Integrated fault estimation and accommodation design for discrete-time Takagi-Sugeno fuzzy systems with actuator faults

Integrated Fault Estimation and Accommodation Design for Discrete-Time Takagi-Sugeno Fuzzy Systems with Actuator Faults

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***Abstract*—This paper addresses the problem of integrated robust fault estimation (FE) and accommodation for discrete-time Takagi-Sugeno (T-S) fuzzy systems. First, a multiconstrained reduced-order FE observer (RFEO) is proposed to achieve FE for discrete-time T-S fuzzy models with actuator faults. Based on the RFEO, a new fault estimator is constructed. Then, using the information of on-line FE, a new approach for fault accommodation based on fuzzy dynamic output feedback is designed to compensate for the effect of faults by stabilizing the closed-loop systems. Moreover, the RFEO and the dynamic output feedback fault tolerant controller are designed separately, such that their design parameters can be calculated readily. Simulation results are presented to illustrate our contributions.**

***Index Terms*—Fault estimation, fault accommodation, T-S fuzzy models, discrete-time systems.**

I. Introduction

The demand for increased productivity leads to more challenging operating conditions for many practical engineering systems. Such conditions would increase the possibility of system failures. Sensor, actuator or component failures may drastically change the system behavior, resulting in degradation or even instability. In order to improve efficiency, the reliability can be achieved by fault detection and isolation (FDI) and fault tolerant control (FTC), so FDI and FTC have been the subjects of intensive investigations over the past two decades. Fruitful results can be found in several excellent books [1]–[3], survey papers [4]–[6] and the references therein.

Since most practical systems are nonlinear in nature, FDI/FTC applications to industrial and commercial processes need nonlinear models to be specifically considered. Takagi-Sugeno (T-S) fuzzy models are based on a set of if-then rules which give a local linear representation of an underlying nonlinear system and it is well-known that such models can describe or approximate a class of nonlinear systems. Therefore, they have attracted considerable attention and given rise to many important results in the past decades [7]–[19]. However, note that most address stability analysis and feedback stabilization, and very few address the issues of fault estimation (FE) and accommodation.

For discrete-time systems, the topic on fault detection has attracted considerable attention [20]–[24], but FDI is only the first step in fault accommodation (FA). FE is utilized to on-line determine the magnitude of the fault, but this is not an easy task. Compared with fault detection, FE is more challenging and has motivated few attention for discrete-time systems. Furthermore, using the obtained fault information, the FA module can be used to compensate for the effect of the fault, so it is shown that the issue of FE is more meaningful and challenging. Meanwhile, most continuous-time control systems being implemented digitally, FE design for discrete-time cases is more practical, but it has motivated few attention. A learning approximation approach proposed in [25], [26] assumed the occurred faults belonged to a special structure and did not consider FE performance. The problem of FE filter was dealt with in [24], [27], but the filter design was only suitable for open-loop stable systems. In practical situations, most of systems are open-loop unstable, so such a constraint limits its application scopes. In [28], [29], one FE method based on a special coordinate transformation was studied, but the assumption that rank(C_iB_i) are of full-column rank was required. Also, the on-line fault estimate at time k needed the output vector at time $k+1$. Due to the introduction of an estimation delay, such a case could be not suitable for practical situations. In [30], [31], the given proportional integral observer could realize constant unknown input estimation by constructing augmented systems, but the estimation performance was not considered.

Based on the above works, our objective of this paper is to analyze and develop a general framework of robust FE and accommodation for discrete-time T-S fuzzy systems with actuator faults. The main contributions of this paper are summarized in three aspects.

–First, a multiconstrained fuzzy reduced-order FE observer (RFEO) including an H_∞ performance level and a regional pole constraint is proposed, not only to guarantee the convergence speed of FE, but also to attenuate the influence of disturbances as much as possible.

–Second, compared with the FE methods in [28], [29], the proposed RFEO design has a wider application range, and uses the current output information to enhance FE performances.

–Third, using the on-line fault estimate, a discrete-time

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fuzzy dynamic output feedback fault tolerant controller (DOFFTC) is designed to guarantee the system stability in the presence of actuator faults.

In the whole design process, the RFEO and the DOFFTC are designed separately and their performances are considered simultaneously, which is convenient to calculate design parameters.

The rest of this paper is organized as follows. Section 2 presents the system description. In Section 3, a fuzzy RFEO design including an H_∞ performance index and a regional pole constraint is proposed to estimate the fault vector. Furthermore, in Section 4, based on the on-line fault estimate, a new FA design is proposed to compensate for the effect of faults. Simulation results of a discrete-time nonlinear truck-trailer model are presented to illustrate the effectiveness of the proposed method in Section 5, followed by some concluding remarks in Section 6.

II. System Description

The T-S fuzzy model is described by fuzzy IF-THEN rules, whose collection represent the approximation of the nonlinear system. The ith rule of the T-S fuzzy model is of the following form.

Plant Rule i:
IF $\vartheta_1(k)$ is π_{i1} and ... and $\vartheta_s(k)$ is π_{is}, THEN

$$x(k+1) = A_i x(k) + B_i\big(u(k) + f(k)\big) + D_i\omega(k), \quad (1)$$
$$y(k) = C_i x(k), \quad (2)$$
$$z_L(k) = C_{Li}x(k), \quad (3)$$

where $x(k) \in \mathbb{R}^n$ is the state, $u(k) \in \mathbb{R}^m$ is the input, $y(k) \in \mathbb{R}^p$ is the measurable output, $z_L(k) \in \mathbb{R}^l$ is the controlled output, $f(k) \in \mathbb{R}^m$ represents the additive actuator fault, and $\omega(k) \in \mathbb{R}^d$ is the disturbance which is assumed to belong to $l_2[0,\infty)$. The number of measurable output channels is greater than or equal to the number of input ones, i.e. $p \geq m$. A_i, B_i, C_i, C_{Li} and D_i are constant real matrices of appropriate dimensions. It is supposed that matrices B_i and C_i are of full rank, i.e. $\text{rank}(B_i) = m$ and $\text{rank}(C_i) = p$, and the pairs (A_i, B_i) and (A_i, C_i) are respectively controllable and observable. $\vartheta_j(k)(j = 1,\ldots,s)$ are the premise variables, $\pi_{ij}(i = 1,\ldots,q; j = 1,\ldots,s)$ are the fuzzy sets that are characterized by membership function, q is the number of IF-THEN rules and s is the number of the premise variables. The overall fuzzy model achieved by fuzzy blending of each individual plant rule (local model) is given by

$$x(k+1) = \sum_{i=1}^{q} h_i\big(\vartheta(k)\big)\Big[A_i x(k) + B_i\big(u(k) + f(k)\big) + D_i\omega(k)\Big], \quad (4)$$

$$y(k) = \sum_{i=1}^{q} h_i\big(\vartheta(k)\big)C_i x(k), \quad (5)$$

$$z_L(k) = \sum_{i=1}^{q} h_i\big(\vartheta(k)\big)C_{Li}x(k), \quad (6)$$

where

$$\vartheta(k) = \big[\vartheta_1(k),\ldots,\vartheta_s(k)\big], \quad h_i(\vartheta(k)) = \frac{\sigma_i(\vartheta(k))}{\sum_{i=1}^{q}\sigma_i(\vartheta(k))},$$
$$\sigma_i(\vartheta(k)) = \prod_{j=1}^{s}\pi_{ij}(\vartheta_j(k))$$

and $\pi_{ij}(\cdot)$ is the grade of the membership function of π_{ij}. We assume

$$\sigma_i(\vartheta(k)) \geq 0, \quad i = 1,\ldots,q, \quad \sum_{i=1}^{q}\sigma_i(\vartheta(k)) > 0 \quad (7)$$

for any $\vartheta(k)$. Hence $h_i(k)$ satisfies

$$h_i(\vartheta(k)) \geq 0, \quad i = 1,\ldots,q, \quad \sum_{i=1}^{q}h_i(\vartheta(k)) = 1 \quad (8)$$

for any $\vartheta(k)$.

For simplicity, we introduce the following notations:

$$h_i = h_i(\vartheta(k)), \quad A(h) = \sum_{i=1}^{q}h_i(\vartheta(k))A_i,$$
$$B(h) = \sum_{i=1}^{q}h_i(\vartheta(k))B_i, \quad D(h) = \sum_{i=1}^{q}h_i(\vartheta(k))D_i,$$
$$C(h) = \sum_{i=1}^{q}h_i(\vartheta(k))C_i, \quad C_L(h) = \sum_{i=1}^{q}h_i(\vartheta(k))C_{Li},$$

then the T-S fuzzy model (4)–(6) can be rewritten as

$$x(k+1) = A(h)x(k) + B(h)\big(u(k) + f(k)\big) + D(h)\omega(k), \quad (9)$$
$$y(k) = C(h)x(k), \quad (10)$$
$$z_L(k) = C_L(h)x(k). \quad (11)$$

Assumption 1. For the measurable output matrices, $C := C_1 = C_2 = \cdots = C_q$.

Remark 1. In many practical systems, since the measurable output usually is a part of the states, which means that the measurable output matrices would be common for each local model. Assumption 1 is not very restrictive and widely used in T-S fuzzy systems [15], [32].

Under Assumption 1, the T-S fuzzy model (9)–(11) can be expressed as

$$x(k+1) = A(h)x(k) + B(h)\big(u(k) + f(k)\big) + D(h)\omega(k), \quad (12)$$
$$y(k) = Cx(k), \quad (13)$$
$$z_L(k) = C_L(h)x(k). \quad (14)$$

Remark 2. In this paper, we only focus on a class of additive actuator faults, for the sake of clarity. However, please note that such kind of additive actuator faults considered in this paper can be readily extended to general class of additive faults [1], [33]. Meanwhile, the additive faults representation is more general than the multiplicative ones, which can be modelled as additive actuator ones [3].

Before ending this section, the following lemma, which will be used to present our main results, is recalled.

Lemma 1 [34]. The eigenvalues of a given matrix $\mathcal{A} \in \mathbb{R}^{n\times n}$ belong to the closed circular region $\mathcal{D}(\alpha,\tau)$ with center $\alpha+j0$ and radius τ if and only if there exists a symmetric positive definite matrix $\mathcal{P} \in \mathbb{R}^{n\times n}$ such that the following condition holds:

$$\begin{bmatrix} -\mathcal{P} & \mathcal{P}(\mathcal{A}-\alpha I_n) \\ * & -\tau^2\mathcal{P} \end{bmatrix} \le 0, \tag{15}$$

where here and everywhere in the sequel, $*$ denotes the symmetric elements in a symmetric matrix.

