

Integrated Guidance-Control Systems for Fixed-Aim Warhead Missiles

By

P. K. Menon*
Optimal Synthesis Inc
Los Altos, CA.

Ernest J. Ohlmeyer†
Naval Surface Warfare Center
Dahlgren, VA.

Abstract

Integrated guidance-control systems have the potential to improve missile system performance by taking advantage of the synergism existing between subsystems. These systems allow the designer to impose unusual performance requirements on the missile. Such requirements may arise out of the new sensor and warhead technologies that may require complex maneuvers at target interception.

Integrated guidance-control techniques of missiles incorporating fixed-aim warheads are discussed in this paper. The fixed-aim warhead technology seeks to reduce the weapon system weight by using a highly directional warhead, together with enhancements to the guidance-control systems. The fixed-aim warhead projects the blast fragments in a direction normal to the missile longitudinal axis. In order to be effective, this warhead requires the missile to achieve a specific roll orientation with respect to the target at interception. Moreover, it is desirable to achieve a near-zero relative lateral velocity vector orientation with respect to the target at interception to minimize the sensitivity of the system to fuze delay. These specifications require the missile to perform a combination of conventional lateral acceleration maneuvers and terminal attitude maneuvers during its operation.

Recently developed nonlinear control system design software is used to synthesize three different integrated guidance-control strategies. These design approaches use a nonlinear, six-degrees-of-freedom air-to-air missile model. Simulation results for sample engagement scenarios are given.

1. Introduction

There has been an increasing interest in the integrated synthesis of guidance and control systems in recent literature [1 – 5]. These techniques have the potential to enhance missile performance by exploiting the synergism between guidance and control subsystems. For instance, guidance laws that have anticipatory capabilities can reduce the autopilot

time response requirements. Low time-constant autopilot can make the guidance system more effective. By establishing additional feedback paths in the control system, the integrated system design methods allow the analyst to realize these beneficial interactions. Resulting improvements in target interception accuracy will allow the use of smaller warheads, leading to a more efficient weapon system.

The focus of the present research is on the development of integrated guidance-control systems for a fixed-aim warhead air-to-air missile. The fixed-aim warhead technology seeks to reduce the weapon system weight by using a highly directional warhead, together with enhancements to the guidance-control systems. Unlike the traditional missile warheads, the fixed-aim warhead projects the blast fragments in a direction normal to the missile longitudinal axis. Thus, in order to be effective, the warhead requires the missile to achieve a specific roll orientation with respect to the target at interception. Moreover, it is desirable to achieve a near-zero relative lateral velocity vector orientation with respect to the target at interception to minimize the sensitivity of the system to fuze delay. These specifications require the missile to perform a combination of conventional lateral acceleration maneuvers and terminal attitude maneuvers during its operation.

Traditional approach to the design of missile control and guidance systems has been to neglect these interactions and to treat individual missile subsystems separately. The missile dynamics is split into translational and rotational components. The translational dynamics is used to synthesize the guidance law, while the rotational dynamics is employed to design an autopilot to stabilize the missile and to track guidance commands. Designs are generated for each subsystem and these designs are then assembled together. If the overall system performance is unsatisfactory, individual subsystems are re-designed to improve the system performance. Due to its iterative nature, this latter part of the design process can be highly time-consuming.

* Research Scientist, Associate Fellow AIAA, menon@optisyn.com

† Research Scientist, Associate Fellow AIAA, ohlmeyerEJ@NSWC.NAVY.MIL

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Figure 1 illustrates the differences between traditional and integrated guidance-control systems. In the conventional approach, the guidance law does not employ the missile body rates or the sensed acceleration components to generate the commands to the autopilot. As a result, in engagement scenarios requiring agile maneuvers, the guidance commands can sometimes exceed the autopilot performance limits. If the autopilot employs integral feedback loops for improved command tracking response, these guidance commands can drive the flight control system unstable. Additionally, since the autopilot does not use target relative missile position and velocity components for feedback, it cannot adjust its response to accommodate for agile target maneuvers.

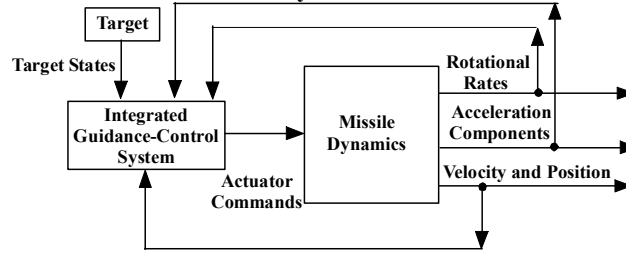
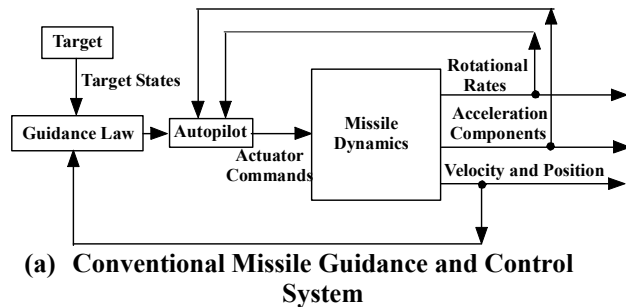


