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Integrated models of capital adequacy – Why banks are undercapitalised

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ABSTRACT

With the majority of large UK and many US banks collapsing or being forced to raise capital over the 2007–9 period, blaming bankers may be satisfying but is patently insufficient; Basel II and Federal oversight frameworks also deserve criticism. We propose that the current methodological void at the heart of Basel II, Pillar 2 is filled with the recommendation that banks develop fully-integrated models for economic capital that relate asset values to fundamental drivers of risk in the economy to capture systematic effects and inter-asset dependencies in a way that crude correlation assumptions do not. We implement a fully-integrated risk analysis based on the balance sheet of a composite European bank using an economic-scenario generation model calibrated to conditions at the end of 2007. Our results suggest that the more modular, correlation-based approaches to economic capital that currently dominate practice could have led to an undercapitalisation of banks, a result that is clearly of interest given subsequent events. The introduction of integrated economic-scenario-based models in future can improve capital adequacy, enhance Pillar 2's application and rejuvenate the relevance of the Basel regulatory framework.

1. Introduction

The Economic Capital (EC) concept is clear from a technical perspective – it is the capital that a financial institution requires in order to operate as a solvent concern at a specified confidence level over a given time horizon. In the banking sector, Pillar 2 of Basel II was specifically intended to focus on the regulatory review and internal risk assessment procedures, examining the extent to which risk management best practices are embedded into bank decision making. Economic capital modelling and the closely related requirement for stress testing have become fundamental planks of Pillar 2 compliance (Alexander and Sheedy, 2008). Moreover, banking institutions are required by Pillar 3 to disclose these risk assessments to external stakeholders. A fundamental problem, however, is that Pillar 2 EC calculation and Pillar 3 disclosure requirements exist without clear regulatory guidance as to the methodology that complex institutional capital models should employ to *integrate* risk effects across asset classes.

Broadly, EC encapsulates the concept of measuring risk across a financial institution and using the model and its outputs in risk-adjusted comparisons of performance to assist strategic decision making and deliver value for shareholders. In this paper, we will argue that the consideration of economic scenarios, their firm-wide effects and the dependencies they induce in asset performances should be the cornerstone of economic capital practice, and that this requirement should be more clearly articulated in regulation. In reviewing the current state of financial regulation Brunnermeier et al. (2009) find that "macro-economic analysis and insight has, in the past, been insufficiently applied to the design of financial regulation…the crisis which began in the US sub-prime mortgage market in early 2007 and then spread broadly and deeply was not the first banking crisis. It was closer to the 100th…".

A central question concerns the nature of integrated risk methodology used by financial institutions for economic capital calculation before and during the current crisis. How were/are risk effects considered across asset classes and then integrated into a coherent capital framework? A summary of methodological practice in the financial sector is presented in a comprehensive pre-crisis survey by the International Financial Risk Institute that included both banks and insurance companies. In this survey, the prevailing

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approach is reported to be assessment of risk through standalone models for broad asset classes (or in many cases crude risk categories like market, credit and operational risk) followed by integration using correlation matrices (see IFRI Foundation and CRO Forum, 2007). This approach to integration was favoured by over 75% of the surveyed banks with the others using simulation approaches or hybrid approaches. In the insurance industry there was more diversity in the approaches used for integration: around 35% of respondents used the correlation approach and about the same number used simulation; the remainder reported the use of copulas or hybrid approaches.

The correlation-based method favoured by so many of the IFRI respondents, and in particular the banks, is a modular calculation approach, widely used for its simplicity. In such an approach capital requirements are estimated on a per asset class basis using an appropriate risk model for that asset class and a risk measure such as Value-at-Risk (VaR). At the simplest level these per-asset-class capital requirements can be added although this tends on the whole (but not always) to overstate capital requirements (Alessandri and Drehmann, 2010; Breuer et al., 2010). Inter-asset diversification is typically superimposed using a matrix overlay of correlation coefficients between asset classes. In this way there is a resultant downward adjustment to the total capital charge applied to the institution as a whole. A good example of a very detailed application of the modular approach is Rosenberg and Schuermann (2006), which also shows how copulas can be used in place of correlations to take better account of dependencies in the tail.

While modular methods, when carried out carefully, may give adequate results in "normal" periods, it has become clear that the modular approach may prove unreliable in crises and that the complex interactions of macro-economic factors, financial risk factors, liquidity effects and asset valuations on which economic capital assessment depends cannot be underpinned by such a simplistic integration approach. Superimposed correlation numbers are hard to justify, subject to sampling error on account of scarce data, and, most importantly, make no attempt to tell the narrative of how correlation arises which is necessary for risk mitigation and management. In fact, it is essential to understand the sources of correlation if one wants to measure inter-asset dependencies and use this to reduce dependencies between different lines of business.

Integration is an extremely important methodological issue that requires urgent global regulatory guidance. In a report of the Financial Stability Forum (2008) supervisors have acknowledged the need for Pillar 2 principles to strengthen banks' risk management practices, to sharpen banks' control of tail risks and to mitigate the build-up of excessive exposures and risk concentrations. Addressing the methodological deficiencies of current treatments of integration is a major part of this challenge. Our contention in this paper is that fully integrated factor models based on scenario generation are the key to addressing this issue. Aggregate risk capital should depend on changes in the valuation of asset positions which are driven by vectors of risk factors calibrated to real-world economic conditions. Capital held to support asset positions should only be reduced by diversification due to differences in risk driver dependencies from position to position. This reflects the fact that, although it may be possible for banks to limit risks by not holding certain asset classes, it is not possible for bank assets to fully avoid the pervasive systematic effects of risk factors describing interest rates, inflation, credit, equity and property risk (Alessandri and Drehmann, 2010; Drehmann et al., 2010).

Although our focus in this paper will be *fully-integrated models at institutional level*, this work is taking place against the backdrop of a wide-ranging review of regulation that raises important questions about the future of so-called micro-prudential regulation. Brunnermeier et al. (2009) suggest that regulation has been excessively focussed on seeking to improve the behaviour and risk management practices of individual banks. However, the fully-integrated approach described in this paper has its counterpart in integrated models of system-wide risk with additional feedback effects that are being developed by central banks to shed light on systemic crises and macro-prudential regulation.

The main contributions of this paper are: (i) to demonstrate the feasibility of fully-integrated economic capital modelling by applying the methodology to a composite balance sheet derived from a sample of European banks from the pre-crisis period; (ii) to show how the results suggest a much higher level of capitalisation would have been desirable than that implied by a typical modular correlation-based approach (i.e. the approach currently used by the majority of institutions – see IFRI Foundation and CRO Forum (2007)); (iii) to show how the fully-integrated approach allows the allocation of this capital to asset classes to gain deeper insights into the issue of diversification. We conclude that there is little surprise that current practice in enterprise risk management failed to insulate the banking sector against the extreme capital losses that were incurred.

The paper is structured as follows. In Section 2 we describe the derivation of an "average" European bank which will be used for the empirical investigation of capital adequacy. In Section 3 we summarise the fully-integrated methodology of the paper, contrast it with more modular approaches, and describe the architecture of the economic scenario generation model that we will use. Results are presented in Section 4 where we devote particular attention to discussions of fully-integrated projection and fair capital allocation at the institutional level. Section 5 concludes.

2. Construction of an average European bank

To provide empirical insights into the differing effects of implementing both modular and fully-integrated approaches to capital, we construct a composite 2006 balance sheet of a representative European bank (EuroBank). Balance sheets for 51 European banks for the year 2006 are selected to provide a cross-sectional assessment of capital adequacy prior to the credit crisis. Summary statistics presented in the Fifth Quantitative Impact Study, (QIS5, Basel Committee, 2006) inform our split of aggregate asset positions by exposure type and credit class, ensuring consistency with asset profiles held by European banks.

The reason we specifically select European banks as at 2006 is that Europe offers a fertile ground for investigating the basic effects of diversification on EC in the context of implementing Basel II Pillar 2 regulations. Our data enables a pre "credit crunch" view of sector capital adequacy.

We reformulate individual bank balance sheets into a format that can be utilized to compare EC approaches. Thomson Worldscope database is used to collect an initial sample of 90 banks whose primary listings are the six largest banking nations in Europe: the United Kingdom (GBR), France (FRA), Germany (GER), Italy (ITA), Spain (ESP) and the Netherlands (NED).We exclude small banks (defined as banks with less than £500 million in total assets), retaining banks which are engaged in at least one of the following activities: investment banking, deposit-taking or loan-making. Institutions classified as Islamic banks are also excluded as their asset accounting information does not allow the use of QIS 5 asset mapping characteristics. After exclusions the sample set is reduced to 51 banks, with the majority of their assets regulated in the UK and the Euro-zone, and therefore subject to Basel II Pillar 2. Categorisation of individual banks' balance sheet items into broader asset classes is informed by notes accompanying the

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Table 1
Geographic data of the sample set in £ million. This table gives the geographical distribution of the 51 European banks in the sample set (Appendix A, Table A.13). It displays the total asset value per country, the total number of banks per country and the percentage of total assets in the entire sample set represented by each respective country and in the final column for the entire sample set.

