Progress of Theoretical Physics, Vol. 60, No. 3, September 1978

Instability of a Gaseous Envelope Surrounding a Planetary Core and Formation of Giant Planets

Hiroshi MIZUNO, Kiyoshi NAKAZAWA and Chushiro HAYASHI

Department of Physics, Kyoto University, Kyoto 606

(Received March 17, 1978)

We consider the evolution of the solar nebula in stages where, as studied previously by Hayashi, Nakazawa and Adachi, protoplanets composed of involatile materials are growing gradually through the capture of planetesimals. When a protoplanet becomes greater than the moon's mass, an appreciable amount of the gas of the solar nebula is attracted by the protoplanet to form a gaseous envelope surrounding it.

We have studied the structure and stability of this envelope, which depend on the mass of the protoplanet, on the assumption that the envelope is spherically-symmetric, in hydrostatic equilibrium and, thermally, isothermal in the outer optically-thin region but adiabatic in the inner region. The existence of the isothermal region is due to a circumstance that the opacity of the gas is very low since almost all of grains have already condensed into planetesimals and protoplanets.

We have found that, when the mass of a protoplanet becomes greater than a certain critical value which depends on the opacity, the envelope can no longer be in hydrostatic equilibrium and begins to collapse. For a roughly estimated value of the opacity, the critical mass is of the order of $15M_E$ and $6M_E$ (M_E being the Earth's mass) for proto-Jupiter and proto-Saturn, respectively. These masses are about one fifth and one tenth of the values found by Perri and Cameron, but they are consistent with recent Slattery's models of the present Jupiter and Saturn.

§ 1. Introduction

According to recent developments of theories on the origin of the solar system, we consider that the planets of the Earth's type as well as the cores of the giant planets were formed through the following processes. At first, grains contained in the disk-like primordial solar nebula sediment to the ecliptic plane, as they agglomerate, and a thin dust layer is formed.[1] When the density of the dust layer becomes great enough, the layer becomes gravitationally unstable and fragments into an enormous number of planetesimals with masses of the order of 10^{18}g.[2),3)] These planetesimals are composed mainly of involatile materials. Through direct collisions between the planetesimals lasting for a period of the order of 10^5 years, only a small number of the planetesimals grow to protoplanet with mass of about 10^{25}g while the mean mass of all the remaining planetesimals is increased to the order of 10^{21}g. Subsequently, a protoplanet grows through the capture of the survived planetesimals into its Hill sphere. According to Hayashi, Nakazawa and Adachi,[4] protoplanets existing in the regions of the present Earth and Jupiter grow from 10^{25}g to $1\ M_E$ ($=6\times10^{27}$g; the Earth's mass) in $10^{6\sim7}$ years and to $7\ M_E$ in $10^{7\sim8}$ years, respectively.

Now, the giant planets contain a large amount of hydrogen and helium[5] and it is important to know at what stage and in what process the above-mentioned protoplanets could acquire such gaseous materials. Perri and Cameron[6] investigated this problem for the first time by considering that a part of the gas of the solar nebula is attracted by a protoplanet and forms a gaseous envelope surrounding it. Henceforth the protoplanet will be called the core for simplicity. According to their result for Jupiter, when the core mass becomes greater than about $70\ M_E$ the envelope can no longer be in hydrostatic equilibrium and collapses onto the core.

Perri and Cameron assumed that the envelope is wholly adiabatic, but this assumption is not valid, at least, for the outer layers of the envelope for the following reason. At stages under consideration, the opacity of the solar nebula is very low since almost all of grains, which are the main sources of the opacity, have already sedimented and condensed into protoplanets and planetesimals. According to our estimate as described in § 4, the opacity is of the order of 3×10^{-4}cm^2/g. Hence, the outer layer of the envelope with a considerable thickness is transparent to radiation and is approximately isothermal instead of being adiabatic.

In the present paper, we will investigate the structure and stability of such an envelope as composed of two layers, where the outer optically-thin layer is assumed to be isothermal while the inner layer is assumed to be adiabatic, and will find the value of the core mass at a critical stage where the envelope begins to collapse. In § 2, we will describe assumptions and basic equations for the envelope as well as the sources of opacity of the gas. In § 3, we will summarize numarical results which reveal that the existance of the isothermal layer reduces the critical core mass considerably. In § 4, by means of these results the process of formation of the giant planets will be discussed.

