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INSTABILITY ANALYSIS OF BRAKE SQUEAL WITH UNCERTAIN CONTACT CONDITIONS

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Brake squeal, as a phenomenon of friction-induced self-excited vibrations, has been a noise, vibration and harshness (NVH) problem for the automotive industry due to warranty-related claims and customer dissatisfaction. Intensive research in the past two decades have provided insight into a number of mechanisms that trigger brake squeal. However, brake squeal is a transient and nonlinear phenomenon and many determining factors are not known precisely such as material properties, operating conditions (brake pad pressure and temperature, speed), contact conditions between pad and disc, and friction. As a result, reliable prediction of brake squeal propensity is difficult to achieve and extensive noise dynamometer testings are still required to identify problematic frequencies for the development and validation of countermeasures. Here, the influence of uncertainties in friction modelling and contact conditions on friction-induced self-excited vibrations of a 3 x 3 coupled friction oscillators model is examined by combining the linear Complex Eigenvalue Analysis (CEA) method widely used in industry with a stochastic approach that incorporates these uncertainties. It has been found that unstable vibration modes with consistently high occurrence of instability independent of the contact area, friction modelling and sliding speed could be identified. Such unstable modes are considered to be robustly unstable and are most likely to produce squeal. An example is given to illustrate how instability countermeasures could be designed by repeating the uncertainty analysis for these robustly unstable modes. These results highlight the potential of reliable prediction of brake squeal propensity in a full brake-system using a stochastic approach with the CEA.

Keywords: Brake squeal, instabilities, complex eigenvalues

1. Introduction

Despite substantial research efforts and some success in the last two decades in understanding the mechanisms of brake squeal, reliable prediction of its occurrence remains as difficult as ever. This is due to a number of difficulties including (a) brake squeal is an acoustic phenomenon but most numerical studies were confined to the prediction of unstable vibration modes and only in recent years acoustic calculations were performed [1-3]; (b) brake squeal is a transient non-linear phenomenon [4, 5] but the most popular numerical prediction method is the linear complex eigenvalue analysis (CEA) conducted in the frequency domain [6] as transient non-linear analysis is still computationally too expensive to be applied to a full brake system in practice [7]; and (c) uncertainties in material properties, operating conditions, friction modelling and contact conditions [8, 9]. Zhang et al. [10] however, have shown that uncertainty analyses can compensate to some degree the effect of nonlinearity even if the linear CEA is used. Significant interest in brake squeal in recent years is reflected in the number of papers published in the English language literature, as shown in Fig. 1.

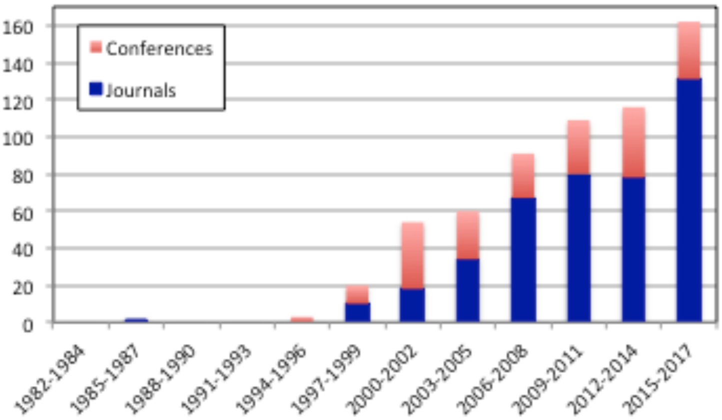

Figure 1: Number of English Language papers published during 1982-2017 with key words "brake squeal" according to ISI Thomson Web of Science as at 20 March 2018.

Although the influence of the uncertainty due to the contact surface topography on brake squeal has been analysed [11, 12], there are no studies attempted to investigate the combined effects of the variability in friction modelling and contact area on squeal propensity. Also, no general conclusion has been made on how the incorporation of an uncertainty analysis into friction modelling and contact conditions enhances instability predictions using linear methods. Therefore, in this study, an analytical model of 3×3 coupled oscillators in randomised frictional contact with a sliding rigid plate is used to investigate the effect of different friction models and contact area on predictions of unstable vibration modes using the CEA.

