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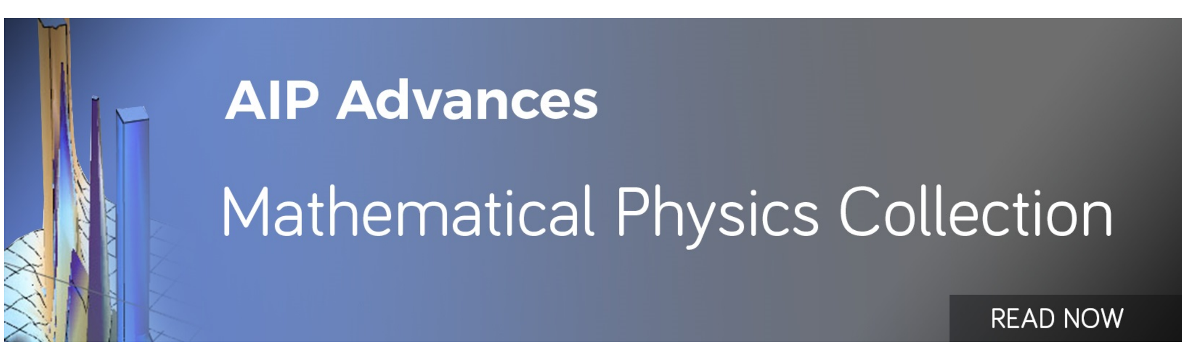

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Instability analysis of an electro-magneto-elastic actuator: A continuum mechanics approach

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The study of advanced artificial electro-magneto-elastic (EME) materials recently connects the material science with the electrodynamics. In particular, EME materials established a new research direction, which provides the fruitful ideas for the advanced engineering and medical field applications. In the present paper, we introduce a continuum mechanics-based method to analyze an electro-magneto-mechanical instability (EMMI) phenomenon of a smart actuator made of an EME material. The proposed method is based on the nonlinear theory of electro-magneto-elasticity followed by the second law of thermodynamics. We develop an analytical EMMI model for a smart actuator through a new amended energy function. This amended energy function accounts the electro-magnetostriction phenomenon for a class of an incompressible isotropic EME material. Additionally, the amended energy function successfully resolves the physical interpretation issue of the Maxwell stress tensor in large deformation. The formulated continuum mechanics-based EMMI model is also compared and validated with an energy-based EMMI model existing in the literature. https://doi.org/10.1063/1.5055793

I. INTRODUCTION

Electro-magneto-elastic (EME) materials which are mechanically flexible, bendable, and stretchable, may able to provide a quick response in the presence of an electromagnetic field. By filling the electro-active and magneto-active particles in a rubber-like matrix, an electro-magneto-mechanical field coupling may be achieved in the soft materials. EME materials are also known as smart materials, which have attracted a significant interest due to their potential use in engineering and medical field applications. Engineering applications include human-like robots, stretchable electronics, actuators, and energy harvesting devices etc.[1,2] Similarly, medical applications include detection of various physiological activities, health-care monitoring, human motion detection, and diagnosis etc.[3,4]

In the literature, a well-known artificial electromagnetic material was discovered in the 1940s, that may able to produce magnetoelectricity[5] with a field application. Liu and Sharma[6] studied an electromagnetic coupling phenomenon in soft material. They[6] found that a very few material support the electromagnetic coupling due to the stringent symmetry and diametric electronic structure of the ferromagnets and the ferroelectrics. But, the magnetoelectricity connects the electromagnetic degrees of freedom through a third order parameter known as the mechanical deformation. In the parallel work, Alameh et al.[7] and Krichen et al.[8] proposed a new mechanism to develop an artificial soft electromagnetic material. An extensive theoretical, as well as experimental works[9–11] related to electro-mechanical instability (EMI) in smart actuators, are presented in the literature. To the best of our knowledge, only Alameh et al.[12] analyzed an EMMI phenomenon of a smart actuator. Alameh et al.[12] studied EMMI phenomenon of smart material within the framework of an energy-based approach. Additionally, they[12] also proposed the design of the wireless energy

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harvesting process due to the remotely applied magnetic fields. An EME material shows an EMI phenomenon in the absence of the magnetic field. In the same way, an EME material may also be considered as a hyperelastic material in the absence of the electromagnetic field. It is assumed that the material is homogeneous, incompressible, and isotropic. With an application of the electromagnetic field, the complete deformation and the stress fields may be obtained from a single energy density function. This necessitates for proposing a new energy density function, which incorporates the corresponding energy contributions of the EME materials.

Large deformation and instability in the soft matter are the two factors, which are interrelated strongly with each other. As the deformation range increases, the requirement of the instability controlling efforts to overcome the failure also increases. These efforts may fall into the category of theoretical advancement or experimental improvements for the instability analysis of smart materials. In line with the theoretical advancement, the modeling methods play a vital role in the analysis of an EME deformation of a continua. Therefore, the primary goal of the present study is to present a new method to analyze an EMMI phenomenon of a smart actuator within the framework of the second law of thermodynamics.

Herein, we present the second law of thermodynamics-based deformation approach in EMMI phenomenon of an EME material through an amended energy function. Incorporation of the second law of thermodynamics-based method is our one of the major contribution in this work. Additionally, we also made an effort to describe the combined electro-magneto-elasticity with least material parameters for a class of an incompressible isotropic EME material. The amended energy function also successfully resolves the difficulty in the physical interpretation of the Maxwell stress tensor existing in the literature.[13] In contrast to the conventional energy harvesting method, the interaction of an additional magnetic field provides an opportunity to enhance the energy harvesting power from the soft dielectrics.

