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Integrated Spatial Reasoning in Geographic Information Systems: Combining Topology and Direction

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by

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Chapter 1

Introduction

This thesis deals with computational methods that exploit qualitative spatial information for making inferences about objects in a geographic database. The motivation is to enhance Geographic Information Systems (GISs), which manage the storage and retrieval of large data sets, with intelligent mechanisms to deal with complex spatial concepts for data selection and integration. One purpose of these intelligent mechanisms is to enable intuitive interaction with the data by capturing and reflecting the user's perception of the world. Intuitive interaction entails providing facilities for the representation of qualitative spatial information and making inferences. *Qualitative spatial information* is characterized by a finite set of symbols that specify distinctions among spatial configurations. For example the symbol set {*North*, *South*, *East*, *West*} denotes a system of qualitative directions and the set {near, far} a system of qualitative distances. *Inference* is the process of combining facts and rules to deduce new facts. We investigate the inference of qualitative spatial information from stored base facts. Thus the core problem is to find those spatial relations that are implied by a particular configuration, from a set of objects and a set of spatial constraints relating these objects. While spatial inferences may appear trivial to humans, they are difficult to formalize for implementation in an automated system. This problem is germane to GISs because spatial reasoning is extremely useful for searching in large databases containing complex geographic datasets.

In the past, work on spatial data models and spatial relations concentrated on defining models for representing spatial objects and on defining spatial relations within these models. Work on defining spatial relations independent of the data model or underlying representation usually tackled specific types of relations or their combinations. For example, considerable work has been done on defining topological relations (Egenhofer and Franzosa, 1991; Egenhofer and Herring, 1990) and using the formalism for consistency checking (Smith and Park 1992; Egenhofer and Sharma 1993a), specifying integrity constraints (Hadzilacos and Tryfona 1992), maintaining a spatial knowledge base (Hernández 1993), and optimizing queries on topological relations (Clementini *et al.* 1994; Papadias *et al.* 1995). Similar efforts have been undertaken for cardinal directions and approximate distances (Frank 1995; Hong 1994). These research efforts have concentrated on specific aspects of spatial reasoning in isolation from each other, partially due to the lack of a comprehensive spatial reasoning framework.

This thesis concentrates on the use of these formalisms for spatial reasoning. In particular, it defines a framework within which reasoning across different formalisms can be integrated such that more precise spatial information is obtained.

1.1 Spatial Reasoning

Since people are particularly skilled in spatial cognition and spatial reasoning, such as wayfinding, computational models for spatial reasoning are guided by research in the cognitive sciences and psychology (Mark and Frank 1991; Mark and Freundschuh 1995; Tversky 1981). Spatial reasoning is the process by which information about objects in space and their interrelationships is gathered by various means, such as measurement, observation, or inference, and used to arrive at valid conclusions regarding the objects' relationships or in determining how to accomplish a certain task. Spatial reasoning is used in inferring all possible spatial relations between a set of objects using a specified subset of the relations. Inference of spatial relations, which is the focus of this thesis, is used in applications such as designing the layout of an office or in navigation. Based on the results of cognitive science research, we build computational models and formalisms of spatial relations that permit the inference of new spatial relations from a specified, but possibly incomplete, set of spatial relations between objects. In particular, the models utilize inferences on combined spatial information about topological and directional relations. The following sections present an overview of cognitive aspects of human spatial reasoning and their implications for developing formal models of spatial reasoning.

1.1.1 Spatial Cognition

A study of human spatial reasoning abilities helps provide answers to questions like: Which are the properties of the spatial domain and objects that are preserved in a mental model? Which properties are discarded? What is the level of abstraction? What is explicit in the mental representation and what is implicit? How is spatial reasoning accomplished in the absence of complete information? Answers to these questions determine the level of detail, the encapsulated properties, the built-in functionality, and the overall organization of information in computational models for spatial reasoning.

Research in cognitive psychology has examined humans' mental organization of a large-scale environment such as a city. Lynch (1960) states that humans' cognitive maps organize a large-scale environment using five mental concepts: landmarks, paths, nodes, districts, and edges. Topological information is encoded in paths and districts while direction and distance information is represented using landmarks, nodes, and edges. Other research has studied humans' use of hierarchies in organizing landmarks in a cognitive map of their environment (Hirtle and Jonides 1985) and in mental representations of spatial knowledge (McNamara 1986). Researchers also investigated whether people use rules of inference and mental models—multiple or unified—for spatial reasoning, and if they use only categorical representations defining classes of spatial relations between objects (Byrne and Johnson-Laird 1989). Examples of categorical representations are connected/disconnected and left/right, whereas coordinate representations determine a division of space based on some unit or resolution of the visual system (Kosslyn *et al.* 1992). Results of this research indicate that people (1) organize cognitive maps hierarchically and (2) use deductive reasoning via simple inferences and non-deductive reasoning via mental imagery.

The research in cognitive issues has implications and utility for Artificial Intelligence (AI). An essential component of any AI system is a knowledge representation scheme that highlights the core issues and constraints of a problem and thereby facilitates its solution. The representation should reflect its intended use and contain information at the appropriate level of granularity. One possible approach is to develop representation and manipulation schemes that mimic the methods humans use in similar problem solving situations. As a result there is considerable interest, in the areas of AI and robotics, in qualitative reasoning and particularly spatial reasoning. The insights that these fields have drawn from the cognitive sciences about human spatial reasoning are (Freksa 1991; Freksa and Röhrig 1993; Hernández 1994; Kuipers 1994):

- The information people store and process is necessarily qualitative, since their cognitive mechanisms are limited in resolution and capacity. Hence only comparisons between features are made possible, whereas details such as size, shape, and location within some grid are disregarded. However topological information such as inclusion, coincidence, and connectivity is retained fairly precisely.

- The number of features and distinctions encoded is just sufficient to make the identification of objects or situations possible within a given context.

- Structural similarities between the represented and representing world are used to capture the constraints and inherent properties of the domain.

- People use multiple representations. Two commonly used kinds of representation are depictional, where the image acts as an analogy of the precept, and propositional, in which the relations between identified entities are stored as facts.

- Within the context of human spatial reasoning the static data structures that encode the qualitative information about the represented world can be viewed as data or information depending on their semantic content. Knowledge can be viewed in terms of the active processes performed on these data structures. Reconstructing a scene from its verbal description or performing inferences are examples of such active processes.

This thesis builds on results from research into human spatial cognition such as: the fact that humans use qualitative spatial information; that the interdependence between different types of spatial relations, for example distances and orientation, is taken into account in human spatial reasoning; and that humans use rules of inference for qualitative spatial reasoning.

1.1.2 Qualitative Spatial Information

Incomplete, imprecise, and qualitative spatial information occur frequently when users want to analyze spatial descriptions. Narrative, as in newspaper articles (Montello 1992), trip descriptions (Golledge 1995), and emergency reports (Welebny 1993), include descriptions of geographic space without the required precise description of the objects involved. Similarly, instructions humans give to guide others through geographic space contain combinations of spatial descriptions without references to coordinates (McGranaghan *et al.* 1987; Golledge 1992). In another domain, biologists collected herbarium specimens, for which they recorded narrative descriptions of the sites where each specimen was found (Futch *et al.* 1992). For such applications, spatial reasoning is needed in order to allow a GIS to fill the gaps and infer missing information. Automated spatial analyses, for instance, about endangered species or the relations to soil types and climate, are severely hampered by the lack of methods to integrate the individual natural language descriptions of geographic spaces to infer and compare the spatial relations among the specimens. In all cases, a presently available commercial GIS would require a user to identify the location of the object and geocode a complete description of the object's geometry, most often in terms of points, lines, and areas and their inter-relationships.

Qualitative information and reasoning, on the other hand, deals with a small set of symbols whose semantics may vary depending on the context or scale. For example, the notion of nearness depends on the person's task—such as walking, driving, or flying—and the scale. One would say that Bangor is near Boston when describing its location to someone in India, which sets the scale implicitly to be the whole of the U.S., but not when the context is the New England region. Qualitative approaches to information handling allow users to abstract from the myriad of details by establishing "landmarks" (Gelsey and McDermott 1990) when "something interesting happens," therefore, they allow users to concentrate on a few, but significant events or changes (Egenhofer and Al-Taha 1992). This working pattern is typical for scientists and relevant for geographic databases in which scientists record the data of their experiments—frequently time series observations—with the goal of subsequently extracting the "interesting" stages. By abstracting details and highlighting significant aspects of a problem, qualitative spatial information facilitates planning an approach to a solution and in determining what further information is needed. For many decision processes qualitative information is sufficient; however, occasionally quantitative measures, dealing with precise numerical values, may be necessary.

Quantitative information is ideally of arbitrary precision and detail, and independent of context. For example, depending on the desired precision one could state that downtown

Orono is 10, 10.4, or 10.438 miles from downtown Bangor. While quantitative representations allow for very powerful and frequently efficient calculations, they fall short when users lack some information about the geometry of the objects involved (Egenhofer and Mark 1995). Quantitative representations always need complete descriptions of the objects' geometry, i.e., they cannot handle partial information, and they have serious problems when geometric information is imprecise. Problems also arise due to finite-precision computations and the resultant error propagation (Hassan 1995). It has been recognized that a quantitative approach is an inappropriate representation of human cognition and spatial reasoning (Kuipers 1978). The remoteness from familiar or intuitive processes makes Euclidean geometry reasoning systems inappropriate for applications with a high level of user interaction, since they deal with different concepts—small set of symbols on an ordinal and nominal scale in a discrete space vs. quantitative calculations in an infinitely precise, continuous space—which have significantly different properties. Analytical geometry and Cartesian coordinates have also been found as inappropriate tools, e.g., for the integrating the biologists' narrative descriptions of geographic locations.

Qualitative and quantitative approaches to spatial reasoning are complementary methods. Quantitative spatial relations include such observations as bearings (150˚ 25'), distances (4.3 miles), and corresponding values derived from coordinates. Such quantitative values are in close relationship with some qualitative spatial relations (Hong *et al.* 1995). For example, if the azimuth to a point (measured clockwise from due north) is 90˚ then this corresponds to the cardinal direction *East*. Likewise, if two regions *meet* topologically, then the distance between their boundaries is 0. Unlike qualitative, quantitative spatial relations depend on precise metric information. Thus qualitative information is concerned with the "what" and quantitative with "how much" as illustrated in the statements, "parcel *A* neighbors parcel *B*" and "parcel *A* shares a 20.5 meter boundary with parcel *B*."

This thesis introduces a comprehensive framework for reasoning about qualitative spatial information within a purely qualitative environment.

1.1.3 Formalisms for Qualitative Spatial Reasoning

Understanding and modeling people's skills in qualitative spatial reasoning requires a formal definition of spatial relations. Spatial relations between objects can be classified as being metrical, directional, or topological relations (Pullar and Egenhofer 1988; Worboys 1992). We identify spatial objects with a distinct identity with the uppercase letters *A*, *B*, *C*, and their spatial relations by symbols r_i, r_j, r_k etc. The topological relations are denoted by the symbols t_i, t_j, t_k, and so on. Similarly the directional relations are denoted by the symbols d_i, d_j, d_k etc. The terminology and notation used here are based on those used by researchers working on *relation algebras*. In particular, we use the notation preferred by Tarski (1941) and Maddux (1993).

A relation algebra is a Boolean algebra with an additional binary operation corresponding to composition, and four distinguished elements: the identity relation, the universal relation, the empty relation, and the diversity relation (Maddux 1994). A relation, in a relation algebra on a set *U*, is an element in a subset of all possible binary relations $U \times U$ on the set *U*. The four distinguished relations are defined as follows:

$1_U = \{\langle x, y \rangle : x, y \in U\}$ universal relation

$0_U = \varnothing$ empty relation

$1'_U = \{\langle x, x \rangle : x \in U\}$ identity relation

$0'_U = \{\langle x, y \rangle : x, y \in U, x \neq y\}$ diversity relation

For example, for the set {*disjoint, meet, overlap, coveredBy, inside, covers, contains, equal*}, of binary topological relations between simply connected regions without holes, *equal* is the *identity relation*, the disjunction of all relations in the set is the *universal relation*, and the disjunction of the set of relations minus *equal* is the *diversity relation*.

