Transactions on Ecology and the Environment vol 48, © 2001 WIT Press, www.witpress.com, ISSN 1743-3541

# Instantaneous residential water demand as stochastic point process

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## Abstract

In this paper a model of indoor residential water demand for describing the instantaneous temporal and spatial variability of flow in a municipal water distribution system is presented. Such a model can be a powerful tool for studying both problems of planning and managing water distribution networks. In particular, as it provides much more accurate information on the real water demands, it makes possible a better design of the system. Moreover a more efficient management of instantaneous water pressure heads is made also possible, allowing the control of water losses and of water quality along the whole distribution system. In the field of water losses control, instantaneous water demand models can also be used to develop simplified techniques of *district metering*. Here following the work of Buchberger and Wu [2], a Poisson rectangular pulse stochastic process has been used to characterise the intensity, duration and frequency of water demand at single and/or multiple family residences. From measured instantaneous water consumption data of a group of 85 single-family residences, the model parameters have been estimated using the method of moments. The monitored residences are part of four different blocks of the same building and their occupants belong to the same socio-economic status. The estimated parameters show a good agreement with the expected values and give interesting information on the temporal features of water demand. Besides, numerical simulations carried out with the proposed stochastic model and using the estimated parameters supply water demand time series that perfectly fit the measured ones.

# 1 Introduction

Residential water use is about 70 percent of system water demand [1] and shows large fluctuations in both time and space. Several factors affect water demand, in particular weather and climate and users' socio-economic status. Most of previous studies aimed at quantifying and forecasting water demand for improving the design of water distribution systems and price policies, and generally produced non dynamic water consumption models. A first work proposing a model for describing water demand at finer temporal scale was presented by Buchberger et al.[2],[3] in the hypothesis that residential water demands occur as a non-homogeneous Poisson rectangular pulse process. This model was formulated mainly to verify distribution system ability to guarantee water quality standards.

The presented work is part of a research activity aimed at identifying and implementing new methodologies for optimal positioning and managing of system control devices, slide gates and valves, for reducing physical water losses in a municipal distribution network. Water losses in Italian water distribution systems represent about 27% of the total amount of the entering water and it is often difficult to locate them, particularly in urban areas. This dynamic consumption model and the methodologies for parameter estimation and analysis represent the first step for describing in detail the dynamic behaviour of a water distribution system, within the formulation of an optimum real time management policy of instantaneous pressure head layout.

# 2 The Poisson rectangular pulses model (PRP)

The model presented in this paper is commonly known as the Poisson Rectangular Pulse (PRP) model. This kind of model has been widely used in hydrology for the analysis of precipitation and in particular for describing the temporal evolution of the rainfall intensity process [4],[5]. In the study of residential water demand, this model leads to the introduction of three main parameters, namely intensity, duration and frequency, which characterise the elementary use.

As described by Buchberger et al.[2], in a single residence, home occupants can be thought as customers using servers represented by water fixtures and appliances. Customers arrive randomly according to a Poisson process with parameter $\lambda$. Each single server use is assumed to be defined by a rectangular pulse of intensity $I$ and duration $d$. Due to the finite duration of each pulse, it is possible that one or more events, with different starting time, overlaps for short time intervals. In the overlapping period the intensity of water demand is the sum of each pulse intensity, (fig.1). It is assumed throughout that water uses occur according to a Poisson process, their duration are mutually independent as well as independent of the occurrence time, and described by an exponential probability function with rate $\eta$ (1). We also assume that their intensities are mutually independent and equally distributed according to a normal law with mean μ (2).

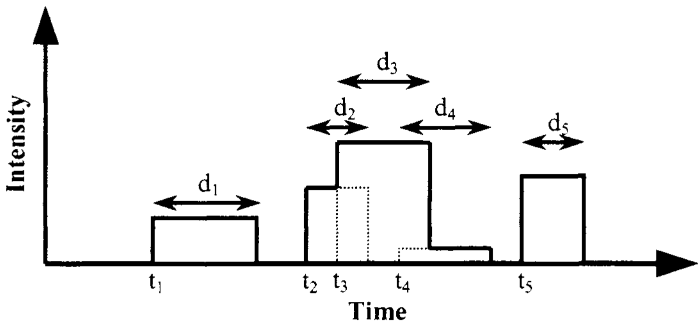


Figure 1 : Rectangular pulse process.