Country	GBR	GER	FRA	ITA	ESP	NED	Europe
Total asset	3654142.88	1314256.21	2401069.29	1111829.91	982681.26	600281.90	10064261.47
Number of banks	9	7	6	18	9	2	51
Percentage (%)	36.31	13.06	23.86	11.05	9.76	5.96	100.00

institutions' annual reports. Table 1 gives a summary overview of the distribution of total asset value for the sample. For simplification we assume that the composition of EuroBank's portfolio does not include proprietary derivative positions, a reasonable assumption given the objective of this work: to compare modular and integrated EC. Our results are robust to the inclusion of these derivative positions, since the use of the modular approach is likely to understate the risk of complex derivatives with non-linear payoff profiles. Likewise, risk characteristics of CDOs and RMBS are not specifically modelled. The real problem is that disclosures for these asset classes are often opaque. We classify structured products as trading book assets and allocate QIS5 type risk characteristics (note that this conservative treatment strengthens the results of this work – more capital would be required to support riskier asset positions).

Credit risky assets are split into five categories dependening on their Basel II exposure type: claims on sovereigns, banks, corporates, retail customers and specialized lending. For example, the capital charge for lending to a corporate is higher than for lending to a government. As shown in Table 2, lending to corporates and retail/mortgage products are EuroBank's core business. To reflect credit asset characteristics, we impute the QIS 5 rating attributes for these classes. Worldscope data disclose nominal figures for each bank's investment and loan portfolio, and so detailed information on the asset composition of each bank's investment and loan portfolio is hand collected from the annual financial statements. The majority of banks supply data that enables the derivation of asset composition for investment and loan portfolios.[1] Based on an evaluation of accounting notes contained in the 2006 Annual Reports we obtain an approximate picture of weighted average asset holdings in each bank's investment and loan portfolios.

The Basel Committee on Banking Supervision has conducted several Quantitative Impact Studies to gather information to assess the effect of the Basel II regulatory framework on capital requirements. In the Fifth QIS (Basel Committee, 2006), the probability distribution of default for every category of credit asset is calibrated and linked to the corresponding credit rating and asset model (see Crouhy et al. (2000) for full discussion of model alternatives). The percentage of exposure in three PD ranges are mapped to external credit rating grades of *A and better*, *BBB*, and *worse than BBB*. Table 3 illustrates the portfolio composition.

We make the simplifying assumption that all sovereign bonds are AAA rated. For group *A and better*, we assume that one-third are AA rated. The category *worse than BBB* is considered as uniformly BB rated, see Table 4. For example, 38.5% of exposure in corporate loan portfolio show a probability of default less than 0.2%,

Table 2
Balance sheet assets (December 2006) of EuroBank in £ million. This table illustrates the arithmetic average of 51 banks' balance sheets which will be used as EuroBank's balance sheet (last two rows of Appendix A, Table A.13). Derivative positions are excluded.

Asset class	Average exposure	% Total assets
Cash	2898.33	1
Claims on government	22716.34	12
Claims on banks	31281.68	16
Claims on corporates	65914.72	33
Retail loans	11441.62	6
ABS	2897.66	1%
Residential loans	37761.59	19
Commercial real estate	6061.13	3
Property	2283.87	1
Equity	14081.54	7
Total assets	197338.46	100

Table 3
The calibration of probabilities of default for three categories of credit assets given in QIS 5 (Basel Committee, 2006). This table displays the calibration of probabilities of default for three categories of credit assets (bank loan, corporate loan and retail loan) and three credit ratings (A, BBB and BBB-) given in QIS 5 (Basel Committee, 2006). For example, 86.2% of claims on banks with PD less than 0.2% are rated as A.

	PD < 0.2% (A)	0.2% ⩽ PD < 0.8% (BBB)	PD ⩾ 0.8%(BBB-)
Banks	86.2%	9.1%	4.7%
Corporates	38.5%	31.8%	29.7%
Retails	30.8%	34.6%	34.6%

which implies a credit rating better than BBB for 38.5% of corporate portfolio. For currency we apply a 70%/30% split to recognise that balance sheet assets are denominated in both GBP and Eurozone currency.

The QIS 5 composition parameters for investment and loan portfolios are applied consistently across all 51 banks to replicate detailed balance sheet attributes (Appendix A, Table A.13). In projecting and simulating EuroBank, we use Table 2 as the initial balance sheet (arithmetic average of Appendix A, Table A.13). Banks across the European region differ in size and asset structure; nonetheless the analysis of EuroBank is representative of QIS 5 asset attributes and is therefore useful for examining the difference between modular and integrated EC calculations.

One very real enterprise risk management (ERM) challenge is how different portfolios perform over different time horizons. As one of the key confidence setting parameters in ERM, the time period for capital management directly affects the choice between conditional (point in time) and unconditional (through the cycle) calibration processes. Using Table 4, we transform EuroBank's original balance sheet into a rating based balance sheet (Table 5), and then compute the EC by the modular approach. For objective comparison with the fully-integrated approach, we use the covariance matrix proposed by Standard and Poor's (2008); firstly, it lacks bank-specific institutional bias and secondly, it is an informed and well-justified approximation of asset class correlations.

[1] Off-balance sheet exposures for credit lines are not specifically modelled. These are usually representative of on-balance sheet asset characteristics and therefore can be considered as having a multiplicative scaling effect on the positions we do consider. Their omission is very unlikely to change our qualitative conclusions with respect to undercapitalisation. However, this simplifying assumption does not account for the proportion of undrawn (relative to drawn) credit lines. These could differ largely across asset portfolios and potentially lead to or reduce concentration effects.

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Table 4
Mapping balance sheet data to asset models using the Fifth Quantitative Impact Study (QIS 5) statistical parameters. This table provides mapping parameters for each asset class in EuroBank's balance sheet (Table 2). These percentage parameters are collected from QIS 5 (Basel Committee, 2006) and adjusted by our assumptions. The second and third columns together represent the asset classes and credit ratings to which balance sheet items are mapped for modelling purposes. For example, 63% and 27% of the claims on government are mapped into domestic and foreign AAA risk-free nominal bonds, respectively with the remaining 10% mapped to AAA risk-free index-linked bonds.

Asset	Modeled as	Credit rating[a]	% Split	Dom. 70%	For. 30%
Claims on government[c]	Risk-free nominal bonds	AAA	90%	63%	27%
	Risk-free index-linked bonds[b]	AAA	10%	–	–
	Nominal sovereign bonds	AA			
	Nominal sovereign bonds	A			
	Nominal sovereign bonds	BBB			
	Nominal sovereign bonds	BB			
			100%		
Claims on bank[d]	Nominal corporate bonds	AA	26%	18%	8%
	Nominal corporate bonds	A	52%	36%	16%
	Nominal corporate bonds	BBB	8%	6%	2%
	Nominal corporate bonds	BB	4%	3%	1%
	Index-linked corporate bonds[b]	A	10%	–	–
			100%		
Claims on corporates[e]	Nominal corporate bonds	AA	12%	8%	3%
	Nominal corporate bonds	A	23%	16%	7%
	Nominal corporate bonds	BBB	29%	20%	9%
	Nominal corporate bonds	BB	27%	19%	8%
	Index-linked corporate bonds[b]	A	10%	–	–
			100%		
Retail loans[f]	Nominal corporate bonds	AA	4%	3%	1%
	Nominal corporate bonds	A	8%	6%	3%
	Nominal corporate bonds	BBB	30%	21%	9%
	Nominal corporate bonds	BB	58%	40%	17%
			100%		
Residential loans[g]	Nominal corporate bonds	AA	4%	3%	1%
	Nominal corporate bonds	A	8%	6%	3%
	Nominal corporate bonds	BBB	30%	21%	9%
	Nominal corporate bonds	BB	58%	40%	17%
			100%		
Commercial real estate	Nominal corporate bonds	BB	100%	70%	30%
ABS	Nominal corporate bonds	BBB	100%	70%	30%
Cash	Fixed Risk-Free bonds	AAA	100%	–	–
Equities	Equities	–	100%	70%	30%
Property	Property	–	100%		

[a] For A rated bonds and better, we assume that one-third are AA rated and two-third are A rated.
[b] The proportions for risk-free/corporate Index-linked bonds are all fixed at 10%, the rest (90%) are assigned rating categories according to QIS 5.
[c] QIS 5 (Basel Committee, 2006), Table 16 Committee of European Banking Supervisors (CEBS) Group 1 gives a different set of estimates with only 30% AAA, so it may not be appropriate to use QIS 5 parameters for Sovereigns bonds. We assume that 90% of Sovereign bonds are all AAA rated.
[d] QIS 5 (Basel Committee, 2006), Table 15 CEBS Group 1.
[e] QIS 5 (Basel Committee, 2006), Table 14 CEBS Group 1.
[f] QIS 5 (Basel Committee, 2006), Table 17 Other non-G10 Group 1. QIS 5 does not provide the full PD calibration for Retail but only a simplified Table 18. The reason we use Table 17 is that it has similar values of average PD and In Default to Table 18.
[g] QIS 5 (Basel Committee, 2006), Table 17 Other non-G10 Group 1. We assume Residential Loans share the same rating parameter with Retail Loans.

3. An economic capital modelling framework

3.1. Economic capital and risk measurement

Our economic capital computation for EuroBank will be based on the application of *suitable* risk measures to the distribution of *unexpected losses* arising from balance sheet positions. These losses are incurred by value changes in the asset portfolio V_t and liabilities B_t due to fluctuations in underlying risk drivers. At the initial time t, EuroBank is considered to be technically solvent $(V_t > B_t)$ with initial equity value $E_t = V_t - B_t$. But E_t needs to be sufficient to maintain solvency over the period $[t, t+1]$.

