Inside-out electromagnetic cloaking

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Abstract: We introduce a novel concept of an inside-out (or inverse) cloak for electromagnetic waves based on the coordinate transformation of Maxwell's equations. This concept can be employed for creating absorbing non-reflecting media as matching layers in numerical simulations. In contrast to the commonly used perfectly matched layers, such absorbing boundaries are characterized by physically meaningful parameters, and the concept can be used in various numerical simulation schemes.

OCIS codes: (260.2110) Electromagnetic Theory; (999.9999) Metamaterials.

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#91869 - $15.00 USD Received 23 Jan 2008; revised 12 Mar 2008; accepted 14 Mar 2008; published 19 Mar 2008

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1. Introduction

Recent pioneering ideas [1, 2] and many subsequent theoretical studies [3, 4, 5, 6, 7, 8, 9, 10, 11, 12] have demonstrated the possibility of creating invisibility cloaks for electromagnetic waves. The development of such unusual devices for manipulation of electromagnetic waves is based on transformation optics [3, 13, 14]. Transformation optics allows calculation of the properties of a hypothetical material which would be required to achieve a desired propagation of electromagnetic waves. Based on this general approach, the invisibility cloaks are designed in such a way that all waves are guided by the cloak around a concealed object without any reflections or scattering. A cylindrical cloak for a partial invisibility of electromagnetic waves has been recently demonstrated experimentally for microwaves [15].

The basic concepts of transformation optics can be employed in a wider range of physical problems, as has been already suggested in Ref. [16] for the example of the so-called electromagnetic field concentrator. Here we suggest another application of transformation optics for the design of a novel type of absorbing material. We demonstrate the use of such materials as matching layers in numerical simulation schemes where it is often desired to make non-reflecting boundaries at the edges of a simulation domain. In most of the cases, such boundaries are designed differently for different numerical techniques, and usually they are represented by some artificial complex conditions imposed on the fields. For example, in the finite-difference time-domain (FDTD) method the commonly used absorbing boundaries are introduced through the so-called perfectly matching layer (PML). The PML conditions have several drawbacks, e.g., they do not work simultaneously for all polarizations of electromagnetic waves, and they can be employed only for waves incident from vacuum. As a result of the latter limitation, if the numerical simulations are performed for a dielectric medium, in order to use the PML conditions one needs to introduce an additional free-space layer between the media and PML which in turn may result in unwanted reflection from an interface between the medium and vacuum.

In this paper, using the concepts of transformation optics we introduce a novel idea of the so-called *inside-out cloak* that can be immediately employed as a novel type of non-reflecting boundary layers in numerical simulations of the electromagnetic wave propagation or for an innovative design of real absorbing non-reflecting composite structures. Such absorbing boundaries are free from all limitations mentioned above, and they can be used in various simulation schemes. Moreover, due to the symmetry of the dielectric and magnetic properties, they work for both polarizations of electromagnetic waves, and they can be designed to make non-reflecting interfaces between media characterized by nontrivial dielectric and magnetic properties. In fact, our matching layers generalize the concept of non-reflecting media introduced long time ago by Efimov [17].

2. Basic concept of the inside-out cloak

Our concept can be termed as ‘inside-out cloak’ (or ‘inverse cloak’) which in some sense is opposite to the original idea suggested by John Pendry for creating invisibility cloaks. For the example of a spherical case, we want to ’compress‘ all the space $\infty > R = (X^2+Y^2+Z^2)^{1/2} > a$ into a finite spherical layer $b > r = (x^2+y^2+z^2)^{1/2} > a$ so that the sphere $R = a$ is transformed

#91869 - $15.00 USD Received 23 Jan 2008; revised 12 Mar 2008; accepted 14 Mar 2008; published 19 Mar 2008