Under this last hypothesis the additional parameter, the variance $\sigma^2$ of the intensities distribution, must be taken into account. Some of the previous assumptions lead to a Markovian dependence structure of the PRP model.

$$f(d_i) = \eta e^{-\eta d} \qquad , \quad \eta > 0. \tag{1}$$

$$f(I_i) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}. \tag{2}$$

The analytical expression of the second order statistics (mean, variance and correlation structure) of the continuous stochastic process are given by Rodriguez-Iturbe et al.[5], [6], under the hypothesis of exponentially distributed intensity. Integrating the water intensities in disjoint time intervals of length T (aggregation time), the aggregated process Y is obtained. Under the hypothesis that the non-aggregated stochastic process is stationary in the interval T and in the case of normally distributed intensity, the second order statistics become:

$$E[Y] = \frac{T\lambda\mu}{\eta}. \tag{3}$$

$$VAR[Y] = \frac{2\lambda}{\eta^3}(\mu^2 + \sigma^2)(T\eta - 1 + e^{-T\eta}). \tag{4}$$

$$COV[Y_1, Y_k] = \frac{\lambda}{\eta^3}(\mu^2 + \sigma^2)(1 - e^{-T\eta})^2 e^{T\eta(k-2)}. \tag{5}$$

$$\rho_Y(1) = \frac{1}{2}\frac{(1 - e^{-\eta})^2}{(\eta - 1 + e^{-\eta})}. \tag{6}$$

It is important to underline, following Rodriguez Iturbe [5], the Markovian property of the correlation structure (6), which depends only on the mean duration of water uses and not on their frequency and intensity. The stochastic integrated process is non-homogeneous, being characterised by a temporal variability of its parameters at different time intervals T. The proposed model can describe either single or multiple residential water consumption in the case of homogeneous users. This means that the spatial aggregation scale has no influence on the model and that the model parameters do not depend on the number of water uses considered.

## 3 Parameters estimation

In the model proposed here four parameters were considered namely frequency $\lambda$, duration $\eta$, intensity with mean $\mu$ and variance $\sigma^2$. Their estimation has been accomplished with the methods of moments equating four statistical properties (moments) of the observed data with their analytical expression previously examined. In particular the mean, the variance, the lag-2 covariance and the lag-1 auto-correlation were considered. The resulting system of equation is non-linear. It has been solved with a non-linear least-squares algorithm. A further estimation procedure was also tested which fits approximately a larger set of sample moments, minimizing the objective function described by Cowpertwait et al.[4]. The results from this procedure are not presented in this paper, but they do not differ substantially from those reported.

## 4 Experimental data

The peculiar aspect of this work is the large number of experimental data obtained measuring water consumption in 85 residences into four blocks of the same building, located in the city of Latina, about 70 km south of Rome. The residences inhabitants are characterised by an homogeneous low social status. In particular the monitored building belongs to IIACP (*Italian Institute of Council House).* In each residence a single-jet, turbine-type water-meter with an extra-dry register has been installed. Each water-meter has been equipped with a remote communication system emitting a pulse every 2.5 litres of water consumption. The clock time of each pulse is recorded by four appropriate data-loggers, one for each block. Recorded data are periodically downloaded on a personal computer by phone connection. In fig.2 a short window of a recorded signal is showed. It is possible to notice its typical pulse shape, reminding the PRP pattern.

The availability of long time series of measured consumption data for such a large number of residences, makes possible to validate the proposed stochastic instantaneous water demand model and to obtain good estimations of its parameters.