We now take the simplifying assumption that EuroBank replicates their liabilities by a portfolio of assets. We assume that (with certainty) between time t and $t+1$, the expected increase in asset value exceeds the increase in the value of liabilities plus any shortfall in income I_{t+1} such that

$$E(V_{t+1} - V_t) \geqslant (B_{t+1} - B_t) - I_{t+1}.$$

For a given confidence level α (say 99% for a century event) and with $\Delta_{t+1} = V_{t+1} - V_t$, the enterprise would be sufficiently capitalised if

$$P(\Delta_{t+1} - E(\Delta_t) + E_t > 0) = \alpha$$

or equivalently, expressed in terms of losses with $L_{t+1} = -\Delta_{t+1}$, if

$$P(L_{t+1} - E(L_t) < E_t) = \alpha.$$

Now $L_{t+1} - E(L_t)$ is simply the so-called unexpected loss so this argument justifies setting capital at the α-percentile of the distribution of the unexpected loss.

In general, if we denote the cumulative distribution function of a generic loss L by $F_L(l) := \mathrm{P}(L \leqslant l)$, all risk measures we consider are statistical measures computed from F_L; in particular we consider Value-at-Risk (VaR), and expected shortfall (ES). The former is usually defined as the α-quantile of F_L for an appropriate choice of $0 < \alpha < 1$, i.e. the measure

$$\mathrm{VaR}_\alpha(L) := \inf\{l \in \mathbb{R} : F_L(l) \geqslant \alpha\};$$

see McNeil et al. (2005, Definition 2.10). For economic capital calculation, α is typically chosen to match the target credit rating of the enterprise (e.g. 99.97% for a AA-rating). The 99.97% VaR is interpreted as indicating that there is a 0.03% chance that the portfolio loss is at least $\mathrm{VaR}_{99.97\%}$.

Table 5
The transformation of EuroBank balance sheet. This table gives the consolidated balance sheet used in comparing fully-integrated and modular approaches. The fully-integrated approach models all assets in the first column simultaneously with credit-risky assets mapped by credit rating (Table 4). The modular approach models the assets in each of the last six columns separately, where the corresponding sub-portfolios are mapped to credit ratings individually (Table 4); Economic Capital is then computed using a fixed correlation matrix. The sovereign sector consists of all cash and claims on government that appear on EuroBank's balance sheet. The retail sector (fifth column) consists of all retail loans, residential loans, commercial real estate and asset backed securities (ABS). Derivative positions are not considered.

Currency £M	Fully integrated	Sovereigns	Institution	Corporate	Retail	Equity	Property
Fixed risk-free AAA	2898.33	2898.33	–	–	–	–	–
Domestic equities	9857.08	–	–	–	–	9857.08	–
O'Seas equities	4224.46	–	–	–	4224.46	–	
Property	2283.87	–	–	–	–	–	2283.87
AAA (D)[a]	14311.29	14311.29	–	–	–	–	–
AA (D)	12438.39	–	5662.61	5329.21	1446.57	–	–
A (D)	24876.78	–	11325.22	10658.41	2893.15	–	–
BBB (D)	27325.32	–	1793.38	13205.35	12326.59	–	–
BB (D)	37306.63	–	926.25	12333.30	24047.08	–	–
AAA (F)[b]	6133.41	6133.41	–	–	–	–	–
AA (F)	5330.74	–	2426.83	2283.95	619.96	–	–
A (F)	10661.48	–	4853.66	4567.89	1239.92	–	–
BBB (F)	11710.85	–	768.59	5659.44	5282.82	–	–
BB (F)	15988.56	–	396.96	5285.70	10305.89	–	–
Index-linked AAA	2271.63	2271.63	–	–	–	–	–
Index-linked A	9719.64	–	3128.17	6591.47	–	–	–
Total value	197338.46	25614.67	31281.68	65914.72	58161.98	14081.54	2283.87

[a] D is Domestic.
[b] F is Foreign.

Expected shortfall, also used in this paper, is closely related to the VaR. It is defined as the tail average of the loss distribution above a given confidence level α. A formal definition used in Tasche (2002) and McNeil et al. (2005) is

$$\mathrm{ES}_\alpha(L) := \frac{1}{1-\alpha}\int_\alpha^1 \mathrm{VaR}_u(L)du,$$

which for continuous loss distributions reduces to the more common expression

$$\mathrm{ES}_\alpha = \frac{1}{1-\alpha}E(L\mathbf{1}_{[L\geqslant \varrho_\alpha(F_L)]}) = E(L|L \geqslant \mathrm{VaR}_\alpha),$$

the expected loss given that the VaR at level α is exceeded.

The question of suitability of a risk measure has been addressed by Artzner et al. (1999) who propose four axioms which a sound risk measure should satisfy: monotonicity, subadditivity, positive homogeneity and translation invariance. Subadditivity implies that capital charges computed with the risk measure can be reduced by diversification, an important principle in finance. Conversely, if a regulator uses a non-subadditive risk measure to determine the capital charge for a financial institution, the institution is incentivised to split its operations into various subsidiaries in an attempt to reduce the overall capital requirement.

ES, when defined as above, is a coherent risk measure; see McNeil et al. (2005, Chapter 6). VaR however, is not a coherent risk measure in general due to non-subadditivity. For comparison, we compute economic capital requirements under both risk measures in this paper. In both cases we apply the measures to the distribution of the unexpected loss $L_{t+1} - E(L_{t+1})$, which is equivalent to applying them to L_{t+1} and subtracting the expected loss. Note that we do not cap losses at 0; 'negative losses' are interpreted as gains but may still lead to positive unexpected losses if gains fall short of expectations.

3.2. Loss distributions via economic scenario generation

Valuing the portfolio in the present (V_t) and in the future (V_{t+1}) is a significant challenge that has been recognised by IFRS 7. Current practice favours *market-consistent* (or fair-value) valuation. Certain assets are capable of being marked to market while others are required to be marked to model. We assume that liabilities are modelled by a matched replicating portfolio of assets. This is a simplification we make in order to illustrate the asset valuation differences between modular and integrated methodologies. When information on liabilities is fully disclosed we would also be able to model the stochastic fluctuations in liability values.

All asset values at time t can be viewed as being dependent on a high-dimensional vector of underlying risk factors $\mathbf{Z}_t = (Z_{t1}, \ldots, Z_{td})$ consisting of such items as equity returns (index and some single stocks), exchange rates, points on the yield curve, credit spreads and default or rating migration indicators.

The value of the portfolio at time t can be considered as a random variable of the form

$$V_t = f_t(\mathbf{Z}_t, t), \tag{1}$$

where f_t is a function that we will refer to as the *portfolio mapping at time t*. It contains information about the portfolio composition at time t and incorporates the valuation formulas that can be used to value the more complex (derivative) assets with respect to the underlying risk factors $\mathbf{Z}_t$. Note that, in general, it depends not only on the value of the risk factors at time t, but also on the time t itself; this is because the value of a derivative position with maturity/expiry T typically depends on the remaining time to maturity $T - t$. Note also that there is a time subscript on the mapping function f_t to allow for the possibility of dynamic rebalancing which could change the entire composition of the mapping over time.

Projecting forward the underlying risk factors for purposes of valuation at $t + 1$ is the role that can be filled by an economic scenario generator (ESG). We set up a multivariate stochastic process $\mathbf{Z} = (\mathbf{Z}_s,\ s \geqslant t)$ which projects the values of the risk factors into the future and gives us snapshots $\mathbf{Z}_s$ of the economy at future times $s \geqslant t$. An ESG takes a Monte Carlo (simulation) approach and generates a series of realisations or paths $(\mathbf{Z}_s(\omega_i),\ s \geqslant t)$ for $i = 1, \ldots, m$ where each ω_i is in effect the label for a particular economic scenario.

Risk measures such as VaR and expected shortfall are estimated by corresponding empirical quantities derived from the Monte Carlo samples, such as sample quantiles. As such, they are prone to Monte Carlo error, which diminishes with the number of paths m. Errors and runtimes can be further reduced by employing standard Monte Carlo variance reduction techniques such as the use of antithetic variates (Robert and Casella, 1999).

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3.3. Implementing the modular and fully-integrated approaches

The asset portfolio of our representative EuroBank may be divided by asset class into d sub-portfolios. For each sub-portfolio $j = 1, \ldots, d$ we have to consider possible losses

$$L_{j,t+1} = -\Delta_{j,t+1} = -(V_{j,t+1} - V_{j,t}),$$

which aggregate by simple summation to give the overall value change of the enterprise

$$L_{t+1} = -(V_{t+1} - V_t) = -\left(\sum_{j=1}^{d} V_{j,t+1} - \sum_{j=1}^{d} V_{j,t}\right) = \sum_{j=1}^{d} L_{j,t+1}\,.$$

3.3.1. The modular approach

In the modular approach to capital adequacy individual risks at sub-portfolio level are transformed into capital charges $EC_1, \ldots, EC_d$. These are then combined to calculate the overall economic capital EC, usually by using a correlation matrix approach.