Figure 1. Conventional and Integrated Guidance-Control Systems

Consequently, the traditional design approach requires the autopilot to have a small time constant when compared with the guidance system to assure the stability and performance of the overall flight control system. In fact, the autopilot time constant often dictates the achievable interception accuracy of missiles equipped with conventional flight control systems [6, 7]. While the small autopilot time constant requirement can be easily met when the missile is far away from the target, it becomes increasingly difficult as the missile gets closer to the target. This is due to the fact that most guidance laws are functions of time-to-go, which makes their responses faster as the missile gets close to the target.

On the other hand, in the integrated approach, the guidance and autopilot functions use all the available measurements. As a result, the system is less likely to encounter saturation and stability problems. Moreover, the iterative design steps

required to ensure the compatibility between the guidance and autopilot systems are eliminated process.

While there are definite operational advantages in using integrated guidance-autopilot systems, their design is complicated. This is due to the fact that the increased dimension of the nonlinear control problem makes it difficult to apply the conventional gain-scheduling [8] design methodology. These high-order designs may require gain scheduling not only with respect to the airframe performance variables, but also with respect to the engagement geometry. Although nonlinear control system design techniques [9, 10] can make these problems more tractable, symbolic manipulations required for their development can be formidable. Recent advancements in computer-aided nonlinear control system design methods [11] offer more direct approaches for integrated system design, and avoids the need for any symbolic manipulations.

Another difficulty in integrated guidance-control system design arises from the fact the problem has to be posed as a finite interval control problem. Most design techniques available for linear system design assure stability and performance only when used in an infinite time-interval setting [12, 13]. While it is awkward to adapt linear control system design methods to formulate and solve the nonlinear integrated guidance-control problem, numerical nonlinear control techniques [11] can be readily employed for this purpose.

With the foregoing as the background, the present paper will discuss three different formulations for the integrated design of fixed-aim warhead missile guidance and control systems. The first method employs the yaw and pitch components of the zero effort miss (ZEM) vector [7, 14] as the primary state variables. The second formulation employs the pitch-yaw components of the missile relative target position as the primary state variables. This formulation is termed as the *pure pursuit integrated guidance strategy* in this paper. The third formulation is an integrated guidance-control analog of the classical proportional navigation algorithm [7].

Integrated guidance-control systems were designed using a six degree-of-freedom nonlinear simulation model of an air-to-air missile. The following section will briefly discuss the fixed-aim warhead missile model. Each of the designs was generated using a computer-aided nonlinear control system design software [11], using the feedback linearization method [9, 10] in conjunction with the *LQR* [12] technique. Sample engagement scenarios with a nonmaneuvering target will be given to illustrate integrated guidance-control system performance.

2. Fixed-Aim Warhead Missile Model

A six degree-of-freedom nonlinear dynamic model of an air-to-air fixed-aim warhead missile is employed in the present research. The missile is controlled using four tail-mounted fins, and includes the logic to distribute the pitch, yaw, and roll commands to the appropriate fins. The missile equations of motion are expressed in the body coordinate system X_B, Y_B, Z_B illustrated in Figure 2. The reference frame for the fixed-aim warhead is X_W, Y_W, Z_W . The warhead projects the fragments in a direction normal to the missile longitudinal axis, and is effective if the target is located within the *cone of effectiveness* illustrated in the figure. Successful operation of the warhead requires the missile to approach the target as close and as parallel as possible, while maintaining a specific roll orientation to direct the warhead fragments towards the target.

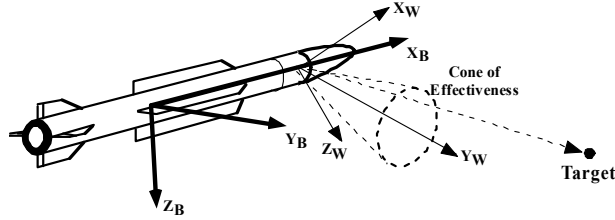


Figure 2. Fixed-Aim Warhead Missile Coordinate System

The equations of motion used in the present research are:

$$\begin{aligned}\dot{p} &= \frac{L}{I_x}, & \dot{q} &= \frac{M}{I_y} - \frac{(I_x - I_z)}{I_y} pr, \\ \dot{r} &= \frac{N}{I_z} - \frac{(I_y - I_x)}{I_z} pq \\ \dot{u} &= \frac{F_{xa} + F_{xg}}{m} - wq + vr, \\ \dot{v} &= \frac{F_{ya} + F_{yg}}{m} - ur + wp, \\ \dot{w} &= \frac{F_{za} + F_{zg}}{m} - vp + uq\end{aligned}$$

