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The original version of this article unfortunately contained a mistake in the main title. Instead of “Integer codes correcting burst asymmetric within a byte and double asymmetric errors” the title should read “Integer codes correcting burst asymmetric errors within a byte and double asymmetric errors”.

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Integer codes correcting burst asymmetric within a byte and double asymmetric errors

Aleksandar Radonjic [1] Vladimir Vujicic [1]

Abstract
This paper presents a class of integer codes capable of correcting l-bit burst asymmetric errors within a b-bit byte ($1 \leq l < b$) and double asymmetric errors within a codeword. The presented codes are constructed with the help of a computer and have the potential to be used in unamplified optical networks. In addition, the paper derives the upper bound on code length and shows that the proposed codes are efficient in terms of redundancy.

Keywords Integer codes · Error correction · Asymmetric errors · Optical networks · Theoretical decoding throughput

1 Introduction

In most communication networks, it is impossible to predict the error behaviour. More precisely, one cannot know in advance which type of errors will be more frequent (1 → 0 or 0 → 1 errors) nor whether they will dominantly affect individual bits or several adjacent bytes.

However, there are networks where error types are known in advance. The best-known example are unamplified optical networks (UONs) (e.g. local and storage area networks) [1]. In these networks, photons (represented by binary 1's) may fade or fail to be detected, but new photons cannot be generated. So, if the receiver operates correctly, only asymmetric (1 → 0) errors may occur [2, 3]. An additional characteristic of these errors is that they always affect a small number of bits. This was first observed in [4] and then confirmed in [5–9]. In all these studies it was shown that 99.99% of all errors are either random errors (single or double errors) or bursts of length up to eight bits.

✉ Aleksandar Radonjic
sasa_radonjic@yahoo.com

Vladimir Vujicic
vujicicv@yahoo.com

[1] Institute of Technical Sciences of the Serbian Academy of Sciences and Arts, Belgrade, Serbia

Aware of this fact, researchers have developed a few classes of codes capable of correcting asymmetric errors [10–13]. Compared to their symmetric counterparts, these codes had simpler structure and lower redundancy. For this reason, they were more suitable for implementation in dedicated hardware. However, it is interesting to mention that none of these codes was ever considered to be used in UONs. The reason for this probably lies in the inability of [10–13] to correct both burst and random asymmetric errors. The mixtures of these errors are not rare, especially in cases when the error rate is high. So, if the codes from [10–13] were used, it would be impossible to provide high throughput and high reliability simultaneously.

Bearing this in mind, in this paper, we present a new class of integer codes called integer $B_{l/b}$AEC–DAEC codes. The proposed codes, like those in [14–20], have several desirable properties, including the capability to be interleaved without delay. However, unlike [14–20], the proposed codes can correct two types of errors: l-bit burst asymmetric errors within a b-bit byte (l/b BA errors) ($1 \leq l < b$) and double asymmetric (DA) errors within a codeword. Owing to this, they are better suited for protection of data in modern UONs.

The organization of this paper is as follows: Section 2 deals with the construction of integer $B_{l/b}$AEC–DAEC codes. The error correction procedure and theoretical decoding throughputs for these codes are described and evaluated in Section 3, while Section 4 concludes the paper.

2 Codes construction

Definition 1 [20] *Let $Z_{2^b-1}=\{0, 1,\dots, 2^b-2\}$ be the ring of integers modulo 2^b-1 and let $B_i=\sum_{n=0}^{b-1} a_n \cdot 2^n$ be the integer representation of a b-bit byte, where $a_n \in \{0, 1\}$ and $1 \leq i \leq k$. Then, the code $C(b, k, c)$, defined as*

$$C(b,k,c)=\left\{(B_1,B_2,\dots,B_k,B_{k+1}) \in Z_{2^b-1}^{k+1} : \sum_{i=1}^{k} C_i \cdot B_i \equiv B_{k+1} \left(\bmod 2^b-1\right)\right\} \quad (1)$$

is an $(kb+b, kb)$ integer code, where $c=(C_1, C_2, \dots, C_k, 1) \in Z_{2^b-1}^{k+1}$ is the coefficient vector and $B_{k+1} \in Z_{2^b-1}$ is an integer.