To fulfill our objectives the present work is organized as follows. In section II, an electro-magneto-elastic deformation theory is presented with a new amended form of energy function. In section III, an electro-magneto-mechanical-instability (EMMI) model is developed for a smart actuator within the framework of the second law of thermodynamics. Next, in Section IV, the effect of the controlling parameters, namely, pre-stretch, electric field, and magnetic field on an EMMI phenomenon of the smart actuator are discussed in comparison with the continuum mechanics-based and an existing energy-based method. Finally, Section V explains some concluding remarks.

II. NONLINEAR ELECTRO-MAGNETO-ELASTICITY

In this section, a brief overview of the fundamental field equations of physics and the constitutive theory related to an EME material deformation[14] are developed.

A. Kinematics

Consider a stress-free configuration of an EME material in the material space β_0. The material point is represented by the position vector $\mathbf{X}$, with respect to an arbitrarily chosen origin in reference configuration β_0. During deformation, the material point $\mathbf{X}$ deforms with an application of the electro-magneto-mechanical loading. Now, the same material point $\mathbf{X}$ may be represented with a new position vector $\mathbf{x} = \kappa(\mathbf{X})$; in the current configuration β, wherein κ denotes a one-to-one deformation mapping. Therefore, the deformation gradient tensor $\mathbf{F}$ for an incompressible isotropic material and its determinant J may be defined as follows

$$\mathbf{F} = \mathrm{Grad}\kappa = \frac{\partial \mathbf{x}}{\partial \mathbf{X}}, \quad J = \det\mathbf{F} = 1, \tag{1}$$

wherein Grad represents the gradient operator with respect to the position vector $\mathbf{X}$ in the reference configuration β_0.

B. Electromagnetic field balance equations

1. Eularian form

The electric field variables for an EME deformation in the current configuration β are denoted by $\mathbf{E}$, $\mathbf{D}$, and $\mathbf{P}$; the electric field vector, the electric induction or electric displacement vector, and the polarization density, respectively. Similarly, the magnetic field variables are denoted by **H**, **B**, and **M**; the magnetic field vector, magnetic induction vector, and magnetization density vector, respectively. Now, for the condensed matter, these electromagnetic field variables are related as follows

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}, \quad \mathbf{B} = \mu_0[\mathbf{H} + \mathbf{M}], \tag{2}$$

wherein ϵ_0, μ_0 are the electric permittivity and the magnetic permeability of free space. The widely used form of the equation (2) in an isotropic media may be represented as follows

$$\mathbf{D} = \epsilon_0 \epsilon_r \mathbf{E}, \quad \mathbf{B} = \mu_0 \mu_r \mathbf{H}, \tag{3}$$

wherein ϵ_r, μ_r are the dielectric and the diamagnetic constants. Now, considering the quasi-static case in the absence of free currents and the free electric charge, the electric field **E** and the electric displacement **D** satisfy Maxwell's equations[15] as follows

$$\begin{aligned} \mathrm{curl}\mathbf{E} = 0, &\quad \mathrm{div}\mathbf{D} = 0, \\ \mathrm{curl}\mathbf{H} = 0, &\quad \mathrm{div}\mathbf{B} = 0, \end{aligned} \tag{4}$$

wherein curl and div represent the curl and divergence operators, with respect to the position vector **x** in the current configuration β.

2. Lagrangian form

In the preceding sub-section, the relations are formulated in the Eulerian form with the operators div and curl. Now, we reformulate the same relations in the reference configuration β_0, and the new operators Div, Curl are connected with respect to the independent spatial variable **X**. The electric field variables in the Lagrangian form may be represented as $\mathbf{E}^l$, $\mathbf{D}^l$, and $\mathbf{P}^l$. Similarly, the magnetic field variables in the Lagrangian form may be represented as $\mathbf{H}^l$, $\mathbf{B}^l$, and $\mathbf{M}^l$. The relationships between these electromagnetic field variables may be obtained as follows

$$\begin{aligned} \mathbf{E}^l = \mathbf{F}^T\mathbf{E}, &\quad \mathbf{D}^l = \mathbf{F}^{-1}\mathbf{D}, \\ \mathbf{H}^l = \mathbf{F}^T\mathbf{H}, &\quad \mathbf{B}^l = \mathbf{F}^{-1}\mathbf{B}. \end{aligned} \tag{5}$$

These above relations (5) ensure that the equation (4) is equivalent to

$$\begin{aligned} \mathrm{Curl}\mathbf{E}^l = 0, &\quad \mathrm{Div}\mathbf{D}^l = 0, \\ \mathrm{Curl}\mathbf{H}^l = 0, &\quad \mathrm{Div}\mathbf{B}^l = 0. \end{aligned} \tag{6}$$

C. Constitutive relations

The constitutive relations for an incompressible isotropic EME material are formulated through the independent electromagnetic field variables **F**, **E**, and **H**. In an isothermal condition, the free energy density function for an incompressible isotropic EME material depends on these independent field variables as follows

$$\varphi = \varphi(\mathbf{F}, \mathbf{E}, \mathbf{H}). \tag{7}$$

The inter-relations between these electromagnetic field variables in Lagrangian form may be represented as follows

$$\begin{aligned} \mathbf{E}^l = \mathbf{F}^T\mathbf{E}, \quad \mathbf{D}^l = \mathbf{F}^{-1}\mathbf{D}, \quad \mathbf{P}^l = \mathbf{F}^{-1}\mathbf{P}, \\ \mathbf{H}^l = \mathbf{F}^T\mathbf{H}, \quad \mathbf{B}^l = \mathbf{F}^{-1}\mathbf{B}. \end{aligned} \tag{8}$$