III. Fault Estimation Design

A. State transformation

Note that since C is of full-row rank, there always exists a matrix $C^{\perp} \in \mathbb{R}^{(n-p)\times n}$ such that $\begin{bmatrix} C^{\perp} \\ C \end{bmatrix} \in \mathbb{R}^{n\times n}$ is nonsingular (indeed $C^{\perp}$ can be chosen as an orthogonal basis of the null-space of C). Then under the coordinate transformation $x(k) = Tz(k)$, where $T = \begin{bmatrix} C^{\perp} \\ C \end{bmatrix}^{-1}$, the dynamics (12) and (13) can be decomposed into the following form:

$$\begin{bmatrix} z_1(k+1) \\ z_2(k+1) \end{bmatrix} = \sum_{i=1}^{q} h_i \left\{ \begin{bmatrix} A_{11i} & A_{12i} \\ A_{21i} & A_{22i} \end{bmatrix} \begin{bmatrix} z_1(k) \\ z_2(k) \end{bmatrix} + \begin{bmatrix} B_{1i} \\ B_{2i} \end{bmatrix} \big(u(k)+f(k)\big) + \begin{bmatrix} D_{1i} \\ D_{2i} \end{bmatrix} \omega(k) \right\}, \tag{16}$$

$$y(k) = \begin{bmatrix} 0_{p\times(n-p)} & I_p \end{bmatrix} \begin{bmatrix} z_1(k) \\ z_2(k) \end{bmatrix}, \tag{17}$$

where $z_1(k) \in \mathbb{R}^{n-p}$ and $z_2(k) \in \mathbb{R}^{p}$ are new state vectors, and

$$z(k) = \begin{bmatrix} z_1(k) \\ z_2(k) \end{bmatrix}, \quad T^{-1}A_iT = \begin{bmatrix} A_{11i} & A_{12i} \\ A_{21i} & A_{22i} \end{bmatrix},$$
$$T^{-1}B_i = \begin{bmatrix} B_{1i} \\ B_{2i} \end{bmatrix}, \quad T^{-1}D_i = \begin{bmatrix} D_{1i} \\ D_{2i} \end{bmatrix},$$
$$CT = \begin{bmatrix} 0_{p\times(n-p)} & I_p \end{bmatrix}.$$

Let

$$A_{11}(h) = \sum_{i=1}^{q} h_i A_{11i}, \quad A_{12}(h) = \sum_{i=1}^{q} h_i A_{12i},$$
$$A_{21}(h) = \sum_{i=1}^{q} h_i A_{21i}, \quad A_{22}(h) = \sum_{i=1}^{q} h_i A_{22i},$$
$$B_1(h) = \sum_{i=1}^{q} h_i B_{1i}, \quad B_2(h) = \sum_{i=1}^{q} h_i B_{2i},$$
$$D_1(h) = \sum_{i=1}^{q} h_i D_{1i}, \quad D_2(h) = \sum_{i=1}^{q} h_i D_{2i},$$

then it follows that

$$\begin{bmatrix} z_1(k+1) \\ z_2(k+1) \end{bmatrix} = \begin{bmatrix} A_{11}(h) & A_{12}(h) \\ A_{21}(h) & A_{22}(h) \end{bmatrix} \begin{bmatrix} z_1(k) \\ z_2(k) \end{bmatrix} + \begin{bmatrix} B_1(h) \\ B_2(h) \end{bmatrix} \big(u(k)+f(k)\big) + \begin{bmatrix} D_1(h) \\ D_2(h) \end{bmatrix} \omega(k), \tag{18}$$

$$y(k) = \begin{bmatrix} 0_{p\times(n-p)} & I_p \end{bmatrix} \begin{bmatrix} z_1(k) \\ z_2(k) \end{bmatrix}, \tag{19}$$

which can be rewritten as

$$\begin{aligned} z_1(k+1) &= A_{11}(h)z_1(k) + A_{12}(h)y(k) + B_1(h)u(k) + \\ &\quad B_1(h)f(k) + D_1(h)\omega(k), \end{aligned} \tag{20}$$
$$\begin{aligned} y(k+1) &= A_{21}(h)z_1(k) + A_{22}(h)y(k) + B_2(h)u(k) + \\ &\quad B_2(h)f(k) + D_2(h)\omega(k). \end{aligned} \tag{21}$$

By introducing the virtual input and output vectors $\eta(k)$ and $\rho(k)$

$$\eta(k) = A_{12}(h)y(k) + B_1(h)u(k),$$
$$\rho(k) = y(k+1) - A_{22}(h)y(k) - B_2(h)u(k),$$

one gets

$$\begin{aligned} z_1(k+1) &= A_{11}(h)z_1(k) + \eta(k) + B_1(h)f(k) + \\ &\quad D_1(h)\omega(k), \end{aligned} \tag{22}$$
$$\rho(k) = A_{21}(h)z_1(k) + B_2(h)f(k) + D_2(h)\omega(k). \tag{23}$$

B. RFEO design

For the dynamics (22) and (23), we construct the following fuzzy RFEO:

$$\begin{aligned} \hat{z}_1(k+1) &= A_{11}(h)\hat{z}_1(k) + \eta(k) + B_1(h)\hat{f}(k) - \\ &\quad G(h)\big(\hat{\rho}(k) - \rho(k)\big), \end{aligned} \tag{24}$$
$$\hat{\rho}(k) = A_{21}(h)\hat{z}_1(k) + B_2(h)\hat{f}(k), \tag{25}$$
$$\hat{f}(k+1) = \hat{f}(k) - F(h)\big(\hat{\rho}(k) - \rho(k)\big), \tag{26}$$

where $\hat{z}_1(k) \in \mathbb{R}^{n-p}$ is the reduced-order observer state, $\hat{\rho}(k) \in \mathbb{R}^{p}$ is the reduced-order observer output, $\hat{f}(k) \in \mathbb{R}^{m}$ is an estimate of $f(k)$, and $G(h) \in \mathbb{R}^{(n-p)\times p}$, $F(h) \in \mathbb{R}^{m\times p}$ are reduced-order observer gain matrices to be designed, $G(h) = \sum_{i=1}^{q} h_iG_i$, $F(h) = \sum_{i=1}^{q} h_iF_i$.

Let $e_{z1}(k) = \hat{z}_1(k) - z_1(k)$ and $e_f(k) = \hat{f}(k) - f(k)$, then the error dynamics is given by

$$\begin{aligned} e_{z1}(k+1) &= \big(A_{11}(h) - G(h)A_{21}(h)\big)e_{z1}(k) + \\ &\quad \big(B_1(h) - G(h)B_2(h)\big)e_f(k) + \\ &\quad \big(G(h)D_2(h) - D_1(h)\big)\omega(k), \end{aligned} \tag{27}$$
$$\begin{aligned} e_f(k+1) &= \hat{f}(k) - F(h)A_{21}(h)e_{z1}(k) - F(h)B_2(h)e_f(k) + \\ &\quad F(h)D_2(h)\omega(k) - f(k+1) \\ &= \hat{f}(k) - f(k) + f(k) - F(h)A_{21}(h)e_{z1}(k) - \\ &\quad F(h)B_2(h)e_f(k) + F(h)D_2(h)\omega(k) - f(k+1) \\ &= e_f(k) - F(h)A_{21}(h)e_{z1}(k) - \\ &\quad F(h)B_2(h)e_f(k) + F(h)D_2(h)\omega(k) - \triangle f(k) \\ &= -F(h)A_{21}(h)e_{z1}(k) + \\ &\quad \big(I_m - F(h)B_2(h)\big)e_f(k) + F(h)D_2(h)\omega(k) - \\ &\quad \triangle f(k), \end{aligned} \tag{28}$$

where $\triangle f(k) = f(k+1) - f(k)$ is the fault increment at time k. Using (27) and (28), the following augmented system is obtained:

$$\begin{aligned}&\begin{bmatrix} e_{z1}(k+1) \\ e_f(k+1) \end{bmatrix} = \\ &\begin{bmatrix} A_{11}(h) - G(h)A_{21}(h) & B_1(h) - G(h)B_2(h) \\ -F(h)A_{21}(h) & I_m - F(h)B_2(h) \end{bmatrix} \times \\ &\begin{bmatrix} e_{z1}(k) \\ e_f(k) \end{bmatrix} + \\ &\begin{bmatrix} G(h)D_2(h) - D_1(h) & 0_{(n-p)\times m} \\ F(h)D_2(h) & -I_m \end{bmatrix} \begin{bmatrix} \omega(k) \\ \triangle f(k) \end{bmatrix}. \end{aligned} \tag{29}$$

Denote

$$\begin{aligned}\bar{e}(k) &= \begin{bmatrix} e_{z1}(k) \\ e_f(k) \end{bmatrix}, \quad \nu(k) = \begin{bmatrix} \omega(k) \\ \triangle f(k) \end{bmatrix}, \\ \bar{A}_{11}(h) &= \begin{bmatrix} A_{11}(h) & B_1(h) \\ 0_{m\times(n-p)} & I_m \end{bmatrix}, \\ \bar{A}_{21}(h) &= \begin{bmatrix} A_{21}(h) & B_2(h) \end{bmatrix}, \quad \bar{G}(h) = \begin{bmatrix} G(h) \\ F(h) \end{bmatrix}, \\ \bar{D}_1(h) &= \begin{bmatrix} D_1(h) & 0_{(n-p)\times m} \\ 0_{m\times d} & I_m \end{bmatrix}, \\ \bar{D}_2(h) &= \begin{bmatrix} D_2(h) & 0_{p\times m} \end{bmatrix},\end{aligned}$$

then the augmented system becomes

$$\begin{aligned}\bar{e}(k+1) = &\big(\bar{A}_{11}(h) - \bar{G}(h)\bar{A}_{21}(h)\big)\bar{e}(k) + \\ &\big(\bar{G}(h)\bar{D}_2(h) - \bar{D}_1(h)\big)\nu(k).\end{aligned} \tag{30}$$

Next, a RFEO design method under an H_∞ performance index and a regional pole constraint is proposed to achieve robust FE.

Theorem 1. Let a prescribed H_∞ performance level γ_1 and a circular region $\mathcal{D}(\alpha_1, \tau_1)$ be given. If there exists a symmetric positive definite matrix $\bar{P} \in \mathbb{R}^{(n+m-p)\times(n+m-p)}$; matrices $\bar{Y}_i \in \mathbb{R}^{(n+m-p)\times p}\,(i=1,\ldots,q)$; matrices with appropriate dimensions $\mathbb{M}_{iii}, \mathbb{N}_{iii}\,(i=1,\ldots,q)$; $\mathbb{M}_{jii} = \mathbb{M}_{iij}^{\mathrm{T}}$, $\mathbb{M}_{iji}$, $\mathbb{N}_{jii} = \mathbb{N}_{iij}^{\mathrm{T}}$, $\mathbb{N}_{iji}$ $(i,j=1,\ldots,q,\ i\neq j)$ and $\mathbb{M}_{ijg} = \mathbb{M}_{gji}^{\mathrm{T}}$, $\mathbb{M}_{igj} = \mathbb{M}_{jgi}^{\mathrm{T}}$, $\mathbb{M}_{jig} = \mathbb{M}_{gij}^{\mathrm{T}}$, $\mathbb{N}_{ijg} = \mathbb{N}_{gji}^{\mathrm{T}}$, $\mathbb{N}_{igj} = \mathbb{N}_{jgi}^{\mathrm{T}}$, $\mathbb{N}_{jig} = \mathbb{N}_{gij}^{\mathrm{T}}$ $(i=1,\ldots,q-2,\ j=i+1,\ldots,q-1,\ g=j+1,\ldots,q)$ such that the following conditions hold:

$$\Phi_{ii} \le \mathbb{M}_{iii}, \quad i=1,\ldots,q, \tag{31}$$

$$\begin{aligned}\Phi_{ii} + \Phi_{ij} + \Phi_{ji} \le \mathbb{M}_{iij} + \mathbb{M}_{iji} + \mathbb{M}_{iij}^{\mathrm{T}}, \\ i,j=1,\ldots,q,\ i\neq j,\end{aligned} \tag{32}$$

$$\begin{aligned}\Phi_{ij} + \Phi_{ji} + \Phi_{ig} + \Phi_{gi} + \Phi_{jg} + \Phi_{gj} \le \mathbb{M}_{ijg} + \\ \mathbb{M}_{igj} + \mathbb{M}_{jig} + \mathbb{M}_{ijg}^{\mathrm{T}} + \mathbb{M}_{igj}^{\mathrm{T}} + \mathbb{M}_{jig}^{\mathrm{T}}, \\ i=1,\ldots,q-2,\ j=i+1,\ldots,q-1, \\ g=j+1,\ldots,q,\end{aligned} \tag{33}$$