The first step to construct integer $B_{l/b}$AEC–DAEC codes is to determine the integer values of l/b BA and DA errors. When it comes to an l/b BA error, its integer value is already known [19]: it is equal to $e' = -2^r \cdot (2m-1)$, where $0 \leq r \leq b-l$, $1 \leq m \leq 2^{v-1}$ and $1 \leq v \leq l$. On the other hand, it is known that a DA error may corrupt one or two b-bit bytes. In the first scenario, its integer value will be equal to $e'' = -2^s - 2^r = -2^r \cdot (2^{s-r} + 1)$, where $0 \leq r < s \leq b-1$. In contrast, if a DA error corrupts two b-bit bytes, it will change their integer values by $e''' = -2^u$, where $0 \leq u \leq b-1$. On the basis of this and previous conclusions, we can give the following definitions.

Definition 2 *Let $x=(B_1, B_2,\dots, B_k, B_{k+1}) \in Z_{2^b-1}^{k+1}$, $y=(\underline{B}_1, \underline{B}_2,\dots, \underline{B}_k, \underline{B}_{k+1}) \in Z_{2^b-1}^{k+1}$ and $e=y-x=(\underline{B}_1-B_1, \underline{B}_2-B_2,\dots, \underline{B}_k-B_k, \underline{B}_{k+1}-B_{k+1})=(e_1, e_2,\dots, e_k, e_{k+1}) \in Z_{2^b-1}^{k+1}$ be respectively, the sent codeword, the received codeword and the error vector. Then, an*

$(kb + b, kb)$ *integer code is said to be a* $(B_lAEC)_b$*–DAEC code if it can correct error vectors from the set* $E = \{(e', 0, ..., 0, 0),..., (0, 0, ..., e', 0), (0, 0, ..., 0, -e'), (e'', 0, ..., 0, 0),..., (0, 0, ..., e'', 0), (0, 0, ..., 0, -e''), (e''', e''', ..., 0, 0), ..., (e''', 0, ..., e''', 0), (e''', 0, ..., 0, -e'''),..., (0, e''', ..., e''', 0), (0, e''', ..., 0, -e'''), ..., (0, 0, ..., e''', -e''')\}$, *where* $e' \in \{-2^r \cdot (2m-1): 0 \le r \le b-l, 1 \le m \le 2^{v-1}, 1 \le v \le l\}$, $e'' \in \{-2^r \cdot (2^{s-r}+1): 0 \le r < s \le b-1\}$ and $e''' \in \{-2^u: 0 \le u \le b-1\}$.

Definition 3 *The error set corresponding to l/b BA errors is defined as*

$$\varepsilon_1 = s_1 \cup s_2 \tag{2}$$

where

$$s_1 = \left\{-2^r \cdot (2m-1) \cdot C_i \left(\text{mod } 2^b - 1\right) : 0 \le r \le b-l, 1 \le m \le 2^{v-1}, 1 \le v \le l, 1 \le i \le k\right\} \tag{3}$$

$$s_2 = \left\{2^r \cdot (2m-1) \left(\text{mod } 2^b - 1\right) : 0 \le r \le b-l, 1 \le m \le 2^{v-1}, 1 \le v \le l\right\} \tag{4}$$

Definition 4 *The error set corresponding to DA errors corrupting one b-bit byte* (DA_1 *errors*) *is defined as*

$$\varepsilon_2 = s_3 \cup s_4 \tag{5}$$

where

$$s_3 = \left\{-2^r \cdot (2^{s-r}+1) \cdot C_i \left(\text{mod } 2^b - 1\right) : 0 \le r < s \le b-1, 1 \le i \le k\right\} \tag{6}$$

$$s_4 = \left\{2^r \cdot (2^{s-r}+1) \left(\text{mod } 2^b - 1\right) : 0 \le r < s \le b-1\right\} \tag{7}$$

Definition 5 *The error set corresponding to DA errors corrupting two b-bit bytes* (DA_2 *errors*) *is defined as*

$$\varepsilon_3 = s_5 \cup s_6 \tag{8}$$

where

$$s_5 = \left\{-2^r \cdot C_i - 2^s \cdot C_j \left(\text{mod } 2^b - 1\right) : 0 \le r, s \le b-1, 1 \le i < j \le k\right\} \tag{9}$$

$$s_6 = \left\{-2^r \cdot C_i + 2^s \left(\text{mod } 2^b - 1\right) : 0 \le r, s \le b–1, 1 \le i \le k\right\} \tag{10}$$

If we compare the sets ε_1 and ε_2, we will note that they have some common elements. The reason for this is that DA_1 errors spaced less than *l* bits can be treated as *l/b* BA errors. Hence, to avoid confusion, we will define the following set of syndromes.