In Lagrangian form, we consider **F**, $\mathbf{E}^l$, and $\mathbf{H}^l$ as the independent field variables for an EME deformation of a continua, and the free energy function $\phi(\mathbf{F}, \mathbf{E}^l, \mathbf{H}^l)$ may be defined with the relations (8) as follows

$$\phi(\mathbf{F}, \mathbf{E}^l, \mathbf{H}^l) = \varphi(\mathbf{F}, \mathbf{F}^{-T}\mathbf{E}^l, \mathbf{F}^{-T}\mathbf{H}^l). \tag{9}$$

Following the previous studies,[16–18] the total stress tensor **T** based on the concept of Maxwell stress for an EME material deformation may be obtained as follows

$$\mathbf{T} = \mathbf{S} + \mathbf{P} \otimes \mathbf{E} + \epsilon_0[\mathbf{E} \otimes \mathbf{E} - \frac{1}{2}(\mathbf{E}.\mathbf{E})\mathbf{I}] + \mu_0^{-1}[\mathbf{B} \otimes \mathbf{B} - \frac{1}{2}(\mathbf{B}.\mathbf{B})\mathbf{I}], \tag{10}$$

wherein **S** represents the Cauchy stress tensor. This direct approach for obtaining the total stress tensor through superposition of stresses may lead to a conceptual inaccuracy, especially in large deformation of a continua.[19,20] Additionally, the stress superposition is physically irrelevant in terms of the definition of stress. There is an issue related to the physical interpretation of the Maxwell stress tensor in large deformation as well. The deformation dependency on the electric permittivity and the magnetic permeability plays a vital role at large deformation condition.[21] Therefore, a new amended form of energy density function $\Omega = \Omega(\mathbf{F}, \mathbf{E}^l, \mathbf{H}^l)$, which incorporates the Maxwell stress contribution for the EME materials may be defined as follows

$$\Omega(\mathbf{F}, \mathbf{E}^l, \mathbf{H}^l) = \rho\phi(\mathbf{F}, \mathbf{E}^l, \mathbf{H}^l) - \frac{1}{2}\epsilon_0 \mathbf{E}^l.(\mathbf{b}^{-1}\mathbf{E}^l) + \frac{1}{2}\mu_0^{-1}\mathbf{B}^l.(\mathbf{b}\mathbf{B}^l), \tag{11}$$

wherein $\mathbf{b} = \mathbf{F}\mathbf{F}^T$ represents the left Cauchy green deformation tensor. In addition, the above amended energy function (11) represents a general form of the combination of electrical energy, magnetic energy, and the interaction energy. In other words, this shows the representation of superposition of the possible forms of energies contributed through individual as well as interaction aspects. Unlike the superposition of stress contributions from the mechanical and the electromagnetic domains as described in (10), we prefer the superposition of energy. This amended energy function $\Omega(\mathbf{F}, \mathbf{E}^l, \mathbf{H}^l)$ also successfully overcomes the hurdle of the physical interpretation of the Maxwell stress in smart material in large deformation.[11] For the detail discussions on this issues related to the physical objectivity of the Maxwell stress, we refer to the previous studies[19,20] and the references therein. The thermodynamically consistent constitutive relations for an incompressible isotropic EME material may be obtained from the Clausius-Duhem inequality. Assuming an isothermal conditions, the inequality may be written in terms of the amended energy density function $\Omega(\mathbf{F}, \mathbf{E}^l, \mathbf{H}^l)$ as

$$\left(\mathbf{T} - \mathbf{F}\frac{\partial\Omega}{\partial\mathbf{F}}\right) : \dot{\mathbf{F}} - \left(\mathbf{D}^l + \frac{\partial\Omega}{\partial\mathbf{E}}\right) : \dot{\mathbf{E}} - \left(\mathbf{B}^l + \frac{\partial\Omega}{\partial\mathbf{H}}\right) : \dot{\mathbf{H}} \geq 0. \tag{12}$$

Now, the set of constitutive laws for an incompressible isotropic EME material may be obtained as follows

$$\mathbf{T} = -p\mathbf{I} + \mathbf{F}\frac{\partial\Omega}{\partial\mathbf{F}}, \quad \mathbf{D}^l = -\frac{\partial\Omega}{\partial\mathbf{E}^l}, \quad \mathbf{B}^l = -\frac{\partial\Omega}{\partial\mathbf{H}^l}, \tag{13}$$

wherein p is the indeterminate hydrostatic pressure, equivalent to the Lagrange multiplier associated with the incompressibility constraint.

The amended energy density function Ω for an incompressible isotropic EME material may also be represented in the invariant form $\Omega(I_1, I_2 \ldots I_9)$ as well. These invariants may be obtained from the three isotropic tensors namely the left Cauchy green deformation tensor $\mathbf{b} = \mathbf{F}\mathbf{F}^T$, $\mathbf{E}^l \otimes \mathbf{E}^l$, and $\mathbf{H}^l \otimes \mathbf{H}^l$ as follows

$$\begin{gathered}
I_1 = \mathrm{tr}\mathbf{b}, \quad I_2 = \frac{1}{2}[(\mathrm{tr}\mathbf{b})^2 - \mathrm{tr}(\mathbf{b}^2)], \quad I_3 = \det\mathbf{b} = 1, \\
I_4 = [\mathbf{E}^l \otimes \mathbf{E}^l] : \mathbf{I}, \quad I_5 = [\mathbf{E}^l \otimes \mathbf{E}^l] : \mathbf{b}^{-1}, \\
I_6 = [\mathbf{E}^l \otimes \mathbf{E}^l] : \mathbf{b}^{-2}, \quad I_7 = [\mathbf{H}^l \otimes \mathbf{H}^l] : \mathbf{I}, \\
I_8 = [\mathbf{H}^l \otimes \mathbf{H}^l] : \mathbf{b}^{-1}, \quad I_9 = [\mathbf{H}^l \otimes \mathbf{H}^l] : \mathbf{b}^{-2}.
\end{gathered} \tag{14}$$

