Instantaneous Measurement of field quadrature moments and entanglement

P. Lougovski,[1] H. Walther,[1] and E. Solano[1,2]
[1] *Max-Planck-Institut für Quantenoptik, Hans-Kopfermann-Strasse 1, D-85748 Garching, Germany*
[2] *Sección Física, Departamento de Ciencias, Pontificia Universidad Católica del Perú, Apartado 1761, Lima, Peru*

We present a method of measuring expectation values of quadrature moments of a multimode field through two-level probe "homodyning". Our approach is based on an integral transform formalism of measurable probe observables, where analytically derived kernels unravel efficiently the required field information at zero interaction time, minimizing decoherence effects. The proposed scheme is suitable for fields that, while inaccessible to a direct measurement, enjoy one and two-photon Jaynes-Cummings interactions with a two-level probe, like spin, phonon, or cavity fields. Available data from previous experiments are used to confirm our predictions.

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State reconstruction of a bosonic field is an important issue in fundamentals of quantum physics that has been studied extensively, both theoretically and experimentally, in the last two decades [1]. Its main concern is to measure either the density matrix of an unknown quantum field state or, equivalently, any of its phase-space representations. Among them, Wigner function reconstructions seem to be the most promising avenue for measuring completely, for example, an intracavity microwave field [2] or the motion of a trapped ion [3]. In the case of a propagating field, usually accessible to direct measurement, homodyne techniques are currently used in the lab [4]. Typically, a complete state reconstruction with standard techniques demand great experimental efforts, is strongly affected by decoherence mechanisms, and, frequently, the obtained information exceeds our requirements. In those cases, techniques for extracting efficiently the required partial information are most welcomed and even necessary. The problem is even harder when the field is not directly accessible and a quantum probe has to be used for the purposes of an indirect measurement [5]. Therefore, the following question arises: how to derive accurately the expectation value of a field observable, through an efficient measure of a probe, with minimal resources and with an outcome that is minimally affected by decoherence mechanisms? In this article, we answer this question for the case of a multimode bosonic field, interacting with a two-level probe, by means of a practical integral transform method. These conditions are naturally fulfilled by several physical systems, like a cavity field interacting with two-level atoms, the motion of an ion interacting, through laser coupling, with two of its internal levels, or even several spins, in a mode approximation, interacting with a single spin, like in NMR or quantum dot systems.

We consider a general picture in which an inaccessible bosonic field is measured through an interacting probe, following the interaction Hamiltonian

$$H = \hbar \sum_{i,j} g_{i,j}(p_i f_j^\dagger + p_i^\dagger f_j), \tag{1}$$

where p_i and f_j are probe and field operators, respectively, and $g_{i,j}$ are coupling strengths. We postulate the existence of an analytical kernel $\kappa(\tau)$ such that

$$\langle F \rangle = \int_{-\infty}^{\infty} \kappa(\tau)\langle P\rangle(\tau) d\tau, \tag{2}$$

where F and P are operators associated with field and probe observables, respectively, and τ is the dimensionless probe-field interaction time. Later, it will be clear why it is possible to include a negative axis for τ in the integration limits of Eq. (2). $\langle P\rangle(\tau)$ will be replaced by experimental measured data and, for the method to be useful, we should be able to formally invert the integral transform and derive an analytical expression for $\kappa(\tau)$. We will show below that this inversion is possible for several important field observables, like quadrature moments of a multimode field, unravelling important information on squeezing and entanglement. Note that we aim at measuring efficiently partial field information without the requirement of full state reconstruction, even though integral techniques can also provide us with complete Wigner function reconstructions [6].

We consider a two-level probe interacting with a single-mode field through a resonant Jaynes-Cummings (JC) Hamiltonian, in the interaction picture,

$$H_{\rm JC} = \hbar g(\sigma^\dagger a + \sigma a^\dagger), \tag{3}$$

where g is a coupling strength, $\{\sigma, \sigma^\dagger\}$ are lowering and raising probe operators, and $\{a, a^\dagger\}$ are annihilation and creation field operators. We assume that, given that the initial probe-field density operator is $\rho^{\rm in}(0)$, we can measure, after a dimensionless interaction time $\tau \equiv gt$, the population of the excited probe level $|e\rangle$

$$P_{\rm e}^{\rm in}(\tau) \equiv {\rm Tr}[\rho(\tau)|e\rangle\langle e|] = {\rm Tr}[U(\tau)\rho^{\rm in}(0)U^\dagger(\tau)|e\rangle\langle e|], \tag{4}$$