Since a relation algebra is a Boolean algebra, the laws of associativity, distribution, and De Morgan's laws hold for the relations. These laws, and the operations of converse and composition in particular, can be used for spatial inference.

The *composition* of spatial relations, denoted by ";", is an inference mechanism that permits the derivation of a spatial relation between two objects *A* and *C* based on their relation with a common object *B*. The composition of $A\ r_1\ B$ with $B\ r_2\ C \Rightarrow A\ r_3\ C$ is denoted by $r_1 ; r_2 \Rightarrow r_3$. The composition may result in a set of relations, for example $r_1 ; r_2 \Rightarrow \{r_3, r_4, r_5\}$, implying that any one of them can be the relation between objects *A* and *C*. The smaller the set, the more precise the inference result. An empty set indicates a contradiction, while the set of all relations—that is the universal relation—indicates that no information can be obtained by the inference. Consistency requirements dictate that the inferred set of relations between objects *A* and *C* be the set intersection of the compositions over common objects. For example, if $A\ r_1\ B ; B\ r_2\ C \Rightarrow \{r_3, r_4, r_5\}$ and $A\ r_1\ D ; D\ r_2\ C \Rightarrow \{r_3, r_5, r_6\}$ then the set of possible relations between *A* and *C* is $\{r_3, r_5\}$. Such spatial inferences require a definition of the spatial relations involved and the corresponding composition tables that define the results of each possible composition among all relations involved. Composition tables have been defined for topological (Egenhofer 1991) and directional relations and qualitative distances (Frank 1992; Hong *et al.* 1995). The formalism for topological relations permits the inference that *A* is *inside C* from the facts (1) *A* is *inside B*, and (2) *B* is *inside C*. Similarly, knowing that *A* is *North* of *B* and *B* is *Northeast* of *C* allows the inference that *A* is *North* or *Northeast* of *C*. The formalization of the relations and their compositions taken together form the model for spatial reasoning used in this thesis.

This thesis defines composition tables for pairs of topological and directional relations, such as overlap and North. These composition tables enable the construction of a comprehensive framework for qualitative spatial reasoning that is capable of reasoning about individual spatial relations of each type, or about combinations of spatial relations of different types.

1.2 Heterogeneous and Integrated Spatial Reasoning

The mere categorization of spatial relations is a useful tool for organizing different types of spatial information, developing formalisms, and providing a match with such terms and prepositions as *adjacent*, *in*, and *left of* used in natural language. In some situations, however, spatial information of different types must be considered together. In contrast with topological and directional relations, which have useful individual composition tables, conclusive reasoning about qualitative distances requires considering information of the relative orientation of the objects involved (Frank, 1995; Hong, 1994). For example, if *A* is *near B* and *B* is *near C* then *A* could be *near C* or at a *medium* distance from it depending on the orientation of *AB* and *BC*. If *AB* and *BC* have opposite orientations then *A* is *near C*, whereas if they have the same orientation then *A* is at a *medium* distance from *C*. The information on the relative orientation enhances the information regarding relative distances. Reasoning about such combinations of spatial relations will be called *integrated* spatial reasoning when all the relations are used in conjunction. In the previous example both the distance and orientation information were used. In such cases the spatial relations between objects are given as tuples, for example *A* [*near*, *left*] *B*. *Heterogeneous* spatial reasoning differs from integrated spatial reasoning in that combinations of single spatial relations of different types are considered at each step of the reasoning process. For example, heterogeneous spatial reasoning is used in inferring that *A* is *North* of *C* from *A North* of *B* and *B contains C* since a directional and topological relation are involved in the inference. Heterogeneous and integrated spatial reasoning enhance the capabilities of an automated spatial reasoning system given the appropriate composition tables.

1.2.1 Motivation

In a pictorial representation or natural language description of a scene all types of spatial relations coexist and their coexistence illustrates the artificiality of the categorization of spatial relations into topological, directional, and metrical relations. Often both topological and directional information is available about objects in a scene and it is necessary to use a combination of the two types of information in order to infer new facts that could not be inferred by considering individual types of relations in isolation. For example, an appropriate formalism would allow the inference of the facts *A disjoint D*, and *A West* of *D* from the specified facts *A* is *West* of *B*, *B overlap C*, and *C West* of *D* (Figure 1.1).

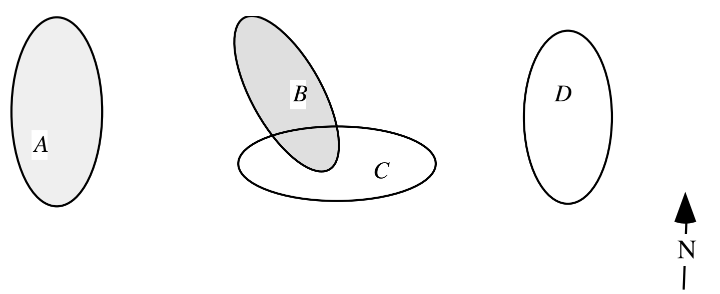


Figure 1.1 An example that requires heterogeneous spatial reasoning.

It is evident by visual inspection that object *A*, in Figure 1.1, is *West* of *D*; however this fact cannot be inferred using pure symbolic manipulations and composition tables for topological and direction relations independently. The directional relation between objects *A* and *C* and objects *B* and *D* are unknown and hence neither composition ($A\ r_d\ B\ ;\ B\ r_d\ D$) or ($A\ r_d\ C\ ;\ C\ r_d\ D$) is feasible. Since the directional relation *West* implies the topological relation *disjoint*, the composition of topological relations suggests itself. The result of a composition of topological relations, however, is a non-empty set of topological relations and hence the directional relation between objects *A* and *D* would remain unknown. In order to infer the directional relation the reasoning process must include the deduction that since B and C overlap they have some part in common, say C'. Therefore, A is *West* of C' since it is a part of B, and C' is *West* of D, and hence A is *West* of D. A comprehensive formalism for the composition of combinations of various types of spatial relations would facilitate this reasoning process.

For an example of a situation where both the topological and directional relation information must be used in conjunction, consider the scene depicted in Figure 1.2. Suppose that the spatial relationships *A disjoint B*, *A Northwest B*, *B disjoint C*, and *B Northwest C* are specified and the relations between *A* and *C* must be inferred.

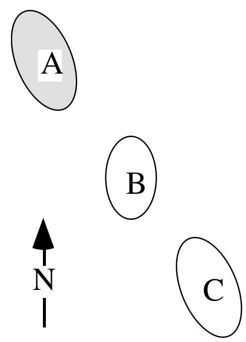


Figure 1.2 An example that requires integrated spatial reasoning.

The composition *A disjoint B*; *B disjoint C* results in the set of all possibilities, i.e., *A* {*disjoint, meet, equal, overlap, inside, coveredBy, contain, covers*} *C*, and therefore provides no topological information. If, however, the topological and directional relations are considered in conjunction, i.e., (*A disjoint* and *Northwest B*); (*B disjoint* and *Northwest C*), it is evident that the result should be *A disjoint* and *Northwest C*.

The above examples indicate the usefulness of heterogeneous and integrated spatial reasoning. We present a formal description of various types of spatial reasoning in the following section.

1.2.2 Formal Definition of Heterogeneous and Integrated Spatial Reasoning

A comprehensive automated spatial reasoning system must be able to deal with each type of relation individually, or with various combinations of relations of different types, or with tuples of relations that form a composite or integrated spatial relation.

- *Homogeneous* spatial reasoning involves the derivation of a single type of spatial relation between objects given two spatial relations of the same type (Equation 1.1a and 1.1b). For example, inferring *A disjoint C* given (1) *A disjoint B* and (2) *B contains C*, where *disjoint* and *contains* are both topological relations.

$$t_i \, ; t_j \Rightarrow \{t_k\} \tag{1.1a}$$

and

$$d_i \, ; d_j \Rightarrow \{d_k\} \tag{1.1b}$$

- *Heterogeneous* spatial reasoning involves the derivation of a spatial relation of either type given two spatial relations of different types (Equation 1.2a and 1.2b). An example is the inference of *A North C* from (1) *A inside B* and (2) *B North C*, where *inside* is a topological relation and *North* a directional relation.

$$t_i \, ; d_j \Rightarrow \{t_k\} \tag{1.2a}$$

and

$$t_i \, ; d_j \Rightarrow \{d_k\} \tag{1.2b}$$

- *Mixed* spatial reasoning involves the derivation of a spatial relation of one type from the composition of two spatial relations of a different type. An instance of mixed spatial reasoning is the inference of *A disjoint C* from the facts (1) *A North B* and (2) *B North C*.

$$d_i \,;\, d_j \Rightarrow t_k \tag{1.3a}$$

$$t_i \,;\, t_j \Rightarrow t_d \tag{1.3b}$$

- *Integrated* spatial reasoning involves the derivation of each type of spatial relation given two sets of identical types of spatial relations between objects (Equation 1.4). The instances of the spatial relations in each set may differ, but both sets will have the same number and types of spatial relations. An example is inferring *A disjoint* and *North C* from (1) *A meet* and *North B* and (2) *B meet* and *North C*.

$$[t_i, d_i] \,;\, [t_j, d_j] \Rightarrow \{[t_k, d_k]\} \tag{1.4}$$

Any spatial inference mechanism can be used depending on the completeness of the available information. For the example illustrated in Figure 1.1 some of the topological and directional relations between the objects concerned is missing and hence the heterogeneous spatial inference mechanism is required.

The three mechanisms of homogeneous, heterogeneous, and mixed spatial reasoning will together be considered as a *combined* spatial inference mechanism that works with individual spatial relations. *Combined* and *integrated* spatial reasoning can be considered as two categories of spatial reasoning. Each category has its own elements, namely combined and integrated spatial relations, and its own composition operator defined for those elements. The interesting question is whether the reasoning power of the two categories is the same, i.e., whether there exists an isomorphism between the two categories such that mapping the elements of one category onto another and then performing a composition gives the same results as first performing a composition and then mapping elements.

1.2.3 The Hypothesis

The hypothesis of this work is that both spatial reasoning mechanisms are equivalent. That is, the set of inferred relations obtained by composing tuples of spatial relations is equal to the set obtained by combination of homogeneous, mixed, and heterogeneous compositions. *The goal of this thesis is to determine if the hypothesis is valid for all inferences involving topological and directional spatial relations*. This thesis focuses on topological and directional relations and the combinations of these two types of relations only.

The validity of the hypothesis implies that a unified approach to spatial reasoning, which uses one canonical form, is possible and that valid inferences can be drawn using partial information. For example, only topological or only directional relations need be used depending on their availability. If that were not the case then inferences such as the one described above would be impossible. This thesis systematically examines the cases for which inferences over combinations of relation types give useful results. In addition, we compare the results of using inferences over tuples and over combinations of relations.

Since this thesis is largely concerned with reasoning about topological and directional relations all future unqualified usage of the terms heterogeneous, mixed, or integrated spatial reasoning should be read as referring to the specific case of combining topology and direction only.

1.2.4 Objective

The overall goal is the construction of a comprehensive formalism of qualitative spatial relations and their interactions. With this formalism as a basis we build a spatial reasoning system, tailored for structured geographic spaces, that integrates spatial concepts about topology, cardinal directions, and qualitative distances.

The specific goal of this thesis is to design a framework that supports heterogeneous and integrated spatial reasoning in geographic databases. The framework permits reasoning about individual types of relations and combinations of two or more types of relations. A prototype implementation demonstrates the utility and benefits of our approach, showing that new inferences can be drawn using heterogeneous or integrated spatial reasoning that are impossible with homogeneous spatial reasoning.