The economic scenario generation approach gives us the framework for a fully-integrated model of economic capital, but clearly it also allows us to derive economic capital estimates for individual asset classes by considering them one at a time. In this way, we have the opportunity to compare a modular, correlation-based approach to economic capital with a fully-integrated approach. Companies without fully-integrated, enterprise-wide models have no choice in the matter; they require a method for combining the capital charges that they compute for individual asset classes using a variety of different models and approaches. The overall EC is generally computed to be

$$EC = \sqrt{\sum_{i=1}^{d}\sum_{j=1}^{d} \rho_{ij} EC_i EC_j}, \tag{2}$$

where ρ_{ij} are the correlations between the asset classes.

The modular method of aggregation is only justified when underlying losses in different asset classes have a joint elliptical distribution and when capital is set using a positive homogeneous, translation-invariant risk measure, such as VaR or expected shortfall (see McNeil et al., 2005). However, the distributional assumption is hardly ever met in practice and, even if it were, the difficulty of calibrating the correlations and of taking into account tail dependence, is a serious limitation.

In this paper, we use economic scenario generation to calculate capital requirements for each asset class using our two risk measures. In other words we set $EC_i = VaR_\alpha(L_{i,t+1}) - E(L_{i,t+1})$ and $EC_i = ES_\alpha(L_{i,t+1}) - E(L_{i,t+1})$ in turn and use (2) to compute overall economic capital. Standard and Poor's (2008) also adopt a modular approach and provide a calibration for the correlation matrix, which we will use in our analysis. Table 6 shows the correlation matrix between various credit exposure classes. Standard and Poor's (2008) judges that the correlation coefficient between the credit and equity markets is equal to 80%.

The S&P correlation matrix is part of their "Risk-adjusted capital framework for financial institutions." This document appeared in April 2008 and summarises the methodology used by S&P to calculate an independent assessment of capital adequacy for financial institutions; the methodology is similar to the Basel II modular methodology with some adaptations and changes that S&P justify in the document. Calibration is reported to take a three year perspective, but the correlation matrix has a large element of expert judgement, as is evident from the round numbers. This matrix is typical of the kind of correlation matrix used in the modular approach in the pre-crisis period of 2005–07.

We also compute the value of the EC requirement for the case where there is no diversification as a special case of the modular approach (referred to as simple additive approach) with $EC = \sum_{i=1}^{d} EC_i$.

In the modular approach, diversification could also be measured by using correlations to calibrate a copula model to join the marginal models together, and to allocate using the composite model. However, the "correct" copula will be difficult to obtain and calibrate and we would be sceptical of the value of the results so obtained.

Table 6
The Standard and Poor's (2008) correlation matrix For objectivity we use the correlation matrix used in Standard and Poor's (2008) own modular approach to capital calculation. This table gives the correlation coefficients ρ_{ij} which are used in Eq. (2).

ρ	Sovereigns	Institutions	Corporates	Retails
Sovereigns	100%	–	–	–
Institutions	75%	100%	–	–
Corporates	50%	50%	100%	–
Retails	25%	25%	25%	100%

3.3.2. Fully integrated approach

Losses in sub-portfolios depend on value changes ($L_{j,t+1} = -\Delta_{j,t+1} = -(V_{j,t+1} - V_{j,t})$) and future valuations are driven by fundamental risk factors $\mathbf{Z}_{t+1}^{(j)}$ according to $V_{j,t+1} = f_{j,t+1}(\mathbf{Z}_{t+1}^{(j)}, t+1)$. Many of these risk factors, for example those describing the structure of the yield curve or the average performance of equity markets, are common to many sub-portfolios of assets.

This is the origin of dependence in a fully-integrated model: correlation arises from the mutual dependence of future values across an enterprise on a set of common risk drivers. Fully integrated models are common factor models. The risk factors $\mathbf{Z}_{t+1}^{(j)}$ that enter into the future valuation of sub-portfolio j contain a subset in common with the risk factors $\mathbf{Z}_{t+1}^{(k)}$ that enter into the future valuation of sub-portfolio k. These common factors are the drivers of dependence between $V_{j,t+1}$ and $V_{k,t+1}$ and consequently between $L_{j,t+1}$ and $L_{k,t+1}$. The dependence arises endogenously through the specification of the model.

In practical terms we treat the enterprise as a single portfolio and simulate overall losses for all asset classes and compute capital using the two risk measures of interest.

3.4. An illustrative example

Suppose we consider a simple balance sheet with three asset classes: an investment in a stock index, a BBB-rated corporate bond portfolio; a AAA-rated government bond portfolio. Suppose that the total portfolio value is 1000 and the initial values of these three asset classes are 300 for AAA-rated bonds, 600 for BBB-rated bonds and 100 for equity. Further suppose that each bond portfolio consists of 100 zero-coupon bonds with a common maturity of 10 years.

3.4.1. Model set-up

We assume the equity index S_t follows a standard geometric Brownian motion. The valuation of the equity investment is straightforward, the value function in (1) taking the form $V_t^{\text{equity}} = f_1(S_t)$ where f_1 is a simple linear scaling function reflecting the size of the investment. The valuation of the bond portfolios in terms of underlying risk factors is more complicated.

We adopt a ratings-based approach to credit risk in which the annual default and rating migration probabilities for bonds are summarised in a matrix P; an element P_{ij} gives the probability of migrating from rating i to rating j in the course of a year and the final column represents default probabilities. The reduced form Markov-model approach of Jarrow et al. (1997) (the JLT model) is used to relate the real-world transition matrix P to a dynamically changing set of market-implied default probabilities $q_{it}(T)$ which can be understood as the market's implicit assessment of the probability that a bond rated i at time t will default before maturity T. Following a suggestion of Lando (2004), the JLT model is extended to incorporate a stochastic credit risk premium process (π_t) following a Cox–Ingersoll–Ross (CIR) model; this process can be thought of as capturing the complex relationship between real-world default rating migration probabilities and credit spreads.

The value of a zero-coupon bond rated i at time t and maturing at time T is given by $p_{it}(T) = p_t(T)(1 - \delta q_{it}(T))$ where $p_t(T)$ is the price at time t of a default-free zero-coupon bond maturing at T and δ is the loss given default (LGD), which is assumed to be constant. This means that to value a bond portfolio with maturity T at time t we essentially have a valuation formula of the form $V_t^{\text{bond}} = f_2(p_t(T), \pi_t, \mathbf{r}(t))$ where we introduce the vector $\mathbf{r}(t)$ as a rating state indicator for all the bonds in the portfolio at time t. If we know the current price of default-free bonds, the current ratings of the bonds and the value of the credit risk premium process, we can value the defaultable bonds. To value the default-free bond we use a 2-factor Black–Karasinski model.

The dependence between equity assets and bond assets is modelled using the popular one-factor approach of Vasicek (1997). Ratings transitions for bond issuer k are considered to be driven by latent asset value process of the form $A_{kt} = \sqrt{\rho}\widetilde{S}_t + \sqrt{1-\rho}\epsilon_{kt}$ where $(\widetilde{S}_t)$ is a standardised version of the stock index process above, the (ϵ_{kt}) are idiosyncratic noise processes for each bond issuer and ρ is the parameter known as asset correlation. A series of deterministic thresholds are created such that the relative value of A_{kt} with respect to the thresholds dictates the rating of obligor k at time t. The thresholds are chosen to give the correct matrix of transition probabilities P. In this way the process of rating state variables $\mathbf{r}(t)$ is driven by the systematic factor S_t and the vector of shock variables $\boldsymbol{\epsilon}_t$.

Writing $\boldsymbol{\epsilon}_t^{\text{AAA}}$ and $\boldsymbol{\epsilon}_t^{\text{BB}}$ for the shocks effecting the government bond and corporate bond portfolios, respectively, we can summarise the three mapping functions for our asset positions as follows:

$$
\begin{aligned}
V_t^{\text{equity}} &= f_1(S_t),\\
V_t^{\text{AAA bond}} &= f_2(p_t(T), \pi_t, S_t, \boldsymbol{\epsilon}_t^{\text{AAA}}),\\
V_t^{\text{BBB bond}} &= f_2(p_t(T), \pi_t, S_t, \boldsymbol{\epsilon}_t^{\text{BBB}}).
\end{aligned}
$$

Note that the equity index risk factor S_t is common to the equity and bond portfolios and induces dependence across all three. The price of default-free bonds $p_t(T)$ and the credit risk premium process π_t induce dependence between the different bond portfolios.

3.4.2. Calibration

Calibration of this model involves a number of tasks. First a geometric Brownian motion model for (S_t) must be calibrated to historical equity data. We need to choose values for real-world migration probabilities P using through-the-cycle rating agency data. We need to calibrate the CIR process for (π_t) using data on corporate bond spreads. The values for the loss-given default δ and the asset correlation ρ have to be chosen. We also have to calibrate the 2-factor Black–Karasinski interest rate model that will give $p_t(T)$ for any combination of t and T. In addition some correlation between the two main principal components in the equity model and the Brownian motions that drive the interest rate model is assumed. More details are available on request.

3.4.3. Results

The results are shown in Table 7. We see that the standalone economic capitals based on Value-at-Risk at the 99% level for the three positions are 52 for the AAA government bonds, 109 for the BBB corporate bonds and 40 for the equity position. Summation gives the additive economic capital of 201. On the other hand use of Standard & Poor's published correlation numbers and formula (2) gives a reduction in the capital to 179; this is the figure we refer to as modular economic capital.

Table 7
Illustrative example.

