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INSTABILITY REGIONS OF A PRESTRESSED COMPOUND COLUMN SUBJECTED TO A FOLLOWER FORCE

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The influence of prestressing and bending rigidity distribution between rods of a non-linear column on its natural vibrations and stability is studied. The perturbation method is used for solving the problem. Regions of divergence and flutter instabilities for a column have been determined on the basis of courses of eigencurves as a function of bending rigidity ratio and prestress rate. Discontinuities of the critical force have been observed for certain values of the investigated parameters leading to instability of the column, even for small values of the external load. Although each prestress reduces the critical force, it can be applied to passive vibration control.

Key words: natural vibrations, prestressing, divergence instability, flutter instability, compound column

1. Introduction

Prestressing of elastic structures plays an important role im moving natural vibration frequency far enough from the excitation band. That kind of passive control of vibration was proposed by Holnicki-Szulc and Haftka (1992) in the case of antenna truss structure, modes of which were reshaped to get small amplitudes at the desired points. Kwan and Pellegrino (1993) studied a few problems of location of actuators, their required extension as well as the best actuator adjustments to improve the incorrect prestress rate arising in 3D prestressed structures. Przybylski et al. (1996) demonstrated the influence of prestress, axial force as well as distribution of both the axial and flexural rigidities on the natural frequency of a non-linear two-member frame basing on results of both the numerical and experimental investigations. The obtained results indicated that each prestress reduced the natural frequency of the system. Tomski et al. (1994) found the effect of both the prestressing force and concentrated mass on the natural frequency of a compound beam.

The stability of nonconservative systems has been extensively studied during last three decades. Leipholz (1975) presented the state-of-the-art stability of elastic systems providing classification of stability problems, indicating the necessity to use the dynamic approach to investigation of nonconservative structures. Pedersen (1976) studied the cantilever under a follower force extended to cover a three-parameter case, including a concentrated mass, a linear elastic spring and a partially follower force at the free end. He stated that generally it was necessary to obtain the characteristic lines of instability in the load-frequency co-ordinate system to determine the stability of nonconservative systems. Kounadis (1983) discussed the presence of regions of divergence instability for an elastically restrained column under a follower compressive force applied at its end. He found a discontinuity (a jump) in the critical load, which as he stated, could be evaluated only by using the dynamic stability criterion. Bogacz and Janiszewski (1986) presented a comprehensive review of the methods of analysis and optimal design of columns subjected to follower forces. Sugiyama et al. (1995) described the effect of intermediate concentrated mass on the dynamic stability of cantilevered column subjected to a rocket thrust. Although the internal structural damping may stabilise or destabilise a nonconservative system it can be neglected as it was done by the authors, in the structures for which it is very small. Sugiyama et al. (1995) presented the experimental results that agreed well with the theoretical flutter predictions. Kurnik (1997) in his book presented a overal introduction to bifurcation in 1D and 2D problems introducing its application to divergence and flutter instability phenomenon in engineering. Kounadis (1997) described the occurrence of flutter instability through Hopf bifurcation before static buckling in the regions of divergence in nonconservative nonself-adjoint systems, to show that a practically nondissipative model under certain conditions may lose its stability via flutter for a load smaller than that of divergence instability. In this work the dynamic approach is used to investigate the influence of prestress and bending rigidity ratio of both rods of a geometrically non-linear column upon its regions of instability. The column is loaded by a follower force what makes the problem a nonconservative one.

2. Solution to the problem

The scheme of deformed axes of both rods of the column under investigation is given in Fig.1.

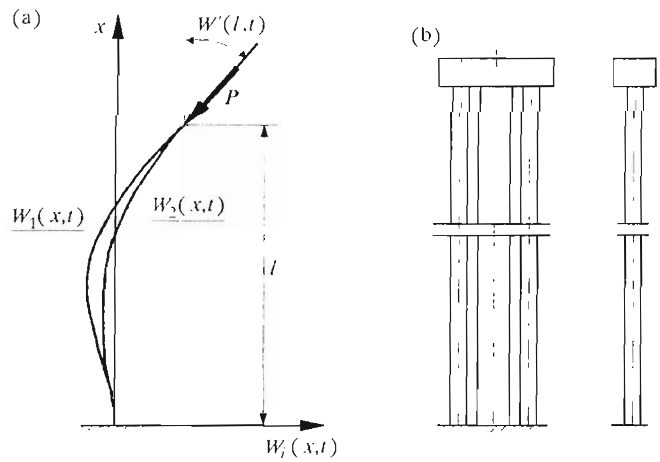

Fig. 1. (a) Scheme of deflected axis of two-member column rods, (b) physical model of a column