§ 2. Formalism

Let us consider that in the solar nebula a protoplanet or the so-called core, which is composed of metallic, rocky and ice constituents, is growing very gradually with a time-scale of 10^6 or 10^7 years.[4] The core attracts the neighboring gas of the solar nebula to form an envelope surrounding it. With the growth of the core, both the density and the mass of the envelope increase and finally the self-gravity of the envelope becomes important. Now, we study the structure of this envelope to find a relation between the core mass and the envelope mass.

a) *Hydrostatic equations*

We assume that, in regions fairly distant from the Hill sphere of a protoplanet considered, the gas of the solar nebula is in circular motion around the Sun with the uniform density ρ_0. Our problem is to find the distribution of the gas density in regions inside the Hill sphere, where the gravity of the planet is dominant. Recently, in the same frame as that of the restricted three-body problem, i.e., in a case where the self-gravity of the gas is negligible, Nakazawa and Hayashi[7] made hydrodynamical calculations of a stationary gas flow, which satisfies the above-mentioned boundary conditions, for a case of a polytropic gas and a non-rotating planet with a Keplerian orbit of zero eccentricity and zero inclination. Their results indicate that, as far as the inner region of the Hill sphere (where the distance from the planet is smaller than one half of the Hill radius) is concerned, the gas velocity relative to the planet is much smaller than the escape velocity and, consequently, the density distribution can be approximated by that of a gas which fills the Hill sphere hydrostatically with a boundary condition such that the density is ρ_0 at the surface of this sphere. Furthermore, it is to be noticed that this static approximation holds also for a case of a growing planet since a time-scale for the sound wave to travel through the Hill sphere is extremely smaller than the growth time of the planet itself.

On the basis of the above results, we assume in the following that the envelope has spherical symmetry about the center of the core and is in hydrostatic equilibrium, i.e.,

$$\frac{1}{\rho}\frac{dP}{dr}=-\frac{GM_r}{r^2} \tag{1}$$

and

$$\frac{dM_r}{dr}=4\pi r^2\rho\,, \tag{2}$$

where P and ρ are the gas pressure and the density, respectively, and M_r is the mass contained inside a sphere of radius r. The boundary conditions to be imposed on Eqs. (1) and (2) are given by

$$\rho=\rho_0\,,\ T=T_0 \quad \text{and} \quad M_r=M_t \quad \text{at} \ r=h\,, \tag{3}$$

where ρ_0 and T_0 are the density and the temperature, respectively, of the solar nebula in regions sufficiently distant from the Hill sphere, $M_t\,(=M_c+M_e)$ is the sum of the core mass M_c and the envelope mass M_e, and h is the radius of the Hill sphere*) given by

$$h=a\left(\frac{M_t}{3M_\odot}\right)^{1/3}, \tag{4}$$

where a is the distance from the Sun to the planet. It is to be noticed that, according to Eq. (3), the mean density of matter (including both the core and the gas) in the Hill sphere is equal to $\rho_R/4.93$ where $\rho_R\,(=3.53\,M_\odot/a^3)$ is the Roche density which depends only on the distance a. Perri and Cameron[6] employed the

*) Here the radius of the Hill sphere is defined as the distance between the planet and the inner Lagrangian point.

same equations as (1) and (2) but adopted different boundary conditions. Instead of $r=h$ in Eq. (3), they put $r=a(M_t/M_\odot)^{1/2}$, i.e., they adopted a point where the gravity of the planet is equal to that of the Sun. However, the resultant differences will not be large since both the density and the temperature change only slightly in a region between $r=h$ and $r=a(M_t/M_\odot)^{1/2}$.