2. Coupled friction oscillators on a sliding rigid plate

To study the combined effects of randomised friction parameters of different friction models and uncertainties in the contact conditions, an analytical model of 3×3 coupled friction oscillators as shown in Fig. 2 is adopted from the study of friction modelling on instability analysis [13]. Here, the 3×3 coupled masses (m_i (i = 1…9)) together represent a pad and each mass is in point contact through springs with stiffness $k^z_{1\sim9}$ with the plate sliding in the x-direction at a constant speed v, with elasticity and damping modelled by springs (k^x_p, k^y_p, k^z_p) and viscous damping dashpots (c^x_p, c^y_p $, c^z_p$). The friction forces f^x_{1-9} and f^y_{1-9} in Fig. 2(a) and contact force of each oscillator can be varied so that their local effects on the global dynamics can be investigated. The friction force acting on each mass in the x-direction is modelled by either the Amonton-Coulomb model with a constant friction coefficient [14], the velocity-dependent model [15] with the friction coefficient dependent on the relative velocity, or the rate-dependent LuGre model [16]. As the plate's translational velocity does not have a component in the y-direction, the friction force in the y-direction is governed by the Amonton-Coulomb model. In the x-y plane, the oscillators are interconnected via springs with stiffness k^x_{1-12} and k^y_{1-12} and viscous dashpots with damping coefficients c^x_{1-12} and c^y_{1-12}. In the z-direction, the oscillators are connected to a foundation via dashpots (damping constant c^z_{1-9}). To trigger an instability, other springs k^a_{1-9} must be inclined [17]. Here, the chosen inclination angles are φ=60°=θ, where φ is measured from the x-axis in the x-y plane and θ is measured from the x-y plane as shown in Fig. 2(b). The equations of motion in matrix form are

$$\mathbf{M}\ddot{\mathbf{U}}+\mathbf{C}\dot{\mathbf{U}}+\mathbf{K}\mathbf{U}=\mathbf{F}, \tag{1}$$

where **U** is the displacement vector**,** **M** is the mass matrix, **C** is the damping matrix, **K** is the stiffness matrix, **F** is the pre-load force vector, and the single and the double dots represent the first and second time derivatives, respectively. Details of elements of various matrices, the identification of parameters of the three friction models, formulation of complex eigenvalue analysis and friction work can be found in [13].