Definition 6 *The set of syndromes corresponding to DA_1 errors excluding l/b BA errors is defined as*

$$\varepsilon_4 = s_7 \cup s_8 \tag{11}$$

where

$$s_7 = \left\{-2^r \cdot (2^{s-r}+1)\cdot C_i \left(\text{mod } 2^b-1\right) : l \le r+l \le s \le b-1, 1 \le i \le k\right\} \tag{12}$$

$$s_8 = \left\{2^r \cdot (2^{s-r}+1) \left(\text{mod } 2^b-1\right) : l \le r+l \le s \le b-1\right\} \tag{13}$$

Now we can prove the next theorem.

Theorem 1 *The codes defined by (1) can correct l/b BA errors and DA errors iff there exist k mutually different coefficients $C_i \in Z_{2^b-1} \backslash \{0,1\}$ such that*

1. $|\varepsilon_1| = \left[2^{l-1}\cdot(b-l+2)-1\right]\cdot(k+1)$
2. $|\varepsilon_3| = \dfrac{b^2 \cdot k}{2}\cdot(k+1)$
3. $|\varepsilon_4| = \dfrac{(b-l+1)\cdot(b-l)}{2}\cdot(k+1)$
4. $\varepsilon_1 \cap \varepsilon_3 \cap \varepsilon_4 = \varnothing$

where $|A|$ denotes the cardinality of A.

Proof The proof for Condition 1 is the same as that given in [19]. Hence, it will be omitted. As far as Condition 2 is concerned, it states that DA_2 errors generate $(b^2 \cdot k/2)\cdot(k-1)$ nonzero syndromes. To prove this, observe that the set ε_3 can be expressed as

$$\varepsilon_3 = \bigcup_{x=1}^{2k} X_x$$

where

$$\begin{aligned}
X_1 &= \left\{-2^r\cdot C_1 - 2^s \cdot C_j \left(\text{mod } 2^b-1\right) : 0 \le r,s \le b-1, 2 \le j \le k\right\}\\
X_2 &= \left\{-2^r\cdot C_1 + 2^s \left(\text{mod } 2^b-1\right) : 0 \le r,s \le b-1\right\}\\
X_3 &= \left\{-2^r\cdot C_2 - 2^s \cdot C_j \left(\text{mod } 2^b-1\right) : 0 \le r,s \le b-1, 3 \le j \le k\right\}\\
X_4 &= \left\{-2^r\cdot C_2 + 2^s \left(\text{mod } 2^b-1\right) : 0 \le r,s \le b-1\right\}\\
&\vdots\\
X_{2k-3} &= \left\{-2^r\cdot C_{k-2} - 2^s \cdot C_j \left(\text{mod } 2^b-1\right) : 0 \le r,s \le b-1, k-1 \le j \le k\right\}\\
X_{2k-2} &= \left\{-2^r\cdot C_{k-1} + 2^s \left(\text{mod } 2^b-1\right) : 0 \le r,s \le b-1\right\}\\
X_{2k-1} &= \left\{-2^r\cdot C_{k-1} - 2^s \cdot C_j \left(\text{mod } 2^b-1\right) : 0 \le r,s \le b-1, j=k\right\}\\
X_{2k} &= \left\{-2^r\cdot C_k + 2^s \left(\text{mod } 2^b-1\right) : 0 \le r, s \le b-1\right\}
\end{aligned}$$

The elements of the above sets will be nonzero and mutually different only if there exist k coefficients $C_i \in Z_{2^b-1} \backslash \{0, 1\}$ such that

$$\begin{aligned}
&\bigcap_{x=1}^{2k} X_x = \varnothing \\
&|X_1| = b^2 \cdot (k-1) \\
&|X_2| = b^2 \\
&|X_3| = b^2 \cdot (k-2) \\
&|X_4| = b^2 \\
&\quad \vdots \\
&|X_{2k-3}| = b^2 \cdot 2 \\
&|X_{2k-2}| = b^2 \\
&|X_{2k-1}| = b^2 \cdot 1 \\
&|X_{2k}| = b^2
\end{aligned}$$