Basing on the strain-displacement relations for a beam undergoing the moderately large deflection described by von Karman and applied by Woinowsky-Krieger (1950), by using the Hamilton's principle for nonconservative systems proposed by Levinson (1966), the governing equations for this problem can be presented in the form:

— for the lateral vibration of the ith column rod

$$\frac{\partial^4 w_i(\xi,\tau)}{\partial\xi^4} + k_i\frac{\partial^2 w_i(\xi,\tau)}{\partial\xi^2} + \varpi_{ni}^2\frac{\partial^2 w_i(\xi,\tau)}{\partial\tau^2} \qquad i=1,2 \tag{2.1}$$

— for the longitudinal displacement $u_i(\xi,\tau)$ of the ith rod

$$u_i(\xi,\tau) = \frac{k_i}{\lambda_i}\xi - \frac{1}{2}\int\limits_0^{\xi}\Big(\frac{\partial w_i(\zeta,\tau)}{\partial\zeta}\Big)^2\,d\zeta \qquad i=1,2 \tag{2.2}$$

where

$$w_i(\xi,\tau) = \frac{W_i(x,t)}{l} \qquad k_i = \frac{S_i l^2}{E_i I_i} \qquad \varpi_{ni} = \Omega_n l^2\sqrt{\frac{\rho_i A_i}{E_i I_i}} \tag{2.3}$$

denote the non-dimensional transverse displacements, load parameters and non-dimensional frequency parameter, respectively, and

$$\xi = \frac{x}{l} \qquad \tau = \Omega_n t \qquad \lambda_i = \frac{A_i l^2}{I_i} \qquad u_i(\xi,\tau) = \frac{U_i(x,\tau)}{l} \tag{2.4}$$

l – length of the column
Ω_n – nth natural frequency
E_iI_i – bending stiffness of the ith rod
$\rho_i A_i$ – mass per the unit length of the ith rod.

In Eqs (2.1) and (2.2) only von Karman's nonlinearity has been taken into account. When considering the large deflection theory the influence of other geometrical nonlinearities should be examined after Kurnik and Pękalak (1992).

By using the perturbation (small parameter) method, the relevant quantities are expanded into an exponential series with respect to the amplitude parameter ε ($\varepsilon \ll 0$) (cf Evansen, 1968)

$$\begin{aligned} w_i(\xi,\tau) &= \sum_{j=1}^{N} \varepsilon^{2j-1} w_{i2j-1}(\xi,\tau) + 0(\varepsilon^{N+1}) \\ k_i &= k_{i0} + \sum_{j=1}^{N} \varepsilon^{2j} k_{i2j}(\tau) + 0(\varepsilon^{N+1}) \\ \varpi_{ni}^2 &= \omega_{ni}^2\Big(1 + \sum_{j=1}^{N} \varepsilon^{2j}\nu_{2j}\Big) + 0(\varepsilon^{N+1}) \end{aligned} \tag{2.5}$$

where ν_{2j} stands for the frequency correction coefficients, and

$$\begin{aligned} w_{i1}(\xi,\tau) &= w_{i1}^{(1)}(\xi)\cos\tau \\ w_{i3}(\xi,\tau) &= w_{i3}^{(1)}(\xi)\cos\tau + w_{i3}^{(3)}(\xi)\cos 3\tau \\ w_{i5}(\xi,\tau) &= w_{i5}^{(1)}(\xi)\cos\tau + w_{i5}^{(3)}(\xi)\cos 3\tau + w_{i5}^{(5)}(\xi)\cos 5\tau \\ k_{i2}(\tau) &= k_{i2}^{(0)}\cos 2\tau \\ k_{i4} &= k_{i4}^{(0)} + k_{i4}^{(2)}\cos 2\tau + k_{i4}^{(4)}\cos 4\tau \end{aligned} \tag{2.6}$$