In the above equations, u, v and w are the velocity components measured in the missile body axis system. The variables p, q and r are the components of the body rotational rate. F_{xg}, F_{yg} and F_{zg} are the gravitational forces acting along the body axes. Gravitational forces will not be included in the integrated guidance-control law design, but will be included in the simulation evaluations. F_{xa}, F_{ya} and F_{za} are the aerodynamic forces; m is the vehicle mass and I_x, I_y and I_z are the vehicle moments of inertia. Note that the pitch moment of inertia I_y is generally equal to the yaw moment of inertia I_z . L, M and N

are the roll, pitch, and yaw aerodynamic moments. Aerodynamic forces and moments are computed using the expressions:

$$\begin{aligned}F_{xa} &= C_A \bar{q} s, & F_{ya} &= C_Y \bar{q} s, & F_{za} &= C_N \bar{q} s \\ L &= C_l \bar{q} s d, & M &= C_m \bar{q} s d, & N &= C_n \bar{q} s d\end{aligned}$$

The aerodynamic coefficients are specified as functions of angle of attack α , angle of sideslip β , Mach number and the roll, pitch, yaw fin deflections $\delta p, \delta q, \delta r$.

As a first step in the integrated guidance-control system design process, the missile model is used to set up a simulation. The computer-aided design software uses this simulation model, together with advanced numerical algorithms to construct the nonlinear controllers. The following sections will discuss the development of three different integrated guidance-control schemes using this software package.

3. Integrated Guidance-Control System Design

The integrated guidance-control system has the task of stabilizing the missile airframe while meeting the terminal conditions required for successful target interception. As in the conventional guidance-control systems, integrated flight control system must deliver low miss distances. Additionally, the fixed-aim warhead requires the missile to maintain a specific roll orientation with respect to the target at interception.

Three integrated guidance-control formulations that meet these requirements will be discussed in the following sections. Each of these formulations employs the feedback linearization technique [9 - 11] in conjunction with the *LQR* method [11, 12] for control system design. A computer-aided nonlinear control system design software [11] is used to enable the use of the missile simulation model in the integrated guidance-control system design. Since a large body of literature is available on these control techniques, they will not be discussed in this paper. The following sections will focus on the problem formulation and simulation results.

3.1. Zero Effort Miss Guidance Strategy

The zero effort miss guidance strategy attempts to drive the zero effort miss components [7, 14] along the y and z inertial axes to zero, while stabilizing the airframe. As formulated in the present research, the roll attitude is continuously adjusted to orient the warhead in the direction of the target throughout the engagement.

The zero effort miss is defined in the following manner. Let y_T^I be the current position of the target and \dot{y}_T^I be the velocity of the target. Assuming that

the velocity remains constant for the remaining time-to-go t_{go} , the target position at interception is

$$y_{Tf}^I = y_T^I + \dot{y}_T^I t_{go}$$

Similarly, if the current position of the missile is y_M^I and its velocity is \dot{y}_M^I , the missile position after time t_{go} will be

$$y_{Mf}^I = y_M^I + \dot{y}_M^I t_{go}$$

The zero effort miss in the yaw direction is then defined as:

$$zem_y = (y_{Tf}^I - y_{Mf}^I) + (\dot{y}_T^I - \dot{y}_M^I) t_{go}$$

The zero effort miss in the pitch direction can be defined in a similar manner as:

$$zem_z = (z_{Tf}^I - z_{Mf}^I) + (\dot{z}_T^I - \dot{z}_M^I) t_{go}$$

The variables z_T^I and z_M^I are the components of the target and missile position vectors resolved along the inertial z axis, and \dot{z}_T^I and \dot{z}_M^I are the components of the target and missile velocity vectors resolved along the inertial z axis.

As discussed in the foregoing, if the target does not maneuver, the expression for zero effort miss suggests that missile velocity can be adjusted such that no further control effort is required for driving the miss distance to zero. In the case of a maneuvering target, the control can be adjusted in each guidance interval to drive the zero effort miss close to zero as t_{go} tends to zero.

Note that the zero effort miss distance computation requires the time-to-go. Following the literature [7], an approximate estimate of time-to-go can be obtained in terms of range and range rate as:

$$t_{go} = -\frac{R^2}{\dot{R}_1 + \dot{R}_2 + \dot{R}_3}$$

$$\dot{R}_1 = (x_T^I - x_M^I)(\dot{x}_T^I - \dot{x}_M^I)$$

$$\dot{R}_2 = (y_T^I - y_M^I)(\dot{y}_T^I - \dot{y}_M^I)$$

$$\dot{R}_3 = (z_T^I - z_M^I)(\dot{z}_T^I - \dot{z}_M^I)$$

Where, the range is given by:

