LT})} \right\}. \tag{2.49}$$

The optimal cut-offs are therefore determined by

$$\overline{\rho}^{ST} = \frac{\overline{\rho}^{LT}}{r_2} = c. \tag{2.50}$$

Let us now show how this analysis connects to that of Diamond and Dybvig. Because their results hinge on the stationarity of preferences (the utility function u is the same for impatient and patient consumers), a proper comparison requires that in the PDL model the distribution of shocks be identical:

$$f^{ST}(\rho) = f^{LT}(\rho) \equiv f(\rho) \qquad \text{for all} \quad \rho.$$

For the optimal investment level I, let a denote net savings, and c_1 and c_2 the expected date-1 and date-2 consumptions of the good:

$$\begin{cases} a & \equiv & A - I \\ c_1 & \equiv & [\int_0^{\overline{\rho}^{ST}} \rho f(\rho)d\rho]I \\ c_2 & \equiv & [\int_0^{\overline{\rho}^{LT}} \rho f(\rho)d\rho]I \end{cases}$$

With this notation, the representative firm's budget constraint (2.48) is

$$(1-\alpha)c_1 + \frac{\alpha c_2}{r_2} \leq a$$

as in the CDL model. The entrepreneur's payoff is

$$[(1-\alpha)F(\overline{\rho}^{ST}) + \alpha F(\overline{\rho}^{LT})]\rho_1 I.$$

This suggests that we define a pseudo-utility function $u(\cdot)$ implicitly by

$$u(c(\overline{\rho})) \equiv F(\overline{\rho})\rho_1 I \tag{2.51}$$

where

$$c(\overline{\rho}) = [\int_0^{\overline{\rho}} \rho f(\rho) d\rho] I \tag{2.52}$$

Recalling that nothing is pledgeable, note that $u(c_t)$ is the expected surplus for expected consumption c_t when the liquidity need arises at date t. Differentiating (2.51) with respect to $\overline{\rho}$ we get

$$u'(c(\overline{\rho})) = \frac{\rho_1}{\overline{\rho}} > 0 \quad \text{and} \quad u''(c(\overline{\rho})) = -\frac{\rho_1}{\overline{\rho}^3 f(\overline{\rho}) I} < 0.$$

The entrepreneur exhibits risk aversion over consumption (expected liquidity demand) despite being risk-neutral.

Next, we investigate the existence of cross-subsidies and whether the analog of the Jacklin critique holds in the PDL model. Cross-subsidies exist provided that

$$r_2 c_1 > c_2$$

or, from (2.52),

$$r_2[\int_0^{\overline{\rho}^{ST}} \rho f(\rho) d\rho] > \int_0^{r_2 \overline{\rho}^{ST}} \rho f(\rho) d\rho.$$

Recall that cross-subsidies exist in the CDL model if the utility function exhibits a degree of relative risk aversion in excess of 1:

$$-\frac{u''(c)c}{u'(c)} > 1.$$

Using (2.51) and (2.52), the analog of this condition for the PDL model is:[19]

$$\frac{\int_0^{\overline{\rho}} \rho f(\rho) d\rho}{\overline{\rho}^2 f(\overline{\rho})} > 1 \tag{2.53}$$

Fixing $\overline{\rho}$, consider the function

$$H(r_2) \equiv r_2 \int_0^{\overline{\rho}} \rho f(\rho) d\rho - \int_0^{r_2 \overline{\rho}} \rho f(\rho) d\rho.$$

Clearly $H(1) = 0$ and

$$H'(r_2) = \int_0^{\overline{\rho}} \rho f(\rho) d\rho - \overline{\rho}^2 r_2 f(r_2 \overline{\rho}).$$

To show that $H(r_2) > 0$ for $r_2 > 1$, it suffices to show that $H'(r_2) > 0$ whenever $H(r_2) = 0$, or

$$\int_0^{r_2 \overline{\rho}} \rho f(\rho) d\rho > r_2^2 \overline{\rho}^2 f(r_2 \overline{\rho}),$$

which is nothing but (2.53). There are cross-subsidies in the PDL model exactly under the same conditions as in the CDL model.

This completes the demonstration that when the firms are net lenders, the PDL model is isomorphic to the CDL model.

Does the Jacklin critique apply to the PDL model? Note that the notion of cross-subsidy studied above refers to an *average* cross-subsidy; under (2.53) this cross-subsidy takes the following form:

$$\int_0^{\overline{\rho}^{ST}} \rho f(\rho) d\rho > \frac{\int_0^{\overline{\rho}^{LT}} \rho f(\rho) d\rho}{r_2}.$$

From (2.50) by contrast, there is no subsidy at the margin:

$$\overline{\rho}^{ST} = \frac{\overline{\rho}^{LT}}{r_2}.$$

Suppose that the firm contracts with a bank for a credit line with the option of withdrawing up to $\overline{\rho}^{ST} I$ at date 1 or up to $\overline{\rho}^{LT} I$ at date 2. A firm that learns at date 1 that it will need liquidity only at date 2 cannot obtain more liquidity by withdrawing at date 1 and investing in a mutual fund at rate of return r_2.

[19] Condition (2.53) is rather strong: in particular it is not satisfied by uniform or increasing densities.

Thus, if the firm has no alternative investment opportunity (of the type considered in Appendix 2.1), the Jacklin's critique does not apply.

Of course, the Jacklin critique of the CDL model rests on the idea that the consumer's balance sheet is not monitored by his liquidity provider. If, in the PDL model, the entrepreneur can claim that the firm has been hit by the highest permissible shock, draw down the full credit line, invest the balance in the market at rate r_2 and consume the proceeds (i.e., not return the money to investors) at date 2, then the credit-line scheme analyzed here is not immune to the Jacklin critique.

Firms' accounts however are probably more easily monitored than those of consumers. The extreme form of diversion considered in the previous paragraph is not the ordinary pattern of corporate behavior. But firms certainly have some leeway to make self-serving use of extra cash. It would be interesting in this context to consider imperfect opportunities for diversion, as we did in section 2.3.

Part II

COMPLETE MARKETS

Chapter 3

Aggregate liquidity shortages and liquidity premia

3.1 Introduction

Much of the agency based models of corporate finance are occupied with the problem of funding investments because a firm may have difficulties assuring its investors that they will get their money back. The model in Chapter 2 is just one example illustrating the constraints and distortions stemming from this credibility problem. Much less attention has been paid to the converse credibility problem: making sure that the suppliers of liquidity – the lenders, insurers and other investors who explicitly or implicitly commit to fund a firm in the future – will actually be able to deliver on that promise.

This issue presents itself starkly in the model in Chapter 2, where investors are being asked to commit to making date-1 investments that have a negative net present value (for them). We suggested instruments that could be used to implement the optimal second-best policy, including credit-lines, bonds and other financial vehicles such as credit default swaps (CDS). But sometimes banks may not be able to honor credit lines, bonds may default and counter-party risks in all kinds of transactions may materialize. The sub-prime crisis has revealed the vulnerability of modern financial markets that rely on extremely complicated chains of derivative instruments where the links to the underlying real assets are much of the time impossible to trace. Investment banks have failed, repo markets have frozen and monoline insurers that were supposed to protect against subprime portfolio losses have defaulted. By now it is evident that we do not understand sufficiently well the determinants of the supply of liquidity or collateral.

In this chapter we will look at the adequacy and cost of liquidity supply from a particular perspective. Our starting point is that all financial commitments ultimately have to be backed up by claims on real assets that produce consumption or services of value to individuals and firms. Intermediaries and investors have to have enough capital, that is, own (directly or indirectly) claims on real assets that can secure pledges to fund firms in the future. We ask: when is this base on which all financial claims have to rest, sufficient to support the second-best policies that Chapter 2 laid out?

As we will see in this chapter, the same factors that constrain firms from obtaining funding for all positive NPV projects, also constrain the supply of liquidity. Limited pledgeability is a problem for those demanding liquidity as well as those supplying liquidity – it puts a double wedge between investors and entrepreneurs. The smaller the share of pledgeable income, the lesser the base on which financial claims can be written. The much discussed "global savings glut" could be seen as a manifestation of these twin problems. Poorly developed financial markets and a booming economy in countries like China led to high savings rates and a search for safe financial assets.

The analysis of liquidity supply can be broken down into two steps. In the first step, one looks at the maximum pledgeable amount of income that is available to back up promises of future liquidity. We call this the economy's *aggregate liquidity.* Aggregate liquidity depends on the state of nature as well as on the investment plans of the firms, so there is a feedback loop running from limited demand of liquidity to limited supply of liquidity and back.

In the second step one studies whether the corporate sector can coordinate the use of available aggregate liquidity sufficiently well. Financial markets including the plethora of intermediaries do much of the coordination, but we will not get into the institutional details of the process. Our notion of aggregate liquidity is premised on perfect coordination in the sense that arbitrary state-contingent contingent claims on pledgeable income can be written. Studying aggregate liquidity in this manner is very abstract, of course, but as in general equilibrium theory, it is not without interest to understand such an idealized world. Among other things, it is useful because it suggests factors that affect the supply of liquidity and it provides a lower bound on the severity of liquidity supply problems.

We make a distinction between *inside (aggregate) liquidity* and *outside (aggregate) liquidity*, depending on the source of the pledgeable income.[1] When the pledgeable income is generated by the corporate sector the claims on it constitute inside liquidity. All claims on goods and services outside the corporate sector constitute outside liquidity.

There are three notable sources of outside liquidity.[2]

- First, consumers own assets unrelated to the corporate sector. Home ownership is by far the biggest component of household wealth (in the United States, 32.3 per cent in 2000 and 41.2 per cent in 2002; see Gottschalk, 2008) and housing has for this reason always played a special role in the supply of liquidity. The volume of mortgages, home equity loans and mortgage backed securities grew dramatically in

[1] Our notion of inside and outside liquidity closely parallels that of Gurley and Shaw (1960)'s notion of inside and outside money, as we earlier pointed out. It is also closely related to that adopted by modern economics: Blanchard and Fischer (1989, chapter 4, section 6) state:

"*Any money that is on net an asset of the private economy is an outside money. Under the gold standard, gold coins were outside money; in modern fiat money systems currency and bank reserves, high-powered money and the money base, constitute outside money. However, most money in modern economies is inside money, which is simultaneously an asset and a liability of the private sector.*"

Lagos (2006) defines inside and outside money in a similar manner:

"*Outside money is money that is either of a fiat nature (unbacked) or backed by some asset that is not in zero net supply within the private sector of the economy. Thus, outside money is a net asset for the private sector. ...Inside money is an asset representing, or backed by, any form of private credit that circulates as a medium of exchange. Since it is one private agent's liability and at the same time some other agent's asset, inside money is in zero net supply within the private sector.*"

Our usage of inside and outside liquidity is consistent with these definitions; our emphasis on liquidity will lead us to take a broad view of outside stores of values that do not originate in the corporate sector and that can be used to face liquidity shortfalls: see chapters 5 and 6.

[2] We will in this book ignore the possibility that asset bubbles, while they last, augment the stock of liquidity in the economy and thereby boost investment. See Farhi and Tirole (2009a) for an analysis of the extent to which bubbles can add to the supply of liquidity.

the period 2000-2008 and, as so often before, real estate was at the center of the subprime crisis. When a home owner takes a loan with her house as collateral it tends to increase the supply of liquidity by creating an asset in which others can invest. How much liquidity (and in which states) is de facto provided depends on the structure of financial markets and its ability to utilize the mortgage asset efficiently. It also depends on what the homeowner does with the funds raised. Whether there is a net increase in the supply of outside liquidity depends on several factors that we will touch on in this chapter.

Of course, off-setting the consumer supply of liquidity is the fact that consumers also have sizable demands for liquidity. For the most part we will ignore the consumer's role both as a supplier and a demander of liquidity even if it is very important. The original analysis of liquidity by Diamond and Dybvig (1983) reviewed in Appendix 2.2 did just the opposite: it focused solely on consumer demand for liquidity.

- Second, the government can issue claims backed by its exclusive right to tax consumers and producers. We will take a closer look at government bonds and government supplied liquidity in Chapter 5, where the government is seen as a financial intermediary between consumers who cannot pledge their human capital as collateral and firms that would like consumers to provide insurance against certain kinds of liquidity shocks.

- Third, international financial markets can offer liquidity in the form of claims on international goods and services. In principle, international markets could supply all the insurance that a small country needs. However, access to international insurance is limited by the amount of tradable goods that a country can offer to international investors (Caballero and Krishnamurthy, 2001). Were this not the case, the question of aggregate liquidity shortages would be rather academic except for the largest countries. We will discuss international liquidity in Chapters 6 and 8.

Let us give a quick overview of this chapter.

As we already noted, a key observation is that the very problem that leads to a *demand* for liquidity, namely a wedge between pledgeable and non-pledgeable income, also limits the *supply* of inside liquidity. Insurance within the corporate sector relies on claims issued by the corporate sector. If firms had no pledgeable income, there would be no corporate claims and therefore no inside liquidity to back up promises of funding or insurance. At the other extreme, if all corporate income were pledgeable (as in the Arrow-Debreu model) there would be no need for insurance, since all continuation decisions would be self-financing and therefore efficient.

We start by asking when the corporate sector will provide enough liquidity on its own, that is, when inside liquidity suffices to support second-best production plans. The demand and supply of inside liquidity are intertwined both through the amount of pledgeable income and the stochastic dependencies between firms. We show that if liquidity shocks are idiosyncratic and the corporate sector is a net borrower ($I - A > 0$), there will always be enough aggregate inside liquidity to meet corporate demand. The more important point, however, is that liquidity shortages can occur unless liquidity is coordinated in the right way, for instance by having intermediaries that offer credit lines to individual firms or by firms holding shares in the market index and making commitments to pay back excess liquidity through dividends or other means.

At the other extreme, when all firms are hit by the same aggregate liquidity shock, inside liquidity will always be in short supply. With a shortage of aggregate liquidity some source of outside liquidity is necessary. Competition among firms will bid up the price of this liquidity, resulting in a liquidity premium. We will provide a simple analysis of how the liquidity premium is determined and how it affects the investment decisions of firms when the outside liquidity is fixed (e.g. government bonds). The supply-demand analysis is consistent with the empirical evidence on the behavior of the liquidity premium of government bonds presented by Krishnamurthy and Vissing-Jorgensen (2006), which we discuss in 3.3.3.[3]

Section 3.4 analyzes a model featuring both consumer and producer demand for liquidity by merging the Diamond-Dybvig model of consumer liquidity with our model of producer liquidity. We end by discussing some ways in which the private sector, including consumers, can expand on the supply of liquidity (section 3.5).

3.2 Inside liquidity and aggregate shortages

Consider a three-period economy with a single good ("corn") and a unit mass of identical firms, each with a technology of the sort described in Chapter 2. Consumers are risk neutral and value consumption according to

$$c_0 + c_1 + c_2. \tag{3.1}$$

Consumers have large endowments of corn in each period, but have no way of storing corn from one period to the next. Equivalently, they have labor endowments that can be used to produce corn, which must be eaten in the period it is produced. There is no outside liquidity in the economy for now; all liquidity is embedded in the returns of the corporate sector. In particular, consumers cannot promise to fund future investments without backing up their promises with claims on pledgeable corporate returns; the consumers' future endowments are not pledgeable.

3.2.1 Net lending: the case of certainty

We begin with a very simple example that illustrates why there may be a shortage of liquidity. There is a continuum of ex ante identical firms of mass 1, endowed with technologies of the type described in Chapter 2 and run by entrepreneurs each with a date-0 endowment A. The representative firm's initial investment I is variable and we assume

[3]Chapter 4 on the Liquidity Asset Pricing Model (LAPM) offers a more general analysis of liquidity premia and their implications for asset pricing.

that it must be augmented with a *deterministic* date-1 continuation investment $\rho i(\rho)$, where $i(\rho) \leq I$ and $\rho > \rho_0$. (This is equivalent to assuming that in the model with two liquidity shocks, analyzed in section 2.2, we have $f_H = 1$ and $\rho_H = \rho$.) Furthermore, suppose $1 + \rho < \rho_1$, implying that the investment would always be worth undertaking from a net-present-value point of view. If there were no liquidity problems, an entrepreneur with funds A would invest as follows at date 0. He would choose I as the initial scale of the project and invest $(\rho - \rho_0)I > 0$ into a liquid asset or a credit line, where I is chosen to exhaust the budget: $(1 + \rho - \rho_0)I = A$.[4] With these initial investments, the entrepreneur will be able to cover exactly the deterministic liquidity shock ρ at date 1. He can raise $\rho_0 I$ by issuing shares against his pledgeable date-2 income and add to it his investment in liquidity $(\rho - \rho_0)I$.

This investment plan presumes that there is a liquid asset, or a credit line backed up by a liquid asset, that allows the entrepreneur to save $(\rho - \rho_0)I$ from date 0 to date 1. However, in the economy just described, the only available assets are claims on the continuation value of the very firms looking to save. Suppose, hypothetically, that all firms were able to meet the date-1 liquidity need ρI and therefore to continue at full scale. In that case, the date-1 continuation value of the corporate sector would be $\rho_0 I$. But this is less than the liquidity needed, ρI. Since the *net* continuation value of the corporate sector, $(\rho_0 - \rho)I$, would be negative, claims on the corporate sector written at date 0, would be liabilities rather than assets. Evidently, the corporate sector can neither act as a store of value nor provide collateral for future funding.

The problem here is that the corporate sector would like to be a *net lender*, but this is not possible because there are no external instruments (no outside liquidity) for transferring wealth from date 0 to date 1. The same problem was originally brought

[4] Note that the constant returns to scale technology will allow the firm to invest up to

$$I = \frac{A}{1 + \rho - \rho_0} < A.$$

up by Woodford (1990) in his analysis of government debt as net wealth. Woodford studied an infinite-horizon model, described in Figure 3.1, where two parties have the opportunity to invest in alternate periods and the investments pay off one period later in the form of a non-storable consumption good. None of the output is pledgeable, and so a party's investment is bounded above by his available cash at the investment date. In the absence of outside stores of value, parties do not invest as their investment needs and cash flow receipts are completely asynchronized. The main point of Woodford's paper is that government bonds (the "liquid asset" in his model) can be valuable as a way of transferring part of the returns from yesterday's investment to tomorrow when they can be reinvested again. This raises the overall welfare in the economy.

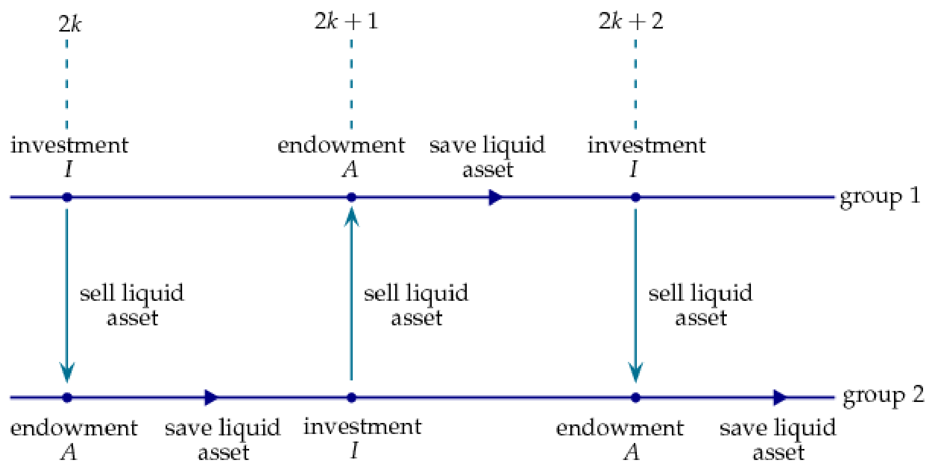


Figure 3.1: Net lending (Woodford's version)

Apart from a finite rather than infinite horizon, one main difference between Woodford's model and our example is that we allow part of the project's income to be pledgeable ($\rho_0 > 0$). If enough of the income were pledgeable in Woodford's model, there would be no problem transferring wealth from one period to the next.[5] Investors whose firms pay off today can store the appropriate fraction of their income for reinvestment tomorrow

[5] In the constant-returns-to-scale model, this requires capping the investment, otherwise investment would be infinite.

by buying a financial stake in firms that invest today and pay off tomorrow; an equity market would be a perfect substitute for government bonds (this conclusion extends to uncertain payoffs.) However, if the pledgeable income drops below the cost of investment ($\rho_0 < 1 < \rho_1$) there will again be a role for outside liquidity in Woodford's model: an equity market will not work if the pledgeable income is too small.

The general message here is that the problem of partial pledgeability, which limits the amount of funds that an entrepreneur can attract, also makes it harder for investors to supply liquidity. The specific observation that the corporate sector cannot provide sufficient aggregate liquidity whenever it is a *net lender* is very general. Even if the liquidity shocks were uncertain as in section 2.2, as long as $I - A < 0$ the budget constraint (2.4) implies that the expected date-1 net return of the corporate sector must be negative, and therefore that the corporate sector does not have sufficient assets to support the proposed plan.

3.2.2 Net borrowing: aggregate shocks

Assume now that the corporate sector is a *net borrower*, $I - A > 0$, where A is the aggregate endowment of the entrepreneurs.[6] Also, assume that there are only two liquidity shocks satisfying $\rho_L < \rho_0 < \rho_H$ with the additional property that the date-1 continuation investments should take place both when the liquidity shock is high and when it is low (condition (2.10) is satisfied). Because $I - A > 0$, the *expected* net return at date 1 must be strictly positive or else the budget constraint (2.4) would be violated. However, it is quite possible that the *realized* net return is negative in some states and therefore that there are states with insufficient inside liquidity. Whether this will be the case depends critically on the correlation of liquidity shocks across firms.

Consider first the extreme case where all firms are technologically identical and hit

[6] The corporate sector is typically a net borrower, see Mayer (1988). Recently, though, the corporate sectors in a number of countries (US, UK and Japan in particular and only a bit less in the euro area) have cut back on total borrowing causing them to run net financial surpluses several years in a row.

by the same shock – either all firms experience a high shock $\rho_H > \rho_0$ or all of them experience a low shock $\rho_L < \rho_0$. The high shock occurs with probability f_H and the low shock with probability f_L. Of course, when the aggregate shock is low all firms can continue, because continuation investments are self-financing in that case. For instance, investors can contractually agree to let a firm in need of liquidity, dilute the initial investors *pro rata* so that the amount $\rho_L I$ can be raised from new investors; alternatively firms may not contract for liquidity, but rather negotiate with investors for fresh funds when the liquidity shock arises as discussed in section 2.2. By contrast, no firm can continue when the high shock hits, because just as in the deterministic case discussed above, every firm now has a negative continuation value. There are no assets to back up investor promises to cover a high liquidity shock. The only option is for all firms to close down in the high-shock state.

3.2.3 Net borrowing: independent shocks

Consider next another extreme case, one where each firm is hit by an idiosyncratic liquidity shock at date 1. With a continuum of ex ante identical firms the law of large numbers allows us to treat aggregate variables as deterministic. Assume tentatively that all firms are able to continue at full scale. The aggregate budget constraint for the corporate sector is then

$$f_L(\rho_0 - \rho_L) I + f_H(\rho_0 - \rho_H) I \geq I - A > 0,$$

implying that

$$f_H(\rho_H - \rho_0)I < f_L(\rho_0 - \rho_L)I. \tag{3.2}$$

The left-hand side represents the aggregate demand for liquidity by "distressed firms" (negative NPV firms), while the right-hand side is the aggregate supply of liquidity by "intact firms" (positive NPV firms). Both terms are deterministic, because shocks are independent. We conclude that with independent shocks and a continuum of firms that are net borrowers at date 0, the productive sector is self-sufficient with regard to liquidity.[7]

There are many institutional arrangements that support the efficient continuation of firms when shocks are independent. For instance, one could set up a financial intermediary that held all the shares of the productive sector and issued claims on this aggregate market portfolio to individual investors. The financial intermediary would provide liquidity to firms on a contingent basis: firms with a high shock would get $\rho_H I$, and firms with a low shock would get $\rho_L I$.

Another way of ensuring that the firms with a liquidity shortage (facing shock ρ_H) benefit from the liquidity provided by the other firms is to have each hold a sufficiently large share of the stock index.[8] At date 0, let each firm buy a fraction β of the stock index. Let the one-hundred-per-cent date-0 value of the stock index be V_β. A firm could borrow $[I - A] + \beta V_\beta$ to finance its date-1 investments. Alternatively, a firm could issue equity and use the proceeds (equal to V_β because of unit mass) to pay both for the net investment $I - A$ and its purchase of a share β in the stock index.

At date 1, the representative firm sells its share in the stock index back to the consumers, using the proceeds to cover its liquidity shock ρI. We need to show that β can be chosen so that the firm can always cover the high shock $\rho = \rho_H$. Any surplus from the sale of the index is distributed as a dividend to the firm's shareholders at date 1. Adding the expected date-2 dividend $\rho_0 I$ to the contingent date-1 dividend provides date-0 investors with a total payout

$$\beta V_\beta - \rho I + \rho_0 I.$$

[7]This conclusion also holds when firms are heterogenous, as long as they are net borrowers at date 0 and face idiosyncratic shocks. However, if there is a finite number rather than a continuum of firms, there will always be a positive probability that the supply of liquidity is insufficient. For instance, with two firms, each facing a high shock with probability 1/2, there is an aggregate liquidity shortage if both experience a high shock simultaneously, which happens with probability 1/4.

[8]In Holmström and Tirole (1998), we wrongly claimed that holding shares in a stock index would not always ensure suffient liquidity for individual firms, because some of it would be wasted. We are very grateful to Ivan Werning for correcting this error.

Averaging across firms we get

$$V_\beta = \beta V_\beta - (f_H \rho_H + f_L \rho_L)I + \rho_0 I = \beta V_\beta + (I - A).$$

The date-0 shares held by investors ($1-\beta$ of the index) are therefore worth

$$(1-\beta)V_\beta = I - A, \tag{3.3}$$

per firm. Their initial investment is covered so they are indeed willing to get on board.

Finally, note that (3.3) implies that V_β goes to infinity as β approaches 1.[9] Consequently, we can always choose β so that the distressed firms can cover their liquidity shock:[10]

$$\beta V_\beta \geq \rho_H I.$$

This reasoning extends to the more general case in which continuation is not second-best for all shocks. For example, in the continuous version of the model, where the optimal cut-off ρ^* exceeds ρ_0, but is strictly less than ρ_1, some shocks will not be met. We encourage the reader to check that the reasoning above applies by having firms hold a share β of the index satisfying

$$\beta V_\beta = \rho^* I$$

which allows them to cover all shocks up to ρ^*. This way, firms with a shortage of liquidity

[9] The smaller the outsiders' share per firm, $1-\beta$, the larger is the one-hundred-per-cent-value of the firm (V_β).

[10] Equivalently, the entrepreneurs can buy a fraction β of the stock market such that they just have enough liquidity when facing a high shock when they have secured the right to dilute initial investors:

$$(\rho_H - \rho_0)I = \beta V_\beta.$$

Then the date-1 stock market value corresponds to the payoff from intact entrepreneurs:

$$V_\beta = f_L\left[(\rho_0 - \rho_L)I + \beta V_\beta\right],$$

or equivalently:

$$\begin{aligned} V_\beta &= f_L\left[(\rho_0 - \rho_L)I + \beta V_\beta\right] + f_H\left[(\rho_0 - \rho_H)I + \beta V_\beta\right] \\ &= I - A + \beta V_\beta. \end{aligned}$$

($\rho \in (\rho_0, \rho^*]$) benefit from the excess liquidity of those who either shut down ($\rho > \rho^*$) or are able to continue without using outside liquidity ($\rho \leq \rho_0$).

There are various ways to provide for liquidity when there is no aggregate risk (shocks are independent). The key is to make sure that there is a mechanism whereby those firms that do not need all their date 1 liquidity (because of a low shock) will transfer their excess liquidity to those firms that do need it (because of a high shock). Intermediaries can handle the job with credit lines and equity markets can do it assuming that firms pay their dividends and sell their shares of the stock index in the right way.[11] By contrast, suppose that firms just borrow from consumers and do not secure a credit line, nor invest in shares of other firms. Then distressed firms (with the high liquidity shock) cannot continue, while the unused liquidity $(\rho_0 - \rho_L)I$ of the intact firms (with a low liquidity shock) is wasted.

The smaller the pledgeable income ρ_0, the less there is inside liquidity and the more important it is to coordinate liquidity efficiently. The reverse may be more common in reality: the same problems that cause pledgeable income to be low (e.g. poor corporate governance and regulatory oversight over financial firms) tend to lead to poor coordination of liquidity as well (due to underdeveloped financial markets).[12]

3.3 Outside liquidity and liquidity premia

3.3.1 Fixed supply of outside liquidity

Let us return to the case of aggregate liquidity shocks discussed in section 3.2.2. All firms are identical and each gets hit by the same liquidity shock, which can be either high

[11] Cross-holdings are common in some countries (e.g. in Europe and Asia) as a way of maintaining control of companies with a smaller number of shares than cash flow rights, a practice that many consider bad corporate governance. Independent companies do not seem to hold shares for liquidity reasons, probably because the stock index is rather volatile, unlike here. Indeed the stock index is likely to be low precisely when firms need cash.

[12] There is a large literature linking financial development and growth; see, e.g., Levine (1997), Pagano (1993) and Rajan and Zingales (1998). Empirical work considers various measures of financial development in an attempt to capture the ease of access to funding liquidity.

($\rho_H > \rho_0$) or low ($\rho_L < \rho_0$). We concluded that all firms could continue in the low-shock state, but no firm could continue in the high-shock state, because there was no inside liquidity in that state nor (by assumption) any outside liquidity.

Suppose now that there is an external, deterministic source of liquidity, an asset or a set of assets that produce L_S units of corn regardless of the state of nature (we let the aggregate payoff be deterministic for expositional purposes.) It will not matter whether the asset produces corn at date 1 or date 2 or some in both, because consumers are indifferent between consumption in the two states. In a more liberal interpretation of "corn", the asset could be anything that produces value for the consumer, such as real estate, land, or a natural resource like oil. As we have mentioned before, the outside liquidity here could also represent a fixed amount of government bonds that are backed up by the government's ability to tax consumers.

With the help of outside liquidity, entrepreneurs can save some of their date-0 endowment to insure, at least partially, against a high liquidity shock at date 1. Let q be the price of a unit of liquidity; that is, the date-0 price of a unit of corn delivered in each of the two states at date 1. If the supply of outside liquidity, L_S, is so large that it can support the second-best plan described in Chapter 2, q will equal 1. The price of liquidity can never go below 1, because consumers (who are assumed to have big aggregate endowments in each period) would then want to postpone all their consumption to date 1. This would drive the price of the liquid asset to its date-1 value, which is 1. By contrast, the date-0 price q can go above 1, because for firms the value of transferring wealth to date 1 may well be worth more than 1. The difference $q - 1 \geq 0$ is the *liquidity premium*. When the liquidity premium is strictly positive ($q > 1$), only firms will buy liquidity at date 0. Consumers do not have a demand for liquidity (but see section 3.4) The spot price of liquidity at date 1 is of course 1.

The budget constraint of a representative firm that buys ℓ units of liquidity at date 0 is

$$I - A + (q-1)\,\ell \le f_L(\rho_0 - \rho_L)\, I + f_H(\rho_0 - \rho_H)\, i_H \tag{3.4}$$

The net cost of liquidity is $(q-1)\,\ell$, since ℓ units of liquidity is worth ℓ at date 1. In the low-shock state, the firm will and can continue at full scale ($i_L = I$) regardless of ℓ, because continuation is self-financing. In the high-shock state, a firm needs outside liquidity to continue.

A firm that buys ℓ units of liquidity can in the H-state continue at a scale $i_H \le I$ that satisfies the liquidity constraint

$$i_H(\rho_H - \rho_0) \le \ell. \tag{3.5}$$

Constraint (3.5) assumes that, when necessary, the firm can raise up to $i_H\rho_0$ at date 1 by diluting its initial stock (issuing claims that are senior to the initial claims; see section 2.3).

Let us determine the firm's demand for liquidity. Given the price of liquidity q, the firm simultaneously chooses its level of liquidity ℓ, the initial investment I, and the continuation scale $i_H \le I$ subject to the budget constraint (3.4) and the liquidity constraint (3.5). Suppose the firm decides not to continue in the high state ($i_H = 0$). Since it does not need any external liquidity in that case the maximum initial investment level is

$$I = \frac{A}{1 - f_L(\rho_0 - \rho_L)}. \tag{3.6}$$

This plan gives the entrepreneur a net payoff

$$U_0 = (\rho_1 - \rho_0)\, f_L I - A = \frac{[f_L(\rho_1 - \rho_L) - 1]}{1 - f_L(\rho_0 - \rho_L)} A. \tag{3.7}$$

Suppose instead that the firm in the high-shock state decides to continue at full scale $i_H = I$. To do so, it has to buy outside liquidity $\ell = (\rho_H - \rho_0)\, I$. Substituting ℓ into the budget constraint (3.4) gives the maximum initial investment

$$I(q) = \frac{A}{1 + (q-1)(\rho_H - \rho_0) - f_L(\rho_0 - \rho_L) - f_H(\rho_0 - \rho_H)}. \tag{3.8}$$

The entrepreneur's net payoff is then

$$U_1(q) = (\rho_1 - \rho_0)\, I(q) - A = \frac{\rho_1 - (1 + (q-1)(\rho_H - \rho_0) + \overline{\rho})}{(1 + (q-1)(\rho_H - \rho_0) + \overline{\rho}) - \rho_0} A. \tag{3.9}$$

Here $\overline{\rho} = f_L \rho_L + f_H \rho_H$ is the average liquidity shock at date 1.

The firm will continue in the high state as long as $U_1(q) > U_0$. Our earlier assumption $f_L(\rho_H - \rho_L) < 1$ implies that $U_1(1) > U_0$ (see (2.13)). If $q = 1$, it is optimal to continue. Note that $U_1(q)$ is a strictly decreasing function. Let $q_{\max}$ be the value at which the firm is indifferent between continuing and not continuing in the H-state:

$$U_1(q_{\max}) = U_0 \tag{3.10}$$

Using the expressions for U_0 and $U_1(q)$, we can write this equality as

$$\frac{f_L}{f_H}(q_{\max} - 1)(\rho_H - \rho_0) + f_L(\rho_H - \rho_L) = 1 \tag{3.11}$$

We can interpret equation (3.11) in the following way. Define the *effective unit cost of investment* as the expected cost of bringing one unit of investment to completion. This value depends on the continuation policy. For a firm that continues in both states the effective unit cost is

$$c_1(q) \equiv 1 + (q-1)(\rho_H - \rho_0) + \overline{\rho} \tag{3.12}$$

Because the firm always continues, $c_1(q)$ is the sum of the initial investment, the net cost of liquidity per continuation unit in the high state and the expected reinvestment cost at date 1.

For a firm that does not continue in the H-state the effective unit cost is

$$c_0 \equiv \frac{1 + f_L \rho_L}{f_L} \tag{3.13}$$

Since only a fraction f_L of projects are continued, the effective cost in this case is obtained by scaling the expected cost of a unit investment by the fraction f_L.

Expression (3.11) equalizes the effective unit costs of investment under the two policies, that is, it shows that

$$c_1(q_{\max}) = c_0. \tag{3.14}$$

Since all firms are identical, the aggregate demand for liquidity is[13]

$$L_D(q) = \frac{(\rho_H - \rho_0)\, A}{1 + (q-1)(\rho_H - \rho_0) + \overline{\rho} - \rho_0}, \quad \text{for } 1 \leq q < q_{\max}. \tag{3.15}$$

If $q > q_{\max}$, $L_D(q) = 0$. If $q = q_{\max}$, the firms are indifferent among continuation investments $i_H \in [0, I]$ (because of the linear technology). Figure 3.2 graphs the supply and demand of liquidity.

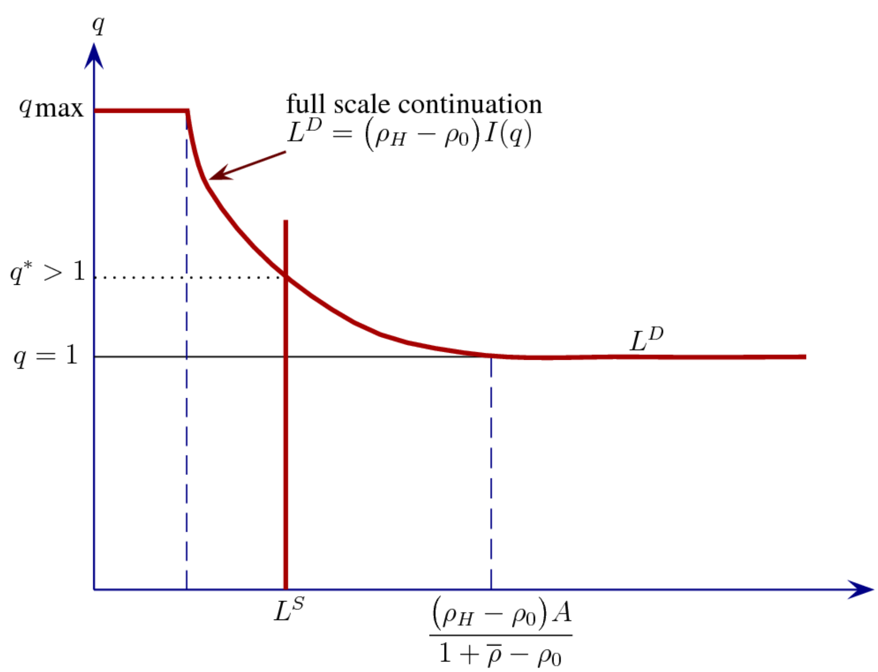


Figure 3.2: equilibrium in the liquidity market

The equilibrium value for liquidity q^* is found by setting demand equal to the inelastic supply

$$L_S = L_D(q^*). \tag{3.16}$$

[13] Note that the individual as well as the aggregate endowment of the entrepreneurs is A since there is a continuum of them with unit mass.

For values L_S such that $1 < q < q_{\max}$, the liquidity premium is

$$q^* - 1 = \frac{A}{L_S} - \frac{1 + \overline{\rho} - \rho_0}{\rho_H - \rho_0}. \tag{3.17}$$

The liquidity premium will be zero if L_S exceeds the maximum demand for liquidity, $A(\rho_H - \rho_0)/(1 + \overline{\rho} - \rho_0)$.

3.3.2 Comparative statics.

Investment scale in response to q.

The firm's initial investment I is non-monotone in the supply L_S. When there is plenty of outside liquidity the price $q = 1$ and the firms continue in both states. As L_S decreases, q increases, causing firms to spend more on buying liquidity and less on the date-0 investment I. At $q = q_{\max}$ firms become indifferent about continuing in the H-state and the supply L_S determines the fraction of firms that continue. The funds no longer used to buy liquidity will instead be used to increase the initial scale of investment, leading to a non-monotone relationship between L_S and the initial investment I. [14]

Entrepreneurial wealth and boom-bust cycles.

The easy money that was available to the US corporate sector in the early 2000s seems to have paved the way for the subsequent liquidity crisis. Our framework suggests a reason why this may have been the case.

In practice, firms with more resources go for an expansion of scale or for more fragile investments. While both can easily be modelled in our framework, consider the case of expanded investment as analyzed in this section. Think of A as the date-0 free (non-collateralized) cash available to firms. From (3.15), we see that an increase in A creates a proportional shift in the demand for liquidity $L_D(q)$ (for $q < q_{\max}$). Figure 3.2 then shows

[14] This non-monotonicity disappears if the firm has two technologies that it can invest in at date 0: a liquidity producing short-term technology in addition to the long-term technology. In that case, as L_S decreases, more of the firm's initial investment will flow into the short-term technology, causing I to decrease.

that the price of liquid assets q increases. When q reaches $q_{\max}$, an increasing fraction of firms start to withdraw their demand for liquidity, implying that they will have to shut down operations in bad times. As a result, a high supply of liquidity today leads to a liquidity shortage tomorrow. This can create recurring periods of booms and busts. (For more on the intertemporal links between liquidity and investment, see Fahri and Tirole, 2009a, who, in an overlapping-generation model, show how firms that currently need stores of value can build on older firms' securities to create liquidity cushions.)

The likelihood of liquidity shortages.

Next let us look at what happens when the probability of the high shock, f_H, increases. We will consider two separate cases: in the first, the change in the probability of a high liquidity shock occurs before the investment I is made, so I will respond to the change. There is a long-term effect. In the second case, news about a change in f_H arrives after the investment I has already been made, so I stays fixed as does the purchase of liquidity. The only impact is on the changed (implicit) price of liquidity after the news.

(a) *Long-term effects* (I variable). Because

$$I(q) = \frac{A}{1 + (q-1)(\rho_H - \rho_0) - (\rho_0 - \overline{\rho})} ,$$

an increase in f_H increases $\overline{\rho}$ and reduces investment. Intuitively the investment is less profitable, so there is less investment for a given q. Let us also examine how the maximal price $q_{\max}$ changes with f_H. Rewriting (3.11) we have

$$\frac{1 + f_L \rho_L}{f_L} = 1 + (q_{\max} - 1)(\rho_H - \rho_0) + \overline{\rho}$$

implying that

$$\text{sign}\left(\frac{dq_{\max}}{df_L}\right) = \text{sign}\left(\frac{d}{df_L}\left(\frac{1}{f_L} + f_L(\rho_H - \rho_L)\right)\right) = \text{sign}\left(-\frac{1}{f_L^2} + (\rho_H - \rho_L)\right) < 0,$$

since by assumption $f_L(\rho_H - \rho_L) < 1$ (that is, there is a liquidity demand when $q = 1$). Therefore $q_{\max}$ increases as the likelihood of a high shock (f_H) increases.

The long-term impact on liquidity demand is depicted in Figure 3.3.

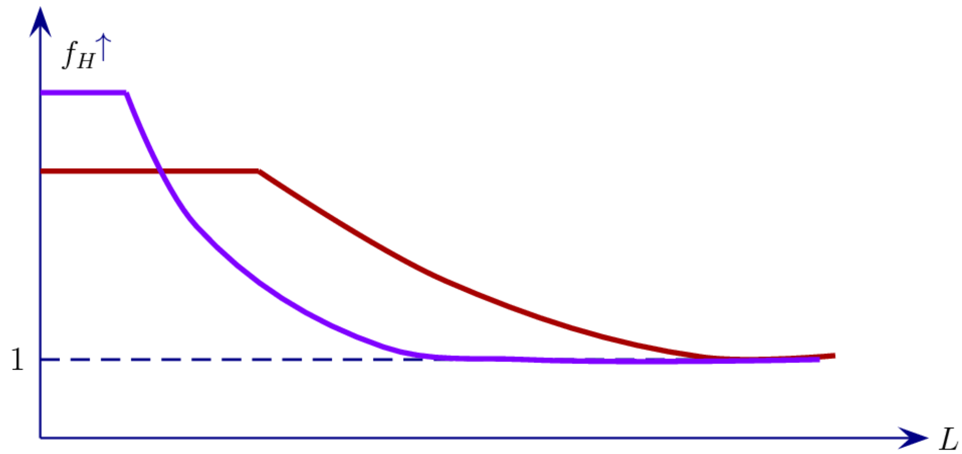


Figure 3.3: long-term impact of growing concerns

Bad news (f_H increases) increases q when there is little liquidity, and decreases it when there is much liquidity.

(b) *Short-term effects* (I fixed).[15] Suppose now that news about the likelihood of a high shock arrives after I has been chosen, but before the shock is revealed as depicted in the time line in Figure 3.4.

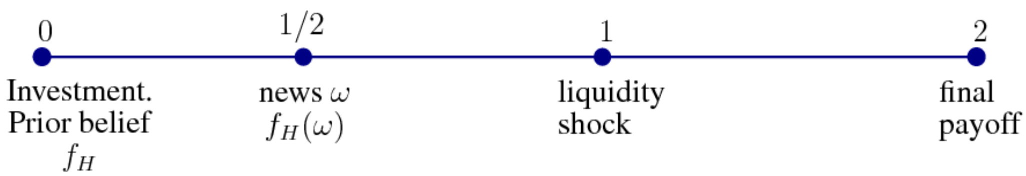


Figure 3.4: news accrual

Let q_0 be the price of liquidity at date 0, and $q(\omega)$ the price at date 1/2, contingent on news ω. At date 0, let the firm choose the initial scale I, an uncontingent supply of liquidity ℓ_0 and a contingent supply of liquidity $\ell(\omega)$, which can be construed as an adjustment to the initial amount of liquidity purchased, ℓ_0. In equilibrium, the contingent price of liquidity $q(\omega)$ delivered on date 1/2 in state ω will, of course, have to be such

[15] This analysis is a special case of the analysis in Holmström and Tirole (2001).

that there is no actual adjustment in liquidity. This is so, because all firms are in identical position also at date 1/2. Therefore,

$$\begin{cases} \ell_0 = L_S, \\ \ell(\omega) = 0, \qquad \forall \omega. \end{cases}$$

To determine $q(\omega)$, consider an individual firm's choice of $\ell(\omega)$. The firm's budget constraint is:

$$I + (q_0 - 1)\ell_0 + E\left[(q(\omega) - 1)\ell(\omega)\right]$$
$$= E\left[f_L(\omega)(\rho_0 - \rho_L)\right] I + E\left[f_H(\omega)(\rho_0 - \rho_H)i(\omega)\right]$$

Its liquidity constraint at date 1, if ω occurred at date 1/2, is

$$\ell_0 + \ell(\omega) \geq (\rho_H - \rho_0)i(\omega).$$

Maximizing with respect to ℓ_0 and $\ell(\omega)$ the entrepreneur's utility

$$(\rho_1 - \rho_0)\left[E\left[f_L(\omega)\right] I + E[f_H(\omega)i(\omega)]\right]$$

subject to the liquidity and budget constraints above yields

$$\begin{cases} q_0 = E\left[q(\omega)\right], \\ q(\omega) - 1 = \dfrac{f_H(\omega)}{f_H}(q_0 - 1). \end{cases}$$

We see that the price follows a martingale. This is because the date-0 price of date-1/2 liquidity, $q(\omega)$, goes up when the updated belief of a high liquidity shock $f_H(\omega)$ is higher than the initial belief f_H and down in the opposite case.