By introducing Eqs (2.5) into the equations of motion (2.1) and axial displacements (2.2), then equating to zero the coefficients of respective ε exponents, one obtains the following set of equations of motion and longitudinal displacements

$$0(\varepsilon^0) \qquad u_{i0}(\xi) = -\frac{k_{i0}}{\lambda_i}\xi \tag{2.7}$$

$$0(\varepsilon^1) \qquad w_{i1}^{IV}(\xi,\tau) + k_{i0}w_{i1}^{II}(\xi,\tau) + \omega_{ni}^2\ddot{w}_{i1}(\xi,\tau) = 0 \tag{2.8}$$

$$0(\varepsilon^2) \qquad u_{i2}(\xi,\tau) = -\frac{k_{i2}(\tau)}{\lambda_i}\xi - \frac{1}{2}\int \Big[w_{i1}^{I}(\zeta,\tau)\Big]^2\,d\zeta \tag{2.9}$$

$$\begin{aligned} 0(\varepsilon^3) \qquad & w_{i3}^{IV}(\xi,\tau) + k_{i0}w_{i3}^{II}(\xi,\tau) + \omega_{ni}^2\ddot{w}_{i3}(\xi,\tau) = \\ & = -k_{i2}(\tau)w_{i1}^{II}(\xi,\tau) - \nu_2\omega_{ni}^2\ddot{w}_{i1}(\xi,\tau) = 0 \qquad i = 1,2 \end{aligned}$$

Roman numerals and dots denote derivatives with respect to ξ and τ, respectively.

In view of Eqs (2.5), Eqs (2.7) ÷ (2.10) are to be solved under the following boundary conditions

— for $j = 1,3,5...$

$$\begin{aligned} & w_{1j}(0,\tau) = w_{2j}(0,\tau) = 0 && w_{1j}(1,\tau) = w_{2j}(1,\tau) \\ & w_{1j}^{I}(\xi,\tau)\Big|_{\xi=0} = w_{2j}^{I}(\xi,\tau)\Big|_{\xi=0} = 0 && w_{1j}^{I}(\xi,\tau)\Big|_{\xi=1} = w_{2j}^{I}(\xi,\tau)\Big|_{\xi=1} \\ & w_{1j}^{II}(\xi,\tau)\Big|_{\xi=1} + \mu w_{2j}^{II}(\xi,\tau)\Big|_{\xi=1} = 0 && w_{1j}^{III}(\xi,\tau)\Big|_{\xi=1} + \mu w_{2j}^{III}(\xi,\tau)\Big|_{\xi=1} = 0 \end{aligned} \tag{2.10}$$

— for $j = 0,2,4,...$

$$\begin{aligned} & u_{1j}(0,\tau) = u_{2j}(0,\tau) = 0 \qquad u_{1j}(1,\tau) = u_{2j}(1,\tau) \\ & k_{1j}(\tau)\mu_1 + k_{2j}(\tau)\mu_2 = p \end{aligned} \tag{2.11}$$

where

$$\begin{aligned} p &= \frac{Pl^2}{E_1I_1 + E_2I_2} & \mu_1 &= \frac{E_1I_1}{E_1I_1 + E_2I_2} \\ \mu_2 &= \frac{E_2I_2}{E_1I_1 + E_2I_2} & \mu &= \frac{E_2I_2}{E_1I_1} \end{aligned} \tag{2.12}$$

and P is the external load applied to the column.

Eqs (2.7) represent the axial displacement-force relation in column members. Substituting these equations into the boundary conditions (2.12) for $j = 0$, yields a linear relationship between the axial forces S_{i0} in each rod due to the axial prestressing with the force P_1 and the external force P in the following form

$$S_{i0} = \pm P_1 + P\frac{E_iI_i}{E_1I_1 + E_2I_2} \qquad i = 1,2 \tag{2.13}$$

The force P_1 is taken positive when compresses a particular rod. For an externally unloaded column $(P = 0)$ when one member is compressed the second must be stretched by the same force P_1. Taking into account different ways of structure loading, the distribution of internal forces can differ, what strongly influences both the vibration frequency and stability of the system.