Strictly speaking, a static gas filling the Hill sphere is not spherically symmetric but instead of Eq. (1) we may have an equation of the form

$$\frac{1}{\rho}\nabla P=-\nabla\left(\phi-\frac{3}{2}\Omega^2x^2+\frac{1}{2}\Omega^2z^2\right), \tag{5}$$

where ϕ is the gravitational potential due to the core and the envelope, $\Omega=(GM_\odot/a^3)^{1/2}$ is the Keplerian angular velocity of the planet and x and z are the coordinates, in a rotating frame where the planet is at rest at the origin, in a direction from the Sun to the planet and in a direction perpendicular to the ecliptic plane, respectively. Errors introduced by the use of Eq. (1) instead of Eq. (5) is not significant in our problem, since the last two potential terms on the right-hand side of Eq. (5) are important only in regions relatively near the surface of the Hill sphere where the variations of density and temperature are both small.

b) *Equation of state*

The equation of state for the gas is approximated by that of an ideal gas which is composed of hydrogen and helium having the solar abundances by mass, i.e., 73 and 27 percent, respectively. The dissociation of hydrogen molecules and the ionization of hydrogen atoms are taken into account in accordance with Hayashi and Nakano.[8] At relatively low temperatures and high densities where the pressure ionization occurs, there is an appreciable departure from an ideal gas. However, this departure has been neglected in the present calculations since, as will be described later in § 3, the mass of the gas contained in such high density regions is very small as compared to the total mass of the envelope.

c) *Temperature distribution*

The envelope is assumed to be isothermal and adiabatic in the regions $r>r_1$ and $r<r_1$, respectively, where r_1 is the radius of a spherical surface where the optical depth measured inwards from the Hill surface is 2/3, i.e.,

$$\int_{r_1}^{h}\kappa\rho dr=\frac{2}{3}, \tag{6}$$

where κ is the opacity of the gas. This assumption is most different from that of Perri and Cameron[6] as mentioned in § 1. It is to be noticed for a solar nebula model of Kusaka, Nakano and Hayashi[1] that, if κ is smaller than 10^{-3} cm^2/g, the nebula itself in Jupiter's region is optically thin if viewed in a direction perpendicular to the ecliptic plane and, consequently, radiation escapes almost freely from the surface at $r=r_1$ into the interstellar space.

d) *Opacity*

The boundary between the isothermal and adiabatic regions is determined by the opacity according to Eq. (6). The temperature of the solar nebula in the regions of Jupiter and Saturn is about 100 K. The opacity of the gas at such a low temperature is given by

$$\kappa=\kappa_g+\kappa_i\,, \tag{7}$$

where κ_g is the opacity due to grains and κ_i is the one due to pressure-induced absorption by hydrogen molecules and helium atoms.

According to Goustad,[9] κ_g is proportional to the total mass of grains contained in a unit volume of the gas, if the sizes of all grains are smaller than the wave length of radiation. Now, at stages under consideration almost all of the grains, which were contained originally in the solar nebula, have already sedimented and condensed into protoplanets and planetesimals. Let the reduction factor for the total mass of grains floating in the nebula be denoted by f. Then κ_g is approximately given by[9]

$$\kappa_g=1.0\times f\,\mathrm{cm^2/g}\,. \tag{8}$$

Since the value of f is not precisely known, we have regarded κ_g as a free parameter in our calculations.

On the other hand, according to Trafton[10),11] the opacity κ_i at low temperatures considered is nearly given by

$$\kappa_i\simeq1.0\times10^2\,\rho\,\mathrm{cm^2/g}\,, \tag{9}$$

where ρ is the gas density in unit of $\mathrm{g/cm^3}$. The results of our calculations, as will be described later in § 3, indicate that the radius r_1 defined by Eq. (6) is determined by κ_g or κ_i, respectively, according to whether κ_g is greater or smaller than $3\times10^{-5}\,\mathrm{cm^2/g}$.

e) *Procedure of integrations and core density*

The procedure of numerical calculations to find the envelope structure is as follows. For given values of M_t, ρ_0 and T_0, Eqs. (1) and (2) are integrated inwards from the outer boundary, $r=h$. Then, if the mean density of the core is given, the inner boundary of the envelope as well as the core mass itself is found from the continuity of M_r at this boundary.

