

Integer and Combinatorial Optimization

GEORGE NEMHAUSER

School of Industrial
and Systems Engineering
Georgia Institute of Technology
Atlanta, Georgia

LAURENCE WOLSEY

Center for Operations Research and Econometrics
Université Catholique de Louvain
Louvain-la-Neuve, Belgium



A Wiley-Interscience Publication

JOHN WILEY & SONS, INC.

New York • Chichester • Weinheim • Brisbane • Singapore • Toronto

Contents

PART I. FOUNDATIONS	1
I.1 The Scope of Integer and Combinatorial Optimization	3
1. Introduction	3
2. Modeling with Binary Variables I: Knapsack, Assignment and Matching, Covering, Packing and Partitioning	5
3. Modeling with Binary Variables II: Facility Location, Fixed-Charge Network Flow, and Traveling Salesman	7
4. Modeling with Binary Variables III: Nonlinear Functions and Disjunctive Constraints	10
5. Choices in Model Formulation	14
6. Preprocessing	17
7. Notes	20
8. Exercises	22
I.2 Linear Programming	27
1. Introduction	27
2. Duality	28
3. The Primal and Dual Simplex Algorithms	30
4. Subgradient Optimization	41
5. Notes	49
I.3 Graphs and Networks	50
1. Introduction	50
2. The Minimum-Weight or Shortest-Path Problem	55
3. The Minimum-Weight Spanning Tree Problem	60
4. The Maximum-Flow and Minimum-Cut Problems	62
5. The Transportation Problem: A Primal–Dual Algorithm	68
6. A Primal Simplex Algorithm for Network Flow Problems	76
7. Notes	82
I.4 Polyhedral Theory	83
1. Introduction and Elementary Linear Algebra	83
2. Definitions of Polyhedra and Dimension	85
3. Describing Polyhedra by Facets	88
4. Describing Polyhedra by Extreme Points and Extreme Rays	92
5. Polarity	98

6. Polyhedral Ties Between Linear and Integer Programs	104
7. Notes	109
8. Exercises	109
I.5 Computational Complexity	114
1. Introduction	114
2. Measuring Algorithm Efficiency and Problem Complexity	117
3. Some Problems Solvable in Polynomial Time	121
4. Remarks on 0-1 and Pure-Integer Programming	125
5. Nondeterministic Polynomial-Time Algorithms and \mathcal{NP} Problems	127
6. The Most Difficult \mathcal{NP} Problems: The Class $\mathcal{NP}^{\mathcal{C}}$	131
7. Complexity and Polyhedra	139
8. Notes	142
9. Exercises	143
I.6 Polynomial-Time Algorithms for Linear Programming	146
1. Introduction	146
2. The Ellipsoid Algorithm	147
3. The Polynomial Equivalence of Separation and Optimization	161
4. A Projective Algorithm	164
5. A Strongly Polynomial Algorithm for Combinatorial Linear Programs	172
6. Notes	180
I.7 Integer Lattices	182
1. Introduction	182
2. The Euclidean Algorithm	184
3. Continued Fractions	187
4. Lattices and Hermite Normal Form	189
5. Reduced Bases	195
6. Notes	201
7. Exercises	202
PART II. GENERAL INTEGER PROGRAMMING	203
II.1 The Theory of Valid Inequalities	205
1. Introduction	205
2. Generating All Valid Inequalities	217
3. Gomory's Fractional Cuts and Rounding	227
4. Superadditive Functions and Valid Inequalities	229
5. A Polyhedral Description of Superadditive Valid Inequalities for Independence Systems	237
6. Valid Inequalities for Mixed-Integer Sets	242
7. Superadditivity for Mixed-Integer Sets	246
8. Notes	254
9. Exercises	256

II.2 Strong Valid Inequalities and Facets for Structured Integer Programs	259
1. Introduction	259
2. Valid Inequalities for the 0-1 Knapsack Polytope	265
3. Valid Inequalities for the Symmetric Traveling Salesman Polytope	270
4. Valid Inequalities for Variable Upper-Bound Flow Models	281
5. Notes	290
6. Exercises	291
 II.3 Duality and Relaxation	 296
1. Introduction	296
2. Duality and the Value Function	300
3. Superadditive Duality	304
4. The Maximum-Weight Path Formulation and Superadditive Duality	308
5. Modular Arithmetic and the Group Problem	312
6. Lagrangian Relaxation and Duality	323
7. Benders' Reformulation	337
8. Notes	341
9. Exercises	343
 II.4 General Algorithms	 349
1. Introduction	349
2. Branch-and-Bound Using Linear Programming Relaxations	355
3. General Cutting-Plane Algorithms	367
4. Notes	379
5. Exercises	381
 II.5 Special-Purpose Algorithms	 383
1. Introduction	383
2. A Cutting-Plane Algorithm Using Strong Valid Inequalities	386
3. Primal and Dual Heuristic Algorithms	393
4. Decomposition Algorithms	409
5. Dynamic Programming	417
6. Notes	424
7. Exercises	427
 II.6 Applications of Special-Purpose Algorithms	 433
1. Knapsack and Group Problems	433
2. 0-1 Integer Programming Problems	456
3. The Symmetric Traveling Salesman Problem	469
4. Fixed-Charge Network Flow Problems	495
5. Applications of Basis Reduction	513
6. Notes	520
7. Exercises	526

PART III. COMBINATORIAL OPTIMIZATION	533
III.1 Integral Polyhedra	535
1. Introduction	535
2. Totally Unimodular Matrices	540
3. Network Matrices	546
4. Balanced and Totally Balanced Matrices	562
5. Node Packing and Perfect Graphs	573
6. Blocking and Integral Polyhedra	586
7. Notes	598
8. Exercises	602
III.2 Matching	608
1. Introduction	608
2. Maximum-Cardinality Matching	611
3. Maximum-Weight Matching	627
4. Additional Results on Matching and Related Problems	636
5. Notes	654
6. Exercises	655
III.3 Matroid and Submodular Function Optimization	659
1. Introduction	659
2. Elementary Properties	662
3. Maximum-Weight Independent Sets	666
4. Matroid Intersection	671
5. Weighted Matroid Intersection	678
6. Polymatroids, Separation, and Submodular Function Minimization	688
7. Algorithms To Minimize a Submodular Function	694
8. Covering with Independent Sets and Matroid Partition	702
9. Submodular Function Maximization	708
10. Notes	712
11. Exercises	714
References	721
Author Index	749
Subject Index	755