From the relations (11), (13) and the definitions of invariants in (14), explicit form of $\mathbf{T}$, $\mathbf{D}$, and $\mathbf{B}$ may be obtained as follows

$$\begin{gathered}
\mathbf{T} = -p\mathbf{I} + 2\Omega_1\mathbf{b} + 2\Omega_2[I_1\mathbf{b} - \mathbf{b}^2] - 2\Omega_5\mathbf{E}\otimes\mathbf{E} - 2\Omega_6[\mathbf{b}^{-1}\mathbf{E}\otimes\mathbf{E} \\
+ \mathbf{E}\otimes\mathbf{b}^{-1}\mathbf{E}] + 2\Omega_8\mathbf{b}\mathbf{H}\otimes\mathbf{b}\mathbf{H} + 2\Omega_9[\mathbf{b}\mathbf{H}\otimes\mathbf{b}^2\mathbf{H} + \mathbf{b}^2\mathbf{H}\otimes\mathbf{b}\mathbf{H}], \\
\mathbf{D} = -2[\Omega_4\mathbf{b} + \Omega_5\mathbf{I} + \Omega_6\mathbf{b}^{-1}]\mathbf{E}, \\
\mathbf{B} = -2[\Omega_7\mathbf{b} + \Omega_8\mathbf{b}^2 + \Omega_9\mathbf{b}^3]\mathbf{H},
\end{gathered} \tag{15}$$

wherein notation Ω_i denotes $\Omega_i = \dfrac{\partial\Omega}{\partial I_i}$, $i = 1, 2, 3..., 9$. The above equation (15) represents the standard constitutive relations for an incompressible isotropic EME material.

III. ELECTRO-MAGNETO-MECHANICAL INSTABILITY (EMMI) MODEL FOR AN EME MATERIAL BASED SMART ACTUATOR

A. Geometry and deformation

Consider a smart actuator made of an EME material with the coordinate systems (X_1, X_2, X_3) for the reference configuration β_0, and (x_1, x_2, x_3) for the current configuration β. The compliant electrodes cover the top and bottom of the actuator plates as shown in the FIG. 1. The smart actuator deforms with an electromagnetic field application by switching on the voltage and the magnetic poles intensity in X_1 direction. The original shape is free from any electro-magneto-mechanical load in the reference configuration. However, the current configuration is subjected to an electro-magneto-mechanical load applied through the voltage with a magnetic field source. The domain occupied by an EME material actuator as shown in the FIG. 1, may be represented as follows

$$\begin{aligned} \beta_0 &= \{\mathbf{X} \in \Re^3 : -H \le X_1 \le H, -L_1 \le X_2 \le L_1, -L_2 \le X_3 \le L_2\}, \\ \beta &= \{\mathbf{x} \in \Re^3 : -h \le x_1 \le h, -l_1 \le x_2 \le l_1, l_2 \le x_3 \le l_2\}, \end{aligned} \tag{16}$$

wherein (H, L_1, L_2) and (h, l_1, l_2) are the geometrical dimensions of a smart actuator in the reference and current configuration, respectively.

Further, consider the smart material deforms homogeneously, and assumed to be incompressible and isotropic. Then, the stretch in the principal directions may be defined as follows

$$\lambda_1 = \frac{h}{H}, \quad \lambda_2 = \frac{l_1}{L_1}, \quad \lambda_3 = \frac{l_2}{L_2}. \tag{17}$$

The deformation gradient tensor $\mathbf{F}$, the electric field vector $\mathbf{E}$, and the magnetic field vector $\mathbf{H}$ for an above electro-magneto-elastic deformation may be obtained as follows

$$\mathbf{F} = \lambda_1 \mathbf{e}_{11} + \lambda_2 \mathbf{e}_{22} + \lambda_3 \mathbf{e}_{33}, \quad \mathbf{E} = E_0 \mathbf{e}_1, \quad \mathbf{H} = H_0 \mathbf{e}_1. \tag{18}$$

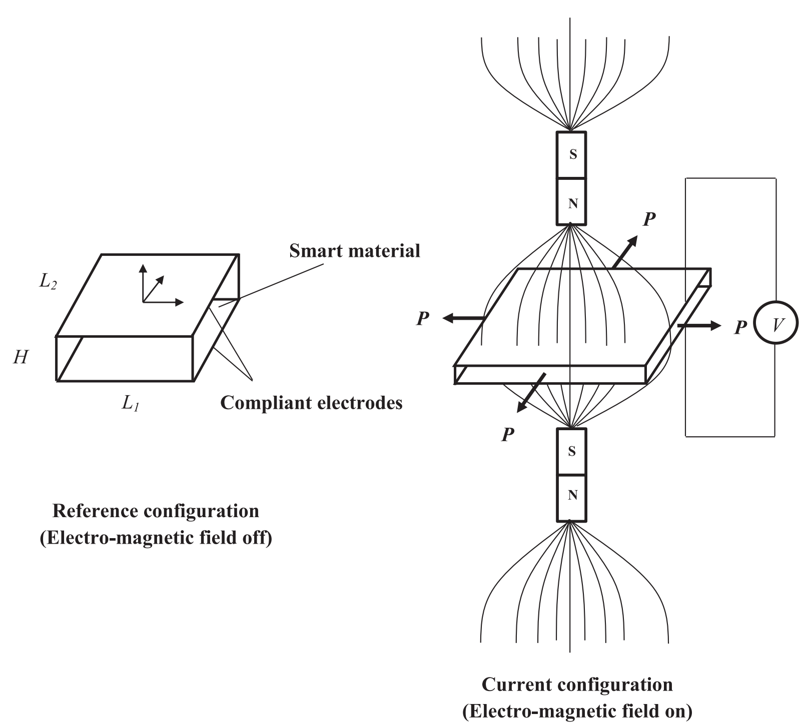

FIG. 1. Actuation of a smart actuator made of EME material under electro-magneto-elastic deformation condition.