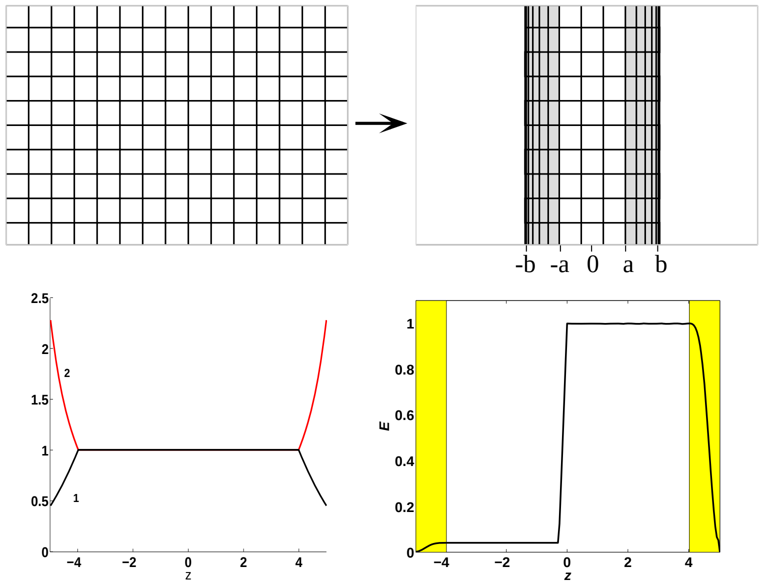

Fig. 1. Top: One-dimensional coordinate transformation. All space for $|z| > a$ is squeezed into the layer $a < |z| < b$. Bottom left: material parameters, curves 1 and 2 show $\varepsilon_{zz} = \mu_{zz} = \varepsilon_{||}$ and $\varepsilon_{\perp}$, respectively. Bottom right: results of the one-dimensional FDTD simulations of the propagation of the electromagnetic wave incident from the source located at $z = 0$ on the right matching layer (shaded). Shown is the magnetic field amplitude vs. the normalized coordinate $\mathrm{z} = z\omega/2\pi c$.

into itself ($r = a$), while infinity is transformed into the sphere $r = b$. Thus, the wave propagation in the transformed spherical layer is equivalent to the wave propagation in infinite space. As a result, for an observer from inside the cloak such a layer will be absolutely ‘black’ (non-reflecting) while for an external observer the layer will be ideally reflecting.

According to the medium interpretation for the coordinate transformation [3] we calculate the tensors of the material parameters which provide the same electromagnetic wave behavior as we get in the transformed space. The tensors of dielectric permittivity and magnetic permeability are equal to each other, $\hat{\varepsilon} = \hat{\mu}$, and they are defined as

$$\varepsilon_{ij} = \frac{J_{ik}J_{kj}}{\det(J_{lm})}, \qquad J_{ik} = \frac{\partial x_i}{\partial X_j}, \tag{1}$$

where $\hat{J}$ is the Jacobian of the coordinate transformation. We note here that closer to the external part of the cloak, $r = b$, some material tensor components diverge resulting in a small effective wavelength of the propagating waves which, in turn, gives an increasing error in the numerical simulations. To avoid this, we introduce gradually changing losses, $\gamma(r)$, which are growing in the matching layer from zero at $r = a$ to infinity at $r = b$. We note that in the original coordinate space this absorption starts from zero at $R = a$ and grows to a constant value, γ_0, $\gamma_{R\to\infty}(R) \to \gamma_0$. A proper selection of the transformation function $r = r(R)$, the loss function $\gamma(R)$, and the matching layer boundaries, a and b, allows us to obtain very low reflection of the waves incident on the absorbing layer from inside the sphere $r < a$.

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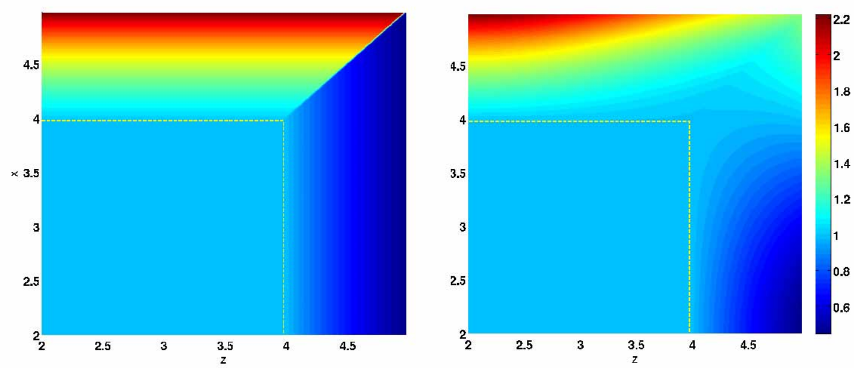

Fig. 2. Sharp (left) and smooth (right) transition between the matching layers for the *xx* component of the inverse tensor of the effective dielectric permittivity.