Asset	$t=0$	$t=1$		
	Exposure	Expected loss	99% VaR loss	99% EC
Standalone				
AAA 10y bonds	300	−21	31	52
BBB 10y bonds	600	−46	63	109
Equity	100	−8	32	40
Portfolio				
Portfolio loss	–	−75	110	185
Capital requirements				
EC additive	–	–	–	201
EC modular	–	–	–	179
EC fully integrated	–	–	–	185

$\rho_{(\text{Sovereign/Institutional debt, Corporate debt})} = 0.5$.
$\rho_{(\text{Equity, Bond})} = 0.8$.

In a fully-integrated economic scenario analysis of the whole portfolio the computed value for economic capital is 185, which is slightly larger than the modular figure. However, it is important to note that the economic scenario generator has in no way been calibrated to match the Standard & Poor's correlations. The ESG is calibrated at the level of the fundamental interest rate, equity, credit spread and credit rating migration models as described in the previous sections.

However, the fully-integrated analysis can be used in a couple of different ways to inform the choice of modular correlations. It is possible to search for values of the correlations ρ_{ij} that give equality between the modular and fully-integrated approaches (for a fixed VaR level). These could be used and justified for modular calculations in situations where banks did not have access to the tools for fully-integrated modelling. Alternatively, the generated data on losses in the three asset classes could be used to estimate a three-dimensional correlation matrix for asset class losses which could be used in a modular calculation, although there is no reason why this would match the economic capital number coming from a fully-integrated analysis, because the three-dimensional loss distribution is not elliptical (see Fig. 1). When we perform this calculation we obtain an economic capital of 174, which does indeed show the non-elliptical behaviour.

Given this non-elliptical behaviour, we control for possible error in the estimation of these correlations by using a robust correlation estimation method based on Kendall's tau (see McNeil et al., 2005, pp. 97–98 and 215–217). In fact the correlation numbers are very similar and the value obtained for economic capital is again 174.

As an aside, in the event of a bank being (partially) funded with long term liabilities it is possible to 'earn spread risk' on, say, bonds issued by the bank itself and which hedge asset side risks. However, the objective of this work is to address the methodological shortcomings of Basel II and practitioner frameworks by comparing modular vs fully integrated approaches to economic capital. So, while we certainly accept that a part of EC could be attributed to spread risk, this would not affect the comparability of modular and integrated results since both capital computation methods are applied to a 'matched asset view'. Under full asset liability modelling it would certainly be important that the spread effect, noted above, be accounted for.

3.5. The economic scenario generator

The main study in this paper and the simple example of the previous section are carried out with the Barrie & Hibbert economic

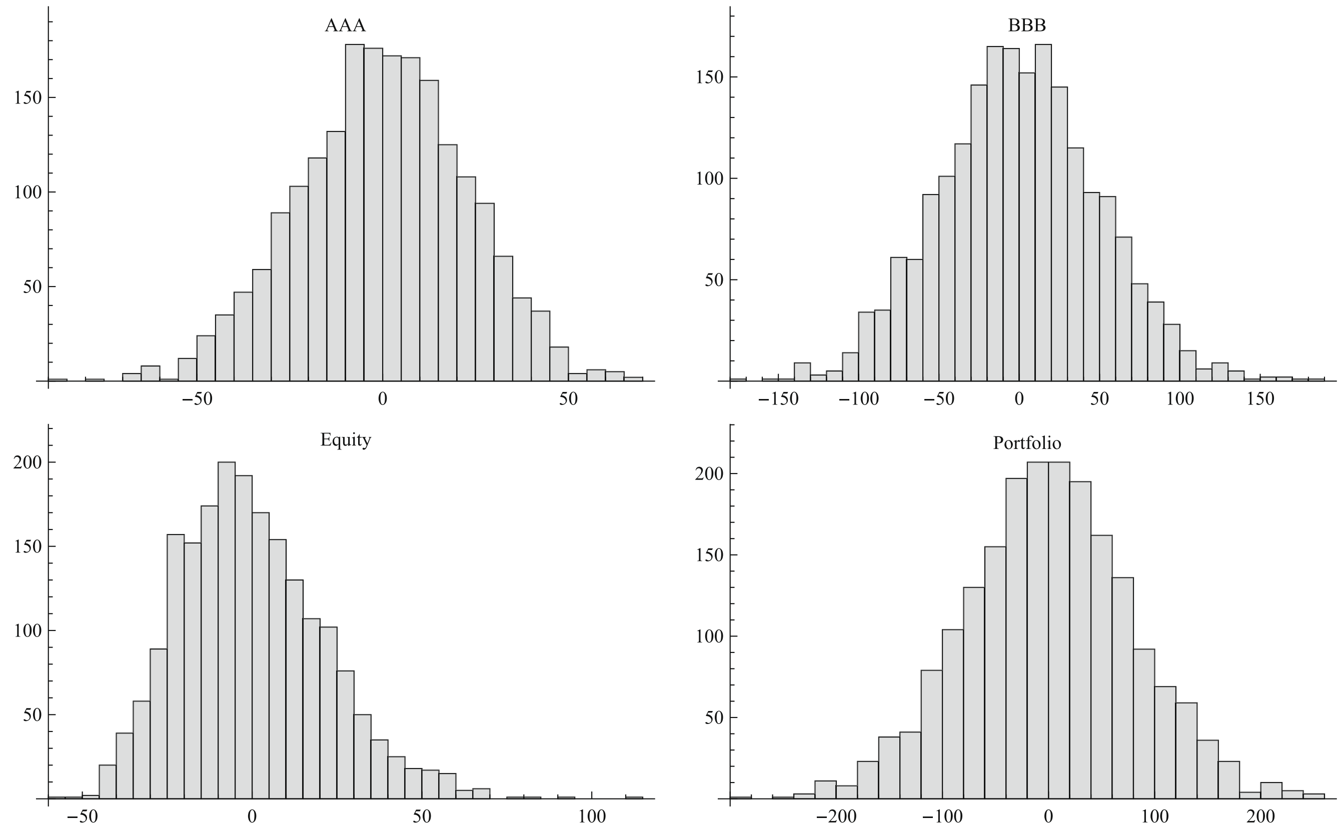

Fig. 1. Loss distributions for illustrative example (mean corrected).

scenario generator (B&H ESG). Fig. 2 shows the main features of the model.

The previous section has given some idea of the credit risk modelling approach in the B&H ESG. In this section we give a non-technical overview of further model choices and model calibrations that have been made to address the economic capital questions which are of central interest in this paper (See Fig. 2).

3.5.1. Models

Interest rate models are at the core of the ESG and, for the analyses of this paper, a 2-factor Black–Karasinski model for nominal interest rates has been used, as its logarithmic structure guarantees positive nominal rates. Real interest rates are assumed to follow a standard 2-factor Vasicek model, which allows for positive and negative real rates, while *inflation* is not explic-

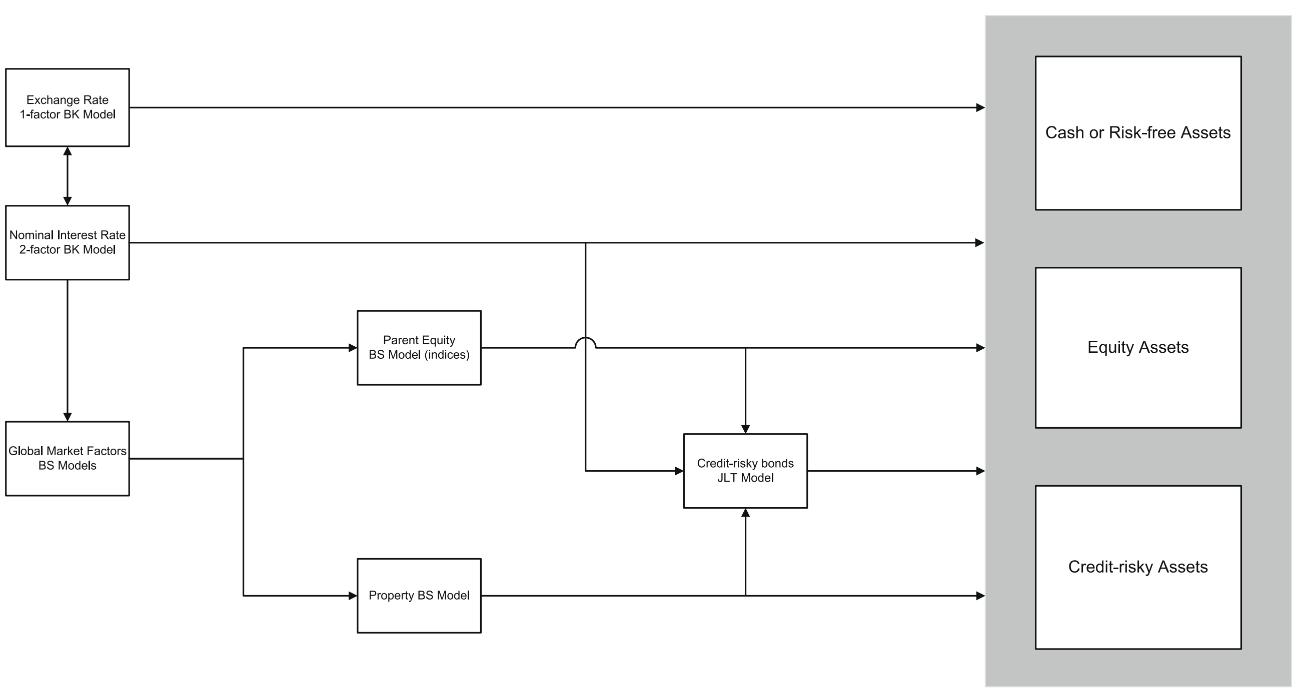

Fig. 2. Diagram of model tree used for fully-integrated calculation of EC.