We have assumed for simplicity that the core is a rigid sphere with a density of $5.5\,\mathrm{g/cm^3}$, i.e., the mean density of the Earth. According to Slatterly,[12] the present Jupiter has a core with mean density lying in the range between 13 and $22\,\mathrm{g/cm^3}$. Accordingly, we have recalculated a part of our models for the core density which ranges from 3.0 to 100 g/cm^3 but we have found that the resultant change of the envelope mass is less than only four percent.

f) *Stability*

We have examined the dynamical stability of the envelope against small adiabatic perturbations in the following way. First, we have checked that perturbations can be treated as adiabatic even for the outer isothermal layer since the dynamical time-scale, τ_d, for a sound wave to travel through this layer is smaller than the cooling time-scale, τ_c. For typical models calculated by us, we have $\tau_c \simeq 10^2$ years and $\tau_d \simeq 1$ year.

Now, let $\delta r = r\xi(r)\exp(i\omega t)$ be an infinitesimal displacement of matter from the equilibrium position r. Then, the equation for $\xi(r)$ is written in the form[13)]

$$\frac{1}{\rho r^4}\frac{d}{dr}\left(\Gamma P r^4 \frac{d\xi}{dr}\right) + \frac{1}{\rho r}\frac{d}{dr}[(3\Gamma - 4)P]\cdot\xi + \omega^2\xi = 0\,, \tag{10}$$

where $\Gamma = (\partial \ln P/\partial \ln \rho)_s$. The boundary conditions to be imposed on Eq. (10) are given by

$$\xi = 0 \text{ at } r = r_c \quad \text{and} \quad \delta P = 0 \text{ at } r = h\,, \tag{11}$$

where r_c is the radius of the rigid core.

Mathematically, Eq. (10) is an eigen-value equation of Strum-Liouville type. In practice, we are interested only in the stability of the envelope, i.e., in the sign of ω^2 but not in the value itself. Hence, on the basis of the well-known argument on the eigen-value problem,[14)] we take the following procedure. First, putting $\omega^2 = 0$, we integrate Eq. (10) inwards from $r = h$ and find a solution which satisfies only one boundary condition at $r = h$ but does not necessarily satisfy the other boundary condition at $r = r_c$. Next, we count the number of nodes contained in this solution. If the number of nodes is zero, the sign of ω^2 for the most dangerous mode is positive and the envelope is stable. On the other hand, if the number is one or greater, then ω^2 is negative and the envelope is unstable.

§ 3. Numerical result

Computations have been made for the regions of Jupiter, Saturn and the Earth. The density and temperature of the solar nebula in these regions, i.e., the values of ρ_0 and T_0 given in Eq. (3) have been taken from the model of Kusaka, Nakano and Hayashi.[1)] These values are listed in Table I. In view of uncertainties involved in this model, computations have been made also for the cases where the densities in the regions of Jupiter and Saturn are both lowered by a factor of 10. Computations have been made for several values of the grain's opacity κ_g which are smaller than 0.1 cm^2/g and also, in order to compare with the results of Perri and Cameron,[6)] for a wholly adiabatic case which corresponds to $\kappa_g = \infty$.

Table I. Physical quantities of the solar nebula in the regions of the Earth, Jupiter and Saturn, which are adopted from the model of Kusaka et al.

Region	Distance from the Sun a(a.u.)	Half-thickness of the nebula z(a.u.)	Density $\rho_0(g/cm^3)$	Temperature T_0(K)
Earth	1.0	0.042	5.7×10^{-9}	225
Jupiter	5.2	0.33	1.5×10^{-10}	97
Saturn	9.5	0.70	6.2×10^{-12}	73

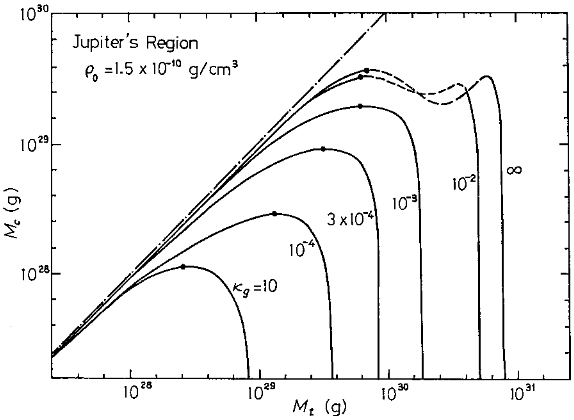

Fig. 1. The relation between the total mass and the core mass in Jupiter's region with $\rho_0=1.5\times10^{-10}$g/cm^3. The numbers attached to the curves denote the grain's opacity. The solid and dashed curves denote the stable and unstable models, respectively, and the dots denote the critical models.