where $|e\rangle\langle e| = \sigma^\dagger\sigma$ and $U(\tau) = \exp(-i\tau H_{\rm JC}/\hbar g)$ is the evolution operator. In Eq. (4), and throughout this work, upper and lower indices stem from initial and measured probe states, respectively. We consider the initial state $\rho^{+\phi} = |+_\phi\rangle\langle +_\phi| \otimes \rho_f$, where $|\pm_\phi\rangle = (|g\rangle \pm e^{i\phi}|e\rangle)/\sqrt{2}$ are the eigenvectors of $\sigma_x^\phi = \sigma^\dagger e^{i\phi} + \sigma e^{-i\phi}$ with $\sigma_x^\phi|\pm_\phi\rangle = \pm|\pm_\phi\rangle$. When $\phi = 0$, σ_x^ϕ turns into σ_x, the usual spin-1/2

Pauli operator. Replacing $\rho^{+\phi}$ in Eq. (4) gives

$$P_{\rm e}^{+\phi}(\tau) = \frac{i}{4}\sum_{n=0}^{\infty} \sin(2\tau\sqrt{n+1})(e^{i\phi}\rho_{n,n+1} - e^{-i\phi}\rho_{n+1,n}) + \frac{1}{2}\sum_{n=0}^{\infty}(\cos^2(\tau\sqrt{n+1})\rho_{n,n} + \sin^2(\tau\sqrt{n+1})\rho_{n+1,n+1}), \tag{5}$$

where the initial field state $\rho_f = \sum_{n,m}\rho_{n,m}|n\rangle\langle m|$ has been written in terms of its matrix elements. Through the knowledge of $P_{\rm e}^{+\phi}(\tau)$, we aim at measuring the field quadratures $X_\phi = \frac{1}{2}(ae^{-i\phi} + a^\dagger e^{i\phi})$ and $Y_\phi = X_{\phi+\pi/2} = \frac{1}{2i}(ae^{-i\phi} - a^\dagger e^{i\phi})$ with expectation values

$$\langle X_\phi\rangle = \frac{1}{2}\sum_{n=0}^{\infty}\sqrt{n+1}(e^{i\phi}\rho_{n,n+1} + e^{-i\phi}\rho_{n+1,n}) \tag{6}$$

$$\langle Y_\phi\rangle = \frac{i}{2}\sum_{n=0}^{\infty}\sqrt{n+1}(e^{i\phi}\rho_{n,n+1} - e^{-i\phi}\rho_{n+1,n}). \tag{7}$$

We choose the kernel in Eq. (2) as an odd function, $\kappa(-\tau) = -\kappa(\tau)$, so that the integral of the second sum in Eq. (5) vanishes, while the integral of the first sum should reproduce Eq. (7). In consequence, replacing operators $P \to |e\rangle\langle e|$ and $F \to Y_\phi$ in Eq. (2), the condition for this ansatz to be true is

$$\int_{-\infty}^{\infty}\kappa(\tau)e^{i\omega_n\tau}d\tau = i\omega_n, \tag{8}$$

where $\sin(2\tau\sqrt{n+1})$ has been rewritten in complex form with $w_n = 2\sqrt{n+1}$. The inverse Fourier transform of Eq. (8) provides us with the kernel

$$\kappa(\tau) = \frac{i}{2\pi}\int_{-\infty}^{\infty}e^{-i\omega_n\tau}\omega_n d\omega_n = -\delta'(\tau), \tag{9}$$

where $\delta'(\tau)$ is the first derivative of a delta function. Note that even if, physically, w_n is a function of discrete n's, it can be treated formally as continuous for the sake of the inverse transform. Then, Eq. (2) can be written as

$$\langle Y_\phi\rangle = -\int_{-\infty}^{\infty}\delta'(\tau)P_{\rm e}^{+\phi}(\tau)d\tau, \tag{10}$$

yielding

$$\langle Y_\phi\rangle = \frac{d}{d\tau}P_{\rm e}^{+\phi}(\tau)\bigg|_{\tau=0}, \tag{11}$$

where the continuity of the first derivative of $P_{\rm e}^{+\phi}(\tau)$ at $\tau = 0$ has been considered. Similarly,

$$\langle X_\phi\rangle = \langle Y_{\phi-\frac{\pi}{2}}\rangle = \frac{d}{d\tau}P_{\rm e}^{+\phi-\frac{\pi}{2}}(\tau)\bigg|_{\tau=0}. \tag{12}$$

Eqs. (11) and (12) show that $\langle X_\phi\rangle$ and $\langle Y_\phi\rangle$ are fully contained in the first derivative, at $\tau = 0$, of the measured probe population, offering a remarkably simple way of obtaining quadrature information. Note that knowing the first derivative at $\tau = 0$ requires knowing the function in a vicinity. However, no necessity of full state reconstruction or lengthy time integration over Rabi oscillations are needed, in contrast to standard methods. Needless to say, the influence of decoherence is minimized.