$$R = \sqrt{(x_T^I - x_M^I)^2 + (y_T^I - y_M^I)^2 + (z_T^I - z_M^I)^2}$$

Nonlinear integrated guidance-control problem of driving zem_y and zem_z to zero is solved in the present paper using a computer-aided nonlinear control system design software package [11]. The first step in synthesizing a nonlinear controller using this software package is that of specifying the main dependencies between the state and the control variables in the problem. In the present case, the roll fin deflection influences the roll rate, which in turn

influences the roll attitude. In the pitch channel, the pitch fin deflection influences the pitch rate. The pitch rate generates the angle of attack, which results in normal acceleration that influences the zero effort miss in the pitch axis. Similarly, in the yaw axis, the yaw fin deflection influences the yaw component of the zero effort miss through the yaw rate and angle of sideslip. These dependencies can be symbolically expressed as:

$$\delta_p \rightarrow p \rightarrow \phi$$

$$\delta_q \rightarrow q \rightarrow \alpha \rightarrow zem_z$$

$$\delta_r \rightarrow r \rightarrow \beta \rightarrow zem_y$$

The computer-aided nonlinear control system design software uses these relationships to construct feedback linearizing transformations. The next step in the design process is that of using the feedback linearized model to construct a linear quadratic regulator. The designer has the responsibility for specifying the state and control weights. For the present problem, these were specified as:

State weighting matrices:

$$Roll : \begin{bmatrix} 100 & 0 \\ 0 & 10 \end{bmatrix}, Pitch : \begin{bmatrix} 1e4 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$Yaw : \begin{bmatrix} 1e4 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Control weighting matrix:

$$FinDeflections : \begin{bmatrix} 0.001 & 0 & 0 \\ 0 & 0.001 & 0 \\ 0 & 0 & 0.001 \end{bmatrix}$$

Using the foregoing data, the computer-aided nonlinear control system design package can synthesize a nonlinear controller that can be coupled to the missile simulation to obtain closed-loop responses.

3.1.1. Simulation Results

Closed-loop simulation results for the zero effort miss integrated guidance strategy in a beam shot engagement scenario are given in this section. A nonmaneuvering target in level flight at 10,000 feet altitude is used for this engagement. The target speed is set to 900 ft/s, the heading angle is chosen to be 90 degrees. Pitch attitude θ and the roll attitude ϕ are initialized to zero. The missile altitude is set at 10,000 ft and the speed is set to Mach 4.5. The missile is initially located at -6500 ft along the x -axis of the inertial frame. All the other states of the missile are initialized to zero.

The missile and target positions with respect to the inertial frame are shown in Figure 3. It may be observed in this figure that the missile continuously turns towards the target to reduce the zero effort miss distance. The miss distance for this engagement was 1.3 feet. The angle of attack and angle of sideslip histories are shown in Figure 4. The angle of attack initially increases to initiate the climb towards the target and rapidly decreases at the end to capture the target. A large angle of sideslip is generated for a short duration to turn the missile towards the target. For the rest of the flight, a small but increasing magnitude of the sideslip angle is used to maneuver the missile towards the target.

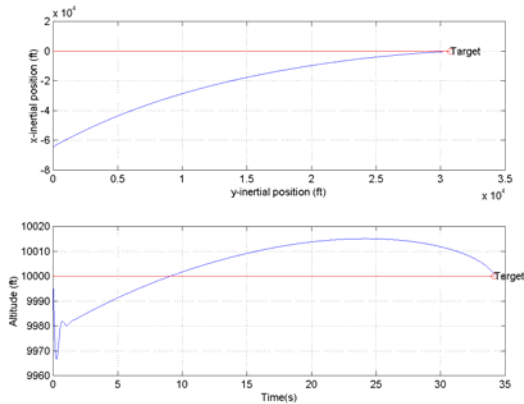


Figure 3. Position and Altitude Time Histories

The roll angle history and the missile relative pitch and yaw angles of the target are shown in Figure 5. Since the target and the missile are essentially flying at the same altitude, the fixed-aim warhead required the maintenance of zero roll attitude. After the initial perturbation, the integrated guidance-control system maintained the roll angle near zero till the very end. The terminal transient in this figure arises from the indefinite nature of the zero effort miss as time-to-go tends to zero. The relative pitch angle is also maintained within two degrees. Starting with an initial value of 90 degrees, the relative yaw angle is gradually lowered to 20 degrees. The roll, pitch and yaw components of the angular velocity vector of the missile are shown in Figure 6. This figure also shows that the initial perturbation quickly settles down and there is little body motion until the very end of the engagement. Thus, the integrated guidance-control law has stabilized the missile airframe. As in the roll attitude history, the terminal transients arise due to the indefinite nature of the zero effort miss variable at very small values of time-to-go. The roll, pitch and yaw fin deflections are shown in Figure 7.