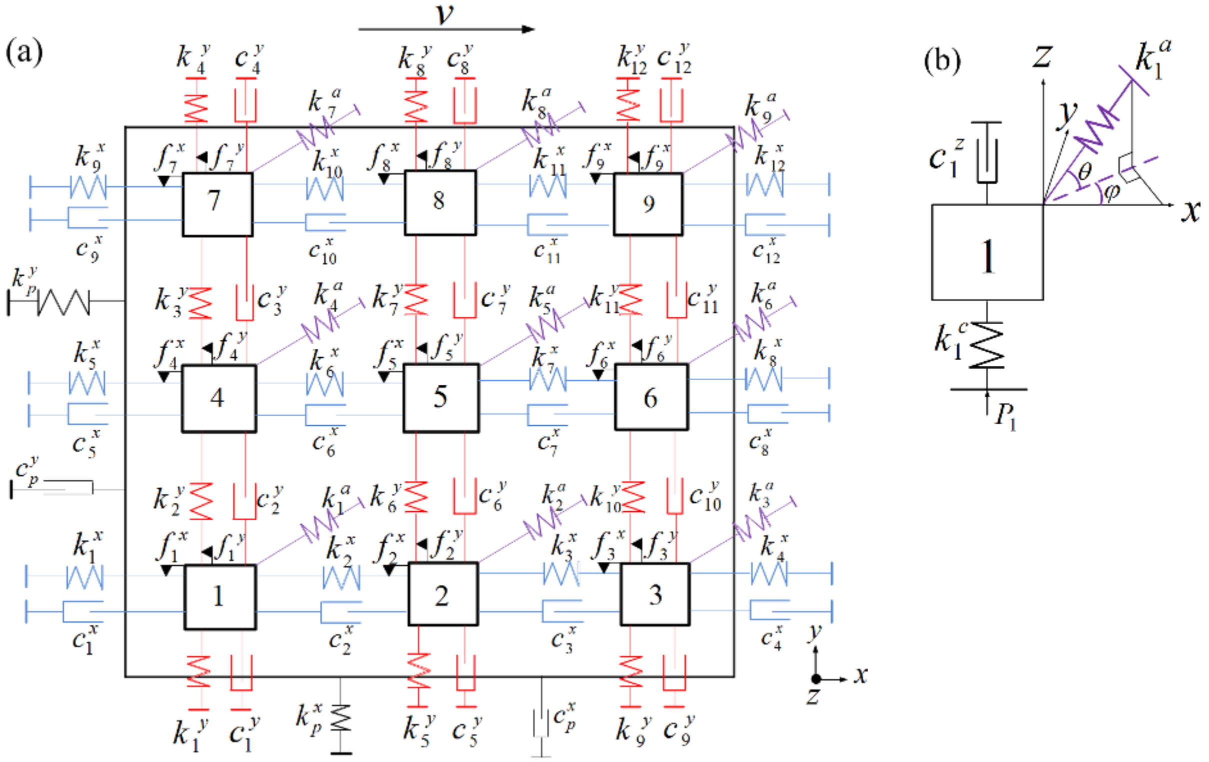

Figure 2 An analytical model of 3×3 oscillators coupled with a sliding plate via the friction force.

Table 1 Stiffness of springs and damping coefficients of dashpots for the 3×3 coupled oscillator model.

Parameter	$k^x_{1,4,5,8,9,12}$	$k^x_{2,3,6,7,10,11}$	$k^y_{1,4,5,8,9,12}$	$k^y_{2,3,6,7,10,11}$	k^z_{1-9}
Value	2.45×10^7 N/m	1.05×10^7 N/m	3.05×10^7 N/m	3.01×10^7 N/m	3.47×10^7 N/m
Parameter	k^a_{1-9}	k^x_p	k^y_p	k^z_p	c^x_{1-12}
Value	3.10×10^7 N/m	4.87×10^8 N/m	4.87×10^8 N/m	4.25×10^8 N/m	1.81×10^2 Ns/m
Parameter	c^y_{1-12}	c^z_{1-12}	c^x_p	c^y_p	c^z_p
Value	1.81×10^2 Ns/m	1.81×10^2 Ns/m	1.55×10^3 Ns/m	1.55×10^3 Ns/m	1.55×10^3 Ns/m

The mass of the plate was set to be identical to that of a realistic brake rotor (9.05 kg) while the mass of each oscillator was set to be 1/9 of the mass of a realistic pad (0.39 kg). By adopting the parametric values of the coupled oscillators model listed in Table 1, the lowest/highest natural frequencies of the oscillators' and plate's modes are 1,772/19,700 Hz and 1,063/19,926 Hz respectively, which are within 1% of the modal testing results reported in [18].

3. Method of Analysis

The combined effects of uncertainties in the contact conditions and the friction modelling on instability predictions is analysed by conducting Monte Carlo simulations using the CEA. The simulations follow a hierarchical structure as described in Fig.3 which shows the choice of oscillators not in contact with the rigid sliding plate by setting the stiffness of the contact springs connecting the corresponding oscillators with the plate to zero. If there are six oscillators in contact, the number of different arrangements is C_6^9=84. For each arrangement, the CEA is conducted 1,000 times with random friction model parameters drawn from constructed statistical distributions described in [13].

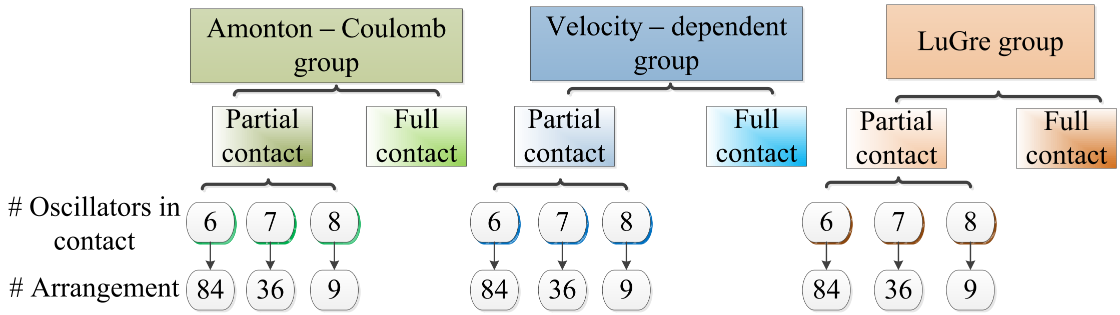

Figure 3 The hierarchy of simulations.