1.2.5 Significance

The significance of this work has three main components. The first is that the formal approach clearly identifies the different types of qualitative spatial reasoning that can be performed. This clear identification helps determine which composition tables are required for a comprehensive qualitative spatial reasoning framework that is capable of handling different types of spatial relations. The second is that this thesis contributes to Naive Geography (Egenhofer and Mark 1995) by showing how and when simpler spatial inference mechanisms can be combined to give results equivalent to those obtained with a more complex spatial inference mechanism. The third component is the systematic derivation of composition tables for heterogeneous and integrated spatial reasoning about topological and directional spatial relations which leads to new insight about models for directional relations among extended spatial objects.

1.3 Scope of the Thesis

Our investigations focus on large-scale geographic space, which is defined as space that is beyond the human body and cannot be observed from any single viewpoint (Kuipers 1978; Kuipers and Levitt 1988). Unlike small-scale or table-top space as used in the context of qualitative kinematics or mechanical parts, geographic space is commonly subject to incomplete and imprecise information for human spatial reasoning (Egenhofer and Mark, 1995); therefore, in the absence of more precise geographic information, purely qualitative geographic reasoning may be used as a substitute. While these reasoning processes may provide only approximate, sometimes crude solutions, they are frequently the only means available to infer new information that may still be sufficient to solve a particular geographic problem.

The objects of concern in this thesis are limited to simply connected, homogeneously 2-dimensional regions without holes. These objects may be real entities such as lakes or administrative districts, or temporary constructions such as buffer zones. The objects are not limited to physical entities and therefore "interesting" spatial relations may occur due to objects sharing space (Varzi 1993; Casati and Varzi 1994).

1.3.1 The Methods

In order to compare the two inference methods (Equations 1.3 and 1.4) we need additional composition tables for individual combinations of topological and directional relations, i.e., t_i ; d_j and d_i ; t_j, and for tuples of relations, i.e., [t_i, d_i] ; [t_j, d_j]. These composition tables are based on the formal definitions of topological and directional relations. Topological relations are described by the 4-intersection model (Egenhofer and Franzosa 1991), which is based on elementary concepts of boundary and interior in point-set topology. The values, empty or non-empty, of the four intersections between the boundary and interior of the objects are used to specify the relations. Directional relations are described by the projection-based system (Frank 1995), which segments the space surrounding an object's minimum bounding rectangle (MBR) into nine zones: the eight cardinal directions, {*North, Northeast, East, Southeast, South, Southwest, West, Northwest*}, and a neutral zone denoting the relation "at the same location" or "same direction."

Since the directional relations are defined using interval relations, which determine the possible relationships between objects, we map topological and directional relations onto pairs of interval relations. We use existing composition tables for interval relations (Allen 1983) to determine the composition tables for combinations of topological and directional relations. The composition of interval pairs results in a set of interval pairs that specifies the possible relations between the MBRs of the objects. We map this set of possibilities onto topological and directional relations. After mapping we use pairwise composition of intervals to determine the composition of combined or individual pairs of topological and directional relations.

1.3.2 Summary of the Results

The main findings of this thesis are:

- Heterogeneous spatial reasoning about topological and directional relations is useful when there is a containment relationship between one or more pairs of objects. This is because the contained object has the same directional relationships with other objects as the containing object. Hence a topological relation implies a directional relation.

- Integrated spatial reasoning about topological and directional relations is useful when the topological relations among objects are either *disjoint* or *meet*. In such a situation the directional relation helps localize the positional relationship and therefore the inferences result in a smaller set of possibilities than the set obtained using topological or directional relations in isolation.

- Integrated and combined spatial reasoning give the same set of inferred relations. Hence integrated spatial reasoning can be used in place of combined spatial reasoning whenever all the necessary spatial information is available. This is computationally more efficient since the predetermined and stored composition tables for integrated spatial relations simplify the inference process.

1.3.3 Complementary Issues

Although the aim of this thesis is a comprehensive formalism for qualitative spatial reasoning it excludes certain related issues:

- Modeling and reasoning about the dynamic aspects of spatial relations, that is, the changes in location or geometry of the objects over time. The scope of this thesis is reasoning about static spatial relations. Dynamic spatial reasoning requires, at the very least, a temporal reasoning capability (Egenhofer and Golledge 1996) and reasoning about motion (Zimmermann and Freksa 1993). Both these aspects of reasoning are the subject of extensive study by various research groups.

- How humans would map natural language expressions onto a formal model of space and spatial relations (Mark and Egenhofer 1992). Hence the question of multiple frames of reference and transformations between these frames is not tackled directly. Rather we assume a mechanism is in place that transforms all orientation relations to a fixed extrinsic reference frame (Hernández, 1991). Interpreting natural language expressions and providing a parametric characterization of various interpretations is the subject of an ongoing doctoral dissertation (Shariff 1996) and hence excluded from the scope of this thesis.

- We focus purely on the topology and geometry of the objects and leave the semantics of the objects (their attributes and non-spatial operations) for future investigations. The semantics will likely enhance qualitative reasoning capabilities by providing additional constraints that limit the type or number of spatial relations among objects. A lake may only *contain* and not *overlap* or *cover* an island, for example. These constraints can be used after the composition-based inference mechanism to further reduce the size of the inferred set of spatial relations. The additional constraints derived from the semantics of the objects are supplementary to the qualitative spatial reasoning using the formal definition of qualitative spatial relations.

- This thesis focuses on reasoning about qualitative spatial information and defers the integration of quantitative information for future work. The integration of quantitative and qualitative information requires a formal method for dealing with qualitative spatial information.

- Imprecision or uncertainty about an object's boundary adds complexity in terms of the number and nature of spatial relations among objects. For example, a new topological spatial relation becomes possible in which one crisply defined object can be contained in the region of uncertainty of another object's boundary. These additional relations and the mixing of crisply and non-crisply defined objects add considerable complexity to the qualitative spatial reasoning mechanism. A discussion of research questions in formalisms for dealing with geographic objects with undetermined boundaries can be found in (Burrough and Frank 1996).

These issues are very relevant and are being studied by various research groups, however, for our purposes they are largely extensions to the core problem of integrated and heterogeneous spatial reasoning.

1.4 Intended Audience

The intended audience of this thesis constitutes any persons interested in spatial reasoning in general and the design and development of Geographic Information Systems software in particular. This includes researchers and practitioners from the fields of geographic and scientific databases, artificial intelligence, constraint processing, and spatial information science. It particularly addresses the needs of designers of next-generation spatial databases, with an emphasis on the semantics of spatial information in spatial constraint databases.

1.5 Thesis Organization

The remainder of this thesis is organized into six chapters.

Chapter 2 discusses the requirements, approaches, and methods for spatial reasoning. A brief statement of the problem and its application is followed by a description of the requirements that must be met. The following section outlines possible approaches to a solution and their relationship to the requirements. Next, alternative methods are described and discussed in the context of a larger framework, which categorizes the core ideas behind various approaches to the problem of spatial reasoning.

The relation algebras and inference mechanisms for topological, directional, and qualitative distance relations are the subject of Chapter 3. The inference mechanism for each relation type is based on a composition table and a means of expressing the relations as a constraint network. The composition table defines the relations that can exist between two objects given their relation with a common third object. The constraint network helps reduce the size of the set of inferred relations by imposing local and global consistency requirements. These consistency requirements basically ensure that the direct relation between two objects is the same as the derived relation obtained using compositions over one or more intermediate objects.

While individual reasoning mechanisms for spatial relations are quite powerful, they fail to take advantage of additional information obtainable when two or more types of relations are given. For example, the fact that *A meets B* loosely constrains the location of objects. Adding the fact *A North* of *B* constrains their location even further. The methods and results of using combined knowledge of spatial relations are described in Chapter 4.

Chapter 5 extends the formalism developed in the previous chapter to derive the composition tables for mixed spatial reasoning. These composition tables give: the topological relations inferred from the composition of directional relations; the directional relations inferred from the composition of topological relations; and the qualitative distances inferred from the composition of topological relations.

Chapter 6 derives composition tables for integrated topological and directional relations. These composition tables, along with the tables for individual pairs of topological and directional relations defined in Chapter 4, are used to confirm our hypothesis. These composition tables constitute a major contribution of this thesis.

Chapter 7 presents a framework for integrated spatial reasoning, which permits a multi-level approach to the problem. The two primary concepts detailed in this chapter are that (1) spatial relations can be represented as a knowledge structure and hence can be organized, related, and constrained and (2) spatial relations are objects in their own right with properties and interdependencies. Based on these findings we propose a design for a qualitative spatial reasoner that supports combined and integrated spatial reasoning.

Chapter 8 summarizes the major contributions of this thesis and analyzes their implication for future work. The primary contribution of this thesis is a framework for integrated and heterogeneous spatial reasoning that exploits the power of using individual or combined knowledge of spatial relations between objects. Further work is required in order to address the issues of integrating quantitative and qualitative constraints, and reasoning with multiple levels of granularity.

Chapter 2

Related Work in Qualitative Spatial Reasoning

Spatial reasoning is concerned with solving problems involving entities that occupy space (Kak 1988). The motivation behind research in spatial reasoning is twofold. The first is the scientific motivation to gain a better understanding of human performance. The second is the need to devise formalisms that allow automated systems to tackle spatial reasoning problems. Examples of applications of automated spatial reasoning include visual object recognition, intelligent image information systems, wayfinding for robots or autonomous vehicles, and query processing in geographic databases.

- Visual object recognition involves reasoning about the shapes and possibly textures of objects and is useful in applications such as automated assembly or testing of electronic components and circuit boards (Fleck 1990).

- Intelligent image information systems utilize spatial reasoning for query processing and similarity-based retrieval or matching of images (Chang and Hsu 1992). The information system stores a symbolic representation of the image, which encodes knowledge about the objects in the image and the spatial relationships between these objects. Queries are then processed using the symbolic representation rather than the actual images by transforming a query into the same symbolic representation and a pattern match between the symbolic representations of the query and images (Arya *et al.* 1994; Faloutsos 1995).

- Wayfinding or navigation requires spatial reasoning abilities since the autonomous agent must construct a cognitive map of its environment in order to solve the problem of getting from one location to another (Kuipers and Levitt, 1988).

- Spatial query processing requires a formal definition of spatial relationships and their properties in order to enable consistency checking and inferences (Egenhofer 1991). Consistency checks determine whether a specified query can be satisfied by a physically realizable configuration of spatial objects. Inferences permit the derivation of all implied relations from the explicitly stated ones.

This thesis is primarily concerned with spatial reasoning from the point of view of spatial query processing.

2.1 Approaches in Spatial Reasoning

Any model devised for spatial reasoning must include a representation system that is particularly suited for dealing with spatial information and a mechanism for making inferences from this information. Thus the representation should preserve certain fundamental properties of the spatial domain, while the reasoning system should utilize the conceptual structure of the spatial relations that result from these properties. The fundamental properties of the domain are:

Uniqueness: Distinct objects occupy distinct locations in physical space and at any instant in time an object is at one and only one location. A location is specified by a set of points in space.

Spatial arrangement: The spatial relationships among objects result from their arrangement in space. These relationships include topological relations, orientation relations, and distances. Orientation relations imply a frame of reference, which should be included in the representation.

Neighborhood: Under continuous change movement occurs only between neighboring locations in space and hence change in spatial arrangement has a neighborhood structure. For example, if two objects are disjoint and either one or both are moved towards each other then the objects must be adjacent at some point before they can overlap.

The following analysis and review examines various approaches to spatial reasoning with respect to their utilization of the above properties of the spatial domain.

There are basically two approaches when constructing a model suitable for spatial reasoning: (1) one may choose to model physical space and the objects within it or (2) one may model the relationships between the objects (Frank and Mark 1991). These approaches are analogous to the raster and vector spatial data models. A model of physical space captures the locations of objects, for example, as a set of coordinates in some reference grid, and the relationships are computed using this metric information. Models that encode the relationships between objects often ignore information on the locations of the objects. The original image or picture is encoded in some symbolic representation, from which the spatial relationships are derived using a set of transformation or manipulation rules. Regardless of the approach used, each spatial data model requires a formal definition of spatial relations that identifies and characterizes the relations describable within that model.