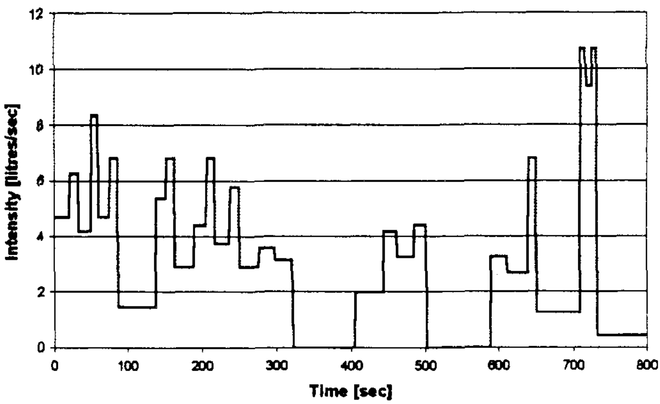


Figure 2: Water demand for a single user.

## 5 Parameters estimation and simulation

For each monitored residence, measured data were firstly stored on a daily base. The cumulated water consumption in one minute (aggregation time) was also evaluated obtaining time series of data on which the estimation procedure was performed. Then each day was divided in 24 hourly time intervals, in which the parameters of the stochastic process can be assumed stationary. Here we present the results of the parameters estimation from two different sets of data, belonging to two different periods of the year, namely January and April 2001. These two different periods of the year were chosen to investigate if the peculiarities of the season, particularly temperature, affect water residential consumption and the estimated parameters. For each of these periods two weeks data have been employed, in order to obtain in each of the 24 time intervals considered, an average estimation of parameters. It is important to highlight here, that the parameters of the proposed model, that is the frequency $\lambda$, the duration $\eta$, the intensity (with mean $\mu$ and variance $\sigma^2$) of the single water use are physically based and they can be easily compared with common water use practise. In fig.3 to fig. 5 estimated frequency, duration and intensity of the water demand of all 85 families are reported. The frequency of events, that is the number of uses per hour, shows an oscillating behaviour with two main peaks around 9.00-10.00 a.m. and 3.00 p.m., and two secondary peaks late in the evening. These peaks can be related to typical domestic duties. Also intensity and duration show an oscillating course even if in a narrower range. In particular it is interesting to notice the one hour shift between the duration behaviour of January and April, perhaps due to the different number of sun light hours. In both cases it should be noted a peak of duration in the morning, 7.00-8.00 a.m., and other secondary peaks distributed throughout the whole day. In both cases intensity shows two different mean values in the night and during the day. The former is about 3 l/m , the latter is about 6 l/m. Similar parameters behaviours has been obtained from single user consumption data. In this case the mean value of duration and intensity are comparable for all single users and for the aggregated demand. The number of event for each user is different in relation to specific habits. In tab.1 the mean values for estimated frequency, duration and intensities are reported together with a climatic parameter (temperature). It should be noted that only frequency is the critical parameter.

Table 1: Mean parameters values estimated for two different periods.

| | January | April | |
|---|---|---|---|
| *Mean temperature* | 11 | 16 | °C |
| *Mean intensity recorded* | 26 | 30 | $m^3$/day |
| *Mean frequency estimated* | 92.3 | 129.5 | uses/min |
| *Mean intensity estimated* | 5.6 | 5.2 | litres/min |
| *Mean duration estimated* | 2.0 | 2.0 | [min] |

Using the estimated parameters numerical simulations have been carried out. The results show good agreement with measured data. In fig. 6 to fig. 9 the second order statistics are reported for measured and simulated data in both cases of single and multiple users. In particular it should be noted the excellent fitting for the mean water demand, litres per minute, figs. 6 and 7.

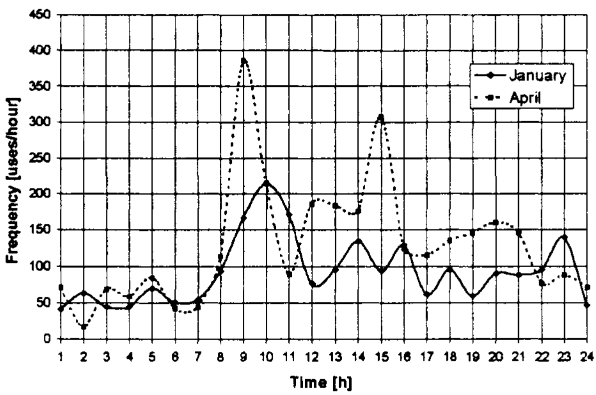


Figure 3: Estimated frequency of water uses.