The total mass-core mass relation (the M_t-M_c curve) for Jupiter's region with $\rho_0=1.5\times10^{-10}$ g/cm^3 is shown in Fig. 1, where the solid and dashed curves denote stable and unstable models, respectively. When M_t is small, the envelope mass is small as compared to M_c and the self-gravity of the envelope is not important. With the increase of M_t, the M_t-M_c curve deviates more and more from the line, $M_c=M_t$, and soon M_c takes a maximum value. With the further increase of M_t, M_c decreases rapidly except for the cases $\kappa_g=\infty$ and 1×10^{-2} cm^2/g, where M_c takes a second maximum before decreasing rapidly. Hereafter, stable models with maximum values of M_c will be called the critical models and the values of M_c and M_t for these models will be denoted by M_c^* and M_t^*, respectively.

The critical core mass M_c^* increases with the grain's opacity κ_g (see also Fig. 4) and this tendency will be understood as follows. Let us compare models with the same value of M_t but corresponding to different values of κ_g. Generally, most of the envelope mass is contained in the adiabatic region. If the opacity is smaller, the isothermal layer extends deeper into the interior and the entropy of the gas in the adiabatic region is smaller. Consequently, the density and also the mass of the gas contained in the adiabatic region are greater and, correspondingly, the core mass is smaller.

The density-temperature curves for three typical models are shown in Fig. 2, where the model of $\log M_t=30.0$ and $\kappa_g=1\times10^{-2}$ cm^2/g is an example of unstable models, that of $\log M_t=29.5$ and $\kappa_g=3\times10^{-4}$ cm^2/g is a critical model and that of

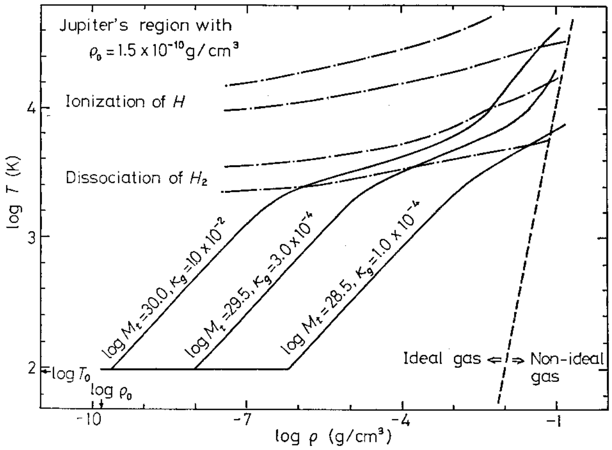

Fig. 2. The density-temperature diagrams for three typical models of the envelope in Jupiter's region with $\rho_0=1.5 \times 10^{-10}$ g/cm^3. The dashed line represents the boundary between the ideal and the non-ideal gas.

log $M_t=28.5$ and $\kappa_g=1\times10^{-4}$ cm^2/g represents a case where the self-gravity of the envelope is not important. The ρ-T curves for the three models start from the same point, $\log \rho_0 = -9.82$ and $\log T_0 = 1.99$, but run into the three different adiabats. At temperatures above 1000 K, the gradient of the adiabats becomes small because of the dissociation of hydrogen molecules.

Now, let us consider the innermost region of the envelope, where the density is as high as 0.1 g/cm^3. In this region, the approximation of an ideal gas is fairly good for the former two models since the temperature is so high that hydrogen molecules dissociate almost completely and hydrogen atoms ionize partially. This is not the case for the last model, but the mass contained in this high-density region is so small that any significant errors will not have been introduced by the approximation of an ideal gas.