The following sections provide fairly comprehensive overview of research in qualitative spatial relations and spatial reasoning. Secion 2.2 presents formalisms for qualitative sptial relations that have gained acceptance and used in subsequent research efforts. We present formalisms for topological and directional relations which are the focus of this thesis and, for the sake of completeness, formalisms for qualitative distances since it is a core element of spatial reasoning. This thesis uses well-defined formalisms for individual types of spatial relations in devising a framework for integrated spatial reasoning and therefore uses the second of two approaches described in the previous paragraph. Considerable research effort, however, has been successfully expended in devising specialized methods for qualitative spatial reasoning using the first approach. In order, therefore to place the results of this thesis in the appropriate perspective, Section 2.3 is a broad and comprehensive review of research in qualitative spatial reasoning.

2.2 Formalisms for Qualitative Spatial Relations

Various categorizations of qualitative spatial relations are possible based on common spatial concepts such as proximity, connectivity, adjacency, and containment. We use the more common classification of spatial relations into topological relations, directional relations, and distances. Other definitions, and categorization, of spatial relations have been made based on partial orders (Pullar and Egenhofer 1988; Kainz 1990; Kainz *et al.* 1993) and set-theoretic concepts (Freeman 1975; Worboys 1992).

Topological spatial relations are those relations that are invariant under continuous transformations with continuous inverses, such as rotation or scaling. *Directional relations* are defined between a reference object and a primary object with respect to a fixed frame of reference, usually determined by a predefined entity such as the North Pole. While direction relations are easily defined for points objects, they have many alternate definitions between extended objects. *Qualitative distances* refer to the use of such terms as near, far, or very far for specifying the distance between two objects. The following is a review and discussion of proposals for the definition of qualitative spatial relations.

2.2.1 Topological Spatial Relations

If the relative orientation and distances between objects are disregarded, there remain some distinctions in the spatial arrangement that can be identified. The objects may overlap, or touch, or one may contain the other, or they may be disjoint. These relations can be defined by topological means, by considering only the two objects involved and their fundamental geometric properties, namely the set of points in space that they occupy.

The requirements for the formal definition of these distinctions are twofold. First, the set of identified relations should cover all possible situations and second, each situation should correspond to one and only one relation definition. Mutual exclusiveness is required because it simplifies the process of constructing compound relations from the primitive ones. The only logical operator required is the OR since the primitive relations are pairwise distinct. The set of relations *disjoint* or *touch* or *contain* or *overlap*, however, does not satisfy both requirements since there is no way to describe the situation where two objects are equal and no means of distinguishing between containment with some border points in common and total containment.

A systematic identification and characterization of topological relations is needed in order to satisfy the requirements of completeness and mutual exclusiveness. The result of this systematic derivation is a relation algebra (Tarski, 1941), that is, a formal definition of topological relations that specifies the minimum set of binary relations that are possible between objects *A* and *B*.

The relation algebra for topological relations is based on the concepts of boundary and interior in point-set topology. The boundary and interior have no points in common and their union forms the closure of the set *A* representing that object. The 4-intersection, which analyzes the intersections of the two objects' boundaries (∂) and interiors ($^\circ$) (Egenhofer and Herring 1990; Egenhofer and Franzosa 1991), contains the entries Ø (empty) or ¬Ø (non-empty) for each of the set intersections $\partial A \cap \partial B$, $\partial A \cap B^\circ$, $A^\circ \cap \partial B$, and $A^\circ \cap B^\circ$. We illustrate the use of this approach defining the topological relations for region objects. A region is a non-empty, connected point-set homeomorphic to a unit disk in $\Re^2$. The same method is applicable to topological relations between any combination of regions, lines, and points by using the 9-intersection, which includes also the intersections with the objects' exteriors (Egenhofer and Herring 1991).

If we consider arbitrary point-sets then sixteen distinct relations are possible, since each one of the four intersections can be empty or non-empty, that is, the tuples can range from (Ø, Ø, Ø, Ø) to (¬Ø, ¬Ø, ¬Ø, ¬Ø). Our interest, however, is in spatial regions and the definition of spatial regions imposes certain restrictions on the point-sets such that not all sixteen relations are possible for regions. Since regions are connected and always have a non-empty boundary and interior, the following condition must hold:

- If the boundary of one object intersects the interior of the other then the two interiors must also intersect.

This condition rules out seven possibilities. An eighth possibility is excluded by the requirement that regions have connected boundaries since they are homeomorphic to a 2-disk. Excluding these eight possibilities results in the set of eight distinct topological relations illustrated in Figure 2.1.

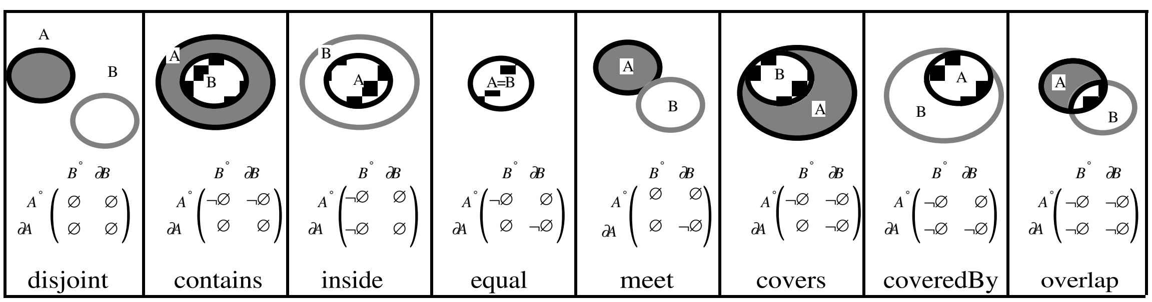


Figure 2.1 Examples of the 8 topological relations between regions in $\Re^2$ (Egenhofer and Franzosa 1991).

An alternative approach to the systematic derivation of binary topological relations was developed by Randall *et al.* (1992) using Clarke's (1981) calculus of individuals. Their Region Connected Calculus (RCC) theory is intended as a model of physical reality that facilitates qualitative reasoning and simulation of real world entities. The basic philosophy is that each physical entity occupies a certain area or region in space at any given time. This region is an atomic unit that cannot be subdivided into an interior and a boundary in the manner permitted by point-set topology. The contention is that attempting to define concepts such as boundaries leads to unnecessary complexities and problems particularly when modeling relations such as adjacency between physical objects. Using point-set topology two regions are adjacent if they share some common boundary points. However, two physical objects, such as a desk and a wall, may be adjacent and not have any points in common since they are physically distinct entities. The RCC theory is designed for modeling such situations.

The theory is developed on a calculus of individuals based on connection. The primitive dyadic relation is "x connects with y," denoted by C(x, y), which holds when the regions x and y share a common point. A refinement of the relation C(x, y) gives a set of eight pairwise disjoint and complete base relations for regions that are the same as the binary topological relations defined using the 4-intersection method based on point-set topology.

The RCC set of eight base relations has one subtle, but important difference from the 4-intersection set. The point-set topology based relation *meet* corresponds to the relation EC(*x, y*), that is, *x* is externally connected with *y*. The two definitions differ in that *meet* implies that the two regions have at least one boundary point in common, while EC implies that there exists at least one point in space such that no third distinct region can be placed between *x* and *y*. Such a distinction is useful when modeling binary topological relations in a discrete or raster space (Egenhofer and Sharma 1993b; Winter 1995). The relation EC(*x, y*) corresponds to the situation where *x* and *y* have pixels that are adjacent, but none in common.

Abdelmoty and El-Geresy (Abdelmoty 1995; Abdelmoty and El-Geresy 1995) developed a set-theoretic approach to the definition of topological relationships between objects of arbitrary dimension and shape. Their method is an extension of the 4-intersection formalism. The objects and space are decomposed into representative components and the combinatorial intersection of the components are used to characterize the spatial relationships. Unlike the 9-intersection formalism, however, the size of the intersection is not fixed and varies with the number of components defined for each object. Also since the nature or characteristics of the components are not predetermined, it is potential source of confusion when used for pure symbolic reasoning. The ramifications of this requirement are that additional information must be stored about the shape of each object in the database and its components.

We simplify the requirements of a qualitative spatial reasoning system and assume that the 9-intersection formalism adequately defines binary topological relations for the purpose of this thesis.

2.2.2 Directional Relations

The direction between two objects is defined by the orientation relationship between a line connecting the objects and a fixed reference line. The components that define an orientation relation are the point of view, the primary object, and the reference object. The viewpoint and reference object together establish a reference frame, which helps determine the orientation of the primary object relative to the reference object. The reference line is defined by a reference frame and determines the orientation of the objects of interest. The specified direction is that of the primary object with respect to the reference object. For example, in the statement "*A* is *North* of *B*", *A* is the primary object and *B* the reference object. The relation *North* is one of four primitive cardinal directions that are specified with respect to a fixed external reference frame determined by a reference meridian and the equator. The reference frame determining the orientation relations can be established in one of three ways (Retz-Schmidt 1988):

- Using the *intrinsic* orientation of the reference object. For example a building or a car has an intrinsic orientation, which is used implicitly in statements like, "The restrooms are at the rear of the building."

• Imposing an *extrinsic* orientation. Examples are (1) the use of a reference meridian and parallel to define compass directions, and (2) defining the vertical axis using gravitation.

• The orientation is imposed by a point of view, usually that of an observer. This is called a *deictic* frame of reference, and is defined by the line connecting the point of view and reference object. Deictic use is often found in route descriptions such as, “If you are at the Library steps facing the Gym then Boardman is the third building on your right.”

Hernández (1994) carefully and clearly outlined the importance of reference frames and their use in spatial reasoning. For example, a primary object *A* is in front of reference object *B* if the viewpoint is from *B* towards *A*. A different viewpoint might result in the orientation relation being *A left of B* or even *A behind B*. Figure 2.2 shows how a data structure, called the *relative orientation node* (`ron`), can be used to represent the orientation relation between objects. Each `ron` corresponds to one object and hence the leftmost pair represent the fact that *A* is left of *B* and *B* is right of *A*. A rotation in the viewing angle is captured by a corresponding rotation of the `rons` and hence the rightmost pair of `rons` represent the fact that *A* is to the *left-back* of *B* and *B* is to the *right-front* of *A*.

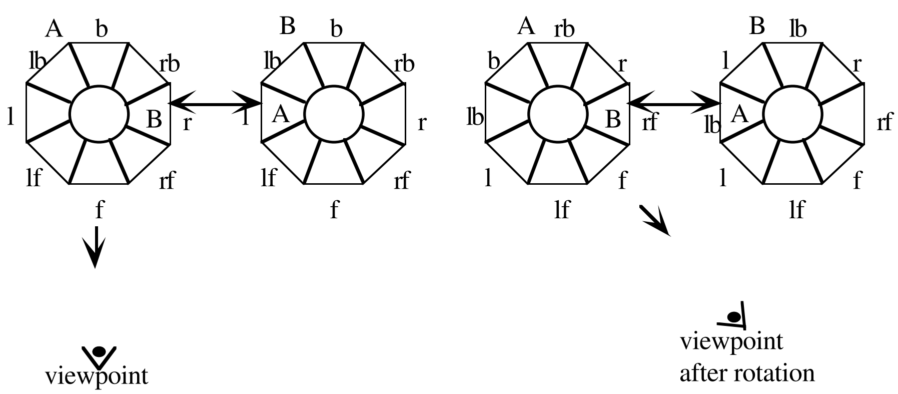


Figure 2.2 Effect of the viewpoint on the orientation relation (Hernández 1994).