$$\begin{bmatrix} \mathbb{M}_{1i1} & \mathbb{M}_{1i2} & \cdots & \mathbb{M}_{1iq} \\ \mathbb{M}_{2i1} & \mathbb{M}_{2i2} & \cdots & \mathbb{M}_{2iq} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbb{M}_{qi1} & \mathbb{M}_{qi2} & \cdots & \mathbb{M}_{qiq} \end{bmatrix} \le 0, \quad i=1,\ldots,q, \tag{34}$$

$$\Psi_{ii} \le \mathbb{N}_{iii}, \quad i=1,\ldots,q, \tag{35}$$

$$\begin{aligned}\Psi_{ii} + \Psi_{ij} + \Psi_{ji} \le \mathbb{N}_{iij} + \mathbb{N}_{iji} + \mathbb{N}_{iij}^{\mathrm{T}}, \\ i,j=1,\ldots,q,\ i\neq j,\end{aligned} \tag{36}$$

$$\begin{aligned}\Psi_{ij} + \Psi_{ji} + \Psi_{ig} + \Psi_{gi} + \Psi_{jg} + \Psi_{gj} \le \mathbb{N}_{ijg} + \\ \mathbb{N}_{igj} + \mathbb{N}_{jig} + \mathbb{N}_{ijg}^{\mathrm{T}} + \mathbb{N}_{igj}^{\mathrm{T}} + \mathbb{N}_{jig}^{\mathrm{T}}, \\ i=1,\ldots,q-2,\ j=i+1,\ldots,q-1, \\ g=j+1,\ldots,q,\end{aligned} \tag{37}$$

$$\begin{bmatrix} \mathbb{N}_{1i1} & \mathbb{N}_{1i2} & \cdots & \mathbb{N}_{1iq} \\ \mathbb{N}_{2i1} & \mathbb{N}_{2i2} & \cdots & \mathbb{N}_{2iq} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbb{N}_{qi1} & \mathbb{N}_{qi2} & \cdots & \mathbb{N}_{qiq} \end{bmatrix} \le 0, \quad i=1,\ldots,q, \tag{38}$$

where

$$\begin{aligned}\Phi_{ij} &= \begin{bmatrix} -\bar{P} & \bar{P}\bar{A}_{11i} - \bar{Y}_j\bar{A}_{21i} & \bar{Y}_j\bar{D}_{2i} - \bar{P}\bar{D}_{1i} & 0 \\ * & -\bar{P} & 0 & \bar{I}_m \\ * & * & -\gamma_1 I_{(d+m)} & 0 \\ * & * & * & -\gamma_1 I_m \end{bmatrix}, \\ \Psi_{ij} &= \begin{bmatrix} -\bar{P} & \bar{P}\bar{A}_{11i} - \bar{Y}_j\bar{A}_{21i} - \alpha_1\bar{P} \\ * & -\tau_1^2\bar{P} \end{bmatrix}, \\ \bar{I}_m &= \begin{bmatrix} 0_{(n-p)\times m} \\ I_m \end{bmatrix},\end{aligned}$$

then the RFEO (24)–(26) results in the H_∞ performance index $\|e_f(k)\|_2 \le \gamma_1\|\nu(k)\|_2$ and the eigenvalues of $\big(\bar{A}_{11}(h) - \bar{G}(h)\bar{A}_{21}(h)\big)$ belong to $\mathcal{D}(\alpha_1, \tau_1)$, and the RFEO gain matrices are given by $\bar{G}_i = \bar{P}^{-1}\bar{Y}_i$.

Proof. We start with the proof of (31)–(34), and (35)–(38) will be considered subsequently.

Constraints (31)–(34): Consider the following Lyapunov function:

$$V(k) = \bar{e}^{\mathrm{T}}(k)\bar{P}\bar{e}(k). \tag{39}$$

Its difference $\triangle V(k) = V(k+1) - V(k)$ along the error dynamics (30) is

$$\begin{aligned}\triangle V(k) &= V(k+1) - V(k) \\ &= \bar{e}^{\mathrm{T}}(k+1)\bar{P}\bar{e}(k+1) - \bar{e}^{\mathrm{T}}(k)\bar{P}\bar{e}(k) \\ &= \bar{e}^{\mathrm{T}}(k)\big(\bar{A}_{11}(h) - \bar{G}(h)\bar{A}_{21}(h)\big)^{\mathrm{T}}\bar{P}\times \\ &\quad \big(\bar{A}_{11}(h) - \bar{G}(h)\bar{A}_{21}(h)\big)\bar{e}(k) + \\ &\quad 2\bar{e}^{\mathrm{T}}(k)\big(\bar{A}_{11}(h) - \bar{G}(h)\bar{A}_{21}(h)\big)^{\mathrm{T}}\bar{P}\times \\ &\quad \big(\bar{G}(h)\bar{D}_2(h) - \bar{D}_1(h)\big)\nu(k) + \\ &\quad \nu^{\mathrm{T}}(k)\big(\bar{G}(h)\bar{D}_2(h) - \bar{D}_1(h)\big)^{\mathrm{T}}\bar{P}\times \\ &\quad \big(\bar{G}(h)\bar{D}_2(h) - \bar{D}_1(h)\big)\nu(k) - \bar{e}^{\mathrm{T}}(k)\bar{P}\bar{e}(k).\end{aligned} \tag{40}$$

Now, let us define

$$J_1 = \sum_{k=0}^{N-1}\left[\frac{1}{\gamma_1}e_f^{\mathrm{T}}(k)e_f(k) - \gamma_1\nu^{\mathrm{T}}(k)\nu(k)\right]. \tag{41}$$

Under zero initial condition, we get

$$\begin{aligned}J_1 &\le \sum_{k=0}^{N-1}\left[\triangle V(k) + \frac{1}{\gamma_1}e_f^{\mathrm{T}}(k)e_f(k) - \gamma_1\nu^{\mathrm{T}}(k)\nu(k)\right] \\ &= \sum_{k=0}^{N-1}\left[\triangle V(k) + \frac{1}{\gamma_1}\bar{e}^{\mathrm{T}}(k)\bar{I}_m\bar{I}_m^{\mathrm{T}}\bar{e}(k) - \gamma_1\nu^{\mathrm{T}}(k)\nu(k)\right].\end{aligned} \tag{42}$$

It follows from (40) and (42) that

$$
\begin{aligned}
&\triangle V(k)+\frac{1}{\gamma_1}\bar{e}^{\mathrm{T}}(k)\bar{I}_m\bar{I}_m^{\mathrm{T}}\bar{e}(k)-\gamma_1\nu^{\mathrm{T}}(k)\nu(k)\\
&=\bar{e}^{\mathrm{T}}(k)\big(\bar{A}_{11}(h)-\bar{G}(h)\bar{A}_{21}(h)\big)^{\mathrm{T}}\bar{P}\times\\
&\big(\bar{A}_{11}(h)-\bar{G}(h)\bar{A}_{21}(h)\big)\bar{e}(k)+\\
&2\bar{e}^{\mathrm{T}}(k)\big(\bar{A}_{11}(h)-\bar{G}(h)\bar{A}_{21}(h)\big)^{\mathrm{T}}\bar{P}\times\\
&\big(\bar{G}(h)\bar{D}_2(h)-\bar{D}_1(h)\big)\nu(k)+\\
&\nu^{\mathrm{T}}(k)\big(\bar{G}(h)\bar{D}_2(h)-\bar{D}_1(h)\big)^{\mathrm{T}}\bar{P}\times\\
&\big(\bar{G}(h)\bar{D}_2(h)-\bar{D}_1(h)\big)\nu(k)-\bar{e}^{\mathrm{T}}(k)\bar{P}\bar{e}(k)+\\
&\frac{1}{\gamma_1}\bar{e}^{\mathrm{T}}(k)\bar{I}_m\bar{I}_m^{\mathrm{T}}\bar{e}(k)-\gamma_1\nu^{\mathrm{T}}(k)\nu(k)\\
&=\zeta^{\mathrm{T}}(k)\Omega(h,h)\zeta(k), \qquad (43)
\end{aligned}
$$

where

$$
\zeta(k)=\begin{bmatrix}\bar{e}(k)\\ \nu(k)\end{bmatrix},\quad \Omega(h,h)=
$$

$$
\begin{bmatrix}
\big(\bar{A}_{11}(h)-\bar{G}(h)\bar{A}_{21}(h)\big)^{\mathrm{T}}\bar{P}\big(\bar{A}_{11}(h)-\bar{G}(h)\bar{A}_{21}(h)\big)-\bar{P}+\frac{1}{\gamma_1}\bar{I}_m\bar{I}_m^{\mathrm{T}} & \big(\bar{A}_{11}(h)-\bar{G}(h)\bar{A}_{21}(h)\big)^{\mathrm{T}}\bar{P}\big(\bar{G}(h)\bar{D}_2(h)-\bar{D}_1(h)\big)\\
* & \big(\bar{G}(h)\bar{D}_2(h)-\bar{D}_1(h)\big)^{\mathrm{T}}\bar{P}\big(\bar{G}(h)\bar{D}_2(h)-\bar{D}_1(h)\big)-\gamma_1 I_{(d+m)}
\end{bmatrix}.
$$

Using the Schur complement, $\Omega(h,h)\leq 0$ is equivalent to

$$
\begin{bmatrix}
\big(\bar{A}_{11}(h)-\bar{G}(h)\bar{A}_{21}(h)\big)^{\mathrm{T}}\bar{P}\big(\bar{A}_{11}(h)-\bar{G}(h)\bar{A}_{21}(h)\big)-\bar{P} & \big(\bar{A}_{11}(h)-\bar{G}(h)\bar{A}_{21}(h)\big)^{\mathrm{T}}\bar{P}\big(\bar{G}(h)\bar{D}_2(h)-\bar{D}_1(h)\big) & \bar{I}_m\\
* & \big(\bar{G}(h)\bar{D}_2(h)-\bar{D}_1(h)\big)^{\mathrm{T}}\bar{P}\big(\bar{G}(h)\bar{D}_2(h)-\bar{D}_1(h)\big)-\gamma_1 I_{(d+m)} & 0\\
* & * & -\gamma_1 I_m
\end{bmatrix}\leq 0. \qquad (44)
$$

Using the Schur complement again, we obtain

$$
\Phi(h,h):=\begin{bmatrix}
-\bar{P} & \bar{P}\big(\bar{A}_{11}(h)-\bar{G}(h)\bar{A}_{21}(h)\big) & \bar{P}\big(\bar{G}(h)\bar{D}_2(h)-\bar{D}_1(h)\big) & 0\\
* & -\bar{P} & 0 & \bar{I}_m\\
* & * & -\gamma_1 I_{(d+m)} & 0\\
* & * & * & -\gamma_1 I_m
\end{bmatrix}\leq 0, \quad (45)
$$

which can be rewritten as

$$
\begin{aligned}
\Phi(h,h)&=\sum_{i=1}^{q}\sum_{j=1}^{q}h_ih_j\Phi_{ij}=\left(\sum_{i=1}^{q}h_i\right)\sum_{i=1}^{q}\sum_{j=1}^{q}h_ih_j\Phi_{ij}\\
&=\sum_{i=1}^{q}h_i^3\Phi_{ii}+\sum_{i=1}^{q}\sum_{\substack{j=1\\ j\neq i}}^{q}h_i^2h_j(\Phi_{ii}+\Phi_{ij}+\Phi_{ji})+\\
&\sum_{i=1}^{q-2}\sum_{j=i+1}^{q-1}\sum_{g=j+1}^{q}h_ih_jh_g(\Phi_{ij}+\Phi_{ji}+\Phi_{ig}+\\
&\qquad\Phi_{gi}+\Phi_{jg}+\Phi_{gj}) \qquad (46)
\end{aligned}
$$