The interest rates r_0 from date 0 to date 1 and $r(\omega)$ from date 1/2 to date 1 are defined by

$$q_0 - 1 = 1/(1 + r_0) - 1 = -r_0/(1 + r_0) \quad \text{and} \tag{3.18}$$

$$q(\omega) - 1 = -r(\omega)/(1 + r(\omega)), \tag{3.19}$$

respectively. Note that r_0 and $r(\omega)$ are negative. For small changes in the probability, hence small changes in the interest rates, we see from the equilibrium prices that

$$|r(\omega)| \simeq \frac{f_H(\omega)}{f_H} |r_0| .$$

We see that the date-1/2 interest rate moves in proportion to the updated probability of an unfavorable liquidity shock.

3.3.3 Evidence of a liquidity premium in corporate bond spreads

There is a lot of suggestive evidence consistent with a liquidity premium, but with so many factors affecting interest rates, it has been hard to make a strong case for a premium. A study by Krishnamurthy and Vissing-Jorgensen (2006) tries to get around this problem by analyzing the spread of corporate bond yields over Treasury yields and how that spread varies with the supply of Treasury bonds. Figure 3.5, imported from their paper, describes the main finding. The horizontal axis measures the logarithm of the ratio of U.S. government debt to GDP. The vertical axis measures the spread between AAA rated corporate bond yields and Treasury yields. The data points are September values from 1925 to 2005.

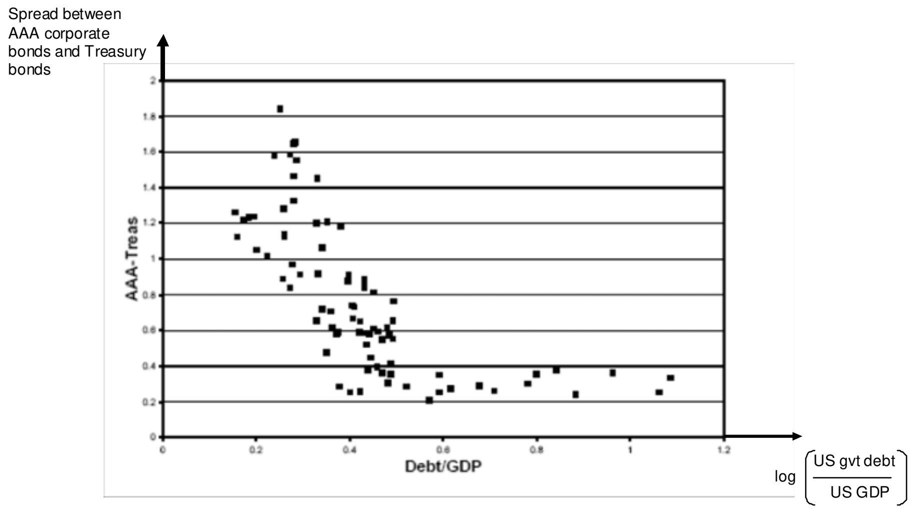


Figure 3.5: Liquidity premium in corporate bond spreads

Using corporate bond spreads rather than absolute yields filters out common effects that influence all yields of AAA bonds. The ratio of debt to GDP is a good proxy of the amount of outstanding Treasury bonds (relative to the overall economic activity). The flat tail of observations with high debt/GDP ratios are from the 1940s and 1950s and evidently due to war spending. This suggests that an important part of the variation in the debt/GDP ratio is exogenous.

Figure 3.2 may then be interpreted as tracing out the demand for Treasury bonds in response to a liquidity premium (or convenience yield in the authors' language) The resemblance between Figure 3.2 and 3.5 is intriguing. There is an initial downward sloping section as the increase in the supply of Treasuries brings down the premium. Then the demand curve turns flat suggesting that the liquidity premium has disappeared. Note, however, that the vertical axis in Figure 3.5 measures yield spreads and not the liquidity premium as in Figure 3.2. In the next chapter we will show that if corporate bonds default are independently of the supply of Treasury bonds, the yield spread can be decomposed into two parts. It is the sum of a liquidity premium (determined by the supply of

Treasuries) and a risk premium due to the difference in default risk between corporate bonds and Treasuries (which are essentially default free). Thus, the fact that Treasuries have a zero liquidity premium in Figure 3.2 when the supply is ample, is fully consistent with a remaining baseline spread visible in the high debt/GDP observations of Figure 3.5.

3.4 Competition for liquidity between consumers and firms

Our workhorse model assumes that consumers are patient. Obviously consumers may themselves demand some liquidity. To consider this and see how our theory works out a mix of consumer and corporate liquidity demands, this section adds Diamond-Dybvig consumers (see Appendix 2.2) to the equilibrium analysis as follows. Because consumers save in the Diamond-Dybvig model, the corporate sector will be a net borrower. The long-term return that secures the consumers' future consumption will come from the pledgeable income of the firms' illiquid investments. Consumers can be treated as firms that are net lenders and cannot offer any pledgeable income (their ρ_0 is equal to 0).

There are three dates: $t = 0, 1, 2$.

- Consumers: They are exactly as in Diamond and Dybvig (1983). They invest their unit endowment at date 0 and receive utility $u(c_1)$ from date-1 consumption if they are impatient (which has probability $1-\alpha$) and $u(c_1+c_2)$ if they turn out to be patient (which has probability α).

- Firms: There is a continuum of identical firms with mass 1. The representative firm has assets A at date 0 and invests a variable amount I in the long-term project. It faces a liquidity shock ρ per unit of investment, drawn from a continuous distribution $F(\rho)$. There is no aggregate uncertainty (the firms' shocks are i.i.d.).

- Firms can also invest in short-term (liquid) assets. By investing $g(\ell)$ at date 0, ℓ units of the good at date 1 can be created, where $g' > 0$, $g'' > 0$, and $g'(0) = 1$.

Letting $\widehat{\rho}$ denote the cutoff for continuing with the long-term project at date 1, the economy's resource constraints at date 0, 1 and 2 are:

$$I - A + g(\ell) \leq 1 \tag{3.20}$$

$$(1-\alpha)c_1 + \left[\int_0^{\widehat{\rho}} \rho f(\rho) d\rho\right] I \leq \ell \tag{3.21}$$

$$\alpha c_2 \leq F(\widehat{\rho})\rho_0 I. \tag{3.22}$$

Condition (3.20) reflects the fact that the economy's total endowment, $1 + A$, is invested in illiquid (I) and liquid ($g(\ell)$) assets. Condition (3.21) highlights how consumers and firms compete for liquidity at date 1. Finally, (3.22) states that the consumers' long-term consumption comes from firms with limited pledgeable income (unlike the model in Appendix 2 2, where all income was pledgeable).

A Pareto optimum among consumers and entrepreneurs can be found by maximizing the representative entrepreneur's rent:

$$\max_{\{\ell, I, \widehat{\rho}\}} \{(\rho_1 - \rho_0) F(\widehat{\rho}) I\} \tag{3.23}$$

subject to the resource constraints (3.20)-(3.22) and the representative consumer's expected utility exceeding some level $\bar{U}$:

$$(1-\alpha)u(c_1) + \alpha u(c_2) \geq \bar{U} \tag{3.24}$$

Let $\lambda_0, \lambda_1, \lambda_2, \lambda_3$ be non-negative Lagrange multipliers corresponding to the constraints (3.20)-(3.24), respectively. The first-order conditions imply:

$$\begin{gathered} \frac{u'(c_1)}{u'(c_2)} = \frac{\lambda_1}{\lambda_2}, \\ F(\rho^*) - \lambda_1 \left[\int_0^{\rho^*} \rho f(\rho) d\rho\right] + \lambda_2 F(\rho^*)\rho_0 - \frac{\lambda_1}{g'(\ell)} = 0, \\ 1 - \lambda_1 \rho^* + \lambda_2 \rho_0 = 0. \end{gathered} \tag{3.25}$$

The first condition shows that the consumer's marginal utilities of consumption may not be set equal, because the investment technologies allow only limited consumption smoothing, while satisfying entrepreneurial objectives. The second condition determines the initial investment scale I, taking into account that ℓ is chosen so that the marginal cost $g'(\ell) = \lambda_1/\lambda_0$. The last condition determines the optimal $\widehat{\rho} = \rho^*$.

Straightforward manipulations lead to the following condition for the optimal cutoff ρ^*:

$$g'(\ell)\rho^* = \frac{1 + g'(\ell)\int_0^{\rho^*} \rho f(\rho)d\rho}{F(\rho^*)} \tag{3.26}$$

Condition (3.26) generalizes the standard optimal cutoff rule ($\rho^* = c$) to the case of costly liquidity hoarding. The average cost of bringing one unit of investment to completion for an arbitrary cutoff $\widehat{\rho}$ becomes

$$c(\widehat{\rho}, q) = \frac{1 + q\int_0^{\widehat{\rho}} \rho f(\rho)d\rho}{F(\widehat{\rho})},$$

which generalizes condition (2.22) to the case of costly liquidity ($q \geq 1$). Letting

$$q \equiv g'(\ell),$$

condition (3.26) can be written in a form analogous to the standard cutoff rule

$$q\rho^* = c(\rho^*, q),$$

since $q\rho^*$ is the cost of raising the cutoff level marginally above ρ^*.

Patient consumers help firms "bridge" dates 1 and 2, as they do not demand cash at date 1. And so the discount factor $1/q$ between dates 0 and 1 is the only modification to the formula obtained in Chapter 2.

The Pareto optimal production plan can be decentralized in the following sense. Let $Q = \lambda_2/\lambda_0$ be the consumer's marginal valuation of date-2 consumption relative to the marginal valuation of the date-0 endowment. The representative firm then solves:

$$\max_{\{\ell, I, \widehat{\rho}\}} \{(\rho_1 - \rho_0) F(\widehat{\rho}) I\}$$
$$\text{subject to}$$
$$I + q\ell \leq A + QF(\widehat{\rho})\rho_0 I,$$
$$[\int_0^{\widehat{\rho}} \rho f(\rho) d\rho] \; I \leq \ell.$$

which yields the same first-order conditions as in the Pareto program and therefore also (3.26).

A corresponding decentralization holds for consumers, but requires that the demand for liquidity be intermediated by banking contracts of the Diamond-Dybvig type (i.e. contracts that provide limited insurance to consumers). The maximization of $(1-\alpha)u(c_1) + \alpha u(c_2)$ subject to the budget constraint:

$$1 \geq (1-\alpha) q c_1 + \alpha Q c_2$$

yields

$$\frac{u'(c_1)}{u'(c_2)} = \frac{q}{Q}$$

which is consistent with the first condition in (3.25).

Our basic model is thus a special case of this model, in which

- consumers are always patient: $\alpha = 1$

  (and so $\frac{\ell}{I} = \int_0^{\rho^*} \rho f(\rho) d\rho$)

- the consumers' marginal rate of substitution between dates 0 and 2 is normalized at $Q = 1$.

An interesting extension of this framework would consist in assuming that the fraction, $1 - \alpha$, of consumers, who face a liquidity shock, and the distribution of the corporate liquidity shocks are co-determined, i.e., are driven by a common factor. For example, as in the stylized model of section 5.2 below, workers from closed plants enter unemployment and face liquidity needs. We leave this extension for future research.

3.5 Endogenous supply of liquidity

So far, we have assumed that the supply of liquidity is exogenous. In reality, the private sector can produce liquidity through a variety of technologies, for instance by investing in projects with safer and shorter term payoffs. As the liquidity premium rises with increased demand, the private sector will respond by shifting more resources towards supplying liquidity.

To illustrate, suppose there is a second sector, the S-sector (for short-term), consisting of a continuum of identical firms with unit mass. Assume that each entrepreneur in this sector has an individual endowment A_S and access to a linear storage technology that transforms one unit of corn at date 0 into $\rho_{0S} < 1$ units of corn at date 1. As well, assume the short-term project provides a private benefit to the entrepreneur, $\rho_1 - \rho_0$, such that $\rho_1 > 1$. The budget constraint for each firm in this second sector is

$$l_S \leq A_S + q\rho_{0S} l_S. \tag{3.27}$$

The plan to invest ℓ_S at date 0 must be covered by the sum of the entrepreneur's own capital A_S and the date-1 return $\rho_{0S}\ell_S$ valued at unit price q. Because $\rho_1 > 1$, the entrepreneur will invest in the storage technology for all values $q \geq 1$. The maximum investment scale, implied by the budget constraint, gives the individual (and aggregate) supply function

$$\ell_S (q) = \frac{\rho_{0S} A_S}{1 - q\rho_{0S}}. \tag{3.28}$$

This supply function is strictly increasing and convex with $q = 1/\rho_{0S}$ as an asymptote. The higher is q, the more liquidity the economy will produce. The marginal cost of production is decreasing in q. At $q_{\max} = 1/\rho_{0S}$, the firm becomes self-financing at which point it can supply an infinite amount of liquidity. For prices below this maximum, the supply is constrained by the initial capital the entrepreneur has.

Note that the distribution of capital between the entrepreneurs in the two sectors matters for the amount of liquidity that will be produced as well as for the equilibrium price of liquidity. As the ratio of A_S/A decreases, the price of liquidity will rise and the leverage of A_S will grow while the leverage of A will decline.

By contrast, if all entrepreneurs have access to both technologies, the equilibrium price of liquidity will be such that each entrepreneur is indifferent between operating the storage technology and selling liquidity to others or operating the long-term technology and buying liquidity from others (or being self-sufficient by operating both technologies).

If liquidity-producing technologies have faster payoffs, something our model is silent on,[16] the above logic will have the simple prediction that tighter liquidity, as measured by a higher liquidity premium $q-1$, will cause the private sector to reallocate investment funds from long-term to short-term ones. This is one potential explanation for why, in times of tight liquidity, the maturity of debt shortens.

Expanding private sector liquidity

There are other ways in which the private sector can expand liquidity supply in times of high liquidity demand. One important means is by expanding pledgeable income ρ_0, which we have so far taken as fixed. There are many ways in which the pledgeable income can be expanded. For instance, a bank, a venture capitalist or some other intermediary may be able to monitor the firm, reduce the cost of moral hazard and thereby the illiquid stake that the entrepreneur needs to hold to keep him from cheating (see Holmström and Tirole 1997). Or a country may institute a better corporate governance system with regulatory changes that improve the firm's reporting or incentives more generally. Increased transparency and better regulatory mechanisms that protect investors make it easier for them to recover investments and therefore raises ρ_0 for most firms. In a dynamic model, reputation effects are often strengthened, reducing the need for financial incentives. This, too, raises the pledgeable income.

[16] It does not matter whether the units of corn produced by the firms in the second sector accrue at date 1 or date 2. The key property is that they are fully pledgeable (liquid).

One might conjecture however that shorter investments, either because they involve less uncertainty and therefore less adverse selection or because they offer less scope for diversion, have a higher pledgeability than longer-term ones.

Another important determinant of aggregate liquidity is the degree of financial market development and the extent to which corporate income and private wealth are turned into tradable assets. When a private firm goes public and its shares become liquid (in the traditional financial sense), the portfolio of tradable securities expands. Note that even if the pattern of returns from the new shares can be replicated with available securities on the market, a public offering can add value if the shares pay off in a state where liquidity is in short supply. This contrasts with standard financial models where there is no such effect.

Increased securitization does not necessarily add to the net supply of liquidity, because the gross increase in the supply of liquidity may be associated with a simultaneous increase in the demand for liquidity.

Consider the home equity loan market (and other mortgage markets), which offers a way in which liquidity can be expanded. Mortgaging a fully-owned house through a home equity loan, makes a non-tradable claim tradable. This adds a savings and investment instrument to the market, which is valuable if there is a shortage of aggregate liquidity. The net addition to aggregate liquidity depends on how the mortgage affects the demand for liquidity. If the home owner invests all of the proceeds from the loan into Treasury bonds or some similar instruments, little if anything has been added to aggregate liquidity. Using the proceeds for consumption or real investments, on the other hand, does expand net liquidity.

Similarly, an IPO may have no consequences for net supply and prices of liquidity, nor production plans. Suppose the entrepreneur all along knew that he could issue equity to outsiders and therefore considered the shares of the company a future source of liquidity. If so, the value of the *potentially* pledgeable income would be part of his date-1 liquidity in our model. Taking the company public at date 0 would be a matter of indifference, if the entrepreneur were to hold equal amounts and an equal distribution of liquidity after the IPO as before it. If the IPO reduces the efficiency of the firm (because of moral hazard, say), it could even be better to keep the firm private until some event forces an IPO.

The general point here is that the net addition to liquidity is typically smaller than the gross increase from the IPO, because even privately held assets provide liquidity. After the IPO, the entrepreneur will likely purchase liquidity to compensate for the loss of the liquidity embedded in the firm when it was private. Assuming the IPO has no direct payoff consequences, a public offering in the type of model we have studied, never makes the liquidity supply worse and typically makes it better. Once the private returns are turned into publicly tradable claims, some of the liquidity will likely be used to support other projects than those of the entrepreneur. The market will make more efficient use of liquidity than if it is privately held.

3.6 Summary

The purpose of this chapter has been to illustrate, in the simplest setting, how a shortage of liquidity can naturally arise from limited pledgeability of income. This links the familiar limitations on the debt capacity of firms to the much less discussed limitations on the funding capacity of investors. The supply problem arises because production of goods also serves a collateral function.[17] The most basic question to ask is whether the corporate sector generates enough collateral to support the second-best production plans. Put differently, when is there enough aggregate liquidity to meet corporate needs in each state of nature?

We have shown that when the productive sector is a net lender, there is always a shortage of aggregate liquidity. When the corporate sector is a net borrower, the presence of aggregate liquidity shortages depends on the stochastic structure of the firms' liquidity shocks. If all uncertainty is idiosyncratic and there is a continuum of firms, there is always sufficient aggregate liquidity. In this case, firms can meet their demand for liquidity simply

[17] Kiyotaki and Moore's (1997) model of credit cycles turns on this same dichotomy of capital, but in their model the dual use is within a single firm. Our emphasis is on collateral production for the rest of the economy.

by holding a fraction of the market portfolio. At the other extreme, when there is only aggregate uncertainty and all firms are hit by the same shock, there is always a shortage of aggregate liquidity. This creates a demand for outside liquidity, such as government bonds, which will command a liquidity premium. The liquidity premium declines as the supply of bonds increases in a manner broadly consistent with the evidence in Krishnamurthy and Vissing-Jorgensen (2006).

We will later on analyze the role of government and international financial markets in alleviating liquidity shortages. There are also many ways in which the private sector can respond to liquidity shortages and high liquidity premia. Firms can change their production plans; private firms can go public; consumers can increase the leverage on their homes through home equity loans; and new pledgeable assets can be built. The subprime crisis may in part have been caused by global shortages of collateral that led to imaginative, but in the end fragile ways to increase the aggregate supply of liquidity, a topic we will come back to in the Epilogue.

Chapter 4

A liquidity asset pricing model (LAPM)

In this chapter we extend the simple model from the previous chapter to a general liquidity asset pricing model (LAPM) with heterogenous firms employing linear technologies of the sort we have been studying. Uncertainty can affect all the parameters of these technologies. The model shows how liquidity premia are determined in such a setting, how they affect bond yields and firm values and how firms plan investments and optimally manage liquidity risks in light of such premia. We start with the case where there are two aggregate shocks, before moving on to the general model.

4.1 LAPM with two aggregate shocks

4.1.1 Liquidity premia and bond yields

With heterogenous firms it is more convenient to work with state prices. All asset prices can readily be recovered from state prices, since markets for claims on date-1 liquidity (the pledgeable part of output) are complete.

Let $s_H f_H$ be the date-0 price of one unit of liquidity delivered only in the high-shock state and let $s_L f_L$ be the corresponding price of liquidity delivered only in the low-shock state; here f_H and f_L are the probabilities of a high and a low liquidity shock, respectively ($f_L + f_H = 1$). Because consumers are indifferent about the timing of consumption, s_H and

$s_L \geq 1$. If there is an aggregate shortage of liquidity in the high-shock state, we will have $s_H > 1$. We assume that there is no aggregate shortage of liquidity in the low-shock state (the corporate sector is self-financing in that state), and therefore $s_L = 1$. The date-0 price q of a unit of unconditional liquidity at date 1 (e.g., a risk-free unit bond, expiring at date 1) can then be written

$$q = f_L \cdot 1 + f_H \cdot s_H \tag{4.1}$$

and the liquidity premium

$$q - 1 = f_H(s_H - 1). \tag{4.2}$$

The yield on a risk-free bond is

$$\kappa = \frac{1-q}{q} = \frac{-f_H(s_H - 1)}{1 + f_H(s_H - 1)} \leq 0. \tag{4.3}$$

The yield is zero whenever the liquidity premium is zero, otherwise it is negative since our risk neutral consumers do not discount the future. The yield on a bond that can default is strictly larger than κ. To illustrate, consider a bond, indexed by k, with face value 1 that in expectation pays $\theta_{kH} < 1$ in state H and $\theta_{kL} = 1$ in state L, that is, the bond defaults with probability f_H. Such a bond commands the date-0 price

$$q_k = f_L + f_H \theta_{kH} s_H. \tag{4.4}$$

The yield on the bond is

$$\kappa_k = \frac{f_L + f_H\theta_{kH} - q_k}{q_k} = \frac{f_H\theta_{kH}(1 - s_H)}{1 - f_H(1 - \theta_{kH}s_H)}. \tag{4.5}$$

Note that $\kappa_k - \kappa > 0$, because $\theta_{kH} < 1$.[1] We can interpret Figure 3.5, describing the spread between AAA rated corporate bonds and Treasury bonds (Krishnamurthy and Vissing-Jorgensen, 2006) through the lens of this simple example. For bonds that may default, the

[1] We have:

$$\kappa_k - \kappa = \frac{f_H(1 - s_H)(\theta_{kH} - 1)(1 - f_H)}{[1 - f_H(1 - \theta_{kH}s_H)][1 - f_H(1 - s_H)]} > 0.$$

average spread should stay strictly positive regardless of the liquidity premium. Variations in default risk (more generally $f_H \theta_{kH}$) would move the spread around for a given liquidity premium. The spread should decrease as the price of liquidity goes down (other things equal) and reach a constant positive value when the liquidity premium is 0. This is the sense in which the theoretical Figure 3.2 and the empirical Figure 3.5 match, as suggested in section 3.3.3.

4.1.2 Asset prices

Liquidity premia will also influence the pricing of firms. With two states, each firm, indexed by j, is characterized by four numbers $\{\rho_{j0}, \rho_{j1}, \rho_{jL}, \rho_{jH}\}$, where ρ_{j0} is the pledgeable payoff, ρ_{j1} is the total payoff, ρ_{jL} is the liquidity shock in the low aggregate state, and ρ_{jH} is the liquidity shock in the high aggregate state; all the numbers are per unit of investment. For simplicity, we assume that there are only two types of firms, "regular" firms and "contrarian" firms, and that $\rho_{jL} = 0$ for both types. Regular firms demand liquidity in the high aggregate state H:

$$0 = \rho_{jL} < \rho_{j0} < \rho_{jH} < \rho_{j1}.^{2} \tag{4.6}$$

Contrarian firms, in contrast, supply liquidity in the high aggregate state (for them $\rho_{jH} < \rho_{j0}$). The fraction of contrarian firms is small enough that there still is a shortage of liquidity in the H-state.

At date 0, each firm j chooses an initial scale of investment I_j and the scale of date-1 continuation investments i_{jL}, $i_{jH} \leq I_j$. Since a regular firm is self-sufficient in state L, such a firm will choose $i_{jL} = I_j$. We assume, as before, that $(\rho_{jH} - \rho_{jL})f_L = \rho_{jH} f_L < 1$ holds for regular firms, so that continuation at full scale is optimal for them also when the high shock hits if the liquidity premium is zero (see (2.13)). For contrarian firms, $i_{jL} = i_{jH} = I_j$, since they never demand liquidity.

[2] Recall that these parameters must be such that investment is not infinite. A sufficient condition for this to be the case is that $f_L(\rho_{j0} - \rho_{jL}) = f_L \rho_{j0} < 1$.

To simplify notation, define the *net (financial) payoff of firm j in state* ω

$$y_{j\omega} \equiv (\rho_{j0} - \rho_{j\omega})i_{j\omega} \qquad \omega = L, H. \tag{4.7}$$

This is also the amount of liquidity that firm j supplies in state ω (if $y_{j\omega}$ is negative, the firm demands liquidity in state ω).

Firm j's budget constraint is

$$I_j - A_j \leq f_L y_{jL} + f_H y_{jH} s_H \equiv E_j(y \cdot s). \tag{4.8}$$

The right-hand side of the budget constraint, $E_j(y \cdot s)$, is the *date-0 market value of the firm*. The *investor's expected date-1 payoff from firm* j is

$$E_j(y) \equiv f_L y_{jL} + f_H y_{jH}. \tag{4.9}$$

The *liquidity premium of firm* j is

$$\nu_j \equiv f_H y_{jH}(s_H - 1). \tag{4.10}$$

The liquidity premium is negative for regular firms ($y_{jH} < 0$) and positive for contrarian firms ($y_{jH} > 0$). Since the budget constraint must bind, we have

$$I_j - A_j = E_j(y \cdot s) = E_j(y) + \nu_j, \tag{4.11}$$

or equivalently

$$(I_j - A_j) - \nu_j = E_j(y \cdot s) - \nu_j = E_j(y). \tag{4.12}$$

The right-hand side of (4.12) is the investors' gross expected return at date 1. The left hand side is their total investment, showing that for a regular firm, which demands liquidity in the high state, the total investment consists of two parts: the investors' contribution $I_j - A_j$ towards the initial investment plus the payment of the liquidity premium $-\nu_j > 0$ for securing the desired date-1 liquidity.

Define a *unit claim* on the firm as a claim that in expectation pays 1 at date 1. *The date-0 value of a unit claim on the firm* is

$$q_j = \frac{E_j(y \cdot s)}{E_j(y)} = \frac{f_L y_{jL} + f_H y_{jH} s_H}{f_L y_{jL} + f_H y_{jH}}. \tag{4.13}$$

When we compare firm values, q_j is the natural measure to use. It tells us how much investors are willing to pay for the asset per unit of return at date 1 net of the liquidity premium/discount.

Finally, we introduce *normalized net state payoffs for firm j*

$$\bar{y}_{j\omega} \equiv \frac{y_{j\omega}}{E_j(y)}, \qquad \omega = L, H. \tag{4.14}$$

This normalization makes net state payoffs have unit expected value. The value of a unit claim on the firm can then be written

$$q_j - 1 = f_H \bar{y}_{jH}(s_H - 1). \tag{4.15}$$

Regular firms demand liquidity in the high shock state, so their unit claim features a liquidity discount ($q_j - 1 < 0$). For contrarian firms, the converse is true ($q_j - 1 > 0$).

The interpretation of the value of a unit claim as an asset price requires some elaboration. At first glance, it would appear that a unit claim cannot sell at a discount, because consumers (who have limitless wealth) would compete for such claims until the discount would be driven to zero. So, how can we make sense of (4.15)? The explanation is that, in the case of a regular firm, the price q_j comes with the additional financing obligation to pay the firm the amount $-\bar{y}_{jH} > 0$ in state H at date 1. To secure this commitment, the investor needs to buy this much liquidity at date 0 at a liquidity premium $s_H - 1$, which just offsets the liquidity discount $q_j - 1$. The total cost of buying a unit claim of a regular firm is therefore 1. (Recall that consumer income is non-pledgeable so commitments must be backed up by tradable assets.)

One could interpret this arrangement as a venture capital deal with two stages of investment. At the initial stage, investors pay q_j and at the second stage (date 1), they pay $-\overline{y}_{jH}$ if needed, which in expectation costs them $1-q_j$.[3] An alternative arrangement is to have investors pay 1 per unit for a regular firm at date 0 with the firm investing q_j of it in productive assets and $1-q_j$ in shares of contrarian firms (or other liquid assets). If the aggregate state turns out to be low, the firm turns the shares of contrarian firms over to its investors (or sells the shares and pays out the proceeds as a date-1 dividend). If the state is high, it sells the contrarian shares and uses the proceeds to cover the liquidity shock. Either way, the expected return on the consumer-investor's total investment is zero. However, the price of a share per unit of investment scale at date 0 differs in the two cases. When everything is paid up front, the price paid at date 0 is 1. When there is a remaining investor liability, the price at date 0 is given by (4.15), reflecting a liquidity discount.

Kocherlakota (1996) has argued that in a world where a representative consumer holds equity either directly or indirectly (through mutual funds, for instance), prices will be determined by the consumers' marginal rate of substitution, which in our model always equals one. This conclusion need not hold when there is segmentation among investors, as illustrated here.[4] If investors pay the full cost of investment up front, letting regular firms arrange their own liquidity needs by buying shares of contrarian firms, say, the price of regular firms will equal the marginal rate of substitution of consumers, while the price of contrarian firms will feature a liquidity premium and consequently no consumer will buy these shares.

4.1.3 A numerical example

Before proceeding, it may be helpful to go through a numerical example that illustrates the above analysis. Let the net payoffs of a regular firm j be $y_{jL} = 2$, $y_{jH} = -1$ and assume

[3] For expositions of the staging of venture capital financing, see, e.g., Gompers and Lerner (1999), Kaplan and Strömberg (2003), and Sahlman (1990).

[4] He and Krishnamurthy (2009) present a model where investment banks are the only ones that can buy equity and therefore asset prices do not reflect consumer rates of substitution. In empirical work, consumer segmentation has been used to reconcile equity premia with consumer marginal rates of substitution (see Vissing-Jorgensen (2002)).

each state has equal probability so that $E_j(y) = (0.5)$. Let the entrepreneur's date-0 endowment be $A = 10$. Without aggregate liquidity shortages, the maximum investment scale would be $I_j = A/[1-(0.5)] = 20$, of which the investors would contribute 10.

Assume now that there is a liquidity shortage in the H-state and that the liquidity premium in this state is $s_H - 1 = 0.2$. The liquidity premium adds $f_H(s_H - 1) = (0.5)(0.2) = 0.1$ to the cost of each unit of investment at date 0, reducing the maximum scale of investment to $I_j = 10/[(1+0.1)-(0.5)] = 16\frac{2}{3}$. The expected net financial return at date 1 is therefore $E_j(y) = (16\frac{2}{3}) \cdot (.5) = 8\frac{1}{3}$. This equals the sum of what the investors contribute to the initial investment, $I_j - A = 16\frac{2}{3} - 10 = 6\,\frac{2}{3}$, and what they pay as a liquidity premium: $-\nu_j I_j = -(.5)(-1)(.2) \cdot 16\,\frac{2}{3} = 1\frac{2}{3}$.

Against a date-2 expected pledgeable return of $8\frac{1}{3}$, the firm is able to raise $6\frac{2}{3}$ units from investors at date 0 (all used for the initial investment) with the additional covenant that investors will promise to cover the liquidity shock at date 1. This covenant costs investors $8\frac{1}{3}-6\frac{2}{3} = 1\frac{2}{3}$, since they have to buy liquid instruments (e.g. shares in contrarian firms) to secure their pledge. They could also pay in $8\frac{1}{3}$ at date 0 and let the firm buy the requisite liquidity. In either case, the date-0 value of the firm is $q_j = (6\frac{2}{3})/(8\frac{1}{3}) = 0.8$ per unit of expected date-1 return. The firm sells at a discount $q_j - 1 = -0.2$, because the purchase of liquidity costs 0.2 per unit of expected date-1 return.

4.1.4 The LAPM formula with two states

In the two-state economy, with a liquidity shortage only in the H-state, we find, using equation (4.15), the following simple relationship between the value of any pair (j, k) of assets:[5]

$$\frac{q_j - 1}{q_k - 1} = \frac{\bar{y}_{jH}}{\bar{y}_{kH}} \tag{4.16}$$

This valuation formula shows that with a single liquidity shock, the ratio of the liquidity premia (or discounts as the case may be) of any two assets is independent of the

[5] This valuation formula can be found in Holmström and Tirole (2001).

probability of the state as well as the cost of liquidity and is simply the ratio of the payoffs in those two states. In particular, we can express the value of every asset in terms of the price q of a bond that pays one unit in each of the two states. From (4.16) we have

$$q_j = 1 + (q - 1)\bar{y}_{jH}. \tag{4.17}$$

4.1.5 Equilibrium determination with two states

In section 3.3.1 we described how the bond price q is established in equilibrium when firms are homogenous and there are two states. The equilibrium in the case of heterogeneous firms can be similarly found. Expressed as a function of s_H, the demand (or supply) function for state-contingent liquidity by each firm looks identical to the demand (or supply) function for the case of homogenous firms. As the price s_H rises, a regular firm, because it demands liquidity ($y_{jH} < 0$), will reduce its level of investment until it reaches the point of indifference between continuing in both states or continuing just in the L-state. A contrarian firm, because it supplies liquidity ($y_{jH} > 0$), will be able to expand its investment scale in response to an increase in s_H until s_H reaches a point where some firm becomes self-financing. This firm can then supply everyone with liquidity, because of the constant returns to scale technology.

We can ask which characteristics determine the order in which regular firms will drop out in the liquidity shortage state as the price of liquidity rises (i.e. the order in which firms switch from $i_{jH} = I_j$ to $i_{jH} = 0$ as s_H increases). We can also ask which contrarian firm will be the one that first becomes self-financing.[6]

In order to determine the critical value s_H at which a regular firm drops out in the H-state (contrarian firms never do), we equate the entrepreneur's net payoffs when the firm chooses $i_{jH} = I_j$ and when it chooses $i_{jH} = 0$. Recall that we assumed $\rho_{jL} = 0$ for all

[6] Note that these questions relate to comparisons across firms, whose parameters are fixed. It is not a comparative statics exercise. The only things changing are the price s_H and the firm's investment policies in response to this price change.

firms. It is easy to verify that a regular firm j will continue in both states if and only if:[7]

$$s_H \leq s_{jH}^{reg} \equiv \frac{(1 - f_L \rho_{j0})}{f_L(\rho_{jH} - \rho_{j0})} \text{ for all } j \text{ (regular).} \tag{4.18}$$

Here s_{jH}^{reg} is the highest value of s_H such that a regular firm j is willing to continue in state H. A regular firm drops out at a lower value s_H the higher is ρ_{jH} and the lower is ρ_{j0}; a firm with a small pledgeable income and/or a large demand for liquidity in the high state, will drop out early.[8] Note that neither the entrepreneur's wealth A_j, nor his private benefit $\rho_{j1} - \rho_{j0}$ influence the point at which the firm's demand for liquidity drops to zero, because of the linear technology.

As long as a regular firm continues in both states (the inequality in (4.18) holds), the initial investment I_j decreases with s_H. Contrarian firms always continue in both states. They increase their initial investment as s_H increases, since the increased value of supplying liquidity relaxes the budget constraint. For a high enough liquidity premium, the budget constraint of a contrarian firm will no longer bind and it becomes self-financing. This occurs when $s_H = s_{jH}^{con}$satisfies:

$$f_L \rho_{j0} + f_H(\rho_{j0} - \rho_{jH}) s_{jH}^{con} = 1. \tag{4.19}$$

As soon as one contrarian firm j becomes self-financing, it can supply all firms with liquidity in the H-state. This puts a second cap on the price of s_H:

$$s_H \leq s_{jH}^{con} \equiv \frac{1 - f_L \rho_{j0}}{f_H(\rho_{j0} - \rho_{jH})} > 0. \text{for all } j \text{ (contrarian).} \tag{4.20}$$

A contrarian firm becomes self-financing at a lower price s_H, the more it supplies liquidity in state H and the higher is ρ_{j0}.

[7] The right hand side of (4.18) can be derived from condition (3.11) by setting $\rho_{jL} = 0$, substituting $q_{max} = f_L + f_H s_H$ and solving for the maximum value of s_H such that firm j will continue in both states.

[8] Recall that we must have $f_L \rho_H < 1$, else it would not be worthwhile for the firm to continue in both states when $s_H > 1$.

4.2 LAPM – the general case

In this main section we extend the two-state model to a general equilibrium model with a finite number of states and a finite number of heterogenous firms, whose linear technology parameters all can depend on the state of nature. The analysis highlights the close similarities of our model and the standard Arrow-Debreu general equilibrium model as well as some key differences.[9]

Let ω be the state of nature revealed at date 1, $f(\omega)$ the probability of ω, and $s(\omega)f(\omega)$ the date-0 price of liquidity delivered in state ω (at date 1). As before, all agents are risk neutral and indifferent about the timing of consumption, which implies $s(\omega) \geq 1$, since we use the value of a unit of consumption at date 0 as our numéraire. There are J firms indexed $j = 1, ..., J$ (or more precisely, J types of firms, each of measure $1/J$, with j a representative firm). Firm j's investment plan is $\{I_j, i_j(\omega)\}$, where I_j is the initial investment, and $0 \leq i_j(\omega) \leq I_j$ is the continuation investment in state ω. Firm j's technology is fully described by the tuple $\{\rho_{j0}(\omega), \rho_{j1}(\omega), \rho_j(\omega)\}$, where $\rho_{j0}(\omega)$ is the pledgeable return at date 2, $\rho_{j1}(\omega)$ is the total return at date 2 and $\rho_j(\omega)$ is the reinvestment shock at date 1 – all per unit of continuation investment $i_j(\omega)$. There is a strictly positive wedge between the firm's total output and what can be paid out to investors in each state, that is, $\rho_{j1}(\omega) > \rho_{j0}(\omega)$. For technical convenience, we assume that

$$\rho_{j0}(\omega) \neq \rho_j(\omega), \text{ for all } j \text{ and } \omega. \tag{4.21}$$

We also assume that

$$0 < \sum_{\omega \in \Omega_j^+} [\rho_{j0}(\omega) - \rho_j(\omega)]f(\omega) < 1 < \sum_{\omega \in \Omega_j^+} [\rho_{j1}(\omega) - \rho_{j0}(\omega)]f(\omega), \quad \text{for all } j, \tag{4.22}$$

[9] In Holmström and Tirole (2000), we study a liquidity asset pricing model using a different approach than in this section. We solve for an equilibrium by treating the corporate sector as a single entity that maximizes the aggregate entrepreneurial welfare. The technology is quite general, and the analysis richer, but the approach requires a particular distribution of entrepreneurial endowments for the maximization to match a competitive equilibrium.

Anther model combining asset pricing with corporate finance is Dow et.al. (2005).

where $\Omega_j^+ = \{\omega | \ \rho_{j0}(\omega) - \rho_j(\omega) > 0\}$ is the set of states in which firm j supplies liquidity. Assumption (4.22) guarantees that initial investments will be bounded and that the entrepreneur always prefers to invest his full endowment into the project.[10] To see this, consider an investment plan where firm j continues only in states where it supplies liquidity. Suppose that $s(\omega) = 1$ for all ω. Then the first term in (4.22) is the maximum share of the firm's investment that investors are willing to finance. The balance, which is strictly positive by (4.22), must be paid out of the endowment A_j. This limits the initial scale of the firm's investment, giving the entrepreneur the expected utility

$$\frac{\sum_{\omega \in \Omega_j^+} [\rho_{j1}(\omega) - \rho_{j0}(\omega)] f(\omega)}{1 - \sum_{\omega \in \Omega_j^+} [\rho_{j0}(\omega) - \rho_j(\omega)] f(\omega)} A_j > A_j. \tag{4.23}$$

We see that it is better for the entrepreneur to invest his full endowment in the firm and continue in all states where the firm supplies liquidity, than to consume his endowment.

Now, suppose that $s(\omega) > 1$ for some ω. In that case the entrepreneur's expected utility, using the same strategy as above, can be obtained by replacing $f(\omega)$ in the denominator with $s(\omega)f(\omega) > f(\omega)$. We see that the expected utility is higher than in (4.23), because investors are willing to pay a larger share of the initial investment, if the firm only invests in states where it supplies liquidity and some such states carry a liquidity premium. It is not in general optimal for the firm to invest only in states where it supplies liquidity. Private benefits may be high enough to warrant continuation even in states where the firm demands liquidity. However, the entrepreneur will always invest his full endowment in the firm, because of the linear technology.

Finally, let us introduce a source of outside liquidity in the form of assets $\{L_k\}, k = 1, ..., K$, each providing liquidity $L_k(\omega) \geq 0$ in state ω. For the equilibrium analysis, we treat outside liquidity as exogenous, but later on, when we discuss government policy, the supply of outside liquidity is endogenous.

[10] Assumption (4.22) is much stronger than needed, but alleviates notation without sacrificing insights.

The *aggregate supply of liquidity in state* ω is

$$L_S(\omega) = \sum_j \rho_{j0}(\omega) i_j(\omega) + \sum_k L_k(\omega). \tag{4.24}$$

The *aggregate demand for liquidity in state* ω is

$$L_D(\omega) = \sum_j \rho_j(\omega) i_j(\omega). \tag{4.25}$$

Equilibrium is achieved when prices $s(\omega) \geq 1$, and the firms' plans $\{I_j, i_j(\omega)\}$ are such that the net aggregate demand for liquidity by the corporate sector satisfies:

$$\sum_j [\rho_j(\omega) - \rho_{j0}(\omega)] i_j(\omega) \leq \sum_k L_k(\omega), \quad \forall\ \omega. \tag{4.26}$$

with an equality whenever $s(\omega) > 1$.

Given the prices $\{s(\omega)\}$, firm j solves the following problem:

$$\max_{\{I_j, i_j(\cdot)\}} \quad \sum_\omega [\rho_{j1}(\omega) - \rho_j(\omega)] i_j(\omega) f(\omega) \tag{4.27}$$

subject to

$$(i) \quad \sum_\omega [\rho_{j0}(\omega) - \rho_j(\omega)] i_j(\omega) s(\omega) f(\omega) \geq I_j - A_j \tag{4.28}$$

$$(ii) \quad 0 \leq i_j(\omega) \leq I_j, \quad \forall \omega.$$

The budget constraint (i) incorporates the purchase of sufficient liquidity to make the continuation plan $i_j(\omega)$ feasible in each state ω. As long as the aggregate supply constraint is satisfied, as it will be in equilibrium according to (4.26), all the firms' demands for liquidity can be met. For this reason, there is no need to include a date-1 liquidity constraint in the firm's program. We have written the objective function as the entrepreneur's expected payoff, because it equals the social surplus when the budget constraint binds. Assumption (4.22) guarantees that this will be the case, as we discussed above.

Prices. The date-0 price for a unit of the exogenous asset L_k, where the unit is defined as the amount of asset that in expectation delivers one unit of liquidity at date 1, is given by

$$q_k = \frac{\sum_\omega f(\omega)L_k(\omega)s(\omega)}{\sum_\omega f(\omega)L_k(\omega)} \tag{4.29}$$

Because $s(\omega) \geq 1$ and outside sources of liquidity, by assumption, deliver non-negative liquidity in each state of nature, the price of the exogenous asset k will be such that $q_k - 1 \geq 0$, with a strict liquidity premium whenever the asset supplies a strictly positive amount of liquidity in some liquidity-constrained state.

We define the date-0 price q_j of a unit claim on firm j's equity, as the amount that the firm can raise for investment purposes at date 0 per unit of expected return at date 1:

$$q_j = \frac{\sum_\omega f(\omega)[\rho_{j0}(\omega) - \rho_j(\omega)]i_j(\omega)s(\omega)}{\sum_\omega f(\omega)[\rho_{j0}(\omega) - \rho_j(\omega)]i_j(\omega)} = \frac{I_j - A_j}{\sum_\omega f(\omega)[\rho_{j0}(\omega) - \rho_j(\omega)]i_j(\omega)}. \tag{4.30}$$

As described in the two-state case, the logic behind this definition can be understood as follows. Take the case where a firm always demands liquidity in the states in which there is an aggregate liquidity shortage ($s(\omega) > 1$). In that case $q_j < 1$. For each unit that the firm raises from outside investors at date 0, it spends $I_j - A_j < 1$ on the initial scale of the investment and $\sum_\omega[(\rho_j(\omega) - \rho_{j0}(\omega))i_j(\omega)(s(\omega) - 1)f(\omega)] > 0$ on state-contingent claims that provide date-1 liquidity for continuation investments. The price q_j only reflects the return on the investment in scale. An alternative implementation has the investors (or their representatives, such as banks) promise to supply the liquidity that the firm needs at date 1. To back up that promise, investors must go out and buy the necessary amounts of state-contingent claims at date 0. The firm sets the date-0 issue price sufficiently below 1 so that investors are able to recoup their additional investment in state-contingent liquidity. In both of these implementations, the investors' net return is 0, despite the fact that $q_j < 1$.