Given a reference frame, a direction is determined by the location of the two objects involved. This thesis is concerned only with cardinal directions and hence the observer's point of view is not used in the reasoning process.

Cardinal directions can be expressed using numerical values specifying degrees in the half-open interval [0˚, 360˚) from the zero direction determined by the North-South meridian, or by using qualitative values or symbols, such as *North* and *South*, that have an associated region of acceptance. The regions of acceptance for qualitative directions can be projections (also known as half planes) or cone-shaped regions (Frank 1992). Figure 2.3 shows acceptance regions for North-South and East-West using projections.

Figure 2.3 Definition of North-South and East-West using half-planes (Frank 1992).

Figure 2.4 shows cone-shaped regions defining *North*, *South*, *East*, and *West*, and a combination of projections to define NW, NE, SE, and SW. A special characteristic of the cone-shaped system is that the region of acceptance increases with distance, which makes it suitable a basis for the definition of directional relations between extended objects (Peuquet and Ci-Xiang 1987).

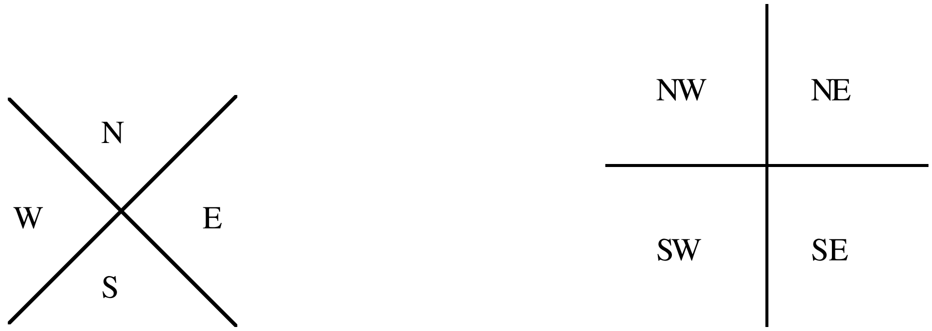


Figure 2.4 Cone-shaped and two half-planes direction systems.

The cone-shaped region definition has the additional benefit of permitting increasingly finer resolutions of relations, and hence there can be eight, or even sixteen different qualitative directions. Figure 2.5 illustrates a definition of eight direction relations.

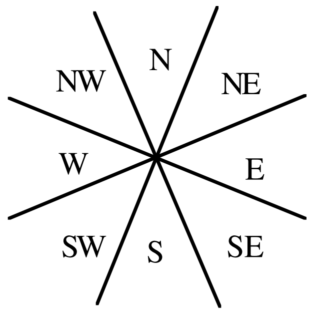


Figure 2.5 Cone shaped eight direction system.

Direction relations between extended objects are not as simply or clearly defined as directions between points. We outline two approaches to their definition, one based on projections (Frank 1992) and the other on a triangular model (Peuquet and Ci-Xiang 1987).

The projection-based system consists of nine acceptance areas, one for each of the directions N, E, S, W, NE, SE, SW, and NW, and one neutral zone. The neutral zone serves as a definition of the concept of “here” or “at the same location as.” The acceptance areas are delimited by extending the bounding lines of an objects’ MBR and hence the MBR forms the neutral zone (Figure 2.6).

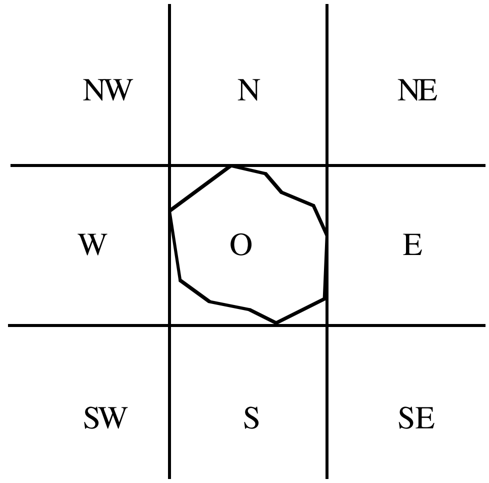


Figure 2.6 Projection based system for directions (Frank 1992).

The triangular model (Evans 1968; Haar 1976) uses triangular acceptance areas that are drawn from the centroid of the reference object towards the second object. Perceptual prominence is used to determine the reference object (Figure 2.7).

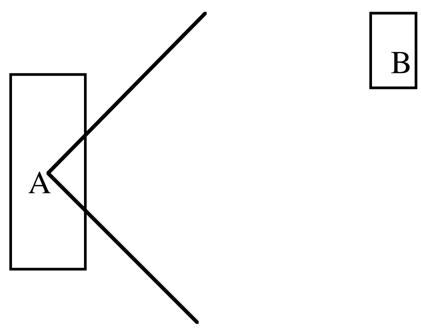


Figure 2.7 Triangular model for directions (Peuquet and Ci-Xiang 1987).

If the objects are close to each other relative to their sizes, then the vertex of the triangular area is moved backward and forward such that the rays touch the end points of the side of the object's frame facing the second object (Figure 2.8). These definitions are useful for non-overlapping or non-intertwining objects. If the objects intertwine then the definition of the directional relation is based on the satisfaction of a compound condition.

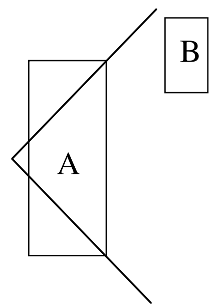


Figure 2.8 Triangular model of directions for close objects.

2.2.3 Qualitative Distances

A distance between two points is a measure of the effort required to reach one point from the other. Thus distances can be specified in numerous ways, for example, as a travel time, a length measurement, or economic cost. Viewed from the perspective of human cognition or perception, distance is a measure of expenditure of effort. Hence even a metric quantity is mapped onto some qualitative indicator or order of magnitude such as close, very close, far, or very far for human commonsense reasoning. This section describes the main properties of a distance and proposals for sets of qualitative distance relations.

The most commonly used distance is defined by Euclidean geometry and Cartesian coordinates. In a 2-dimensional Cartesian system the distance between two points is given by:

$$d_{AB} = \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2} \tag{2.2}$$

where (x_A, y_A) and (x_B, y_B) are the Cartesian coordinates of points *A* and *B*, respectively.

The Euclidean distance is a specific instance of a family of metrics known as the Minkowski or L_p-metrics (Preparata and Shamos 1985) for an *m*-dimensional space (Equation 2.3),

$$d_p(P_i, P_j) = \left[\sum_{k=1}^{m} \left| x_{ik} - x_{jk} \right|^p \right]^{1/p} \tag{2.3}$$

where $P_i = (x_{i1}, x_{i2}, \ldots, x_{im})$ P_j are points in that space. The case $p = 2$ gives the expression for Euclidean distance. Another special case occurs when $p = 1$. This gives a distance known as the taxicab or Manhattan distance (Equation 2.4), which is the shortest path between two points along a path of segments that are parallel to one the of axes.

$$d_{AB} = \left| x_A - x_B \right| + \left| y_A - y_B \right| \tag{2.4}$$

L_p-metrics satisfy the three conditions, or axioms, that define the concept of distance in a metric space (Equations 2.4-2.6).

reflexivity: $d(P_1, P_1) = 0$ (2.4)

symmetry: $d(P_1, P_2) = d(P_2, P_1)$ (2.5)

triangle inequality: $d(P_1, P_2) + d(P_2, P_3) \geq d(P_1, P_3)$ (2.6)

In a qualitative framework the commonsense interpretation is: (1) a point is at a zero distance from itself, e.g., Orono downtown is at Orono downtown; (2) distances are symmetric, for instance, if the UMaine campus is close to Orono downtown then Orono downtown is close to the UMaine campus as well; and (3) the distance between two points is less than or equal to the sum of the distances from *A* to *B* via an intermediate point *C*. In actuality these commonsense interpretations do not always hold in geographic space. Two basic reasons are: (1) the notion of distance as a measure of effort and (2) the existence of anisotropic surfaces. An anisotropic surface is one on which the cost of movement from a point is different depending on the direction. For example, in a road network it is often quicker to take a longer route using highways rather than a more direct route involving minor roads. Similarly a road traveler may opt for a circuitous route around a major city, rather than a direct one through it during rush hours.

These implications of the three axioms when considered as a basis for a qualitative definition of distances, however, are the following:

- There must be a zero element or some means of specifying that an object is at a zero distance from itself.
- The definition of the level of distinctions, such as very close or close, must ensure that the proposition Close(A, B) implies and is implied by Close(B, A).
- The triangle inequality requires a careful definition of the qualitative addition or composition operation in order to prevent inconsistencies.

For example, the propositions Far(A, C), Close(A, B), and Close(B, C) can be simultaneously true if and only if the addition Close + Close = Far, and are inconsistent if Close + Close = Close. This aspect of the definition of a system of qualitative distances will be considered in greater detail in Chapter 4.

Since the number of distinctions made using a qualitative system is very small, each qualitative distance must correspond to a range of quantitative distances specified by an interval. The qualitative distances should also be ordered so that comparisons are possible. Finally the lengths of the interval defining each successive qualitative distance should be greater than or equal to the length of the previous one.

Two groups have proposed a fairly similar qualitative model for distances that supports qualitative spatial reasoning (Hernández *et al.* 1995; Hong *et al.* 1995). The essential conception in both proposals is that in accordance with a cognitive view of distance finer distinctions are made near the reference point and coarser ones further away. Figure 2.9 illustrates a partition of the space surrounding a reference point into five distance distinctions.

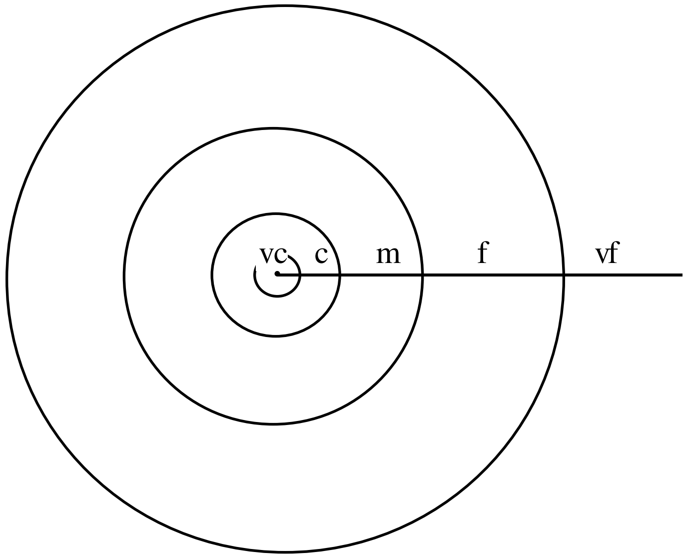


Figure 2.9 Qualitative distances.

For a formal definition of qualitative distances let P be a set of points and D = $\{dist_0, \ldots, dist_n\}$ be n + 1 distance intervals. A qualitative distance between a primary point P_1 and a reference point P_0 is a function d : P x P $\rightarrow$ D, which identifies the qualitative distance, or symbol, associated with the distance from P_0 to P_1. The distance symbols are totally ordered, that is, $dist_0 < dist_1 < \ldots < dist_n$. From the ordering of symbols, the successor and predecessor of a symbol can be defined: $Succ(dist_i) = dist_{i+1}$ for $i < n$ and $succ(dist_n) = dist_n$. Similarly, $pred(dist_i) = dist_{i-1}$, for $i > 1$, and $pred(dist_1) = dist_1$.

Each distance symbol has an acceptance area associated with it. This area specifies the region surrounding the reference point *A*, such that for all points *B* within that region the distance d_{AB} maps onto the same distance symbol $dist_i$. For an isotropic space the acceptance areas are annular regions and hence can be specified by a set of non-overlapping intervals, $\{I_0, I_1, \ldots, I_n\}$, which form a complete partition of the space.