4. Results and Discussions

4.1 Occurrence of instability

Figures 4 to 6 show the occurrence of instabilities for different friction models and different numbers of oscillators in contact with the rigid sliding plate and two different sliding speed ranges. The CEA results indicate that the occurrence of instabilities is proportional to the number of oscillators in contact. The number of unstable modes of the velocity-dependent model and the LuGre model is greater than that of the Amonton-Coulomb model because additional negative damping mechanisms are introduced [16]. Also, when the plate's sliding speed increases, the occurrence of instabilities decreases.

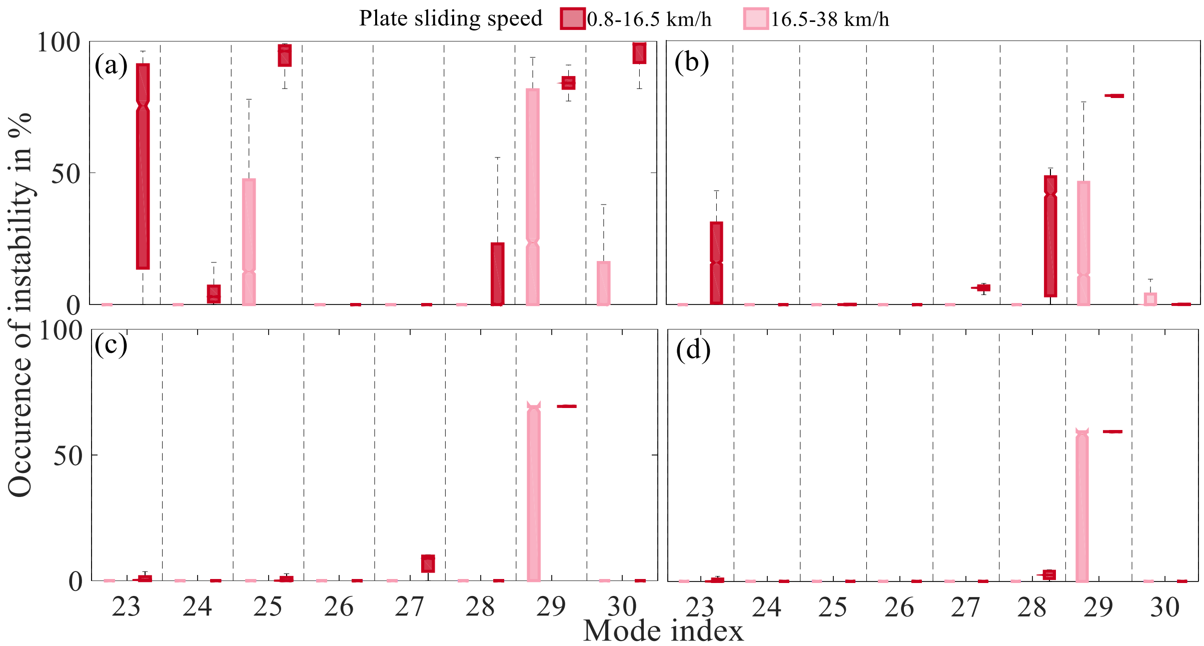

Figure 4 Boxplot of frequency of occurrence of each unstable mode for Amonton-Coulomb friction model (a) nine, (b) eight, (c) seven, (d) six oscillators in contact.