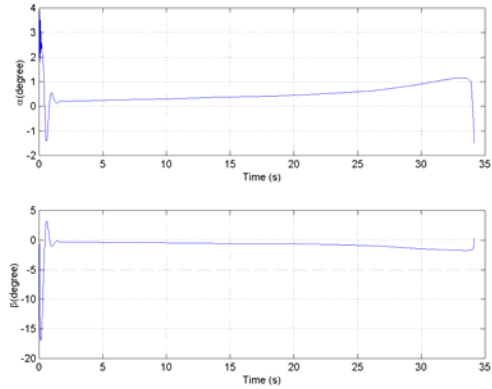


Figure 4. Angle-of-attack and Angle of Sideslip Time Histories

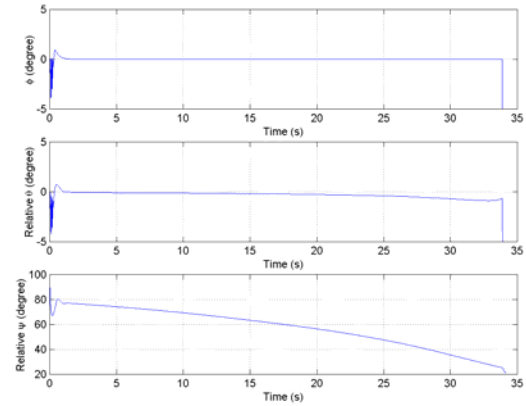


Figure 5. Roll Angle and Relative Pitch and Yaw Angle Histories

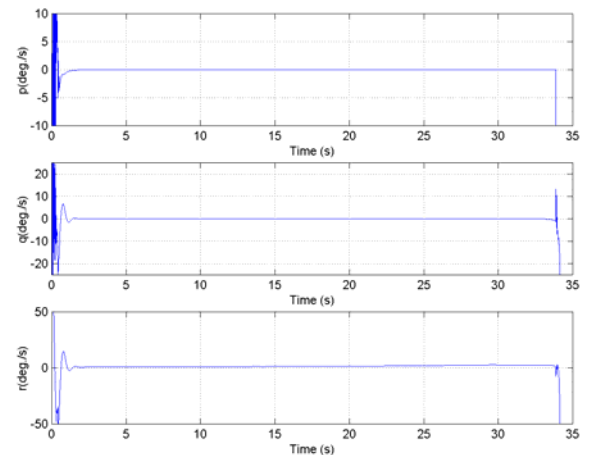


Figure 6. Roll, Pitch and Yaw Body Rate Time Histories

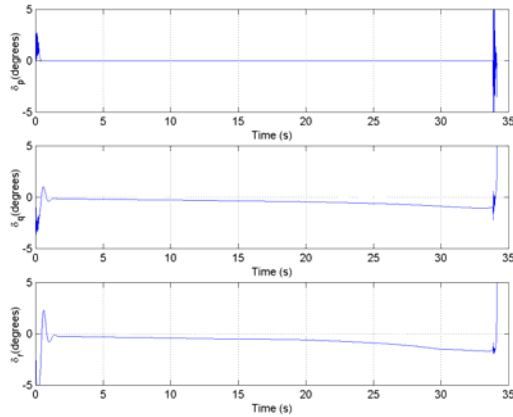


Figure 7. Fin Deflection Time Histories

3.2. Pure Pursuit Guidance Strategy

The pursuit guidance strategy attempts to obtain a favorable differential heading for the missile at the final time by driving the pitch and yaw components of the missile relative target position vector, y_R^M and z_R^M to zero. Note that the x_R^M component is not controllable. The missile relative target position vector is obtained by transforming the instantaneous target-missile relative position in the inertial frame. Note that this transformation requires the missile attitudes with respect to the inertial frame.

The control chain is next defined to facilitate the use of the computer-aided nonlinear control system design software, as described in Reference 11. The control chain defines how the control and state variables influence each other. Only the major relationships need to be defined. Other parasitic and coupling terms are automatically computed by the software. The control chains for the pure pursuit guidance problem are of the form:

$$\begin{aligned}\delta_p &\rightarrow p \rightarrow \phi \\ \delta_q &\rightarrow W_R^M \rightarrow z_R^M \\ \delta_r &\rightarrow V_R^M \rightarrow y_R^M\end{aligned}$$

Here, V_R^M and W_R^M are the yaw and pitch components of the relative velocity vector of the target with respect to the missile. The computer-aided nonlinear system design software uses this control chain to construct feedback linearized form of the missile model. It then applies the *LQR* technique to synthesize the nonlinear guidance-control law. The state and control weights used in the *LQR* synthesis were as follows:

State weighting matrices:

$$\text{Roll} : \begin{bmatrix} 1e3 & 0 \\ 0 & 1e-4 \end{bmatrix}, \quad \text{Pitch} : \begin{bmatrix} 1e6/R & 0 \\ 0 & 1e-3 \end{bmatrix},$$

$$\text{Yaw} : \begin{bmatrix} 1e6/R & 0 \\ 0 & 1e-3 \end{bmatrix}$$

Control weighting matrix:

$$\text{Fin Deflections} : \begin{bmatrix} 0.001 & 0 & 0 \\ 0 & 0.001 & 0 \\ 0 & 0 & 0.001 \end{bmatrix}$$

Note that the state weighting matrices in the pitch and yaw directions have been formulated to be inversely proportional to range. This ensures that the integrated guidance-control system delivers increasingly tighter control as the missile approaches the target.