The intervals have a well defined comparative relationship between successor and predecessor since they characterize orders of magnitude. The two conditions that determine the relationship between intervals are:

Monotonicity. The lengths of intervals increase monotonically, that is, $I_0 \leq I_1 \leq \ldots \leq I_n$.

Range restriction. Let S_i denote the sum of lengths of all preceding intervals, $S_i = \sum_{j=0}^{i} I_j$, then $I_i \geq S_{i-1}$ (Hernández *et al.* 1995). The length of interval *i* is greater than or equal to the sum of the lengths of preceding intervals. Hong (1995) proposed the more restrictive requirement based on a series of systematic tests and simulation experiments with increasing integral ratios between successive intervals, which showed that composition results were consistent geometric computations if the ratio of successive interval lengths was greater than or equal to three.

These formal properties of the distance system are independent of the level of distinctions made and of the actual lengths of the intervals. The granularity of the distinctions and the mapping between quantitative intervals and qualitative distances is often determined by the scale and nature of the application. For example, downtown Bangor is far from the UMaine campus by bicycle, but close by car.

For the purpose of this study we assume that (1) the space is isotropic, (2) the number of distinctions and their symbols is predetermined, and (3) the scale is fixed.

2.3 Methods of Spatial Reasoning

Methods for spatial reasoning were motivated by developments in image information systems, robotics, artificial intelligence, and geographic information systems. The primary goal of each proposed method is a symbolic representation of the original image or data, which captures sufficient information to facilitate spatial reasoning tasks such as path finding, similarity retrieval, inference of spatial relations, and consistency checking. The various proposals can be broadly classified as being based on projections, combinations of orientation and topology or distance, and order relations.

2.3.1 Projection–Based Methods

The projection–based methods represent spatial relationships using n independent dimensions of the embedding space. Spatial reasoning tasks are carried out in each of the n dimensions and the final result is a conjunction of n results. These approaches are all enhancements or variations of the basic concepts developed in Allen's (1983) work on reasoning about temporal intervals. Techniques developed for temporal reasoning are applicable to individual dimensions in the spatial domain since time and one-dimensional space share the properties of order and continuity (Valdés-Pérez 1986).

2.3.1.1 Interval Relations

Based on the observation that human knowledge of time is most often concerned with relative comparisons, Allen developed a temporal interval logic for qualitative reasoning in the temporal domain. An event has a duration, which is represented as a 1-dimensional interval. Using an exhaustive analysis of the relations between the start and end points of two intervals, Allen identified thirteen mutually exclusive relations between two intervals, which form a complete set. Figure 2.10 shows the geometric interpretation of the interval relations for one-dimensional objects A and B. The transitivity law holds for all of these relations with the exception of *overlap*, *overlapped-by, meets*, and *met-by.* A 13×13 transitivity table, defined using the transitivity property and a detailed analysis of the possibilities, lists the result of the composition of any two interval relations. The composition of interval relations determines the relation, or set of possible relations, between intervals A and C based on the knowledge of the relations between A and B, and B and C. For example, *A meets B*; *B contains C* $\Rightarrow$ *A before C*.

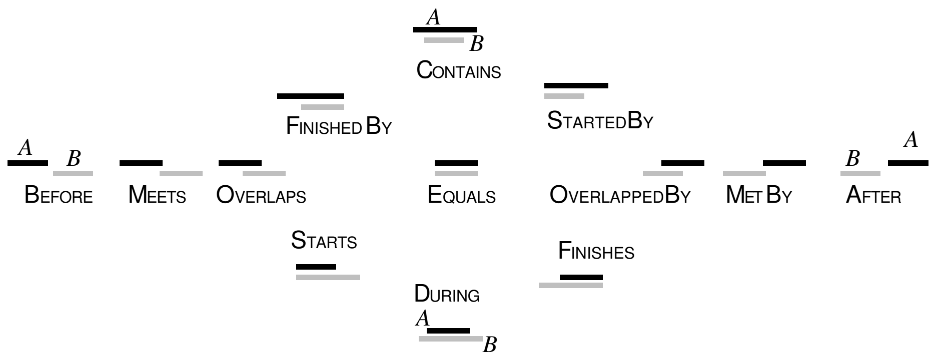


Figure 2.10 Allen’s interval relations for 1-dimensional intervals.

The observation that Allen's interval relations are essentially topological relations in 1-dimension enhanced by the distinction of the order of the space, leads to the possibility of using them as a canonical model for performing heterogeneous reasoning about direction and topological spatial relations between extended objects (Chapter 4).

2.3.1.2 Order Relations

Order relations are reflexive, antisymmetric, and transitive. A set S with a binary relation ≤ is a partially ordered set if and only for every *x, y, z* in S the following conditions are satisfied:

$$\forall x \in S: x \leq x \qquad \text{(reflexivity)}$$

$$x \leq y \wedge y \leq x \Rightarrow x = y \qquad \text{(antisymmetry)}$$

$$x \leq y \wedge y \leq z \Rightarrow x \leq z \qquad \text{(transitivity)}$$

The relation ≤ can be interpreted as “is contained in” and the inverse relation ≥ as “contains.” Two useful concepts, the greatest lower bound and the least upper bound, can be defined for such sets. A *greatest lower bound* is the largest element in the set that is contained in both the elements *x* and *y*. The *least upper bound* is the smallest element in S that contains both *x* and *y*. Adding this refinement gives a *lattice*, which is a partially ordered set such that for every pair of elements, *x* and *y*, there exists a greatest lower bound and a least upper bound. The concepts of *inclusion* and *containment* can hence be represented using partially ordered sets and lattices (Pullar and Egenhofer 1988; Kainz 1990; Kainz *et al.* 1993), thereby providing the capability of answering questions such as:

• Which regions are contained in region *A*?

• Does region *B* contain region *A*?

• Which is the largest region contained in a given set of regions?

• Which is the smallest region that contains a given set of regions?

These queries can be answered without recourse to geometric computations by simply finding the lower bounds of *A*, the greatest lower bound of *A* and *B*, and the least upper bound.

2.3.1.3 Interval Relation Based Definition of 2-Dimensional Spatial Relations

Guesgen (1989) used a straightforward extension of Allen's work for reasoning in two or more dimensions, by partitioning the relationships between objects into components along the axes for each dimension. Allen's interval logic can be applied for relations on individual axes resulting in a tuple of relations. Guesgen did not use all thirteen interval relations defined by Allen, rather he defined eight relations—*left, attached to, overlapping, inside*, and their converse relations—based on combinations of Allen's relations. A composition table for the eight relations is also defined based on Allen's transitivity tables for intervals. The process of performing inferences for each axis independently and subsequently combining the results to form a tuple leads to a combinatorial explosion of possibilities. The other major concern is that while topological relations disjoint and overlap can be represented easily, defining and representing orientation relations is cumbersome.

2.3.1.4 Projection and Order–Based Definition of Spatial Relations

Encoding an *n*–dimensional scene as sets of relations between the projections of the objects onto each axis was effectively used by Chang *et al*. (1988) to represent an image as a symbolic picture. The symbolic picture consists of a grid whose elements are filled by objects or empty space. The grid is created by partitioning the embedding space by lines tangent to extreme points of the objects and parallel to one of the axes. The extreme points are either concave or convex object-points. Once this cutting has been performed a 2-dimensional image, for example, becomes a gridded picture made up of vertical and horizontal strips. The strips have a left-right bottom-up order and represent sets of location along each axis. Thus objects or object segments that lie in each strip may either share an edge or a set of locations. Another possibility is that one object's extent ends within one strip and the next object's extent begins within some subsequent strip. These situations are captured by three operators {<, =, |} where "<" is the left-right or below-above relation, "=" stands for the relation "in the same location", and "|" represents the fact that objects are edge-to-edge. Using object names, the three operators, and the cutting mechanism, a 2-dimensional symbolic picture is encoded into two strings, *u* and *v*, known as 2D-G strings. The relations along the horizontal axis are represented by the *u* string and the vertical axis relations are represented by the *v* string. The *u*- and *v*-strings for the 2-D picture with cutting lines shown in Figure 2.11 are:

u: A|B=A|B<C<D

v:D|B=D|B<C|A=C|A

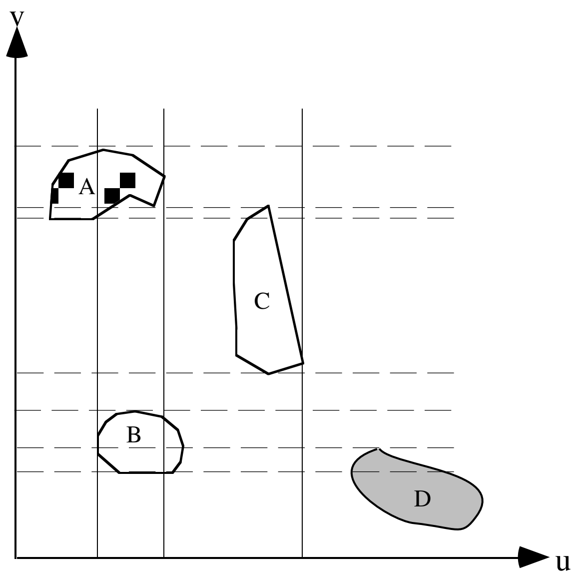


Figure 2.11 Cutting lines for creating a 2D-G string symbolic representation.

In addition to the three operators the separators, “(” and “)” are used for compound objects or to group objects that form a local composite object. For example, the *u*-string A(B|C) implies that the compound object BC is at the same location as A on the x-axis.

Taken together the strings encode the spatial arrangement of objects and are thus a symbolic index useful for similarity retrieval. Finding images that match a specified description of spatial relations becomes a matter of coding that description as 2D-G strings and performing substring matches on the symbolic index of images.

Symbolic projections and 2D strings, have been used for more than just symbolic indexing of images in information systems. Holmes and Jungert (1992) demonstrate their use for navigation or finding the shortest path in digitized maps. Chang and Jungert (1990) show how symbolic projections can derive directional relations, such as *North* or *Northwest*, between objects from the 2D strings for an image. They also describe how 2D strings can be used to retrieve images containing objects that satisfy specified spatial conditions, for example, “hiking trails crossing streams,” or in query processing. Examples of queries answerable using 2D string descriptors for a picture P are; “find all objects South of object *B*,” “what is the spatial relation between *A* and *C,*” and “which objects are adjacent to object *X*.”

An improvement on 2D-G strings using fewer cutting lines is possible if Allen’s interval relations are used in place of the three operators {<, |, =}. Maintaining the left-right bottom-up order the 2D-C string (Lee and Hsu 1990) records the relations between the extents of pairs of objects along each dimension. The directional relations *North*, *South*, *East*, *West* and the topological relations *disjoint*, *overlap*, *meet*, *contains*, and *inside* can be defined from 2D-C strings. Since there are thirteen interval relations in one dimension there are 169 possible relations between rectangles. The limitation of using bounding rectangles for defining the topological relation is that situations can occur when the relation between MBRs differs from the relation between the objects.

The 2D Projection Interval Relationship (2D-PIR) (Nabil *et al.* 1995) is one solution for this problem. For all object pairs in the picture the topological relation as well as the interval relations along each axis are computed and stored. The representation scheme is a labeled connected digraph, whose nodes correspond to objects and its edges are labeled with the 2D-PIR for that object pair. This representation is particularly suited for exact or similarity retrieval of pictures, because identifying pictures or subpictures that match a given one is done by testing graph isomorphism. In general detecting graph isomorphism is NP-Hard (Garey and Johnson 1979), but for this specific problem the complexity is reduced due to the graph construction process.

2.3.2 Symbolic Arrays

Symbolic arrays, like symbolic projections, are a means for qualitative representation of spatial information (Glasgow and Papadias 1992; Papadias and Sellis 1994).

Representative points are used to identify or distinguish the objects in the original image and these points occupy distinct cells in the symbolic arrays. The arrays, therefore, capture the topological and orientation information but may exclude details like relative distances, sizes, and shapes. Symbolic arrays can also represent hierarchies or part_of relationships since they can be nested with each nested level containing finer details (Figure 2.12).