As a result, it follows that

$$|\varepsilon_3| = \sum_{x=1}^{2k} |X_x| = b^2 \cdot k + b^2 \cdot \sum_{n=1}^{k-1} n = \frac{b^2 \cdot k}{2} \cdot (k+1).$$

In a similar way, Condition 3 says that DA_1 errors excluding l/b BA errors generate $(b-l+1) \cdot (b-l) \cdot (k+1)/2$ syndromes that are nonzero. To prove this, note that the set ε_4 can be expressed as

$$\varepsilon_4 = \bigcup_{y=0}^{2(b-l-1)} Y_y$$

where

$$\begin{aligned}
&Y_0 = \left\{-2^r \cdot \left(2^l + 1\right) \cdot C_i \left(\text{mod } 2^b - 1\right) : 0 \le r \le b-l-1, 1 \le i \le k\right\} \\
&Y_1 = \left\{2^r \cdot \left(2^l + 1\right) \left(\text{mod } 2^b - 1\right) : 0 \le r \le b-l-1\right\} \\
&Y_2 = \left\{-2^r \cdot \left(2^{l+1} + 1\right) \cdot C_i \left(\text{mod } 2^b - 1\right) : 0 \le r \le b-l-2, 1 \le i \le k\right\} \\
&Y_3 = \left\{2^r \cdot \left(2^{l+1} + 1\right) \left(\text{mod } 2^b - 1\right) : 0 \le r \le b-l-2\right\} \\
&\quad \vdots \\
&Y_{2(b-l-1)-3} = \left\{-2^r \cdot \left(2^{b-2} + 1\right) \cdot C_i \left(\text{mod } 2^b - 1\right) : 0 \le r \le 1, 1 \le i \le k\right\} \\
&Y_{2(b-l-1)-2} = \left\{2^r \cdot \left(2^{b-2} + 1\right) \left(\text{mod } 2^b - 1\right) : 0 \le r \le 1\right\}. \\
&Y_{2(b-l-1)-1} = \left\{-2^r \cdot \left(2^{b-1} + 1\right) \cdot C_i \left(\text{mod } 2^b - 1\right) : r = 0, 1 \le i \le k\right\} \\
&Y_{2(b-l-1)} = \left\{2^r \cdot \left(2^{b-1} + 1\right) \left(\text{mod } 2^b - 1\right) : r = 0\right\}.
\end{aligned}$$

Obviously, if the coefficients $C_i \in Z_{2^b-1} \backslash \{0, 1\}$ have values such that

$$\begin{aligned}
&\bigcap_{y=0}^{2(b-l-1)} Y_y = \varnothing \\
&|Y_0| = (b-l) \cdot k \\
&|Y_1| = b-l \\
&|Y_2| = (b-l-1) \cdot k \\
&|Y_3| = b-l-1 \\
&\quad \vdots \\
&|Y_{2(b-l-1)-3}| = 2 \cdot k \\
&|Y_{2(b-l-1)-2}| = 2 \\
&|Y_{2(b-l-1)-1}| = 1 \cdot k \\
&|Y_{2(b-l-1)}| = 1.
\end{aligned}$$

then

$$|\varepsilon_4| = \sum_{y=0}^{2(b-l-1)} |Y_y| = (k+1)\cdot \sum_{y=0}^{b-l-1} (b-l-y) = \frac{(b-l+1)\cdot(b-l)}{2}\cdot(k+1).$$

Finally, Condition 4 is a necessary condition for distinguishing l/b BA errors from DA errors. So, $(kb+b,\ kb)$ integer $B_{l/b}$AEC–DAEC codes must satisfy all the conditions 1 to 4. Conversely, the codes satisfying conditions 1 to 4 allow us to distinguish l/b BA errors from DA errors. Then we can correct all l/b BA errors and all DA errors. Therefore, these codes are $(kb+b,\ kb)$ integer $B_{l/b}$AEC–DAEC codes. □

Theorem 2 *Let* $\xi = \varepsilon_1 \cup \varepsilon_3 \cup \varepsilon_4$ *be the error set for* $(kb+b,\ kb)$ *integer* $B_{l/b}$*AEC–DAEC codes. Then,*

$$\begin{aligned}|\xi| &= |\varepsilon_1| + |\varepsilon_3| + |\varepsilon_4| \\ &= \left\lfloor \frac{2^l\cdot(b-l+2) + b^2\cdot k + (b-l+1)\cdot(b-l) - 2}{2} \right\rfloor \cdot (k+1).\end{aligned}$$

Proof This theorem follows directly from Theorem 1. □

Now, by knowing the cardinality of ξ, we can derive the upper bound on code length.