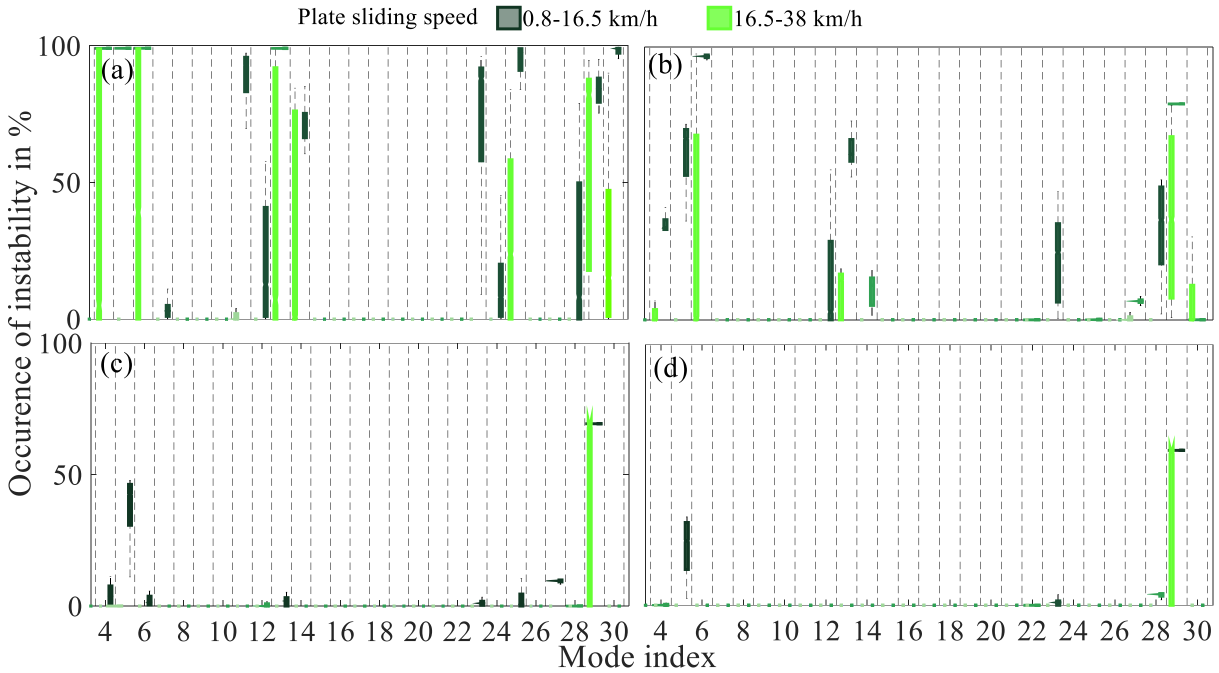

Figure 5 Boxplot of frequency of occurrence of each unstable mode for velocity-dependent friction model (a) nine, (b) eight, (c) seven, (d) six oscillators in contact.

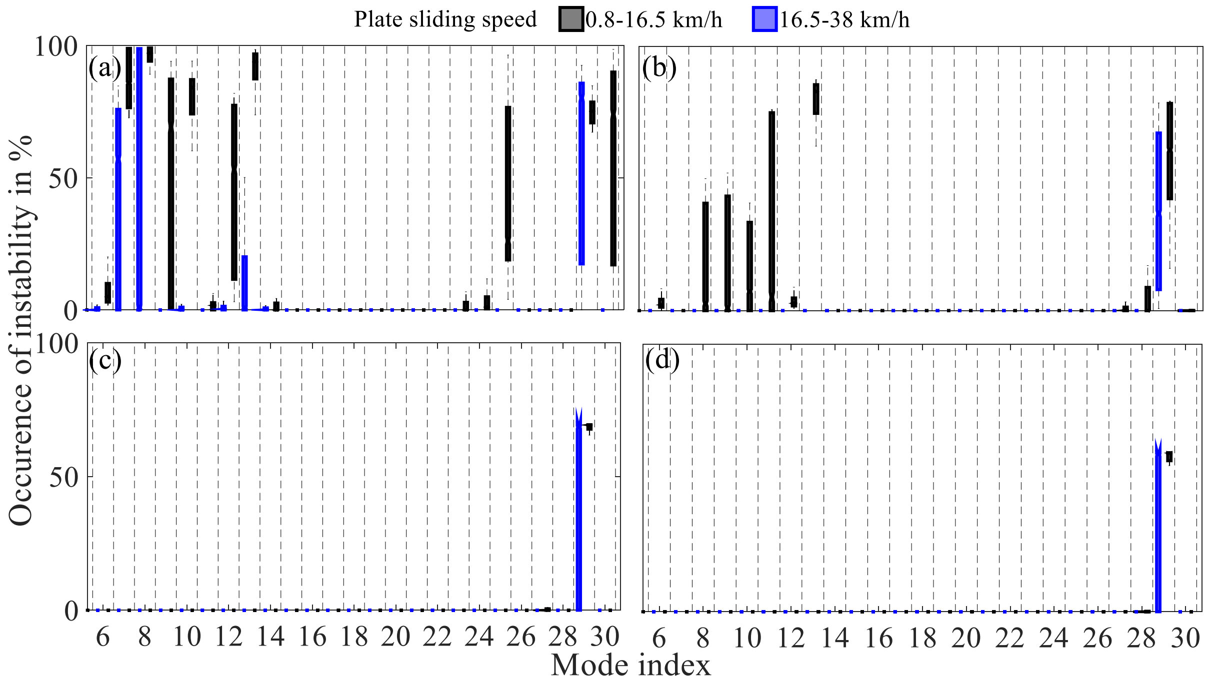

Figure 6 Boxplot of frequency of occurrence of each unstable mode for LuGre friction model (a) nine, (b) eight, (c) seven, (d) six oscillators in contact.