Suppose now that the firm supplies liquidity in every state where there is an aggregate shortage and hence $q_j > 1$. The firm benefits from supplying liquidity by being able to raise more funds for investment than it would in a world where there are no liquidity shortages. The firm could issue separate claims on its date-1 return and the liquidity service that its assets provide. Or it could have investors invest in both claims and deal with the allocation of liquidity through contracts with other firms. However, investors do not invest in assets that command a liquidity premium for their own sake. Investors earn a zero net return, while the liquidity premium is determined by the intertemporal marginal rate of substitution of firms.

Equilibrium. The special structure of the model allows us to recast the equilibrium analysis in terms of an exchange economy without production. This is useful for understanding the underlying structure and convenient for proving existence and efficiency.

The key idea is simple. Instead of viewing the firm as choosing a continuation scale $i_j(\omega)$ in state ω, we can think of it as choosing its demand for liquidity $\ell_j(\omega)$ in that state, as defined by:

$$\ell_j(\omega) \equiv [\rho_j(\omega) - \rho_{j0}(\omega)] i_j(\omega). \tag{4.31}$$

In states where $\rho_j(\omega) - \rho_{j0}(\omega) < 0$, the firm supplies liquidity.

Recall that we assumed that $\rho_j(\omega) - \rho_{j0}(\omega) \neq 0$ for technical convenience (see (4.21)). With the change of variables (4.31), the constraint $0 \leq i_j(\omega) \leq I_j$ can then be expressed as

$$0 \leq \frac{\ell_j(\omega)}{\rho_j(\omega) - \rho_{j0}(\omega)} \leq I_j. \tag{4.32}$$

Note that this constraint forces $\ell_j(\omega)$ to have the same sign as $\rho_j(\omega) - \rho_{j0}(\omega)$ as required by (4.31).

Firm j's payoff in state ω becomes

$$U_j(I_j, \ell_j(\omega), \omega) = [\rho_{j1}(\omega) - \rho_{j0}(\omega)] i_j(\omega) = \frac{\rho_{j1}(\omega) - \rho_{j0}(\omega)}{\rho_j(\omega) - \rho_{j0}(\omega)} \ell_j(\omega). \tag{4.33}$$

For every firm j we assume that there is an upper bound $\bar{I}_j$ on the initial investment level. This assumption guarantees the compactness of the set of feasible investments. In the Appendix we will show that $\bar{I}_j$ can be chosen so that it does not bind in equilibrium. We define firm j's investment set as:

$$\Phi_j \equiv \{I_j, \ell_j(\cdot) |\ 0 \leq I_j \leq \bar{I}_j, \quad 0 \leq \frac{\ell_j(\omega)}{\rho_j(\omega) - \rho_{j0}(\omega)} \leq I_j \text{ for every } \omega\}. \tag{4.34}$$

Note that Φ_j is determined by primitives alone (including $\bar{I}_j$ as defined later).

The exogenously given liquid assets $L_k(\omega) \geq 0$ could be owned by the government and hence indirectly by the consumers and entrepreneurs. For simplicity, we assume that all of the outside liquidity is owned by the consumers and that the total amount of outside liquidity is strictly positive:

$$L(\omega) \equiv \sum_k L_k(\omega) > 0. \tag{4.35}$$

Firm j's choice problem is

$$\max_{\{I_j, \ell_j(\cdot)\}} \quad \{\sum_\omega U_j(I_j, \ell_j(\omega), \omega) f(\omega)\}, \tag{4.36}$$

subject to

$$I_j + \sum_\omega \ell_j(\omega) s(\omega) f(\omega) \leq A_j, \tag{4.37}$$

and

$$\{I_j, \ell_j(\cdot)\} \in \Phi_j. \tag{4.38}$$

With all consumers identical, the representative consumer solves

$$\max_{\{c_0, c_1(\cdot)\}} \quad \left\{ c_0 + \sum_\omega [c_1(\omega) + c_2(\omega)] f(\omega) \right\} \tag{4.39}$$

subject to

$$c_0 + \sum_\omega [c_1(\omega) + c_2(\omega)] f(\omega) \leq A^0 + A^1 + A^2 + \sum_\omega L(\omega)(s(\omega) - 1) f(\omega), \tag{4.40}$$

$$c_0, \ c_1(\omega), \ c_2(\omega) \geq 0. \tag{4.41}$$

Here (A^0, A^1, A^2) are the representative consumer's endowed incomes in the three periods. These endowments are sufficiently large so that the price of consumption at each date is equal to 1 (the normalized price of date-0 consumption). The endowed incomes are non-pledgeable. By contrast, the liquidity $L(\omega)$ that consumers own earns rents from securing commitments to fund reinvestments. The value of the rent in the budget constraint is $(s(\omega)-1)f(\omega)$ per unit. This assumes that $L(\omega)$ is not consumed (or rather, it is capital with a predetermined allocation of consumption benefits, which cannot be reallocated). The reason we use this formulation is that consumption out of $L(\omega)$, which also can act as collateral, would have to be treated separately from consumption out of non-pledgeable endowmed income. This would add notation, without changing anything materially.

The economy's resource constraints are:

At date 0:

$$c_0 + \sum_j (I_j - A_j) \leq A^0. \tag{4.42}$$

At date 1:

$$c_1(\omega) + \sum_j \rho_j(\omega) i_j(\omega) \equiv c_1(\omega) + \sum_j \rho_j(\omega) \frac{\ell_j(\omega)}{\rho_j(\omega) - \rho_{j0}(\omega)} \leq A^1, \quad \text{for every } \omega, \tag{4.43}$$

At date 2:

$$c_2(\omega) - \sum_j \rho_{j0}(\omega) i_j(\omega) \equiv c_2(\omega) - \sum_j \rho_{j0}(\omega) \frac{\ell_j(\omega)}{\rho_j(\omega) - \rho_{j0}(\omega)} \leq A^2, \quad \text{for every } \omega. \tag{4.44}$$

The consumers' date-0 endowment is allocated between consumption and the firms' initial investments. The consumers' date-1 endowment is allocated between consumption and the firms' reinvestments. Finally, at date 2, the consumers consume their date-2 endowment and the pledgeable income of firms. We have not included in these constraints the entrepreneurs' consumption, since it is a private benefit that cannot be reallocated.

In addition to the standard resource constraints, there is a constraint on the amount of aggregate liquidity (collateral) that is available in the economy in each state:

$$\sum_j \ell_j(\omega) \leq L(\omega), \quad \text{for every } \omega. \tag{4.45}$$

The liquidity constraints limit date-1 reinvestments, which have to be backed up by collateral.

We have transformed our original economy with production into a relatively standard exchange economy, in which firms solve the program (4.36)-(4.37), the representative consumer solves the program (4.39)-(4.40) and the economy's aggregate resource constraints are (4.42) through (4.45).

Because the prices of consumption at dates 0, 1 and 2 are all equal to 1, the only nontrivial equilibrium prices, $\{s(\omega)\}$, are associated with the aggregate liquidity constraints (4.45). An equilibrium is achieved when complementary slackness holds: For every ω, (i) $s(\omega) = 1$, if constraint (4.45) is slack and (ii) if $s(\omega) > 1$, constraint (4.45) binds. We can solve for the equilibrium prices $\{s(\omega)\}$ without explicitly considering consumer decisions, since consumers do not care about the timing of consumption. Their consumption is simply determined by the resource constraints (4.42)-(4.44) once we know $\{s(\omega)\}$.

The existence of equilibrium is proved in the Appendix. The main step in the existence proof is to show that prices and investments are bounded. We have $s(\omega) \geq 1$, because of the particular consumer preferences and our choice of units (date 0 unit of consumption has value 1). On the other hand, for each ω, there is a price $\bar{s}(\omega)$ such that whenever $s(\omega) > \bar{s}(\omega)$, all firms stop demanding liquidity in state ω (set $\ell_j(\omega) \leq 0$) *regardless of the other prices* $s(\omega')$, $\omega' \neq \omega$. We also show that the initial investments I_j are bounded by the amount that could be invested by firm j, if all of the economy's resources were devoted to maximizing I_j. This amount is finite, because we have assumed that every unit invested in a firm requires some amount of entrepreneurial endowment (see (4.22)).

In the Appendix we also show that the equilibrium is *constrained efficient* in the sense that there does not exist a plan of consumption and investment that satisfies the aggregate resource constraints (4.42) - (4.45), in which every entrepreneur and the representative consumer are as well off as in the price equilibrium, with at least one of them strictly better off. Note, especially, that even the social planner's allocations have to satisfy the aggregate liquidity constraint (4.45); the social planner cannot circumvent the fact that contingent reinvestment decisions must be backed collateral.

More general technologies. It is straightforward to extend the model and proofs of existence and efficiency to the case where each firm has access to more than one linear technology. A firm would then have to solve a more complicated linear program to determine how it should allocate its funds among the available technologies. Because the optimization takes place against a single resource constraint – the budget constraint – the solution will generically allocate all the capital to a single technology. The active technology will vary as the price vector $s(\cdot)$ varies. Since each technology will be associated with its own private benefit, private benefits as well as the resource costs will determine how much liquidity a firm demands or supplies. In response to an increased liquidity premium in a state ω, a firm will switch towards a technology that produces more liquidity in that state, giving up private benefits for the value of liquidity. Technologies with a smaller wedge between pledgeable and private income – for instance, technologies with more established (or earlier) cash flows or tangible assets would be favored.

4.3 Risk management

Traditional models of asset pricing, such as the CAPM, cannot explain why firms buy insurance against fires or other casualties, nor can these models provide much guidance on how firms should manage risk in general. The basic problem is that risk can just as well be managed by individual investors. Of course, there might be some scale advantages from having a firm deal with some of the risks it faces, but it seems implausible that these advantages would warrant the extensive attention to risk management that firms devote.

Firms do not buy only casualty insurance. They also spend large amounts of time and money on hedging against adverse financial risks like defaults, exchange rate shocks and other price fluctuations. Decisions on how much cash to keep on hand to cover unexpected liquidity needs and how to deal with fluctuations in the stock of cash are matters that the management agonizes over a lot. In the traditional theory they need not do that.

The liquidity asset pricing model developed in this chapter is a small step away from complete markets, but big enough to explain why firms care about liquidity and to give some insights into the way risk should be managed. However, as will become clear below, a full account of hedging and other risk management strategies must necessarily address implementation problems that depart even further from the standard model.

The perspective underlying LAPM is that firms should consider risk management a part of liquidity management. This means that risk should be managed jointly with the firm's investment decisions so that sufficient liquidity is assured throughout the planning horizon in the most cost effective way, and so that the cost of capital in future states guides the firm's investment plan. This perspective is not new. Froot et al (1993) have also analyzed the joint determination of financing and investment using an explicit agency model. One element that is new in our approach is that LAPM places the discussion in an equilibrium context where the price of liquidity influences risk management and conversely. Also, much of the Froot et al (1993) analysis is devoted to imperfect implementation of optimal risk management strategies rather than to fully contingent policies as discussed here.[11]

Before looking at the analytics, let us first consider a concrete question faced by many firms: Should a firm insure its production facilities against fire or other damage and if so, how extensively? The answer presumably depends on the cost of insurance. Let us assume insurance is provided competitively at market determined prices. Given actuarially fair prices, should the firm buy full coverage? Given that firms are effectively risk averse, as

[11] Holmström and Tirole (2000) addresses some implementation issues in the same set-up as here.

discussed in Appendix 2.2, one might think full coverage is optimal. On the other hand, perhaps one should embrace risk, given that a mean preserving spread in the date-1 liquidity shock increases welfare (as shown in section 2.4).

In practice, risk management policies typically recommend that "external" risks like a fire be fully insured or hedged. However, when one cannot find instruments that are perfectly correlated with the risk, hedging should be partial.[12]

The message from LAPM is rather different. The amount of hedging depends on whether risk affects the productivity of the firm or just the liquidity constraint.[13] Events that only influence the date-1 income of the firm, but not continuation investments, should be fully insured provided that current income is uncorrelated with future income.[14] On the other hand, an event that affects the productivity of the firm's date-1 investment will in general be partially rather than fully insured. A fire may fall in either category as discussed below.

To explain these conclusions let us return to the model with general shocks described in section 2.5.[15] We start by adding state-contingent liquidity prices to the model and after that introduce shocks to productivity. The basic problem is as before to decide the level of the date-0 investment I and the date-1 continuation scale $i(\omega)$ in all states ω. These decisions are solved by the program

[12] For a list of reasons why hedging should be partial rather than complete, see chapter 5 in Tirole (2006).

[13] We are assuming here that firms have access to insurance in every state, albeit at a potentially high price.

[14] Suppose that a high profit at date 1 is good news about the date-2 profit (as in de Marzo et al., 2009, or Tirole, 2006, pp.217-218). Then date-1 reinvestment is more desirable when date-1 profit is high. This pattern may be implemented in a variety of ways. A natural one is to partially hedge at date 0 the date-1 profit so as to provide the firm with more liquidity when its profits are high. Alternatively, the date-1 profit shock may be neutralized (fully hedged), provided that the firm receives a credit line that is indexed to the realization of profit.

[15] Note that the model in section 2.5 is more general than the formulation we have used here, as it includes date-1 income of two kinds and also a positive value of liquidation. The proof in the Appendix can be extended to cover existence of equilibrium and constrained efficiency in this more general set-up.

$$\max_{\{I, i(\omega)\}} \int b(\omega) i(\omega) dF(\omega), \tag{4.46}$$

$$\text{subject to}$$

$$\int [(\rho_0(\omega) - \rho(\omega)) i(\omega) + w(\omega)(I - i(\omega))] s(\omega) dF(\omega)$$
$$+ \int [r(\omega) I + y(\omega)] s(\omega) dF(\omega) \geq I - A, \tag{4.47}$$
$$0 \leq i(\omega) \leq I, \quad \text{for every } \omega. \tag{4.48}$$

As in section 2.5, we have replaced the parameter $\rho_1(\omega)$ with the entrepreneur's private benefit $b(\omega) = \rho_1(\omega) - \rho_0(\omega)$, to separate the effect that $\rho_0(\omega)$ has on the objective function from its effect on the budget constraint. Recall that the variable $w(\omega)$ is the liquidation value of a unit of initial investment at date 1, $r(\omega)$ is the date-1 income per unit of investment, and $y(\omega)$ as the income from old assets in place, which therefore does not depend on the initial investment.

Let $1 + \hat{\lambda} \geq 1$ be the shadow price of the budget constraint. The choice of $i(\omega)$ is then determined by the sign of

$$\hat{\Psi}(\omega) = b(\omega) - (1 + \hat{\lambda}) s(\omega) [\rho(\omega) + w(\omega) - \rho_0(\omega)] \tag{4.49}$$

as follows:

$$i(\omega) = 0, \text{ if } \hat{\Psi}(\omega) < 0$$
$$i(\omega) = I, \text{ if } \hat{\Psi}(\omega) > 0$$
$$i(\omega) \in [0, I], \text{ if } \hat{\Psi}(\omega) = 0.$$

Shocks to income at date-1 take the form $r(\omega) I + y(\omega)$. Since income shocks only affect the budget, they do not directly affect the continuation rule $i(\omega)$. There is an indirect effect through the budget and the return to the initial investment I, but these effects only depend on the expected values $E[s(\omega) r(\omega)]$ and $E[s(\omega) y(\omega)]$. All income shocks with the same *weighted* expected costs will lead to the same investment decisions and continuation rules. We interpret this as a case where income shocks are treated as if they are fully hedged, that is, income is as if it were constant.

Compared with a situation where there is ample liquidity in all states ($s(\omega) = 1$ for all ω), the full distribution of (the components of) the income shock, including their correlation with $s(\omega)$ will matter for firm decisions, because it affects the cost of capital $\hat{\lambda}$. However – and this is the key point – the change in $\hat{\lambda}$ will affect every contingent decision, not just the decision in state ω. There is no sense in which one can associate the continuation decision in a state with the total income in that state. Continuation decisions are determined by $\hat{\Psi}(\omega)$, which does not directly depend on the income.

The continuation index $\hat{\Psi}(\omega)$ measures the entrepreneurial value of continuing in state ω. The only change in the continuation rule compared to the case where there are no liquidity shortages comes from a positive liquidity premium $s(\omega) - 1 > 0$. The cost of capital now varies with the state ω, whereas before it was constant. Economically, liquidity premia are of course critically important. They influence the firm's investments as well as repayment plans and therefore what kinds of securities the firm should issue to the investors. Other things equal, firms want to repay investors in states where liquidity premia are high and continue investments in states where they are low.[16] As before, only the sum $\rho(\omega)+w(\omega)-\rho_0(\omega)$ of the liquidity shock, the liquidation value and (minus) the pledgeable income matters for the continuation decision, but now the importance of this sum relative to the private benefit depends on $s(\omega)$ with the weight on the private benefit $b(\omega)$ reduced in states with liquidity premia. The liquidation value is an opportunity cost exactly on par with the liquidity shock or a shock to pledgeable income.

Let us return to the fire insurance question, looking at it through the lens of (4.49). If a fire destroys some property, a building say, it is different from a shock to income,

[16] In Froot et al (1993), the a firm's optimal state-contingent plan manages liquidity so that the marginal value of future investments (in our model, continuation investments) are equalized across states.

because in addition to repairing the building there is the option to abandon it. The financial impact of a fire could take many forms, each affecting differently the decision to continue. Suppose the building is essential for production and the cost of repairing it is proportional to the scale of reinvestment. In that case there would be an additional liquidity shock per unit of investment $\widetilde{\rho}(\omega)$ so that the total per-unit cost of continuing would become $\rho(\omega) + \widetilde{\rho}(\omega)$. In states where there is no fire we would have $\widetilde{\rho}(\omega) = 0$. Assuming, for simplicity, that the cost of abandoning the building is zero, this case is then equivalent to a change in the distribution of the liquidity shock. The structure of the continuation rule would be as in (4.49) but with a change in $\hat{\lambda}$ due to the expected cost of covering the fire in some of the states. Even when the cost of the fire is constant ($\widetilde{\rho}(\omega) = c_{\mathtt{fire}} > 0$, when there is a fire), there may be states in which the building will be abandoned if there is a fire, because it happens to occur in a state with a high cost of capital. The optimal continuation rule would not insulate the firm against a fire.

The upshot is that, in contrast to income shocks, shocks to productivity will in general not be fully insured. Correlations between $\widetilde{\rho}(\omega)$ and $s(\omega)$ matter for this reason, but not because of any desire to hedge risks as such. Of course, one can also imagine a situation where rebuilding is always desirable and therefore insurance against a fire will be complete.

The conclusions above, while providing some insights into risk management and insurance, also suggest that risk management is linked fundamentally to implementation problems of the sort discussed in section 2.3. When investors cannot observe the shocks experienced by firms, insurance against fires, exchange rate fluctuations and related "external" shocks may become more desirable as a way to control the amount of liquidity the firm has available for reinvestments at date 1. To illustrate the point, let us go back to our basic model with a continuum of liquidity shocks (section 2.4), where there never was a shortage of liquidity (so $s(\omega) = 1$ for all ω), where liquidation was worth zero, where there was no kind of date-1 income, and where the only uncertainty concerned the cost of reinvestment $\rho(\omega)$. In this basic model, the second best was characterized by a cut-off rule ρ^* $(0 < \rho^* < \rho_1)$ such that the firm continued at full scale if and only if $\rho \leq \rho^*$.[17] Even if the investors did not observe the liquidity shock, the second best could be implemented by providing the firm with a credit line up to ρ^*; since the firm had no alternative use for funds, it would only use the credit line up to the amount needed to continue at full scale. Now add an income shock $y(\omega)$ to this set up. A cut-off rule ρ^* will still be optimal (though not necessarily the same cut-off as without $y(\omega)$), because the optimal decision rule fully insures against variations in the income $y(\omega)$, as we saw earlier in the chapter. However, if investors cannot contract on $y(\omega)$, the entrepreneur would invest $\rho(\omega)i(\omega) = \max\{(\rho^* I + y(\omega), \rho(\omega)I\}$ at date 1, since he always wants to continue at the maximum feasible scale (constrained either by the initial investment or the available funds). In particular, he would invest a positive amount even if $\rho(\omega) > \rho^*$, provided $y(\omega) > 0$ and he would be unable to go ahead at full scale even if $\rho(\omega) < \rho^*$, if $y(\omega) < 0$.

4.4 Concluding remarks.

The essence of our approach to liquidity is based on the twin assumptions that (i) there is a wedge between total income and pledgeable income and (ii) fully state-contingent contracts can be written on the pledgeable part of the income. This chapter has shown how these two assumptions make it both natural and relatively straightforward to carry out an asset pricing analysis that closely parallels that of an Arrow-Debreu economy. Despite a close formal analogy, the LAPM equilibrium differs from an Arrow-Debreu equilibrium in interesting ways. Notably, asset prices can exhibit liquidity premia. Since we assumed that consumers are risk neutral and indifferent about the timing of their consumption, liquidity premia are entirely driven by the corporate demand for liquidity. This segments investors into two: corporate entrepreneurs, who are willing to pay a

[17] The continuum of states assumption is inconsequential for the point we want to make. Even with a finite number of states, as we have in this chapter, the second best is characterized by a cut-off rule if the other assumptions of the basic model in section 2.4 hold.

premium, and investor-consumers, who are not.[18]

Liquidity premia affect the way corporations hedge risks and plan their investments, as discussed in the section on risk management. The most important message here is that, provided that income shocks are serially uncorrelated and independent of the firm's actions – that is, there is no moral hazard at date 0[19] – income shocks should be fully hedged, because for the budget only the expected value of income matters. By contrast, shocks to productivity are not fully neutralized. It is states of low productivity rather than low income that determine whether investments should be continued. As we noted, these conclusions are critically dependent on fully contingent contracting. If markets are incomplete, then hedging will typically be incomplete also for income shocks.[20]

Liquidity premia and the way they influence corporate investment and funding decisions (the issuing of securities) will play an important role when we go on to study the optimal use of outside liquidity both by the government and by international investors.

4.5 APPENDIX. Existence and efficiency of equilibrium.

We prove existence without consumers, since consumers play no material role in determining equilibrium prices. The consumer is only relevant for proving constrained efficiency, because of the rents enjoyed from owning $L(\omega)$.

[18] This claim needs a qualification. As we have discussed in this chapter, there are implementations of a firm's investment plan, where investor-consumers buy liquidity at a premium in order to back up promises of future payments. But this is not driven by a consumption demand for liquidity, but rather by the firm's demand for liquidity, that is, by segmentation.

[19] See Rochet and Tirole (1996) for an extension of our model to one with date-0 moral hazard affecting date-1 profitability.

[20] For more on this, see Froot et al (1993) and Holmström and Tirole (2000).

4.5.1 Existence.[21]

Our approach to existence is standard. We first define compact, convex choice sets, then a mapping from the product space of choices and prices into itself, which has a fixed point by Kakutani's theorem. Finally, we show that the fixed point is an equilibrium.

Define the price space $S = \{s(\cdot)|1 \leq s(\omega) \leq \bar{s}(\omega)$, for every $\omega\}$, where the upper bound $\bar{s}(\omega)$ is chosen so that

$$\bar{s}(\omega) > \max_{j} \frac{\rho_{j1}(\omega) - \rho_{j0}(\omega)}{\rho_{j}(\omega) - \rho_{j0}(\omega)}. \tag{4.50}$$

Note that S only includes the state-contingent prices at date 0, since the price of the date-0 good was normalized to 1. The upper bound $\bar{s}(\omega)$ ensures that aggregate demand for liquidity in state ω will be zero if $s(\omega) = \bar{s}(\omega)$, regardless of the other components of $s(\cdot)$. To see this, suppose that firm j demands a positive amount of liquidity $\ell_j(\omega) > 0$ when $s(\omega) = \bar{s}(\omega)$. Consider an alternative plan where: (i) the firm reduces the demand for liquidity in state ω to 0; (ii) the entrepreneur reduces his investment in the firm by the amount $\ell_j(\omega)s(\omega)f(\omega)$ and consumes it instead; and (iii) the investors increase their date-0 investment so that it exactly off-sets the entrepreneur's reduced investment. With these changes the budget will continue to balance, since the amount the entrepreneur has withdrawn is exactly the date-0 cost of the amount that investors were expected to invest in state ω in the original plan. Entrepreneurial consumption at date 0 is not optimal, of course, but it gives a lower bound on the value of the alternative plan. The benefit of the alternative plan is the entrepreneur's date-0 consumption value $\ell_j(\omega)\bar{s}(\omega)f(\omega)$. The cost of the alternative plan is the entrepreneur's loss of the private benefit in state ω. The expected net benefit of the alternative plan is therefore:

$$\ell_j(\omega)\bar{s}(\omega)f(\omega) - \frac{[\rho_{j1}(\omega) - \rho_{j0}(\omega)]\ell_j(\omega)f(\omega)}{\rho_{j}(\omega) - \rho_{j0}(\omega)} > 0, \tag{4.51}$$

[21] We are grateful to Philip Reny for comments on this section.

where the inequality comes from (4.50) and the assumption that $\ell_j(\omega) > 0$. We conclude that a strictly positive liquidity demand by firm j (hence any firm) cannot be optimal if $s(\omega) = \bar{s}(\omega)$.

Let the set $I = \Phi_1 \times ... \times \Phi_J$ be the space of investments and liquidity demands by firms, where

$$\Phi_j \equiv \{I_j, \ell_j(\cdot) | \ 0 \leq I_j \leq \bar{I}_j, \quad 0 \leq \frac{\ell_j(\omega)}{\rho_j(\omega) - \rho_{j0}(\omega)} \leq I_j \text{ for every } \omega\}.$$

Let us define the upper bounds on initial investments, which were left unspecified earlier. For firm k, let $\tilde{I}_k$ be the highest level of investment that it could achieve if all of the economy's resources were devoted to maximize I_k.That is, let $\tilde{I}_k$ solve

$$\max_{\{I_j, \ell_j(\cdot)\}} I_k \tag{4.52}$$

$$\text{subject to} \tag{4.53}$$

$$-\sum_j \sum_\omega \ell_j(\omega) f(\omega) \geq \Sigma_j (I_j - A_j), \tag{4.54}$$

$$0 \leq i_j(\omega) \leq I_j \quad \text{for all } .j. \tag{4.55}$$

Note that this program does not impose any liquidity constraints, just the *aggregate budget constraint* for the corporate sector (4.54) and the technological restrictions on continuation investments $i_j(\cdot)$. The aggregate budget constraint for the corporate sector does not include $L(\omega)$, which the consumers own. Given assumption (4.22), this program has a maximum. In fact, no firm other than k will be assigned a positive level of investment in the above program. The optimal plan will devote all of the aggregate endowment to firm k's investment. Any investment into a firm $j \neq k$ consumes some of the aggregate endowment, reducing resources for I_k.

We will see that the aggregate budget constraint cannot be violated in equilibrium, guaranteeing an interior solution if we choose as the upper bounds so that $\bar{I}_j > \tilde{I}_j$ for all j.

Define the mapping $\varphi: S \times I \rightarrow S \times I$ in the standard way. For every selection of prices $s(\cdot)$ in S, firms solve program (4.36) - (4.38), which determines the range of φ in I. And for every selection of investment decisions $\{(I_j, \ell_j(\cdot)) \in \Phi_j\}$ in I, the range of φ in S is defined by the set of prices $s(\cdot)$ that maximize the value of the excess demand for liquidity $[\Sigma_j \ell_j(\omega) - L(\omega)]s(\omega)$. This mapping is upper hemi-continuous, since firms solve a linear program with choices from a compact, convex set.

Let $s^*(\cdot), \{I_j^*, \ell_j^*(\cdot)\}$ be a fixed point of φ. Such a point exists according to Kakutani's theorem.

We already argued that if $s^*(\omega) = \bar{s}(\omega)$ for some ω, aggregate demand for liquidity would be zero in state ω, implying that there would be an excess supply of liquidity, since $L(\omega) > 0$. Given the definition of φ, this is inconsistent with $s^*(\omega) = \bar{s}(\omega)$ being part of a fixed point. While prices at the fixed point cannot be at their upper bound, some or all of them can be at their lower bound 1. The definition of φ guarantees that $s(\omega) = 1$ whenever there is an excess supply of liquidity in state ω. Only if the demand for liquidity equals the supply of liquidity can we have an interior price $s^*(\omega) > 1$. We conclude that at a fixed point, complementary slackness holds, as required by an equilibrium:

$$[\sum_j \ell_j(\omega) - L(\omega)][s(\omega) - 1] = 0, \quad \text{for every } \omega. \tag{4.56}$$

Finally, we need to verify that for each firm j, the initial investment I_j is strictly below the upper bound $\bar{I}_j$, when it chooses an optimal plan given prices $s^*(\cdot)$. To do so, we check that the firms' optimal plans at the fixed point must satisfy the aggregate budget constraint (4.54). Multiply the complementary slackness condition (4.56) by $f(\omega)$ and sum over ω. This gives:

$$\sum_\omega [\sum_j \ell_j(\omega) - L(\omega)][s(\omega) - 1]f(\omega) = 0. \tag{4.57}$$

We have then

$$-\sum_j \sum_\omega \ell_j(\omega)f(\omega) = -\sum_j \sum_\omega \ell_j(\omega)s(\omega)f(\omega) + \sum_\omega [s(\omega) - 1]L(\omega) \geq \sum_j (I_j - A_j), \tag{4.58}$$

The last inequality follows, since the first term in the middle expression is the sum of each firm's budget constraint in (4.37) and therefore greater than or equal to $\Sigma_j(I_j - A_j)$, while the second term in the middle expression is non-negative. The economic intuition is that when $s(\omega) > 1$, some of the budget will be spent on paying for liquidity and therefore the aggregate budget constraint will be satisfied with slack (the liquidity premium will go to the representative consumer).

Because the firms' investment plans at the fixed point satisfy the aggregate budget constraint, we must have $I_j < \bar{I}_j$ for every j. Therefore, firms would choose the same optimal plans without the imposed upper bound on I_j in the feasible set Φ_j. This concludes the proof of existence.

4.5.2 Constrained efficiency.

For constrained efficiency, we need to show that a plan that strictly Pareto improves on an equilibrium plan $\{I_j^*, \ell_j^*(\cdot)|j = 1, ...J\}$, given prices $s^*(\cdot)$, cannot be a feasible plan; it must violate some of the economy's four resource constraints (4.42) - (4.45). Consider an alternative plan in which the representative consumer consumes $\{\hat{c}_0, \hat{c}_1(\cdot), \hat{c}_2(\cdot)\}$ and the firms's invest $\{\hat{I}_j, \ell_j(\cdot)|j = 1, ...J\}$. Suppose it Pareto improves on the equilibrium plan. In the equilibrium plan, the budget constraint for each firm must bind, else the entrepreneur could do better by increasing the initial scale along with some continuation investments. The budget constraints for consumers must also bind, since otherwise consumers would do better by increasing their date-0 consumption. Since the alternative plan makes everyone as well off and some better off, it must violate the sum of the producer and the consumer budget constraints at the equilibrium prices $s^*(\cdot)$. Formally,

$$\begin{aligned} & \sum_j (\hat{I}_j - A_j) + \sum_j \sum_\omega \ell_j(\omega) s^*(\omega) f(\omega) + \hat{c}_0 + \sum_\omega (\hat{c}_1(\omega + \hat{c}_2(\omega)) f(\omega) \\ > \; & A^0 + A^1 + A^2 + \sum_\omega L(\omega)[s^*(\omega) - 1] f(\omega). \end{aligned} \tag{4.59}$$

Consider now the economy's resource constraints (4.42) - (4.45). Suppose that they are all satisfied in the alternative plan. Multiply each constraint by the corresponding equilibrium price (which is 1 for all constraints, except the liquidity constraint (4.45) for which it is $s^*(\omega)$) and add up the constraints. The result is an expression that is identical to (4.59) except that the inequality runs in the opposite direction. The assumption that the alternative plan satisfies all of the economy's resource constraints is inconsistent with the plan being Pareto improving. We conclude that the equilibrium allocation must be constrained efficient. A planner, who must the economy's resource constraints, including the liquidity constraints, cannot find an allocation that would be a Pareto improvement on an equilibrium allocation.

Part III

PUBLIC PROVISION OF LIQUIDITY

Chapter 5

Public provision of liquidity in a closed economy

Chapter 3 demonstrated that the wedge between total returns and pledgeable returns on investments can create a shortage of instruments for transferring wealth from one period to the next and thereby make it more costly or even impossible for firms to insure against future liquidity shocks through credit lines or other forms of advance funding. The problem reflects an imperfection in the insurance market between consumers and entrepreneurs. Consumers have the funds needed for insurance payments at date 1, but the corporate sector may not offer enough financial claims with which consumers can back up funding commitments at date 0. In the language of the book's title, there may be a shortage of inside liquidity (corporate backed claims) in some states of nature. This creates a demand for outside liquidity. But there is also a waste of potential liquidity in that consumers who have income may not necessarily dispatch it to the corporate sector when needed, because the latter is not profitable enough (private insurance has to be arranged ex ante and not ex post).

In this chapter, we will study the role of government as the sole source of outside liquidity. The government's policy can, at an abstract level, be viewed as remedying the waste of liquidity or, equivalently, as making up for the missing contracts between consumers and firms.

The government supplies liquidity in a variety of forms. Treasury bonds are the most obvious example. Consumers, firms and especially financial intermediaries use bonds as savings instruments, as risk sharing instruments and as collateral for complex state-contingent contracts such as credit swaps, repurchase agreements, and the like. Less direct, but equally important, are all the ways in which the government provides consumer insurance such as social security, health and unemployment insurance. The government also takes an active role in securing the functioning of financial markets. Deposit insurance, the discount window and various refinancing facilities are examples of on-going forms of liquidity supply. The recent subprime crisis shows that the government is prepared to provide substantial amounts of contingent liquidity, not just by lowering interest rates and easing monetary policy, but also by creating new, temporary credit facilities in an attempt to alleviate systemic disturbances in financial markets. These interventions can be interpreted as implicit forms of insurance. Our aim is to show that they can be rationalized within the logic of our model.

As always, one has to ask what enables the government to do more than the private sector can do on its own. In our view, the key feature that sets the government apart is its exclusive right to certain decisions, most importantly, the right to tax its citizens. As a result, the government can make commitments on behalf of consumers, including generations that are not yet born or actively part of the market. Put simply, the government can act as an insurance broker between consumers and firms, transferring funds from consumers to producers in states where the net returns from such transfers are high and making the corporate sector pay for this insurance either ex ante through liquidity premia on government securities or ex post with corporate taxes. Of course, if there is a fear that the government may default on its debt or inflate it away, it may be in a poor position to issue debt, or the debt may end up carrying an unreasonably high price, making the government's supply of outside liquidity unreliable or very costly. We will assume that the government is credible, but that taxation is costly.[1]

Another way to look at this is to note that consumers are like firms without pledgeable income (their ρ_0 is equal to 0), because they are unable to borrow against their future income. The state then acts like an intermediary. Through its regalian taxation power, it is able to transform some of the future income of consumers into pledgeable income (make their ρ_0 positive).[2]

When and how much funds should be transferred from consumers to producers depends also on the consumers' liquidity needs. Current generations of consumers may need liquidity precisely in the states that firms need liquidity. A case in point is unemployment insurance, which in most countries is taken up by governments and belongs to the set of automatic stabilizers; were unemployment insurance provided by firms, the (direct plus indirect) corporate liquidity demand in recessions would be even larger. For the most part, we will assume that consumer liquidity demand is unrelated to corporate liquidity demand (or that consumers represent future generations) and leave consumer demand for liquidity out of the picture, because it complicates the analysis. The exception is section 5.2, which analyzes a highly stylized model of risk sharing between consumers and firms with the government as a broker.

The chapter is divided into three parts. Section 5.1 focuses on government bonds and how they should be structured, viewing the issue purely from a liquidity point of view (the supply of government bonds is of course in the first instance determined by public finance considerations). Section 5.2 studies optimal risk sharing between firms and risk averse consumers, mediated by government taxes and subsidies. Section 5.3 gives two examples of indirect supply of government liquidity – in the first example through

[1] Kocherlakota (2001) takes exception with the view that the government has special contracting powers that private parties could not replicate. Instead, he analyzes a model, in some ways similar to ours, in which the government can improve social welfare by insuring, not the firm, but the financiers of the firm (they are all depositors, because debt is optimal due to private information). He views this as a rationale for bailouts, using the banking crisis in Japan as an example.

[2] In the same way that banks raise the pledgeability of borrowers' income in Holmström and Tirole (1997), say.

unemployment insurance, in the second example by supporting collateral values to avert a bad equilibrium.

5.1 Public supply of liquidity

5.1.1 Government bonds

The analysis of government supply of liquidity follows closely our analysis of private supply of liquidity in chapter 3. We continue to use the two-state, variable investment-scale model from section 2.2, focusing on the case of pure aggregate uncertainty. All firms are identical and hit by the same shock ρ, which can be high or low with $\rho_L < \rho_0 < \rho_H$. We assume for simplicity that there are no additional private stores of value. When the high shock ρ_H occurs, it hits all firms and no firm can continue without outside liquidity. Firms will nevertheless invest, because the benefits from continuing in the low-shock state are sufficiently valuable (recall that continuation is always feasible and desirable because it is Pareto optimal and therefore self-financing in the low-shock state).

We start with the case where the government issues non-contingent bonds at date 0. The face value of the bond is one unit of corn paid at date 1 regardless of the state of nature. To be specific, assume that when the government sells a bond at date 0, the bond is acquired by a firm using the funds available to the entrepreneur at date 0 (this includes his own funds A and what can be raised from the market). At date 1, if the liquidity shock is low, the bond goes unused and its value is returned to the investors, who hold the rights to all pledgeable income. This will allow the entrepreneur to maximize the initial investment and thus his private benefit. If the liquidity shock is high, the entrepreneur uses the proceeds from selling the bond to cover the liquidity shock[3]. Regardless of the shock, the government will redeem the bond at date 1 by taxing the consumers and paying the bond holders the face value of their bonds. At date 0, when the bond is issued, the

[3] An alternative interpretation with the same net outcome has consumers either directly or through intermediaries buy up bonds to back up pledges to cover a firm's high liquidity shock.

government receives $q \geq 1$ per bond and distributes the proceeds to the consumers. The net effect of these transactions is that consumers will transfer corn to firms in the high-shock state, but not in the low shock state. There will also be a net loss of corn due to the deadweight loss of taxation.

How is the price of the bond determined and how many bonds should the government issue? The answers to these questions depend on what objective function the government uses. Credit rationing models raise well-known conceptual problems for welfare analysis. Even though parties are risk neutral and have identical time preferences over consumption, Pareto optimal allocations cannot simply be determined by total surplus maximization. We assume that the government maximizes producer surplus subject to the constraint that consumers are not made worse off than they would be without the bond issue. This means that the deadweight cost of taxation will in the end be borne by the corporate sector and therefore the supply function of the government is determined by the marginal deadweight cost of carrying out a bond transaction.

Let $g(L)$ be the date-1 cost for the consumer as a function of the amount of bonds L that the government issues. One should think of the marginal deadweight loss, $g' - 1$, as the marginal cost of public funds, i.e., the cost of distortions embodied in taxation. For now, assume that the marginal cost g' is constant. Since the government's objective is to keep the consumers' welfare unchanged, the price of government bonds is set at $q = g'$.[4] At this price, firms can buy as many bonds as they wish. The outcome will be as in Figure 3.2, but with the equilibrium price fixed by q and the equilibrium quantity purchased by firms determined by the intersection between the government's horizontal supply curve and the firms' aggregate demand curve.

To avoid general equilibrium effects, we assume that the deadweight loss is associated with date-1 household production that is discouraged by government taxes. At date 0,

[4] In the general case, with convex g(L), the government will run a surplus when setting the price q equal to the marginal cost g'. This surplus would have to be distributed back to the firms in some manner.

when the government distributes the proceeds from the bond issue to consumers, there are no distortionary effects, because there is no production at that date. This makes the deadweight loss of taxation increase from date 0 to date 1. There is no reason why it should. Furthermore, it is unlikely to be optimal to shift the full burden to date-1 taxpayers. In general, it will be better to spread the cost of providing liquidity over many periods and across states of nature (for more on this, see section 5.2). The broad picture, however, remains the same: when the economy experiences major adverse liquidity shocks, it may be desirable to transfer funds from the consumers to the corporate sector. These transfers operate through coercive and distortionary recourse to taxpayer money. While the government should consider the overall deadweight loss in its fiscal and debt policy, it can create liquidity that the private sector cannot.

5.1.2 State-contingent bonds

If we were to reintroduce a storage technology, or other short-term private sector technology, it would appear from the previous discussion that private and public supply of liquidity would be qualitatively very similar. The two would generally co-exist and the equilibrium price would be determined by the intersection of the aggregate supply and demand curves of liquidity. The composition of the aggregate supply would be determined by the marginal cost of public and private supply, respectively.

This view is misleadingly simple. The government's actions will naturally influence state prices as determined in the equilibrium model of Chapter 4. Our arguments do not take into account these general equilibrium effects. They only reflect the value of marginal changes.

The view also overlooks a potentially significant advantage that government supplied liquidity has over privately supplied liquidity. When the private sector invests in a storage technology that yields a lower composite return than the long-term technology (taking into account both pledgeable and non-pledgeable income), the value of that liquidity will be wasted if the liquidity shock at date 1 turns out to be low. The problem is that the private sector often has to decide on the supply of liquidity before the state is known, whereas the government can wait and see whether the aggregate shock is high or low before taxing consumers. Only if the aggregate shock is high does the government need to step in and offer liquidity. This reduces the cost of providing liquidity without losing any of its insurance benefits. For this reason, the public sector may be able to produce liquidity more efficiently than the private sector.[5]

The superiority of contingent over non-contingent bonds.

First, suppose that the deadweight loss of taxing consumers (at date 1) is constant across states of nature and let λ be the loss per unit of tax raised. We show that the most effective way for the government to supply liquidity is to issue a *state-contingent bond* that pays

$$\begin{aligned} &1 \text{ if the aggregate shock is high, } \rho = \rho_H \text{, and} \\ &0 \text{ if the aggregate shock is low, } \rho = \rho_L. \end{aligned}$$

To make consumers as well-off as in the absence of a bond, the date-0 price of the contingent bond should be

$$q^c = f_H(1+\lambda). \tag{5.1}$$

Firms will demand contingent bonds as long as the state-contingent price of liquidity in the bad state in the absence of such bonds satisfies $s_H > 1+\lambda$ (see Chapter 4).

Compare this with the case of a non-contingent bond. Consumers' constant welfare condition requires

$$q^{nc} = 1+\lambda. \tag{5.2}$$

Let $\ell \equiv (\rho_H - \rho_0)I$ denote the amount of liquidity needed in state H (firms do not need any liquidity in state L). The net cost for the firms is $f_H(1+\lambda)\ell$ with a contingent bond

[5]Throughout this chapter we will set aside commitment, incentive, political economy and other organizational problems associated with government. To some extent, these could be incorporated into the shadow price of government funds.

and $(1+\lambda)\ell - f_L\ell$ with a non-contingent bond.[6] Thus, contingent bonds allow firms to save

$$f_L\lambda\ell. \tag{5.3}$$

More generally, if λ_L is the marginal deadweight loss of taxation in the L state, non-contingent bonds bring an extra surplus $f_L\lambda_L\ell$ to the firms. It is only when taxation is non-distortionary in the low state (relative to date 0 – see the discussion in subsection 5.1.1) that non-contingent bonds have no advantage over contingent ones.

The government's comparative advantage in providing contingent liquidity.

With these preliminaries, let us get back to the question whether the government can provide liquidity more efficiently than the private sector. Suppose additional liquidity in the private sector requires a physical investment, a silo for corn, say. Then the private sector's marginal cost of supplying liquidity will be determined by the cost of building additional silos, a cost analogous to the non-contingent bond. Let private investment cost c^{silo} and assume, for the sake of argument that $c^{silo} < q^{nc} = 1+\lambda$; if the government only could issue non-contingent bonds, it would be more efficient for the private sector to build silos than to use government supplied liquidity. Now assume that the government can issue state-contingent bonds. The public sector's cost of supplying liquidity is then $q^c = f_H(1+\lambda)$. This cost is proportional to the likelihood of the high state occurring. Assuming that the value of government supplied liquidity is strictly positive in the high-shock state $(1+\lambda < s_H)$, the government will be able to supply liquidity more efficiently than the private sector whenever

$$f_H < \frac{c^{silo}}{(1+\lambda)}. \tag{5.4}$$

We see that the government has a comparative advantage in supplying liquidity in sufficiently rare states of liquidity shortage.