In order to provide a more precise orientation of the fixed-aim warhead, an alternate attitude control strategy was introduced in the pure pursuit integrated guidance-control formulation. When the range to the target falls below 200 feet, the integrated guidance-control system is used to execute pitch, yaw, roll attitude maneuvers to orient the warhead towards the target and to make the missile body axis as closely parallel to the target body axis as possible. For the present study, the warhead direction was arbitrarily assumed to require a roll attitude of 44.5 degrees.

3.2.1. Simulation Results

As in the simulation results given in Subsection 3.1.1, the pure pursuit integrated guidance strategy is next evaluated in a beam shot engagement. In this scenario, the target is in level flight at 11,000 feet altitude, at 900 ft/s airspeed. It has a constant heading angle of 90 degrees with respect to the missile launch point, and is initially located at 24,000 ft along the x-axis and 4500 ft along the y-axis in the inertial frame. The missile is launched at an altitude of 10,000 ft and is flying at Mach 4.5. All the other states of the missile are initialized to zero.

Figure 8 shows the missile and target position with respect to the inertial frame. The target continues to fly at the same altitude from left to right while the missile continuously turns towards the target by pointing its longitudinal axis at the target. The missile adjusts its trajectory so as to meet the terminal aspect angle requirement. The angle of attack and angle of side-slip histories are shown in Figure 9. The integrated guidance-control system initially increases the angle of attack to initiate the climb towards the target. A large angle of sideslip can be observed at the beginning of the engagement, and is used to turn the missile towards the target. The effect of terminal attitude maneuvers can also be seen in Figure 9. Both the angles of attack and sideslip

increase during the terminal attitude maneuver and quickly return to zero after the desired attitudes have been achieved.

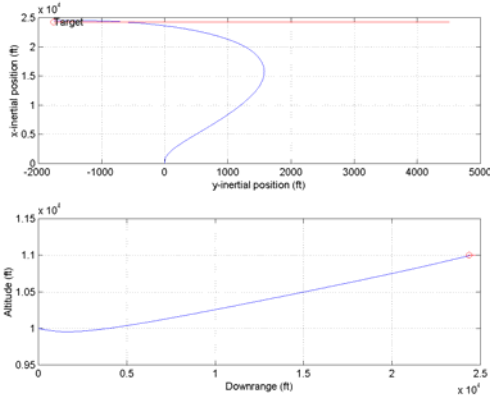


Figure 8. Missile and Target Position Time Histories

The roll attitude and relative pitch and yaw attitude histories of the missile are shown in Figure 10. The effect of the terminal attitude maneuver can be clearly seen in Figure 10. The roll attitude angle is maintained close to zero until the initiation of the terminal attitude maneuver that rolls the missile to 44.5 degrees to point the warhead towards the target. The relative yaw attitude increases from -90 degrees to 43 degrees as the missile approaches the target. The terminal maneuver changes the yaw attitude of the missile to match the yaw attitude of the target, thus driving the relative yaw attitude to zero.

The roll, pitch and yaw rates of the missile body are presented in Figure 11. During the terminal maneuver, large body rates are required to quickly align the missile with respect to the attitude of the target. It may be observed that the integrated guidance-control system preserved airframe stability throughout the engagement.

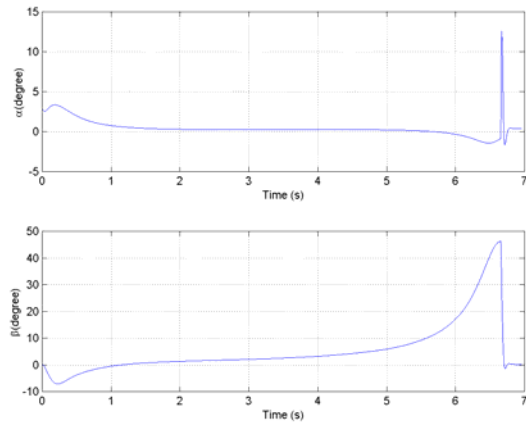


Figure 9. Angle of Attack and Angle of Sideslip Histories

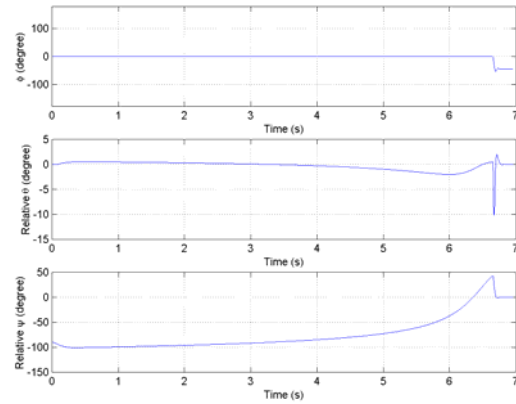


Figure 10. Roll, Pitch and Yaw Attitude Histories.