General solution of Eqs (2.8), after separation of the ξ and τ variables in accordance with Eq $(2.6)_1$ is as follows

$$w_{i1}(\xi) = A_{i1}\cosh(\alpha_{i1}x) + B_{i1}\sinh(\alpha_{i1}x) + C_{i1}\cos(\beta_{i1}x) + D_{i1}\sin(\beta_{i1}x) \qquad (2.14)$$

where

$$\alpha_{i1} = \sqrt{-\frac{1}{2}k_{i0} + \sqrt{\frac{1}{4}k_{i0}^2 + \omega_{ni}^2}} \qquad \beta_{i1} = \sqrt{\frac{1}{2}k_{i0} + \sqrt{\frac{1}{4}k_{i0}^2 + \omega_{ni}^2}} \qquad (2.15)$$

By substituting Eqs (2.15) into boundary conditions (2.11) for $j = 1$ one obtains the system of eight homogenous equations with unknown integration constants A_{i1}, B_{i1}, C_{i1} and D_{i1}, $(i = 1,2)$. The determinant of coefficient matrix of the system must be equal to zero to get a nontrivial solution to the problem. In this way the relationship between the load and the natural frequency is obtained and solved numerically.

From the results obtained by Przybylski et al. (1996) it follows that for the vibration amplitudes small enough, i.e., when the frequency correction coefficient is close to zero, this problem can be satisfactorily solved when taking into account only two terms (ε^0 and ε^1) of Eqs $(2.5)_3$.

3. Results of numerical calculation

All the results are presented as functions of dimensionless quantities to allow for comparison with the results obtainable for a single rod column. These quantities are as follows

- p – external load parameter, $p = Pl^2/(E_1I_1 + E_2I_2)$
- p_c – critical value of p ($p_c^{(B)}$ is the critical parameter for Beck's column equal to 20.0509)
- p_{m1} – internal prestress parameter, $p_{m1} = P_1l^2/(E_1I_1 + E_2I_2)$ (when the positive the rod 1 is compressed and the rod 2 is stretched)
- ω_n – natural vibration frequency parameter, $\omega_n = (\rho_1A_2 + \rho_2A_2)\Omega_n^2l^4/(E_1I_1 + E_2I_2)$
- w_{EI} – coefficient describing the relation between the bending rigidity of both rods, $w_{EI} = E_2I_2/(E_1I_1)$.

During calculations the sum of bending rigidities was constant $(E_1I_1 + E_2I_2 =$ const), identical values of mass per unit length for each rod, as well as identical values of longitudinal rigidity of each rod were assumed.

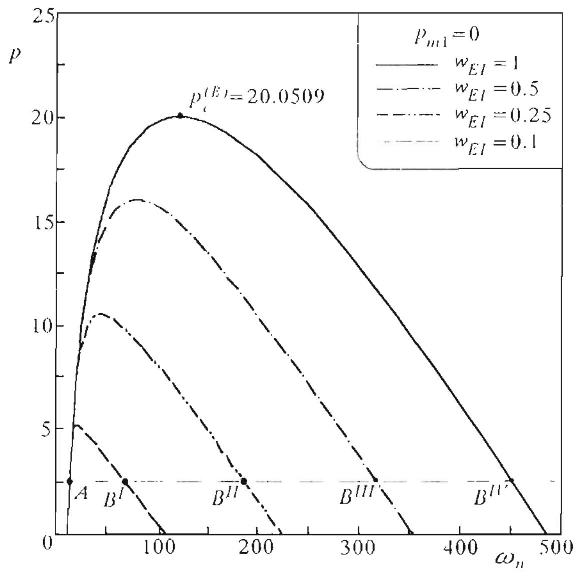

Fig. 2. Eigenvalue curves for unprestressed column with different bending rigidity ratios

Fig.2 shows the natural frequency curves obtained for zero prestress and different bending rigidity relations. For $w_{EI} = 1$ the flutter critical load arises for exactly the same value as for Beck's column $(p_c^{(B)} = 20.0509)$, because both identical rods due to equality of their rigidities and masses per unit length vibrate with the same frequency and mode shape as a single column. The greater asymmetry in rigidity the smaller is the value of critical load. That phenomenon is connected with the decrease in the second natural frequency. The flutter critical load is the value of load to which the two smallest eigenvalue courses approach each other until they join. As it can be seen from Fig.2 all branches of the eigenfrequencies representing the second frequency are parallel to each other, while those for the first frequency overlap each other. Since the second frequencies decrease with the decrease in w_{EI}, all points of joint of adequate eigencurves depend on the course of the second eigenfrequency curves. The decrease in the second vibration frequency may be understood when one observes the second mode shapes – Fig.3. The normalization condition for each computation was the same $(w_{i1}(1) = 1)$. The second modes obtained at the points B^I to B^{IV} (Fig.2) are different for each w_{EI}, especially for the rod 2.