[6] The net cost refers to the cost of the liquidity used less the value of unused liquidity. With a contingent bond, all of the liquidity is used. With a non-contingent bond, liquidity is unusued in the L-state, but the cost λ is still incurred.

This conclusion rests on the assumption that the private sector cannot offer state-contingent liquidity[7]. Why should that be the case? There are two reasons. First, and as we already noted, if there is too little pledgeable income in some state of nature, the corporate sector has to generate additional income in some manner. Typically, this involves an ex ante, real investment, which like the metaphorical silo will deliver liquidity not just in the desired state, but in a broad range of states. Liquidity will likely be wasted in a number of these states. The government, by contrast, can act ex post rather than ex ante. Ex post taxation does not involve an ex ante investment. Secondly, because the government can act ex post, it can avoid the difficulty of having to identify the state in advance. It can call a recession when it sees it. Note that even if the private sector is much more efficient than the government in building silos ($c^{silo} << 1 + \lambda$), the fact that the government can step in, even at a high cost, only when liquidity is needed, can give it a substantial advantage (see (5.4)). Below we will give some illustrations of ex post interventions by the government.

The logic of the two-state case carries over to many states. In general, the government's cost of delivering liquidity will vary with the state. One reason is that consumers may also have a demand for liquidity, resulting in a state-contingent deadweight loss of taxation. The relevant comparison will be between the consumer's marginal cost of taxation (or marginal value of liquidity) $1+\lambda(\omega)$ versus the firms' value of liquidity $s(\omega)$. The government should provide liquidity in state ω if and only if $s(\omega) > 1+\lambda(\omega)$.[8] With no deadweight losses from taxation and no consumer demand for liquidity, the government

[7] In the wake of the financial crisis, Kayshap et. al. (2008) have recommended that financial intermediaries should be required to carry private insurance against adverse events. To be credible, such insurance should be backed by Treasury bonds held in escrow by the insurer. This proposal can be viewed as a private market variant of our government insurance scheme (see Rochet 2008a). Note, however, that our equilibrium (see chapter 4) already employs this form of private insurance to the fullest. Our interest here is in the case where Treasury bonds are scarce, as the high liquidity premia experienced during the subprime crisis suggested they were.

[8] We should note again that the government's actions will naturally influence state prices as determined in the equilibrium model of Chapter 4. Our arguments do not take into account these general equilibrium effects. They are merely showing the value of marginal changes.

should provide liquidity in all states with a liquidity shortage (more on this in section 5.2). On the other hand, if the marginal cost of a tax dollar is already so high that it exceeds the private cost of supplying liquidity in a state, then the government should not provide any additional liquidity.

5.1.3 Ex post public liquidity provision

In practice, we rarely observe the government issuing explicit, state-contingent bonds. One interesting exception occurred at the turn of the millennium when the Federal Reserve Bank of New York decided to issue call options on the right to access the discount window (see Sundaresan and Wang, 2004). The Fed decided to offer state-contingent liquidity, because it feared that the computer systems might break down due to the so-called "Y2K problem", causing chaos in the financial markets. Indeed, the liquidity premia for dates shortly after January 1, 2000 were exceptionally high (about 150 basis points before the government intervened). The Fed's response to elevated liquidity premia in the market is consistent with the logic of state-contingent delivery of liquidity as discussed above. Optimal risk sharing (subject to budget and liquidity constraints) suggests that the Fed probably was right in using taxpayers' funds to provide insurance against exceptional aggregate liquidity shortages. Of course, we never got to observe how well the provision of liquidity would have worked had there been a major crisis, but the significant reduction in liquidity premia suggests that market participants thought the problem was alleviated by the Fed's intervention.

Typically, government bonds are not explicitly state-contingent; the Y2K problem is a notable exception. One reason government bonds are not state-contingent as the model prescribes is that it is hard to identify the right contingencies in advance. Instead, the government manages liquidity by following state-contingent policies that one can view as emulating a state-contingent bond.

Consider, for instance, monetary policy in the face of an adverse shock to the productive sector. A looser monetary policy drives bond prices up (interest rates down) helping entrepreneurs that bought government bonds to weather the shock. A loose monetary policy also reduces the cost of capital, especially for those firms and financial institutions with a substantial maturity mismatch (a very short average maturity of liabilities) and therefore a higher need for refinancing.[9] Consumers holding bank deposits and comparable assets are worse off. In effect, a loose monetary policy represents a transfer of wealth from the consumers to the producers, an outcome that is broadly consistent with what our analysis suggests the government should do.

The primary application of our analysis is to large tail risks such as those realized in the subprime crisis. The crisis is an example of an exceptional state, where the government was in a unique position to deal with the tail risk (ex post) by supplying badly needed liquidity. But the analysis also provides foundations for a fine-tuned, continuously adjusting liquidity management policy by the government. The magnitude of government intervention should be commensurate with the liquidity shortage. The latter may not be directly measurable, but a good indicator is the liquidity premium observed in markets.

The Federal Reserve Bank's attempt to deal with the potential Y2K problem as well as major financial crises, are fairly transparent examples of how the government engages in contingent liquidity supply. There are many other government programs with significant, indirect liquidity effects: the social security system, mandated or government-run unemployment insurance, use of deposit insurance premia that are not indexed to the banking industry's solvency, industry bailouts, and a host of welfare programs. We will come back to discuss two examples of these kinds of liquidity schemes in more depth after the next section. But first we show that the date-0 contingent-bond intervention studied earlier has a counterpart in terms of a date-1 ex-post liquidity support/bailout.

Suppose, as earlier, that the corporate sector's shortfall is $(\rho_H - \rho_0)I$ in the bad state

[9] To capture this in our framework, suppose that consumers demand a return R – normalized to 1 in this book – between dates 1 and 2; then for continuation investment i at date 1, the firm can raise up to $\rho_0 i/R$ at that date by issuing securities. Refinancing is thus easier, the lower the rate of interest.

of nature and that the shadow cost of public funds in that state of nature is λ. Let the government

- levy a tax $T = (\rho_H - \rho_0)I$ at date 1 on consumers in the bad state of nature and transfer it to corporations, possibly by taking a negative NPV stake in them;
- demand a "deposit insurance" premium $\tau \equiv f_H(1+\lambda)(\rho_H-\rho_0)$ per unit of investment at date 0. Note that τ exactly compensates the consumers for their date-1 expected loss.

The firms' budget constraint is then given by:

$$(1+\tau)I - A = f_L(\rho_0 - \rho_L)I,$$

which, given the choice of τ, amounts to

$$I - A + f_H s_H(\rho_H - \rho_0)I = f_L(\rho_0 - \rho_L)I,$$

where $s_H = 1 + \lambda$. Letting q be defined by $q - 1 \equiv f_H(s_H - 1)$, one gets back to equation (3.4):

$$I - A + q\,(\rho_H - \rho_0)\,I = f_L\,(\rho_H - \rho_L)\,I.$$

This demonstrates the formal equivalence between a contingent bond and an ex-post bailout in our stylized description of government provision of liquidity.[10]

5.2 Optimal supply with risk averse consumers

So far, we have assumed that consumers are risk neutral and therefore that the cost of government supplied liquidity stems from the deadweight loss of taxation. In this section

[10] Many economists have criticized the Federal Reserve for engaging in ex post redistribution (see, e.g. Meltzer, 2009). This criticism either overlooks or disagrees with the feasibility of government provided insurance as described here.

we will show that if consumers are risk averse, then even without a deadweight loss of taxation, the supply of government liquidity is limited.

The starting point is an economy where the corporate sector supplies its own liquidity and in the high state H, there is a shortage of liquidity. We assume that the state takes the liquidity premium determined in the private market, $s_H - 1$, as given. This assumption is fine provided that either public liquidity supply is small, or that the liquidity premium is fixed by a private, constant-return-to-scale, short-term savings technology.

The setting is as follows. There are two types of non-corporate agents:

- *Investors* are, as before, risk neutral, with preferences $c_0+c_1+c_2$. Their endowments can be hidden away (say abroad) and therefore cannot be taxed. By contrast, their investments in and returns from the firms can be taxed or subsidized. Taxation takes place at the firm level using lump-sum taxes. Let τ_0 , τ_L and τ_H denote the lump-sum taxes (subsidies if negative) at date 0 and at date 1, in states L and H respectively. The firm is protected by limited liability. The entrepreneur distributes all pledgeable income net of taxes. The firm's budget constraint is then

$$I - A + f_H s_H(\rho_H - \rho_0)i_H + \tau_0 \leq f_L[(\rho_0 - \rho_L)I - \tau_L] - f_H s_H \tau_H, \tag{5.5}$$

where $f_H s_H$ is the state price for liquidity at date 1 and $i_H \leq I$ is the continuation scale in the bad state of nature. Because of the firm's limited liability we have

$$\tau_L \leq (\rho_0 - \rho_L)I \tag{5.6}$$

and

$$\tau_H \leq 0. \tag{5.7}$$

In the low-shock state the tax on capital is bounded above by the return on capital; similarly in the high-shock state, investors receive no return and only capital subsidies are feasible.

- *Consumers* are risk-averse and do not invest (for example they could be successive generations, each living for a single period). Let T_0, T_L and T_H denote the taxes (subsidies if negative) that they pay. The government's budget constraint is:

$$[T_0 + \tau_0] + f_L[T_L + \tau_L) + f_H s_H [T_H + \tau_H] \geq 0.$$

The consumers have expected utility

$$U_0(-T_0) + f_L U_L(-T_L) + f_H U_H(-T_H),$$

where U_k is increasing and concave.

We look for a Pareto optimum, in which the state maximizes the consumer's welfare subject to a given level of utility for the representative entrepreneur.[11] For a given s_H, keeping the entrepreneurs' utility constant, requires keeping the total corporate tax $\tau_0 + f_L\tau_L + s_H f_H \tau_H$ constant. We can therefore rewrite the government's budget constraint as

$$T_0 + f_L T_L + s_H f_H T_H \geq \overline{T}_0. \tag{5.8}$$

The maximization of the consumers' utility

$$\max_{\{T_0, T_L, T_H\}} \{U_0(-T_0) + f_L U_L(-T_L) + f_H U_H(-T_H)\}$$

subject to the constraints (5.5) through (5.8) yields

$$U_0' = U_L' < U_H' = s_H U_0' = s_H U_L'. \tag{5.9}$$

Assuming that utility functions are not date- or state-contingent and $\overline{T}_0 = 0$, (5.9) implies that

$$(1 + f_L)T_0 + s_H f_H T_H = 0,$$

[11] Alternatively, we could as in section 5.1 maximise the entrepreneurs' utility subject to consumers being as well off as before government intervention (i.e. solve the dual program). The results would be the same.

and so

$$T_0 < 0 < T_H.$$

In the optimal insurance arrangement, consumers are asked to contribute in case of an adverse macroeconomic shock. More generally, if the consumer's utility is state-dependent (which is reasonable since workers may be laid off in bad times; see the next section), a sacrifice in the sense of a lower marginal utility is demanded from consumers in bad times. Even with a state-dependent utility function, consumers should share macroeconomic risks.

The extent to which consumers participate in supplying contingent liquidity depends on the cost of liquidity in the private sector. The optimal risk sharing solution indicates that the higher is the corporate sector's value from liquidity, as measured by the liquidity premium $s_H - 1$, the higher will be the participation by the consumers, that is, the more the government will supply liquidity.

Finally, we note that as long as corporate taxes (τ_0, τ_L, τ_H) are lump-sum and therefore do not distort the firm's behavior, their allocation is irrelevant, since the investors care only about the total pledgeable income net of taxes.

5.3 Other forms of government supplied liquidity

5.3.1 Short-term savings: unemployment insurance as liquidity

Let us return to the assumption that the corporate sector makes efficient use of liquidity, but question the premise that it does not influence the aggregate supply of liquidity. This premise is reasonable for assets such as Treasuries whose supply and state-contingent payoffs can be considered exogenous from the point of view of the corporate sector. But as we will show by means of simple examples the corporate sector's date-1 policy will in general affect aggregate liquidity and do so in ways that are quite relevant for policy making. In the first example, analyzed in this subsection, the aggregate supply of liquidity depends on the labor contracts agreed on in the corporate sector. In the second example, considered in the next subsection, asset prices depend on how much corporations sell assets to meet liquidity shocks. In both cases, coordination failures may occur and may occasion government intervention.

Assume, for the first example only, that consumers, whose mass is also 1, must consume at least a "subsistence level" $\underline{c}_1$ (of food, education, housing, etc.) at date 1. That is, we replace the utility

$$c_0 + c_1 + c_2$$

from consumption flow (c_0, c_1, c_2) by the utility

$$c_0 + u(c_1) + c_2,$$

where

$$u(c_1) = \begin{cases} c_1 & \text{,if } c_1 \geq \underline{c}_1 > 0 \\ -\infty & \text{,otherwise.} \end{cases} \tag{5.10}$$

Consumers thus care about the timing of their consumption as well as its overall level.

The representative firm has a fixed-sized investment opportunity at date 0 that costs I. It has no assets $(A = 0)$. As before, assume that consumers have enough of the non-storable good at date 0 to help finance the initial investment I. The firm's income at date 1 has two possible values: a high income y_H with probability α (the good state), and a low income y_L with probability $1 - \alpha$ (the bad state), where[12]

$$y_H > y_L = \underline{c}_1. \tag{5.11}$$

Continuation at date 1 requires, in addition to the entrepreneur, one worker/consumer

[12] That y_L is exactly equal to $\underline{c}_1$ facilitates the analysis, but is not crucial: y_L could be (at least a bit) above $\underline{c}_1$.

per unit of investment.[13] A worker must be paid an efficiency wage w, where[14]

$$w > \underline{c}_1. \tag{5.12}$$

Assumptions (5.10) and (5.12) imply that consumers do not value liquidity at date 1 provided that they know that they will have a job at date 1. Note also that the liquidity shock is here a random date-1 income rather than a random reinvestment need (the "investment" here is the non-stochastic wage w that the firm has to pay). There is of course no significant difference between the two approaches. We assume that there are no stores of value in this economy and so liquidity shocks must be met through retained earnings.

Continuation yields a private benefit $\rho_1 - \rho_0$ to the entrepreneur, and a pledgeable income ρ_0. Assume $\rho_1 > w$ so that continuation is efficient from the point of view of the entrepreneur.

As well, assume that

$$\rho_0 - w < 0, \tag{5.13}$$

$$y_L + \rho_0 - w \geq 0, \tag{5.14}$$

and

$$-I + \alpha(y_H + \rho_0 - w) \geq 0 \tag{5.15}$$

These conditions can be interpreted as follows: continuation is not self-financing ($\rho_0 < w$), but retained earnings combined with pledgeable income/dilution always enables continuation ($y_L + \rho_0 > w$). Finally, condition (5.15) implies that investors are willing to finance the initial investment as their return in the good state can cover the initial investment

[13] Since the measure of worker-consumers is the same as the measure of firms, there is full employment when all firms continue.

[14] We can invoke a standard efficiency wage argument where the worker can "steal" w. Suppose the worker's decision is verifiable ex post, that the worker is protected by limited liability and that stealing has disastrous consequences for production so that it is optimal to prevent it. Then a firm must pay its worker at least w.

outlay. With these parameter restrictions there is a feedback effect between aggregate liquidity and the maturity of savings such that multiple equilibria can arise:

- *A long-maturity, high-liquidity, high-employment equilibrium.* Suppose, first, that all firms continue at date 1 in both states of nature. All consumers then have a job at date 1 and receive a wage w. By (5.10) and (5.12) they have no demand for liquidity and are thus willing to defer all payments on their date-0 investment to date 2. Since the corporate sector need not meet any short-term payment obligations, it always has enough liquidity to pay the date-1 wage w, as $y_L + \rho_0 \geq w$ from (5.14). That is, even in the bad state of nature (state L), the firms can use their retained earnings and dilute somewhat their investors in order to cover the wage bill. Condition (5.15) then implies that the firms can repay date-0 investors out of the pledgeable income ρ_0 since $-I + [\alpha y_H + (1 - \alpha) y_L] + \rho_0 - w \geq 0$. In this efficient equilibrium, the corporate sector issues long-term claims and does not lay off workers.

- *A short-maturity, low-liquidity, low-employment equilibrium.* Suppose instead that firms are unable to continue in the bad state of nature (state L). Consumers become unemployed and, because they have to consume $\underline{c}_1$, they will insist on receiving at least $\underline{c}_1$ in this bad state of nature. Therefore, they want to hold short-term claims on firms. Condition (5.11) guarantees that this is indeed feasible, while (5.13) implies that given that workers are paid $\underline{c}_1$, firms no longer have any cash to finance the reinvestment. Firms continue only in the good state of nature, and equation (5.15) ensures that such a plan can be financed at date 0.

The latter equilibrium exhibits a coordination failure and is inefficient.[15] It illustrates in a stark manner that the liquidity available to the corporate sector depends on the

[15] One may wonder whether this coordination failure could be avoided if workers invested solely in their own firm at date 0 and signed a contract with their employer specifying that no cash will be withdrawn at date 1 as long as they are not laid off. This arrangement is not robust to minor perturbations of the model such as job mobility or idiosyncratic liquidity shocks. Suppose for example that with a small

liability side as well as the asset side. In this respect, it is interesting to note that public decision makers occasionally call for "well-oriented savings," that is, for a switch from short-term market investments by the households towards long-term, equity investments that will benefit the productive sector.[16]

A couple of additional remarks can be made. First, the *government can create aggregate liquidity by offering unemployment insurance*. Here, unemployment insurance (at or above $\underline{c}_1$) eliminates the consumers' demand for liquidity and restores the efficient equilibrium. This raises the issue of where the money for unemployment insurance comes from. If unemployment insurance is financed through a levy on corporations, it may not eliminate the bad equilibrium. Suppose, for instance, that all firms lay off their employees in the bad state and that the only source of cash for the government is a tax on firms. Then each firm must pay a tax equal to (at least) $\underline{c}_1$ and so in state L is unable to cover the efficiency wage plus the tax by issuing securities and using its short-term income. For deposit insurance to be effective, the cash must come from an external source such as a tax on future consumers (or of current consumers not affected by liquidity needs).

Second, multiple equilibria could not arise in this economy in the absence of agency problems: firms would be able to pledge their entire date-2 income, which would be enough to pay workers at date 1 in both states.

5.3.2 Prevention of fire sales

Events such as the recessions of the late '80s and early '90s or the recent subprime crisis often leave the financial institutions burdened with depreciated commercial real estate.

probability each firm receives no income at all at date 1. Then the workers, who want to secure $\underline{c}_1$, must diversify their portfolio (have claims at least worth $\underline{c}_1$ in other firms), and the coordination failure may still occur. (Technically, one must assume that y_L exceeds $\underline{c}_1$ slightly so as to offset the absence of income in this small number of firms and allow all consumers to "survive" in the bad state of nature.)

[16]See former French finance minister Jean Arthuis' January 14, 1997 speech at the parliamentary hearings on savings. The speech discussed several ways of encouraging equity investments, such as the creation of pension funds and the reform of the tax system (equity investments in France were taxed at the personal income rate; this implied at the time of the speech an overall tax rate of 61.7% for the highest tax bracket. In contrast, money market funds were taxed in a lump-sum, with withholdings ordinarily not exceeding 20%).

While banks, badly in need of liquidity, would like to divest their real estate holdings, they realize collectively that dumping real estate assets on the market simultaneously, would have a disastrous impact on prices in a state of low demand for commercial real estate. Cartel-like restraints on the sale of real estate or government support to stabilize real estate prices may prevent prices from falling too far.[17] In the eyes of an industrial organization economist, such price-fixing would appear wrong and in need of corrective measures. This section argues that there is more to it than just collusion.

An example with multiple equilibria

We return to our basic paradigm in which preferences are linear on the consumer side: $c_0 + c_1 + c_2$ (consumers have no liquidity needs). Suppose that, as in section 5.3.1, continuation at date 1 requires paying for an input, but this time, let the input be commercial real estate rather than labor. In case of continuation, one unit of commercial real estate is needed. The date-0 investment yields each firm a date-1 income $y_L = 0$ (the bad state) with probability $(1 - \alpha)$ and $y_H > 0$ (the good state) with probability α. Income shocks across firms are perfectly correlated. As in section 5.3.1, reinvestment yields a private benefit $\rho_1 - \rho_0$ at date 2, but no pledgeable income ($\rho_0 = 0$). The reinvestment cost is $\rho + \upsilon$, where $\rho > 0$ is a fixed cost and υ is the price of commercial real estate and depends on the income shock. We assume that $\rho < y_H$. Commercial real estate construction is part of the initial investment. Each firm builds one unit of real estate per unit of investment. Firms invest in commercial real estate at date 0, because they want to be able to produce at least in the good state. Real estate fully depreciates at the end of date 1. Letting ν_L and ν_H denote the commercial real estate prices in the bad and good income state, respectively, the overall liquidity need is

$$\rho \quad \text{in the bad state (probability } 1 - \alpha))$$

[17] Such cartels are sometimes organized by the Central Bank.

and

$$\rho - y_H < 0 \quad \text{in the good state (probability } \alpha\text{).}$$

Divested real estate is costlessly converted into residential real estate at date 1 on a one-to-one basis. To make our main point in the starkest way, suppose that there is a fixed (residual) demand for residential real estate at the price υ; the total demand for residential real estate is $\zeta < 1$. Thus, if less than ζ units of commercial real estate is converted, the price on the residential real estate market is υ; if more than ζ is converted, there is excess supply and the price on that market drops to 0 (or, to be precise, some small ε so that the sale of assets is a strictly preferred option).

$$\upsilon > (1 - \zeta)(\rho + \upsilon). \tag{5.16}$$

Again, there are two possible equilibria, which differ in the low-income state. In the high-income state firms can cover the reinvestment cost ρ without selling real estate.

- *Low-price, low-production equilibrium.* Suppose that in the bad state the corporate sector dumps all its commercial real estate onto the residential market. The price drops to 0, and since there is no other liquid asset besides real estate and pledgeable income ($\rho_0 = 0$) is smaller than the reinvestment cost (ρ), all firms are liquidated. Even though the commercial real estate is now free, firms are unable to continue.

- *High-price, high-production equilibrium.* Suppose instead that only a fraction $\hat{\zeta}$ of the assets are liquidated, where

$$\upsilon = (1 - \hat{\zeta})(\rho + \upsilon). \tag{5.17}$$

  Conditions (5.16) and (5.17) imply that $\hat{\zeta} < \zeta$, and so the market price of real estate is υ. The proceeds of the sale of a fraction $\hat{\zeta}$ of the firms' assets yields $\hat{\zeta}\upsilon$, which must be high enough to cover the firms' cost of reinvestment $(1-\hat{\zeta})\rho$ corresponding to the real estate that they retain, $(1-\hat{\zeta})$; (the firms own their real estate so that part of this liquidity need is already provided). From (5.17)

$$\hat{\zeta}\upsilon = (1-\hat{\zeta})\rho.$$

In this equilibrium, consumers (involuntarily) provide liquidity to the corporate sector by paying a higher date-1 price for residential real estate, and the restraint on sales provides insurance to the corporate sector against a complete price collapse and total credit rationing at date 1.[18] This implicit insurance raises ex-ante social surplus.

To conclude (and as illustrated in Figure 5.1), if dumping real estate on the market provokes a steep fall in real estate prices, there may be multiple equilibria. Measures that prevent a fire-sale of assets can be ex ante Pareto improving in such a case (provided that the consumers are appropriately compensated).

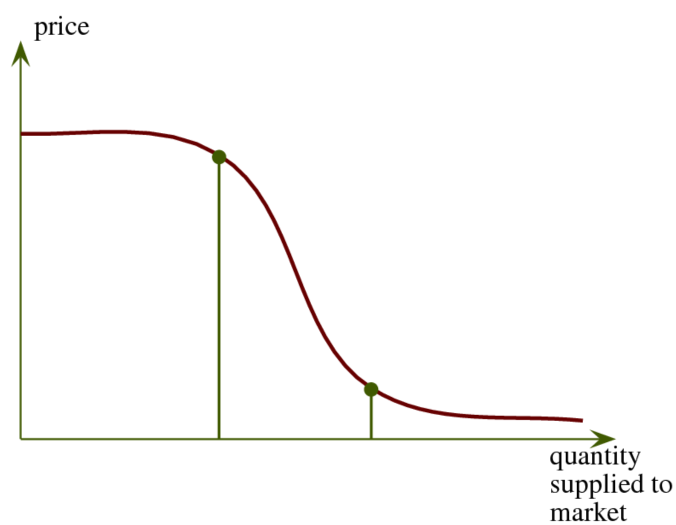


Figure 5.1

[18] This example is in the spirit of Kiyotaki and Moore's (1997) analysis of the dual role of assets as stores of value and inputs into production. Our analysis differs both in the key drivers (our treatment relies on the existence of aggregate shocks while theirs does not) and in the emphasis (they stress the possibility of business cycles while we emphasize market power and liquidity creation through price support policies.)

Remark: Allen and Gale (e.g.,1994, 1998) have provided early analyses of fire sales in the context of the Diamond-Dybvig model of consumer liquidity demand reviewed in Appendix 2.2. In those papers investors invest in an uncoordinated fashion in liquid (short-term) assets, that yield a safe return at the intermediate date, and in higher-yield illiquid (long-term) assets. A fraction of consumers, the "impatients", want to consume at the intermediate date; they use the returns on the short-term assets and also resell their long-term assets. How much these long-term assets fetch in the secondary market depends on the realized number of consumers who desire to consume early; so there is aggregate uncertainty. Consumers who desire to consume late use the proceeds of their short-term assets to purchase the long-term assets not wanted by the consumers who desire to consume early. The former – the buyers – have limited cash on hand, and so the asset price decreases when more consumers – the sellers – want to dispose of their long-term assets in the market. This phenomenon is called "cash-in-the-market pricing" by Allen and Gale.[19]

Soft pricing: selling assets to consumers at the intermediate stage.

Finally, we turn to a brief analysis of price stabilization using restricted trade. Assume that the representative firm invests a fixed amount I at date 0, and faces a random shock ρ per unit of continuation investment at date 1, where ρ is drawn from the distribution $F(\rho)$. It can then reinvest $i(\rho) \in [0, I]$. The shock is the same for all firms so there is only aggregate uncertainty. Each identical entrepreneur has endowment $A \geq I$ at date 0 and no endowment thereafter.

The only store of value in the economy is the investment itself, which can be sold to consumers (liquidated) at date 1 at a market determined price p. When facing a shock

[19] Allen and Gale then allow intermediaries to pool liquidity, while still offering non-contingent deposit contracts. The lower the resale price, the more long-term assets the intermediary needs to sell in order to honour its commitment towards depositors. This, together with the intermediaries' limited liability, adds a discontinuity in the resale price of the secondary asset. If the resale price is too low, the intermediary goes bankrupt and then its entire holdings of long-term assets are dumped on the market, creating a "crisis". Related work on fire sales has recently been done by Diamond and Rajan (2009).

$\rho > \rho_0$, firms must sell a fraction of their initial investment if they wish to continue. Let m be the amount of assets placed on the market at date 1. The consumers' demand for assets at date 1 is described by a downward sloping demand curve $p(m)$, which is derived from the marginal consumer surplus $S'(m)$. The revenue collected by the representative firm is $R(m) \equiv p(m)m$. For purposes of illustration, we will focus on the *linear case* $p(m) = 1 - m$.

(i) *Free market*

Provided that the value ρ_1 is "large enough" (see below), firms choose to reinvest as much as they can by selling in the secondary market. Given the state of nature ρ and the market price $p(m)$, the maximum scale of investment i is given by

$$i = \frac{p(m)I}{(\rho - \rho_0) + p(m)}.$$

In equilibrium we must have

$$(\rho - \rho_0)(I - m) = R(m) = p(m)m.$$

This equation defines the equilibrium amount of assets put on the market $m = m^*(\rho)$, or equivalently the continuation investment $i^*(\rho)$, since $i^*(\rho) = I - m^*(\rho)$. In general, the equilibrium will not be unique; in the linear case there is either one or two equilibria.

Continuing with the investment at date 1 has a positive net present value provided that $\rho_1 \geq p + \rho$. For example, with $I = 1$, we have $(\rho - \rho_0)(1 - m) = m(1 - m) \Longrightarrow m^*(\rho) = \rho - \rho_0$ (a unique equilibrium). In this case, the net present value will be positive if $\rho_1 \geq 1 + \rho_0$.

(ii) *Price stabilization*

Let us now show that a state-contingent cap on sales m can sometimes increase total surplus, which together with a date-0 transfer from entrepreneurs to consumers results in a Pareto improvement. To this end, imagine that consumers and entrepreneurs agree at date 0 on a transfer $T \leq A - I$ and date-1 policies $i(\rho)$ and $m(\rho)$. Given the consumers' gross surplus $S(m)$ from acquiring m units of assets at date 1, the optimal insurance contract solves:

$$\max_{\{T,i(\cdot),m(\cdot)\}} \{-T + E[(\rho_1 - \rho_0)i(\rho)]\}$$

subject to

$$T + E[(\rho_0 - \rho)i(\rho) + \mathcal{S}(m(\rho))] \geq \underline{U},$$

$$(\rho - \rho_0)i(\rho) \leq R(m(\rho)), \quad \text{for all } \rho,$$

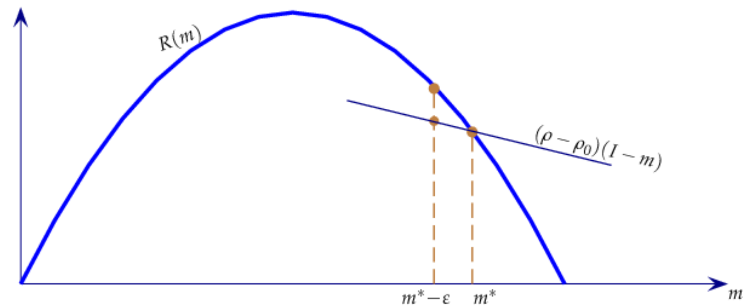
$$i(\rho) + m(\rho) \leq I,$$

$$I + T \leq A.$$

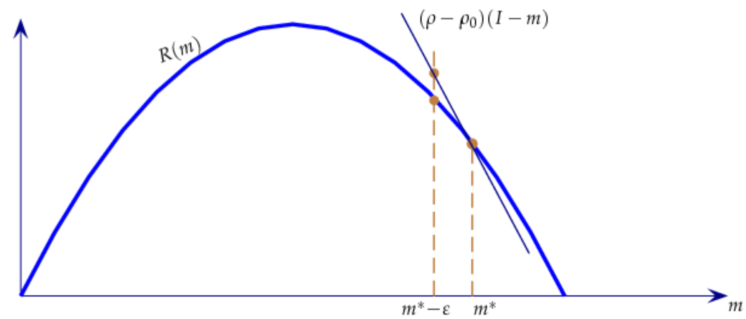
In this Pareto program, the representative entpreneur's expected utility is maximized subject to consumers being guaranteed some level of utility $\underline{U}$. The reinvestment $i(\rho)$ may be constrained either by the date-1 liquidity constraint $(\rho - \rho_0)i \leq R(m)$ or by the technological constraint $i + m \leq I$.

Rather than solve the entire program, we will investigate whether reducing m slightly by capping sales below the market level can improve welfare. To simplify matters we assume that the entrepreneur's endowment A is so large that the shadow value of the consumers' budget constraint is 1. In that case, we can substitute the budget constraint into the objective function to obtain the maximand $E[(\rho_1 - \rho)i(\rho) + \mathcal{S}(m(\rho))]$. Note that if the market equilibrium $m^*(\rho)$ lies below the monopoly level (i.e. $R'(m^*(\rho)) \geq 0$) it cannot be improved by capping m, because a decrease in m reduces the revenue $R(m)$ and, through the liquidity constraint, the reinvestment i. Therefore, we are only interested in levels of m beyond the monopoly level.

Two cases, depicted in Figure 5.2, need to be considered:



Case 1: technology constraint binds when m is capped



Case 2: liquidity constraint binds when m is capped

Figure 5.2

In case 1, the revenue curve $R(m)$ has a higher slope at the selected market equilibrium (lying to the right of the monopoly price) than the liquidity demand $(\rho - \rho_0)(I - m)$. We see from the figure that a cap at $m^* - \varepsilon$ increases revenue by more than enough to finance an additional reinvestment ε as permitted by the technological constraint $i(\rho) + m(\rho) \leq I$; the technology constraint will bind, while the liquidity constraint will be slack with the cap. The increase in total surplus (the objective in the Pareto program) is $(\rho_1 - \rho - S'(m^*(\rho))\varepsilon = (\rho_1 - \rho - p(m^*(\rho)))\varepsilon$, which is strictly positive. A cap improves welfare, provided that consumers are compensated through an increase in T.

In case 2, a restraint on sales generates little income and, as the figure indicates, the liquidity constraint will be binding, $(\rho - \rho_0)i = R(m)$, while the technology constraint stays slack, $i < I - m$. This means that some of the initial investment will be completely wasted. Nonetheless, the cap *may* generate an increase in surplus. This occurs when

$$(\rho_1 - \rho)di + p\,dm = \left[(\rho_1 - \rho)\left(-\frac{R'}{\rho - \rho_0}\right) - \rho\right]\varepsilon > 0.$$

For example when $I = 1$ (and $p(m) = 1 - m$), this condition becomes $\rho_1 > \rho_0 + \frac{m^2}{2m-1}$. Near the monopoly price, very little extra revenue is generated when m is reduced, implying that i cannot be increased much and instead most of the reduction in m goes to waste. In case 2, an equilibrium that is close to the monopoly price cannot be improved upon using a cap.

We conclude that efforts to stabilize the market price (perhaps with the help of government), by reducing the amount of asset sales when liquidity is scarce, can be beneficial. Of course, price stabilization is only one instrument to transfer income from consumers to firms. For policy purposes, the relative merits/inefficiencies of the various instruments need to be compared, but we have not attempted to do that.

Remark: Lorenzoni (2008) motivates the consumers' downward sloping demand for second-hand corporate assets in a different way (consumers are endowed at date 1 with an alternative, decreasing-returns-to-scale storage technology), but this is inconsequential. He also shows that there is over-borrowing (I is too high).

5.4 Concluding remarks

The main message of this chapter is that the government can play an important role in supplying and managing liquidity. When there is a wedge between total and pledgeable income, privately supplied liquidity may not be sufficient for supporting second-best production plans. Put differently, optimal risk sharing within the productive sector, when all contracts have to be backed up claims on real output, may fall short of second-best, because of a shortage of aggregate liquidity. Consumers, directly or through intermediaries, would be willing to provide insurance that firms want, but there are not enough claims to back up such promises. We take the view that the government is in a position to back up such promises because of its unique right to tax. We have shown that granted this ability, government intervention, especially in rare states of nature where liquidity is exceptionally short, is warranted. We have also shown that the basic logic behind the optimal supply of government liquidity follows the logic of state-contingent pricing of the kind seen in traditional general equilibrium models.

One of the many open issues that deserves serious attention is the optimal channel for the government to use in distributing liquidity. It is reasonable to conjecture that banks and related intermediaries are the natural channels to use, because they have the expertise to know which entrepreneurs are deserving of more liquidity. Another important issue is what price signals the central bank or the government might use to decide when and how to intervene. The central bank is in a unique position to signal or certify states of liquidity shortage by making announcements or more convincingly by acting.[20] The YK2 intervention as well as the interventions triggered by the subprime crisis were based on price signals such as liquidity premia and unusual spreads. It is likely that other signals could have proved useful, too. Also, the government should be, and seems to be, letting the shape of the yield curve determine the duration of bond issues. The optimal use of the yield curve could be approached from the point of view of aggregate liquidity shortages as well (see Holmström and Tirole, 2001).

We have also hinted at the possibility that liquidity depends in complex ways on the equilibria in the labour and asset markets. Again, this suggests some scope for government intervention. Viewed more broadly, the government can either create some "general purpose liquidity" in the form of stores of value that any economic agent in the economy can make use of. Or it can provide more "directed liquidity", that aims at resolving a particular source of liquidity shortage. Unemployment insurance makes consumers more willing to invest their savings in long-term securities, thereby creating more liquidity for

[20] For a specific proposal, see Caballero and Kuralt (2009).

the corporate sector. Asset price stabilization makes the corporate sector less dependent on outside stores of value. In a similar vein, the state often targets its provision of liquidity to the banking sector, that then dispatches the liquidity to the non-financial sector. We know very little about the optimal *structure* of liquidity provision, and this is definitely a key subject for future investigation.

Throughout this chapter, we have assumed that the government ccan costlessly commit to actions, such as state-contingent interventions. We have done so, to highlight the potential benefits of government interventions. In reality, problems of commitment are serious and will constrain what types of interventions are worthwhile. The political economy issues related to government interventions is a large and fruitful area of research.

Chapter 6

Is there still scope for public liquidity provision when firms have access to global capital markets?

In this chapter we study an economy with free access to global financial and goods markets. We have two objectives in mind with this extension. The first is to suggest that the model we have developed, appropriately extended to an international context, can offer a useful perspective on worldwide events such as the 1990s financial crises in Thailand and Mexico and the current concerns about global imbalances. Our second, closely related objective is to address a conceptual question of great relevance for our study: How can a shortage of savings/insurance instruments in a small country like Thailand play any significant role given the enormous depth, scope and liquidity of today's global financial markets? Why would Thai companies, banks and the Thai government not be able to meet their liquidity needs on international financial markets using foreign exchange swaps, dollar or euro loans and lines of credit and foreign bond and equity investments? Our theory rests on a shortage of aggregate liquidity within a country. If a national shortage of liquidity could easily be overcome through international financing, our theory would be of little relevance.

There is a simple reason why international markets cannot meet a country's liquidity needs despite an abundance of international financial instruments for saving and insurance. Foreign investments and foreign debts have to be paid with (or be backed up by) the country's pledgeable income internationally, namely its net production of tradable goods. Therefore the amount of foreign liquidity that a country can access – the amount of international insurance that it can buy – is constrained by the amount of pledgeable *tradable* income that it has. Thus, the problem is not that international markets have limited instruments for transporting wealth from one period to the next or for securing insurance across states, but rather that the country that seeks insurance may not produce enough tradable goods to pay for them.

The theory of aggregate liquidity shortages has interesting applications in international finance. Following Caballero and Krishnamurthy (2001, 2002, 2003a,b), we distinguish between a country's international collateral and its domestic collateral. International collateral consists of claims backed up by tradable goods, while domestic collateral consists of claims backed up by non-tradable goods.[1] Our objective is to indicate, through two simple examples, how the implications of aggregate liquidity shortages get enriched and modified when firms have access to international financial markets.

We show that, if international collateral is scarce, the need for domestic collateral and government intervention remains relevant as in our earlier analysis without international financial markets, but the prescriptions are somewhat different. We analyze how the task of liquidity supply is ideally shared between the three providers of liquidity: (i) the corporate sector; (ii) the domestic government; and (iii) the international financial markets. We conclude with some thoughts on the role of multilateral organizations such as the IMF in dealing with national liquidity shortages.

[1] Domestic consumers will be indifferent between consuming tradable and non-tradable goods. For this reason, we could alternatively have defined domestic liquidity as the value of all pledgeable goods.

6.1 A model with domestic and international liquidity

We extend our basic model to a small open economy facing aggregate shocks that on the international financial markets can be considered idiosyncratic and therefore are priced like riskless assets. We are interested in how the corporate sector makes optimal use of international insurance and how the government supplies and manages liquidity in this situation, assuming that arbitrarily rich contracts contingent on domestic shocks can be written.

- *Goods and preferences.*

  There are two kinds of goods:

  - *Tradable goods*, which are consumed by foreigners as well as domestic residents. These goods will at times be called *dollar goods* or simply dollars.

  - *Nontradable goods,* which only domestic residents consume. These goods are called *peso goods* or pesos.

All variables referring to dollars are indexed by a \$-sign, while those referring to pesos are non-indexed.

There are three periods, $t = 0, 1, 2$. Economic agents only care about the sum of their consumption at the three dates, and therefore demand a zero expected rate of return on investments. We further assume, mainly for convenience, that domestic residents view tradables and nontradables as perfect substitutes. Thus, a foreigner's utility from the consumption stream $\left\{c_t^{\$}\right\}_{t=0,1,2}$ is

$$\sum_{t=0}^{2} c_t^{\$},$$

while a domestic resident's utility from the consumption stream $\left\{\left(c_t^{\$}, c_t\right)\right\}_{t=0,1,2}$ is

$$\sum_{t=0}^{2} \left[c_t^{\$} + c_t\right].$$

We use date-0 pesos as our numeraire.

All uncertainty is realized at date 1. Let ω be the realized state of nature, with ex ante density $f(\omega)$. In line with the general model in chapter 4, we let $s(\omega)f(\omega)$ be the date-0 price of a peso delivered in state ω at date 1 (or date 2 – it does not matter) and $s^{\$}(\omega)f(\omega)$ the corresponding date-0 price of a date-1 dollar.

The model has a real exchange rate, defined as the relative price of tradables over nontradables. The date-0 exchange rate, denoted e_0, is the peso price of a dollar at date 0. The date-0 forward exchange rate for dollars at date 1 in state ω is denoted $e_1(\omega) = s(\omega)/s^{\$}(\omega)$. In equilibrium, $e_0, e_1(\omega) \geq 1$, because domestic consumers are indifferent between pesos and dollars delivered in state of nature ω.

Technologies and timing.

There is a continuum of ex ante identical domestic firms with mass 1, each run by an entrepreneur. The representative firm has a peso endowment A and a dollar endowment $A^{\$}$. With little loss of generality, we assume that domestic consumers have no dollar endowments, but they have, as before, non-binding amounts of peso goods at all dates. In addition, there is an external supply of date-1 liquidity in the form of tradable claims on peso goods L and dollar goods $L^{\$}$.[2] We do not specify the source of the external liquidity, but later on assume that the government can affect the overall level of external liquidity.

We will use technologies of the following generic form (see Figure 6.1).

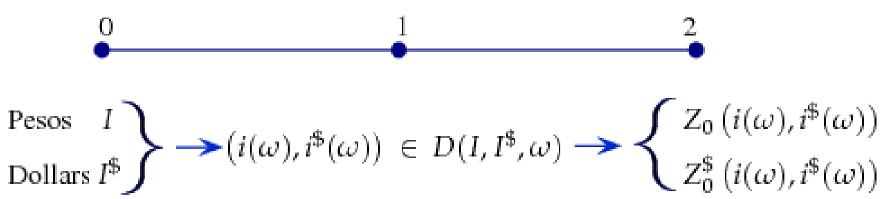


Figure 6.1: International liquidity

[2] Alternatively, we could assume that there are storable goods at date 0 in the amounts L and L^*.

Date 0: Firms invest I, $I^\$$.

Date 1: Conditional on the realized state of nature ω firms make reinvestments $i(\omega)$ and $i^\$(\omega)$. The total cost of reinvestment in state ω is $Z(i(\omega), i^\$(\omega))$ pesos and $Z^\$(i(\omega), i^\$(\omega))$ dollars. We use capital letters to distinguish total payoffs from per-unit-of-scale payoffs. Reinvestments $(i(\omega), i^\$(\omega))$ are constrained by the initial investments and the state ω, described abstractly at this point by a compact set $D(I, I^\$, \omega)$, that is, $(i(\omega), i^\$(\omega)) \in D(I, I^\$, \omega)$.

Date 2: The pledgeable peso and dollar payoffs $Z_0(i(\omega), i^\$(\omega))$ and $Z_0^\$(i(\omega), i^\$(\omega))$ as well as the total payoff $Z_1(i(\omega), i^\$(\omega))$, are realized. We assume the non-pledgeable private benefit $Z_1 - Z_0 - Z_0^\$$ is always strictly positive. Note that we write the total payoff without a **$**-sign, because it makes no difference whether private benefits are peso or dollar goods.

Because international investors only consume tradables, all promises to international investors have to be backed up by claims on dollar goods. The amount of *international collateral* produced by the corporate sector is $Z_0^\$(i(\omega), i^\$(\omega))$. Since domestic investors are indifferent between consuming tradables and non-tradable goods the total amount of collateral available for transactions on the domestic market is $Z_0(i(\omega), i^\$(\omega)) + Z_0^\$(i(\omega), i^\$(\omega))$, the sum of *domestic collateral* $Z_0(i(\omega), i^\$(\omega))$ and international collateral $Z_0^\$(i(\omega), i^\$(\omega))$.

The firm's decision problem.

The representative firm has to decide on the level of initial investments I and $I^\$$ and the date-1 choice $(i(\omega), i^\$(\omega))$. The firm also has to make sure it has enough liquidity to implement this investment plan. For this purpose the firm buys domestic and international liquidity $\ell(\omega)$ and $\ell^\$(\omega)$ at the going prices $s(\omega)f(\omega)$ and $s^\$(\omega)f(\omega)$. These variables are negative if the firm sells liquidity (i.e. promises net payments to investors). When buying liquidity the firm takes into account that international liquidity can substitute for domestic liquidity, but not the other way around. Let $t^{\$}(\omega) \geq 0$ denote the amount of international liquidity that the firm decides to "transform" into domestic liquidity – that is, the amount of dollar goods that will be paid domestic agents. This variable is constrained to be non-negative, because domestic investors accept one-for-one dollar goods for peso goods in consumption, while international investors do not accept any peso goods.