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itly modelled but inferred as the differential between nominal and real rates.

Since we are limited to financial data from annual reports, without full disclosure of the different currencies of assets, we simply group assets in two economies, domestic and overseas. *Exchange rates* between the two economies are modelled based on the assumption of purchasing power parity (PPP). Over time, real exchange rates are allowed to fluctuate around a long term target; the deviation of nominal exchange rates from real exchanges rates is driven by the inflation differences between economies.

For *equities*, we adopt a multi-factor modelling approach where factors are statistical and derived by principal component analysis (PCA) from equity index return data. By inversion of the PCA, an equity index model for the performance of both domestic and overseas equities is inferred. *Property* returns are modelled according to the same underlying factor model with appropriate factor sensitivities derived from empirical analysis.

As indicated in the previous section, the *credit risk model* in the ESG combines a Jarrow–Lando–Turnbull (JLT) reduced-form model ratings-based model with a one-factor Vasicek (or Gaussian copula) model (Jarrow et al., 1997). The extended JLT model allows for stochastic defaults and migrations of credit-risky assets and is also able to produce stochastic credit spreads by assuming that credit risk premia follow a Cox–Ingersoll–Ross (CIR) process.

The ESG modelling suite is used to project the underlying fundamental risk factors for possible future states of the economy. These are then used to value all balance sheet assets using appropriate valuation models. For simplicity, and because balance sheet disclosures do not give the information necessary for a more detailed analysis, we treat all credit-risky assets as bond-like assets. This means that we have the relatively simple task of valuing risk-free cash flows, equity-like assets (including property) and defaultable bond-like assets. Thus the approach for the whole balance sheet is in effect a real-world balance sheet expansion of the example of the previous section.

In the case of the credit-risky assets, these are assumed to be either nominal or index-linked bonds (see Table 4). The index-linked coupon bond yields are semi-annually compounded. All spot rates are continuously compounded.

3.5.2. Calibration

For the calibration process, model parameters of the ESG can essentially be separated into two sets. Firstly, some "long term unconditional targets" are set, based on the statistical analysis of long series of historical data. These targets generally relate to the evolution of economic variables and fundamental asset prices such as volatility, speed of mean reversion and mean reversion level. These targets are typically updated every quarter or, in some cases, once per year; they are parameters that are not expected to change materially from one quarter to the next. Secondly, the remaining parameters are initialised in a way that is consistent with market prices where available, at the calibration date. For our study this calibration date was September 2007, reflecting market conditions at the start of the credit crisis.

Note that the use of long term unconditional targets ensures long term stability of the suite of models and dampens the effect of short-term volatility. As a consequence, the model calibrations for the period prior to the onset of the financial crisis, say March 2007, would not be significantly different from the first "in crisis" calibration for September 2007.

The long term targets include variables such as interest rate volatility, equity volatility and equity risk premia, credit spread volatility and long term average levels of spreads, dividend yield volatility and long terms average levels, the credit rating transition matrix, exchange rate volatilities and correlations between exchange rates, loss given default and the correlation parameter in Vasicek's one-factor model of portfolio credit risk. The remaining "short-term parameters" are calibrated to available market data at the calibration date; these data include nominal and real yield curves, equity dividend yields, current spreads and exchange rates. All market data are based on mid prices.

In addition it is possible to set correlations between the various Brownian motions that drive the equity, interest rate and other models. For example, the two main principal components in the equity model are usually correlated with the Brownian motions in the interest rate models to better model the observed dependence between these risk factors.

More precise details of calibrated parameter values for some of the key ESG components which were used in the illustrative example are available on request. The full analysis of the balance sheet of the average EuroBank of Section 2 uses a larger suite of models which also includes an exchange rate model (to model the assumed split of assets into domestic and foreign) and a real interest rate model (to allow the valuation of inflation-index-linked bonds). Full calibration details of these additional models are omitted, but are available on request.

3.6. EuroBank capital allocation

The advantage of using an integrated risk framework is that economic capital calculated for an asset portfolio or enterprise can be broken up into pieces that are attributable to sub-portfolios or business units. While this is not performed for EuroBank, this process of capital allocation can be used as the basis of risk-adjusted performance comparison across sub-portfolios and there is now a considerable literature on the theory of fair allocation of capital including Tasche (2008), Denault (2001), Kalkbrener (2005). The generic principle that is commonly adopted is known as Euler allocation.

Writing $L = L_{t+1}, L_i = L_{i,t+1}$ for $i = 1, \ldots, d$ it can be shown that under some technical assumptions on the distribution of $(L_1, \ldots, L_d)$ (fulfilled, for example, by the existence of a joint probability density) the Euler contribution to the loss $\varrho(L_i|L)$ from asset class i takes the following forms in the case of VaR and expected shortfall:

$$\mathrm{VaR}_\alpha(L_i|L) = E(L_i|L = \mathrm{VaR}_\alpha(L)),$$
$$\mathrm{ES}_\alpha(L_i|L) = E(L_i|L \geqslant \mathrm{VaR}_\alpha(L)).$$

The allocated capital is $\varrho(L_i|L) - E(L_i)$. The forms of these expressions reveal how the economic capital contribution may be estimated using the Monte Carlo output from an economic scenario generator. For example, in the case of expected shortfall, we would average the losses in each sub-portfolio over all scenarios where the total portfolio loss exceeded the Value at risk. In practice, the problem of rare event simulation arises, and long run times may are necessary to get accurate results. But the main point is that the necessary prerequisite for computing allocations is a fully-specified joint model for $(L_1, \ldots, L_d)$ and this is delivered by a fully integrated model but not by a modular model and correlation matrix.

A further development, described in Tasche (2006), is the calculation of diversification scores to give a measure of the extent of diversification in the total portfolio of an enterprise. A global diversification index can be calculated as

$$\mathrm{DI} = \frac{\varrho(L) - E(L)}{\sum_{i=1}^{d} \varrho(L_i) - E(L)} = \frac{\mathrm{EC}}{\sum_{i=1}^{d} \mathrm{EC}_i}.$$

This is simply the total economic capital for the portfolio divided by the sum of standalone economic capital amounts for the sub-portfolios.

A sub-portfolio diversification index can be calculated as

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$$\mathrm{DI}_i = \frac{\varrho(L_i|L) - E(L_i)}{\varrho(L_i) - E(L_i)} = \frac{\mathrm{AC}_i}{\mathrm{EC}_i}.$$

This shows the reduction in capital that the sub-portfolio enjoys through being part of the enterprise. Where the ratio is small, this is an indication that sub-portfolio i is well-diversified with respect to the rest of the enterprise. If the global diversification is less impressive, it may be possible to gain a global improvement by increasing the size of sub-portfolio i at the expense of other sub-portfolios.

4. Projecting EuroBank's balance sheet and computing EC requirements

4.1. Projected portfolio loss distributions

Following the modular approach, we first estimate the loss distribution for sub-portfolios covering credit-risky asset class by simulation. Descriptive statistics of loss distributions for sub-portfolios of sovereign bonds, interbank lending, corporate bonds and retail products are shown in the Table 8, describing distributions for simulated portfolio loss over a one and five year projection horizon. Note that every distribution shows a different degree of skewness. In particular, the loss distribution of retail assets has the heaviest upper tail out of the four credit risk exposures types. By contrast, the right hand tail of sovereign bonds is the 'lightest'. Results imply that the riskiness of purchasing sovereign bonds is much lower than mortgage business as would clearly be expected. We also simulate total returns of equity and property assets over one and five years based on an equity multi-factor model to obtain two corresponding loss distributions. Both loss distributions have skewed non-normal shapes, which coincides with our expectation for parent equities, Table 8

4.2. Projected economic capital requirements and capital attribution under different risk measures

Risk measures VaR and ES for each asset class are computed based on portfolio loss distributions. Tables 9 and 10 show EC requirements for every asset class for one-year through five-year projections. For the modular approach, we obtain the total EC by aggregating individual risk measures with and without diversification benefit. The special case of aggregation without diversification benefit is referred to as an "additive" approach where modular EC is calculated using a correlation matrix overlay. Tables 9 and 10 illustrate EC results under additive and modular approaches with risk measures 99% VaR, 99% ES and 95% ES. With a fully-integrated approach, we simulate the total value of the portfolio over one and five years and obtain the loss distribution which exhibits a high degree of skewness. EC is computed under three measures of VaR and ES and given in Tables 9 and 10.

Table 8
Descriptive statistics for loss distributions. This table shows statistical sample parameters for the simulated one-year and five year loss distributions. All distributions fail the Jarque–Bera test of normality according to the p-values.