If conditions (31)–(33) hold, one obtains

$$
\begin{aligned}
\Phi(h,h)\leq&\sum_{i=1}^{q}h_i^3\mathbb{M}_{iii}+\sum_{i=1}^{q}\sum_{\substack{j=1\\ j\neq i}}^{q}h_i^2h_j(\mathbb{M}_{iij}+\mathbb{M}_{iji}+\mathbb{M}_{iij}^{\mathrm{T}})+\\
&\sum_{i=1}^{q-2}\sum_{j=i+1}^{q-1}\sum_{g=j+1}^{q}h_ih_jh_g(\mathbb{M}_{ijg}+\mathbb{M}_{igj}+\mathbb{M}_{jig}+\\
&\qquad\mathbb{M}_{ijg}^{\mathrm{T}}+\mathbb{M}_{igj}^{\mathrm{T}}+\mathbb{M}_{jig}^{\mathrm{T}})\\
=&h_1\begin{bmatrix}h_1I\\ h_2I\\ \vdots\\ h_qI\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}\mathbb{M}_{111}&\mathbb{M}_{112}&\cdots&\mathbb{M}_{11q}\\ \mathbb{M}_{211}&\mathbb{M}_{212}&\cdots&\mathbb{M}_{21q}\\ \vdots&\vdots&\ddots&\vdots\\ \mathbb{M}_{q11}&\mathbb{M}_{q12}&\cdots&\mathbb{M}_{q1q}\end{bmatrix}\times\\
&\begin{bmatrix}h_1I\\ h_2I\\ \vdots\\ h_qI\end{bmatrix}+h_2\begin{bmatrix}h_1I\\ h_2I\\ \vdots\\ h_qI\end{bmatrix}^{\mathrm{T}}\times\\
&\begin{bmatrix}\mathbb{M}_{121}&\mathbb{M}_{122}&\cdots&\mathbb{M}_{12q}\\ \mathbb{M}_{221}&\mathbb{M}_{222}&\cdots&\mathbb{M}_{22q}\\ \vdots&\vdots&\ddots&\vdots\\ \mathbb{M}_{q21}&\mathbb{M}_{q22}&\cdots&\mathbb{M}_{q2q}\end{bmatrix}\begin{bmatrix}h_1I\\ h_2I\\ \vdots\\ h_qI\end{bmatrix}\\
&+\cdots+h_q\begin{bmatrix}h_1I\\ h_2I\\ \vdots\\ h_qI\end{bmatrix}^{\mathrm{T}}\times\\
&\begin{bmatrix}\mathbb{M}_{1q1}&\mathbb{M}_{1q2}&\cdots&\mathbb{M}_{1qq}\\ \mathbb{M}_{2q1}&\mathbb{M}_{2q2}&\cdots&\mathbb{M}_{2qq}\\ \vdots&\vdots&\ddots&\vdots\\ \mathbb{M}_{qq1}&\mathbb{M}_{qq2}&\cdots&\mathbb{M}_{qqq}\end{bmatrix}\begin{bmatrix}h_1I\\ h_2I\\ \vdots\\ h_qI\end{bmatrix}\\
=&\begin{bmatrix}h_1I\\ h_2I\\ \vdots\\ h_qI\end{bmatrix}^{\mathrm{T}}\times\\
&\left(\sum_{i=1}^{q}h_i\begin{bmatrix}\mathbb{M}_{1i1}&\mathbb{M}_{1i2}&\cdots&\mathbb{M}_{1iq}\\ \mathbb{M}_{2i1}&\mathbb{M}_{2i2}&\cdots&\mathbb{M}_{2iq}\\ \vdots&\vdots&\ddots&\vdots\\ \mathbb{M}_{qi1}&\mathbb{M}_{qi2}&\cdots&\mathbb{M}_{qiq}\end{bmatrix}\right)\begin{bmatrix}h_1I\\ h_2I\\ \vdots\\ h_qI\end{bmatrix} \qquad (47)
\end{aligned}
$$

It follows that the error dynamics (30) is robustly stable with an H_∞ performance index $\|e_f(k)\|_2\leq\gamma_1\|\nu(k)\|_2$ provided (34) holds true.

Constraints (35)–(38): Not considering $\nu(k)$, and setting $\big(\bar{A}_{11}(h)-\bar{G}(h)\bar{A}_{21}(h)\big)\rightarrow\mathcal{A}$ and $\bar{P}\rightarrow\mathcal{P}$ in Lemma 1, one gets

$$\Psi(h,h):=$$

$$\begin{bmatrix} -\bar{P} & \bar{P}\big(\bar{A}_{11}(h)-\bar{G}(h)\bar{A}_{21}(h)\big)-\alpha_1\bar{P} \\ * & -\tau_1^2\bar{P} \end{bmatrix} \leq 0, \quad (48)$$

which can be rewritten as

$$\Psi(h,h)=\sum_{i=1}^{q}\sum_{j=1}^{q}h_ih_j\Psi_{ij}. \quad (49)$$

Then it follows from the proof of (31)–(34) that if (35)–(38) hold, then the eigenvalues of $\big(\bar{A}_{11}(h)-\bar{G}(h)\bar{A}_{21}(h)\big)$ belong to $\mathcal{D}(\alpha_1,\,\tau_1)$. □

Remark 3. Note that the FE filter problem for discrete-time systems proposed in [24], [27] was only suitable for open-loop stable systems. While, achieving the estimation of actuator faults by the system decomposition proposed in [28], [29], would require the constraints rank(C_iB_i) are of full-column rank, to be satisfied. From (30), a necessary condition for the proposed fuzzy RFEO design is that the pairs $(\bar{A}_{11i},\bar{A}_{21i})$ are observable. Our approach allows to deal with systems that do not satisfy these conditions, an example of which will be given in Section 5.

Remark 4. In Theorem 1, the purpose of introducing the regional pole constraint (35)–(38) is to guarantee system stability and control the FE transient performance. Note that other pole placement constraints, such as $\alpha-$stability, vertical strips, sectors and the intersection thereof can also be considered [35], [36].

Remark 5. Under the given regional pole constraint, the minimum H_∞ attenuation level of Theorem 1 can be obtained by solving the following semidefinite programming problem [37]:

$$\text{minimize } \gamma_1 \text{ subject to } (31)-(38).$$

This remark also applies to the other H_∞ performance calculations in this paper.

Remark 6. From the RFEO design (24)–(26), the on-line fault estimate is obtained via the following augmented system:

$$\begin{aligned}
\begin{bmatrix} \hat{z}_1(k+1) \\ \hat{f}(k+1) \end{bmatrix} &= \begin{bmatrix} A_{11}(h) & B_1(h) \\ 0_{m\times(n-p)} & I_m \end{bmatrix}\begin{bmatrix} \hat{z}_1(k) \\ \hat{f}(k) \end{bmatrix} - \\
&\quad \begin{bmatrix} G(h) \\ F(h) \end{bmatrix}\big(\hat{\rho}(k)-\rho(k)\big)+ \\
&\quad \begin{bmatrix} I_{n-p} \\ 0_{m\times(n-p)} \end{bmatrix}\eta(k) \\
&= \left(\begin{bmatrix} A_{11}(h) & B_1(h) \\ 0_{m\times(n-p)} & I_m \end{bmatrix} - \right. \\
&\quad \left.\begin{bmatrix} G(h) \\ F(h) \end{bmatrix}\begin{bmatrix} A_{21}(h) & B_2(h) \end{bmatrix}\right)\begin{bmatrix} \hat{z}_1(k) \\ \hat{f}(k) \end{bmatrix} + \\
&\quad \begin{bmatrix} G(h) \\ F(h) \end{bmatrix}\rho(k) + \begin{bmatrix} I_{n-p} \\ 0_{m\times(n-p)} \end{bmatrix}\eta(k) \\
&= \big(\bar{A}_{11}(h)-\bar{G}(h)\bar{A}_{21}(h)\big)\begin{bmatrix} \hat{z}_1(k) \\ \hat{f}(k) \end{bmatrix} + \\
&\quad \bar{G}(h)\rho(k)+\bar{I}_{n-p}\eta(k), \quad (50)
\end{aligned}$$

where $\bar{I}_{n-p}=\begin{bmatrix} I_{n-p} \\ 0_{m\times(n-p)} \end{bmatrix}$. Then substituting the vectors $\eta(k)$ and $\rho(k)$ into (50), one obtains

$$\begin{aligned}
\begin{bmatrix} \hat{z}_1(k+1) \\ \hat{f}(k+1) \end{bmatrix} &= \big(\bar{A}_{11}(h)-\bar{G}(h)\bar{A}_{21}(h)\big)\begin{bmatrix} \hat{z}_1(k) \\ \hat{f}(k) \end{bmatrix} + \\
&\quad \bar{G}(h)y(k+1)+ \\
&\quad \big(\bar{I}_{n-p}A_{12}(h)-\bar{G}(h)A_{22}(h)\big)y(k)+ \\
&\quad \big(\bar{I}_{n-p}B_1(h)-\bar{G}(h)B_2(h)\big)u(k). \quad (51)
\end{aligned}$$

In fact, the advantage of the fault estimator (51) lies in using the current output information to achieve FE. To show this point clear, we introduce a new variable $\chi(k+1)=\begin{bmatrix} \hat{z}_1(k+1) \\ \hat{f}(k+1) \end{bmatrix}-\bar{G}(h)y(k+1)$, and one gets

$$\chi(k)=\begin{bmatrix} \hat{z}_1(k) \\ \hat{f}(k) \end{bmatrix}-\bar{G}(h_-)y(k),$$

where $\bar{G}(h_-)$ is the value at time $k-1$. Then it follows from (51) that

$$\begin{aligned}
\chi(k+1) &= \big(\bar{A}_{11}(h)-\bar{G}(h)\bar{A}_{21}(h)\big)\big(\chi(k)+\bar{G}(h_-)y(k)\big) \\
&\quad + \big(\bar{I}_{n-p}A_{12}(h)-\bar{G}(h)A_{22}(h)\big)y(k)+ \\
&\quad \big(\bar{I}_{n-p}B_1(h)-\bar{G}(h)B_2(h)\big)u(k) \\
&= \big(\bar{A}_{11}(h)-\bar{G}(h)\bar{A}_{21}(h)\big)\chi(k)+ \\
&\quad \Big(\big((\bar{A}_{11}(h)-\bar{G}(h)\bar{A}_{21}(h)\big)\bar{G}(h_-)+ \\
&\quad \big(\bar{I}_{n-p}A_{12}(h)-\bar{G}(h)A_{22}(h)\big)\Big)y(k)+ \\
&\quad \big(\bar{I}_{n-p}B_1(h)-\bar{G}(h)B_2(h)\big)u(k). \quad (52)
\end{aligned}$$

Further, it follows that

$$\begin{bmatrix} \hat{z}_1(k) \\ \hat{f}(k) \end{bmatrix}=\chi(k)+\bar{G}(h_-)y(k), \quad (53)$$

and finally, the on-line fault estimate is given by

$$\hat{f}(k)=\bar{I}_m^{\mathrm{T}}\begin{bmatrix} \hat{z}_1(k) \\ \hat{f}(k) \end{bmatrix}, \quad (54)$$

which readily shows that the fault estimator contains the current output information, and allows to enhance the FE performance.

