

Instanton partition functions and M-theory^{*}

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Abstract. We discuss instanton partition functions in various spacetime dimensions. These partition functions capture some information about the spectrum of the supersymmetric gauge theories and their low-energy dynamics. Some of these theories can be defined microscopically only through string theory. Remarkably, they even know about the M-theory. Our conjectures include the identities between the generalization of the MacMahon formula and the character of M-theory, compactified down to $0 + 1$ dimension.

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