The models for $\kappa_g=\infty$ and 1×10^{-2} cm^2/g, which are denoted by the dashed curves in Fig. 1, are all unstable. This instability is due to the fact that, in regions where a significant fraction of the envelope mass is contained, Γ is small because of the dissociation of hydrogen molecules. On the other hand, all the other models in Fig. 1 are stable because the entropy of the envelope is so small that Γ is small only in a region with a relatively small mass.

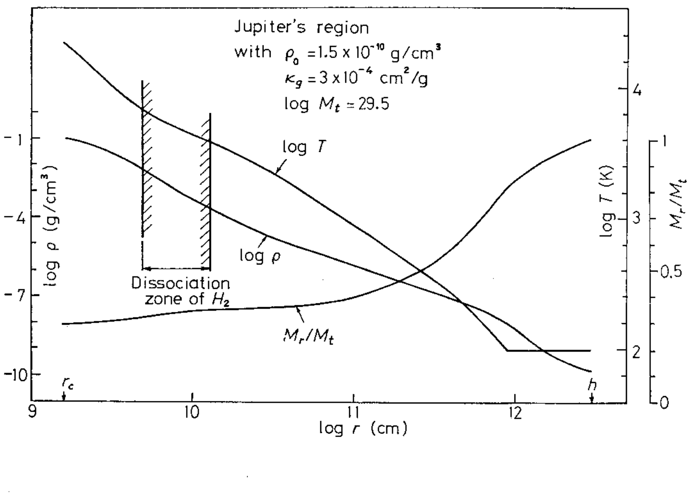

Fig. 3. The distributions of the density, the temperature and the mass fraction as functions of the radius in the model of $\log M_t=29.5$ and $\kappa_g=3\times10^{-4}$ cm^2/g as shown in Fig. 2.

In Fig. 3 the distribution of ρ, T and M_r are plotted as functions of the radius for the critical model of $\log M_t = 29.5$ and $\kappa_g = 3\times 10^{-4}\,\mathrm{cm^2/g}$. It is to be noticed that the core radius is as small as 10^{-3} times the Hill radius and most of the envelope mass is contained in the adiabatic region. The inner boundary of the isothermal region has a radius of about one third of the Hill radius. This radius is, of course, strongly dependent upon the values of M_t and κ_g; it is in the range between 1 and 10^{-2} times the Hill radius for all the models calculated.

The M_t-M_c curves for the regions of the Earth and Saturn, including the cases where the density ρ_0 is lowered by a factor of 10, are all very similar to the curves shown in Fig. 1. In Fig. 4, the critical core mass M_c^* is plotted as a function of the grain's opacity κ_g for the regions of the Earth, Jupiter and Saturn. The values of M_c^* for $\kappa_g = 0$ and ∞ differ only very slightly from these for $\kappa_g = 1\times 10^{-5}$ and 1×10^{-1} $\mathrm{cm^2/g}$, respectively.*) In the wholly adiabatic case (i.e., the case $\kappa_g = \infty$), M_c^* is 60 and 100 M_E in the regions of Jupiter and Saturn, respectively. This value of M_c^* for Jupiter's region differs only slightly from the value, 70 M_E, found by Perri and Cameron.[6] The small difference may be due to the difference in the boundary conditions as mentioned in § 2.

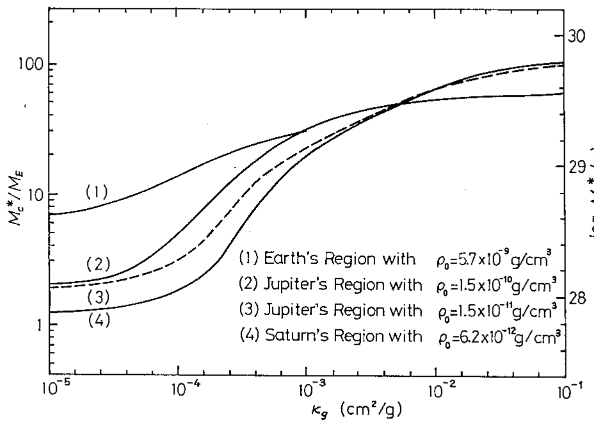

Fig. 4. The critical core mass as a function of the grain's opacity.