If we use a single RFEO gain matrix to deal with all fuzzy subsystems, i.e. $\bar{G}:=\bar{G}_1=\bar{G}_2=\cdots=\bar{G}_q$, then it results in the following Corollary 1, which can be treated as a special case of Theorem 1.

Corollary 1. Let a prescribed H_∞ performance level γ_1 and a circular region $\mathcal{D}(\alpha_1,\,\tau_1)$ be given. If there exists a symmetric positive definite matrix $\bar{P}\in\mathbb{R}^{(n+m-p)\times(n+m-p)}$ and a matrix $\bar{Y}\in\mathbb{R}^{(n+m-p)\times p}$ such that the following conditions hold:

$$\Lambda_i \leq 0, \quad i=1,\ldots,q, \quad (55)$$
$$\Delta_i \leq 0, \quad i=1,\ldots,q, \quad (56)$$

where

$$\Lambda_i=\begin{bmatrix} -\bar{P} & \bar{P}\bar{A}_{11i}-\bar{Y}\bar{A}_{21i} & \bar{Y}\bar{D}_{2i}-\bar{P}\bar{D}_{1i} & 0 \\ * & -\bar{P} & 0 & \bar{I}_m \\ * & * & -\gamma_1 I_{(d+m)} & 0 \\ * & * & * & -\gamma_1 I_m \end{bmatrix},$$

$$\Delta_i = \begin{bmatrix} -\bar{P} & \bar{P}\bar{A}_{11i} - \bar{Y}\bar{A}_{21i} - \alpha_1\bar{P} \\ * & -\tau_1^2\bar{P} \end{bmatrix},$$
$$\bar{I}_m = \begin{bmatrix} 0_{(n-p)\times m} \\ I_m \end{bmatrix},$$

then the RFEO (24)–(26) results in the H_∞ performance index $\|e_f(k)\|_2 \leq \gamma_1\|\nu(k)\|_2$ and the eigenvalues of $\big(\bar{A}_{11}(h) - \bar{G}\bar{A}_{21}(h)\big)$ belong to $\mathcal{D}(\alpha_1, \tau_1)$, and the RFEO gain matrix is given by $\bar{G} = \bar{P}^{-1}\bar{Y}$.

Therefore, using Corollary 1, (52) becomes

$$\begin{aligned}\chi(k+1) = &\big(\bar{A}_{11}(h) - \bar{G}\bar{A}_{21}(h)\big)\big(\chi(k) + \bar{G}y(k)\big)+ \\ &\big(\bar{I}_{n-p}A_{12}(h) - \bar{G}A_{22}(h)\big)y(k)+ \\ &\big(\bar{I}_{n-p}B_1(h) - \bar{G}B_2(h)\big)u(k) \\ = &\big(\bar{A}_{11}(h) - \bar{G}\bar{A}_{21}(h)\big)\chi(k)+ \\ &\Big(\big((\bar{A}_{11}(h) - \bar{G}\bar{A}_{21}(h)\big)\bar{G}+ \\ &\big(\bar{I}_{n-p}A_{12}(h) - \bar{G}A_{22}(h)\big)\Big)y(k)+ \\ &\big(\bar{I}_{n-p}B_1(h) - \bar{G}B_2(h)\big)u(k). \end{aligned} \tag{57}$$

Finally, the on-line fault estimate is given by

$$\hat{f}(k) = \bar{I}_m^{\mathrm{T}}\begin{bmatrix} \hat{z}_1(k) \\ \hat{f}(k) \end{bmatrix} = \bar{I}_m^{\mathrm{T}}\big(\chi(k) + \bar{G}y(k)\big). \tag{58}$$

IV. FAULT ACCOMMODATION DESIGN

On the basis of the obtained on-line FE information, we design a fault tolerant controller to guarantee stability in the presence of faults. Since the state $x(k)$ is unmeasurable, we use the fuzzy dynamical output feedback controller scheme [15], [38] to construct the fuzzy DOFFTC for T-S fuzzy models as

$$\xi(k+1) = A_K(h,h)\xi(k) + B_K(h)y(k), \tag{59}$$
$$u(k) = C_K(h)\xi(k) + D_Ky(k) - \hat{f}(k), \tag{60}$$

where $\xi(k) \in \mathbb{R}^n$ is the state, $A_K(h,h) \in \mathbb{R}^{n\times n}$, $B_K(h) \in \mathbb{R}^{n\times p}$, $C_K(h) \in \mathbb{R}^{m\times n}$ and $D_K \in \mathbb{R}^{m\times p}$ are the designed DOFFTC matrices, and

$$A_K(h,h) = \sum_{i=1}^{q}\sum_{j=1}^{q} h_ih_jA_{Kij}, \quad B_K(h) = \sum_{i=1}^{q} h_iB_{Ki},$$
$$C_K(h) = \sum_{i=1}^{q} h_iC_{Ki}.$$

Substituting (13) into (59) and (60), one obtains

$$\xi(k+1) = A_K(h,h)\xi(k) + B_K(h)Cx(k), \tag{61}$$
$$u(k) = C_K(h)\xi(k) + D_KCx(k) - \hat{f}(k). \tag{62}$$

Then substituting $u(k)$ into (12), we further obtain

$$\begin{aligned}x(k+1) = &A(h)x(k) + B(h)C_K(h)\xi(k)+ \\ &B(h)D_KCx(k) - B(h)\hat{f}(k)+ \\ &B(h)f(k) + D(h)\omega(k) \\ = &A(h)x(k) + B(h)C_K(h)\xi(k)+ \\ &B(h)D_KCx(k) - B(h)e_f(k) + D(h)\omega(k) \\ = &\Big(A(h) + B(h)D_KC\Big)x(k) + B(h)C_K(h)\xi(k) \\ &+ D(h)\omega(k) - B(h)e_f(k). \end{aligned} \tag{63}$$

It follows that

$$\tilde{x}(k+1) = \tilde{A}(h,h)\tilde{x}(k) + \tilde{D}(h)\mu(k), \tag{64}$$
$$z_L(k) = \tilde{C}_L(h)\tilde{x}(k), \tag{65}$$

where

$$\tilde{x}(k) = \begin{bmatrix} x(k) \\ \xi(k) \end{bmatrix}, \quad \mu(k) = \begin{bmatrix} \omega(k) \\ e_f(k) \end{bmatrix},$$
$$\tilde{A}(h,h) = \begin{bmatrix} A(h) + B(h)D_KC & B(h)C_K(h) \\ B_K(h)C & A_K(h,h) \end{bmatrix},$$
$$\tilde{D}(h) = \begin{bmatrix} D(h) & -B(h) \\ 0_{n\times d} & 0_{n\times m} \end{bmatrix}, \quad \tilde{C}_L(h) = \begin{bmatrix} C_L(h) & 0_{l\times n} \end{bmatrix}.$$

Theorem 2 gives a FA design based on dynamic output feedback control, in which FA performances are specified by an H_∞ performance index and a regional pole constraint.

Theorem 2. Let a prescribed H_∞ performance level γ_2 and a circular region $\mathcal{D}(\alpha_2, \tau_2)$ be given. If there exist two symmetric positive definite matrices $X, Y \in \mathbb{R}^{n\times n}$; matrices $\hat{A}_{ij} \in \mathbb{R}^{n\times n}$, $\hat{B}_i \in \mathbb{R}^{n\times p}$, $\hat{C}_i \in \mathbb{R}^{m\times n}$, $\hat{D} \in \mathbb{R}^{m\times p}$ $(i,j = 1,\ldots,q)$; matrices with appropriate dimensions $\mathbb{U}_{iii}, \mathbb{V}_{iii}\,(i = 1,\ldots,q)$; $\mathbb{U}_{jii} = \mathbb{U}_{iij}^{\mathrm{T}}$, $\mathbb{U}_{iji}$, $\mathbb{V}_{jii} = \mathbb{V}_{iij}^{\mathrm{T}}$, $\mathbb{V}_{iji}$ $(i,j = 1,\ldots,q,\ i \neq j)$ and $\mathbb{U}_{ijg} = \mathbb{U}_{gji}^{\mathrm{T}}$, $\mathbb{U}_{igj} = \mathbb{U}_{jgi}^{\mathrm{T}}$, $\mathbb{U}_{jig} = \mathbb{U}_{gij}^{\mathrm{T}}$, $\mathbb{V}_{ijg} = \mathbb{V}_{gji}^{\mathrm{T}}$, $\mathbb{V}_{igj} = \mathbb{V}_{jgi}^{\mathrm{T}}$, $\mathbb{V}_{jig} = \mathbb{V}_{gij}^{\mathrm{T}}$ $(i = 1,\ldots,q-2,\ j = i+1,\ldots,q-1,\ g = j+1,\ldots,q)$ such that the following conditions hold:

$$\Xi_{ii} \leq \mathbb{U}_{iii}, \quad i = 1,\ldots,q, \tag{66}$$
$$\Xi_{ii} + \Xi_{ij} + \Xi_{ji} \leq \mathbb{U}_{iij} + \mathbb{U}_{iji} + \mathbb{U}_{iij}^{\mathrm{T}}, \quad i,j = 1,\ldots,q,\ i \neq j, \tag{67}$$
$$\begin{aligned}&\Xi_{ij} + \Xi_{ji} + \Xi_{ig} + \Xi_{gi} + \Xi_{jg} + \Xi_{gj} \leq \mathbb{U}_{ijg}+ \\ &\mathbb{U}_{igj} + \mathbb{U}_{jig} + \mathbb{U}_{ijg}^{\mathrm{T}} + \mathbb{U}_{igj}^{\mathrm{T}} + \mathbb{U}_{jig}^{\mathrm{T}}, \\ &i = 1,\ldots,q-2,\ j = i+1,\ldots,q-1, \\ &g = j+1,\ldots,q,\end{aligned} \tag{68}$$
$$\begin{bmatrix} \mathbb{U}_{1i1} & \mathbb{U}_{1i2} & \cdots & \mathbb{U}_{1iq} \\ \mathbb{U}_{2i1} & \mathbb{U}_{2i2} & \cdots & \mathbb{U}_{2iq} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbb{U}_{qi1} & \mathbb{U}_{qi2} & \cdots & \mathbb{U}_{qiq} \end{bmatrix} \leq 0, \quad i = 1,\ldots,q, \tag{69}$$
$$\Pi_{ii} \leq \mathbb{V}_{iii}, \quad i = 1,\ldots,q, \tag{70}$$
$$\Pi_{ii} + \Pi_{ij} + \Pi_{ji} \leq \mathbb{V}_{iij} + \mathbb{V}_{iji} + \mathbb{V}_{iij}^{\mathrm{T}}, \quad i,j = 1,\ldots,q,\ i \neq j, \tag{71}$$
$$\begin{aligned}&\Pi_{ij} + \Pi_{ji} + \Pi_{ig} + \Pi_{gi} + \Pi_{jg} + \Pi_{gj} \leq \mathbb{V}_{ijg}+ \\ &\mathbb{V}_{igj} + \mathbb{V}_{jig} + \mathbb{V}_{ijg}^{\mathrm{T}} + \mathbb{V}_{igj}^{\mathrm{T}} + \mathbb{V}_{jig}^{\mathrm{T}}, \\ &i = 1,\ldots,q-2,\ j = i+1,\ldots,q-1, \\ &g = j+1,\ldots,q,\end{aligned} \tag{72}$$
$$\begin{bmatrix} \mathbb{V}_{1i1} & \mathbb{V}_{1i2} & \cdots & \mathbb{V}_{1iq} \\ \mathbb{V}_{2i1} & \mathbb{V}_{2i2} & \cdots & \mathbb{V}_{2iq} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbb{V}_{qi1} & \mathbb{V}_{qi2} & \cdots & \mathbb{V}_{qiq} \end{bmatrix} \leq 0, \quad i = 1,\ldots,q, \tag{73}$$