Theorem 3 *For any* $(kb+b,\ kb)$ *integer* $B_{l/b}$*AEC–DAEC code it holds that*

$$k \le \left\lfloor \frac{\sqrt{\left(2^{b+1} - z - 4\right)\cdot 4b^2 + \left(b^2+z\right)^2} - b^2 - z}{2b^2} \right\rfloor,$$

where $z = 2^l\cdot(b-l+2) + (b-l+1)\cdot(b-l) - 2.$

Proof From Definition 1 we know that the total number of nonzero syndromes is $2^b - 2$. On the other hand, from Theorem 2 we know that the set ξ has $|\xi| = (z + b^2\cdot k)\cdot(k+1)/2$ nonzero elements. Consequently, we have the inequality

$$\frac{\left(z + b^2\cdot k\right)\cdot(k+1)}{2} \le 2^b - 2$$

from where it follows that

$$k \le \left\lfloor \frac{\sqrt{\left(2^{b+1} - z - 4\right)\cdot 4b^2 + \left(b^2+z\right)^2} - b^2 - z}{2b^2} \right\rfloor. \square$$

From the above it is obvious that the coefficients C_i cannot be generated without using a computer. For this reason, it was necessary to perform computer-based searches. For the purpose of this paper, we have restricted ourselves to the codes with parameters $8 \le$

$l \leq 9$, $b = 32$ and $k \leq 32$ (Table 1). Besides this, we have partially investigated how the number of C_i's depends on the values of b and l. Although the obtained results are far from the theoretical maximum (Table 2), the proposed codes are very efficient in terms of redundancy. This can be seen from the fact that two integer $B_{l/b}$AEC–DAEC codes have $b = 2l$ check bits (the (32, 16) integer $B_{8/16}$AEC–DAEC code and the (36, 18) integer $B_{9/18}$AEC–DAEC code), while l-bit burst error correcting Fire codes use at least $3l + 1$ check bits [21].

3 Error control procedure and theoretical decoding throughput

The error correction procedure for the proposed codes is similar to that described in [15]. In short, it consists of two steps: obtaining the error correction data from the syndrome table and executing one of the following operations:

- *for l/b BA and DA_1 errors*

$$B_i = \underline{B}_i + E_1 \left(\bmod 2^b - 1\right), 1 \leq i \leq k + 1; \tag{14}$$

$E_1 \in \{2^r \cdot (2m - 1)\colon 0 \leq r \leq b - l,\ 1 \leq m \leq 2^{v-1},\ 1 \leq v \leq l\} \cup \{2^r \cdot (2^{s-r} + 1)\colon 0 \leq r < s \leq b - 1\}$

- *for DA_2 errors*

$$B_i = \underline{B}_i + E_1 \left(\bmod 2^b - 1\right), 1 \leq i \leq k; \tag{15}$$

$$B_j = \underline{B}_j + E_2 \left(\bmod 2^b - 1\right), i < j \leq k + 1; \tag{16}$$

$E_1, E_2 \in \{2^u\colon 0 \leq u \leq b - 1\}$.

To generate the syndome table it is necessary to substitute the values of l, b and C_i into (2)–(4) and (8)–(13). In this way, exactly $|\xi|$ relationships (Theorem 2) between the syndrome (element of the set ξ), error location (i, j) and error vector (E_1, E_2) will be

Table 1 First 32 coefficients for some integer $B_{l/32}$AEC-DAEC codes

$l = 8$							
515	533	553	603	719	1153	1263	1317
4159	4747	5811	7557	9121	13679	18557	19741
23951	30511	31223	44615	45017	49263	52075	54421
56299	69621	80371	102001	105277	112425	114387	144093
$l = 9$							
1027	1037	1081	1167	1217	1385	1483	2213
2551	3339	5295	7411	10997	11365	13233	16795
18617	26351	27609	30417	39257	43611	46701	55825
64389	77159	89799	99699	104851	107481	115583	122463