3. One-dimensional FDTD simulations

As an immediate application of this concept of the inverse cloak, we employ it for creating a new type of perfectly matching layer for the simulations. First, we consider a one-dimensional problem for which the idea of the coordinate transformation is shown in the top of Fig. 1. Similar to the spherical case discussed above, here we apply the transformation along one coordinate z only, and compress all the space from the regions $|z| > a$ into the layers $a < |z| < b$. For our particular case, we consider the simulation domain equal to ten wavelengths ($L_z = 10\lambda$), and place a harmonically oscillating ($\exp(-i\omega t)$) source at the origin ($z = 0$); the source emits waves in the $z > 0$ direction. In the absence of reflection from the edges of the simulation domains, the magnetic field in the wave can be written as $h_y = A_0 \exp[i\omega(z/c - t)]$, and the absolute value of h_y is constant for $0 < z < a$, $|h_y|_{0<z<a} = A_0$. In the presence of reflection, the absolute value of h_y is modulated with the relative amplitude R, $|h_y|_{0<z<a} = A_0[1 + R\cos(z\omega/c - \phi_0)]$, where R is the reflection coefficient from the matching layer and ϕ_0 is a phase shift. We have chosen the transformation in the following form,

$$\rho = \frac{a^2 + r(b-2a)}{(b-r)}, \qquad (2)$$

which maps the finite coordinate range $r = [a,b]$ onto the infinite interval $\rho = [a,\infty]$. The actual coordinate z is related to r as $r = b + |z| - L_z/2$. We note that the transformation is only applied to the region $|z| = (L_z/2 - \Delta : L_z/2)$. In order to avoid singularities in simulations, we use a numerical multiplier of 1.05 in front of b in Eq. (2).

To absorb electromagnetic waves in the matching layer, we introduce a loss function $\gamma = \gamma_0(\rho/r - 1)$, where we take $b = 40\lambda$, $a = b - \lambda$, $\gamma_0 = 80\omega$. We have also shifted the matching layer inside the simulation domain. Figure 1(bottom left) shows the coordinate dependence of the real parts of the corresponding components of the layer material tensors $\hat{\varepsilon}$ and $\hat{\mu}$ calculated from Eqs. (1,2). We note that the resulting tensors posses the same symmetry as those predicted for anisotropic non-reflecting crystals [17, 18], namely $\varepsilon_{||} = 1/\varepsilon_{\perp}$. A specific case of such media but with constant parameters was used earlier for numerical simulations [19]. Figure 1(bottom right) shows the results of our numerical simulations. Discretization is taken with 20 points per wavelength. Calculated reflection coefficient is of the order of $2 \cdot 10^{-3}$.

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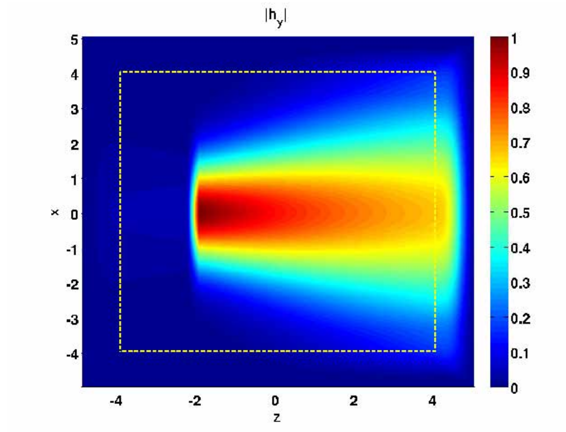

Fig. 3. Magnetic field distribution for the problem of the Gaussian beam incident on the edge of simulation domain.