§ 4. Formation of giant planets

As studied by Hayashi et al.,[4] the core mass grows gradually through the capture of planetesimals. When the core mass becomes greater than the critical value as shown in Fig. 4, the envelope can no longer be in hydrostatic equilibrium and the envelope begins to contract due to the lack of the pressure force compared with gravity. Since this leads to the inversion of the pressure gradient near the Hill radius, the further evolution of the envelope, generally, is not quasi-static but dynamical and is to be solved in the future. However, it is certain that the envelope collapses onto the core to form a tightly bound envelope which is, more or less, isolated as a whole from the gas of the solar nebula although the time scale of the collapse is not known precisely at present.

*) For $\kappa_g \lesssim 3\times 10^{-5}\,\mathrm{cm^2/g}$, the radius r_1 defined by Eq. (6) is determined by κ_i rather than by κ_g.

In the case of the Earth, such a collapse never occured since the growth of the core mass was stopped before reaching the critical value as shown in Fig. 4. We consider that at later stages the gas of the solar nebula as a whole escaped gradually from the solar system and, at the same time, the envelope surrounding the Earth disappeared. After this stage, the present atmosphere was formed out of the gases contained originally in the core.

Now, we consider the cases of Jupiter and Saturn where the circumstances are quite different from that of the Earth as mentioned above. Generally, the critical core mass depends on the grain's opacity or on the amount of grains which are floating in the solar nebula at stages under consideration. In order to know a precise value of this opacity, we have to calculate the space and time variations of the mass spectrum of the floating grains, which are due to the growth of grains by thermal collision, to the sedimentation to the ecliptic plane of the disk, to the mixing of grains caused by turbulent gas flows and so on. At present, however, we are not in a position to be able to solve this difficult problem.

Accordingly, we will make a very rough estimate of the amount of the floating grains by using both of the results of Kusaka et al.[1] and those of Hayashi and Nakagawa.[15] Kusaka et al. studied both the growth and the sedimentation of grains but in the approximation that at any time all the grains have the same mass, while Hayashi and Nakagawa calculated the time variation of the mass spectrum of grains growing without sedimentation and found the shape of the final spectrum.

Let t be the time measured from the formation of the disk of the solar nebula. According to Kusaka et al., the sedimentation begins to be effective at a time, $t_1=2\times10^5$ years, in Jupiter's region. We assume that the spectrum of the grains at this time has the final shape as found by Hayashi and Nakagawa. On the other hand, the sedimentation time of each grain is known as a function of mass (see Kusaka et al.[1]). Then, we assume that, among the grains floating at $t=t_1$, large grains whose sedimentation time is smaller than 1.0×10^6 years (i.e., whose radius is greater than 8×10^{-4} cm) have already sedimented and the remaining smaller grains are still floating at a stage under consideration. In this way, we have found that the total mass of the floating grains has been reduced to a fraction of about 3×10^{-4} and, from Eq. (8), κ_g is about $3\times10^{-4}\,\text{cm}^2/\text{g}$. This value is, of course, very rough since we have not exactly calculated the time variation of the mass spectrum and also there is an uncertainty in the above adoption of the time, 1×10^6 years.

In the following we adopt the value, $\kappa_g=3\times10^{-4}\,\text{cm}^2/\text{g}$. The values of M_c^* and M_t^* for this value of κ_g are listed in Table II and the structure of the corresponding model for Jupiter's region has already been shown in Figs. 2 and 3. It is to be noticed that the values of M_c^* are about one fifth and one tenth of Perri and Cameron's values in the regions of Jupiter and Saturn, respectively. Furthermore, the two values of M_t^* given in Table II are considerably smaller than the present masses of Jupiter and Saturn, respectively. Accordingly, we have

Table II. The critical core mass $M_c{}^*$ and total mass $M_t{}^*$ for the grain's opacity $3\times10^{-4}\text{cm}^2/g$. The masses of the Earth, Jupiter and Saturn are denoted by M_E, M_J and M_S, respectively.