Induced by the structure of Eq. (5), and aiming at cancelling the population while keeping the off-diagonal (phase) information, we could find a similar result by subtracting rotated populations

$$P_{\rm e}^{+\phi}(\tau) - P_{\rm e}^{-\phi}(\tau) = \frac{i}{4}\sum_{n=0}^{\infty}\sin(2\tau\sqrt{n+1})(e^{i\phi}\rho_{n,n+1} - e^{-i\phi}\rho_{n+1,n}). \tag{13}$$

Following a similar procedure as before, we can write

$$\langle Y_\phi\rangle = \frac{1}{2}\left(\frac{d}{d\tau}P_{\rm e}^{+\phi}(\tau) - \frac{d}{d\tau}P_{\rm e}^{-\phi}(\tau)\right)\bigg|_{\tau=0}. \tag{14}$$

This result has evident resemblance to the known technique of field homodyning [4]. There, an unknown field is mixed in a 50-50 beam splitter with a local oscillator, and the difference of field intensities (rate of photon clicks) at the output gives us quadrature information. Based on this similarity, the proposed method could be called after two-level probe "homodyning".

X_ϕ and Y_ϕ happen to be relevant observables in a wide range of physical systems where current experiments enjoy probe rotations and JC-like interactions, like cavity QED (CQED), trapped ions, BEC, and different solid-state systems. In CQED, the quadrature information can be obtained by sending an excited atom through a Ramsey zone before crossing the cavity mode [7], and finally measuring the population of the excited state at the cavity output. For trapped ions, $\langle X\rangle$ and $\langle Y\rangle$ represent, literally, expectation values of position and momentum operators, that will be obtained by measuring the internal state statistics, where the efficiency can reach $\sim 100\%$, after a JC-like sideband excitation [8]. In the case of solid-state devices, there are several systems enjoying two-level probes interacting through JC interactions with cavity, phonon or spin fields. It is noteworthy to mention that in all these examples a probe is needed due to the lack of a direct measurement.

In the rest of this article, for the sake of simplicity, we will use the language of cavity QED, where a two-level atom probes an intracavity electromagnetic field.

Another important field observable that can be obtained straightforwardly with a JC interaction is the mean photon number $\langle n\rangle = \langle a^\dagger a\rangle$. Considering the initial state $\rho^{\rm e} = |e\rangle\langle e| \otimes \rho_f$, we can derive the kernel

$$\bar{\kappa}(\tau) = -\delta''(\tau) \tag{15}$$

for measuring

$$\langle n\rangle = \frac{1}{2}\frac{d^2P_{\rm g}^{\rm e}(\tau)}{d^2\tau}\bigg|_{\tau=0} - 1\ . \tag{16}$$

Note that measuring $\langle n\rangle$ does not require Ramsey zones for rotating the atom, as was the case before. Given the available experimental data, expression in Eq. (16) is the only one that could be presently tested. For example, using the experimental data associated with the experiments at ENS, see Figs. 2 (A), (B) in Ref. [5], we predict $\langle n\rangle \approx 0.14$ and 0.81, respectively. These values are quite close to the ones obtained via integration or fitting long Rabi oscillations, 0.06 and 0.85, respectively. We made similar estimations for the experiments at NIST, obtaining $\langle n\rangle \approx 1.6$ and 3.1 for the experiments associated with Figs. 2 and 3 in Ref. [9], to be compared with 1.5 and 2.9, respectively. Clearly, our predictions could only be better if specific experiments are performed, aiming at first and second derivatives at very short interaction times.