where

$$\Xi_{ij} = \begin{bmatrix} -X & -I_n & A_iX + B_i\hat{C}_j & A_i + B_i\hat{D}C & D_i & -B_i & 0 \\ * & -Y & \hat{A}_{ij} & YA_i + \hat{B}_jC & YD_i & -YB_i & 0 \\ * & * & -X & -I_n & 0 & 0 & XC_{Li}^{\mathrm{T}} \\ * & * & * & -Y & 0 & 0 & C_{Li}^{\mathrm{T}} \\ * & * & * & * & -\gamma_2 I_d & 0 & 0 \\ * & * & * & * & * & -\gamma_2 I_m & 0 \\ * & * & * & * & * & * & -\gamma_2 I_l \end{bmatrix},$$

$$\Pi_{ij} = \begin{bmatrix} -X & -I_n & A_iX + B_i\hat{C}_j - \alpha_2 X & A_i + B_i\hat{D}C - \alpha_2 I_n \\ * & -Y & \hat{A}_{ij} - \alpha_2 I_n & YA_i + \hat{B}_jC - \alpha_2 Y \\ * & * & -\tau_2^2 X & -\tau_2^2 I_n \\ * & * & * & -\tau_2^2 Y \end{bmatrix},$$

then the system dynamics (64) and (65) satisfy the H_∞ performance index $\|z_L(k)\|_2 \leq \gamma_2\|\mu(k)\|_2$ and the eigenvalues of $\tilde{A}(h,h)$ belong to $D(\alpha_2, \tau_2)$, and the parameter matrices of the DOFFTC are given by
$D_K = \hat{D}$,
$C_{Ki} = (\hat{C}_i - D_KCX)M^{-\mathrm{T}}$,
$B_{Ki} = N^{-1}(\hat{B}_i - YB_iD_K)$,
$A_{Kij} = N^{-1}\big(\hat{A}_{ij} - Y(A_i + B_iD_KC)X\big)M^{-\mathrm{T}} - B_{Kj}CXM^{-\mathrm{T}} - N^{-1}YB_iC_{Kj}$,
where $M, N \in \mathbb{R}^{n\times n}$ satisfy $MN^{\mathrm{T}} = I_n - XY$.

Proof. We start with the proof of (66)–(69), and (70)–(73) will be considered subsequently.

Constraints (66)–(69): Consider the following Lyapunov function

$$V(k) = \tilde{x}^{\mathrm{T}}(k)\tilde{P}\tilde{x}(k). \tag{74}$$

Its difference $\triangle V(k) = V(k+1) - V(k)$ along the error dynamics (64) is

$$\begin{aligned} \triangle V(k) &= \tilde{x}^{\mathrm{T}}(k+1)\tilde{P}\tilde{x}(k+1) - \tilde{x}^{\mathrm{T}}(k)\tilde{P}\tilde{x}(k) \\ &= \tilde{x}^{\mathrm{T}}(k)\tilde{A}^{\mathrm{T}}(h,h)\tilde{P}\tilde{A}(h,h)\tilde{x}(k) + \\ &\quad 2\tilde{x}^{\mathrm{T}}(k)\tilde{A}^{\mathrm{T}}(h,h)\tilde{P}\tilde{D}(h)\mu(k) + \\ &\quad \mu^{\mathrm{T}}(k)\tilde{D}^{\mathrm{T}}(h)\tilde{P}\tilde{D}(h)\mu(k) - \tilde{x}^{\mathrm{T}}(k)\tilde{P}\tilde{x}(k). \end{aligned} \tag{75}$$

Let us introduce

$$J_2 = \sum_{k=0}^{N-1}\left[\frac{1}{\gamma_2}z_L^{\mathrm{T}}(k)z_L(k) - \gamma_2\mu^{\mathrm{T}}(k)\mu(k)\right]. \tag{76}$$

It can be shown that

$$J_2 \leq \sum_{k=0}^{N-1}\left[\triangle V(k) + \frac{1}{\gamma_2}z_L^{\mathrm{T}}(k)z_L(k) - \gamma_2\mu^{\mathrm{T}}(k)\mu(k)\right]. \tag{77}$$

Substituting (75) into (77), one obtains

$$\begin{aligned} &\triangle V(k) + \frac{1}{\gamma_2}z_L^{\mathrm{T}}(k)z_L(k) - \gamma_2\mu^{\mathrm{T}}(k)\mu(k) \\ &= \tilde{x}^{\mathrm{T}}(k)\tilde{A}^{\mathrm{T}}(h,h)\tilde{P}\tilde{A}(h,h)\tilde{x}(k) + \\ &\quad 2\tilde{x}^{\mathrm{T}}(k)\tilde{A}^{\mathrm{T}}(h,h)\tilde{P}\tilde{D}(h)\mu(k) + \mu^{\mathrm{T}}(k)\tilde{D}^{\mathrm{T}}(h)\tilde{P}\tilde{D}(h)\mu(k) - \\ &\quad \tilde{x}^{\mathrm{T}}(k)\tilde{P}\tilde{x}(k) + \frac{1}{\gamma_2}z_L^{\mathrm{T}}(k)z_L(k) - \gamma_2\mu^{\mathrm{T}}(k)\mu(k) \\ &= \tilde{x}^{\mathrm{T}}(k)\tilde{A}^{\mathrm{T}}(h,h)\tilde{P}\tilde{A}(h,h)\tilde{x}(k) + \\ &\quad 2\tilde{x}^{\mathrm{T}}(k)\tilde{A}^{\mathrm{T}}(h,h)\tilde{P}\tilde{D}(h)\mu(k) + \mu^{\mathrm{T}}(k)\tilde{D}^{\mathrm{T}}(h)\tilde{P}\tilde{D}(h)\mu(k) - \\ &\quad \tilde{x}^{\mathrm{T}}(k)\tilde{P}\tilde{x}(k) + \frac{1}{\gamma_2}\tilde{x}^{\mathrm{T}}(k)\tilde{C}_L^{\mathrm{T}}(h)\tilde{C}_L(h)\tilde{x}(k) - \gamma_2\mu^{\mathrm{T}}(k)\mu(k) \\ &= \varsigma^{\mathrm{T}}(k)\Theta(h,h)\varsigma(k), \end{aligned} \tag{78}$$

where

$$\varsigma(k) = \begin{bmatrix} \tilde{x}(k) \\ \mu(k) \end{bmatrix},$$

$$\Theta(h,h) = \begin{bmatrix} \tilde{A}^{\mathrm{T}}(h,h)\tilde{P}\tilde{A}(h,h) - \tilde{P} + \frac{1}{\gamma_2}\tilde{C}_L^{\mathrm{T}}(h)\tilde{C}_L(h) & \tilde{A}^{\mathrm{T}}(h,h)\tilde{P}\tilde{D}(h) \\ * & \tilde{D}^{\mathrm{T}}(h)\tilde{P}\tilde{D}(h) - \gamma_2 I_{(d+m)} \end{bmatrix}.$$

Using the Schur complement, $\Theta(h,h) \leq 0$ is equivalent to

$$\begin{bmatrix} -\tilde{P} & \tilde{P}\tilde{A}(h,h) & \tilde{P}\tilde{D}(h) & 0 \\ * & -\tilde{P} & 0 & \tilde{C}_L^{\mathrm{T}}(h) \\ * & * & -\gamma_2 I_{(d+m)} & 0 \\ * & * & * & -\gamma_2 I_l \end{bmatrix} \leq 0. \tag{79}$$

Then we express the symmetric positive definite matrix $\tilde{P}$ and its inverse matrix $\tilde{P}^{-1}$ as

$$\tilde{P} = \begin{bmatrix} Y & N \\ N^{\mathrm{T}} & W \end{bmatrix}, \quad \tilde{P}^{-1} = \begin{bmatrix} X & M \\ M^{\mathrm{T}} & Z \end{bmatrix}.$$

Due to $\tilde{P}\tilde{P}^{-1} = I_{2n}$, one gets $\tilde{P}\begin{bmatrix} X \\ M^{\mathrm{T}} \end{bmatrix} = \begin{bmatrix} I_n \\ 0_n \end{bmatrix}$ and $\tilde{P}\begin{bmatrix} X & I_n \\ M^{\mathrm{T}} & 0_n \end{bmatrix} = \begin{bmatrix} I_n & Y \\ 0_n & N^{\mathrm{T}} \end{bmatrix}$.

Denote

$$F_1 = \begin{bmatrix} X & I_n \\ M^{\mathrm{T}} & 0_n \end{bmatrix}, \quad F_2 = \begin{bmatrix} I_n & Y \\ 0_n & N^{\mathrm{T}} \end{bmatrix}.$$

then it follows that $\tilde{P}F_1 = F_2$. Pre- and post-multiplying by $\mathrm{diag}(F_1^{\mathrm{T}}, F_1^{\mathrm{T}}, I_{(d+m)}, I_l)$ and its transpose in (79), one gets

$$F_1^{\mathrm{T}}\tilde{P}F_1 = \begin{bmatrix} X & I_n \\ I_n & Y \end{bmatrix},$$

$$F_1^{\mathrm{T}}\tilde{P}\tilde{A}(h,h)F_1 = \begin{bmatrix} A(h)X + B(h)\Big(D_K CX + C_K(h)M^{\mathrm{T}}\Big) & A(h)+B(h)D_K C \\ Y\Big(A(h)+B(h)D_K C\Big)X + NB_K(h)CX + YB(h)C_K(h)M^{\mathrm{T}} + NA_K(h,h)M^{\mathrm{T}} & YA(h) + \Big(YB(h)D_K + NB_K(h)\Big)C \end{bmatrix},$$

$$F_1^{\mathrm{T}}\tilde{P}\tilde{D}(h) = \begin{bmatrix} D(h) & -B(h) \\ YD(h) & -YB(h) \end{bmatrix},$$

$$F_1^{\mathrm{T}}\tilde{C}_L^{\mathrm{T}}(h) = \begin{bmatrix} XC_L^{\mathrm{T}}(h) \\ C_L^{\mathrm{T}}(h) \end{bmatrix}.$$

Denote

$$\begin{aligned} \hat{A}(h,h) &= Y\Big(A(h)+B(h)D_K C\Big)X + NB_K(h)CX + \\ &\quad YB(h)C_K(h)M^{\mathrm{T}} + NA_K(h,h)M^{\mathrm{T}}, \\ \hat{B}(h) &= YB(h)D_K + NB_K(h), \\ \hat{C}(h) &= D_K CX + C_K(h)M^{\mathrm{T}}, \\ \hat{D} &= D_K. \end{aligned}$$

then (79) can be expressed as

$$\Xi(h,h) := \begin{bmatrix} -X & -I_n & A(h)X + B(h)\hat{C}(h) & A(h)+B(h)\hat{D}C & D(h) & -B(h) & 0 \\ * & -Y & \hat{A}(h,h) & YA(h)+\hat{B}(h)C & YD(h) & -YB(h) & 0 \\ * & * & -X & -I_n & 0 & 0 & XC_L^{\mathrm{T}}(h) \\ * & * & * & -Y & 0 & 0 & C_L^{\mathrm{T}}(h) \\ * & * & * & * & -\gamma_2 I_d & 0 & 0 \\ * & * & * & * & * & -\gamma_2 I_m & 0 \\ * & * & * & * & * & * & -\gamma_2 I_l \end{bmatrix} \leq 0, \tag{80}$$

Therefore, $\Xi(h,h) \leq 0$ can be rewritten as

$$\Xi(h,h) = \sum_{i=1}^{q}\sum_{j=1}^{q} h_i h_j \Xi_{ij}. \tag{81}$$

Then it follows from the proof of Theorem 1 that if (66)–(69) hold, the system dynamics (64) and (65) are robustly stable with an H_∞ performance index $\|z_L(k)\|_2 \leq \gamma_2 \|\mu(k)\|_2$.