Independent of contact conditions and friction models, the relative frequency of occurrence of having an instability generated for mode 29 is consistently high and is termed here the *most robustly unstable mode*. The occurrence of instability of other modes is sensitive to either the contact condition or friction modelling. Therefore, the characteristics of mode 29 are further investigated by considering the contribution of each oscillator to this instability for countermeasure design.

4.2 Characteristics of the most robustly unstable mode

The mode shape of mode 29 for nine oscillators in contact obtained with the median values of parameters for different friction models in Fig. 7 shows that the displacement of the four corner oscillators (1st, 3rd, 7th and 9th , cf. Fig. 2) in the *x*-direction is dominant. The mode shape vector for each oscillator is given in the brackets in Fig. 7(a),(b) and (c) with the three values corresponding to the displacement in the *x*-, *y*-, and *z*- directions respectively. The median of the net work (the sum of the work done by friction and damping) of each oscillator for mode 29 is normalised by the largest median net work for the oscillator. Results confirm that the four corner oscillators make the largest contribution to its instability, independent of the friction model used (see Fig. 7(d), (e), (f)).

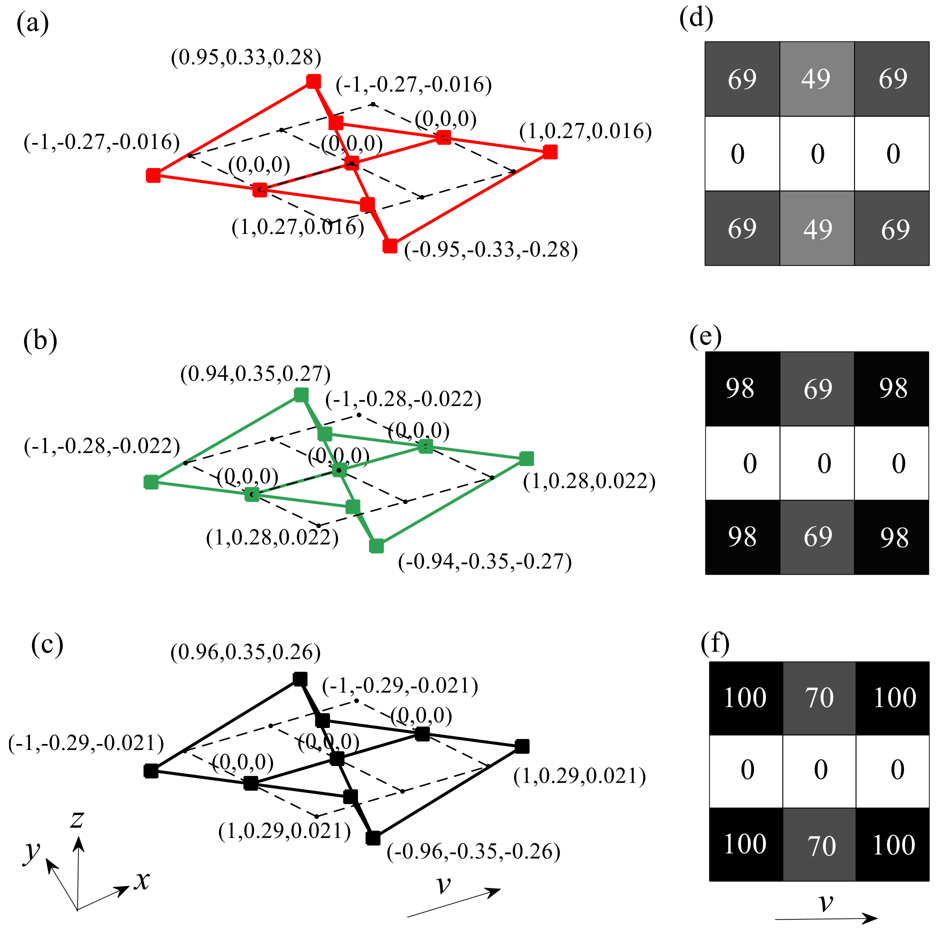

Figure 7 Characteristics of mode 29 with 9 oscillators in contact: mode shape and normalised median of net work, (a), (d) Amonton –Coulomb; (b),(e) Velocity-dependent; (c),(f) LuGre friction model