The firm's decision problem is the following:

$$\max_{\{I, I^{\$}, i(\cdot), i^{\$}(\cdot), \ell(\cdot), \ell^{\$}(\cdot), t^{\$}(\cdot)\}} E_{\omega}\{Z_1(i(\omega), i^{\$}(\omega)) - Z_0(i(\omega), i^{\$}(\omega)) - Z_0^{\$}(i(\omega), i^{\$}(\omega))\} \tag{6.1}$$

subject to

(i) $\quad (I - A) + e_0(I^{\$} - A^{\$}) + E_{\omega}[\ell(\omega)s(\omega) + \ell^{\$}(\omega)s^{\$}(\omega)] \leq 0$

(ii) $\quad Z_0(i(\omega), i^{\$}(\omega)) - Z(i(\omega), i^{\$}(\omega)) + \ell(\omega) + t^{\$}(\omega) \geq 0,$ for every ω

(iii) $\quad Z_0^{\$}(i(\omega), i^{\$}(\omega)) - Z^{\$}(i(\omega), i^{\$}(\omega)) + \ell^{\$}(\omega) - t^{\$}(\omega) \geq 0,$ for every ω

(iv) $\quad t^{\$}(\omega) \geq 0$ and $(i(\omega), i^{\$}(\omega)) \in D(I, I^{\$}, \omega),$ for every ω.

Constraints (ii) and (iii) define the firm's demand for peso and dollar liquidity, respectively, in each state. These constraints are always binding, else the budget constraint could be relaxed by reducing the levels of liquidity. Given this, we see that the budget constraint (i) presumes that all of the pledgeable income, domestic ($Z_0(i(\omega), i^{\$}(\omega))$) and international ($Z_0^{\$}(i(\omega), i^{\$}(\omega))$), is paid to investors. This is optimal, since the rate of return on entrepreneurial capital is higher than the market rate (normalized to 0). With (i) binding, the total surplus equals the entrepreneur's expected private benefit, which is the objective function that is being maximized. The auxiliary transfer variable $t^{\$}(\omega)$ is strictly positive whenever it is desirable to augment domestic liquidity with international liquidity and non-negative, because internationational investors will not accept pesos.

Equilibrium prices.

The prices for contingent peso liquidity, $s(\omega) \geq 1$, have to be such that the exogenous date-1 supply of peso liquidity $L(\omega)$ in state ω covers the representative firm's demand $\ell(\omega)$:

$$\ell(\omega) \leq L(\omega), \qquad \text{for every } \omega, \text{ with } s(\omega) = 1, \text{ if the constraint is slack.} \tag{6.2}$$

There is no corresponding condition for dollar liquidity, because foreign investors are not collateral constrained. By assumption, the international markets are deep enough to back up any dollar pledges that international investors make. Instead, the relevant constraint is the availability of domestic dollars. Foreign investors need to get paid the expected dollar return required in international markets (0, because domestic risk is orthogonal to world risk). This gives the equilibrium condition for the date-0 exchange rate e_0:

$$I^{\$} + E_{\omega}\ell^{\$}(\omega) \leq A^{\$} + E_{\omega}L^{\$}(\omega), \text{ with } e_0 = 1, \text{ if the constraint is slack.} \tag{6.3}$$

What about prices $s^{\$}(\omega)$? Contingent date-1 dollars must cost the same as date-0 dollars, so

$$e_0 = s^{\$}(\omega) \geq 1, \quad \text{for every } \omega. \tag{6.4}$$

To see why, recall that the price $s^{\$}(\omega)$ is the *peso* price for contingent dollars. Also, date-1 contingent dollars can be bought at a zero premium in international markets using dollars. Now, if $1 \leq e_0 < s^{\$}(\omega)$, a firm could buy date-0 dollars in the domestic market for the price e_0 and exchange these dollars for contingent dollars in the international market. This would be cheaper than buying contingent dollars using pesos in the domestic market eliminating the price premium $s^{\$}(\omega) > e_0$. In the reverse case – $e_0 > s^{\$}(\omega) \geq 1$ – the arbitrage runs in the opposite direction. A firm could buy contingent dollars in the domestic market, sell them in the international market in exchange for date-0 dollars that would cost less than e_0. There would be no demand for domestic date-0 dollars to sustain a price differential.

Finally, because dollar goods – either consumed or used as collateral – are perfect substitutes for peso goods, their value has to be at least as high as that of peso goods. We conclude that

$$e_0 = s^{\$}(\omega) \geq s(\omega) \geq 1, \text{ for every } \omega. \tag{6.5}$$

International investors do not earn any rents. Rents (in addition to the non-pledgeable income) go to entrepreneurs with pledgeable income in states where liquidity is scarce as well as to domestic agents who have date-0 dollar endowments.

Rather than analyzing the general model, we highlight its key features with two illustrative examples.

6.2 All output tradable

We start with the case where all pledgeable output is tradable, because it provides a useful benchmark. When all pledgeable output is tradable, international markets can meet the firms' liquidity demands at zero cost. Consequently, there is no need for domestic liquidity supply; the solution is the same as when domestic liquidity is so plentiful that it commands a zero liquidity premium.

It will not matter in the end, but for now, let us assume that all inputs as well as the endowment of the representative firm are in peso goods ($A^{\$} = I^{\$} = Z^{\$} \equiv 0$). To simplify further, consider our standard constant returns-to-scale technology. Firms initiate projects at date 0 by choosing an initial investment scale I measured in pesos. At date 1, an additional amount of pesos ρI is needed to continue the projects at full scale, where ρ is the liquidity shock, which we assume has a continuous distribution. The date-1 shock ρ represents the state ω, so we will use ρ instead of ω to denote the state of nature. The date-1 decision concerns the extent of downsizing from I to i. A smaller continuation scale $i(\rho) \leq I$ can be chosen by reinvesting $Z(\rho) = \rho i(\rho)$. Furthermore $Z^{\$} = 0$. At date 2, projects produce no pledgeable peso goods ($Z_0 = 0$), a pledgeable amount of dollar goods

$Z_0^\$ = \rho_0^\$ i(\rho)$ and a private benefit $(\rho_1 - \rho_0^\$)i(\rho)$, consumed by the entrepreneur.

Assume tentatively that there is no shortage of liquidity at date 1. This is equivalent to positing equilibrium prices

$$e_0 = s^\$(\rho) = s(\rho) = 1. \tag{6.6}$$

With these prices the representative firm chooses the optimal investment I and the optimal continuation rule $\{i(\rho)\}$ by solving

$$\max_{\{I, i(\cdot)\}} E[(\rho_1 - \rho_0^\$)i(\rho)] \tag{6.7}$$

subject to

$$\text{(i)} \quad I - A \leq E[(\rho_0^\$ - \rho)i(\rho)],$$

$$\text{(ii)} \quad 0 \leq i(\rho) \leq I, \text{ for all } \rho.$$

This is the same program as we analyzed in Section 2. The optimal continuation rule takes the form $i(\rho) = I$ if $\rho \leq \rho^*$ and $i = 0$ otherwise, where the optimal cut-off level ρ^* satisfies

$$\rho_0^\$ < \rho^* < \rho_1. \tag{6.8}$$

Is there enough liquidity to implement this second-best plan at a zero premium as posited in (6.6)? The answer is yes. We have assumed that international financial markets are willing to offer domestic firms insurance at a zero premium up to their pledgeable dollar income. Since all the firm's pledgeable income is in dollars and dollars are as good collateral as pesos, the only constraint on the firm's access to liquidity is its budget constraint (6.1)(i). This confirms that (6.6) is indeed an equilibrium.

The simple but important point here is that when all the pledgeable output is in dollars, there is no role for government supplied domestic liquidity. If a small open economy has enough export income, international markets can in principle eliminate any domestic liquidity shortages. This conclusion holds quite generally. It holds for the model we described in Section 6.1 with the added restriction that pledgeable output is in dollars. It is also true if firms like to save at date 0, that is, $I - A < 0$, since savings can be done on international markets at no cost.

In the example firms choose incomplete insurance ($\rho^* < \rho_1$), even though international investors offer insurance on actuarially fair terms. This is the key implication of optimal risk management with limited pledgeability: both initial investments and continuation investments will be credit rationed. Placed in an international context, such credit rationing suggests that a country, even if it could do otherwise, will leave its corporate sector exposed to extreme liquidity shocks. Put a bit provocatively: financial crises can be part of an optimal insurance plan even in the most favorable of circumstances (when all collateral is acceptable to international investors). With our linear specification, output drops to zero when ρ is above the cutoff ρ^*, making a "crisis" appear very dramatic. It would look less dramatic with a concave date-1 production function. However, as shown in Rampini-Viswanathan (2010), if there are decreasing returns to scale at the initial investment stage, then the amount of insurance bought will depend on entrepeneurial endowments. If these endowments are small, firms (and the nation) will choose to buy very little or even no insurance due to the same trade-off that drives our cut-off ρ^*. Poor countries will have to choose plans that make them prone to crises.

6.3 Tradable and non-tradable outputs

In the previous section, there was no role for government supplied domestic liquidity. We now show that when a country produces limited amounts of tradable goods so that its international collateral is scarce, there is a role for government supplied domestic liquidity. This addresses the important concern we raised earlier that international markets might resolve a small country's liquidity problems, because international markets are so deep.

We continue to use a constant-returns-to-scale technology and assume that both the initial investment and the date-1 (per unit) liquidity shock ρ, are in peso goods. As in the previous section, the date-1 decision concerns the scale $i(\rho)$ of the continuation investment, all paid in pesos ($Z = \rho i(\rho)$, $Z^{\$} = 0$). But now, in addition to the pledgeable dollar output $Z_0^{\$} = \rho_0^{\$} i(\rho)$ there is also a pledgeable peso output $Z_0 = \rho_0 i(\rho)$. The maximum amount of income that can be pledged to foreign investors is therefore $\rho_0^{\$} i(\rho)$, while the maximum amount of income that can be pledged to domestic investors is $(\rho_0^{\$} + \rho_0) i(\rho)$. The representative entrepreneur has a date-0 endowment A of peso goods and $A^{\$}$ dollar goods. Figure 6.2 illustrates the set-up.

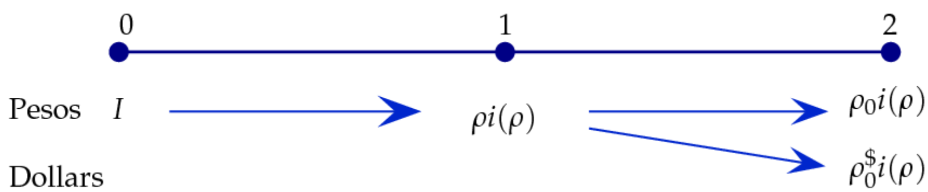


Figure 6.2: Domestic inputs, international collateral

Let the government supply L units of one-period, non-contingent peso bonds and $L^{\$}$ units of one-period, non-contingent dollar bonds (we will discuss state-contingent supply of liquidity at the end). The representative firm chooses its initial investment I, its *net* demands for liquidity $\ell(\rho)$ and $\ell^{\$}(\rho)$, the amount of dollar liquidity, $t^{\$}(\rho)$, that it transforms into domestic liquidity, and its planned continuation investments $i(\rho)$, to solve the following program:

$$\max_{\{I, i(\cdot), \ell(\cdot), \ell^{\$}(\cdot), t^{\$}(\cdot)\}} \{E[(\rho_1 - \rho_0 - \rho_0^{\$}) i(\rho)]\} \tag{6.9}$$

subject to

(i) $I - A - e_0 A^{\$} + E[\ell(\rho) s(\rho) + \ell^{\$}(\rho) s^{\$}(\rho)] \leq 0,$

(ii) $(\rho_0 - \rho) i(\rho) + \ell(\rho) + t^{\$}(\rho) \geq 0, \quad$ for all ρ,

(iii) $\rho_0^{\$} i(\rho) + \ell^{\$}(\rho) - t^{\$}(\rho) \geq 0, \quad$ for all ρ,

(iv) $t^{\$}(\rho) \geq 0$ and $0 \leq i(\rho) \leq I \quad$ for all ρ.

This program is also a special case of the general program (6.1). Constraints (ii) and (iii) are the liquidity constraints for pesos and dollars at date 1. These constraints assure that the demand for peso liquidity $\ell(\rho)$ and dollar liquidity $\ell^{\$}(\rho)$ are consistent with the investment plan $(I, i(\rho))$ and the planned transfer $t^{\$}(\rho)$ of dollar liquidity into peso liquidity. These constraints always bind. Negative values for $\ell(\rho)$ and $\ell^{\$}(\rho)$ imply that the firm supplies liquidity. The auxiliary variable $t^{\$}(\rho)$ is constrained to be non-negative, because international liquidity cannot be augmented with the help of peso liquidity. Constraint (i) is the date-0 peso budget constraint. There is no date-0 dollar budget, since dollar goods can be exchanged into peso goods at date 0 using the date-0 peso prices $\{e_0, s^{\$}(\rho)\}$.

It bears emphasizing that the sole purpose of international liquidity is to alleviate the *domestic* shortage of liquidity (or collateral) in this example. To isolate the role of international markets in supplying stores of value, we have designed the example so that dollars are not needed for production (there is no international liquidity shock $\rho^{\$}$). Equilibrium prices are determined by equalizing the supply and demand of domestic liquidity in each state (see (6.2)) and dollars at date 0 (see (6.3)). In this example, the equilibrium is characterized by

$$\ell(\rho) \leq L, \quad \text{for every } \rho, \text{ with equality whenever } s(\rho) > 1,$$

and

$$E[\ell^{\$}(\rho)] \leq A^{\$} + L^{\$}, \quad \text{with equality whenever } e_0 > 1.$$

We know that $e_0 = s^{\$}(\rho) \geq s(\rho) \geq 1$ for all ρ. The key difference between this example and the previous one is that prices are not necessarily unity. We can have $e_0 > 1$, because dollar liquidity can be scarce and we can have $s^{\$}(\rho) > s(\rho)$ in some states, because dollars are potentially more valuable than pesos: they are as good as pesos in consumption, but unlike peso liquidity, dollar liquidity can be distributed across states through $t^{\$}(\rho)$.

Given that all firms are identical, it is easiest to find an equilibrium by solving the central planner's problem, which maximizes the representative firm's utility. In doing so, we assume, for simplicity, that the available peso liquidity L and the dollar liquidity $L^{\$}$ have been distributed equally across firms by the government. The representative firm's program is

$$\max_{\{I,i(\cdot),t^{\$}(\cdot)\}} E[(\rho_1 - \rho_0 - \rho_0^{\$})i(\rho)] \tag{6.10}$$

subject to

(i) $I - A + E[(\rho - \rho_0)i(\rho) - t^{\$}(\rho)] \leq 0$,

(ii) $A^{\$} + E[(\rho_0^{\$} i(\rho) + L^{\$})] \geq E[t^{\$}(\rho)]$,

(iii) $(\rho - \rho_0)i(\rho) - t^{\$}(\rho) \leq L$, for all ,

(iv) $t^{\$}(\rho) \geq 0$ and $0 \leq i(\rho) \leq I$ for all ρ.

Constraint (i) is the peso budget constraint. The dollar-peso exchange constraint (ii) will always hold as an equality. As we have (re)formulated the problem here, the expected value of the pledgeable dollar income is transferred into the peso budget constraint. The cost of reinvestments is explicitly accounted for in the budget. The dollar liquidity can be used to relax the peso liquidity constraints (iii) in an arbitrary state-contingent manner. This highlights the insurance role of the international investors. They only care about the expected repayment, not about the states in which repayments are made (see (ii)). Without access to international markets, there would be no insurance at all across states ρ, since all firms are hit by the same shock ρ. In each state ρ, the domestic market would have to make do with the outside domestic liquidity supply $L + L^{\$}$ plus the net inside liquidity $(\rho_0 - \rho)i(\rho)$. For small shocks ρ there would be excess liquidity and for high shocks ρ there would be a shortage of liquidity. This inefficiency is reduced by the presence of international investors, who can provide insurance across states.

The extent to which international markets can provide insurance depends on the amount of pledgeable dollar income that firms generate. We are of course interested in the case where there is a limited amount of dollar income so that even with insurance through the international market, the liquidity constraint (iii) will be constrained by available dollars for high enough liquidity shocks ρ. The main questions of interest are how the foreign and domestic liquidity will optimally be used and how the state prices will behave.

Let λ_i, λ_{ii} and λ_{iii}, be non-negative Lagrangian multipliers for the three first constraints. The first-order conditions for $i(\rho)$are:

$$\rho_1 - \rho_0 - \rho_0^{\$} - (\lambda_i + \lambda_{iii}(\rho))(\rho - \rho_0) + \lambda_{ii}\rho_0^{\$} \geq 0, \text{ whenever } i(\rho) = I. \tag{6.11}$$

When $0 < i(\rho) < I$, this constraint holds as an equality. The first-order conditions for $t^{\$}(\rho)$ are:

$$\lambda_i - \lambda_{ii} + \lambda_{iii}(\rho) = 0, \quad \text{whenever } t^{\$}(\rho) > 0. \tag{6.12}$$

When $t^{\$}(\rho) = 0$, the value of additional liquidity is zero, so $\lambda_{iii}(\rho) = 0$.

From these first-order conditions we can deduce that the solution to the representative firm's program has four regions, described formally below and graphically in Figure 6.3.

$$
\begin{aligned}
&\text{Region I:} && \text{if } \rho \leq \overline{\rho}_{\mathrm{I}}, \quad \text{where } (\overline{\rho}_{\mathrm{I}} - \rho_0)I = L, \\
& && \text{then } i(\rho) = I \text{ and } t^{\$}(\rho) = 0, \\
&\text{Region II:} && \text{if } \overline{\rho}_{\mathrm{I}} < \rho \leq \overline{\rho}_{\mathrm{II}}, \text{ where} \\
& && \textstyle\int_{\overline{\rho}_{\mathrm{I}}}^{\overline{\rho}_{\mathrm{II}}} [(\rho - \rho_0)I - L] f(\rho) d\rho = \rho_0^{\$} I + L^{\$} + A^{\$}, \\
& && \text{then } i(\rho) = I \text{ and } t^{\$}(\rho) = (\rho - \rho_0)I - L, \\
&\text{Region III:} && \text{if } \overline{\rho}_{\mathrm{II}} < \rho \leq \overline{\rho}_{\mathrm{III}} < \rho_1, \\
& && \text{then } i(\rho) = \tfrac{L}{\rho - \rho_0} < I \quad \text{and} \quad t^{\$}(\rho) = 0, \\
&\text{Region IV:} && \text{if } \rho > \overline{\rho}_{\mathrm{III}}, \\
& && \text{then } i(\rho) = 0 \quad \text{and} \quad t^{\$}(\rho) = 0.
\end{aligned}
\tag{6.13}
$$

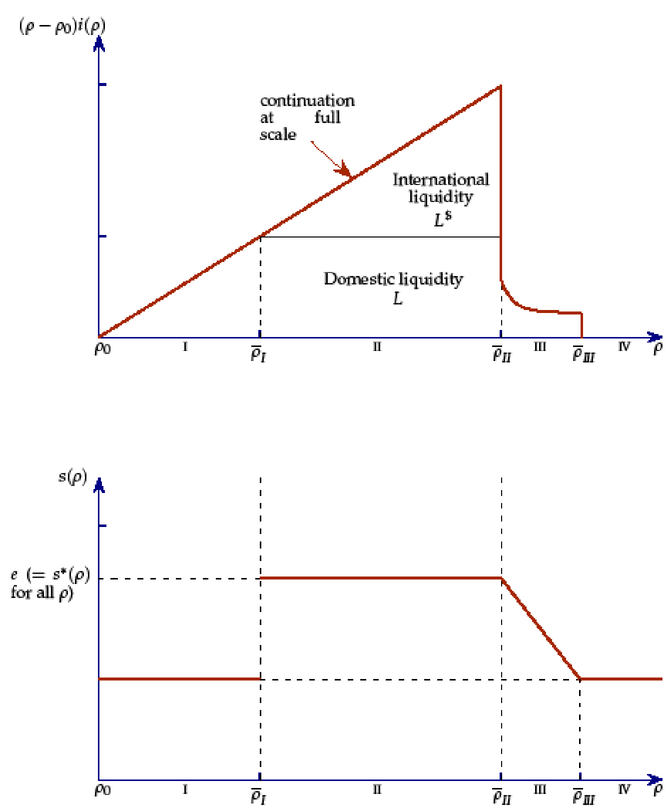


Figure 6.3: Demand for liquidity

Consider first the top panel in Figure 6.3. In region I, the liquidity shock ρ is so low that a firm can continue at full scale $i(\rho) = I$ with peso liquidity alone; no dollar liquidity is needed nor will it be used, since it has a negative shadow price in constraint (ii) (the shadow price of constraint (iii) is zero and that of (i) is constant, so it is strictly better to wait until constraint (iii) is binding). This is just saying that there is no point in wasting precious international liquidity if there is enough domestic liquidity. In region II, continuation is still at full scale, but requires the addition of dollar liquidity; constraint (iii) is binding and $\lambda_{iii}(\rho) = \lambda_{iii} > 0$ is constant, as seen from condition (6.12). As ρ increases, full scale continuation is possible until the cumulative amount of dollar liquidity used in region II (the top triangle in the figure) reaches the total amount of available dollar liquidity. The economic intuition is straightforward: dollars, being scarce, will be allocated to states in decreasing order of productivity, which means in increasing order of ρ. In region III the firm reverts back to using only peso liquidity, because all dollar liquidity has been used up. The scale of continuation investment is now below I and decreasing in ρ. As in the model without international liquidity, there is credit rationing. Finally, beyond $\overline{\rho}_{III} < \rho_1$ it no longer pays to continue.

What about state prices? The prices that will support the centralized solution as an equilibrium are shown in the lower panel of Figure 6.3.

Start with the price of peso liquidity. In the unconstrained region I, $s(\rho) = 1$, since there is an excess of peso liquidity even at full scale $i(\rho) = I$. In region III, only pesos are used, but now the continuation investment is below full scale. The price of liquidity must be such that the firm is persuaded to choose an interior value $i(\rho) < I$. Since the marginal value of a peso in region III is declining in ρ, so must its price $s(\rho)$[3]. At the left end of region III we have $s(\overline{\rho}_{II}) = s^{\$}(\overline{\rho}_{II})$; the value of a peso equals the value of a dollar, since the next best use of a marginal dollar (in state $\overline{\rho}_{II}$) is to augment pesos in state

[3]If one goes back to the firm's program (6.9), one finds that the first-order condition for an interior solution for $i(\rho)$ requires that $s(\rho) = \varphi/(\rho - \rho_0)$, where φ is a constant.

$\overline{\rho}_{II} + \varepsilon$. At the right end of region III we have $s(\overline{\rho}_{III}) = 1$; since the firm is indifferent between continuing or not, the marginal peso does not command a premium. Note that throughout region III, the transfer constraint $t^{\$}(\rho) \geq 0$ is binding and the shadow price increasing. In region II both dollar liquidity and peso liquidity are employed so $t^{\$}(\rho) > 0$. Since the shadow price on the transfer constraint is zero, both forms of liquidity must have equal value. In fact,

$$s(\rho) = s^{\$}(\rho) = e_0 > 1 \text{ throughout region II.} \tag{6.14}$$

The price of liquidity is constant in region II, because a unit of liquidity, regardless of the state and the currency, will be used in the same manner, namely to increase the date-0 investment I, because the continuation investment equals I in this region. Furthermore, liquidity prices must equal the exchange rate e_0, because exchanging e_0 pesos for a dollar at date-0 and using the proceeds to buy insurance on the international market against shocks in region II must return the same as buying insurance against these states in the domestic market.

The date-1 spot exchange rate, defined as the cost of a dollar good in terms of peso goods, equals 1 in all states, because domestic consumers are equally happy to consume either good. If $e_0 > 1$, the exchange rate appreciates from date 0 to date 1. This is an artifact of the three-period model where the need for insurance only occurs in period 0. The (contingent) forward exchange rate $e_1(\rho) = s(\rho)/s^{\$}(\rho) = s(\rho)/e_0$ will follow the pattern of $s(\rho)$.

Discussion (public provision of liquidity): We assumed a fixed, non-contingent amount of government supplied peso and dollar liquidity, L and $L^{\$}$. Referring to the analysis in the two previous chapters, we can derive some principles for how the government should optimally participate in the supply of liquidity considering the limited access to international liquidity. This depends on the government's objective and the implied cost of supplying liquidity. The simplest case occurs if consumers have no demand (or a non-contingent demand) for liquidity and the cost of supplying liquidity (taxing consumers) is constant in every state. In that case, the government should dispatch a constant amount of liquidity, just as in the figure, but the cut-off $\overline{\rho}_{II}$ will be determined by the marginal cost of liquidity supply instead of the exogenously given supply (e.g. it may be optimal for the government not to supply any liquidity if the marginal cost is high enough). A less obvious difference is that no outside liquidity will be supplied in region III, which is eliminated. The reason for this is that partial continuation cannot be optimal other than at the boundary of region II.[4] This is consistent with the use of a state-contingent bond as discussed in the previous chapter (see also Holmström and Tirole, 1998).

More realistically, the government's supply function $g(L;\omega)$ is increasing and also varies by state of nature, because the cost of taxation will depend on the consumer demand for liquidity. The logic behind our analysis in this chapter gives a good indicationn of how this general case works out. It will still be the case that international liquidity is most valuable, because it is most flexible. Also, the price of international liquidity will be constant as a function of the state, else it would be reallocated to its highest value. Therefore, the price of domestic liquidity will also be constant, except in states where no international liquidity is used. The amount of domestic liquidity will, however, vary with the state to reflect the marginal cost $\partial g(\cdot;\omega)/\partial L$. The supply of corporate liquidity would respond to the international price when international liquidity is used and to the government price of liquidity where such is used. Of course, in the end all of these prices and decisions are determined jointly in a more complicated equilibrium than we have studied (i.e. one where the government's decisions are a part of the equilibrium.)

The main point is that with some of the pledgeable output non-tradable, liquidity management by the government is again relevant. International investors make insurance

[4] Suppose we had a region III. There would be partial continuation for all states ρ such that $\rho_{II} < \rho < \rho_{III}$. Since the cost of government supplied liquidity, q, is constant, it would be more efficient to move government liquidity from the high-ρ states in region III to the low-ρ states in that region, bringing the low states to full scale. Consequently, region III cannot exist.

cheaper, but they cannot provide all of it when international collateral is scarce. International liquidity is utilized most efficiently by having international investors supply the marginal liquidity needed in high demand states. As before, there is capital rationing at date 1 (a "planned crisis"), but with access to international markets the rationing will be less severe. Thus*, the analysis of the value of domestic outside liquidity in the previous chapter will be robust to the introduction of international capital markets.* This finding holds as long as some inputs are non-tradable. Had we assumed instead that all inputs at date 1 were tradable, domestic liquidity supply would be entirely irrelevant. The reason for this disparity is again that dollars can be substituted for pesos, but not the other way around.

6.4 Conclusion

This chapter has studied liquidity management when firms have access to international financial markets and coordinate ex ante on the use of liquidity (second-best insurance). The main findings are the following:

- International markets will provide adequate insurance if there is sufficient international collateral. In that case domestic creation of liquidity will be unnecessary. In general, however, access to international markets does not obviate the need for domestic creation of liquidity, because the amount of insurance that can be bought is limited by the amount of international collateral (tradable assets and income).

- Insurance will be incomplete (some states will not be covered) even when there is full parity between international and domestic liquidity. Optimal risk sharing, with limited entrepreneurial capital, leaves firms and therefore the country exposed to the possibility of financial crises. Financial crises are part of an optimal plan even in the best of circumstances.

- When international collateral is scarce, the analysis of domestic government policy is similar to that without international markets: the decision to supply liquidity in a state is determined by comparing state-contingent liquidity premia with the opportunity cost of providing liquidity in that state. International liquidity will be used as a last resort, in states where the cost of domestic outside liquidity exceeds the (domestic) cost of international liquidity. The cost of international liquidity is strictly positive on the domestic market, whenever domestic dollar output is scarce, this despite the assumption that international liquidity is supplied at zero cost on international markets.

Even though this chapter is short and the analysis is presented in the form of simple examples, it reflects both the richness and the idiosyncratic character of our complete market approach. At its core, it is a study of optimal risk sharing under limited pledgeability of income, reflecting the view that the demand for liquidity is a demand for insuring risks. Our focus is on how different parties should optimally participate in the supply of risk sharing. In our analysis there are three parties: the corporate sector, which optimizes the use of inside liquidity, the government, which intermediates between consumers and the corporate sector in supplying outside liquidity and finally, the international investors, who also provide outside liquidity. International insurance is limited by the international investors' willingness to accept only tradable goods (or in an alternative interpretation, by their limited information about and ability to monitor firms in a foreign country). At the same time, international investors are the most flexible and therefore the most valuable suppliers of liquidity. By assumption, they do not care about local risks, because they can diversify away such risk. Both the scarcity of international insurance and the indifference it displays to local risks, lead it to be used when the domestic, less flexible sources of insurance, are in short supply or too expensive. The conclusions of this chapter are all a simple consequence of this general perspective and can be readily extended to include shocks to pledgeable and non-pledgeable income as well as liquidation values and income shocks along the lines discussed in section 4.3.

In reality, international investors do not seem to carry the most expensive domestic risk as suggested by our point of view. A possible reason for this is that while international investors have the capacity to insure, they do not have enough information to take on significant foreign risks. Another reason is that it is hard to write narrowly targeted, contingent insurance plans ex ante (see section 5.1.2). As a result, international investors often invest in instruments guaranteed by the government, explicitly or implicitly (e.g. they invest through domestic banks), or they fund the government directly by buying government bonds. Even in this case, however, concerns about moral hazard problems – national governments can take advantage of foreign funding in a variety of ways – limit the sharing of exceptional risks.[5]

Unlike private investors, the International Monetary Fund (IMF), has significant monitoring capacity and also leverage over national governments (largely because private investors would pull out without the IMF). Therefore it can play an important role as a provider of insurance against sovereign tail risk. During the subprime crisis the IMF has gone well beyond orthodox interventions (just like central banks) and offered a variety of liquidity facilities to countries in trouble. These include special drawing rights, contingent credit lines and systemic liquidity facilities, for instance. Nonetheless, the role played by the IMF seems rather different from that played by a treasury or a central bank. The latter, both in practice and in theory (see Chapter 5), act as lenders of last resort, when there is a shortage of aggregate domestic liquidity. The problem for a country, by contrast, is not that there are not enough stores of value in the world, but that the country may have a shortage of tradable goods, which can be pledged to international investors. Also,

[5] For models of how the level and structure of sovereign and private borrowing affect government policies and for their implications for capital controls and the policies toward "original sin" features of borrowing (the issuance of debt claims, that are short-term and denominated in foreign currency), see e.g., Amador (2008) and Tirole (2003); for an interesting alternative perspective on state opportunism, see Broner et al (2010).

the IMF expects to get reimbursed, at least for its largest loans, so it does not provide liquidity directly.

The IMF would provide liquidity directly only if its claim on the country were de facto junior to that of public and private lenders and thus involved some substantial risk taking. Instead, the IMF provides liquidity indirectly, by writing and monitoring the covenants that will make it more credible that the country reimburses its (sovereign and private) debts to foreigners. In the parlance of our model, most IMF interventions occur at date 1 when the country has faced a shock to its income or endowment. The covenants increase the country's date-2 international collateral at a cost which it can recoup by getting access to international capital markets, provided that the intervention is credible. And indeed this is how it should be if a more explicit contract were written between the country and the international community at date 0.[6] The IMF has also tried to venture into "ex ante monitoring", which in our model would correspond to writing covenants at date 0. In 1999 it introduced "contingent credit lines", that were meant to give a country automatic access to a credit line provided that certain criteria were met; unfortunately, as interesting as this idea was, no country ever took up those credit lines for fear of being stigmatized by the signal to the market that it may encounter difficulties in the future.[7]

We conclude that the cost to monitor what is going on in foreign countries limits the amount of international risk sharing. The monitoring perspective can also be applied within a country as we have done in Holmström and Tirole (1997). In that paper, intermediaries are better informed than the public about the doings of firms and can more effectively limit the firms' ability to engage in moral hazard. To some extent, and as the recent Eurozone crisis demonstrated, the same applies much more broadly: The will-

[6] A thorny issue, though, is that the government may not behave in the citizens' best interests, and that hardships imposed by covenants – however well justified for the country as a whole at date-0 – may unduly affect the poor at date 1.

See Tirole (2002) for further discussion of the raison d'être of the IMF.

[7] Accordingly, the IMF in 2009 replaced these contingent credit lines by a new facility, the flexible credit line: see http://www.imf.org/external/np/sec/pr/2009/pr0985.htm

ingness to share risk depends on the information and monitoring abilities of all parties involved. Much of the liquidity is local for this reason. For instance, what goes for good collateral in one country or one region of a country, may not be worth much in another country or a distant region within the same country. The approach in this chapter could be used to study the role of such "local liquidity" more broadly.

Part IV

WASTE OF LIQUIDITY AND PUBLIC POLICY

Chapter 7

Financial muscle and overhoarding of liquidity

So far our study of aggregate liquidity shortages has assumed that the corporate sector can make efficient use of the available liquidity in each state of nature at date 1. One way in which efficient use of liquidity can be assured is to have a complete market of state-contingent claims on pledgeable income available for trade at date 0. This was illustrated by the LAPM model in Chapter 4. As we discussed earlier, there may be many institutional solutions for achieving the same outcome, but our model is not rich enough to distinguish between them. In reality, banks, conglomerates and a variety of other intermediaries play an important part in trying to allocate state-contingent aggregate liquidity to its best use. How close these arrangements come to efficiency is an empirical matter. We view our model of complete contingent contracting as a helpful benchmark and reference point for such investigations.

In this fourth and final part of the book we depart from the benchmark model and analyze settings in which firms cannot pool their liquidity to achieve the complete market outcome. Instead, they have to rely on *self-provision of liquidity.* In order to meet liquidity shocks that cannot be financed by issuing new claims at date-1 (funding liquidity) – that is, liquidity shocks that result in a negative net pledgeable income – firms have to invest in short-term assets that can be liquidated in the date-1 spot market (market liquidity). Firms operate in isolation, without coordinating either the use or the acquisition of liquidity. We also assume that firms cannot hold stakes in each other. These assumptions are somewhat arbitrary, a problem shared by most models of incomplete markets. Our choices have been guided by an interest in understanding when investments in short-term assets will be insufficient and when they will be excessive.

This chapter focuses on reasons why firms may hoard too much liquidity. We will consider two models. In the first model (section 7.1), firms hoard liquidity to put themselves in a position to compete aggressively for the long-term assets of distressed firms. Our analysis of such "*predatory hoarding*" makes two main points: (a) The financial strength of the potential buyers increases the value of the distressed assets, contributing to liquidity at date 1; (b) uncoordinated purchases of short-term assets at date 0 may result in excessive hoarding of liquidity. During the subprime crisis (as in earlier crises) banks have kept large amounts of excess reserves at the central bank (the Fed and the ECB). This is consistent with the idea that they want to be in a strong position to profit from fire-sales of distressed assets.[1]

We will analyze when lack of coordination leads to waste of liquidity and whether the government can remedy the situation using the following standard instruments:

- *Liquidity regulation.* The state may mandate minimum or maximum levels of liquidity. For example, some policy proposals following the subprime crisis recommend a floor on the level of liquidity hoarded by financial institutions. Our analysis suggests that a cap (or tax) on liquidity hoarding may also be warranted, at least when the anticipation of fire sales makes it likely that financial institutions will stockpile on liquidity for predatory purposes.

- *Public provision of liquidity*: As in Chapter 5, the state can change the supply of

[1] Of course, there are other reasons for holding excess reserves. Banks may be concerned about their own survival in a worsening liquidity squeeze. They may also be waiting for the resolution of uncertainty that would reduce adverse selection.

liquidity. The reason here is not the sharing of aggregate risk, since there are no aggregate shocks in the model. Instead, the state can influence the way the private sector manages its liquidity because it can control the price. The public supply of liquidity has an ambiguous impact if no separate liquidity regulation is in place. As in Chapter 5, it lowers the cost of liquidity; however it facilitates rent seeking.

The second model (section 7.2) focuses on "*precautionary hoarding*" of liquidity. In this framework, firms can use the secondary market to cover their future cash needs; for instance, they can sell assets that they have hoarded on their balance sheets. If the market for such assets is marred by asymmetric information, though, it may freeze, depriving the firms from access to market liquidity. To avoid such a fate, firms may instead hoard liquidity using safe assets that are not not subject to adverse selection, but the yield on safe assets, such as Treasury bonds, is low. Interestingly, the tradeoff between investing in risky assets versus safe assets can lead to multiple equilibria.[2] If firms invest mostly in safe assets, they are less dependent on being able to sell assets in the secondary market. This creates adverse selection: assets are primarily sold because their owners do not value them highly and want to get rid of them. In anticipation of such a date-1 "lemons market" firms indeed want to invest in safe safe assets at date 0.

Conversely, if firms do not expect a market freeze, they prefer to invest in higher yielding assets that they will sell if they need liquidity. There is less adverse selection, since asset sales are driven by liquidity shocks rather than shocks to the value of the assets. This "liquid market" equilibrium dominates the "precautionary hoarding" equilibrium. Firms would be better off collectively if they could agree not to hoard safe assets.

7.1 Predatory hoarding of liquidity and fire sales

To put our analysis in perspective, it is useful to recall some earlier work on asset re-deployability. Shleifer and Vishny (1992), building on Williamson (1988), made several

[2] See Malherbe (2009).

insightful observations about the limited ability of distressed firms to resell assets. To illustrate their reasoning, consider the following parable.

Parable of the farmer and his land: A farmer may have expertise in cultivating a specific crop, but must borrow cash from a bank in order to purchase the land. Ex post, the farmer may prove competent or incompetent; or, equivalently, the crop he has expertise in may turn out to be well adapted to the soil (or in high demand) or ill adapted (in low demand). If things go poorly, the land, which optimally is pledged as collateral to the bank, must be resold on the market. One may distinguish three types of sales:

- *Alternative use* : The land may be transformed by the buyer into, say, a baseball field. The issue with this type of sale is that the new usage does not correspond to the best use of the land if keeping it for farming purposes yields a higher social value.
- *Deep-pocket investor* : The land is sold to investors with no farming expertise, who hire a farmer to plant and harvest a new crop. This approach has the advantage of maintaining the land in its best use, but faces an agency cost; and so it creates a wedge between the total value created and the price that the investors are willing to pay for the land.
- *Specialist* : To avoid this agency cost, it would seem more natural to resell the land to another farmer. Prospective buyers in this category however may not have the resources to buy the land themselves, and if they do not they must borrow from non-specialists, namely the deep-pocket investors considered in the second type of sale.

Shleifer and Vishny argue that, when assets must be managed by specialists, the wedge between the value of assets in specialist hands and the price that can obtained by selling the assets in the market is particularly high for industries in recession. Specialists tend to be found within the same industry and if that industry is in recession, many of them are likely to be strapped for cash, pushing the market price below its value in the best use.

This section develops a formal analysis of the three types of sales.

7.1.1 Coordinated investment in liquidity

Consider a continuum of entrepreneurial firms with unit mass.

Firms are identical at date 0. Each has initial wealth A. At date 1, a fraction α of entrepreneurs is revealed to be *competent* and a fraction $1-\alpha$ *incompetent*. "*Intact firms*" in this chapter are firms run by competent entrepreneurs, "*distressed firms*" are firms run by incompetent entrepreneurs. No one knows at date 0 which firms will be intact. At date 1, the entrepreneurs' types are privately revealed. The fraction α is known at date 0, so there is no aggregate uncertainty.

As before, all agents are risk neutral and indifferent over the timing of consumption, valuing their consumption stream

$$c_0 + c_1 + c_2.$$

Thus, the consumer's required rate of return must be at least zero.

At date 0 the representative firm can invest in two types of assets:

- *Long term (LT) assets*: If the firm invests I in LT assets at date 0 and its entrepreneur is revealed to be competent at date 1, the constant-returns-to-scale technology of section 3.2 applies: The LT asset delivers nothing at date 1. At date 2 it delivers total income $\rho_1 I$ and pledgeable income $\rho_0 I$, where

  $$\rho_0 < 1 < \rho_1.$$

  For simplicity, we assume that no reinvestment is needed at date 1.

  If an entrepreneur turns out to be incompetent, the LT asset yields nothing ($\rho_0 = \rho_1 = 0$) under his management. Competent entrepreneurs can manage any number of assets and LT assets may be transferred from incompetent to competent entrepreneurs in a date 1 spot market. Assets transferred to competent entrepreneurs yield the same return as the entrepreneurs' original assets. LT assets may also be employed in an alternative use (converted to a "baseball field"), with a total return equal to the pledgeable income $\underline{\rho}I$ where $\underline{\rho} < 1$ can exceed or be smaller than ρ_0. An incompetent entrepreneur's assets will either be converted to the alternative use or be put under a competent entrepreneur's management, depending on the spot market price at date 1.

- *Short term (ST) assets*: A firm has also access to a storage technology. To obtain ℓ units of goods at date 1, the firm must invest $g(\ell)$ at date 0 in the ST asset, where

$$g'(0) = 1 \quad , \quad g'(\ell) > 1 \quad , \quad \text{and} \quad g''(\ell) > 0 \quad \text{for all } \ell > 0.$$

  Note that the the firm must pay a higher marginal cost the more it hoards liquidity.

For the purpose of this model, one can think of $g(\ell) - \ell$ as the physical depreciation of the storage good. But we have an alternative interpretation in mind, in which the firms in this industry compete with other industries for scarce (inside and outside) liquidity in an economy in which liquid assets command a premium $q - 1$, where in equilibrium $q = g'(\ell)$.[3]

Coordinated solution. As a benchmark, consider the second-best case in which, at date 0, the entrepreneurs can agree among themselves and with investors (by forming a conglomerate or through contracting) on the levels of long-term and short–term investments and on the date-1 policy for redeploying the assets. Because there is no reinvestment need at date 1 and because liquidity is costly, the optimal coordinated policy is not to invest in ST assets at all:

$$\ell^* = 0. \tag{7.1}$$

[3]For this interpretation q should be taken as exogenous at the firm level.

Instead, firms agree in advance that the LT assets of the incompetent entrepreneurs will be turned over to the competent entrepreneurs. Because firms are identical ex ante and the entrepreneurs are risk neutral, there is no need for financial payments. In this case, the optimal investment in LT assets, I^*, is given by the usual budget constraint equating investors' outlays with the pledgeable income:

$$I^* - A = \rho_0 I^*,$$

or

$$I^* = \frac{A}{1-\rho_0}.$$

7.1.2 Self-provision of liquidity and the market for distressed assets

Assume now that, at date 0, each firm selects the pair (I, ℓ) independently, without coordinating its plan with the other firms.[4] At date 1 a spot market for assets opens, in which the entrepreneurs can sell (if they are incompetent) or buy (if they are competent) long-term assets. The price in the spot market is denoted p. Because a distressed firm can choose to put its assets in the alternative use, the date-1 price must satisfy $p \geq \underline{\rho}$. If $p > \underline{\rho}$, all the distressed assets are sold in the spot market; if $p = \underline{\rho}$ some of the distressed assets may go into the alternative use. In either case the distressed firm will earn pI at date 1 from its LT investment. The model thus focuses on *local liquidity* in the sense that a limited set of players has the knowledge necessary to manage the assets or to buy the securities. Furthermore, this set is financially constrained.

Intact firms have two potential sources of funds for purchasing distressed assets. They can sell their short-term assets ℓ and they can issue new, senior securities at date 1.[5] By

[4] Recall that we also assume that a firm cannot own shares of the other firms.

[5] A distressed firm has no incentive to issue new shares to buy more assets, since the incompetent entrepreneurs obtain no private value from selling assets or putting assets into the alternative use. In contrast, permitting dilution by intact firms is valuable from an ex ante point of view, since it is a cheaper form of liquidity than buying short-term assets. Contingent transfers from investors to firms would of course also be cheaper than using short-term assets. Such transfers are ruled out as part of the assumption that firms cannot coordinate the use of liquidity.

fully diluting the initial shareholders, an intact firm can raise $\rho_0(I+j)$ at date 1, where I is the initial investment and j is the amount of assets purchased in the spot market. Together with the ST assets, the new issue has to cover the purchase of new assets, so the firm's investment plan has to satisfy the following date-1 *liquidity constraint*:

$$pj \leq \rho_0(I+j) + \ell. \tag{7.2}$$

The firm's investment plan also has to satisfy the following date-0 *budget constraint:*

$$I - A + g(\ell) \leq \alpha\,[\rho_0(I+j) + \ell - pj] + (1-\alpha)\,[\ell + pI] \tag{7.3}$$

On the left-hand side are the costs of investments in LT and ST projects. On the right-hand side are the date-1 net returns. An intact firm will spend pj of its pledgable income on buying assets. A distressed firm will pay out to investors the returns from its ST assets plus what it gets by selling the LT assets in the date-1 spot market (or, alternatively, the returns it gets from putting the LT assets into the alternative use). When the liquidity constraint (7.2) binds, the first term on the right-hand side of (7.3) is zero. In that case, initial investors will only cash in on the liquidation of the distressed firms.