Table 2 Number of coefficients for some integer $B_{l/b}$AEC-DAEC codes

		$b=16$	$b=17$	$b=18$	$b=19$	$b=20$	$b=21$	$b=22$	$b=23$	$b=24$
Theoretical bound	$l=8$	17	24	35	48	67	92	126	173	236
	$l=9$	14	21	31	45	63	89	123	169	233
Computer-search result	$l=8$	1	2	3	5	7	11	14	17	22
	$l=9$	0	0	1	3	4	6	9	14	19

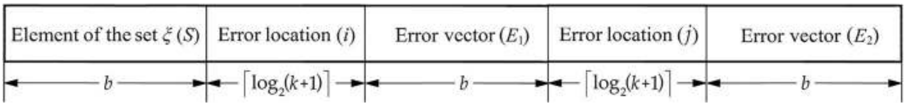


Fig. 1 Bit-width of one syndrome table entry

established (Fig. 1). So, when $S \neq 0$, the decoder's task is to find the entry with the first b bits as that of the syndrome S. If the syndrome table is sorted (according to the elements of ξ), this task will be completed after n_{TL} table lookups $(1 \leq n_{TL} \leq \lfloor log_2|\xi| \rfloor + 2)$ [22].

To illustrate the effectiveness of this approach, suppose that the network nodes (computers, servers, routers, etc.) are equipped with four-core processors having the same specifications as in [19]. In that case, each node will require one second to decode

$$G = \frac{\left(3.5 \cdot 10^9\right) \cdot 128 \cdot k}{9 \cdot k + 29 \cdot n_{TL} + 4} \tag{17}$$

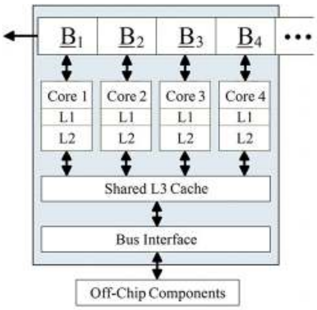
data bits [19]. In addition, in [19] it was shown that each core uses the operation.

$$S_t = \sum_{i=1}^{k} C_i \cdot \underline{B}_{4\cdot(i-1)+t} - \underline{B}_{4\cdot k+t} \ \left(\bmod 2^{32}-1\right), t = 1, 2, ..., 4. \tag{18}$$

to calculate the syndrome S_t. Applying these results to the presented theory, we can easily assure that, for smaller values of k, all considered codes have the potential to be used in 10G networks (Fig. 2) [1]. In addition, from Fig. 2 we also see that the codes with code rate 1024/1056 have theoretical throughput of 15.91 Gbps. Hence, they could be candidates for use in 16G Fibre Channel networks [1]. Finally, from (18) it is evident that all analyzed codes are interleaved at the byte level. Thanks to this, they are able to correct various types of errors, including t DA errors and t BA errors less than $l+1$ bits $(1 \leq t \leq 4)$.

4 Conclusion

In this paper, we have presented a new class of integer error control codes. We have shown that the presented codes have three characteristics: first, they can correct l-bit burst asymmetric errors within a b-bit byte and double asymmetric errors within a codeword, second, they use processor-friendly operations, and third, they can be interleaved without delay and without using dedicated hardware. Owing to these



(a)

Integer $B_{l/32}$AEC–DAEC Code	k	l	Memory Requirements for Storing the Syndrome Table	Number of Table Lookups	Minimum Theoretical Decoding Throughput
(544, 512)	16	8	2.66 MB	$1 \le n_{TL} \le 19$	10.25 Gbps
(544, 512)	16	9	3.35 MB	$1 \le n_{TL} \le 19$	10.25 Gbps
(800, 768)	24	8	5.27 MB	$1 \le n_{TL} \le 20$	13.44 Gbps
(800, 768)	24	9	6.28 MB	$1 \le n_{TL} \le 20$	13.44 Gbps
(1056, 1024)	32	8	8.91 MB	$1 \le n_{TL} \le 21$	15.91 Gbps
(1056, 1024)	32	9	10.27 MB	$1 \le n_{TL} \le 21$	15.91 Gbps

(b)

Fig. 2 **a** Block diagram of four-core processor and **b** theoretical decoding throughputs for some four-byte interleaved integer $B_{l/32}$AEC–DAEC codes

features, the presented codes can be transformed into codes capable of correcting various mixtures of asymmetric errors. Such constructed codes could have great potential for practical use, especially in optical networks for short-haul applications.

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