	Mean	Median	Std. dev.	Skewness	Kurtosis	Max	Min	p-value
Panel A: One-year loss distribution								
Sovereign	0.09	−24.94	1209.13	0.07	−0.23	3390.84	−3572.74	0.00
Institutional	−362.28	−467.17	1891.23	0.09	0.03	5951.95	−6560.55	0.00
Corporate	−1080.56	−1246.79	4478.40	0.11	0.20	14669.39	−15,432.77	0.00
Retail	−1287.55	−1581.66	4706.91	0.11	0.32	15917.12	−16397.23	0.00
Equity	−637.16	−309.73	2947.55	−0.51	0.31	6284.98	−12062.11	0.00
Property	−74.18	−45.45	380.65	−0.46	0.35	902.29	−1592.42	0.00
Fully integrated	−3441.62	−4030.40	13993.46	0.09	0.20	45990.98	−48027.38	0.00
Panel A: Five-year loss distribution								
Sovereign	62.51	61.21	2547.93	−0.02	0.13	7824.31	−8632.50	0.00
Institutional	−2028.07	−2057.58	4077.70	−0.01	−0.01	10497.51	−14795.54	0.00
Corporate	−5917.73	−5960.25	9740.86	−0.02	−0.05	25339.82	−36,061.48	0.00
Retail	−6957.87	−7067.96	10493.26	−0.07	−0.04	26855.20	−40430.02	0.00
Equity	−4008.44	−2291.68	9361.30	−1.43	3.57	12829.63	−59179.59	0.00
Property	−463.22	−288.62	1143.18	−1.06	2.08	1844.88	−6525.12	0.00
Fully integrated	−19168.12	−18107.17	31864.71	−0.16	0.05	81244.36	−132269.84	0.00

Table 9
One year economic capital requirements and risk measures for the enterprise and individual asset classes. The original balance sheet of EuroBank's portfolio in €million. The one-year standalone risk measures for every asset class calculated in modular approach. The one-year Euler risk contributions $\varrho(L_i|L) - E(L_i)$ based on three risk measures for every asst class together with their sum $\sum_{i=1}^{d}\varrho(L_i|L) - E(L_i)$. EC Modular is calculated using a variance–covariance matrix overlay. Figures are in € million.

Asset	Balance sheet	99% 1 year VaR		99% 1 year ES		95% 1 year ES	
	$t = 0$	Standalone	Contribution	Standalone	Contribution	Standalone	Contribution
Sovereign	25614.67	2806.85	3214.85	3110.95	2254.84	2523.13	2068.33
Institutional	31281.68	4129.21	4602.81	4694.06	4480.61	3665.38	3534.87
Corporate	65914.72	10032.46	10115.11	11448.57	11394.58	8609.56	8550.03
Retail	58161.98	9950.04	10312.50	12163.98	12021.01	8970.62	8931.69
Equity	14081.54	4946.09	1980.55	5677.10	5305.00	4483.16	3565.85
Property	2283.87	712.10	−192.54	770.53	286.01	617.68	134.60
Sum	197338.46	32576.75	30033.28	37865.19	35742.05	28869.52	26785.36
EC additive	–	32576.75	–	37865.19	–	28869.52	–
EC modular	–	24756.55	–	28744.45	–	21927.07	–
EC fully integrated	–	–	30037.00	–	35742.06	–	26785.38
EC modular (2)	–	–	29580.48	–	34476.02	–	26188.86
EC modular (3)	–	–	29601.87	–	34501.01	–	26208.24

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Table 10
Five year economic capital requirements and risk measures for the enterprise and individual asset classes. The original balance sheet of EuroBank's portfolio in €million. The five-year standalone risk measures for every asset class calculated in modular approach. The five-year Euler risk contributions $\varrho(L_i|L) - E(L_i)$ based on three risk measures for every asst class together with their sum $\sum_{i=1}^{d}\varrho(L_i|L) - E(L_i)$. EC Modular is calculated using a variance–covariance matrix overlay. Figures are in € million.

Asset	Balance sheet	99% 5 year VaR		99% 5 year ES		95% 5 year ES	
	$t = 0$	Standalone	Contribution	Standalone	Contribution	Standalone	Contribution
Sovereign	25614.67	5901.87	3368.26	6898.74	5082.56	5356.10	4290.52
Institutional	31281.68	7407.95	5835.90	8614.26	7714.31	6345.27	5897.07
Corporate	65914.72	15600.69	15048.47	18785.63	18103.49	13896.11	13607.96
Retail	58161.98	15833.99	16112.26	18739.28	18465.21	1397.84	13696.57
Equity	14081.54	10232.94	6955.03	11131.77	7434.16	9020.38	6031.08
Property	2283.87	1401.99	819.32	1595.18	508.80	1283.42	-4.69
Sum	197338.46	56379.43	48139.25	65764.85	57308.52	49876.12	43518.51
EC additive	–	56379.43	–	65764.85	–	49876.12	–
EC modular	–	43006.76	–	50062.16	–	38059.94	–
EC fully integrated	–	–	48926.24	–	57041.64	–	43763.47

Comparing the EC calculated under all three methods, the additive capital requirements are highest, in line with our expectations for all three risk measures and both projection horizons (Tables 9 and 10). Under a modular approach, the lowest Economic Capital number is computed. Modular EC is primarily driven by the correlation matrix used to derive cross asset class diversification benefit. For our study, we used the correlation matrix specified by Standard and Poor's (2008) as a benchmark.

We conclude that the dependence between various assets in global financial markets should be much higher than conventional assumptions applied in correlation-based calculations. In particular, the equity market and credit market have a strong dependence on each other, as evidenced by the current credit crisis. Importantly, economic capital calculated over a one year projection horizon shows an undercapitalisation of around 18% for the modular correlation-based approach compared to the fully-integrated model (Table 9). Modular economic capital remains more than double the average amount of regulatory capital required under Basel II Pillar 1 when compared to Bank Tier 1 capital average across 51 banks.

As in the example of Section 3.4, we now calculate the inter-asset-class correlation matrix implied by the fully integrated model by computing an empirical correlation matrix from the generated losses. We repeat the modular EC calculation for the one-year time horizon using this correlation matrix. The results are found in Table 9 in the row marked "EC modular (2)". We also use the robust method of correlation estimation based on Kendall's tau, as described in Section 3.4; the results for a modular calculation based on this matrix are shown in the row marked "EC modular (3)". As can be seen, the numbers for the two correlation estimation methods are quite close, suggesting that the robust method of correlation estimation adds little. In all cases the modular figures are lower than the EC figures coming from a fully-integrated analysis, revealing the non-elliptical nature of the simulated multivariate asset loss distribution. However, the discrepancy is much less than for the S&P correlation matrix. This suggests that output from a fully integrated analysis can be used to set correlation parameters for use in a modular calculation, as long as we are aware that there may still be an inherent tendency to underestimate capital due to the non-elliptical shape of the underlying loss distribution.

4.3. Capital allocation and diversification

As noted earlier, overall capital requirements computed under a fully integrated approach can be allocated down to sub-portfolios using Euler Allocation (Tasche, 2008). Euler risk contributions are calculated for three risk measures and all asset class. The sum of all attributions is equal to total capital requirement for the fully integrated approach. Capital requirements attributed to sub-portfolio include respective shares of the total diversification benefit implicit in the fully-integrated EC projection.

The differences between modular sub-portfolio EC calculation and fully integrated illustrates the diversification benefit effect on a sub-portfolio level and the diversification indices of Tasche (2008) can be computed. Results over one and five year horizons for all three risk measures are shown in Tables 11 and 12. From these table, it is apparent that there is a discount for the contribution to overall capital as measured on a standalone vs contribution basis. With reference to Table 9, and using 99% ES over 1 year for example, property requires standalone capital of 770.53 but only 286.01 of capital when it is part of EuroBank's balance sheet structure. Naturally this computation is sensitive to model choice and

Table 11
One year diversification indices. The one-year marginal $DI_\varrho(L_i|L)$ of every asset class with respect to three risk measures. The one-year "absolute" $DI_\varrho(L)$ of whole portfolio with respect to three risk measures.

Asset	Risk measure		
	99% VaR	99% ES	95% ES
Panel A: Marginal diversification index			
Sovereigns	1.145	0.725	0.820
Institutions	1.115	0.955	0.964
Corporates	1.008	0.995	0.993
Retails	1.036	0.988	0.996
Equity	0.400	0.934	0.795
Property	−0.270	0.371	0.218
Panel B: Absolute diversification index			
Total portfolio	0.922	0.944	0.928

Table 12
Five year diversification indices. The five-year marginal $DI_\varrho(L_i|L)$ of every asset class with respect to three risk measures. The five-year "absolute" $DI_\varrho(L)$ of whole portfolio with respect to three risk measures.

Asset	Risk measure		
	99% VaR	99% ES	95% ES
Panel A: Marginal diversification index			
Sovereigns	0.195	0.737	0.801
Institutions	0.473	0.896	0.929
Corporates	0.794	0.964	0.979
Retails	0.917	0.985	0.980
Equity	1.233	0.668	0.669
Property	0.778	0.319	-0.004
Panel B: Absolute diversification index			
Total portfolio	0.867	0.867	0.877

calibration, but unlike the covariance approach it is possible to isolate the source and cause of the capital diversification.