It may seem surprising that investors get something only when bad news is received. In other models of liquidity demand, such as the one considered in the next chapter, bad news means that more money must be injected in order to avoid downsizing or closure; in this case, investors provide funding liquidity by agreeing in advance to let their stakes be diluted and market liquidity by letting the firm's ST assets be sold. Good news (no liquidity shock), in these models, make entrepreneurs and investors both better off. Here, by contrast, good news means an opportunity to purchase distressed assets, which creates a liquidity need. Funding liquidity and market liquidity are both employed in the good state, which ex post is beneficial to the entrepreneur, but not to the investors.

For any given price p the representative firm will choose $\{I, j, \ell\}$ to maximize expected net utility:

$$U = \quad \alpha\,[\rho_1(I+j) - pj] + (1-\alpha)pI - I - [g(\ell) - \ell]\,, \tag{7.4}$$

subject to the the liquidity constraint (7.2) and the budget constraint (7.3).[6]

An equilibrium in the date 1 spot market obtains when the demand for assets does not exceed the supply

$$\alpha j \leq (1-\alpha)I \tag{7.5}$$

and $p = \underline{\rho}$ if the inequality is strict.

It follows immediately from the firm's maximization problem that

$$p > \rho_0. \tag{7.6}$$

Suppose, to the contrary, that $p \leq \rho_0$. The firm could then purchase boundless amounts of assets j; neither the liquidity constraint nor the budget constraint would constrain it. Therefore, the purchase of date-1 assets would be self-financing and entrepreneurial rents could be increased without bounds. This is inconsistent with equilibrium.

Note also that since $p \geq \underline{\rho}$, the option to put distressed assets into the alternative use is relevant only if

$$\underline{\rho} > \rho_0. \tag{7.7}$$

Otherwise the purchase of distressed assets would be self-financing. We will assume that (7.7) holds for now.

Next we argue that p must satisfy[7]

$$p \leq 1. \tag{7.8}$$

To see why, consider the following two investment options:

(i) at date 1, the firm buys an extra unit of distressed assets on the spot market at price

[6]Alternatively (from (7.3)):

$$U = \quad \alpha(\rho_1 - \rho_0)(I + j) - A.$$

[7]At first, one may think that ρ_1 is the upper bound for p, because that is the social value of an extra unit of the asset in the hands of a competent entrepreneur. However, this reasoning overlooks the alternative ways in which a firm can procure an asset for use at date 1.

p;

(ii) at date 0 the firm increases its investment I by one unit.

Let us ignore for the moment the firm's liquidity constraint and consider maximizing the firm's NPV subject only to the budget constraint (7.3). Letting $\lambda > 0$ be the Lagrange multiplier for the budget constraint, we have the following first-order conditions for choosing interior values of I and j.

$$\alpha\rho_1 + (1-\alpha)p - 1 - \lambda[1 - \alpha\rho_0 - (1-\alpha)p] = 0, \quad \text{for I} \tag{FOC-I}$$

$$(\rho_1 - p) - \lambda(p - \rho_0) = 0, \quad \text{for } j \tag{FOC-j}$$

The first-order condition (FOC-I) is increasing in p while (FOC-j) is decreasing in p. At $p = 1$ the two are equal. Since it is cheaper to buy date-1 assets through the spot market if $p < 1$ but not if $p > 1$, and because reintroducing the liquidity constraint can only work against buying assets through the date-1 spot market, the spot market can be in equilibrium only if $p \leq 1$. Note that this argument rests on the whole plan $\{I, j, \ell\}$ being agreed on between the investors and the firm. [8]

With these preliminaries, we turn to discuss equilibrium outcomes. Figure 7.1 describes the outcome when $\underline{\rho} > \rho_0$. The spot market price p increases monotonically with the fraction α of intact firms, staying within the bounds $\underline{\rho}$ and 1. There are three possible regimes or regions of α. Let us look at the conditions under which each region applies, considering them in reverse order.

[8] If the intact firms could bid for the date-1 assets by diluting shareholders at will, their purchasing power in Region III of figure 7.1, would exceed the supply at $p = 1$ even without any investments in short-term assets. This would force the price up.

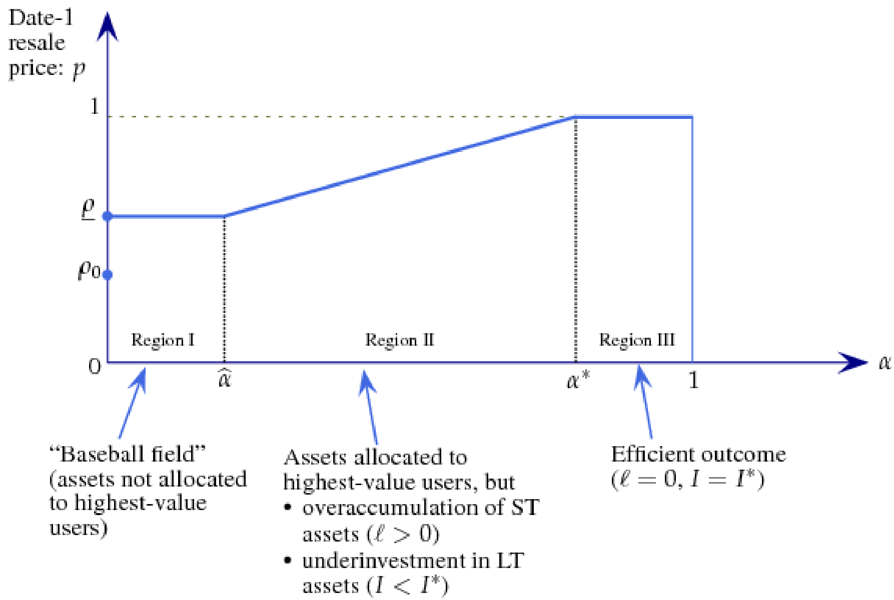


Figure 7.1: Redeployability in the absence of coordination

Region III ($\alpha > \alpha^* \equiv 1 - \rho_0$): *No hoarding of assets (Efficient equilibrium)*

Recall that the outcome is efficient if

– firms do not invest in ST assets ($\ell = 0$), and

– all the assets of distressed firms are transferred to the intact firms (none is put into the alternative use).

When most firms are intact (α is large), few LT assets are put on the market so supply is low. At the same time, demand is high, because the aggregate purchasing power of intact firms (through dilution) is high. One is led to conjecture that the assets put on the market will command the maximum price $p = 1$ and that the firms will invest nothing in outside liquidity ($\ell = 0$). To verify that this is an equilibrium, consider the two conditions that have to be met by the firm's choice $\{I, j, \ell\}$ and the price p. The firm's date-1 liquidity constraint implies

$$j \leq \frac{\rho_0 I + \ell}{(p - \rho_0)}. \tag{7.9}$$

Also, because $p = 1 > \underline{\rho}$, the demand for assets is equal to the supply:

$$j = \frac{1-\alpha}{\alpha} I. \tag{7.10}$$

Recall from (FOC-I) and (FOC-j) that a firm is indifferent between investing in I and j when $p = 1$. Therefore, when $p = 1$, we can adjust I and j so that supply equals demand regardless of α, so (7.9) is the key constraint. (Again, this is done in agreement between the firm and the investors at date 0 – at date 1, the firm would like to expand purchases and full dilution would give enough purchasing power to do that.)

Since no firm hoards liquidity ($\ell = 0$) in Region III, they all invest the same amount as in the coordinated second-best solution:[9]

$$I = \frac{A}{1-\rho_0} = I^*. \tag{7.11}$$

Substituting (7.10) into (7.9), noting that $p = 1$ and $\ell = 0$, we see that the efficient equilibrium (Region III) prevails as long as

$$\alpha \geq \alpha^* \equiv 1 - \rho_0. \tag{7.12}$$

When α falls below α^* the liquidity constraint starts to bind, which moves us into the next regime.

Region II ($\hat{\alpha} \leq \alpha < \alpha^*$): *Hoarding short-term assets to compete for LT assets at date 1 (Rent-seeking equilibrium).*

In Region II, since $\alpha < \alpha^*$, we can no longer have $p = 1$ as we just argued; so we must have $p < 1$. Comparing (FOC-I) and (FOC-j), we see that the firm would now like to allocate all of its date-0 budget to buying distressed assets at date 1, choosing the maximal j and setting $I = 0$. But this would violate the liquidity constraint (7.2). Therefore the firm will maximize (7.4) subject to the liquidity constraint (7.2) and the budget constraint (7.3) both binding.

[9] To obtain this expression, use the investors' breakeven constraint for $p = 1$, $I - a = I[\rho_0(I + j) - j] + (1-\alpha)I$ and the fact that supply equals demand: $\alpha j = (1-\alpha)I$.

The only purpose of hoarding costly liquidity at date 0 is to boost the firm's capacity to acquire distressed assets. Despite the liquidity premium, $g' - 1 > 0$, it is worth hoarding ST assets, since LT assets can be bought at a discount ($p < 1$) in the date-1 market. An intact firm's acquisition capacity j at date 1 is determined by the (binding) liquidity constraint

$$(p - \rho_0)\, j = \rho_0 I + \ell. \tag{7.13}$$

Using the equilibrium condition (7.10), we find that the equilibrium price is

$$p = \frac{\rho_0}{1 - \alpha} + \frac{\alpha \ell}{(1 - \alpha) I}. \tag{7.14}$$

The first term is less than 1, because $\alpha < \alpha^* = 1 - \rho_0$. The second term reflects the fact that precautionary purchases of ST assets increase date-1 liquidity, boosting the price of assets relative to what it would be if $\ell = 0$ (but, of course, not so much that p would be pushed back up to 1).[10]

We can rewrite the firm's program in Region II by substituting the binding liquidity constraint (7.2) into the budget constraint (7.3) to get:

$$\begin{aligned}
&\max_{\{I, \ell, j\}} \ \{\alpha(\rho_1 - \rho_0)(I + j)\} \\
&\text{s.t.} \\
&(1 - \alpha)(\ell + pI) - I + A - g(\ell) \geq 0 \\
&\text{and} \\
&[pI + \ell] - (p - \rho_0)(I + j) = 0.
\end{aligned}$$

This can be further simplified to

$$\begin{aligned}
&\max \ [pI + \ell] \\
&\text{s.t.} \\
&(1 - \alpha)(pI + \ell) - I + A - g(\ell) \geq 0,
\end{aligned}$$

which yields

$$g'(\ell) = \frac{1}{p}. \tag{7.15}$$

[10] Note that if we introduced uncertainty into the model, for example in the form of a random fraction of intact firms α, then the point of Shleifer and Vishny (1992) about price softness would hold: the asset price would fall during an industry recession (α low).

It follows that $\ell > 0$. In Region II, entrepreneurs engage in wasteful competition for entrepreneurial rents by acquiring positive amounts of short-term assets. As we will see, this inefficiency can be reduced by discouraging liquidity hoarding.

A firm's maximum scale of investment, given that it purchases liquidity ℓ, is given by the budget constraint (7.3). Substituting expression (7.14) for p into the budget constraint, we see that a firm invests in equilibrium

$$I = \frac{A - [g(\ell) - \ell]}{1 - \rho_0} < I^* . \tag{7.16}$$

The inequality follows from $\ell > 0$. When the fraction of distressed firms is large (low α) the date-1 resale market is awash with assets. The price p falls when α falls while firms respond by increasing ℓ and reducing I (buying assets in the spot market becomes increasingly attractive). To see this, suppose p at some α were to (weakly) decrease with α. Then (7.15) implies that ℓ would (weakly) increase with α. From (7.16) I decreases with ℓ, so ℓ/I would (weakly) increase, which would contradict (7.14). Therefore, p must be increasing in α in Region II.

For some $\hat{\alpha} > 0$, determined by equations (7.13) through (7.16) and the firm's first-order condition for choosing I, the price eventually hits its lower bound:[11]

$$p = \underline{\rho} > \rho_0.$$

Region I ($\alpha < \hat{\alpha}$): *Employing some assets in the alternative use (Second-best equilibrium).*

For values $\alpha < \hat{\alpha}$, a fraction of distressed assets are converted to the alternative use. The situation is somewhat similar to the basic model considered in Chapter 3 in which (intact) consumers prefer to consume rather than supply liquidity to firms. The reason why assets are inefficiently employed here is the same: there is a wedge between pledgeable and total income.

[11] At the lower bound, we must have $\ell > 0$ or else (7.13) would not hold for $p = \underline{\rho}$.

We summarize the analysis of this section in:

Proposition 7.1. *Suppose that the assets of distressed firms are either sold to intact firms or converted to an inferior alternative use at value* $\underline{\rho}$*; and that firms procure their liquidity independently of each other by obtaining the right to issue new claims on their long term assets at date 1 and by investing in costly short term assets at date 0.*

If $\underline{\rho} > \rho_0$*, then the price* p *at which LT assets are traded at date 1 is increasing in* α*. Furthermore, there exist thresholds* α^* *and* $\hat{\alpha}$*,* $1 > \alpha^* > \hat{\alpha} \geq 0$*, such that*

a) if $\alpha \geq \alpha^*$*, the outcome is second-best efficient and* $p = 1$*;*

b) if $\hat{\alpha} \leq \alpha < \alpha^*$*, firms hoard excess liquidity (*$\ell > 0$*) and* $p < 1$*; when* α *decreases,* ℓ *increases and* p *and* I *decrease.*

c) if $\alpha < \hat{\alpha}$*, firms hoard excess liquidity (*$\ell > 0$*) and* $p = \underline{\rho}$*, implying that some fraction of LT assets will be put into the (inferior) alternative use.*

If $\underline{\rho} \leq \rho_0$*, assets are never placed in the alternative use (*$\hat{\alpha} = 0$), but parts (a) and (b) hold as stated.

If we interpret a lower α as reflecting a worse economic outlook, then the fact that the date-1 spot price p increases with α implies that there is more hoarding of liquidity when entrepreneurs become more concerned about the risk of distress and potentially also when they anticipate a less efficient use of LT assets[12]

7.1.3 Policy implications

In the following, we ignore Region III ($\alpha \geq \alpha^*$) since the uncoordinated solution in this case coincides with the coordinated (efficient) solution. For the other regions we study government interventions, for different values of α, keeping in mind that the value of α is fixed and known to the government at the time it decides on any intervention.

[12] This property is obtained from the first-order conditions with respect to ℓ, I and j in Region II. In Region I, set $p = \underline{\rho}$ and use (7.14) and (7.16).

A. *Liquidity regulation: imposing a liquidity cap (or taxing liquidity)?*[13]

- Consider first Region II ($\widehat{\alpha} \leq \alpha < \alpha^*$). In this region all firms invest in liquidity and the intact firms buy all the long-term assets of the distressed firms at date 1. The ex post allocation is efficient and the deadweight loss stems solely from liquidity hoarding. Furthermore, even without purchasing any liquidity, intacts can still buy all the long-term assets on sale, since the spot price will drop accordingly (until it hits its lower bound). This suggests that a liquidity cap of 0 would be an optimal intervention that would eliminate rent-seeking without affecting the efficient transfer of assets.

To check this, note that (using (7.10) and (7.14)) the entrepreneur's net expected utility in this region is

$$U = \rho_1 I - I - [g(\ell) - \ell].$$

Using (7.14), the firm's budget constraint (7.3) can be expressed as

$$I - A + [g(\ell) - \ell] \leq \rho_0 I.$$

Because $g(\ell) - \ell > 0$ is increasing in ℓ, hoarding more liquidity reduces welfare both directly, and indirectly through the reduction in the investment level that investors are willing to grant. Thus as long as the LT asset is efficiently reallocated ex post, it is optimal to cap the use of liquidity as much as possible.

Let us imagine that the government bans liquidity hoarding altogether and sets $\ell = 0$. For which α values would intacts still be able to buy all the assets of the distressed? In Region III of figure 7.1, firms privately choose $\ell = 0$, so the government cap has no effect. When α goes below α^*, firms would like to buy liquidity, but with the government cap they cannot. As a consequence, their purchasing power in the date-1 market is reduced and the spot price is smaller than without government intervention as indicated in figure

[13] A steep tax on excess hoarding, starting at the level of the cap, achieves the same outcome as a cap. By contrast, an uncompensated tax on hoarding reduces entrepreneurial wealth.

7.2. Inserting $\ell = 0$ in (7.14), we see that the equilibrium date-1 price is

$$p = \frac{\rho_0}{1-\alpha} > \underline{\rho}, \tag{7.17}$$

as long as

$$\alpha > \hat{\hat{\alpha}} \equiv \frac{\underline{\rho} - \rho_0}{\underline{\rho}}. \tag{7.18}$$

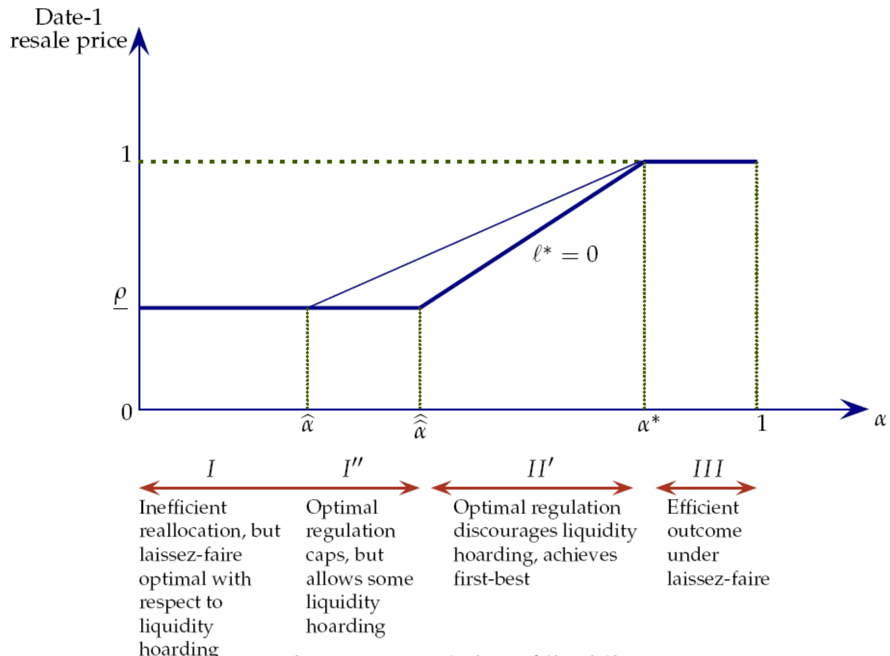


Figure 7.2: regulation of liquidity

Because the market clears when $p > \underline{\rho}$, the bound $\hat{\hat{\alpha}}$ in (7.18) defines the lower bound for α, such that there is a full transfer of assets from distressed to intacts when they cannot hoard any liquidity. Consequently, in Region II$'' = [\hat{\hat{\alpha}}, \alpha^*]$ of figure 7.2, it is optimal (and essential) for the government to ban all liquidity hoarding ($\ell = 0$). Hoarding of liquidity serves no efficiency purpose and merely results in rent-seeking.

Next consider Region I $= [0, \hat{\alpha}]$ from figure 7.1. Firms hoard liquidity in this region, but hoarding is efficient, because it does not affect the date-1 price. Hoarding serves efficiency by reducing the amount of long-term assets that get diverted to the alternative use. While some long-term assets still go to second-best use in Region I, because liquidity hoarding is increasingly costly, the government cannot improve on the situation (other than reducing the cost of liquidity provision – see below). Formally, this can be seen by checking that in Region I, the government's optimal program coincides with each firm's. (This is left to the reader).

Finally, consider Region $I'' = [\hat{\alpha}, \hat{\hat{\alpha}}]$. Note first that we must have $\hat{\alpha} < \hat{\hat{\alpha}}$, because preventing firms from hoarding liquidity reduces their purchasing power in the date-1 market. Hence the date-1 price will be uniformly lower with than without liquidity hoarding. However, if the government sets $\ell = 0$, the long-term assets cannot all be transferred to the intacts and some will go to the alternative use. Below we will show that it is in the government's interest to allow some hoarding of liquidity. Indeed, as long as hoarding does not raise the price in the date-1 market above $p = \underline{\rho}$, hoarding of liquidity will merely improve the transfer of assets from distressed to intacts. If the price rises above $\underline{\rho}$, however, we know that all assets are transferred and some of the liquidity is used for rent-seeking. Hence, the government's optimal policy is to allow enough hoarding of liquidity to permit full transfer of assets without letting the price rise above $\underline{\rho}$.

Let

$$w(\ell, I) \equiv (1-\alpha)I - \alpha j = (1-\alpha)I - \frac{\alpha(\rho_0 I + \ell)}{\underline{\rho} - \rho_0}$$

denote the amount of LT assets placed in alternative use (with $w = 0$ and $\ell = 0$ at $\alpha = \hat{\hat{\alpha}}$). Using the fact that $(1-\alpha)\underline{\rho} > \rho_0$ in this region, one has

$$\frac{\partial w}{\partial \ell} < 0 \text{ and } 1 > \frac{\partial w}{\partial I} > 0.$$

Welfare can be written as a function of the two forms of inefficiency, the waste of LT assets ex post and the inefficient build-up of liquidity ex ante:

$$U = [\rho_1 I - I] - [\rho_1 - \underline{\rho}]w(\ell, I) - [g(\ell) - \ell]. \tag{7.19}$$

The social planner's budget constraint is:

$$I - A + [g(\ell) - \ell] \leq \rho_0 I + (\underline{\rho} - \rho_0)w(\ell, I). \tag{7.20}$$

Sustituting this budget constraint into (7.19) welfare can be written:

$$U = (\rho_1 - \rho_0)[I - w(\ell, I)].$$

Ignoring the constraint $w \geq 0$ for the moment, and substituing out $w(\ell, I)$ from U and the budget constraint (7.20), the social planner's problem can be stated as:

$$\begin{aligned} &\max_{\{I,\ell\}} \left\{ \frac{\alpha}{\underline{\rho} - \rho_0} (\underline{\rho} I + \ell) \right\} \\ &\text{s.t.} \\ I \; &= \; [A - g(\ell) + (1-\alpha)\ell]/[1 - (1-\alpha)\underline{\rho}]. \end{aligned}$$

The solution to this program is given by

$$g'(\ell) = \frac{1}{\underline{\rho}}.$$

Defining $\ell(p)$ by $g'(\ell(p)) = 1/p$, and rewriting the waste as a function of ℓ after substituting for I from the budget constraint, $\widehat{w}(\ell, \alpha) \equiv w\left(\ell, \frac{A-g(\ell)+(1-\alpha)\ell}{1-(1-\alpha)\underline{\rho}}\right)$, a decreasing function of ℓ, one has $\widehat{w}(\ell(p(\alpha)), \alpha) = 0$ for $\alpha \in [\widehat{\alpha}, \widehat{\widehat{\alpha}}]$, and $p(\alpha) > \underline{\rho}$ in that range. Hence $\widehat{w}(\ell(\underline{\rho}), \alpha) < 0$, which violates the constraint $w \geq 0$. Hence at the optimum

$$w = 0.$$

As we announced, the regulator caps ℓ at the level that just prevents wasting assets by converting them to the alternative use.

B. *Public provision of liquidity*

We next investigate whether the state can improve welfare by increasing availabile stores of value. We only consider the impact of a marginal increase $\delta\ell$ in the supply per entrepreneur. Assume that the cost of taxation for the consumers at date 1 is $(1 + \lambda_0)\delta\ell$ and that the state at date 0 charges each entrepreneur $q_0 = (1 + \lambda_0)\delta\ell$. This keeps the welfare of the consumers constant, provided that the corporate tax is transferred to them.

To make things interesting, we assume that $(1 + \lambda_0)$ is smaller than the marginal cost of private liquidity creation, $g'(\ell)$, in the uncoordinated, laissez-faire equilibrium studied above (otherwise there would be no demand for public liquidity). Finally, we assume that there is no direct regulation of liquidity.

- Consider first Region I in which the LT asset is partly put into the alternative use $(\alpha < \widehat{\alpha})$. An increase in the public supply of liquidity shifts the cost function downwards

$$\widehat{g}(\ell) = \begin{cases} g(\ell) \text{ if } g'(\ell) \leq 1 + \lambda_0 \\ g(\ell - \delta\ell) + (1 + \lambda_0)\delta\ell \text{ if } g'(\ell - \delta\ell) \geq 1 + \lambda_0. \end{cases}$$

The cost function $\widehat{g}(\ell)$ is uniformly lower as δ is increased and therefore raises the objective function in the social program above. Public supply of liquidity is (at the margin) welfare-enhancing.

- In Region II $(\widehat{\alpha} \leq \alpha < \alpha^*)$, an increase in the supply of public liquidity has two opposing effects. As in Region I, it reduces the cost of accumulating liquidity; but it also encourages rent seeking. Either effect may dominate as seen from the following two cases:

  - (i) Suppose that $g(\ell) = q\ell$ for $\ell \leq \bar{\ell}$ and $+\infty$ for $\ell > \bar{\ell}$ (where $q > 1 + \lambda_0$), and that, in equilibrium without public liquidity, $\ell = \bar{\ell}$. Adding $\delta\ell$ of public liquidity increases the deadweight loss from $(q - 1)\bar{\ell}$ to $(q - 1)\bar{\ell} + \lambda_0\delta\ell$. Thus public liquidity reduces welfare.

    (ii) Suppose that $g(\ell) = q\ell$, with $q > 1 + \lambda_0$.[14] Now the marginal cost of liquidity, and therefore the choice of ℓ, is unchanged by additional public liquidity. The deadweight loss is reduced from $(q - 1)\ell$ to $(q - 1)(\ell - \delta\ell) + \lambda_0\delta\ell < (q - 1)\ell$, so public liquidity raises welfare in this case.

Proposition 7.2. (i) *In Region I′ some of the LT assets are wastefully redeployed in the alternative use, but firms hoard an optimal amount of liquidity, assuming that any*

[14] This violates our assumption that $g'(0) = 1$, but is inconsequential for our argument.

reallocation of liquidity must be voluntary. Suppressing liquidity reduces welfare because it increases the volume of assets that are turned to the alternative use. By contrast, adding publicly provided liquidity is welfare enhancing.

(ii) *In Region II', firms hoard excess liquidity under laissez-faire, but LT assets are properly reallocated ex post. The tightest possible constraint on liquidity hoarding maximizes welfare. As a potential alternative to regulation, public provision of liquidity has an ambiguous impact: it reduces the firms' cost of hoarding liquidity, but it facilitates rent seeking.*

We can summarize these results more succintly as follows

- A cap on liquidity hoarding increases welfare as long as assets are reallocated efficiently with the cap.
- Firms choose the efficient level of liquidity whenever some of the assets are put into alternative use.

Remark: Let us return to the observation discussed in the introduction, that many banks accumulated large amounts of reserves at the Fed and the ECB, possibly to take advantage of future fire sales, at the same time as other financial institutions and firms were in dire need of liquidity. In response to a near-zero federal funds rate, the Fed decided to raise the interest rate on bank reserves deposited at the central bank. This is the opposite of a policy that would encourage liquidity-rich institutions to invest their extra liquidity in the private sector. The welfare analysis in this section (regarding the benefits of a cap and the provision of liquidity) suggests that the Fed's policy may have been unwise.[15]

[15] The Fed's argument for paying interest on reserves was that this gave the Fed control of an interest rate that influenced credit. The Federal Funds rate had lost that role in the near-zero environment.

7.2 Precautionary hoarding in anticipation of a market freeze

Drawing on Malherbe (2009), this section studies a model where there may be asymmetric information about the value of assets at date 1. Under asymmetric information resale markets may break down as buyers become concerned that lemons are offered for sale (Akerlof 1970). On the other hand, if buyers know that firms face liquidity shocks unrelated to the quality of their assets, and that firms do not have insurance against such shocks, buyers will be less concerned about lemons and date-1 markets will be liquid. Expecting that the secondary market will be liquid, firms do not need to self-insure as much. The reason that assets are sold in the date-1 market will then be mostly due to liquidity shocks, rather than idiosyncratic shocks to the firm's asset. The fraction of lemons will be low, confirming the expectation that markets are liquid.

An analogy may be useful here. Students who are moving abroad and selling the car or the furniture make it clear that they are moving abroad. Similarly, people selling their house would be inclined to reveal that they are moving out of town or that they move for life-cycle reasons that make their current house too small or too large. Sellers try to alleviate adverse selection by disclosing a legitimate reason for selling. A similar effect operates here as the sellers would want buyers to believe that they are selling an asset, because of a fragile balance sheet rather than for opportunistic reasons.

To illustrate how adverse selection affects the hoarding of liquidity, we consider the following model.

Timing and actors. There are three periods, $t = 0, 1, 2$, and a continuum of firms.

Investment opportunity: At date 1, each firm will have a new (fixed sized) investment opportunity that, if undertaken, will deliver ρ_1 at date 2. To simplify notation, we assume that none of this value is pledgeable ($\rho_0 = 0$); this is inconsequential for the points we want to make. With probability α, the investment costs zero and with probability $1 - \alpha$

it costs I. There are idiosyncratic liquidity shocks, but no aggregate uncertainty. We note that investment I could alternatively be interpreted as a reinvestment on a project that started at date 0, in line with our earlier analyses.

Tradable legacy asset: Each firm owns a legacy asset that will deliver θ at date 2. The value θ is drawn from a cumulative distribution function $F(\theta)$ with support $[0, \rho_1]$ and mean $\overline{\theta}$.[16] The value of the legacy asset is independent of the cost of investment. No one knows the true value of θ at date 0. At date 1, the firm will have an opportunity to sell its legacy asset, knowing its true value, while the market will remain uninformed. Later, we will show that the conclusions are unchanged if a firm knows its θ and can sell the asset at date 0.

Assumption 1 Without private information, the firm could finance the date-1 project in all states of nature by selling the legacy asset:

$$\overline{\theta} > I.$$

Stores of value: At date 0, firms can buy stores of value (ST assets) priced $q \geq 1$ per unit, which deliver 1 per unit at date 1. We continue to assume that the firms' capital insurance decisions are separate, that is there is no pooling of liquidity. Let ℓ denote the short-term investment by the representative firm at date 0.

Assumption 2: At date 1, the legacy asset is indivisible; it must be sold as a single unit.

This assumption simplifies the analysis and avoids some technical complications.[17]

Assumption 3: A firm's purchase of ST assets is not observed by outsiders.

[16] The upper bound ρ_1 simplifies expressions, but is not crucial to the analysis.

[17] We are making the same indivisibility assumption as Akerlof did in his seminal paper. When a good for sale is divisible and subject to exclusivity (i.e., the seller must trade with a single buyer or none), a pure-strategy equilibrium in the market for lemons often does not exist, as was shown by Rothschild and Stiglitz (1976). By contrast, the equilibrium in the market for a divisible good under non-exclusive competition (shares may be sold to different buyers) always exists and is the Akerlof equilibrium: see Attar, Mariotti and Salanié (2009).

The third assumption can be motivated in two ways. First, as banking regulators know well, it is difficult to apprehend the liquidity position of a bank, because the liquidity position depends on many hard-to-observe factors: the correlation of the risks on the bank's balance sheet, the quality of its assets, the reliability of its counterparties, the banks's reputation, its contracts in OTC markets, its pledges for liquidity support, and so forth.

Second, and anticipating our analysis, firms would like to prove that they have not hoarded liquidity, since this makes them more reliable trading partners at date 1. However, it can be difficult to demonstrate to the market that one is selling due to liquidity needs rather than strategic reasons. We capture both of these considerations simply by assuming that the firm's choice ℓ is unobserved by the market.

Figure 7.3 summarizes the timing.

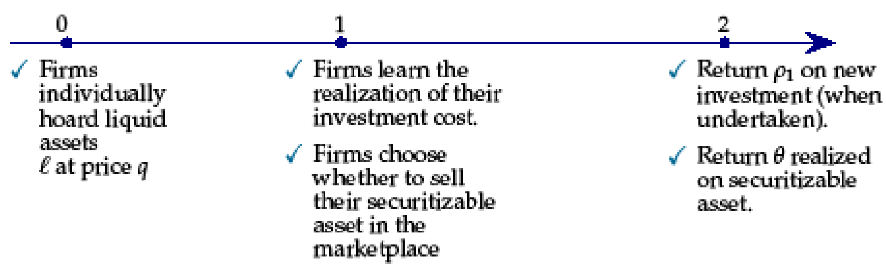


Figure 7.3

We will now demonstrate that market liquidity is endogenous and that multiple equilibria, one with sufficient market liquidity and the other with a market freeze, may co-exist.

A. *Equilibrium with a liquid market (no hoarding)*

Consider an equilibrium with a liquid secondary market, where firms do not hoard any liquidity at date 0:

$$\ell = 0.$$

Instead, firms count on selling their legacy asset if they need funds at date 1. Let $p(\alpha)$ be the equilibrium price of the assets in the date 1 market (α is the fraction of intact firms).

It is defined by:[18]

$$p(\alpha) = \frac{\alpha \int_0^{p(\alpha)} \theta dF(\theta) + (1-\alpha)\overline{\theta}}{\alpha F(p(\alpha)) + (1-\alpha)}. \tag{7.21}$$

The equation above sets the date-1 price equal to the expected value of the assets put on sale, assuming that all distressed firms sell their asset, along with the intact firms that have an asset worth less than $p(\alpha)$, as reflected in the nominator. The denumerator is the total fraction of assets sold.

The value of $p(\alpha)$ in (7.21) is decreasing in α as pictured in figure 7.4. The higher the fraction of intact firms, the more there is adverse selection and the lower is the price.

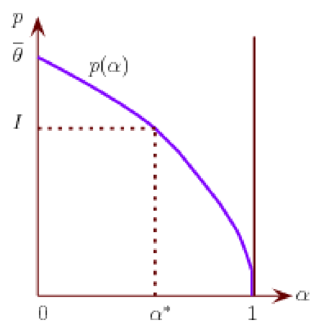


Figure 7.4

There are two premises behind (7.21). The first is that distressed firms sell their legacy asset, regardless of the asset's realized value θ. This will be true if and only if

$$p(\alpha) \geq I \quad \text{or} \quad \alpha \leq \alpha^*, \tag{7.22}$$

where α^* is defined by $p(\alpha^*) = I$. Since we assumed that the unconditional expected value of the legacy asset is higher than I, we have $p(0) > I$. Also, since the market is one with pure adverse selection when $\alpha = 1$, we have $p(1) = 0$. Because $p(\alpha)$ is strictly decreasing it follows that $\alpha^* \in (0, 1)$. Condition (7.22) will hold as shown in figure 7.4.

[18] It can be shown that $p(\alpha)$ is unique.

The second premise behind (7.21) is that firms do not hoard any liquidity at date 0. The firm's net expected utility without liquidity hoarding ($\ell = 0$) is:

$$U^{\text{no capital insurance}} = \rho_1 + \alpha\left[p(\alpha) + \int_{p(\alpha)}^{\rho_1} (\theta - p(\alpha))dF(\theta)\right] + (1-\alpha)(p(\alpha) - I).$$

This utility must exceed the utility that the firm obtains by choosing $\ell = I$ (because of our indivisibility assumption, choosing $\ell \in (0, I)$ is useless, while $\ell > I$ would be wasteful), which is

$$\begin{aligned} U^{\text{capital insurance}} &= -qI + \rho_1 + \alpha\left[I + p(\alpha) + \int_{p(\alpha)}^{\rho_1} (\theta - p(\alpha))dF(\theta)\right] \\ &\quad +(1-\alpha)\left[p(\alpha) + \int_{p(\alpha)}^{\rho_1} (\theta - p(\alpha))dF(\theta)\right]. \end{aligned}$$

Thus the second condition for a liquid market equilibrium is:

$$(q-1)I \geq (1-\alpha)\int_{p(\alpha)}^{\rho_1} [\theta - p(\alpha)]dF(\theta). \tag{7.23}$$

This condition is intuitive. It says that the cost of capital insurance, $(q-1)I$, must exceed the savings, $\theta - p(\alpha)$, associated with not selling an undervalued asset ($\theta > p(\alpha)$) when distressed. Note in particular that a liquidity premium $q - 1 > 0$ is required to sustain an equilibrium without ex ante hoarding of liquidity.

B. *Equilibrium with a market freeze (hoarding of liquidity)*

In an equilibrium with a market freeze, no transactions take place at date 1. The only way for a firm to finance an investment is to self-insure by buying short-term assets at date 0 in the amount $\ell = I$. The condition for an equilibrium with a market freeze is therefore:

$$\begin{aligned} &U^{\text{capital insurance}} = -qI + \rho_1 + \alpha I + \overline{\theta} \\ &\geq U^{\text{no capital insurance}} = \alpha\rho_1 + \overline{\theta}, \end{aligned}$$

or

$$(1-\alpha)(\rho_1 - I) \geq (q-1)I. \tag{7.24}$$

This condition simply says that the net cost of ex ante insurance, $(q-1)I$, must be smaller than the value of the foregone investment opportunity, else firms would not want to hoard liquidity.

Because firms already have all the liquidity they need, there are no gains from trade at date-1. The date-1 market is a pure lemons' market, clearing at a zero price with no trade:

$$\hat{p}(\alpha) = E[\theta|\theta \leq \hat{p}(\alpha)] \Longrightarrow \hat{p}(\alpha) = 0.$$

Multiple equilibria arise whenever $\alpha \leq \alpha^*$ and conditions (7.23) and (7.24) both hold. Combining these two conditions we get

$$\rho_1 - I \geq \int_{p(\alpha)}^{\rho_1} [\theta - p(\alpha)] dF(\theta), \tag{7.25}$$

where $p(\alpha)$ is defined by (3.21). Inequality (7.25) is a necessary condition for having both an equilibrium without any hoarding of liquidity ($\ell = 0$, $p(\alpha) \geq I$) and an equilibrium with a market freeze ($p(\alpha) = 0, \ell = I$).

There may also exist intermediate equilibria between these two extremes. In such equilibria $0 < p < I$ and $0 < \ell < I$, such that $p + \ell = I$. The price is no longer determined by (7.21), since some of the distressed firms will not sell their legacy asset. In particular, only distressed firms with $\theta \leq \rho_1 - \ell = \rho_1 + p - I < \rho_1$ will sell; the rest prefer to keep their legacy asset. As a result, the fraction of assets sold for strategic reaons will be higher, leading to more adverse selection and a price $p < I$. Again, the more firms procure liquidity in advance, the more severe the adverse selection will be and the lower the price. The equilibrium, as long as it exists, is unique for a given α, since the price decline per unit of increase in ℓ is less than one.

Because hoarding liquidity is costly, the Pareto-best equilibrium, given a fixed α, is the one with a liquid market and no hoarding. The worst equilibrium is the one where the market freezes and firms have to arrange all of the liquidity in advance. In between these two is the mixed equilibrium. These comparisons are, of course, relevant only when they exist for a given α.

Discussion (public policy). Government supplied liquidity can help or harm depending on circumstances. To see this, suppose $\alpha > \alpha^*$ so that the market for legacy assets alone cannot support the firm's investment (because $p(\alpha) < I$; see figure 7.4). Firms would then like to hoard liquidity in advance so that they can invest at date 1 as discussed above. If there are no private stores of value, the government can improve matters by supplying liquidity for this purpose. More generally, if there is a fixed but insufficient supply of private stores of value, the ex ante insurance market will push the liquidity premium up to the point where firms become indifferent between hoarding and not hoarding liquidity. If the liquidity premium in this equilibrium is higher than the government's cost of supplying stores of value, the government can improve welfare.

By contrast, when $\alpha \leq \alpha^*$, it is socially optimal not to issue government securities (or more generally to lower the yield on such securities so that (7.23) is violated), because the equilibrium without hoarding is more efficient than any other equilibrium. Issuing more liquidity may make no difference to the equilibrium, but would be wasteful (since there is a dead-weight loss from taxation). If anything, the government may want to reduce the supply of liquidity, raising its price, which could make the no-hoarding equilibrium more likely.

The general point here is that an increased supply of liquidity will lower the cost of liquidity and encourage hoarding. This makes adverse selection in the asset market worse, inhibiting the functioning of the date-1 market. In this light, the policy of central banks to accomodate the flight to quality during the subprime crisis (they welcomed deposits at the central bank) may have had the unintended consequence of delaying the recovery of asset markets. This conclusion is similar to our earlier point about the advisability of paying interest on reserves (see subsection 7.1.3) Our analysis underlines the exceptional informational requirements for a good public policy. Providing cheap public liquidity is desirable if markets do not provide enough credit, but it can also crowd out private markets for liquidity.

Discussion (market versus funding liquidity). Instead of relying on market liquidity by selling assets, as we have assumed so far, firms could rely on funding liquidity by issuing securities. We could have developed our analysis in this alternative context. At date 1, suppose that firms have no assets and hold private information about the value ρ_0 of their date-2 pledgeable income, which is now drawn from a distribution $G(\rho_0)$. Suppose further that the entrepreneurs keep all claims on pledgeable income until date 1 with the intent to raise funds by selling the claims at date 1 (this could be a cheaper way of saving). Then those who sell claims because of liquidity needs will on average offer higher quality claims than those who sell to unload impaired claims. A formula akin to (7.21) obtains: If the market for funding liquidity does not break down, shares will trade at price $p(\alpha)$ such that (with double use of notation)

$$p(\alpha) = \frac{\alpha \int_0^{p(\alpha)} \rho_0 dG(\rho_0) + (1-\alpha)\overline{\rho}_0}{\alpha G(\rho_0) + (1-\alpha)}.$$

By contrast, if firms were to invest in stores of value, the market for funding liquidity would dry up (under conditions similar to those discussed earlier).

Discussion (stigma). One of the potentially counter-intuitive features of the analysis presented above is that a firm benefits from appearing fragile. Having stored little liquidity and being forced to go to the asset market for more, rather than being selective about selling, is reassuring to potential buyers. Yet, common wisdom as well as empirical evidence has it that issuing securities or being forced to sell assets entail a stigma. Financial institutions even avoid turning to a government agency for help, despite the fact that the agency may ignore any stigma when it decides to lend. For instance, banks are often reluctant to borrow at the discount window and countries are often reluctant to apply for contingent credit lines (CCF) from the IMF. How can we reconcile the theory with this evidence?

A potential story has the following two ingredients. First, the bank or the country typically needs to borrow from other lenders than the central bank or the IMF, respectively, so borrowing from the discount window or a CCL may send a signal of fragility to these lenders. Second, we need to explain why borrowing from a public body conveys bad news about an institution. One possible reason is that the need for cash is due to the unvillingness of better informed lenders and counterparties to continue funding the institution. Whatever is the reason, our analysis is clearly picking up only a limited aspect of the story.

Discussion (securitization at date 0).

We have assumed that securities are issued at date 1.[19] Would the outcome be different if the firm could issue securities at date 0, knowing the value of its legacy assets (the realization of θ), but not yet knowing the cost of the new investment opportunity? To address this issue, let p_0 and p_1 denote the date-0 and date-1 equilibrium prices of the legacy asset. Because legacy assets represent a claim that can be resold at date 1, the date-0 price p_0 is equal, if there is a demand for liquidity at price q, to q times the expected dividend conditional on the legacy asset being sold at date 0. We consider possible equilibria in this new situation.

A. *No hoarding at date 0*

An equilibrium without hoarding at date 0 is observationally equivalent to the no-hoarding equilibrium when only date-1 securitization is feasible. As before, $p_1 = p(\alpha) \geq I$. (assuming that $\alpha \leq \alpha^*$, of course). Firms wait until date 1 to sell their legacy asset and do so if they are distressed or if they are intact and face a liquidity shock and $\theta \leq p_1$.

[19] One possible justification is that the firm is in the process of creating an asset that is not yet ready to be traded at date 0 (e.g. because of severe asymmetric information). This raises interesting avenues for extending the model. As has been demonstrated empirically, the prospect of issuing securities creates moral hazard (see Keys, et.al., 2009). So we would expect the distribution $F(\theta)$ to be less favorable in the securitization equilibrium than in the market freeze equilibrium. (For more on this, see Aghion, et al, 2004, who take a mechanism design approach to securitization under moral hazard. See also Faure-Grimaud and Gromb, 2004, as well as Parlour and Plantin, 2008.)

There is no trade of the legacy asset as long as $p_0 - (q-1)I \leq p_1$, that is, as long as there are no gains to trading early even if firms knew that they will sell at date 1, regardless of the liquidity shock. In particular, there is an equilibrium with $p_0 = 0$ and no trade, supported by the off-equilibrium belief that any attempt to sell at date 0 has a purely strategic motive.

Thus, the no-hoarding equilibrium in the earlier model is robust to the possibility of date 0 trading of the legacy asset under adverse selection.

B. *Hoarding at date 0*

In a hoarding equilibrium, $p_0 = p_1 = 0$ without any trading of assets. Firms will hoard their own liquidity (which requires (7.24) to be satisfied). Like the no-hoarding equilibrium, the hoarding equilibrium is robust to early trading.