It is also possible to use these integral methods to measure second-order quadrature moments, providing information about field quadrature squeezing and entanglement of a multimode field. We will use a resonant two-photon JC Hamiltonian that reads

$$H_{2\rm JC} = \hbar g(\sigma^{\dagger} a^2 + \sigma a^{\dagger\,2}) \tag{17}$$

in the interaction picture. This nonlinear interaction has been realized experimentally in the context of microwave CQED [10] and trapped ions [9]. Our aim, here, is to measure expectation values of squared quadratures,

$$\langle X_\phi^2\rangle = \frac{1}{4} + \frac{\langle n\rangle}{2} + \frac{1}{4}\sum_{n=0}^{\infty}\sqrt{(n+1)(n+2)}\left(e^{2i\phi}\rho_{n,n+2} + e^{-2i\phi}\rho_{n+2,n}\right), \tag{18}$$

$$\langle Y_\phi^2\rangle = \frac{1}{4} + \frac{\langle n\rangle}{2} - \frac{1}{4}\sum_{n=0}^{\infty}\sqrt{(n+1)(n+2)}\left(e^{2i\phi}\rho_{n,n+2} + e^{-2i\phi}\rho_{n+2,n}\right). \tag{19}$$

with the help of Eq. (17) and the proposed integral transform techniques. Then, in a close analogy to Eq. (13), now for a two-photon JC interaction, we can calculate

$$P_{\rm g}^{+\phi}(\tau) - P_{\rm g}^{-\phi}(\tau) = \frac{i}{4}\sum_{n=0}^{\infty}\sin(2\tau\sqrt{(n+1)(n+2)}) \times(e^{2i\phi}\rho_{n,n+2} + e^{-2i\phi}\rho_{n+2,n}). \tag{20}$$

With the help of Eqs. (16), (18), and (19), and by deriving and using the corresponding kernels, we arrive at

$$\langle X_\phi^2\rangle = \frac{1}{2i}\left(\frac{dP_{\rm g}^{+\phi}(\tau)}{d\tau} - \frac{dP_{\rm g}^{-\phi}(\tau)}{d\tau}\right)\Big|_{\tau=0} + \frac{1}{4}\frac{d^2P_{\rm g}^{\rm e}(\tau)}{d^2\tau}\Big|_{\tau=0} - \frac{1}{4}, \tag{21}$$

$$\langle Y_\phi^2\rangle = \frac{i}{2}\left(\frac{dP_{\rm g}^{+\phi}(\tau)}{d\tau} - \frac{dP_{\rm g}^{-\phi}(\tau)}{d\tau}\right)\Big|_{\tau=0} + \frac{1}{4}\frac{d^2P_{\rm g}^{\rm e}(\tau)}{d^2\tau}\Big|_{\tau=0} - \frac{1}{4}. \tag{22}$$

The quadrature variances $(\Delta X)^2 = \langle X^2\rangle - \langle X\rangle^2$ and $(\Delta Y)^2 = \langle Y^2\rangle - \langle Y\rangle^2$ contain information about field squeezing and can be calculated straightforwardly by using Eqs. (11), (12), (21), and (22).

It is noteworthy to say that it is not necessary to use a two-photon JC interaction for measuring second-order quadrature moments. For example, it is enough to use a two-atom probe interacting with the tested field through a single-photon JC, whose interaction Hamiltonian reads

$$H^{\rm I} = \hbar g[(\sigma_1^{\dagger} + \sigma_2^{\dagger})a + (\sigma_1 + \sigma_2)a^{\dagger}], \tag{23}$$

where the subindexes are labelling probe atoms "1" and "2". We consider the Bell states $|\phi_\theta^+\rangle = [|g_1g_2\rangle + e^{i\theta}|e_1e_2\rangle]/\sqrt{2}$ and $|\phi_\theta^-\rangle = [|g_1g_2\rangle - e^{i\theta}|e_1e_2\rangle]/\sqrt{2}$ as two probe initial states, and in both cases we measure $|\psi^+\rangle = [|g_1e_2\rangle + |e_1g_2\rangle]/\sqrt{2}$, obtaining

$$P_{\psi^+}^{\phi_\theta^+}(\tau) - P_{\psi^+}^{\phi_\theta^-}(\tau) = \frac{1}{2}\sum_{n=0}^{\infty}\frac{\sqrt{(n+1)(n+2)}}{2n+3} \times\sin^2(\sqrt{2}\tau\sqrt{2n+3})(e^{i\theta}\rho_{n,n+2} + e^{-i\theta}\rho_{n+2,n}). \tag{24}$$