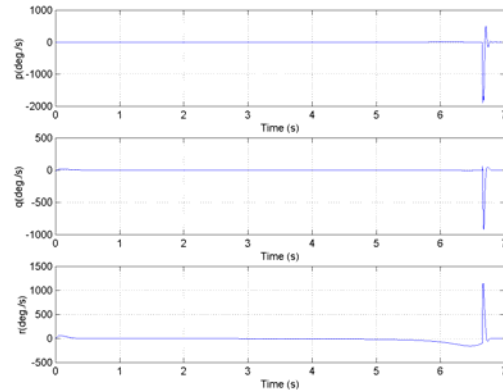


Figure 11. Roll, Pitch and Yaw Body Rate Histories

The roll, pitch and yaw fin deflections δ_p , δ_q and δ_r histories are given in Figure 12. After an initial transient, most of the fin demand occurs during the terminal attitude maneuver. The missile relative target position vector components at the final time were: $x_R^M = 6.7 ft$, $y_R^M = -150.9 ft$, $z_R^M = -153.4 ft$. The larger values of lateral miss arise in this guidance-control strategy from the terminal attitude maneuvers that sacrifice relative position accuracy in favor of attitude accuracy. This tradeoff can be made more favorable by waiting to execute the terminal attitude maneuvers until the missile gets closer to the target. Such investigations will be undertaken during the future research.

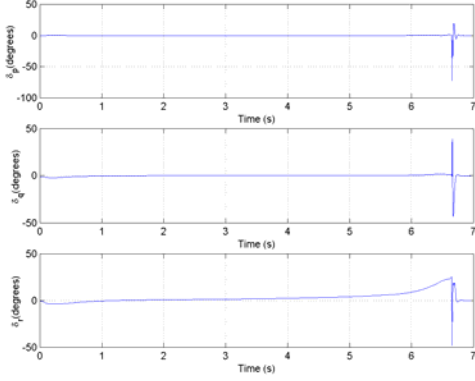


Figure 12. Roll, Pitch and Yaw Fin Deflection Histories

3.3. Integrated Proportional Navigation

The third formulation of the integrated guidance-control problem considered in this paper is an analog of the classical proportional navigation [7]. In this case, the guidance-control objective is to drive the target line-of-sight rate with respect to the missile to zero, while stabilizing the airframe. The yaw and pitch components of the line-of-sight angle can be computed using missile and target position vectors in the inertial frame as:

$$\lambda_y = \tan^{-1} \left(\frac{y_T^I - y_M^I}{x_T^I - x_M^I} \right), \quad \lambda_z = \tan^{-1} \left(\frac{z_T^I - z_M^I}{x_T^I - x_M^I} \right)$$

The corresponding line of sight rates can be computed as:

$$\dot{\lambda}_y = \frac{(x_T^I - x_M^I)(\dot{y}_T^I - \dot{y}_M^I) - (y_T^I - y_M^I)(\dot{x}_T^I - \dot{x}_M^I)}{(x_T^I - x_M^I)^2 + (y_T^I - y_M^I)^2}$$

$$\dot{\lambda}_z = \frac{(x_T^I - x_M^I)(\dot{z}_T^I - \dot{z}_M^I) - (z_T^I - z_M^I)(\dot{x}_T^I - \dot{x}_M^I)}{(x_T^I - x_M^I)^2 + (z_T^I - z_M^I)^2}$$

As in the two previous formulations, the first step in the design process is the definition of the control chains. The control chain in the roll axis is identical to that used in the other two formulations discussed in this paper. In the pitch and yaw axis, the fin deflections are used to generate pitch and yaw rates, which in turn generate the angle of attack and angle of sideslip. The angle of attack and angle of sideslip then influence the line-of-sight rates. These processes can be summarized symbolically as:

$$\begin{aligned} \delta_p &\rightarrow p \rightarrow \phi \\ \delta_q &\rightarrow q \rightarrow \alpha \rightarrow \dot{\lambda}_z \\ \delta_r &\rightarrow r \rightarrow \beta \rightarrow \dot{\lambda}_y \end{aligned}$$

The computer-aided nonlinear control system synthesis software [11] uses these control chains to compute feedback linearizing transformations. The

LQR technique is then employed to compute the feedback linearized integrated guidance-control logic. The state and control weights used in the design are:

$$\text{Roll} : \begin{bmatrix} 100 & 0 \\ 0 & 10 \end{bmatrix}, \quad \text{Pitch} : \begin{bmatrix} 100 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{Yaw} : \begin{bmatrix} 100 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{Fin Deflections} : \begin{bmatrix} 1e-03 & 0 & 0 \\ 0 & 1e-03 & 0 \\ 0 & 0 & 1e-03 \end{bmatrix}$$

3.3.1. Simulation Results

The integrated proportional navigation scheme is next evaluated in a beam-shot engagement scenario close to that discussed in Section 3.2.1. In this scenario, the target is in level flight at 11,000 feet altitude, at 900 ft/s airspeed. It has a constant heading angle of 95 degrees with respect to the missile launch point, and is initially located at 24,000 ft along the x-axis and 4500 ft along the y-axis in the inertial frame. The missile is launched at an altitude of 10,000 ft and is flying at Mach 4.5.