Now, the associated invariants of the given electro-magneto-elastic deformation (16) in terms of principal stretches may be obtained from the relations (14) and (18) as follows

$$\begin{gathered} I_1=\lambda_1^2+\lambda_2^2+\lambda_1^{-2}\lambda_2^{-2}, \quad I_2=\lambda_1^{-2}+\lambda_2^{-2}+\lambda_1^2\lambda_2^2, \\ I_3=\lambda_1\lambda_2\lambda_3=1, \quad I_4=\lambda_1^2E_0^2, \\ I_5={E_0}^2, \quad I_6=\lambda_1^{-2}E_0^2, \\ I_7=\lambda_1^2H_0^2, \quad I_8={H_0}^2, \quad I_9=\lambda_1^{-2}H_0^2. \end{gathered} \tag{19}$$

The stress components in the principal directions may also be obtained from the relation $(15)_1$ as follows

$$\begin{gathered} T_{11}=-p+2\Omega_1\lambda_1^2+2\Omega_2(\lambda_1^2\lambda_2^2+\lambda_1^2\lambda_3^2)-2\Omega_5E_0^2-4\Omega_6\lambda_1^{-2}E_0^2+2\Omega_8\lambda_1^4H_0^2-4\Omega_9\lambda_1^6H_0^2, \\ T_{22}=-p+2\Omega_1\lambda_2^2+2\Omega_2(\lambda_1^2\lambda_2^2+\lambda_2^2\lambda_3^2), \\ T_{33}=-p+2\Omega_1\lambda_3^2+2\Omega_2(\lambda_1^2\lambda_3^2+\lambda_2^2\lambda_3^2). \end{gathered} \tag{20}$$

B. A new electro-magneto-elastic energy density function

We recall the general expression of the nine invariants (14) related to an amended energy function, and we may discard $I_3 = 1$ (incompressibility constraint) essentially. Thus, the generalized energy density function will have the eight such independent invariants. However, our target is to propose a new material model with least material constants for a class of an incompressible isotropic EME material. Therefore, a Mooney-Rivlin type EME material model for an incompressible, isotropic EME material may be generalized through the concept of an amended energy function (11). Our effort is to propose an amended energy function with least material parameters for an isotropic EME material. The energy density function which includes elastic, electric and magnetic contributions is proposed as follows

$$\Omega=C_1(I_1-3)+C_2(I_2-3)-\frac{\epsilon_0}{2}(C_3I_4+C_4I_5)-\frac{\mu_0}{2}(C_5I_7+C_6I_8), \tag{21}$$

wherein $C_1, C_2, \ldots., C_6$ are the material parameters. The elastic, electric and magnetic contributions may also be extended together with the coupled electro-elastic, magneto-elastic, electro-magneto, and EME contributions. These extended coupled contributions may use eight or more material parameters depending on the coupling extensions. It is expected that with the use of eight material parameters, we may represent the coupling phenomenon very effectively. However, the proposed EME material model (21) is the simplest possible form of energy density functions with least material parameters. We have neglected the I_6 and I_9 invariant effect in the proposed energy function. The physical electro-elastic and magneto-elastic interaction represented by these I_6 and I_9 invariants, respectively have already been adopted with the I_5 and I_6 invariants.

It is customary to mention that in the absence of an electromagnetic field, the proposed energy function (21) represents the classical Mooney-Rivlin type strain energy density function, and $C_2 = 0$ retains the Neo-Hookean energy form. The proposed EME material model also successfully fulfills all the fundamental restrictions on the form of energy density function presented by Darrijani et al.[22] The above energy function is only one of the various possible forms of the energy density function (21) for an isotropic EME materials. These possible forms of energy density functions provide a direct road-map for the analytical advancement of the smart material deformation.

C. Classical continuum mechanics-based EMMI model

The EMMI model expression for a pre-stretched smart material actuator in equi-biaxial deformation ($\lambda_2 = \lambda_3 = \lambda$, $T_{22} = T_{33} = T_p$, and $T_{11} = 0$) may be obtained from the relations (20) and (21) as follows

$$T_{22}=T_{33}=2C_1(\lambda^2-\lambda^{-4})+2C_2(\lambda^4-\lambda^{-2})-C_4\epsilon_0E_0^2+C_6\mu_0H_0^2\lambda^{-8}, \tag{22}$$

wherein $T_{22} = T_{33} = T_p$ at $E_0 = H_0 = 0$ represent the pre-stretching effect in the above EMMI model (22). For the sake of convenience, we introduce the following non-dimensional variables as follows

$$T_p^* = \frac{T_p}{C_1}, \quad E^* = \widehat{E}\sqrt{\frac{C_4\epsilon_0}{C_1}}, \quad H^* = \widehat{H}\sqrt{\frac{C_6\mu_0}{C_1}}, \tag{23}$$

wherein $\widehat{E} = \lambda_1 E_0$ and $\widehat{H} = \lambda_1 H_0$ are the nominal electric and magnetic fields, respectively. Now, the above expression (22) with (23) in non-dimensional form may be rewritten as follows

$$2(\lambda^2 - \lambda^{-4}) + 2C_2/C_1(\lambda^4 - \lambda^{-2}) - E^{*2}\lambda^4 + H^{*2}\lambda^{-4} - T_p^* = 0. \tag{24}$$

The above non-dimensional expression (24) represents an EMMI model for a pre-stretched smart material actuator in an equi-biaxial deformation condition. This EMMI model (24) relates three controlling parameters, namely, non-dimensional mechanical load T_p^*, non-dimensional electric field E^*, and non-dimensional magnetic field H^*. The critical value of deformation λ may also be obtained for their corresponding values of controlling parameters, namely, T_p^*, E^*, and H^*.