The greatest lateral displacements of that rod along its length arise for the maximal asymmetry in the bending rigidity, i.e. when $E_2I_2/(E_1I_1) = 0.1$. For the same value of $w_{EI} = 0.1$, the second frequencies take the smallest values. The greater amplitude is connected with the smaller vibration frequency.

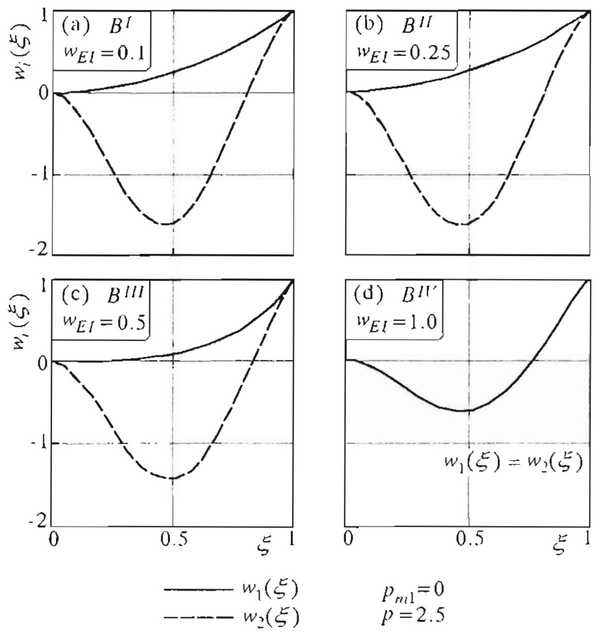

Fig. 3. Mode shapes for unprestressed column loaded by the external load $p = 2.5$ for different w_{EI}

The regions of instability for a column with identical rods internally prestressed (in relation to Beck's load parameter) are presented in Fig.4. There are two regions of instability for the column, the flutter instability region for $0 \leqslant p_{m1}/p_c^{(B)} \leqslant 0.557912$ and the divergence instability region for $0.557912 \leqslant p_{m1}/p_c^{(B)} \leqslant 1$. Between those regions a jump in the critical force occurs from the flutter load $p_c = 4.0787$ to the divergence load $p_c = 0.0$. That results from the courses of eigenvalue curves depicted in Fig.5.

Each prestress up, to its boundary value, introduced into identical rods of the column reduces the flutter load shifting simultaneously the first frequency branch of the eigencurves closer to the origin of co-ordinate system. The divergence critical load is the value of the load to which the smallest square of the eigenfrequencies becomes equal to zero. For both $p_{m1} = 0.557912p_c^{(B)}$ and the external load the first frequency is equal to zero. There is no stable static shape of the column for such a prestress for any external compressing

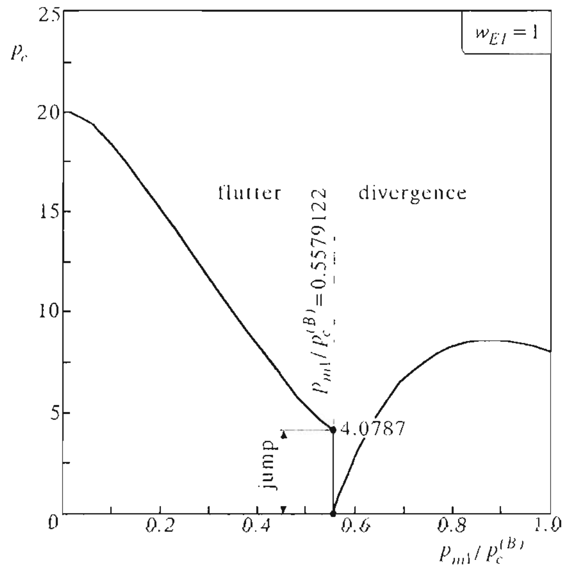

Fig. 4. Regions of instability for different prestressing ratio for a column with symmetrical rods

force. There exist, however, the eigencurves for the same prestress which join at $p_c = 4.0787$, what causes a jump in the critical values. Further increase in the prestress changes the shape of eigencurves so they lose their maximum peaks and cross the y-axis at the points of divergence critical loads; to show it a sample curve for $p_{m1} = 0.7p_c^{(B)}$ has been drawn in Fig.5.