	Jupiter's region with $\rho_0=1.5\times10^{-10}\text{g/cm}^3$	Saturn's region with $\rho_0=6.2\times10^{-12}\text{g/cm}^3$
Critical core mass $M_c{}^*$	$15M_E$	$5.5M_E$
Critical total mass $M_t{}^*$	$53M_E$	$27M_E$
$M_c{}^*/M_J$	0.046	—
$M_c{}^*/M_S$	—	0.055

to consider that, after the envelope begins to collapse onto the core, the gas of the solar nebula existing in some regions outside the Hill sphere is falling continuously into the Hill sphere and accretes onto the protoplanet. This growing process will continue until the gas, which exists presumably in a certain ring-like region around the Sun, is all exhausted to form a present giant planet. It is to be noticed that during the growth of the planet the size of the Hill sphere itself is growing.

The present structure of Jupiter and Saturn, which fits to the observations of mass, radius, rotation period and quadrupole moment, have been studied by Podolak and Cameron,[16),17)] Podolak,[18)] Hubbard and Smoluchowski,[5)] Slattery[12)] and others. Since the equation of state, particularly that for hydrogen molecules at high densities is not precisely known at present, the results of these investigations are not definite. Moreover, for Saturn, observational data are not accurate enough to construct a reliable model. Now, according to recent results of Slattery,[12)] both Jupiter and Saturn have cores with mass of about 15 M_E. The core masses given in Table II are not inconsistent with the models of Slattery, in view of the uncertainties involved in his models and in our models, and also in view of a possibility that the core mass itself grows through the capture of planetesimals during the above-mentioned stages where the gas is flowing into the Hill sphere.

Finally, a remark will be made on the temperature distribution in the envelope, which has been assumed to be isothermal or adiabatic in the present paper. The real distribution may not be approximated by such simple distributions. Indeed, the core is growing by the accretion of planetesimals which are moving more or less rapidly in a relatively dense gas of the envelope. During the motion, the kinetic energy of the planetesimals will be dissipated into the heat of the gas. It will be an important future task to improve the envelope model, as has been found in this paper, by taking account of both the above-mentioned energy source and energy transport by radiation in the envelope.

Acknowledgement

The authors wish to thank Dr. T. Tsuji for his helpful advice on the opacity data. The computations were performed by FACOM M190 of the Data Processing Center of Kyoto University.

References

1) T. Kusaka, T. Nakano and C. Hayashi, Prog. Theor. Phys. **44** (1970), 1580.
2) P. Goldreich and W. R. Ward, Astrophys. J. **183** (1973), 1051.
3) V. S. Safronov, *Evolution of the Protoplanetary Cloud and Formation of the Earth and the Planets*, NASA TTF-677 (U.S. Department of Commerce, Springfield, VA 22151, 1969), p. 45.
4) C. Hayashi, K. Nakazawa and I. Adachi, Publ. Astron. Soc. Japan **29** (1977), 163.
5) W. B. Hubbard and R. Smoluchowski, Sp. Sci. Rev. **14** (1973), 599.
6) F. Perri and A. G. W. Cameron, Icarus **22** (1974), 416.
7) K. Nakazawa and C. Hayashi, to be published.
8) C. Hayashi and T. Nakano, Prog. Theor. Phys. **30** (1963), 460.
9) J. E. Gaustad, Astrophys. J. **138** (1963), 1050.
10) L. M. Trafton, Astrophys. J. **146** (1966), 558.
11) L. M. Trafton, Astrophys. J. **147** (1967), 765.
12) W. L. Slattery, Icarus **32** (1977), 58.
13) J. P. Cox and R. T. Giuli, *Principles of Stellar Structure* (Gordon and Breach, Science Publishers, 1968), p. 1029.
14) See, for example, R. Courant and D. Hilbert, *Methods of Mathematical Physics* (Interscience Pub., New York, 1953), Vol. I, p. 451.
15) C. Hayashi and Y. Nakagawa, Prog. Theor. Phys. **54** (1975), 93.
16) M. Podolak and A. G. W. Cameron, Icarus **22** (1974), 123.
17) M. Podolak and A. G. W. Cameron, Icarus **25** (1975), 627.
18) M. Podolak, Icarus **30** (1977), 155.