4.3 Countermeasure design to suppress instability

The characteristics of mode 29 indicate that constraining the movement of the four corner oscillators in the *x*-direction could suppress this instability. This constraint can be implemented by eliminating the corresponding elements of the original matrix equations. Monte Carlo simulations of CEA and net work calculations were run with the newly formed matrix equations. Figure 8 shows that, compared to the results of the original system in Figs. 4 to 6, the countermeasure is able to reduce the likelihood of the occurrence of instabilities. Only mode 26 has a high probability to be unstable. This mode is not the mode 26 of the original system, because the system has been altered by the countermeasure. The mode shape and the normalized median of net work of mode 26 (not shown here) indicates that the 4th and 6th oscillator are dominant in contributing to the instability. Thus constraining the 4 corner oscillators plus the 4th and 6th oscillator might further reduce the occurrence of the instability. Figure 9 confirms that (1) for the six oscillators in contact, there is no instability irrespective of the plate's sliding speed or the the friction model; and (2) mode 18 is unstable in other conditions and for all friction models but the highest median occurrence of instability is only 34.8% for the LuGre friction model.

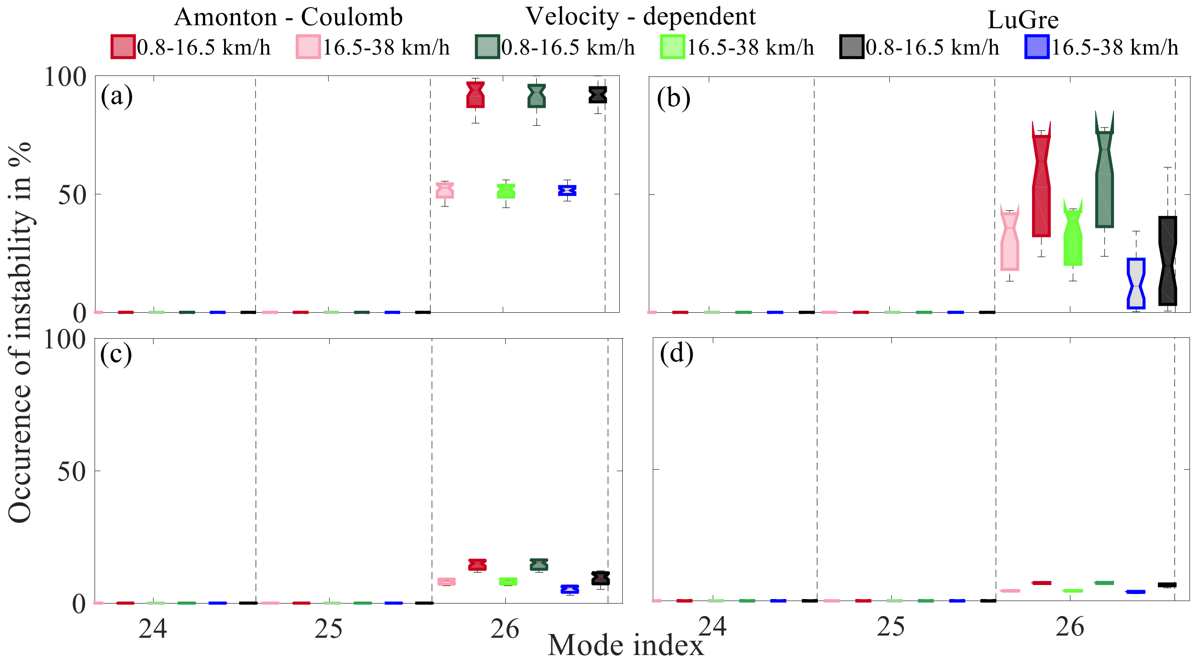

Figure 8 Boxplot of frequency of occurrence of each unstable mode for constraining the 4 corner oscillators in the x-direction (a) nine, (b) eight, (c) seven, (d) six oscillators in contact.