The combined effect of prestress and asymmetry of the bending rigidity of column rods on the eigenvalue curves is shown in Fig.6. The prestress can increase (Fig.6a) or decrease (Fig.6b) the critical flutter force depending on the asymmetry in the bending rigidity. For $w_{EI} = 0.1$ and $p_{m1} = 0.4p_c^{(B)}$ there exists a critical load of the divergence type, however, its value is greater than that of flutter type appearing for the point of join of the second and third eigenvalues. So the value of flutter load must be taken as a critical one for the system.

Taking into account the results presented in Fig.6 for different prestressing and bending rigidities ratio, it seemed to be interesting how asymmetry in the distribution of bending rigidities could influence the stability for a certain prestress. The two values of prestress were chosen for further investigation: $p_{m1} = 0.4p_c^{(B)}$ and $p_{m1} = -0.4p_c^{(B)}$. The first prestress makes the rod 1 compressed and the rod 2 stretched, the second one creates an oposite state

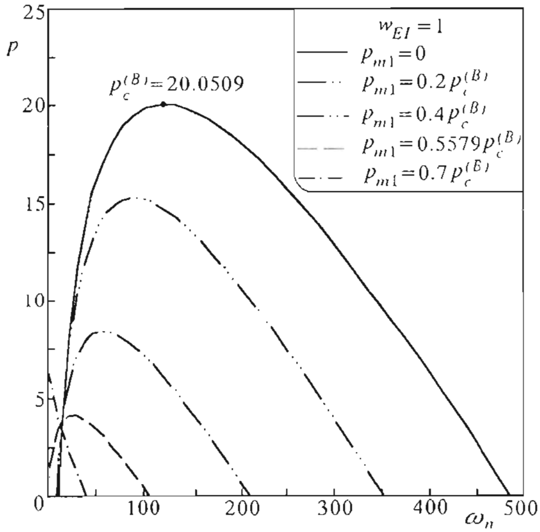

Fig. 5. Eigenvalue curves for a column with symmetrical rods and different prestressing ratio

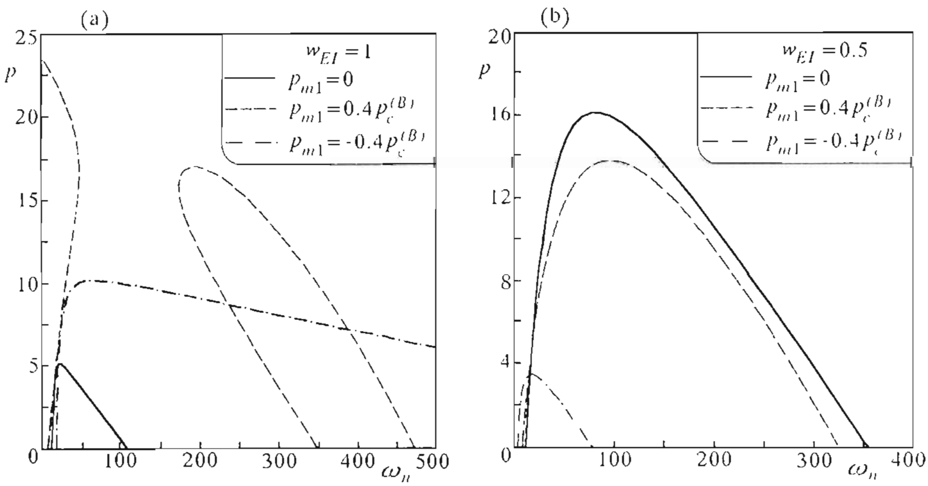

Fig. 6. Comparison of eigencurves for identical prestress rate and different assymetry in bending rigidity; (a) $w_{EI} = 0.1$, (b) $w_{EI} = 0.5$

of strain.