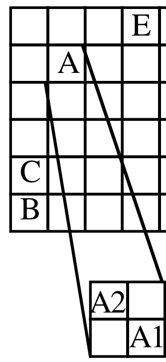


Figure 2.12 Symbolic array representation of a scene (Papadias and Sellis 1994).

Directional information is easily obtained by using a projection-based model for directions. Topological information such as adjacency and neighborhood are easily represented by single points representative per object, whereas containment requires at least two points per object since the spatial extents of the objects must be represented. Papadias (1994) uses the projection-based model for directions, the 4-intersection model for binary topological relations between regions, and symbolic arrays as the basis of a pictorial query-by-example language (Papadias and Sellis 1995). Topological information is encoded by marking arrays cells whenever an intersection of the boundary or interior of two objects occurs or when the bottom-left and top-right corners of the object's bounding rectangle are encountered.

Multidimensional symbolic arrays are particularly useful for computational models of mental imagery that humans use for spatial reasoning (Farah 1988). Mental imagery provides: (1) an implicit encoding of the relations among objects; (2) a preservation of the spatial relations between objects; and (3) a structural equivalence between the perceived scene and the working memory representation of the scene (Finke 1989). Symbolic arrays provide a formal meta-language for mental imagery since they are based on array theory (Jenkins and Glasgow 1989), which is the mathematics of nested collections of data objects that have spatial locations relative to each other. Thus the spatial structure of an image is represented by a multidimensional symbolic array whose elements denote the meaningful parts of the image and implicitly encodes their spatial relationships. These relationships can be interpreted directly from the array representation and the semantics depends on the domain of the application. For example, if the arrays represent mental maps of geographic space then we use cardinal directions, whereas if the mental map is of the layout of a room then we use orientation relations such as *left*, *right*, *front*, or *back*. Since the relations are interpreted, rather than logically inferred from the array representation, updates are handled easily. A change in an object's location requires only a modification of that object's representative point or points in the array. The array representation has the additional advantage of allowing the extraction of propositional information, such as left_of(*A, B*), and the creation of symbolic arrays from propositional information. Inference of spatial relations between objects in different symbolic that have at least one object in common is done using composition tables in manner similar to propositional or predicate representations (Papadias and Sellis 1992). However, symbolic arrays do not have the expressive power of first-order logic since it is not possible to express quantification or disjunction. For example, it is impossible to represent a configuration in which object *A* is either *North* or *South* of object *B*.

2.3.3 Combining Orientation and Topology

Recognizing that topological and orientation relations both serve to identify regions of space in which the primary object may exist and hence together the relations further limit the extent of this region, Hernández (1994) developed a formalism for representing combined information about the topological and orientation relations between objects.

Hernández's work is concerned with building a cognitive model of physical space that is constructed using abstract mathematical structures and incorporates humans' perceptions and descriptions of space. The abstract mathematical structures used are graphs and lattices. Graphs capture the relationships between the objects, whereas lattices capture the partial ordering or hierarchy of the relations involved. Human perception and description is incorporated by using qualitative spatial information.

The innovation lies in the concept of neighborhood of the relations and the use of this structure in the inference mechanism. The composition table reflects the neighborhood structure, because the result of a composition of relations that are not immediate neighbors will contain the relations that are on the shortest path between the two. The neighborhood structure of the relations results from the properties of the physical space in which the objects are placed. Since the space is continuous, changes in an object's position are also continuous rather than discrete. If object *A*'s position changes from location X to location Y the locations must be adjacent or a connected path from X to Y must exist. This connectivity of locations implies that spatial relations also have a neighborhood structure. The neighborhood structure implies if one of two *disjoint* objects is moved towards the other then they will *meet* before they *overlap*. Similarly assume there is a fixed viewpoint and reference object *A*. If the primary object *B*, is initially at the *back* of *A*, then if *B* is moved such that it is currently on the *right* of *A* then at some intermediate points *B* must have been at the *right-back* of *A*. Figure 2.13 shows the combined neighborhood structure of topological and orientation relations.

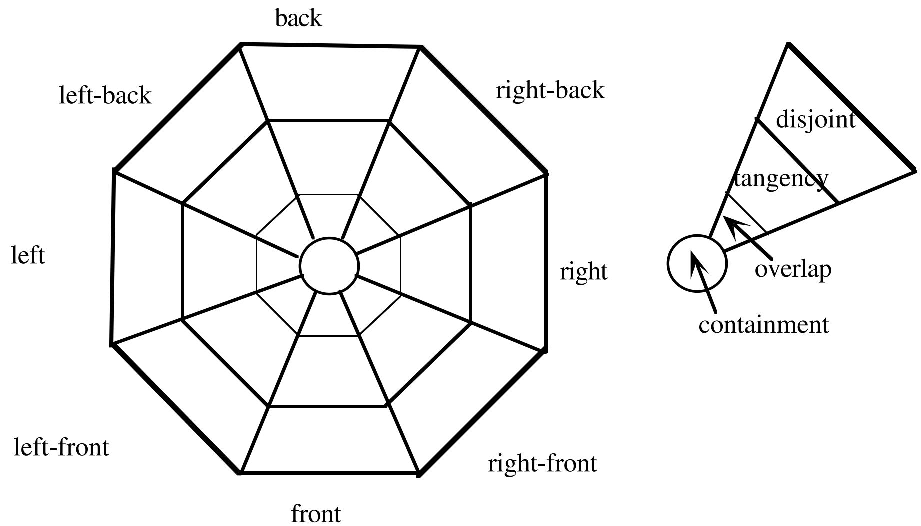


Figure 2.13 Combined neighborhood structure of orientation and topological relations (Hernández 1994).

The only topological relations that are enhanced by orientation information are *disjoint*, *meet*, and *overlap*. However the locational uncertainty for the relation disjoint is still quite large and hence a logical step would be to consider delineating the probable locations by taking the distance between objects into account.

2.3.4 Combining Orientation and Distance

For objects that are disjoint, the addition of qualitative information about distances gives further reasoning capabilities (Frank 1992; Hernández *et al.* 1995; Hong 1994). In many instances inferences can be made regarding the direction and qualitative distance between two objects given their relationships with a common third object. For example, the mere knowledge that *A* is *North* of *B* and *B* is *South* of *C* does not provide any information on the relation between *A* and *C*; however, adding the facts that *A* is *Close* to *B* and *B* is *Far* from *C* allows one to infer that *A* is *South* of *C*.

Figure 2.14 illustrates the delineation of the space surrounding an object into sectors bounded by orientation lines and distance intervals. Assuming *A*, *B*, and *C* are points, the validity of the inference that *A* is *South* of *C* is evident from the figure. The dotted lines indicate the demarcation of the space around object *A*, while the solid lines indicate the same for object *B*. Object *C* could lie anywhere in the sector (f, N), i.e., *far* and *North* of, for *B* and hence *C* must be *North* of *A*; and *C* could be either *far* or at a *medium* distance from *A*.

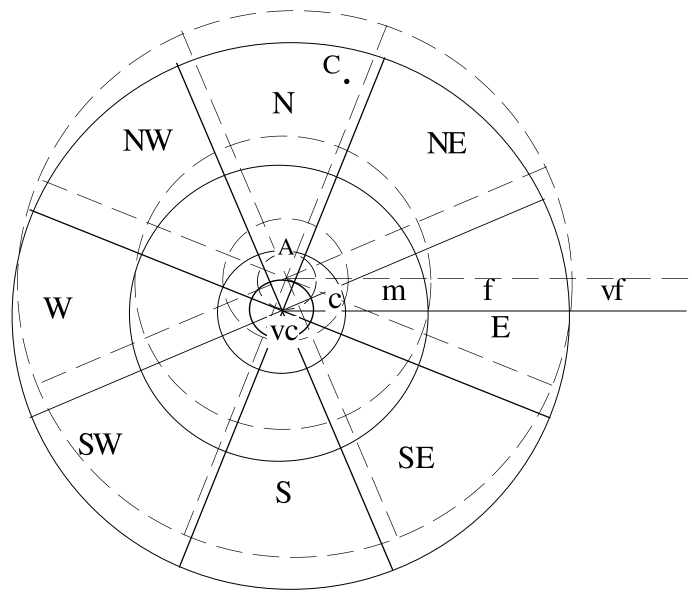


Figure 2.14 Qualitative distances and directions.

The monotonicity and range restriction properties of intervals denoting qualitative distance determine the lower and upper interval bounds of the vector addition of two distances and therefore the result of the composition of two qualitative distances.

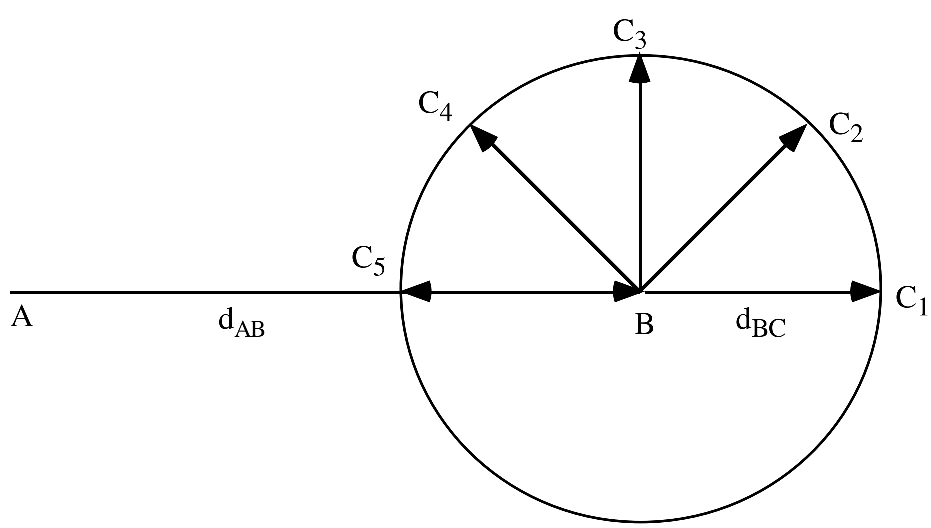


Figure 2.15 Effect of orientation on addition of distances.

The effect of orientation is largely to limit the set of possibilities (Figure 2.15). If the two distances have the same orientation—AB and BC_1 in Figure 2.15—then the composition result is the upper bound. On the other hand, if the orientations are opposite—AB and BC_5—the composed distance is the lower bound. For any other orientation—BC_2, BC_3, or BC_4—the result is between the lower and upper bounds. An exhaustive and systematic analysis of various possibilities results in a composition table for qualitative distances. Using the composition table one can draw such inferences as *A* {*far*, *medium*} *C* from the facts *A North B*, *B South C*, and *B far* from *C*.

2.4 Summary

The various proposals for defining and modeling spatial relationships take one of two approaches. The first is choosing a representation for space and objects within it. Relationships among objects are then defined in terms of the particular representational method such as the 2-D C-string. The second approach is identifying what spatial relationships can exist and how they could be characterized. The definition of spatial relationships depends on the nature of the objects under consideration, but not on the representation of space. One example is the point-set based definition of binary topological relations. We use the latter approach. The following chapter describes how spatial inferences may be performed within each model for qualitative distances and topological and directional spatial relations.

Chapter 3

Homogeneous Spatial Reasoning

High-level spatial information about a scene or collection of spatial objects can be described in terms of a set of qualitative spatial relations between the objects, where the spatial relations are binary predicates, that is, each relation holds between two objects. Unlike a graphical representation that is complete and consistent within itself, a set of qualitative spatial relations may be incomplete and even inconsistent. The consistent integration of such spatial information relies on the algebraic properties of the qualitative spatial relations. Properties such as the converseness of pairs of relations and the composition of relations must be fulfilled, for any combination of relations, in order to guarantee that the description is free of internal contradictions so that a physical realization is possible.