Constraints (70)–(73): Not considering $\mu(k)$, and setting $\tilde{A}(h,h) \to \mathcal{A}$ and $\tilde{P} \to \mathcal{P}$ in Lemma 1, one gets

$$\begin{bmatrix} -\tilde{P} & \tilde{P}\tilde{A}(h,h) - \alpha_2\tilde{P} \\ * & -\tau_2^2\tilde{P} \end{bmatrix} \leq 0, \tag{82}$$

Pre- and post-multiplying by $\mathrm{diag}(F_1^{\mathrm{T}}, F_1^{\mathrm{T}})$ and its transpose in (82), and then using the definitions $\hat{A}(h,h)$, $\hat{B}(h)$, $\hat{C}(h)$ and $\hat{D}$, one gets

$$\Pi(h,h) := \begin{bmatrix} -X & -I_n & A(h)X + B(h)\hat{C}(h) - \alpha_2 X & A(h)+B(h)D_K C - \alpha_2 I_n \\ * & -Y & \hat{A}(h,h) - \alpha_2 I_n & YA(h) + \hat{B}(h)C - \alpha_2 Y \\ * & * & -\tau_2^2 X & -\tau_2^2 I_n \\ * & * & * & -\tau_2^2 Y \end{bmatrix} \leq 0, \tag{83}$$

which can be rewritten as

$$\Pi(h,h) = \sum_{i=1}^{q}\sum_{j=1}^{q} h_i h_j \Pi_{ij}. \tag{84}$$

Then it follows from the proof of Theorem 1 that if (70)–(73) hold, then the eigenvalues of $\tilde{A}(h,h)$ belong to $\mathcal{D}(\alpha_2, \tau_2)$. □

Remark 7. From Sections 3 and 4, we can see that the RFEO and the DOFFTC are designed separately and their performances are considered simultaneously, which can avoid design difficulties caused by the coupling between them and is convenient to calculate their respective design parameters.

Remark 8. In order to reduce conservatism caused by conventional T-S fuzzy design methods, in this paper, we have adopted relaxed quadratic stabilization results obtained in [39]. Moreover, for possible improved results, i.e. a more general case in terms of necessary and sufficient conditions when a design parameter n tends to infinity, please refer to [40]. However, the multiconstrained design, i.e. an H_∞ performance index and a regional pole constraint, is considered in both the RFEO and the DOFFTC, so the conditions of Theorems 1 and 2 are still sufficient even using the method in [40].

Remark 9. In this paper, only discrete-time T-S fuzzy systems are considered, please note that the proposed FE and accommodation design method can be readily extended to continuous-time T-S fuzzy ones. For the implementation procedure, interested readers can refer to this work and [7], [8], [16].

V. Simulation Results

System description. In this section, a discrete-time nonlinear truck-trailer model borrowed from [41], [42] is considered to illustrate the effectiveness of the proposed method.

$$\begin{aligned} x_1(k+1) &= \left(1 - \frac{vt}{L}\right)x_1(k) + \frac{vt}{l}u(k) \\ x_2(k+1) &= \frac{vt}{L}x_1(k) + x_2(k) \\ x_3(k+1) &= x_3(k) + vt\sin(\theta(k)) \\ y_1(k) &= x_2(k), \quad y_2(k) = x_3(k) \\ z_L(k) &= 0.05x_3(k) \end{aligned}$$

where $x_1(k)$ is the angle difference between truck and trailer (rad), $x_2(k)$ is the angle of trailer (rad) and $x_3(k)$ is the vertical position of rear of trailer (m) and $u(k)$ is the steering angle (rad). l is the length of truck, L is the length of trailer, t is sampling time and v is the constant speed of backing up. In all simulations, $l = 2.8$m, $L = 5.5$m, $t = 2$s and $v = -1.0$m/s. $\theta(k) = (vt/2L)x_1(k) + x_2(k)$. Since $x_1(k)$ is unavailable, we use $\hat{x}_1(k)$ as a substitute for $x_1(k)$. The nonlinear system can be modeled as a two-rule fuzzy model.

Rule 1: IF $\theta(k)$ ia about 0, THEN

$$\begin{aligned} x(k+1) &= A_1 x(k) + B_1 u(k), \\ y(k) &= C_1 x(k), \\ z_L(k) &= C_{L1} x(k) \end{aligned}$$

Rule 2: IF $\theta(k)$ ia about $\pm\pi$, THEN

$$\begin{aligned} x(k+1) &= A_2 x(k) + B_2 u(k), \\ y(k) &= C_2 x(k), \\ z_L(k) &= C_{L2} x(k) \end{aligned}$$

where

$$A_1 = \begin{bmatrix} 1 - \frac{vt}{L} & 0 & 0 \\ \frac{vt}{L} & 1 & 0 \\ \frac{(vt)^2}{2L} & vt & 1 \end{bmatrix}, \; B_1 = \begin{bmatrix} \frac{vt}{l} \\ 0 \\ 0 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 1 - \frac{vt}{L} & 0 & 0 \\ \frac{vt}{L} & 1 & 0 \\ \frac{0.01(vt)^2}{2L\pi} & \frac{0.01vt}{\pi} & 1 \end{bmatrix}, \; B_2 = \begin{bmatrix} \frac{vt}{l} \\ 0 \\ 0 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$C_{L1} = \begin{bmatrix} 0 & 0 & 0.05 \end{bmatrix}, \quad C_{L2} = \begin{bmatrix} 0 & 0 & 0.05 \end{bmatrix}.$$

The membership functions are

$$h_1 = \frac{\sigma_1}{\sigma_1 + \sigma_2}, \quad h_2 = \frac{\sigma_2}{\sigma_1 + \sigma_2}$$

where

$$\sigma_1 = e^{-5\theta_k^2}, \quad \sigma_2 = \max\{e^{-5(\theta_k - \pi)^2}, e^{-5(\theta_k + \pi)^2}\}.$$

Membership functions for Rules 1 and 2 are shown in Figure 1.

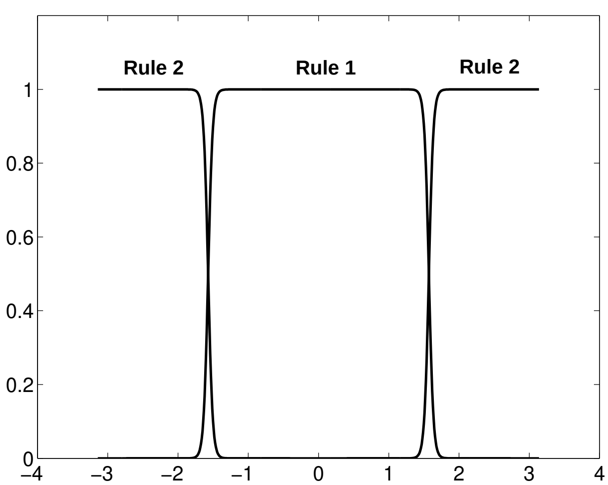

Fig. 1. Membership functions of the two-rule model.

Note that we assume only the last two states to be measured, rather than all states [41], [42], to achieve the actuator faults estimation, thus setting a more restrictive problem. It is assumed that the disturbance distribution matrices are chosen as $D_1 = D_2 = \begin{bmatrix} 0.1 & 0.1 & 0.1 \end{bmatrix}^{\mathrm{T}}$. It is easy to verify that the pairs (A_i, B_i) are controllable and the pairs (A_i, C_i) are observable. Note that it can be verified that rank$(C_i B_i) = 0$, the FE design in [28], [29] can not be used for this system to achieve FE. Meanwhile, from system matrices A_i, it is shown that the system is open-loop unstable, so the FE filter design in [24], [27] is also not suitable for such system.

Fault estimation design. From the system description, it is shown that $C_1 = C_2$, so the state transformation matrix can be chosen as $T = \begin{bmatrix} 1 \quad 0 \quad 0 \\ C \end{bmatrix}^{-1} = I_3$. It follows that the fuzzy RFEO augmented matrices can be constructed as

$$\bar{A}_{111} = \begin{bmatrix} A_{111} & B_{11} \\ 0 & I \end{bmatrix} = \begin{bmatrix} 1.3636 & -0.7143 \\ 0 & 1.0000 \end{bmatrix},$$

$$\bar{A}_{112} = \begin{bmatrix} A_{112} & B_{12} \\ 0 & I \end{bmatrix} = \begin{bmatrix} 1.3636 & -0.7143 \\ 0 & 1.0000 \end{bmatrix},$$

$$\bar{A}_{211} = \begin{bmatrix} A_{211} & B_{21} \end{bmatrix} = \begin{bmatrix} -0.3636 & 0 \\ 0.3636 & 0 \end{bmatrix},$$

$$\bar{A}_{212} = \begin{bmatrix} A_{212} & B_{22} \end{bmatrix} = \begin{bmatrix} -0.3636 & 0 \\ 0.0012 & 0 \end{bmatrix},$$

$$\bar{D}_{11} = \begin{bmatrix} D_{11} & 0 \\ 0 & I \end{bmatrix} = \begin{bmatrix} 0.1000 & 0 \\ 0 & 1.0000 \end{bmatrix},$$

$$\bar{D}_{12} = \begin{bmatrix} D_{12} & 0 \\ 0 & I \end{bmatrix} = \begin{bmatrix} 0.1000 & 0 \\ 0 & 1.0000 \end{bmatrix},$$

$$\bar{D}_{21} = \begin{bmatrix} D_{21} & 0_{2\times1} \end{bmatrix} = \begin{bmatrix} 0.1000 & 0 \\ 0.1000 & 0 \end{bmatrix},$$

$$\bar{D}_{22} = \begin{bmatrix} D_{22} & 0_{2\times1} \end{bmatrix} = \begin{bmatrix} 0.1000 & 0 \\ 0.1000 & 0 \end{bmatrix}.$$

It is seen that the pairs $(\bar{A}_{111}, \bar{A}_{211})$ and $(\bar{A}_{112}, \bar{A}_{212})$ are observable. Solving the conditions in Theorem 1 with the regional pole constraint $\mathcal{D}(0.5, 0.45)$ allows to take into account both the estimation convergence speed and the level of disturbance attenuation, then one obtains the optimal value $\gamma_1 = 2.4020$ with

$$\bar{P} = \begin{bmatrix} 1.9346 & 1.5622 \\ 1.5622 & 1.8103 \end{bmatrix}, \quad \bar{G}_1 = \begin{bmatrix} -0.8687 & 3.7256 \\ 0.7032 & -1.4018 \end{bmatrix},$$

$$\bar{G}_2 = \begin{bmatrix} -4.5824 & 3.7256 \\ 2.1005 & -1.4018 \end{bmatrix}.$$