C. *Other equilibria*

Could there be other equilibria? Intuitively, there are two forces that determine the equilibrium price. First, we would expect "more adverse selection" at date 0, since at date 1 there are at least some potential sellers who experience a liquidity shock. Ceteris paribus the sellers with low-quality assets are more eager to sell at date 0 (i.e. unconditionally) than those who have better quality assets and are more willing to wait to see whether they will end up distressed or intact at date 1. Second, selling claims at date 0 creates stores of value for the buyers and this has social value since $q > 1$. This could favor date-0 trading, though the benefit of saving by not selling also needs to be taken into consideration.

Let us look for an equilibrium in which some trade occurs at date 0. We are led to consider two cases:

Case 1: $p_1 < I$. In this case there are no gains from trade at date 1 so the only market for assets is at date 0. A firm that wants to invest at date 1 will have to buy liquidity at date 0, regardless of whether it sells the asset at date 0 or holds it to maturity. Type θ

decides to sell at date 0 rather than hold the asset to maturity if and only if

$$p_0 - (q-1)I + [\rho_1 - (1-\alpha)I] \geq \theta - (q-1)I + [\rho_1 - (1-\alpha)I] \tag{7.26}$$

or

$$p_0 \geq \theta.$$

The term on the left hand side of (7.26) includes the proceeds from selling at date 0, the cost of buying I units of liquidity to weather a low shock at date 1, and the net benefit of investing at date 1. The right-hand side has a corresponding interpretation, reflecting the value of investing and holding the asset to maturity. Let θ^* be the cutoff for selling the asset. Note that $p_0 = qm^-(\theta^*)$, where $m^-(\theta^*) = E(\theta|\theta \leq \theta^*)$ is the expected the value of the assets put up for sale. The equilibrium cutoff, if it is interior, satisfies:

$$qm^-(\theta^*) = \theta^*.$$

For instance, in the case of a uniform distribution, $m^-(\theta^*)/\theta^* = 1/2$, so trade occurs at date 0 (and involves all types) if and only if $q \geq 2$.

Case 2: $p_1 \geq I$. In this case, there is a need to buy liquidity at date 0 only if the asset is sold at date 0. If it is sold at date 1, the price at date 1 is sufficient to cover the investment need.

Selling at date 0 yields as before

$$p_0 - (q-1)I + [\rho_1 - (1-\alpha)I].$$

On the other hand, the value of holding off on selling until date 1 is

$$[\rho_1 - (1-\alpha)I] + p_1 \quad \text{if} \quad \theta \leq p_1$$

and

$$[\rho_1 - (1-\alpha)I] + \alpha\theta + (1-\alpha)p_1 \quad \text{if} \quad \theta \geq p_1.$$

Again, only types below some cutoff θ^* sell at date 0. Futhermore, a necessary condition for trade at date 0 is

$$p_0 - (q-1)I \geq p_1.$$

Since only types above θ^* can sell at date 1, we must have $p_1 \geq \theta^*$ and therefore an equilibrium of the assumed kind exists only if there is a θ^* such that:

$$qm^-(\theta^*) - (q-1)I = p_1(\theta^*)$$

where $p_1(\theta^*)$ is the solution to

$$p_1 = \frac{\alpha \int_{\theta^*}^{p_1} \theta dF(\theta) + (1-\alpha)\int_{\theta^*}^{\rho_1} \theta dF(\theta)}{\alpha\left[F(p_1) - F(\theta^*)\right] + (1-\alpha)[1-F(\theta^*)]}.$$

Discussion (mixing financial muscle and adverse selection). Bolton, Santos and Scheinkman (2009) argue that adverse selection may increase liquidity hoarding, but for a somewhat different reason. Their model involves two sets of players: long-term arbitrageurs (sovereign wealth funds, pension funds,...), which hoard costly liquidity to buy assets at cash-in-the-market prices (that is, at a discount) and firms, which before the date of reckoning, i.e. before the final return on a project is realized, face a liquidity need. Uncertainty about the final profit of each firm unfolds gradually, and efficient trade involves trading as late as possible before the date of reckoning. Delaying trade reduces the quantity hoarded by long-term investors, as they do not have to acquire intermediate dividends.[20] However late trading may not be feasible as Bolton et al posit that adverse selection increases over time. The prospect of a market freeze then forces early trading and therefore more liq-

[20]The paper assumes all-or-nothing trades, that is, a firm's cash flow cannot be split into securities with different maturities. Otherwise the long-term investors could economize on costly liquidity hoarding by purchasing only the securities' late dividends, and not the early ones.

In the paper, there are four dates: 0, 1, 2, 3. The ST traders have preferences $c_0 + c_1 + c_2 + \delta c_3$ with $\delta < 1$. Thus ST traders do not need to trade before date 2, at which date they need to sell only the claim to the last dividend.

uidity hoarding. Increasing adverse selection provides incentives for early trading, which, if tranching is infeasible, induces an inefficient amount of liquidity hoarding.

7.3 Summary

This chapter has shown that the inability of firms to write state-contingent contracts on the available supply of liquidity at date 1 may lead to excess investments in liquidity at date 0 as well as to a wrong allocation of liquidity at date 1. When liquidity is used to buy distressed assets, investments in liquidity tend to be excessive due to wasteful competition for ex-post rents. However, our analysis of predatory hoarding of liquidity also shows that decentralized choices regarding liquidity do not necessarily cause inefficiencies. With our particular structure, spot markets are often capable of replicating the fully coordinated solution. And even when spot markets cannot duplicate the coordinated solution, the government is not always able to increase welfare by regulating liquidity. If the problem is a reluctance on the part of firms with excess resources to reallocate these resources to distressed firms, then increasing government supplied liquidity may simply lead to more assets being employed suboptimally at date 1.

We also showed that precautionary hoarding of liquidity may have the unintended consequence of a market freeze. Buyers may question the sellers' motives for selling their assets if they know that the sellers have a sufficiently large cushion of cash. Liquidity hoarding encourages market freezes (and conversely). While the laissez-faire outcome may involve excessive liquidity hoarding, in some circumstances precautionary hoarding is desirable. Distinguishing between two cases with such radically different policy implications requires detailed information and makes the policy task very challenging.

An extension that would be interesting and relatively straightforward to explore is to have the fraction of intact firms be random. This would be one way of introducing an aggregate shock.[21] Given that the shock is aggregate, it should be possible to write

[21] Allen and Gale (1994, 2005) allow for such aggregate shocks in their models.

contracts on the state. The characteristics of an optimal supply of liquidity (eg by government) when the idiosyncratic shocks must be handled through spot markets would be interesting to understand.

While the two motives leading to overhoarding of liquidity (predatory and precautionary hoarding) seem realistic, we certainly do not believe overhoarding is the rule. Indeed, the subprime crisis appears to have been triggered in part by underhoarding of liquidity and excess leverage among financial institutions. Many commercial and investment banks (and insurance companies, like AIG) relied on short-term financing for continued funding, instead of investing in short-term assets that could have been used when liquidity got scarce. These institutions, which benefitted from cheap sources of capital thanks to an implicit guarantee from the governments, took increasingly risky gambles with more severe maturity mismatches despite warnings. As Farhi and Tirole (2009b) argue, maturity mismatches are subject to strategic complementarities. Central banks are unlikely to maintain interest rates low in order to keep a single institution alive. By contrast, when faced with a widespread maturity mismatch, the centrals banks have no choice but keeping the interest rate very low, at the cost of substantial distortions to the economy; and so they did.

Chapter 8

Specialized inputs and secondary markets

This chapter continues to analyze the implications of firms relying exclusively on secondary markets for their liquidity needs. As in the previous chapter, we assume that firms do not coordinate their balance sheet choices, but while in section 7.1, the motive is self-insurance. Firms hoard an input, which we will generically call a "widget." This input is needed to keep long-term assets operative, much as in the basic model we studied Parts II and III of the book. A widget could stand for many things. It could literally be a part that is needed for continuing production; it could be a service or knowledge that is needed to make the asset productive; or, as in the application of section 8.2, it could stand for collateral secured on international financial markets.

In section 8.1, ex ante identical firms invest both in illiquid, long-term assets and in widgets. Widgets are used at date 1 to help those firms that end up being distressed to weather the shock. Widgets however need not be in the right hands, and so in the absence of an ex ante coordinated pooling arrangement, a secondary market will open to allow transfers of widgets held by intact firms to distressed firms. As in section 7.1, buyers may not have enough financial resources to acquire the needed widgets at a price that dominates the alternative use of widgets. Therefore, financial constraints may lead to an inefficient reallocation of widgets through the secondary market. One of the central question we will address is whether firms will tend to underhoard those widgets. Intuitively, one might reason that a low resale price should discourage investments in widgets: A firm that ends up on the sell side would have to sell widgets at a discount relative to the social value. A firm that ends up on the buy side would get widgets at a discount. Both forces would seem to favor less hoarding and more reliance on the date-1 market. We will show however that this intuition is wrong in our model. In our equilibrium there is no underhoarding, even though the date-1 widgets sell at a discount.[1]

Section 8.2 applies the model in section 8.1 to an international context. Widgets refer to dollars, i.e., international liquidity. Our analysis of international liquidity in Chapter 6 presumed that the firms optimally coordinate their liquidity plans and focused on the value of international financial markets as a source of insurance. In section 8.2, the focus is on international liquidity (dollars) as an input in the production process. For this reason we assume that there are no macroeconomic shocks. Like the widgets in section 8.1, dollars are needed for re-investment. We are interested in possible inefficiencies arising from the firms' uncoordinated purchase of dollars at date 0 – in particular the possibility that firms will not secure enough dollars at date 0 (or equivalently, that they will borrow excessively on the international markets).

8.1 Secondary markets for specialized inputs

8.1.1 Model and two benchmarks

We assume that firms are ex ante homogenous but ex post heterogenous. Our model is again one where distressed assets are salvaged by intact firms. But we turn the model of section 7.1 on its head by assuming that distressed firms do not sell their assets to intacts, but rather that intact firms sell their assets to distressed firms. Those in distress buy rather than sell. Secondary markets will trade liquid (ST) assets rather than illiquid

[1] Caballero and Krishnamurthy (e.g. 2001, 2002) explain their results using this kind of intuition. We will discuss why the "cheap widget" intuition fails in our setting.

(LT) assets. And liquid assets rather than illiquid ones will sell at a discount relative to their date-0 prices.

At date 0 a representative entrepreneur has wealth A and invests

- I in LT assets, delivering, at date 2, $\rho_1 I$ in total income and $\rho_0 I$ in pledgeable income, provided that the assets are still in use at date 2;

- ℓ in ST assets (widgets), where ℓ is the number of units received at date 1 in return for a date-0 investment $g(\ell)$. For simplicity, we assume that the marginal cost of producing widgets is constant, that is $g(\ell) = q\ell$, where $q \geq 1$. Firms that invest in the LT asset also have access to the ST widget technology.

At date 1, a firm will be

- *intact* with probability α, where "intact" means that the firm does not need any widget. Its long-term investments deliver total value $\rho_1 I$ and pledgeable income $\rho_0 I$ at date 2;

- *distressed* with probability $1-\alpha$, where "distressed" means that it takes one widget per unit of long-term investment to deliver the date-2 returns of intact firms. A distressed unit that does not get a matching widget at date 1 generates no income at date 2.

As in Chapter 7, there is no aggregate uncertainty. Widgets at date 1 can be converted to an alternative use (the "baseball field" of the parable in Chapter 7) with per unit value[2]

$$\underline{\rho} < q.$$

Only an intact firm will face the choice of converting its widgets to the alternative use.

A. Second-best benchmark (full coordination)

[2] In this model, unlike the one in chapter 7, there will be no inefficiencies without an outside outside option.

As a benchmark, we first consider policies in which firms coordinate their date-0 investments and the date-1 use of widgets. Because there is no aggregate uncertainty and the investment and use of widgets are fully coordinated, there is never any excess accumulation of widgets. No widgets are converted to the alternative use (since $\underline{\rho} < q$) and so we can, without loss of generality, assume that widgets are only used by the distressed firms.

How much will the firms invest in widgets (ST assets)? Intuitively, it is worth investing in widgets whenever they are cheap enough (q is small) or the probability of distress $(1-\alpha)$ is high. To demonstrate this formally, consider the following social planning program.

$$\max_{\{I,\ell,m\}}\{(\rho_1 - \rho_0)\,[\alpha I + (1-\alpha)(\ell + m)]\} \tag{8.1}$$

$$\text{s.t.}$$

$$\alpha\ell \geq (1-\alpha)m, \tag{i}$$

$$I + q\ell - A \leq \alpha\rho_0 I + (1-\alpha)\rho_0(\ell + m), \tag{ii}$$

$$\ell + m \leq I. \tag{iii}$$

The choice variables are: the date-0 investments I in LT assets and ℓ in ST assets (widgets) and the date-1 transfer m of widgets from intact firms to distressed firms. The objective is to maximize the representative entrepreneur's gross utility, which is equal to the social surplus, because the budget constraint must bind at an optimum. The first constraint is the date-1 aggregate resource constraint; it requires that the transfer of widgets to the distressed firms cannot exceed the supply of widgets by the intact firms. As noted above, this constraint could be written as an equality, because it would be wasteful to hoard more widgets than will be used by the intacts (recall that there is no aggregate uncertainty). The second constraint is the date-0 resource, or budget constraint. The third constraint is technological: the continuation scale cannot exceed I.

Let the date-1 continuation scale be $i = \ell + m$. Constraint (i), as an equality, implies

$m = \alpha i$ and $\ell = (1-\alpha)i$. Noting that $m = \alpha i$ and $\ell = (1-\alpha)i$, we can then rewrite the social program as

$$\max_{\{I,i\}}\{\alpha I + (1-\alpha)i\} \tag{8.2}$$

s.t.

$$I + q(1-\alpha)i - A \leq \alpha\rho_0 I + (1-\alpha)\rho_0 i, \tag{i}$$

$$i \leq I. \tag{ii}$$

In this program, the firm chooses the initial scale I and the continuation scale i with the continuation i assured by choosing m and ℓ according to $m = \alpha i$ and $\ell = (1-\alpha)i$. The budget constraint, as an equality, allows us to subtitute out I and write the social objective (dropping constants) as

$$\max_{i}\{[\alpha(\rho_0 - q) + (1-\alpha\rho_0)]i\}$$

It follows that it is socially optimal to continue at full scale (choose $i = I$) whenever

$$\alpha q < 1. \tag{8.3}$$

As a weak inequality, the condition is necessary for buying widgets at date 0 ($\ell > 0$). As one would expect, liquidity hoarding is more desirable, the higher the probability of distress and the cheaper the acquisition of liquidity.

B. Another useful benchmark: the autarky case.

Even though we will allow a spot market for reallocating widgets shortly, it is also useful to study the autarky case as a benchmark. Indeed, for very low levels of ρ_0, distressed firms have little with which to pay for widgets and so, even in the presence of a secondary market, trade will be minimal and the outcome will resemble autarky.

In autarky, each firm provides for its own widgets: there is no coordination or reallocation of widgets. Given the linear technology, a firm has two relevant choices. It can bet all its money on being intact and use its capital to maximize the initial scale without any investment in widgets. Alternatively, it can allocate its capital equally between the initial investment I and widgets $\ell = I$ so that it can continue at full scale even when it ends up in distress. The latter strategy will waste part of the initial investment in widgets, because if the firm ends up being intact, it recovers only $\underline{\rho} < q$ per unit. On the other hand, the former strategy will waste the initial investment if the firm ends up in distress.

With the first strategy the maximal scale and the entrepreneur's *gross* payoff are:

$$I_I = \frac{A}{1 - \alpha\rho_0}, \tag{8.4}$$

$$\tag{8.5}$$

$$U_I^g = \frac{(\rho_1 - \rho_0)A}{\frac{1}{\alpha} - \rho_0}. \tag{8.6}$$

The maximal scale associated with investing in widgets (the second strategy) is given by

$$I + qI \leq A + \rho_0 I + \alpha\underline{\rho} I,$$

yielding

$$I = \frac{A}{1 + q - (\rho_0 + \alpha\underline{\rho})}. \tag{8.7}$$

So the *gross* payoff to the entrepreneur is

$$U_{II}^g = \frac{(\rho_1 - \rho_0)A}{1 + q - \rho_0 - \alpha\underline{\rho}}. \tag{8.8}$$

Comparing U_I^g and U_{II}^g we find that hoarding is optimal in autarky if and only if

$$\alpha(1 + q) - \alpha^2\underline{\rho} < 1. \tag{8.9}$$

There is an important difference between the coordinated hoarding criterion (8.3) and the autarky hoarding criterion (8.9). In autarky, the firm is forced to hoard the initial investment ($\ell = I$) in order to continue at full scale, while in the coordinated solution it suffices to hoard $\ell = (1 - \alpha)\, I$, because liquidity is optimally shared. So, hoarding is more costly in autarky and the cut-off criterion for hoarding therefore more stringent. If there is hoarding in autarky, there will be hoarding in the coordinated case as well. Moreover, there is a region of α for which there is no hoarding in autarky even though there is hoarding in the coordinated case.[3]

8.1.2 Self-provision of liquidity with resale

Suppose now that at date 1 a spot market opens in which the intact firms may sell their widgets (for which they have no use) to distressed firms. The distressed firms pay for these widgets by issuing claims on their pledgeable income. In addition, the distressed firms can rescue long-term assets by using widgets procured at date 0. We first investigate the case where widgets have no alternative use and hence are all sold to distressed firms.

- *No alternative use for widgets* ($\underline{\rho} = 0$)

In this section, we show that without an outside option for liquidity (a "baseball field" in the parable of the introduction to Section 7.1), trading in a date-1 spot market always achieves the second-best outcome. We do it by showing that the firm's optimization program is equivalent to the the second-best, full-coordination program described in subsection A above.

Let p be the date-1 spot price for widgets. The firm's problem is solved by the program

$$\max_{\{I,\ell,m\}}\{(\rho_1 - \rho_0)\left[\alpha I + (1-\alpha)(\ell + m)\right]\} \tag{8.10}$$

s.t.

$$pm \leq \rho_0(\ell + m), \tag{i}$$

$$I + q\ell - A \leq \alpha(\rho_0 I + p\ell) + (1-\alpha)[\rho_0(\ell + m) - pm], \tag{ii}$$

$$\ell + m \leq I. \tag{iii}$$

[3]Let $\widetilde{\alpha}(\underline{\rho})$ denote the solution of:

$$\alpha(1+q) - \alpha^2\underline{\rho} = 1.$$

Note that $\widetilde{\alpha}(q) = \frac{1}{q}$ (the cutoff found in the perfectly corrdinated solution), that $\widetilde{\alpha}(0) = \frac{1}{1+q}$, and that for all $\underline{\rho} < q, \widetilde{\alpha}(\underline{\rho}) < \frac{1}{q}$.The region with hoarding only in the coordinated case is the interval $\left(\widetilde{\alpha}(\underline{\rho})\,, \frac{1}{q}\right)$.

Constraint (i) is the firm's date-1 liquidity constraint, which limits purchases m to no more than the pledgeable income. Constraint (ii) is the date-0 budget constraint and constraint (iii) limits the continuation to the initial scale of investment. The objective function maximizes the entrepreneur's gross utility; because the budget constraint will bind, the entrepreneur obtains all the social surplus.

Since an intact firm has no other outlet for its stored widgets than the spot market, all the widgets must in the date-1 equilibrium be transferred from the intact firms to the distressed firms (without aggregate certainty, there is no point in buying more widgets than are needed in equilibrium). This gives the equilibrium condition

$$\alpha \ell = (1 - \alpha) m. \tag{8.11}$$

Adding up the individual firms' liquidity constraints (i) and the aggregate date-1 liquidity constraint (8.11), yields the following condition which p must satisfy:

$$p \leq \frac{\rho_0}{\alpha}. \tag{8.12}$$

To build some intuition for the solution, it is helpful to consider the relative attractiveness of investments in short-term and long-term assets, ignoring the liquidity constraint (i) and the feasibility constraint (iii). That is, we look at the benefit of ℓ, m and I relative to the budget expenditure.

Consider first the choice of acquiring widgets in the *primary* versus the *secondary* market: A firm can procure widgets ℓ at date 0 for price q or widgets m at date 1 for price p. Suppose $p > q$ and $m > 0$ and consider the effect of substituting a unit of ℓ for a unit of m in the firm's program (8.10). This will loosen the firm's budget constraint (ii) by $(p - q) > 0$, loosen the firm's liquidity constraint (i) by p and leave the firm's objective and the technological constraint (iii) unaltered. The entrepreneur could therefore do strictly better by reducing m all the way down to zero. But at zero demand, the date-1 equilibrium price cannot be positive, a contradition. So in equilibrium, we must have $p \leq q$. Running the substitution argument in reverse, we see that $p < q$ implies that the liquidity constraint (8.10)(iii) must bind, else the entrepreneur could do better by substituting m for ℓ. Note also that the liquidity constraint (8.10)(i) can bind only if $p \geq \rho_0$. We have established that the equilibrium price must satisfy:

$$\rho_0 \leq p \leq q. \tag{8.13}$$

With these preliminaries we proceed to analyze the four regions of α that characterize the solution to program (8.10), and show that in the absence of an alternative use for liquidity at date 1, the outcome will be efficient.

Region I $(0 < \alpha \leq \rho_0/q)$*: Widgets are resold at their purchase price*

In this region, there are few intact firms (α is small), and so widgets are in high demand and command a high price in the secondary market. For this reason, we conjecture that the equilibrium price will be at its upper bound, $p = q$, (see (8.13)). We also conjecture that the firm's liquidity constraint is slack at this price and relax the firm's program by ignoring this constraint. We will come back to verify that both conjectures are true in equilibrium in the indicated range of α.

When $p = q$ the firm's budget constraint (8.10) (ii) can be written

$$I + q(1-\alpha)(\ell + m) - A \leq \alpha\rho_0 I + (1-\alpha)\rho_0(\ell + m). \tag{8.14}$$

In the relaxed program, where the firm's liquidity constraint is dropped, only the choices I and $i = \ell + m$ matter. We can therefore impose the additional constraint

$$m = \alpha i \text{ and } \ell = (1-\alpha)i \tag{8.15}$$

and restrict the firm to chosing ℓ and m along this ray. Substituting (8.15) and $p = q$ into the firm's liquidity constraint we get

$$\alpha q \leq \rho_0 < 1 \tag{8.16}$$

We see that on the ray (8.15) the validity of the liquidity constraint does not depend on i. The constraint will be satisfied if and only if $\alpha \leq \rho_0/q$, which is the upper bound of Region I. So, the solution to the relaxed program (without liquidity constraint) satisfies the liquidity constraint in Region I when $p = q$. Finally, it is immediate that the market equilibrium condition (8.11) holds whenever the firm chooses ℓ and m along the ray (8.15); indeed, the ray condition is equivalent to the market equilibrium condition as we noted earlier.

We have verified that the equilibrium in Region I entails $p = q$ with the representative firm choosing $m = \alpha I$, $\ell = (1 - \alpha)I$, where (using the budget constraint)

$$I = \frac{A}{1 + q(1 - \alpha) - \rho_0}. \tag{8.17}$$

In Region I, the firm's liquidity constraint will be slack until α hits its upper bound. Conversely, note that if the liquidity constraint is slack, we must have $p = q$ or else the firm would do better either by buying all its liquidity at date 0 or at date 1, violating the equilibrium condition. The ex post competition for scarce liquidity goes hand in hand with a non-binding liquidity constraint.

It is more surprising that the uncoordinated equilibrium in Region I is actually the same as the second-best solution obtained by coordinating liquidity purchases (as in the benchmark model discussed earlier). This can be verified directly by solving the second-best program (8.1). But it is more instructive to check that the firm's program (8.10) is identical with the second-best program when $p = q$. Note first that the liquidity constraint in the firm's program is irrelevant in Region I. Condition (8.15) assures that the date-1 resource constraint in the second-best program is satisfied regardless of i. When one substitutes $p = q$ and (8.15) into the firm's budget contraint (8.14) and the second-best program (8.1), the two programs can be seen to be identical. Finally, note that $\alpha q < 1$, so the second-best program has $i = I$.

Region II $(\rho_0/q < \alpha < \rho_0/\underline{\rho})$*: Resale at a discount*

When $\alpha > \rho_0/q$, the solution in Region I no longer works, because it violates the firm's liquidity constraint. In Region II, therefore, we expect the liquidity constraint to bind. We proceed with this conjecture and verify that it is indeed true in equilibrium.

The binding liquidity constraint $pm = \rho_0(\ell + m)$, together with the market clearing condition $\alpha\ell = (1-\alpha)m$, implies that the market price must be

$$p = \frac{\rho_0}{\alpha} < q. \tag{8.18}$$

The inequality follows from the lower bound of Region II. If we insert this equilibrium price into the firm's liquidity constraint we get $m = \alpha i$, implying that $\ell = (1-\alpha)i$. So, the liquidity constraint at the equilibrium price coincides with the date-1 resource constraint of the second-best program. Furthermore, the firm's budget constraint becomes

$$I + q(1-\alpha)i - A \leq \alpha\rho_0 i + (1-\alpha)\rho_0 i + [\alpha p\ell - (1-\alpha)pm] = \alpha\rho_0 i + (1-\alpha)\rho_0 i. \tag{8.19}$$

Therefore, the budget constraint in the firm's program is identical to the budget constraint in the second-best program. We conclude that the second-best and first-best programs coincide again and hence the solution in Region II will be efficient. The investment I will be given by (8.17) and $i = I$. In particular, firms will buy a strictly positive amount of widgets at date 0.

The reader may find it surprising that efficiency obtains in Region II despite an undervalued resale price ($p < q$). After all, intact firms sell their widgets at too low a price and distressed firms buy them up at an equally low price. Either way, ex ante firms should have an incentive to underhoard widgets. However, this reasoning misses the point that widgets relax the liquidity constraint of distressed firms: The more widgets distressed firms own, the higher their funding liquidity, and so the more widgets they can purchase: see the firms' liquidity constraint (8.10)(i). And the lower the secondary market price p, the more distressed firms can lever up the pledgeable income that stems from their own widgets. As a result, under constant returns to scale there is neither underhoarding nor overhoarding.[4]

- *Widgets have an alternative use* ($\underline{\rho} > \rho_0 q$)

Without the outside option, Region II would extend all the way to $\alpha = 1/q$ and after that $i = I$. The uncoordinated solution would coincide with the coordinated solution for all α. To introduce a source of inefficiency, we assume that there is an alternative use for widgets (a "baseball field" in the langue of Chapter 7), which intact firms can make use of. The value of this use is assumed to be $\underline{\rho} > \rho_0 q$, so that $\rho_0/\underline{\rho} < 1/q$, which is the point at which it becomes inefficient in the second-best to continue.

In Region II, as α increases, the equilibrium price $p = \rho_0/\alpha$ falls. The alternative use of widgets implies

$$p \geq \underline{\rho} \tag{8.20}$$

This constraint binds whenever

$$\alpha > \frac{\rho_0}{\underline{\rho}}, \tag{8.21}$$

which defines the upper bound of Region II.

Region III ($\rho_0/\underline{\rho} < \alpha < \alpha_0 < 1/q$).[5] *Widgets are in part converted to the alternative use.*

Other things equal, ρ_0 has to be sufficiently small to satisfy (8.21) A small ρ_0 means that the purchasing power of the distressed firms is small; the smaller it is, the more of

[4] This discussion raises the issue of what would happen if relatively cheap stores of value (Treasury bonds) were available as well. Would they crowd out the hoarding of widgets, offering a better yield while allowing firms to obtain refinancing? We have not conducted a full analysis, but we conjecture that the answer is "no". If widgets are expected to be cheap in the secondary market, then firms will hoard substantial amounts of Treasury bonds to take advantage of fire-sale prices. This will raise the price of widgets until the firms are indifferent between hoarding widgets and hoarding Treasury bonds.

When Treasury bonds are costless (their price is equal to 1), the liquidity constraint is never binding as firms have access to costless liquidity. Then regardless of the value of other parameters, the widgets trade at no discount in the secondary market: $p = q$.

[5] The upper limit α_0 will be defined when we discuss Region IV.

the intact firms' widgets will go to the alternative use. To be more precise, the liquidity constraint will allow a firm to purchase widgets (at price $p = \underline{\rho}$) up to the amount

$$m = \frac{\rho_0 \ell}{\underline{\rho} - \rho_0}. \tag{8.22}$$

Consider an equilibrium in which some liquidity is converted to the alternative use and so the market price for liquidity is

$$p = \underline{\rho}.$$

There are two cases to consider:

- *The firm withstands the shock:*

  It then purchases the maximal amount (8.22) and so, as $\ell + m = I$

$$\ell = \frac{\underline{\rho} - \rho_0}{\underline{\rho}} I$$

This yields gross utility:

$$U_{\mathrm{II}}^g = \frac{(\rho_1 - \rho_0) A}{1 + q\frac{\underline{\rho} - \rho_0}{\underline{\rho}} - \alpha \underline{\rho}}.$$

- *The firm does not withstand the shock:*

  This strategy, as usual, yields gross utility:

$$U_{\mathrm{I}}^g = \frac{(\rho_1 - \rho_0) A}{\frac{1}{\alpha} - \rho_0}.$$

For firms to be willing to accumulate liquidity, one must have

$$U_{\mathrm{II}}^g \geq U_{\mathrm{I}}^g$$

or

$$1 + q\frac{\underline{\rho} - \rho_0}{\underline{\rho}} - \alpha \underline{\rho} \leq \frac{1}{\alpha} - \rho_0. \tag{8.23}$$

Let α_0 be the highest α satisfying (8.23), implicitly defined by

$$1 + q\frac{\underline{\rho} - \rho_0}{\underline{\rho}} - \alpha_0 \underline{\rho} = \frac{1}{\alpha_0} - \rho_0.$$

Finally, we check that there is enough liquidity on the market to support the suggested solution. This requires

$$\alpha\ell \geq (1-\alpha)m,$$

which indeed holds, by (8.22), whenever

$$\alpha \geq \frac{\rho_0}{\underline{\rho}}, \tag{8.24}$$

that is, in Region III.

If (8.24) is slack, then there is excess liquidity in the market and some of the widgets will be converted to the alternative use. The wasted amount is found (after some manipulations) to be

$$\begin{aligned} w &= \alpha\ell - (1-\alpha)m = \frac{\alpha\underline{\rho} - \rho_0}{\underline{\rho}} I \\ &= \left(\frac{\alpha\underline{\rho} - \rho_0}{\underline{\rho}}\right) \left(\frac{A}{1 + q\left(\frac{\underline{\rho} - \rho_0}{\underline{\rho}}\right) - \alpha\underline{\rho}}\right). \end{aligned}$$

Simple computations show that

$$\frac{\partial w}{\partial \rho_0} < 0.$$

A smaller pledgeable income implies more waste of liquidity.

Region IV $(\alpha > \alpha_0)$. *No investment in liquidity*

Firms may not find it attractive to hoard liquidity at all when the spot price is $\underline{\rho}$. This arises when (8.23) is violated. Firms will not invest in liquidity when $\alpha > \alpha_0$. Note that

$$\alpha_0 < \frac{1}{q}.$$

Thus the region over which there is investment in liquidity is smaller than that in the coordinated case, giving rise to an inefficiency.

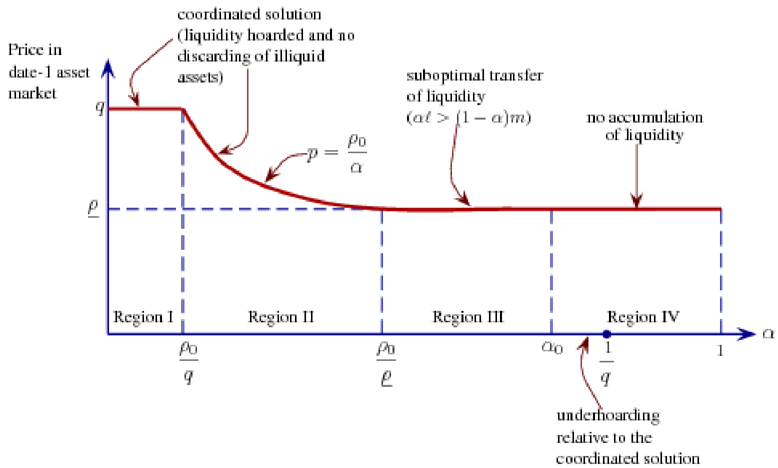


Figure 8.1: Attractive alternative use ($\underline{\rho} > \rho_0$)

We summarize these findings in figure 8.1 and in:

Proposition 8.1. *Suppose that there is a date-1 spot market and that the liquidity (widgets) of intact firms can either be turned to an alternative use (with per-unit value* $\underline{\rho}$*) or be sold to distressed firms. Then*

(i) uncoordinated investment in liquidity (self-provision) will replicate the second-best solution if $\alpha \leq \rho_0/\underline{\rho}$. *The resale price equals the purchase price* $(p = q)$ *if* $\alpha \leq \rho_0/q$. *Otherwise it involves a discount* $(p < q)$.

(ii) Liquidity is wasted (turned into alternative use) when

$$\frac{\rho_0}{\underline{\rho}} < \alpha < \alpha_0, .$$

where α_0 is the highest value of α satisfying (8.23) and hence

$$\alpha_0 < \frac{1}{q}.$$

(iii) There is no hoarding of liquidity when $\alpha > \alpha_0$.

Example: Very little pledgeable income

Let us discuss the case where there is very little pledgeable income, since this is where the coordinated and uncoordinated cases are different.

A firm that wants to continue in the distressed state can count on getting m widgets from the spot market and therefore will buy at most $\ell = I - m$ widgets at date 0. When ρ_0 is very small, m will be very small and ℓ will be close to I. If $\underline{\rho}$ is also small, the proceeds from selling widgets at date 1 will be small. In the non-coordinated case, then, for

$$\rho_0 \simeq 0,$$

the decision whether to hoard liquidity or not at date 0 is very close to the autarky decision, where the firm had to choose between $\ell = I$ and $\ell = 0$. In that case, we found that the firm would not buy any widgets if $\alpha \geq \frac{1}{1+q}$.[6] This is a lower cut-off than the second-best $\alpha \geq \frac{1}{q}$. So, if $\rho_0 \simeq 0$, and $\underline{\rho} \simeq 0$, we will have a range of α such that firms in the spot market choose not to buy any widgets, while firms in a coordinated market would buy $\ell = (1-\alpha)I$. In this limited range $(1/(1+q), 1/\alpha)$, there is "under-hoarding" of widgets relative to the second-best. On the other hand, when $\alpha < \frac{1}{1+q}$, we know that the firm will procure $\ell = I$ widgets in the autarky solution and hence $\ell = I - m \approx I$, when $\rho_0 \simeq 0$. In this case, there is "over-hoarding" of widgets relative to the second best.

8.1.3 Policy

Regions I and II deliver the coordinated solution so no policy intervention is called for. Similarly, the optimal solution ($\ell = 0$) prevails when $\alpha \geq 1/q$, which is a subset of Region IV.

Liquidity is wasted in Region III. However, encouraging firms to hoard more liquidity can only reduce welfare, since the problem is that the secondary market price is too low $(p = \underline{\rho})$. Any extra liquidity will only be wasted as it goes to the alternative use. Indeed, "under-hoarding" and "over-hoarding", as discussed above, refer to equilibrium investments in liquidity relative to the second-best, coordinated benchmark studied in part A. Therefore, liquidity regulation, assuming it is feasible, does not improve welfare.

[6] Note that for $\rho_0 = 0$, (8.23) is equivalent to $\alpha \leq 1/(1+q)$ as $\underline{\rho}$ tends to 0.

Consider the "over-hoarding" case and assume that the government can force firms to hoard less than I, say, the second-best level $(1-\alpha)I$. For ρ_0 small, the distressed firms will not be able to buy liquidity to make up the shortage (for example acquire αI, when the liquidity is constrained to the coordinated level). Similarly, forcing firms to hoard more in the "under-hoarding" case will not help as the extra liquidity will not get transferred at date 1. The firms de facto live in autarky as they cannot count on acquiring liquidity when distressed.

8.1.4 Asymmetries and the emergence of underhoarding

This section studies a two-sector variant of the previous model. Liquidity is being produced by one group of firms at date 0, while another group of firms need liquidity at date 1. Because the latter may have little to offer in exchange for liquidity, the former may produce (or procure) insufficient amounts of liquidity. Due to the asymmetry, the production of liquidity will have no direct benefit for the producers and this eliminates the countervailing effect that rules out underhoarding in the symmetric model of the previous section.

Consider the two-sector economy described in Figure 8.2. Sector 1 consists of a continuum of identical entrepreneurial firms of the type studied in Section 3.2. A representative entrepreneur in this sector has initial wealth A and borrows $I-A$ to invest I in LT assets. At date 1, the firm faces a deterministic demand for liquidity. It needs one widget per unit of investment in order to continue. Investments, with widgets as inputs, yield, per unit of investment, ρ_1 at date 2, of which ρ_0 is pledgeable. The investment unit costs c with $\rho_1 > 1 + c$ and $c > \rho_0$.

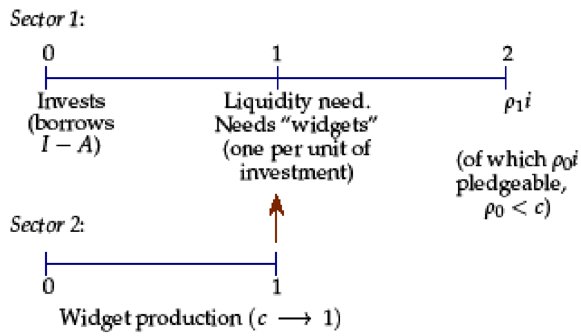


Figure 8.2: The "vertical" example

Sector 2 is composed of a continuum of competitive intermediaries which specialize in the production of widgets. They have a technology that transforms c units of the consumption good at date 0 into 1 widget at date 1. We make the following assumptions:

- widgets have no alternative use at date 1. They are of no value to consumers; neither do they have any value to sector 2 (so inefficient date-1 allocations of the type studied in the previous sections do not arise);
- firms in sector 1 have no access to the widget-producing technology. In particular, widgets in this sector should not be thought of as Treasury bonds or other liquid assets; actually, we will shortly assume that no such store of value is available in the economy. Nor can firms in sector 1 secure in advance a supply of widgets through contracts or cross-ownership of firms in sector 2. Trading of widgets can only occur on a date-1 spot market;
- there is no outside liquidity, and hence no stores of value that firms in sector 1 could use to pay for widgets at date 1.

It is easy to see that no production can take place in this economy. The only way firms in sector 1 can pay for widgets on the date-1 spot market is by offering their securities to sector-2 firms, that is offering claims on their pledgeable income. But because $\rho_0 < c$, a sector-1 entrepreneur cannot pledge enough to make it worthwhile for sector-2 entrepreneurs to invest. In the anticipated absence of widgets, sector-1 entrepreneurs do not invest either. Yet, investment is desirable (and would occur if the two sectors could coordinate their plans at date 0) as long as $\rho_1 > 1 + c$.

Compare this with the case where firms can coordinate plans. The consolidated net present value for the economy is $(\rho_1 - 1 - c)\,I$. Assuming that firms in sector 1 are not constrained by their date-0 endowment, the representative firm in sector 1 invests I, where

$$(1+c)I - A = \rho_0 I \quad , \quad \text{or} \quad I = \frac{A}{1+c-\rho_0}.$$

It finances the date-0 investment in widgets by issuing claims on its future income and by transferring date-0 endowment so as to make up for the shortfall $(c - \rho_0)I$.

Note that the coordination failure highlighted here – the failure to contract at date 0 – differs from the standard, multiple-equilibrium coordination failure developed in the literature on strategic complementarities – e.g., Hart (1980). In our example there is only one equilibrium. Relatedly, if ρ_0 were greater than c, the problem we have identified would be moot, but the standard coordination failure – a bad equilibrium in which sector 1 does not invest because it anticipates that sector 2 will not invest, and conversely – could still arise.

Policy. If date-0 contracting proves infeasible, the creation at date 0 of outside liquidity (stores of value) that can be hoarded by firms in sector 1 and traded at date 1 against the widgets produced by sector 2 improves welfare provided that the standard coordination failure is avoided: firms in sector 2 invest in widgets at date 0 if they expect firms in sector 1 to invest in liquid assets and make up for the income shortfall $c - \rho_0$ per unit; and conversely.

A possible interpretation. The logic of our example can be used to illustrate a key distinction between intermediated and direct finance. A large theoretical and empirical literature emphasizes the need for firms with serious agency problems to resort to costly monitoring by intermediaries as a way to alleviate the agency problem and to create more pledgeable income (see, for instance, Diamond, 1984, Gorton and Winton, 2003 and Holmström and Tirole, 1997) Firms with low agency costs, in contrast, have direct access to (cheaper) market finance. In this context, the situation depicted in Figure 8.2 can be given the following interpretation: At date 1, firms may face a shock that aggravates the agency problem. For example, new opportunities for misbehavior appear, or, due to bad news, the entrepreneur's stake in the firm has decreased, making her more prone to waste resources or to gamble for resurrection. Continuation then calls for monitoring by intermediaries. Monitoring at date 1, however, requires that intermediaries hoard resources ("widgets") at date 0: they must build human capital (c then represents the cost of dedicating personnel that is specialized in sector 1). But, as we have just argued, intermediaries will not have the incentive to invest in monitoring expertise, because firms lack pledgeable income with which to pay for intermediary services at date 1 (we are assuming here that firms cannot arrange for intermediary services in advance, in the same way that sector-1 entrepreneurs could not contract in advance with sector-2 producers of widgets in the more formal treatment above).[7]

The assumption that no production can take place in the absence of monitoring (our interpretation of widgets) is a bit stark. More generally, one could follow Holmström and Tirole (1997) and assume that monitoring reduces moral hazard and thereby increases pledgeable income. Consider the extreme case in which nothing is pledgeable in the absence of monitoring. Then, the entrepreneurs in sector 1 can invest $I = A$ in the absence of monitoring and $I = A/(1 + c - \rho_0)$ with monitoring and coordination. But if a firm waits until date 1 to secure monitoring, it can invest only $I = A$ since, as we have seen, at that point it is unable to pay monitors enough to induce them to invest at date 0.

[7] This interpretation points to a difficulty one is likely to encounter when trying to measure aggregate liquidity in an industry or a country. Here, liquidity is provided by the stock of knowledge held by intermediaries about the industry (or in the context of international crises, by the foreign financial institutions about the country's firms and markets), as well as by their own financial strength. Such variables are by no means easy to measure.

8.2 Application to uncoordinated international liquidity provision

8.2.1 Framework and the coordinated benchmark

Let us first briefly recall the international liquidity model of Chapter 6.

- *Goods and preferences.*

  There are two kinds of goods:

  - *Tradable goods*, which are consumed by foreigners as well as domestic residents. These goods will at times be called *dollar goods* or simply dollars.

  - *Nontradable goods,*which only domestic residents consume. These goods are called *peso goods* or pesos.

All variables referring to dollars have superscript "\$ ", while those without superscript refer to pesos.

There are three periods, $t = 0, 1, 2$. We assume that economic agents only care about the sum of their consumptions at the three dates, and therefore demand a zero expected rate of return on investments. We further assume, mainly for convenience, that domestic residents view tradables and nontradables as perfect substitutes. Thus, a foreigner's utility from the consumption stream $\{c_t^{\$}\}_{t=0,1,2}$ is

$$\sum_{t=0}^{2} c_t^{\$},$$

while a domestic resident's utility from the consumption stream $\{(c_t^{\$}, c_t)\}_{t=0,1,2}$ is

$$\sum_{t=0}^{2} \left[c_t^{\$} + c_t\right].$$

We use date-0 pesos as our numéraire.

All uncertainty is revealed at date 1.

• *Technologies and timing*

There is a continuum of ex-ante identical domestic firms of mass 1. Ex post (date 1) a known fraction α of them remain *intact* and the remaining fraction $(1-\alpha)$ will be *distressed*. To simplify notation, we assume that intact firms do not have to make any additional investments at date 1 ($\rho_L = 0$), while distressed firms are hit by a high liquidity shock normalized to have value $\rho_H = 1$.

The flows of tradable (dollar) and nontradable (peso) goods for a representative firm are depicted in Figure 8.3.

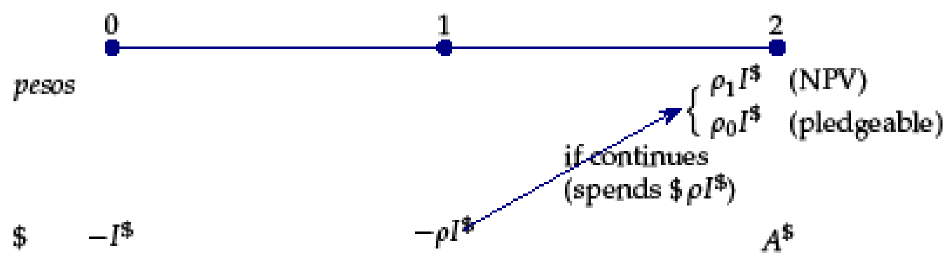


Figure 8.3: production process

The representative firm has an endowment $A^\$ > 0$ of dollar goods at date 2 (it could equivalently be at date 1). All investments are in dollars and all outputs in pesos. The technology is constant returns to scale. At date 0 the firm invests $I^\$$ dollars in a long-term project and $\ell^\$$ in dollar reserves. We will assume that the parameter values are such that it is optimal in the coordinated solution to continue at full scale, $I^\$$, even in the high shock state. At date 2, the firm produces $\rho_1 I^\$$ of which $\rho_0 I^\$$ is pledgeable in pesos and nothing in dollars. We have deliberately chosen a specification in which all inputs are tradable to ensure (as will become evident) that there is no role for government-provided liquidity if firms coordinate their use of liquidity.[8]

Full coordination

[8]This is very different from the model in chapter 6. There we focused on the value of international collateral in supporting second-best insurance.