IV. CONTROLLING PARAMETER EFFECTS ON ELECTRO-MAGNETO-MECHANICAL INSTABILITY (EMMI) PHENOMENON

In this section, the effects of different controlling parameters, namely, pre-stretch, electric field, and magnetic field are discussed on an EMMI and an EMI phenomena of a smart actuator made of an EME material. A comparison of the continuum mechanics-based and the energy-based approaches is presented. These effects are found remarkable on an EMMI and an EMI phenomena as shown in the upcoming discussions.

A. In the absence of external magnetic field

In line with the previous works on the dielectric elastomers, we first study our EMMI model (24) in the absence of the magnetic field. For $H_0 = 0$, our EMMI model (24) represents the Electro-mechanical instability (EMI) phenomenon for a pre-stretched smart material actuator in an equi-biaxial deformation condition existing in the literature.[9–12] We remark that an electric displacement $\mathbf{D}$ may also be used as an independent field variable, instead of an electric field $\mathbf{E}$ for an electro-elastic deformation of dielectric elastomers.[23,24] In this case, the amended energy function $\Omega = \Omega(\mathbf{F}, \mathbf{E}^l, \mathbf{H}^l)$ takes another form $\Omega = \Omega(\mathbf{F}, \mathbf{D}^l)$ for an EMI phenomenon of the dielectric elastomers. The same state variables (λ, D) were also used in the previous energy-based EMI works.[9–12]

In order to analyze the EMI phenomenon from the formulated EMMI model (24) with $H_0 = 0$, the non-dimensional electric field versus stretch curves are plotted in comparison with an energy-based EMI model[12] in the FIG 2. From the stress-stretch and electric permittivity-stretch plots available in the literature[25,26] for VHB 4910 dielectric elastomer, we obtained the material parameters $C_1 = 28$ kPa, $C_2 = 0.00026$ kPa, and $C_4 = 3.36$ corresponding to an amended energy function (21). The different curves with material parameters C_1, C_2, and C_4 in the FIG. 2 represent the theoretical EMI model for an equi-biaxial deformation at different values of the pre-stretching effects. These pre-stretching values for each curve can be read from the intersection between the curve and the horizontal axis. The solid lines in the FIG 2 represent the proposed EMMI model (24), and the dotted lines represent an energy-based Alameh et al.[12] EMMI model with $H_0 = 0$ for a pre-stretched smart actuator.

FIG 2 clearly indicates the comparison between the proposed EMMI model (24) and an energy-based Alameh et al.[12] EMMI model with $H_0 = 0$ in an equi-biaxial deformation condition. The proposed EMMI model (24) is formulated through a new amended form of energy function (21), which accounts the electro-magnetostriction phenomenon. On the other hand, Alameh et al.[12] EMMI model is formulated through an extended Neo-hookean type energy function. It is customary to mention that energy-based EMI model does not account electro-magnetostriction phenomenon accurately unlike the proposed EMMI model (24).

B. Effect of pre-stretch on the EMI phenomenon

Further, in the FIG 2, for a fixed mechanical load, the actuator deforms in a succession of the states of equilibrium, as the electrical field varies. A particular pre-stretch value may have single or multiple

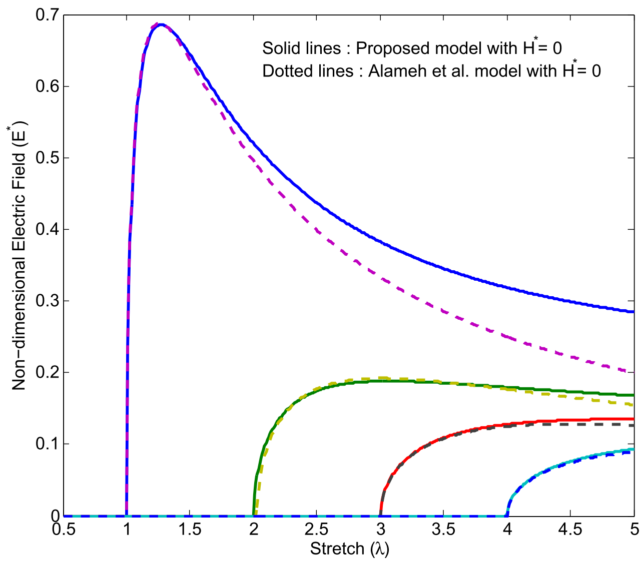

FIG. 2. Comparison of a classical continuum mechanics-based EMMI model (24) with an existing energy-based EMMI model at $H_0 = 0$ for EMI phenomenon.

states of the equilibrium.[27] At low pre-stretch, the curves are initially going up and after that coming down. However, when the pre-stretch is large, the curves become monotonic. These two types of electro-mechanical responses we get due to pull-in instability phenomena at small pre-stretches, and the membrane becomes more thinner with a higher electric field at the larger pre-stretches. Both of the continuum mechanics and energy-based methods are showing the similar results on the pre-stretching effects as shown in the FIG 2. Both of the methods conclude that the EMI limit can be suppressed or specifically saying can be eliminated as we increase the pre-stretching conditions. Therefore, we may conclude that a classical continuum mechanics-based method may also be considered as an alternative method for the EMI analysis of a pre-stretched DE actuator.