From this expression, and following similar steps to previous derivations, it is possible to deduce

$$\langle X_\theta^2\rangle = \frac{1}{8}\left(\frac{d^2P_{\psi^+}^{\phi_\theta^+}(\tau)}{d^2\tau} - \frac{d^2P_{\psi^+}^{\phi_\theta^-}(\tau)}{d^2\tau}\right)\Big|_{\tau=0} + \frac{1}{4}\frac{d^2P_g(\tau)}{d^2\tau}\Big|_{\tau=0} - \frac{1}{4}, \tag{25}$$

$$\langle Y_\theta^2\rangle = -\frac{1}{8}\left(\frac{d^2P_{\psi^+}^{\phi_\theta^+}(\tau)}{d^2\tau} - \frac{d^2P_{\psi^+}^{\phi_\theta^-}(\tau)}{d^2\tau}\right)\Big|_{\tau=0} + \frac{1}{4}\frac{d^2P_g(\tau)}{d^2\tau}\Big|_{\tau=0} - \frac{1}{4}\ . \tag{26}$$

Note that the required Bell states and the measurement procedure have already been implemented in the lab in the case of CQED [11] and trapped ion [12] setups.

The formalism for measuring squeezing can be generalized to a two-mode field (or more), providing us with entanglement information. Accordingly, we define the two-mode quadratures as

$$X_\phi = X_{\phi_1} + X_{\phi_2} = \frac{1}{2}\sum_{j=1}^{2}(a_j^{\dagger}e^{-i\phi_j} + a_je^{i\phi_j}), \tag{27}$$

$$Y_\phi = Y_{\phi_1} + Y_{\phi_2} = \frac{i}{2}\sum_{j=1}^{2}(a_j^{\dagger}e^{-i\phi_j} - a_je^{i\phi_j}), \tag{28}$$

where j labels modes "1" and "2". The quantities $\langle X_\phi\rangle$ and $\langle Y_\phi\rangle$ can be easily calculated and, here, we will concentrate on the second-order quadrature moments

$$\langle X_\phi^2\rangle = \langle X_{\phi_1}^2\rangle + \langle X_{\phi_2}^2\rangle + 2\langle X_{\phi_1}X_{\phi_2}\rangle \tag{29}$$

$$\langle Y_\phi^2\rangle = \langle Y_{\phi_1}^2\rangle + \langle Y_{\phi_2}^2\rangle + 2\langle Y_{\phi_1}Y_{\phi_2}\rangle, \tag{30}$$

In these expressions, we define $\langle X_{\phi_1} X_{\phi_2}\rangle = \frac{1}{2}\langle A\rangle + \frac{1}{2}\langle B\rangle$, $\langle Y_{\phi_1} Y_{\phi_2}\rangle = \frac{1}{2}\langle A\rangle - \frac{1}{2}\langle B\rangle$, with

$$A = a_1^\dagger a_2 e^{-i(\phi_1-\phi_2)} + a_1 a_2^\dagger e^{i(\phi_1-\phi_2)}, \quad (31)$$
$$B = a_1^\dagger a_2^\dagger e^{-i(\phi_1+\phi_2)} + a_1 a_2 e^{i(\phi_1+\phi_2)}. \quad (32)$$

Single-mode quantities $\langle X_{\phi_i}\rangle$, $\langle Y_{\phi_i}\rangle$, $\langle X^2_{\phi_i}\rangle$ and $\langle Y^2_{\phi_i}\rangle$, can be determined using two-level probes as it was shown above. Therefore, the main issue is to calculate the expectation values of A and B, which describe correlations between modes 1 and 2. It has been shown, theoretically [13] and experimentally [10], that the two-photon probe-field interaction Hamiltonian

$$H_{\rm A} = \hbar g(\sigma^\dagger a_1 a_2^\dagger + \sigma a_1^\dagger a_2) \quad (33)$$

can be engineered and controlled. If the probe is prepared initially in the superposition states, $|+_\phi\rangle$ or $|-_\phi\rangle$, with $\phi = \phi_1 - \phi_2$, we can calculate

$$P^+_{\rm e,A}(\tau) - P^-_{\rm e,A}(\tau) = \frac{i}{2}\sum_{n_1,n_2=0}^{\infty} \sin(2g\tau\sqrt{n_2(n_1+1)}) \times(e^{-i\phi}\rho_{n_1,n_2;n_1+1,n_2-1} + e^{i\phi}\rho_{n_1+1,n_2-1;n_1,n_2}), \quad (34)$$

from which we can derive

$$\langle A\rangle = \frac{i}{g}\left(\frac{dP^+_{\rm e,A}(\tau)}{d\tau} - \frac{P^-_{\rm e,A}(\tau)}{d\tau}\right)\Big|_{\tau=0}. \quad (35)$$