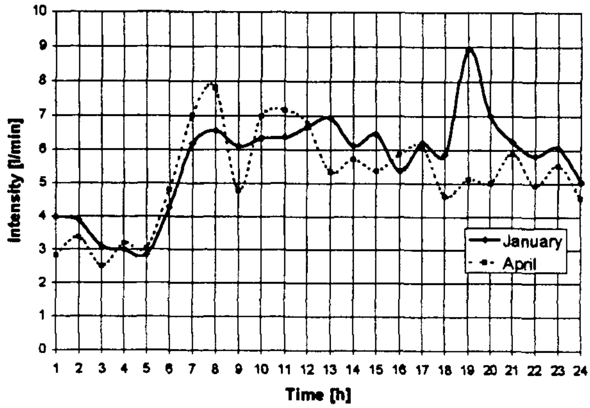


Figure 4: Estimated intensity of water uses.

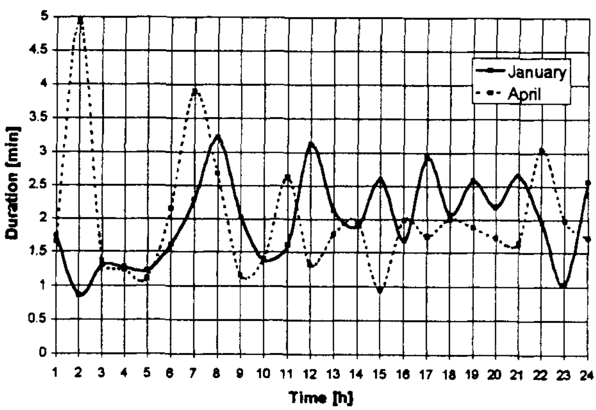


Figure 5: Estimated duration of water uses.

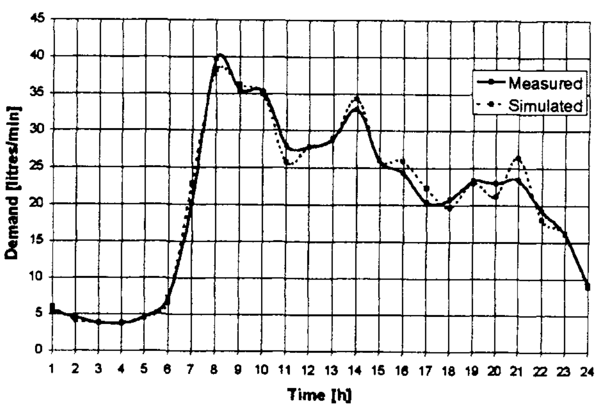


Figure 6: Mean water demand for all users.

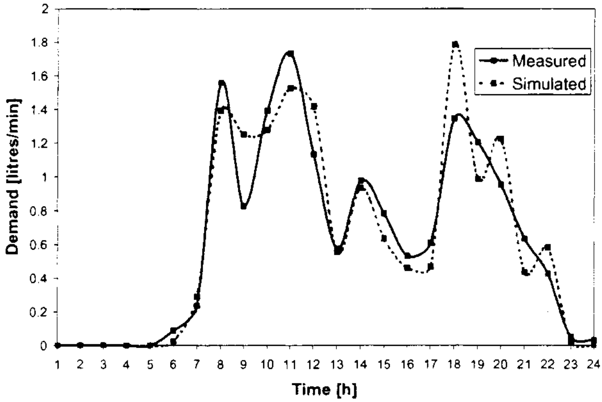


Figure 7: Mean water demand for a single user.

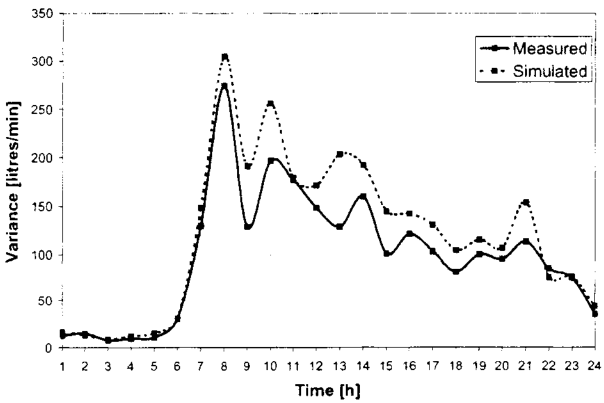


Figure 8: Variance of the water demand for all users.