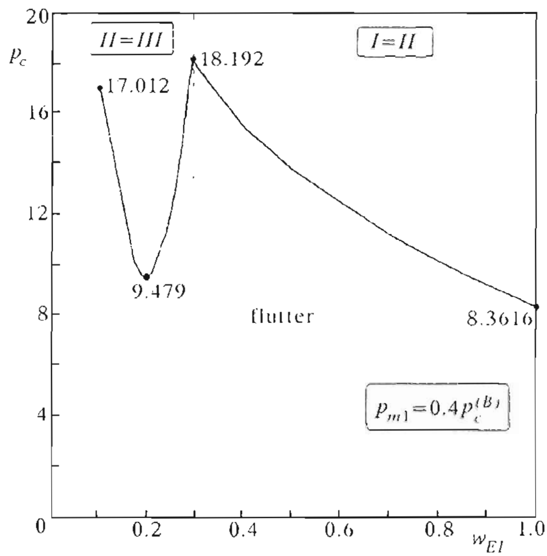

Fig. 7. Flutter critical load as a function of asymmetry of bending rigidity for a column prestressed by $p_{m1} = 0.4p_c^{(B)}$

The column loses its stability via flutter in the entire investigated range of $0.1 \leqslant w_{EI} \leqslant 1$ for the prestress $p_{m1} = 0.4p_c^{(B)}$ – Fig.7. A discontinuity along the curve of critical force appears due to a change in the flutter mechanism shown in Fig.8. For $0.1 \leqslant w_{EI} \leqslant 0.2861$ the critical flutter force occurs for coalescing of the second and third eigenvalues, whereas for the rigidity ratio above 0.2861 for the first and second eigenvalues. In the first range of w_{EI} the critical force decreases to the value of 9.479, and then increases with the increase in w_{EI}. In the second range of w_{EI} the critical force monotonically decreases with the increase in w_{EI}.

When the rod 1 is stretched ($p_{m1} = -0.4p_c^{(B)}$), two changes in the instability mechanism occur for increasing value of w_{EI} – Fig.9. The first jump downwards of the critical force exists for $w_{EI} = 0.119$ and it associates the change in the instability mechanism from flutter to divergence. After that the divergence critical force increases at first up to 14.111 (the maximum value for the system) and then drops down for $w_{EI} = 0.42395$. For $w_{EI} = 0.46$ the mechanism of instability changes for the second time from divergence to flutter. The eigenvalue curves obtained for $p_{m1} = -0.4p_c^{(B)}$ and $0.423 \leqslant w_{EI} \leqslant 0.46$, the courses of which explain one of the changes in the instability mechanism, are presented in Fig.10. From that figure it can be noticed that for $w_{EI} = 0.423$

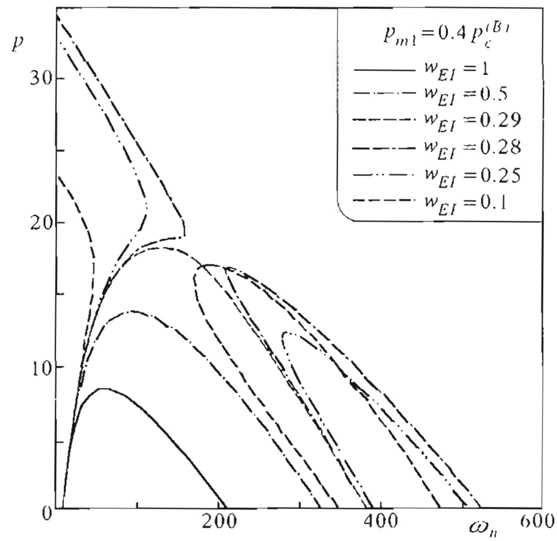

Fig. 8. Eigencurves for a column prestressed by $p_{m1} = 0.4p_c^{(B)}$ and different bending rigidity ratios

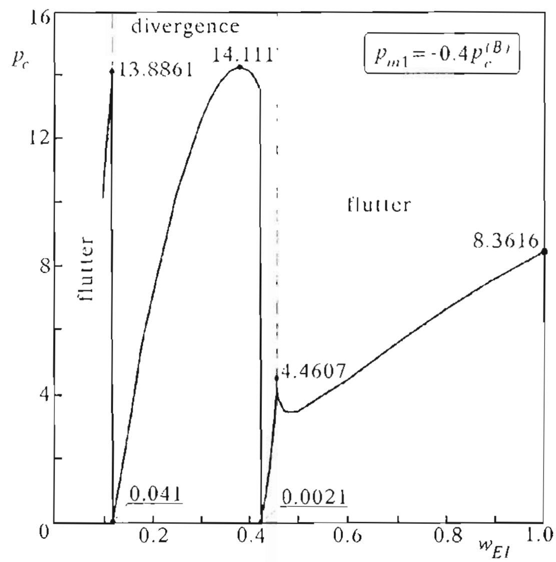

Fig. 9. Regions of instability for a column prestressed by $p_{m1} = -0.4p_c^{(B)}$ for increasing w_{EI}