Similarly, and by using the Hamiltonian

$$H_B = \hbar g(\sigma^\dagger a_1^\dagger a_2^\dagger + \sigma a_1 a_2), \quad (36)$$

we can deduce

$$\langle B\rangle = \frac{i}{g}\left(\frac{dP^+_{\rm e,B}(\tau)}{d\tau} - \frac{P^-_{\rm e,B}(\tau)}{d\tau}\right)\Big|_{\tau=0}. \quad (37)$$

In consequence, we can also estimate two-mode field variances $(\Delta X)^2$ and $(\Delta Y)^2$ in terms of measurable probe observables. Furthermore, by using the same approach, we can compute the variances of EPR-like operators

$$u = a_0 X_1 - \frac{c_1}{|c_1|}\frac{1}{a_0}X_2, \quad (38)$$
$$v = a_0 Y_1 - \frac{c_2}{|c_2|}\frac{1}{a_0}Y_2, \quad (39)$$

where a_0, c_1 and c_2 are constants. For example, it was shown in Ref. [14] that a two-mode Gaussian state ρ is separable if, and only if, $\langle(\Delta u)^2\rangle_\rho + \langle(\Delta v)^2\rangle_\rho \geq a_0^2 + 1/a_0^2$.

In summary, we have shown how expectation values of quadrature field operators can be measured by means of a two-level probe, helped by a practical integral transform method and without the necessity of full state reconstruction. Surprisingly, all relevant information is contained in first and second derivatives of measurable probe observables at interaction time $\tau = 0$, making unnecessary long range probe measurements and minimizing decoherence effects. Also, we showed that a similar technique allows to measure second-order quadrature moments and variances, that is, squeezing and entanglement. These results allow us to conjecture the possibility of realizing full state reconstructing with "instantaneous" measurements that are robust to decoherence.

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[1] W. P. Schleich, *Quantum Optics in Phase Space* (VCH-Wiley, Weinheim, 2001).
[2] P. Bertet, A. Auffeves, P. Maioli, S. Osnaghi, T. Meunier, M. Brune, J. M. Raimond, and S. Haroche, Phys. Rev. Lett. **89**, 200402 (2002).
[3] D. Leibfried, D. M. Meekhof, B. E. King, C. Monroe, W. M. Itano, and D. J. Wineland, Phys. Rev. Lett. **77**, 4281 (1996).
[4] L. Mandel and E. Wolf, *Optical Coherence and Quantum Optics* (Cambrige University Press, 1995).
[5] M. Brune, F. Schmidt-Kaler, A. Maali, J. Dreyer, E. Hagley, J. M. Raimond, and S. Haroche, Phys. Rev. Lett. **76**, 1800 (1996).
[6] P. Lougovski, E. Solano, Z. M. Zhang, H. Walther, H. Mack, and W. P. Schleich, Phys. Rev. Lett. **91**, 010401 (2003).
[7] J. M. Raimond, M. Brune, and S. Haroche, Rev. Mod. Phys. **73**, 565 (2001).
[8] D. Leibfried, R. Blatt, C. Monroe, and D. Wineland, Rev. Mod. Phys. **75**, 281 (2003).
[9] D. M. Meekhof, C. Monroe, B. E. King, W. M. Itano, and D. J. Wineland, Phys. Rev. Lett. **76**, 1796 (1996).
[10] M. Brune, J. M. Raimond, P. Goy, L. Davidovich, and S. Haroche, Phys. Rev. Lett. **59**, 1899 (1987).
[11] S. Osnaghi, P. Bertet, A. Auffeves, P. Maioli, M. Brune, J. M. Raimond, and S. Haroche, Phys. Rev. Lett. **87**, 037902 (2001).
[12] C. A. Sackett, D. Kielpinski, B. E. King, C. Langer, V. Meyer, C. J. Myatt, M. Rowe, Q. A. Turchette, W. M. Itano, D. J. Wineland, C. Monroe, Nature **404**, 256 (2000).
[13] P. A. M. Neto, L. Davidovich, and J. M. Raimond, Phys. Rev. A **43**, 5073 (1991).
[14] L.-M. Duan, G. Giedke, J. I. Cirac, and P. Zoller, Phys. Rev. Lett. **84**, 2722 (2000).