The trajectories of the missile and the target are given in Figure 13. It is instructive to compare this figure with Figure 8. These figures show that the proportional navigation achieves target interception, but fails to provide the desired aspect angle at interception. The miss distance for this engagement was 5.4 feet. The angle of attack and angle of sideslip histories in Figure 14 are typical of proportional navigation laws exhibiting large initial and final transients with fairly modest values in the middle. The pitch, yaw, roll body rates in Figure 15 illustrate the system stability throughout the maneuver.

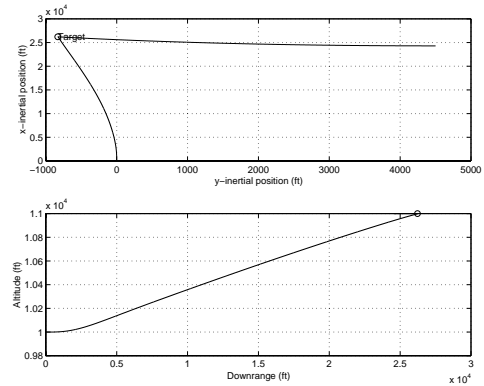


Figure 13. Missile and Target Position Time Histories

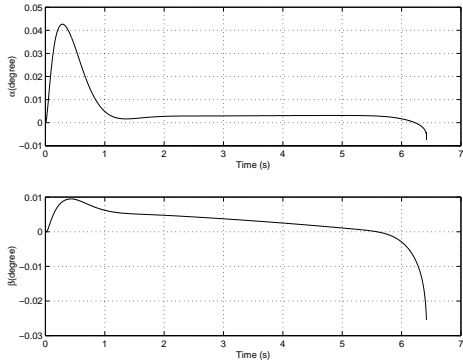


Figure 14. Angle of Attack and Angle of Sideslip Histories

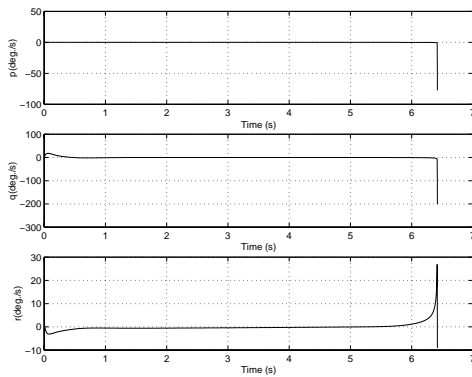


Figure 15. Roll, Pitch and Yaw Fin Deflection Histories

4. Conclusions

This paper discussed the development of three integrated guidance-control systems for an air-to-air Missile incorporating a fixed-aim warhead. The fixed-aim warhead technology seeks to reduce the weapon system weight by using a highly directional warhead, together with enhancements to the guidance-control systems. The fixed-aim warhead projects the blast fragments in a direction normal to the missile longitudinal axis. In order to be effective, this warhead requires the missile to achieve a specific roll orientation with respect to the target at interception. Moreover, it is desirable to achieve a near-zero relative lateral velocity vector orientation with respect to the target at interception to minimize the sensitivity of the system to fuze delay. These specifications require the missile to perform a combination of conventional lateral acceleration maneuvers and terminal attitude maneuvers during its operation.

Integrated guidance-control systems were derived using three different operational strategies. The first strategy minimizes the zero effort miss.

Minimization of the missile relative pitch and yaw components of the target position vector produced the second guidance-control strategy. This strategy was termed as the *pure pursuit guidance* strategy. The last strategy is an integrated system analog of the classical proportional navigation. In each case, the integrated guidance-control system was required to stabilize the airframe while achieving target interception. Integrated system designs were carried out using the feedback linearization-*LQR* methodology, using a computer-aided nonlinear control system design software. This software package avoids the need for complex symbolic manipulations often required to synthesize nonlinear control laws.

For a sample beam shot scenario analyzed in this paper, the zero effort miss strategy produced the smallest miss distance, while the pure pursuit guidance strategy resulted in the most favorable differential heading for the fixed-aim warhead. Integrated proportional navigation produced miss distances comparable to the zero effort miss formulation. All three integrated guidance-control schemes were able to stabilize the airframe and execute the attitude maneuvers required to orient the fixed-aim warhead.

Future research will assess the performance of these integrated guidance-control systems in multiple engagement scenarios, and will examine alternate formulations to satisfy the terminal aspect angle requirements.

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