As a benchmark, let us first calculate the fully coordinated solution. Since shocks are idiosyncratic, date-1 aggregate demand for liquidity is deterministic. Given our special assumptions $\rho_L = 0$ and $\rho_H = 1$, the aggregate demand is $\ell^\$ = (1-\alpha)I^\$$, if firms want to continue at full scale. Foreigners must be paid in dollars. The only source of dollars is the representative firm's initial endowment $A^\$$, which is split between the initial investment and reinvestment. The budget constraint is

$$I^\$ + (1-\alpha)I^\$ = A^\$, \tag{8.25}$$

implying that

$$I^\$ = \frac{A^\$}{1+(1-\alpha)}. \tag{8.26}$$

The representative firm's gross utility, if it holds enough reserves to cover the high liquidity shock, is

$$U^g = \frac{\rho_1}{2-\alpha} A^\$.$$

The policy of holding no reserves would yield investment $I^\$ = A^\$$ and a net present value $U^g = \alpha\rho_1 A^\$$. Assuming that $1 > \alpha(2-\alpha)$, so that it is optimal to hoard reserves.

To meet the high liquidity shock, the corporate sector must have $(1-\alpha)I^\$$ dollars at date 1 and find a way to dispatch liquidity to those firms hit by the high shock. As in the previous chapter, we assume that institutions for coordinating liquidity use are not available.[9] Instead, liquidity will be allocated in a spot market at date 1.[10]

8.2.2 No coordination: A date-1 spot market

When firms act on their own without any advance agreements on how liquidity is to be used, the date-1 spot market determines how excess dollar reserves get reallocated. The market is similar to the spot market for liquidity in section 8.1. Intact firms can do one

[9] Note that even if there is only aggregate risk, so that all firms are hit by the same shock, the second-best solution is the same as for the idiosyncratic case discussed above. However, international investors may find it easier to insure country shocks, since these are more readily identified. If foreign investors can insure country shocks, there is no need for firms to coordinate their liquidity purchases.

[10] Because there is no aggregate uncertainty, there is no need for state-contingent government bonds.

of two things. They can hand over their dollar reserves to distressed firms in exchange for date-2 peso goods (paid out of the $\rho_0 I^\$$ income). Alternatively, they can choose to consume their dollar reserves, which they value the same as peso goods. However, consumption is an inferior use of dollar reserves, just like the "baseball field" was an inferior use of land in Chapter 7. The value of consumption implies that the inferior use of dollars is worth

$$\underline{\rho} = 1 < \rho_1.$$

Let e_1 be the date-1 real exchange rate, defined as the price of the dollar good in terms of the peso good. The real exchange rate must fall in the range

$$1 \le e_1 \le \rho_1 \tag{8.27}$$

The lower bound holds because the entrepreneurs of intact firms would prefer to consume the tradable good if e_1 fell below 1. The upper bound holds because a distressed firm does not want to pay more for salvaging a unit of the project than what that unit can produce at date 2.

Let us next analyze the date-1 spot market:

- *Demand for dollar goods at date 1*

A distressed firm, holding $\ell^\$$ units of dollar reserves, can buy up to $m^\$$ units of dollar goods at date 1, subject to the liquidity constraint

$$e_1 m^\$ \le \rho_0(\ell^\$ + m^\$). \tag{8.28}$$

The left-hand side of this inequality is the total cost, expressed in peso goods, of buying dollar goods on the spot market. The right-hand side is the amount of peso goods that can be promised in exchange, namely the pledgeable income on salvaged assets $(\ell^\$ + m^\$)$. As long as the exchange rate falls within the bounds (8.27), the demand for dollar reserves is given by

$$m^\$ \le \min\left(\frac{\rho_0 \ell^\$}{e_1 - \rho_0},\ I^\$ - \ell^\$\right). \tag{8.29}$$

Note that the right-hand side of (8.29) is well defined, since $e_1 \geq 1 > \rho_0$. For technological reasons, the firm has to choose $m^\$$ so that the reinvestment scale does not exceed the initial investment level

$$i^\$ \equiv \ell^\$ + m^\$ \leq I^\$ \Leftrightarrow m^\$ \leq I^\$ - \ell^\$. \tag{8.30}$$

• *Supply of dollar goods at date 1*

As in section 8.1.4, intact entrepreneurs are the only agents that can rescue distressed firms. They have dollar reserves and are willing to trade dollars for peso consumption if the spot price is at least as high as the value of consumption. By assumption, domestic consumers do not have dollar reserves (had they had dollar reserves at date 0, they would have sold them to the entrepreneurs.) Foreigners do have dollars, but they are unwilling to supply dollars for pesos, because they cannot convert peso claims into dollar goods at date 2.

• *Investment in production and in dollar reserves at date 0*

The date-0 gross utility of a representative firm that chooses investments $I^\$$ and $\ell^\$ \leq I^\$$ at date 0 is[11]

$$U^g = \alpha \left[\rho_1 I^\$ + e_1 \ell^\$\right] + (1 - \alpha) \left[\rho_1 (m^\$ + \ell^\$) - e_1 m^\$\right] \tag{8.31}$$

We have grouped the entrepreneur's utility into two terms. The first term is the return when the firm is intact, which happens with probability α; an intact firm produces $\rho_1 I^\$$ and gets $e_1 \ell^\$$ from selling (or consuming) its dollar reserves, which it does not need to continue operations. The second term is the return when the firm is in distress, which happens with probability $1 - \alpha$; a distressed firm can salvage $\ell^\$$ units using its own dollar reserves and $m^\$$ units by buying dollars in the spot market. From (8.29) we see that the firm can purchase dollars in the amount $m^\$ = \rho_0 \ell^\$ / (e_1 - \rho_0)$ as long as the total does not exceed $I^\$ - \ell^\$$.

[11] Note that the firm does not borrow at date 0, as (a) domestic investors cannot buy dollar goods, (b) foreign investors can lend dollar goods, but will demand them back, a wash in terms of access to tradables.

The entrepreneur will have to pay investments up front, because all the investments are in dollars while all the returns are in pesos. Foreigners will not accept pesos as payment, and domestic investors do not, by assumption, have dollars for the investments. The entrepreneur's date-0 budget constraint is therefore simply

$$I^{\$} + \ell^{\$} \leq A^{\$}. \tag{8.32}$$

- *Equilibrium in the date-1 spot market*

Supply must weakly exceed demand in the date-1 spot market for dollar goods, so

$$\alpha \ell^{\$} \geq (1 - \alpha) m^{\$}. \tag{8.33}$$

An equilibrium in the date-1 spot market is achieved when the price e_1 of dollar goods is set at a level such that when the representative firm maximizes (8.31) subject to (8.29) and (8.32), the inequality (8.33) is satisfied and holds as an equality whenever $e_1 > 1$. It is easy to see that an equilibrium always exists.

At a firm optimum, the budget constraint (8.32) is always binding. Substituting $I^{\$} = A^{\$} - \ell^{\$}$ into the objective function (8.31) we find that the derivatives of the objective function with respect to $\ell^{\$}$ and $m^{\$}$ are:

$$\alpha e_1 + (1 - 2\alpha)\rho_1$$

and

$$(1 - \alpha)(\rho_1 - e_1) \geq 0,$$

respectively. Note that whenever

$$e_1 < \rho_1,$$

the derivative with respect to $m^{\$}$ is positive (purchasing liquidity is cheap), so the liquidity constraint must be binding:

$$(e_1 - \rho_0) m^{\$} = \rho_0 \ell^{\$}. \tag{8.34}$$

Also, there would be no activity in the date-1 market if e_1 were above ρ_1, verifying formally the upper bound of (8.27).

No ex-post waste of liquidity. Suppose $e_1 > 1$. Then (8.33) is an equality as supply must equal demand. Combining the market equilibrium condition with a binding liquidity constraint gives the following price for the equilibrium in the date-1 dollar market:

$$e_1 = \frac{\rho_0}{\alpha}. \tag{8.35}$$

This equilibrium price prevails, provided that e_1 falls in the interval (8.27), that is,

$$1 < \frac{\rho_0}{\alpha} < \rho_1. \tag{8.36}$$

Substituting (8.35) with (8.34) we see that $(1-\alpha)m^{\$} = \alpha\ell^{\$}$ or equivalently that $m^{\$} = \alpha i$ and $\ell^{\$} = (1-\alpha)i^{\$}$, where $i^{\$} \equiv m^{\$} + \ell^{\$}$ is the continuation scale of distressed firms. The technology requires $i^{\$} \leq I^{\$}$. When we insert the expressions for $\ell^{\$}$ and $m^{\$}$ into the firm's objective function we see that it is optimal to choose $i^{\$} = I^{\$}$. The representative firm's gross utility then becomes

$$U^g = \rho_1 I^{\$} = \rho_1(A^{\$} - \ell^{\$}).$$

We note in passing that when the price given by (8.35) prevails, the amount that the intact firms make on selling dollars at date 1 equals the amount that the distressed firms pay for the dollars, regardless of the level of continuation investment $i^{\$}$. This is, of course, always the case in equilibrium if all the dollars get transferred from intact to distressed firms. Because $\ell^{\$} = (1-\alpha)I^{\$}$, the equilibrium allocation coincides with the second-best allocation, in analogy with the analysis in section 8.1.2. There is no waste of dollars.

The price given by (8.35) has to fall within the bounds defined by (8.36). For $\alpha \leq \rho_0/\rho_1$, the date-1 price is at its upper bound

$$e_1 = \rho_1.$$

In this region

$$U^g = \alpha\rho_1 A^{\$} + (1-\alpha)\rho_1\ell^{\$},$$

where $\ell^\$$ is determined by the liquidity and budget constraints after substituting $I^\$ = A^\$ - \ell^\$$. The representative firm's utility is

$$U^g = \frac{\rho_1 A^\$}{2 - \alpha}.$$

Also in this case the uncoordinated solution coincides with the coordinated solution.

Ex-post waste of liquidity. We finally consider the case where the equilibrium occurs on the boundary $e_1 = 1$, in which case some dollar reserves will be consumed rather than transferred to salvage distressed assets. For $\alpha > \rho_0$, the exchange rate given by (8.35) lies below 1 so we must have

$$e_1 = 1.$$

In this price regime it can be optimal for firms not to buy any liquidity at all at date 0. This happens if α is high enough and the pledgeable income low. If firms do buy liquidity, supply will exceed demand:

$$\alpha \ell^\$ > (1 - \alpha) m^\$.$$

The excess supply of liquidity is consumed by the intact entrepreneurs, who are indifferent between selling into the market and consuming when $e_1 = 1$. Putting liquidity to an inferior, alternative use is, of course, wasteful from a social perspective. Note, however, that distressed firms will be able to continue at full scale. Knowing that they cannot afford to buy much in the market (despite the price being as low as it can go), they hoard more liquidity at date 0. In the limit, as ρ_0 goes to zero, and distressed firms have no purchasing power at date 1, the situation will be the same as in autarky – either firms hoard $\ell^\$ = A^\$/2$ of liquidity or they do not buy any. When the distressed firms have a bit of purchasing power ($\rho_0 > 0$) they can reduce $\ell^\$$ a bit.

It is tedious, but straightforward to determine when firms opt to buy liquidity and when they do not. The optimum either occurs at the intersection of the budget and the liquidity constraint or at the origin. We have

(i) $\ell^{\$} = m^{\$} = 0$ and $U^g = \alpha\rho_1 A^{\$}$,
if $\alpha > \dfrac{\rho_1 + (\rho_1 - 1)\rho_0}{2\rho_1 - 1}$,

(ii) $\ell^{\$} = \dfrac{1-\rho_0}{2-\rho_0}A^{\$}$, $m^{\$} \equiv \dfrac{\rho_0}{2-\rho_0}A^{\$}$ and $U^g = \left[\dfrac{\alpha + \rho_1 - \rho_0}{2-\rho_0}\right]A^{\$}$,
if $\dfrac{\rho_1}{2\rho_1 - 1} < \alpha \leq \dfrac{\rho_1 + (\rho_1 - 1)\rho_0}{2\rho_1 - 1}$.

Our results are summarized in Figure 8.4, which is, for obvious reasons, reminiscent of Figure 8.1.

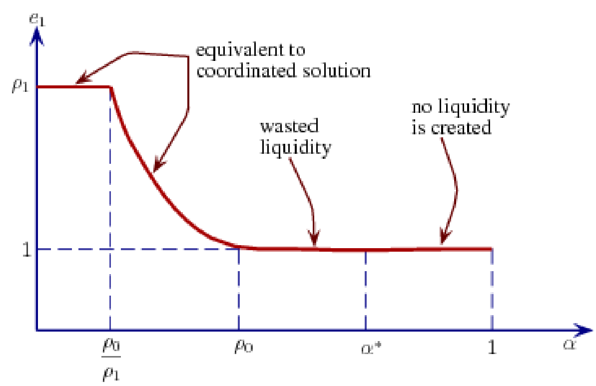


Figure 8.4: Exchange rate when liquidity is not coordinated

8.3 Summary

This chapter has investigated incentives for uncoordinated investments in specialized assets or widgets (physical inputs, knowledge assets or international liquidity, for instance). Because widgets here serve to rescue the operations of financially constrained agents, we should expect widgets to trade at a discount in the secondary market, which is indeed the case. The price discount raises the concern that firms will not have the right incentive to accumulate widgets. After all, if the secondary-market price is low, reselling widgets when one does not need them involves a capital loss. And on the buy side, it is relatively cheaper to purchase widgets in the secondary market than in the primary market. So it would seem that a low resale price would lead to undersupply of widgets relative to the social optimum, a key conclusion in Caballero and Krishnamurthy (2001). The surprising result in this chapter is that this need not be the case when firms are ex ante identical and the only imperfection is their inability to coordinate in advance the purchase and use of liquidity

While it is correct that a low price discourages the hoarding of widgets, this reasoning misses another effect: In the absence of alternative stores of value (say, Treasury bonds), hoarding widgets provides date-1 funding liquidity (the ability to dilute the initial investors – with their ex ante consent – by issuing new securities). This relaxes the liquidity constraint and creates an additional incentive to hoard widgets as a store of value. The incentive is higher, the lower the secondary-market price and in our model the incentive to hoard widgets is in the end socially optimal. The only situation where self-provision of widgets does not replicate the coordinated outcome is when the distressed firms have so little aggregate purchasing power that some of the widgets will be turned into the alternative use. In this case, there is nothing the government can do to improve welfare.

One reason why we get efficiency while Caballero and Krishnamurthy (2001) conclude that investment in liquidity (investment in dollars, to be more specific) is suboptimal, is that our investors have more commitment power. In particular, our investors allow firms to issue new shares when liquidity is needed. Because intact firms have no use for additional funds, this right will not be abused. One can argue that it would be more realistic to assume that intact firms would abuse the right to dilute initial shareholders (as in Appendix 2.1 of Chapter 2 and in Caballero and Krishnamurthy, 2001). On the other hand, our analysis, when juxtaposed with Caballero and Krishnamurthy's analysis, identifies the inability to control the firms' use of funds as a key assumption in their underinvestment result.

Underinvestment occurs, perhaps more naturally, when there is a separation between the producers of widgets and the users of widgets as suggested by the asymmetric case discussed in section 8.1.4. The shortage of pledgeable income of the buyers of widgets discourages the sellers from investing in widgets as in the intuitive reasoning above. The extra effect on funding liquidity is missing. In the case there is scope for policy intervention; in particular, the creation of stores of value makes it possible for the buyers of widgets to pay a high enough price to sellers thanks to higher savings.

The inquiry set forth in this chapter is still rather tentative. In particular, while it has delivered some new insights into policy intervention, it has not delivered clear guidelines. Furthermore, it has not investigated whether the reasons leading to the absence of forward markets could interact with the overall logic of this argument. Our analysis should therefore be read with caution when it comes to drawing policy recommendations.

Part V

EPILOGUE

Summary and concluding thoughts on the subprime crisis.

We started our research on liquidity over fifteen years ago with the aim of understanding liquidity (collateral) shortages in financial crises.[12] We close this book with some reflections on what our approach can tell us about the on-going subprime crisis. We begin by summarizing the key themes and insights of the book, before getting into the crisis itself.[13]

The main ingredients and insights

Our approach is based on two key premises. The first is that income streams are not fully pledgeable. Only part of the pie that a firm creates can be credibly pledged to investors,[14] and consumers cannot pledge any of their future income without collateral or through government taxation.[15] The second premise is that arbitrary, state-contingent contracts can be written on pledgeable income as in the standard Arrow-Debreu general equilibrium model. The critical departure from the Arrow-Debreu model is therefore the first assumption that there is a wedge between total income and pledgeable income. Introducing a wedge extends the standard complete market model to a slightly more

[12] We were inspired by the severe financial crises in Scandinavia in 1991-92 and later in Asia in 1997.

[13] The terms *pledgeable income, liquidity* and *collateral* are used interchangably in this book, including this epilogue.

[14] Because firms will have to give up some of the income to management, workers and monitors (e.g. venture capitalists), who share in the rents created by firms; how much depends on many factors that we lump together into an unpledgeable share of the pie.

[15] This modeling assumption is a caricature of reality, of course. Unsecured consumer debt in the form of credit cards, for instance, is not trivial, but still relatively small in comparison with collateralized debt, such as housing.

realistic setting with surprisingly powerful implications.[16]

The first implication is that firms and financial institutions will have a demand for liquidity despite everyone being risk neutral. In contrast to the Arrow-Debreu model, where firms can fund all positive net present value projects on a pay-as-you-go basis, in our model firms have to arrange funding in advance to avoid credit rationing in states where their pledgeable income is insufficient to fund desirable continuation investments. The firms' demand for liquidity is a demand for insurance against future credit rationing. With a limited net worth, the firm has to choose how much of its capital to spend on insurance (contingent liquidity) and how much on current investment. The trade-off results in the firm being credit rationed both today and in the future. In general, the firm will only buy partial insurance.

A second implication of partial pledgeability of income is that it reduces the supply of liquidity. Investors (or financial institutions) need collateral to back up their promises for future funding of firms. If the corporate sector's pledgeable income in some state is too small, there will be a shortage of aggregate (inside) liquidity in that state. It is the potential for aggregate liquidity shortages and their consequences for liquidity premia, investment decisions, and government policies that constitute the central and most original part of our analysis.

If the private sector is a net lender, or if it is subject to significant aggregate shocks, there is always insufficient inside liquidity. On the other hand, when the private sector is a net borrower and shocks are idiosyncratic, then the private sector is self-sufficient. Nonetheless, the second-best solution requires state-contingent insurance that transfers liquidity from the intact to the distressed firms in an efficient way. Firm-specific credit lines offered by a financial intermediary that holds a well-diversified market portfolio of securities can achieve the required reallocation of liquidity, as can a variety of other state-

[16] We still refer to our model as one with complete markets, because the market is complete with respect to the pledgeable income.

contingent mechanisms, including the option that firms themselves hold claims in the market portfolio.

In states where there is a shortage of aggregate liquidity, there is a liquidity premium on its supply. The price of liquidity and the value of assets can be determined using a liquidity asset pricing model (LAPM). The value of an asset has two components: the consumption value of its cash flow plus the value (or cost) of the collateral that it produces or requires as part of its investment plan. We study asset pricing using techniques familiar from standard general equilibrium models. We find that the equilibrium is efficient subject to the constraint that all contracts must be secured by pledgeable income.

The government can play a key role in making up for aggregate liquidity shortages. Its unique access to current and future consumer income through taxation, allows it to act as an intermediary between consumers and firms. It can raise welfare by transferring liquidity from consumers (current and future) to firms in states where the value of such transfers is higher than the shadow cost of public funds. The marginal value of government transfers can be inferred from the state-contingent prices for liquidity in LAPM.

In practice, the government supplies contingent liquidity through a variety of policies and interventions including the conduct of monetary policy, the provision of deposit insurance, the occasional bailout of commercial banks, investment banks, pension funds and other financial institutions (like Freddie Mac and Fannie Mae). In addition, a whole range of social insurance programs such as unemployment insurance and social security and of implicit catastrophe insurance (earthquakes, nuclear accidents,..) play an important role in influencing the amount of aggregate liquidity in the economy.

It is important to stress that government transfers can be seen as part of an explicit or implicit insurance plan. Having tax payers get involved, because private insurance is constrained by limits on aggregate liquidity makes perfect sense, at least in theory. Relatedly, the government also enjoys an advantage over private parties because it can supply liquidity on a contingent basis. The private sector has to decide ex ante how much liquidity it creates, often by making investments in short-term, physical assets (or just biasing investments in the direction of income that is more readily pledgeable). In contrast, the government can supply liquidity once the need has materialized, saving significantly on the cost of investing in lower-yielding projects. This implies that the state has a particularly strong comparative advantage in offering insurance against unlikely and extreme shocks hitting the economy.

International markets can alleviate domestic liquidity shortages, but even if there is more than enough international liquidity to meet the needs of a country, this will not in general eliminate the country's liquidity shortages. A country has its own pledgeability limitations: Non-tradable output, while exchangeable in markets with tradable goods, must in the end go to domestic consumers and therefore has little appeal to international investors as such. When there is a shortage of international liquidity (tradable output), domestic liquidity supply and liquidity management by the government remain effective just as in a closed economy.

Assuming that contingent claims markets are complete, is of course unrealistic. But in return it offers simplicity and discipline. It also provides an upper bound on the supply of aggregate liquidity. When markets are incomplete, aggregate liquidity shortages tend to become accentuated, as coordination problems prevent the efficient use of available liquidity. The burgeoning literature on financial crises employs almost exclusively incomplete market models. By way of illustration, we studied a few models of this sort in part IV of the book. We found that despite market incompleteness, the equilibrium can sometimes be efficient, replicating the outcome of complete contingent markets. When the equilibrium is inefficient it may exhibit:

- over-hoarding of liquidity before a crisis, because firms invest in liquidity to be able to compete for distressed assets in the future,
- insufficient transfers of liquidity during the crisis to firms whose management cannot be replaced (due to specific knowledge) as firms with excess liquidity prefer to invest in

low-productivity projects rather than in firms that need liquidity, but do not have enough funds to compete for them,

- consumer savings behavior that is too short-termist when consumers are employees of firms,
- fire-sale prices that inhibit an efficient transfer of assets,
- possible under-hoarding of liquidity relative to the complete contingent market case; however, forcing firms to invest more in liquidity may not solve the problem, if the extra reserves can be put to alternative, inferior use (because the spot price is low).

Relating our model to the subprime crisis

Our understanding of the causes of the subprime crisis is still limited. While it is easy to list a variety of factors that must have played some part in the collapse, there is still disagreement on the major drivers. We will focus on four factors that seem to us particularly central and which form a relatively coherent view of the crisis. These are: (i) the global imbalances and the savings glut, (ii) the growth of the U.S. shadow banking system in response to the savings glut, (iii) the collapse of the wholesale funding market that supported shadow banking, and (iv) regulatory deficiencies[17].

The savings glut. There is wide-spread agreement that the increasingly large current account deficits that the U.S. ran up in the new millennium (growing steadily from 4.4 per cent of GDP in 2000 to 6.2 per cent in 2006) were a major cause of the crisis. Why did the account deficits grow so large? The traditional argument would suggest that the U.S. lived beyond its means, needing ever more funds to support its consumption habits. But there is an alternative view, originally put forward by Ben Bernanke, the current chairman of the Federal Reserve. In a widely noticed speech in 2005, he suggested that the problem stemmed from a global savings glut, caused largely, but not exclusively, by

[17] Our interpretation of events draws on Adrian and Shin (2008), Caballero et al (2008), Hellwig (2009), Holmström (2008), Gorton (2008, 2009), chapter 3 in Gylfasson et al, (2010), and Dewatripont et al (2010), among others.

excess savings in the emerging markets of Asia. Foreign investors were looking for a safe place to park their excess funds and found it in the U.S. The view that money was pushing its way into the U.S. rather than being pulled in by a demand for capital, has received wider acceptance lately. One argument in its favor is that interest rates kept falling as the U.S. current account deficit grew. This seems inconsistent with the notion that the U.S. consumer was driving the growth.[18]

The notion that Asian money was looking for safe investment opportunities in the U.S., fits well with our basic premise that a country can have a shortage of aggregate liquidity if pledgeable income is low. In developing economies, poorly developed financial markets and political and legal uncertainties generally make the pledgeable income of firms small. Paired with exceptionally high savings ratios, this may well have caused much of the savings to be channeled towards countries with higher fractions of pledgeable income and greater safety[19].

The savings glut also may have made it easier for the housing bubble to emerge.[20] Bubbles augment the stock of stores of value, albeit in a very imperfect way to the extent that that they are an unreliable method of carrying income over from one period to the next. They risk bursting, and what is worse, they burst at the wrong time for those institutions that seek liquidity from the bubble: When a bubble bursts, not only does

[18] See Caballero, Farhi and Gourinchas (2008). However, Taylor (2009) has argued forcefully that global imbalances played a minor role relative to U.S. monetary policy, which was too lax, keeping interest rates low for too long.

[19] These observations fit well with Forbes (2010)'s empirical demonstration that foreigners hold greater shares of their investment portfolios in the US if they have less developed financial markets. She further shows that foreigners, who invested \$6.2 trillion between 202 and 2006 earned substantially lower returns on their US investments than US investors earned abroad; and that diversification motives do little to explain the patterns of foreign investments in the US.

[20] There is always some difficulty in identifying empirically the existence of a bubble. Take the recent bubble in real estate in the United States. It could be argued that the very large subsidies to house ownership (both direct- such as the tax deductibility of interest- and indirect- for instance, low interest rates, the lenient regulatory treatment of asset backed securities or the implicit insurance given to Freddie Mac and Fannie May) made the fundamentals of real estate very strong if these subsidies were expected to last. See however Shiller (2008) for a rather convincing case for the existence of a bubble. For important historical perspectives on bubbles and financial crises, see Kindelberger (1989) and Reinhart and Rogoff (2009).

it have a negative effect on these institutions' solvency (a wealth effect), it also deprives them of collateral at a moment when it is most valuable (because when liquidity becomes scarcer, interest rates fall, and leveraging one's collateral becomes more attractive).[21]

The rise of shadow banking. Global imbalances, as such, do not explain why the bulk of excess savings flowed into the U.S. rather than into Europe, for instance.[22] One reason could be the U.S. consumer's willingness to go deep into debt, especially in the low interest rate environment maintained by the Fed at the time. Another possibility is that the U.S. was especially well positioned to absorb the global savings glut because its financial markets were (and still are) the most advanced in the world. In particular, the U.S. had come up with a financial innovation that enabled it to absorb huge inflows of capital in a short period of time – the shadow banking system.

The shadow banking system is usually defined as that part of banking (intermediation of credit) that is not explicitly insured by the government.[23] It is comprised of a network of players – investment banks, hedge funds, money market mutual funds (MMMF) and Government Sponsored Entities (GSEs) like Freddie Mac and Fannie Mae among others – all helping to intermediate credit through the market. The shadow banking system is based on wholesale funding rather than deposits, making it potentially more vulnerable to runs.

The liabilities of the shadow banks grew rapidly from less than 10 per cent of total bank liabilities in 1980 to about 60 per cent in 2008.[24] The growth rate was especially strong after 1995. It is suggestive that the yearly growth of gross liabilities was of the same order of magnitude as the yearly U.S. trade deficits. Shadow banks must have played a central role in channeling the vast amounts of money that flowed into the U.S.

[21] See Farhi-Tirole (2009a). Other recent work on bubbles includes Kocherlakota (2009) and Ventura (2010); the latter argues that bubbles can substitute for imperfect capital flows and then appear in countries in which productivity is low relative to the rest of the world.

[22] In fact, the European private sector invested substantial amounts in the U.S.during the same period.

[23] See Potzar et al (2010).

[24] Figure 1 in Potzar et al (2010).

The shadow banking system has been widely implicated in the crisis. Many see the whole system as a scam, driven by excessive short-term incentives for bankers on Wall Street. Some of these accusations are certainly valid. But the original driver of the growth of shadow banking must in the first instance have been its ability to respond to the demand for safe investment opportunities. In particular, the repo market, which is at the core of shadow banking, offered a form of secured lending that emulated many of the features of demand deposits in commercial banking.

To understand how, let us briefly discuss the repo market. A repo (repurchase agreement) consists of two simultaneous transactions. In the first, party A buys a security (almost always debt) from party B at a price that typically is below the value of the security (the difference is called the "haircut"). In the second transaction, party B agrees to buy back the security for a slightly higher price at a later date (the next day, in case of an overnight repo). By rolling over an overnight repo until one or the other party withdraws, the repo becomes similar to a deposit. If party A decides not to roll over an overnight repo, the effect is the same as the withdrawal of the deposit. However, unlike traditional banking, the repo market was designed to secure hugely bigger deposits. – typically millions or even hundreds of millions of dollars in a single trade. Federal deposit insurance would provide minimal protection for investments of this size, whereas the repo market, by selling a security in exchange for the deposit, could provide insurance on the appropriate scale. There is a key legal detail in the repo contract: If party B (the borrower) cannot repurchase a security when party A (the investor) demands it (i.e. pay back A's deposit), party A owns the security and is immediately free to sell it or use it as collateral in another repo transaction. Structuring things as a purchase and repurchase agreement is ingenious, because it avoids having a multi-million dollar deposit tied up in a bankruptcy process if party B defaults. Unlike traditional banking, repo markets were highly scaleable without much (apparent) risk. The only limitation, really, was the supply of securities and the assets underlying them.

The raw material for creating new securities came mostly from housing. There are several reasons for this. Housing is by far the largest consumer asset. But before the era of modern financial engineering, housing was an underdeveloped asset. Keeping mortgages on the books of regional banks was like operating a severely under-utilized financial parking lot: mortgages that could serve as collateral for millions of investors with billions more money, were serving just a fraction of the potential investor demand. And American home owners that had paid off their loans were denying foreign investors the opportunity to park their money in such houses with minimal risk. The enormous growth of home equity loans in the 1990s, unleashed some of the housing potential. Securitization, especially of mortgages, but later on of just about any other kind of asset (car loans, student loans, credit card loans, etc) made the most of this opportunity. Securitization could deliver highly rated (AAA) bonds out of relatively marginal assets using tranching.[25] The combination of the repo market with asset-backed, highly rated securities used to secure repo trades, created a formidable intermediation machine, one that in appearance no other country could match in terms of legal safety and capacity.

How does securitization and shadow banking fit our theory? A plausible interpretation is that the shadow banking system strived to and perhaps came close to replicating the state-contingent use of collateral that is the central ingredient in our model (see chapter 4). By mixing securities to create portfolios that diversified away much of the idiosyncratic risk, parties were left trading aggregate risk. Relatively liquid markets in such risk made it possible to use state-contingent aggregate liquidity more efficiently as the theory suggests one should. In this view, securitization created investment instruments that used collateral much more effectively than if the funds had been parked for long periods in non-traded

[25] The creation of structured products involves two steps: first, pooling of assets (e.g. mortgages) and second, issuing of claims against these assets – mainly debt – with varying credit ratings. The second step is tranching. Gorton (2008, 2010) describes in detail how structured products are created. Coval et al (2009) contains an excellent analysis of how much AAA-rated securities can be created from low subprime mortgages, depending on the correlation of the underlying assets.

instruments.[26]

The tri-party repo system, which is a major part of the repo market, illustrates the contingent use of aggregate liquidity.[27] Each day, the dealers (borrowers) in the tri-party repo market place collateral (securities) with the clearing banks. The clearing banks provide a number of services, including daily optimization of the allocation (and re-allocation) of a dealer's collateral to its investors. There are only two banks, JP Morgan Chase and Bank of New York Mellon, which act as clearing banks for tri-party repos, possibly because economies of scale are critical for the allocation of collateral. Bilateral, over-the-counter repos do not offer the same matching efficiencies. The same trades that are made bilaterally can be made by tri-party repos, but bilateral repos have to be cleared sequentially, so collateral cannot be optimized at the end of the day.[28] Intraday large payment systems, such as the Clearing House Inter-Bank Payment System (CHIPS) the main privately held clearing house for large-value transactions in the United States, similarly are designed to make efficient use of limited collateral while concurrently trying to contain systemic risk .[29]

To sum up, the financial innovations that have been so decried lately should in principle have raised welfare by allowing a much better use of collateral in the economy; so what is being questioned is not the ideas themselves, but how they were and can be implemented in practice. The collateral underlying the state-contingent claims was not what it was supposed to be. We now turn to possible explanations for this failure.

[26] Regulatory arbitrage was another powerful driver. By moving capital off their balance sheets, commercial banks could evade onerous capital constraints. Capital charges were reduced roughly by a factor of ten when assets were moved into Special Purpose Vehicles (SPVs).

[27] For an in-depth analysis of tri-party repos, see the report of the Payments Risk Committee (2010).

[28] The tri-party system is not without its problems, because the clearing banks are highly exposed during the day, effectively providing huge bridge loans to the broker-dealers. This poses big risks for the clearing banks, which may react abruptly and decisively to news about the credit worthiness of their clients. By raising haircuts, or in the extreme, by refusing to roll over a bridge loan for a counter-party, whose risk has increased (such as Lehman), they can do quickly do significant damage to the position of broker-dealers. See Tuckman (2010).

[29] See Rochet-Tirole (1996b) for a description of the working and properties of intraday payment systems.

The panic. We now know that the reliance on wholesale funding, especially the increasing fraction of overnight lending in the shadow banking system, proved much riskier than investors had imagined. In August 2007, the market for asset-backed commercial paper (ABCP) collapsed.[30] While the ABCP market was not big enough to set off a panic, the collapse elevated the level of risk as measured by Libor-OIS spreads, for instance. The much larger repo market, mostly secured by AAA-rated products, initially held up, though haircuts increased and the terms of funding became shorter. But when Lehman fell, in September 2008, the myth of repos as secure deposit-like contracts unraveled. Markets for asset-backed securities, which had been very liquid, froze quickly as banks began to question not just the underlying value of mortgage-backed securities but, more importantly, the counter-party risks caused by impaired mortgage-backed assets. A full-blown panic ensued.

It is difficult to judge whether the bond ratings were flawed from an ex-ante perspective or whether investors were hit by a highly unlikely tail event. Given that many people were concerned about the real estate bubble bursting at some point too little attention was paid to the consequences of an aggregate shock in the housing market that could bring down the value of assets across the nation. That said, a drop in the average nominal price of U.S. housing had not occurred since the Great Depression, making it difficult to assess aggregate risks and forecast a 30% drop in prices. Also, it is clear that the standard mechanics of a balance sheet driven crisis – funding and margin spirals caused by and causing fire sales of assets, flight to quality and strategic hoarding of liquidity – all contributed to the large collapse in prices. This systemic risk, or at least its size, was largely overlooked.

Our complete market model views the panic as an exogenous tail event; it says nothing

[30] See Covitz et al (2009). The ABCP market was much smaller than the repo market and ABCPs were not tranched (only one type of claim was issued). Banks provided liquidity backstops for the ABCP market, which also helped prevent a panic. Subsequently, such backstops contributed greatly to the spread of the crisis into commercial banking.

about how a panic might unfold, since all contracts are made at the beginning.[31] However, it can explain why firms may choose not to buy insurance against rare events. In our model, firms may cut down production dramatically (in the absence of government assistance) when an aggregate shock hits, because liquidity is so scarce and the ex ante returns from investing in private insurance to cover such an eventuality is low. Liquidity could be scarce, because of adverse selection in the market for collateral; it would be easy to incorporate this explicitly into our model, but we have not done so. Liquidity could also be scarce in states where liquidity is very valuable (ex post), but the states are so hard to define that the assets needed to back up the insurance have to be broad, requiring high and costly up-front investments, as we discussed earlier in this epilogue.

The subprime crisis fits both conditions. Arguably, adverse selection, driven by uncertainty about counter-party risks, froze many markets.[32] And even if many saw the possibility of a collapse in house prices, the fact that so few did something about it, suggests that either they believed that the consequences would be relatively benign or, as we discuss next, they deliberately gambled on the tail event. Also, buying insurance against a catastrophic event may be hard, because there is insufficient liquidity (collateral) to back up such insurance. From this point of view, our approach of modeling the collapse as an exogenous event is not an unreasonable starting point for analyzing some aspects

[31] A large literature (e.g. Allen and Gale 2000, Caballero and Simsek 2009) shows how one bank's default may propagate in a financial system in which institutions are interdependent through cross-exposures (in interbank or derivatives markets for example). The magnitude and impact of the resulting contagion effects depend on the completeness of the structure of claims, on the existence of a common understanding of these claims (securities traded in OTC markets can have unforeseen or complex consequences) and on the opaqueness of bilateral exposures (as participants need to know the solvency not only of their counterparties, but also that of their counterparties' counterparties, and so on). While this literature obtains a number of useful insights, cross-exposures are taken as exogenous; in particular, they are unaffected by the regulatory environment (cf. the centralization and transparency of intra-day payment systems – see Rochet and Tirole, 1996b, for a description – or the current regulatory push toward central clearing houses for derivative products). Related, it is interesting to study the benefits of decentralized systems (in Rochet and Tirole, 1996a, the benefit is associated with mutual monitoring). We are not aware of research showing how cross-exposures and shortages in aggregate liquidity feed back on each other, but the work of Caballero and Simsek (2009) for example suggests that fear of propagation may lead liquidity to dry up rather quickly.

[32] See for instance Acharya et al (2009) and Caballero and Simsek (2009).

of the crisis.[33]

One of the key implications of our model is that there is a role for government in redistributing wealth ex post. Transfers from taxpayers (that is, consumers) to the corporate sector – including bailouts of banks, to the extent banks are the efficient channel for such transfers – can be rationalized if the government can make pledges on behalf of consumers (especially consumers who are not yet born). In practice such plans take a variety of forms. A looser monetary policy lowers interest rates and transfers resources from consumers to entrepreneurs, by lowering the cost of reinvestments and by providing capital gains for those who own government debt (entrepreneurs buy government bonds in our model). Also, at a conceptual level, the numerous liquidity facilities provided by the Fed to jump start frozen asset markets in the shadow banking system (TARP, TALF, TLGP, etc.) receive support from our model, though our model does not speak to the relative merits of any given liquidity facility.

Regulatory weaknesses. Of course, government transfers and insurance come with well-known costs. We considered the dead-weight losses from taxation, but did not consider the moral hazard problems associated with subsidized insurance, especially bailouts that the government was forced to undertake ex post. It is clear that the subprime crisis revealed very significant weaknesses in the regulatory systems that were meant to keep in check moral hazard and other dysfunctional actions. We have taken a normative viewpoint – analyzing how the state should manage aggregate liquidity – rather than a positive one – analyzing the political economy of domestic liquidity management. Our analysis therefore misses a crucial element of the recent subprime crisis. Let us briefly mention how we could embody the positive considerations into our analysis.

Consider first the role of the state in the traditional banking sector. One view of

[33] Of course, at a more detailed level, the crisis was the consequence of many factors, including the (deliberately) complex way the securitized contracts were structured. There is every reason to try to gain a good understanding of how the panic actually evolved. Treating the crisis as an exogenous state in the way we do tells us nothing about this.

regulation has it that the state monitors on the behalf of retail depositors, clients of insurance companies, pension funds or MMMFs that the financial institution is adequately capitalized and operates proper risk management processes so as not to leave the economic agents – or the deposit insurance fund – with an empty shell .[34] Yet in many countries, the state, by ignoring off-balance sheet vehicles, by accepting (at the international level) low capital requirements for the trading book, and by permitting (or failing to check) other actions that circumvented capital requirements, allowed regulated institutions to over-lever themselves. In the US, it also provided many subsidies for real estate investments, including the indirect subsidies from Freddie Mac and Fannie Mae. Subsidies promote excess investment, which according to our model sows the seeds for a subsequent liquidity shortage, since financial institutions trade off how much to invest in productive assets and how much in liquidity (insurance).

Consider next the role of the state in the shadow banking sector. Large investment banks were largely unregulated. Yet they and their creditors felt that they could count on a bailout with taxpayer money if the investment banks got into trouble – indeed, the financial markets were flabbergasted when the US authorities let Lehman go under. This implicit access to taxpayers' money has dramatic consequences as it implies that troubled institutions can keep refinancing and gamble for resurrection without any market discipline. Observers sometimes point out that many wealthy managers and investors in the shadow banking system lost their shirt and take this as evidence against the moral hazard argument. The point, though, is that the implicit insurance of short-term funding allowed them to gamble with taxpayer money and to continue receiving financing, when they should have de-levered or even shut down.

But why were states eager to rescue institutions in which no small depositor had invested? The answer is "systemic risk", the fear that a default might propagate through the financial system. The combination of interconnectedness and opaqueness of cross-

[34] This view is developed in more detail in Dewatripont and Tirole (1994).

exposures made a default particularly troubling for the financial system, which after the collapse of Lehman had little clue as to how subsequent episodes would unfold and therefore chose safety over almost any risk. The regulators were on the whole too tolerant when they acquiesced to large exposures in opaque OTC derivative markets and thereby accepted that regulated entities would be heavily exposed if large unregulated institutions defaulted. This led to substantial additional commitments of taxpayers' money, which unregulated institutions took advantage of. It would have been wiser to require that derivative contracts between regulated and unregulated institutions be traded in exchanges with a well-capitalized central clearing house as the counterparty. In short, the state supplied liquidity in a way that created moral hazard. Needless to say, our theoretical conclusions on the role of the state as provider of liquidity presume that liquidity is injected in a way that limits moral hazard.

We mentioned above that repo markets may well have strived to emulate the optimal state-contingent use of aggregate liquidity or at least significantly improved the use of the collateral. This comment applies to orderly times in the market. The crisis suggests that coming close to efficiency in normal times may provide no guarantees against a crisis in exceptional times. Put differently, trading systems that emulate contracts are not as good as contracts, when times get tough. This has been seen many times. Long Term Capital Management collapsed because the trading strategy did not work in all states of nature. Had there been explicit contingent contracts, perhaps LTCM would have protected itself against the collapse. On the other hand, it is possible that it relied on a government bailout. The same was true when all portfolio insurance strategies ran for the exits at the same time in 1987. The programs that functioned so beautifully against idiosyncratic risks in peaceful times, proved worthless or worse in the face of a big aggregate shock.

This raises the question of transparency of financial claims and markets and the role of clearing houses and other financial institutions. With organized markets, there is more transparency and better information about outstanding positions and risks (though the both the LTCM and the portfolio insurance examples suggest that parties can create complex strategies that are completely hidden from the eyes of the market – intentions are not easily inferred from the way people trade, especially in the case of rare events.) However, exactly how and where transparency should be provided is not obvious. There is a fundamental paradox embedded in liquidity provision. In order to function well, liquidity providing markets are designed to minimize the need for information acquisition. Debt is the dominant instrument, because it is information insensitive.[35] Parties that trade in debt that is sufficiently over-collateralized need not worry about adverse selection stemming from private information about the underlying collateral. It is sufficient to know that there is enough collateral. But if the purpose of debt is to provide little incentives for information acquisition (about credit risk), it is hard to see how debt markets can properly reflect the price of systemic risk. The suddeness with which financial crises typically appear is consistent with lack of information about systemic risk. To remedy the situation, the government will either have to produce such information itself (e.g. through periodic stress tests) or come up with some way to make the markets produce better information (e.g. by requiring trade in instruments that are more information sensitive).

In order to understand the role of systemic risk, how it should be measured and what types of transparency will be good for getting information about it, one has to move away from the complete market model to a model that offers a deeper understanding of how

[35] Gorton and Penacchi (1990) argued that banks, by issuing riskless debt, allow depositors to avoid the costs of adverse selection. Dang et. al. (2010) analyze a model where risky debt is optimal, because it is least information sensitive in the sense that parties will have the lowest incentive to collect information. For this reason, debt maximizes the amount of wealth that can be transported from one period to the next. The paper also shows that bad news about the aggregate state of the economy (e.g. lower housing prices) will result in write-downs of debt, sometimes exceeding the reduction in the fundamental value of debt to prevent costly information acquisition and sometimes in adverse selection. In both instances the reduction in trade is amplified by bad news, which is one of the manifestations of a crisis.

The transition from an information insensitive to an information sensitive security helps us understand why trading partners were so ill informed about the complex securities they were trading before the crisis and why they wanted to know so much more about the securities once the crisis broke out. In the logic of the model, this is privately rational.

crises erupt and what kinds of warning signs about the systemic risk one should follow. Building the conceptual foundation for measuring systemic risk remains a challenging task.

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