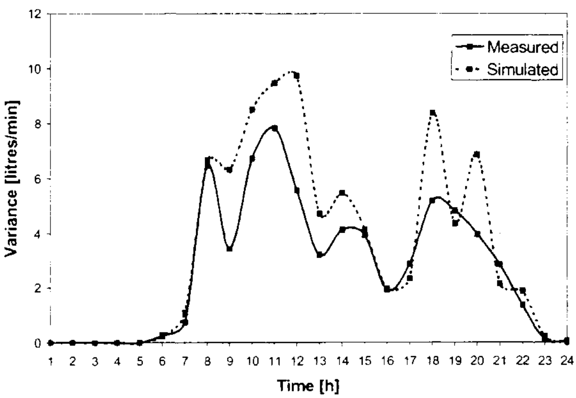


Figure 9: Variance of the water demand for a single user.

Transactions on Ecology and the Environment vol 48, © 2001 WIT Press, www.witpress.com, ISSN 1743-3541



The previous analysis of estimated parameters suggests the possibility of considering a constant mean value for intensity in the night, 3 l/m, and during the day, 6 l/m, and a constant mean value of duration throughout the day equal to 2 minutes. In this way a relevant simplification of the proposed model can be obtained. In fig.10 the mean water demand, litres per hour, obtained from measured data can be compared with the mean water demand resulting from the 4 parameters and 1 parameter numerical models. Results from the 4 parameters model, as already shown, agree very well with the experimental data. The 1 parameter model output shows a little worse fitting, but, considering the strong simplification that we have introduced the result is anyway interesting. In particular in this way the important role played by the frequency $\lambda$ in the residential water demand PRP model is highlighted.

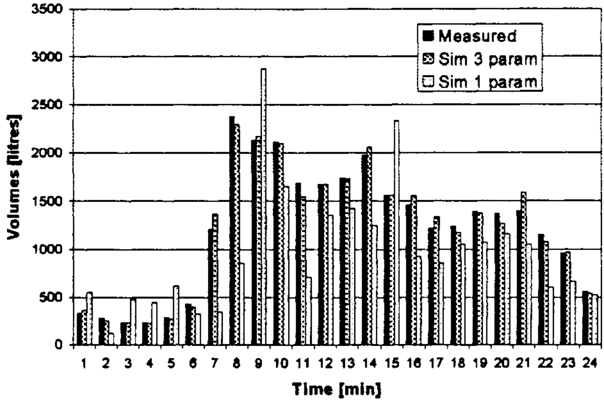


Figure 10: Mean water demand for all users.

## 6 Conclusions

In this paper instantaneous residential water demand has been described as a stochastic Poisson Rectangular Pulse process. The process is characterised by three main parameters: intensity, duration and frequency of demand pulse events. An additional parameter must be introduced having assumed a normally distributed intensity. Parameters estimation has been carried out using the method of moments from time series measured in 85 single family residences of the same building characterised by an homogeneous socio-economic status. Estimation has been performed both on single and multiple residences consumption data and for two different periods of the year. The mean values of the estimated intensity, duration and frequency agree with the typical values usually assumed for these parameters in Italy. Moreover, the estimated parameters are quite similar to those evaluated by Buchberger and Wells [3] monitoring for one year a small number of single-family residences in Ohio. Findings from the estimation presented here show a typical temporal behaviour throughout the day that seems to be also influenced by seasonal features. In particular frequency, that is the number of water uses per unit of time, looks like to be the more oscillating parameter and also the more affected by the period of the year.

Numerical simulations of instantaneous water demand have been also carried out introducing in the PRP model the estimated parameters. Visual comparison of measured and simulated water demand time series and of corresponding statistics reveal the excellent ability of the proposed model to describe the phenomenon. Further numerical simulation carried out with a simplified model in which only frequency is considered variable throughout the day still produce interesting results. Frequency can then be considered the most critical parameter in modeling instantaneous residential water demand.

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