4.4. Summary: A framework for management and regulatory action

In the modular approach, correlations are somewhat arbitrary and hard to justify. The fully-integrated approach gives a more structural and explanatory way to construct dependence of assets on risk factors for which data and policies are capable of being analysed. The fully integrated approach enables a risk-based allocation of capital and facilitates "use" by permitting the isolation of worst case paths for EC. In this way, it provides a clear framework for informing management actions. The main points taken from our results suggest that:

- Fully integrated capital is greater than modular capital but less than additive capital. See Table 9.
- The main contributions to fully-integrated capital come from corporate lending and retail advances; this is a function of the effect of risk factors on balance sheet exposures to assets-and the credit risk rating embodied in our credit risk calibration. See Table 9 (column 2) for capital contributions.
- The overall diversification score is high; see Table 11. This score should be taken as measuring diversification potential rather than absolute diversification. In other words the overall balance sheet is quite concentrated and there is potential to improve diversification by moving into asset classes that have lower diversification scores (and which are thus better diversified). However, we note that the dependence assumptions are quite

Table A.13

Balance sheet assets (December 2006) of 51 European banks in £ million. This table displays 51 European banks' balance sheets, modified using detailed accounting notes. The last three rows summarise the total value of the balance sheets, the average which we will use as the EuroBank's balance sheet and the percentages of the composition. This table does not take account of derivative positions.

Ticker	Country	Cash	Claims on gov.	Claims on banks	Claims on corp.	Retail loans	ABS	Res. loans	Comm. real estate	Prop.	Eq.	Total assets
AABA-AE	NED	8299	52,416	92,955	201,340	39,267	6686	129,597	20,802	4224	32,492	588,077
ACA-FR	FRA	10,637	104,292	112,107	235,869	33,014	13,303	108,960	17,489	4650	64,649	704,971
AL.-LN	GBR	2390	3308	10,177	21,310	5148	422	16,990	2727	556	2051	65,077
BARC-LN	GBR	9753	152,788	137,133	267,327	31,579	19,489	104,223	16,729	2492	94,711	836,224
BB.-LN	GBR	233	1285	7138	15,120	3973	164	13,111	2104	91	796	44,015
BBVA-MC	ESP	8432	17,256	38,969	83,161	18,109	2201	59,768	9593	3175	10,697	251,362
BDB-MI	ITA	70	379	702	1511	302	48	997	160	102	235	4507
BEB2-FF	GER	643	9171	13,663	28,209	5260	1170	17,359	2786	469	5685	84,415
BKT-MC	ESP	367	1951	4593	9465	2164	249	7141	1146	232	1209	28,516
BMPS-MI	ITA	1214	5862	13,876	33,284	6547	748	21,608	3468	1728	3634	91,968
BNP-FR	FRA	7983	136,077	127,131	263,670	31,220	17,358	103,036	16,538	12,318	84,352	799,684
BPE-MI	ITA	482	1152	4473	9610	2368	147	7816	1255	625	714	28,642
BPI-MI	ITA	410	1161	4030	9864	2091	148	6899	1107	638	720	27,069
BPSO-MI	ITA	47	503	1543	3212	780	64	2574	413	87	312	9535
BPVN-MI	ITA	781	1688	6791	14,698	3616	215	11,936	1916	363	1046	43,050
BTO-MC	ESP	282	5439	10,361	21,640	4518	694	14,910	2393	634	3371	64,241
BVA-MC	ESP	83	129	1662	3563	985	16	3250	522	157	80	10,445
CAP-MI	ITA	1076	3241	13,104	29,128	6984	413	23,050	3700	1959	2009	84,663
CBK-FF	GER	3456	35,778	59,916	122,667	24,615	4564	81,239	13,040	935	22,178	368,387
CC-FR	FRA	6208	19,986	18,740	36,038	4627	2549	15,272	2451	939	12,389	119,200
CE-MI	ITA	372	1147	2339	4911	1048	146	3460	555	227	711	14,917
CRG-MI	ITA	395	896	2296	5052	1109	114	3660	588	795	555	15,460
CVAL-MI	ITA	226	272	1441	3144	798	35	2633	423	313	168	9452
DBK-FF	GER	4722	123,463	90,187	200,227	12,744	15,749	42,058	6751	2795	76,533	575,229
DPB-FF	GER	684	12,066	18,031	36,991	6954	1539	22,952	3684	684	7480	111,065
EHY-FF	GER	83	9738	23,124	47,808	10,921	1242	36,045	5786	209	6036	140,992
GLE-FR	FRA	6305	91,368	85,510	174,873	21,054	11,655	69,487	11,153	7373	56,638	535,417
GUI-MC	ESP	115	120	929	1982	533	15	1760	283	84	75	5896
HBOS-LN	GBR	2846	40,670	86,156	181,455	39,158	5188	129,237	20,744	11,264	25,211	541,928
HSBA-LN	GBR	22,068	78,148	131,680	281,316	54,266	9968	179,099	28,747	8393	48,443	842,129
IKB-FF	GER	33	2387	4617	9486	2026	305	6686	1073	161	1480	28,255
ISP-MI	ITA	3645	11,322	28,495	60,369	13,695	1444	45,198	7255	1973	7019	180,414
KN-FR	FRA	513	48,165	38,662	81,407	7126	6144	23,517	3775	1145	29,857	240,311
LANS-AE	NED	35	229	1950	4247	1128	29	3722	597	126	142	12,204
LLOY-LN	GBR	3329	21,510	49,392	105,285	23,080	2744	76,172	12,226	8991	13,333	316,062
MB-MI	ITA	46	2695	4465	9110	1824	344	6021	966	209	1671	27,352
MEL-MI	ITA	14	99	379	810	200	13	661	106	3	61	2347
NRK-LN	GBR	956	1670	15,968	34,112	9306	213	30,714	4930	197	1035	99,101
OLB-FF	GER	43	151	945	2015	533	19	1758	282	74	93	5913
PAS-MC	ESP	570	240	2400	5166	1403	31	4629	743	229	149	15,558
PEL-MI	ITA	30	178	740	1554	396	23	1308	210	114	110	4662
PIN-MI	ITA	31	78	413	900	228	10	754	121	44	49	2628
PMI-MI	ITA	332	1250	3891	8479	1973	159	6513	1045	502	775	24,918
POP-MC	ESP	1012	1191	9436	20,390	5428	152	17,915	2875	477	738	59,614
RBS-LN	GBR	6121	61,181	123,944	266,102	55,400	7804	182,840	29,348	18,420	37,925	789,085
SAB-MC	ESP	610	1083	7060	16,002	3994	138	13,183	2116	662	671	45,519
SAN-MC	ESP	13,734	37,970	78,002	167,329	35,051	4843	115,683	18,568	6812	23,537	501,529
STAN-LN	GBR	3934	9317	18,484	41,136	8194	1188	27,044	4341	1108	5775	120,521
TRNO-FR	FRA	18	16	238	508	142	2	467	75	10	10	1486
UBI-MI	ITA	273	2089	7212	15,069	3737	266	12,334	1980	908	1295	45,163
UC-MI	ITA	11,876	43,965	77,916	163,729	32,904	5608	108,597	17,431	5805	27,253	495,083
Total		147,815	1,158,533	1,595,366	3,361,651	583,522	147,781	1,925,841	309,117	116,477	718,158	10,064,261
Average		2898.33	22716.34	31281.68	65914.72	11441.62	2897.66	37761.59	6061.13	2283.87	14081.54	197338.46
%		1%	12%	16%	33%	6%	1%	19%	3%	1%	7%	100%

conservative in the model. We would expect that the inclusion of complex assets and derivative positions might enhance the picture of diversification.
- Corporate and retail have high diversification indexes (see Table 11) whereas equity and property are lower, reflecting potential to diversify by moving more assets to the latter classes.
- Conclusions are broadly similar for all risk measures: additive capital is the highest with modular the lowest; fully-integrated EC consistently falls between the two.

5. Conclusion

At an institutional level we observe materially different results for Economic Capital computations for identical asset classes under modular and fully-integrated approaches. Both are methods currently permissible under Basel II, Pillar 2. The modular approach uses a correlation matrix overlay to capture dependence between different asset class risks. By contrast, in the fully-integrated approach, correlations are due to mutual dependencies in the driving risk factors in global markets. The comparison of the two approaches shows that, precisely in stress episodes, such as credit contagion, capital derived using a correlation matrix is discrepant with the fully-integrated framework (and can only be accordant by accident). In summary the fully-integrated approach:

- Avoids theoretical pitfalls and practical limitations of more modular approaches.
- Opens the door to capital allocation, risk-adjusted performance comparison and risk-based enterprise steering, as is the ultimate goal of Enterprise Risk Management.
- Provides a framework for rational (probability-based) stress testing. It is possible to identify the risk factors that "correlate" highly with asset value losses and reveal the factors that are particularly influential in the tail, i.e. we can get a proper handle on tail dependence.
- Allows the isolation of model and calibration effects on EC.
- Provides capital results that can allow the consideration of path dependent actions, such as portfolio rebalancing.

Finally, it is clear that different risk measures and different approaches give different risk capital figures, and the "undercapitalisation" implied by a modular approach with respect to a fully-integrated approach, while interesting in the current climate, is not really the main message of this paper. After all, it might be argued that this undercapitalisation can be simply rectified by increasing the correlations. We believe that this is the wrong conclusion and that risk management can never be about the manipulation of poorly understood numbers to obtain the most convenient set of results.

Rather the main message of this paper is that the fully-integrated scenario-based approach offers a powerful explanatory framework for integrated risk management. A regulatory emphasis on the development of such models for calculating economic capital would greatly enhance Pillar 2's application and indeed rejuvenate the relevance of Basel II. A focus on the economic drivers of risk and their systematic (and systemic) effects would be expected to lead to better capitalisation standards in future.

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Appendix A. Balance sheet assets of 51 European banks

See Table A.13.

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