4. Two-dimensional FDTD simulations

Now we demonstrate the application of the inside-out cloak for creating matching layers in two-dimensional FDTD simulations. We consider a TM-polarized Gaussian beam propagating toward an edge of the simulation domain and observe its absorption in the properly designed matching layer. The size of the simulation domain is 200x200 points with the physical size of $L_z = L_x = 10\lambda$. We use identical expressions for the coordinate transformation (where we have included numerical 1.05 multiplier explicitly now)

$$\rho = \frac{a^2 + r(1.05b - 2a)}{(1.05b - r)} \tag{3}$$

as in the one-dimensional case discussed above, with $r = b + |z| - \frac{1}{2}L_z$, for $|z| > |x|$ and $r = b + |x| - \frac{1}{2}L_x$, otherwise. The two-dimensional matching layers introduce additional complications associated with the corners of the rectangular simulation domain. In the standard PML technique additional efforts are required to suppress reflection from corners of the simulation domain [20]. Here, we compare two realizations for the corners in the framework of the coordinate transformation approach. First, we simply make a sharp transition between two transformations in Eq. (3) along the diagonal of the corner of the matching layer [see Fig. 2(left)]. A more complex approach, which can give more accurate matching at the corners, is to make a smooth transition between two transformations in the corner area [see Fig. 2(right)]. In this case we should take the general form of the transformation given by Eq. (2) with $r = (x^2 + z^2)^{1/2}$. This provides, e.g., a smooth transition of ε_{xx}^{-1} from the value $\varepsilon_{\perp}^{-1}$ to the value $\varepsilon_{||}^{-1}$, as shown in Fig. 2(right). This transition generates also non-zero off-diagonal components of the tensors of dielectric permittivity and magnetic permeability in the matching layer.

Our numerical FDTD simulations do not show any significant difference in the matching results for the two cases of the corner realization suggesting that a simpler sharp transition between the material parameters shown in Fig. 2(left) is sufficient for most of the purposes. Figure 3 shows the electromagnetic field for the Gaussian beam incident from the source located in the center of the simulation domain onto the right boundary. The beam is completely absorbed in the matching layer. Figure 4 shows the beam incident on the corner of the simulation domain; in this case the beam is absorbed almost completely with negligible reflection by the corner with sharp transition in matching layer. Drawbacks of our matching layers are larger

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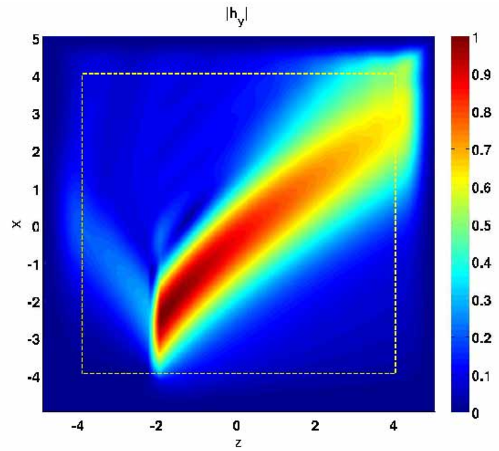

Fig. 4. Same as in Fig. 3 for the beam incident on the corner of simulation domain.

(but reasonable) reflection compared to the PML conditions, and thicker layers are required in order to produce lower reflection.

Apart from the use of transformation optics for creating novel type of boundary conditions in numerical modeling, we believe it can find a much wider range of applications in numerical schemes, e.g. it can be used inside the simulation domain itself. For example, modeling of metallic wedges usually requires either a dense discretization mesh or a non-uniform grid in order to describe correctly the large gradients of the field. Instead of using a non-uniform grid, one can transform (expand) the space around the wedge and use a regular mesh, however in this case the media will be described by modified tensors $\hat{\varepsilon}$ and $\hat{\mu}$. Another problem is the calculation of wave propagation in corrugated waveguides. Instead of directly calculating the complex geometry, one can transform the waveguide so that the simulations will be performed in a straight waveguide filled with inhomogeneous media described by modified dielectric and magnetic tensors.

5. Conclusions

We have introduce the concept of the inside-out (or inverse) cloak and employed it for creating a novel type of boundary matching layers in numerical FDTD simulations and completely absorbing layers in a real space. The absorbing boundaries created with the help of the transformation optics have distinct advantages over conventionally used matched layers in numerical simulations. First, they work simultaneously for all polarizations of electromagnetic waves. Second, they are described by physically meaningful material parameters and thus can be employed in any kind of numerical simulations. Finally, the matching layers can be designed to directly match magneto-dielectric materials without introducing additional vacuum gaps between the simulation domain and a matching layer.

Acknowledgments

The work was supported by the Australian Research Council through Discovery and Federation Fellowship projects. Nina Zharova thanks the Nonlinear Physics Center of the Australian National University for warm hospitality and support during her stay in Canberra, and acknowledges support from the Russian Fund for Basic Research through grant No. 08-02-00379.