C. Effect of the magnetic field on the EMI phenomenon

In order to analyze an EMMI phenomenon by controlling the parameter H^* at dead load ($T_p = 0$) condition, the non-dimensional electric field versus stretch curves are plotted in comparison with an energy-based EMMI model in the FIG 3. In another way, the non-dimensional electric field versus lateral stretch curves are also plotted in comparison with an energy-based EMMI model in the FIG 4. Both of the continuum mechanics-based and energy-based methods are showing the similar results by controlling the magnetic field effects. Both of the methods in the FIGs 3 and 4 assert that the non-dimensional magnetic field H^* enhances the critical value of the non-dimensional electric field E^*.

The effect of the magnetic field on the EMI phenomenon may also be clearly explained through a simple system design shown schematically in the FIG. 5. This system represents a wireless actuation and energy harvesting process proposed by Alameh et al.[12] The system contains the fixed charge on the top and the bottom layers of an EME material film. Initially, film thinning and area expansion with an electric field application result in a large capacitance and a low voltage state. However, with an application of the magnetic field, we may observe increment in the thickness and decrement in the area of the film. These effects result in a lower capacitance with a larger voltage state. A new higher power state is obtained with an application of the magnetic field. That may provide more power to the same connected device as compared to the previous lower power state.

D. Effect of the electric field on the EMMI phenomenon

In order to analyze an EMMI phenomenon by controlling the parameter E^* at dead load ($T_p = 0$) condition, the non-dimensional magnetic field versus stretch curves are plotted in the FIG 6. In line with an existing energy-based EMMI model given by Alameh et al.,[12] we plot a continuum

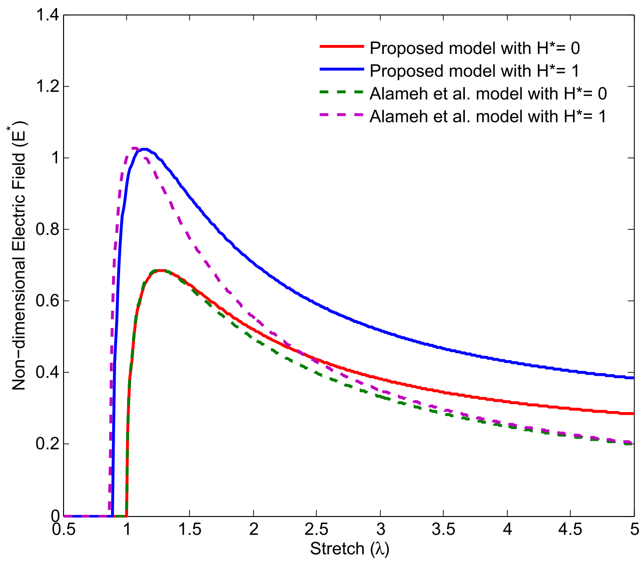

FIG. 3. Comparison of a classical continuum mechanics-based EMMI model (24) with an existing energy-based EMMI model at different magnetic fields for an EMI phenomenon.

mechanics-based EMMI model (24) for an equi-biaxial deformation condition as shown in the FIG 6. Herein, the proposed continuum mechanics-based method is agreeing well with an existing energy-based method[12] on the control of the magnetic field effects. In addition, we may clearly note that the non-dimensional magnetic field versus stretch curves are symmetric about the horizontal axis. The nature of these curves may be expected mathematically due to the quadratic term presents in the EMMI model (24). The non-dimensional magnetic field versus stretch curves are also showing two different types of the natures for the different values of E^*.

Firstly, in an existing energy-based EMMI model,[12] the equilibrium curves are separated on the left and right sides for a E^* parameter below the critical value between 0.5 to 0.75. However, through our continuum mechanics-based EMMI model (24), the equilibrium curves are separated on the left

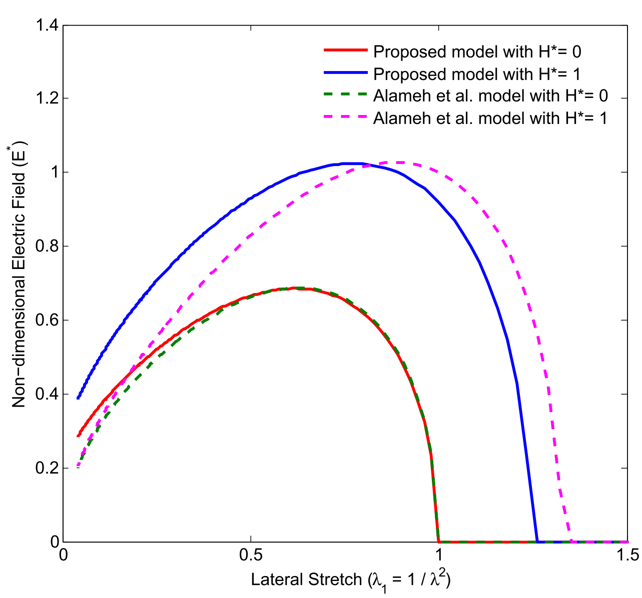

FIG. 4. Comparison of a classical continuum mechanics-based EMMI model (24) with an existing energy-based EMMI model at different magnetic fields for an EMI phenomenon.