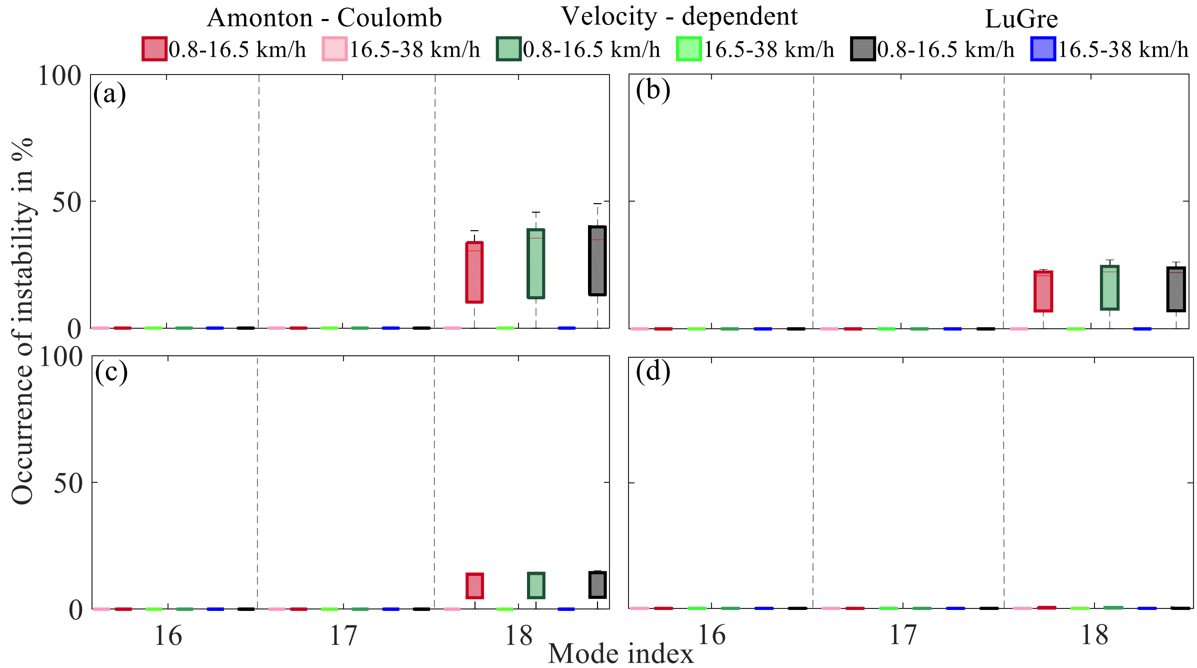

Figure 9 Boxplot of occurrence of instability of each mode being unstable for constraining the 4 corner oscillators and 4th and 6th oscillators in the *x*-direction (a) nine, (b) eight, (c) seven, (d) six oscillators in contact

5. Conclusions

In this study, the influence of the uncertainty in friction modelling and contact conditions on friction-induced self-excited vibration of a 3×3 coupled friction oscillators model has been investigated. One mode has been identified by the uncertainty analysis of the complex eigenvalue analysis to be robustly unstable, independent of the uncertainties in contact condition and friction models. Both the mode shape and the distribution of the net work indicate that the oscillators at the corners are dominant in providing energy to sustain self-excitations. By constraining these corner oscillators, the occurrence of instability is reduced. However, a new unstable mode is produced with a median occurrence of instability of 92% for the full contact condition. For this new mode, the dominant oscillators are the 4th and 6th oscillator and constraining them further reduces the instability occurrence to less than 40% for all contact conditions.

Results here illustrate that by incorporating uncertainty methods into the analysis of friction-induced instabilities, unstable modes with consistently high occurrence of instability independent of contact area, friction modelling and sliding speed could be identified. Such unstable modes are considered to be robustly unstable and are most likely to produce squeal. Hence, instability countermeasures could be designed by repeating the uncertainty analysis for these robustly unstable modes. For other modes with the occurrence of instability sensitive to contact conditions and the choice of friction models, confidence cannot be established to determine whether they are likely to squeal or not. This is no different from a deterministic analysis where it is virtually impossible to establish any confidence in identifying which predicted unstable vibration modes will produce squeal.

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