Fault accommodation design. We set the circle region as $\mathcal{D}(0, 0.999)$ to obtain a smaller H_∞ performance index as much as possible. By solving the conditions in Theorem 2, one obtains the minimum attenuation value $\gamma_2 = 2.0282$ with

$$X = \begin{bmatrix} 27.8504 & 7.0691 & 0.0132 \\ 7.0691 & 2.6140 & 1.2150 \\ 0.0132 & 1.2150 & 2.9169 \end{bmatrix},$$

$$Y = \begin{bmatrix} 2.8289 & 8.8895 & -0.5989 \\ 8.8895 & 43.6326 & -2.6413 \\ -0.5989 & -2.6413 & 2.6398 \end{bmatrix},$$

$$M = \begin{bmatrix} -0.9554 & 0.1769 & 0.2365 \\ -0.2888 & -0.3920 & -0.8734 \\ -0.0618 & -0.9028 & 0.4256 \end{bmatrix},$$
$$N = \begin{bmatrix} 147.1842 & 0 & 0 \\ 583.8634 & 10.3713 & 0 \\ -35.8151 & 6.2819 & -0.0566 \end{bmatrix},$$
$$A_{K11} = \begin{bmatrix} -2.8619 & 0.0409 & 0.0001 \\ 2.4592 & -0.7117 & -0.0045 \\ -156.8730 & -24.7808 & -0.1452 \end{bmatrix},$$
$$A_{K12} = \begin{bmatrix} -2.9847 & -0.0340 & 0.0005 \\ 4.6122 & 0.2621 & -0.0369 \\ 131.9765 & 98.6897 & 2.4439 \end{bmatrix},$$
$$A_{K21} = \begin{bmatrix} -2.8506 & 0.0360 & -0.0005 \\ 3.7602 & -0.3604 & 0.0351 \\ -176.8456 & -110.8960 & -2.5693 \end{bmatrix},$$
$$A_{K22} = \begin{bmatrix} -2.9716 & -0.0410 & -0.0002 \\ 6.2141 & 0.4634 & 0.0015 \\ 114.9717 & 8.7115 & 0.0451 \end{bmatrix},$$
$$B_{K1} = \begin{bmatrix} -0.3601 & 0.0509 \\ 0.5596 & -0.3863 \\ -18.8758 & -0.9996 \end{bmatrix},$$
$$B_{K2} = \begin{bmatrix} -0.3550 & 0.0510 \\ 0.8055 & -0.3698 \\ 14.2606 & -0.6685 \end{bmatrix},$$
$$C_{K1} = \begin{bmatrix} -222.5433 & 2.7100 & 0.0110 \end{bmatrix},$$
$$C_{K2} = \begin{bmatrix} -231.8686 & -2.9150 & -0.0127 \end{bmatrix},$$
$$D_K = \begin{bmatrix} -22.2548 & 3.4606 \end{bmatrix}.$$

Note that, in order to avoid the singular solution, the following additional constraint has been added when solving the conditions of Theorem 2,

$$\left\| \begin{bmatrix} \hat{A}(h,h) & \hat{B}(h) \\ \hat{C}(h) & \hat{D} \end{bmatrix} \right\| \le \delta,$$

where $\delta = 70$, which can be written as

$$\Upsilon_{ii} \ge 0, \qquad i = 1, \ldots, q,$$
$$\Upsilon_{ij} + \Upsilon_{ji} \ge 0, \qquad 1 \le i < j \le q,$$

where

$$\Upsilon_{ij} = \begin{bmatrix} \delta I_n & 0 & \hat{A}_{ij} & \hat{B}_i \\ * & \delta I_m & \hat{C}_i & \hat{D} \\ * & * & \delta I_n & 0 \\ * & * & * & \delta I_p \end{bmatrix}.$$

To solve the above problems we used CVX, a package for specifying and solving convex programs [43], [44].

Simulation results. For simulation, an actuator fault $f(k)$ is created as

$$f(k) = \begin{cases} 0 & \begin{matrix} 0s \le t < 200s \\ (0 \le k < 100) \end{matrix} \\ 2(1 - e^{-0.05(t-200)}) & \begin{matrix} 200s \le t \le 1000s \\ (100 \le k \le 500) \end{matrix} \end{cases}.$$

Under initial value $\begin{bmatrix} 0 & 0 & 5 \end{bmatrix}^{\mathrm{T}}$, simulation results are displayed as follows. In the following simulation results, we apply the proposed design method to the original nonlinear system, instead of T-S fuzzy models, whose purpose is to verify the robustness of the proposed method with respect to modeling errors. Simulation results for the system controlled output responses are shown in Figure 2 (a $10s$ detection delay is considered). Figures 3–5 show simulation results of state responses. Figure 6 illustrates FE simulation result.

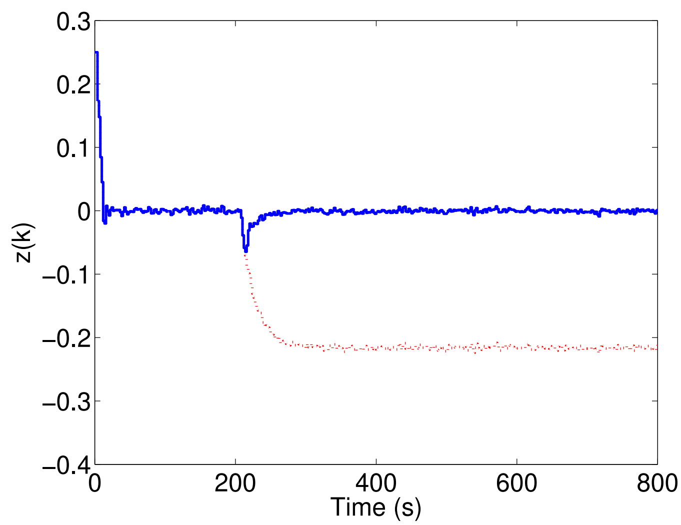

Fig. 2. Controlled output responses of $z_L(k)$ (without FA: dotted line; under FA: solid line).

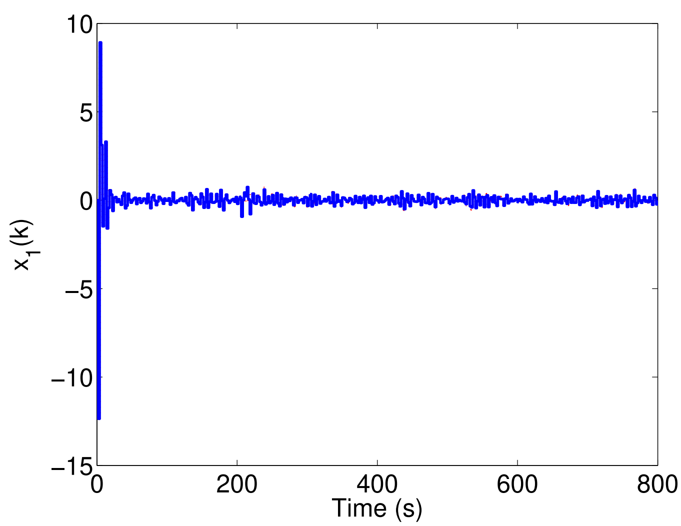

Fig. 3. State responses of $x_1(k)$ (without FA: dotted line; under FA: solid line).

From the above simulation results, we can see that, despite the fact that rank$(C_iB_i) = m$ is not satisfied and the open-loop system is unstable, the proposed design still achieves the performance under actuator faults, and the stability performance of the closed-loop system is guaranteed by the fuzzy DOFFTC. Since the property of the presented example, the effect of the fault on the third state $x_3(k)$ is more obvious

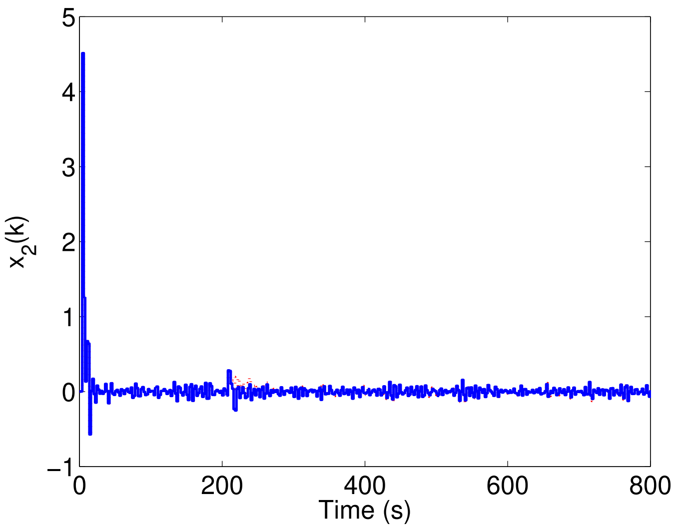

Fig. 4. State responses of $x_2(k)$ (without FA: dotted line; under FA: solid line).

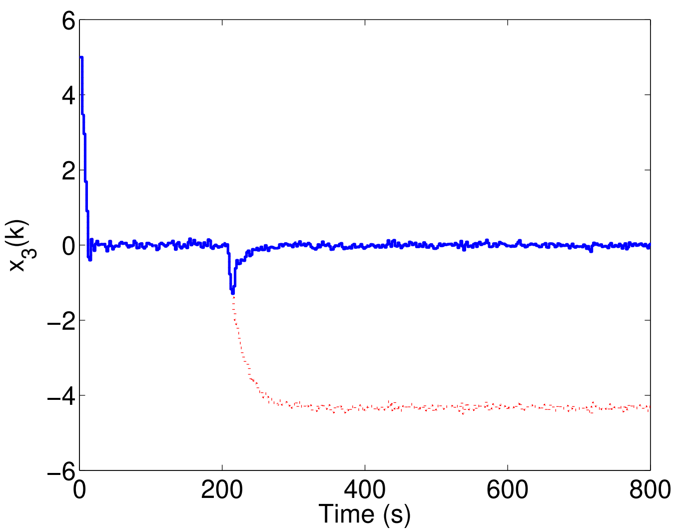

Fig. 5. State responses of $x_3(k)$ (without FA: dotted line; under FA: solid line).

than the others. Note that the proposed fuzzy RFEO design can achieve asymptotical estimation for constant faults.

VI. Conclusions

In this paper, a design framework for integrated robust FE and FA is developed for a class of discrete-time nonlinear systems described by a T-S fuzzy model. The framework includes a reduced-order FE observer and a dynamic output feedback fault tolerant controller to guarantee given stability requirements. Simulation results of a discrete-time nonlinear truck-trailer model are used to show the effectiveness of the obtained results. The issue of T-S fuzzy systems subject to unmeasurable premise variables [9], [45] and the extension of the proposed design to general nonlinear systems [46], [47] are interesting and practical, which will be studied in our future work.

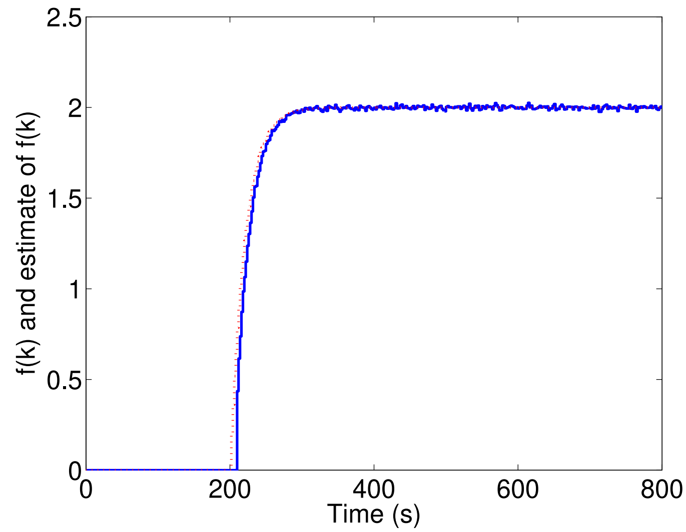

Fig. 6. Fault $f(k)$ (dotted line) and its estimate $\hat{f}(k)$ (solid line).

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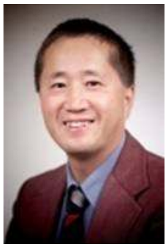

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