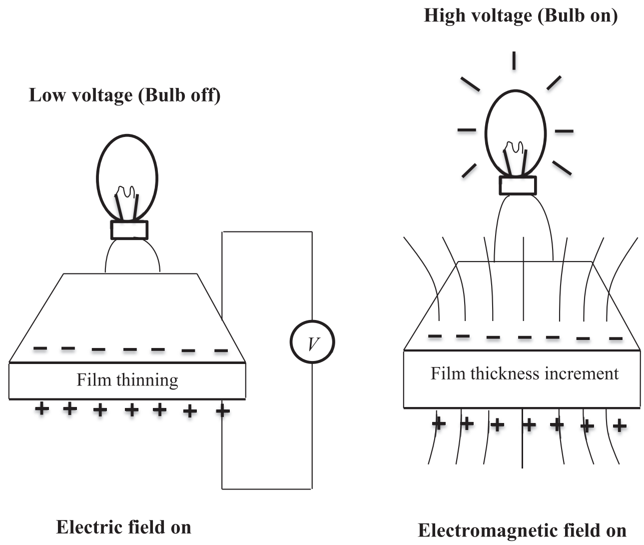

FIG. 5. Schematic diagram of a wireless actuation and energy harvesting system.

and right sides for the same E^* parameter below the critical values between 0.75 to 1 as shown in the FIG 6. Secondly, in both of the continuum mechanics-based (24) and an existing energy-based[12] EMMI models, the equilibrium curves are separated on the top and bottom sides with respect to the horizontal axis for a E^* parameter greater than the critical value.

Finally, in both of the works based on an existing energy-based EMMI model given by Alameh et al.[12] and a new alternative work shown in FIG 6, each curve is having its own turning point. The turning points in the top and bottom curves represent the minimum and maximum magnetic field values, respectively. Similarly, the turning points in the left and right curves represent the maximum and minimum stretch values, respectively.

E. Effect of the magnetic field on the stability

In order to analyze the stability of the actuator by controlling the magnetic field parameter at various dead load conditions, the non-dimensional critical electric field versus dead load curves are

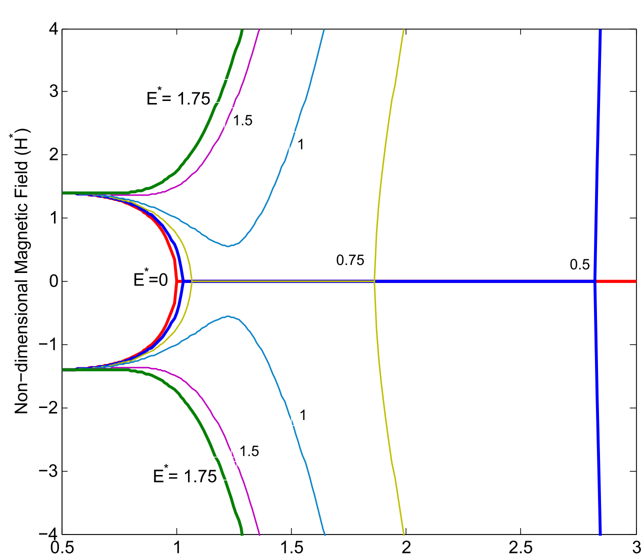

FIG. 6. A classical continuum mechanics-based EMMI model (24) at different electric fields.

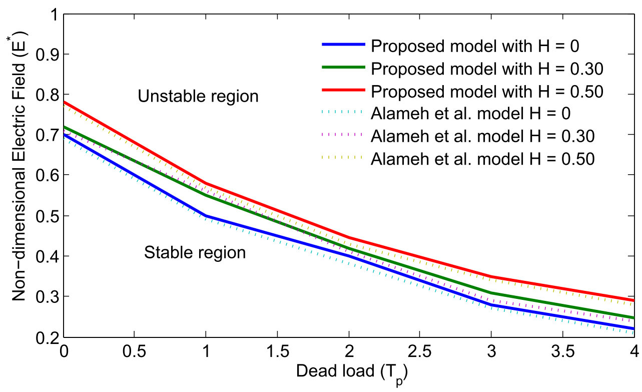

FIG. 7. Stable and Unstable regions through a continuum mechanics-based and an energy-based methods at different magnetic fields.

plotted in the FIG 7. For the sake of convenience, we exclude the electrical breakdown effect. The different stability and instability regions are presented for a film of the smart material subjected to an uni-axial tension with an applied electric field under various external magnetic fields. In the FIG 7, the solid line curves represent the proposed model (24), which is formulated within the framework of the second law of thermodynamic-based continuum mechanics approach. On the other hand, the dotted line curves represent the Alameh et al.[12] model, which is formulated within the framework of the first law of thermodynamics-based energy approach. Additionally, we may clearly notice that an applied magnetic field enhances the electro-magneto-mechanical stability of the system. A higher value of the magnetic field provides a larger stability region. Now, we may conclude that an applied magnetic field allows the film to sustain a higher electric field.

V. CONCLUDING REMARKS

In the present paper, we present a new method to analyze the electro-magneto-mechanical-instability (EMMI) phenomenon of a smart actuator made of an electro-magneto-elastic (EME) material. In line with that, an EME deformation theory is formulated based on the continuum mechanics approach through a new amended energy function for an incompressible isotropic EME material. A new amended energy function (21) with least material parameters is proposed for an electro-magnetostriction phenomenon of an EME material. An analytical EMMI model of a smart actuator under an equi-biaxial deformation condition is developed followed by the second law of thermodynamics. The effect of the controlling parameters, namely, pre-stretch, electric field, and magnetic field on the EMMI phenomenon are also discussed in comparison with the continuum mechanics-based and an energy-based method.

Incorporation of the second law of thermodynamics-based deformation approach in an EMMI phenomenon through a new amended energy function, is one of the major contribution of this work. Additionally, a new amended energy function (21) with least material parameters is also proposed for a class of an incompressible isotropic EME material. The amended energy function successfully overcomes the difficulty in the physical representation of the Maxwell stress tensor in large deformation. Further, the presence of an external magnetic field provides a significant control variable to enhance the energy harvesting power from the smart materials. An immediate extension of the present study also lies to study the post-bifurcation analysis of EMMI phenomenon, that will be presented in our future work.

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