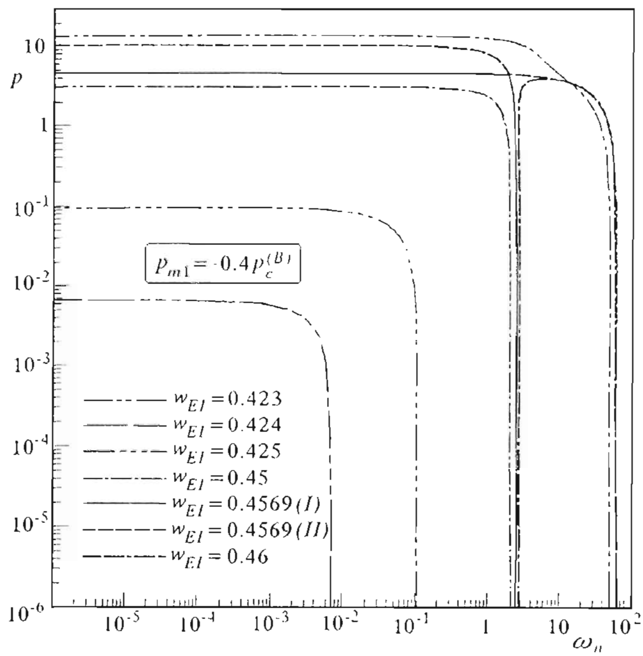

Fig. 10. Eigencurves for a column prestressed by $p_{m1} = -0.4p_c^{(B)}$ and different bending rigidity ratios

the eigencurve crosses the y-axis at the point of $p_c = 13.488$, whereas for $w_{EI} = 0.424$ the divergence critical force p_c is equal to 0.0062. This decrease in the critical load results from the fact that starting from $w_{EI} = 0.42395$ new eigencurves appear for small values of the external load. Then, with the increasing values of w_{EI} the divergence critical load increases up to 4.4607 where the second change of instability from divergence to flutter occurs. There is an evolution of eigenvalue curves observed in Fig.10 with a small change of bending rigidity ratio. Two first eigencurves for $w_{EI} = 0.4569$ approaching very close to each other at one point, evolve to one flutter curve existing for $w_{EI} = 0.46$.

4. Conclusions

On the basis of dynamic analysis, regions of instability as a function of both the initial prestress and distribution of bending rigidity of non-linear two-rods column have been established.

The instability behaviour of the column may be either a divergence or a flutter type depending on values of prestress and the bending rigidity ratio.

The prestress can decrease or increase the critical flutter load depending on the value of bending rigidity ratio.

A column without prestress loaded by a follower force loses its stability via flutter. The greater asymmetry in the bending rigidity the smaller critical flutter force is.

The prestressing of rods for a column composed of identical members can cause flutter or divergence instability of the system with a jump phenomenon reducing the critical load for a certain value of prestress.

The way of prestressing play an important role for the column stability. The same column of identically distributed bending rigidity between rods can lose its stability via divergence or flutter under different critical force, depending on the prestress introduced into a particular rod causing its stretching or compressing.

There are such combination of the values of prestress and bending rigidity ratios for which the column can be unstable when loaded by a small external load.

Passive vibration control of an asymmetrical column is possible by introducing the adequate prestress into rods of the column.

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Rola wstępnego sprężania przy wyznaczaniu obszarów niestateczności dla kolumny nieliniowej

Streszczenie

W pracy zbadano wpływ wstępnego sprężania i asymetrii rozkładu sztywności na zginanie między prętami geometrycznie nieliniowej kolumny dwuprętowej na drgania takiego układu. Do rozwiązania zagadnienia zastosowano metodę małego parametru.

Wyznaczono obszary niestateczności dywergencyjnej i fluterowej na podstawie przebiegów krzywych wartości własnych w odniesieniu do relacji między sztywnościami i stopniem sprężenia. Zaobserwowano skokowe zmiany wartości siły krytycznej dla pewnych wartości obu parametrów prowadzące do niestateczności układu przy niewielkiej sile zewnętrznej. Mimo tego, że każde wstępne sprężenie obniża wartość siły krytycznej, może być wprowadzane w celu pasywnej kontroli częstości drgań.

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