While consistency is implied in any graphical rendering it is difficult to evaluate or enforce it for any symbolic representation. We are primarily interested in whether a specified set of binary spatial relations giving a symbolic description of a scene, is consistent or not. Solutions to this problem have been applied to various aspects of topological spatial reasoning. Some of these are: (1) identifying whether or not a set of topological relations is sufficient to describe a scene uniquely (Egenhofer and Sharma 1992); (2) comparing two sub-scenes, or multiple representations of the same scene, for the same topology (Egenhofer *et al.* 1994); and (3) finding efficient strategies to execute queries over multiple topological constraints (Egenhofer and Sharma 1993a; Clementini *et al.* 1994). This chapter presents the basis of a rigorous computational method, i.e., the composition tables, designed to facilitate reasoning about qualitative spatial relations and to evaluate the consistency of such spatial information.

3.1 Types of Spatial Relations and Their Compositions

Qualitative spatial relations have been the subject of extensive research over the past few years and numerous formalisms and prototype systems exist (Dutta 1989; Egenhofer, 1989; Mukerjee and Joe 1990; Freksa 1992b; Cui *et al.* 1993; Hernández 1993; Papadias and Sellis 1993). These formalisms could be substituted or complemented by any other formalization of spatial relations that follows the guidelines of a relation algebra (Tarski 1941).

This thesis is concerned with three types of qualitative spatial relations namely, topological relations, directional relations, and qualitative distances. The remaining sections describe formalisms for each one of these types of spatial relations and their composition tables.

3.2 Formalisms for Topological Relations

The relation algebra for topological relations is based on the 9-intersection, which analyzes the intersections of the two objects' interiors ($^\circ$), boundaries (∂), and exteriors ($^-$) (Egenhofer and Herring 1990; Egenhofer and Franzosa 1991). We illustrate the use of this algebra with the theory developed for region objects (Figure 2.1, in Section 2.2.1). The same method is applicable to topological relations between any combination of regions, lines, and points (Egenhofer and Herring 1991).

The basis of the reasoning mechanism is the *composition* of topological relations, which is defined by the composition table. The composition table was formally derived based on the transitivity of empty and non-empty intersections between the object parts and gives the results of the 64 compositions of the 8 basic region-region relations

(Egenhofer 1994). Each object has three parts (interior, boundary, and exterior) which have a particular relationship with the three parts of another object. Hence a total of nine intersections between object parts exist and these characterize the topological relationship between the objects. One of the nine intersections, the exterior-exterior, is always non-empty since by definition region objects cannot be half-planes, therefore, only eight intersections are relevant in encoding the information about the topological relation. The eight intersections are set intersections, as the object parts are point-sets, and hence the transitivity properties of set relations can be used to derive the composition of binary topological relations.

Given three region objects A, B, and C, and the 9-intersection for the pairs $\{A, B\}$ and $\{B, C\}$ the 9-intersection for the pair $\{A, C\}$ is derived using rules determined by the values of the set intersections and subset relations between their parts. The symbols A_i, A_j, A_k, B_l, B_m, B_n, C_o, C_p, C_q represent pairwise disjoint point-sets that are components of objects A, B, and C and their complements. Similarly A_x, B_y, and C_z denote any one of the three components. The symbols A', B', and C' denote subsets of the union of objects A, B, and C and their complements, respectively. Thus, C' could represent the complement of C, or union of the boundary and the interior of C, or the whole space, that is, the union of C and its complement. A special subset relation is required for specifying the general rules for deriving the intersections between components of objects A and C. The special subset relation, denoted by p, occurs when one set is a subset of the union of one or more sets and has a non-empty intersection with each one of them. For instance, when $A_i \subseteq (B_l \cup B_m)$ and $A_i \cap B_l = \neg\varnothing$ and $A_i \cap B_m = \neg\varnothing$.

The first rule deals with the propagation of non-empty intersections such that A_i intersects with at least one of the set of components of C that constitute C' (Equation 3.1).

- If $A_i \cap B_l = \neg\varnothing$ and B_l p C' then $A_i \cap C' = \neg\varnothing$ (3.1)

The second rule deals with the propagation of non-empty intersections (Equation 3.2). It holds only under the additional constraints that (1) C' is the union of at most two of the three parts (interior, boundary, exterior) of C and (2) the object part A_i is a subset of the object part B_l such that A_i does not intersect with any of the set of components of C that do not constitute C'.

- If $A_i \cap B_l = \neg\varnothing$ and B_l p C' then $A_i \cap \left(C \cup \overline{C} - C' \right) = \varnothing$ (3.2)

The two rules must be applied to the components of both objects A and C and their relationships with the components of the common object B. Translating the set intersection and subset relations into an equivalent 9-intersection form, and the rules, allow for the derivation of eight relevant object part intersections that determine the topological relation between two objects. The ninth object part intersection is always non-empty since by definition no pair of objects constitute the whole space, therefore, the two exteriors must intersect.

The composition of two topological relations, denoted by $r_1 ; r_2$, could result in a single topological relation, e.g., *meet* ; *contains* = *disjoint*, or more than one, e.g., *contains* ; *meet* = *contains* ∨ *covers* ∨ *overlap*. Table 3.1 gives the 64 compositions of the binary topological relations $A\ r_i\ B$ and $B\ r_j\ C$. It uses iconic representations (Figure 3.1), which are based on the conceptual neighborhood of binary topological relations (Egenhofer and Al-Taha, 1992).

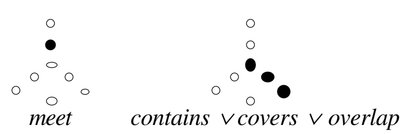


Figure 3.1 Iconic representation of binary topological relations.

The topmost circle, if filled in, represents the relation *disjoint*, the next is *meet*, followed by *overlap* and so on. The disjunction of topological relations is denoted by filling more than one circle in the icon.

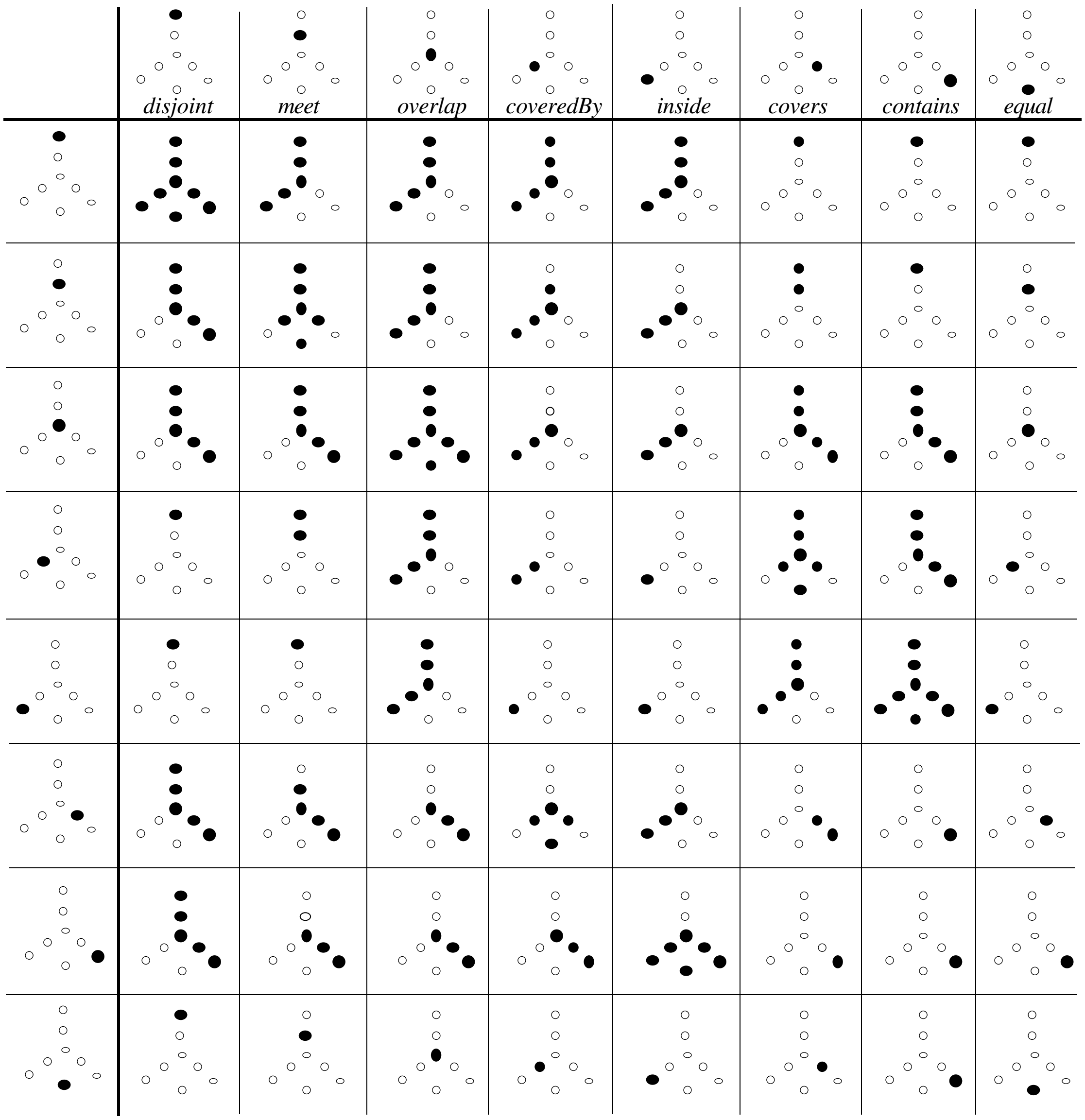


Table 3.1 The composition table for binary topological relations between regions (Egenhofer 1994).

We use the 9-intersection formalism and hence Table 3.1 for homogeneous spatial reasoning about topological relations in this thesis.

3.3 Formalisms for Cardinal Directions

The prototypical concept of cardinal directions comes from the compass, from which the idea of cone-shaped areas each associated with a specific direction has been derived (Figure 2.4a). An equally useful construction is based on projections (Figure 2.4b), where the directions are defined by half-planes. Similar to topological relations as formalized by the 9-intersection, one can construct relation algebras for the different models of cardinal directions (Frank 1992).

The composition of cardinal directions is based on an algebra for paths and a mapping from paths onto directions (Frank 1992). A path is a directed edge from a start point P_1 to an end point P_2. Properties of paths, such as associativity, and operations, such as inverse and addition, are used to determine the composition rules for directions. The inverse of a path from P_1 to P_2 is the path from P_2 to P_1. Composition combines two paths from P_1 to P_2 and from P_2 to P_3 resulting in a path from P_1 to P_3. The result of a composition is thus a single well-defined path. The identity element for paths is the special path from a point to itself and is denoted by the symbol 0_p. These operations are correspondingly defined for cardinal directions.

The inverse of a direction from point P_1 to P_2 is the direction from P_2 to P_1 and must be defined for all qualitative direction symbols. The composition of two directions follows from the composition of paths by defining a mapping, denoted by `dir`, from paths onto directions. Let $;_p$ denoted path composition and $;_c$ denote composition of cardinal directions and p_1, p_2 denote two paths, then the mapping `dir` distributes over the composition (Equation 3.3).

$$\mathtt{dir}(p_1 ;_p p_2) = \mathtt{dir}(p_1) ;_c \mathtt{dir}(p_2) \tag{3.3}$$

The identity element for cardinal directions, denoted by 0_c, is the direction from a point to itself and $\mathtt{dir}(0_p) = 0_c$.

The set of symbols, C_n, used for qualitative directions depends on the specific model used, for example, using two half-planes gives the set $C_4 = \{N, E, S, W\}$ whereas using a cone-shaped model with eight directions gives $C_8 = \{N, NE, E, SE, S, SW, W, NW\}$. The number of symbols is always finite, they are cyclically ordered, and can be mapped onto integers modulo n. The cyclic ordering means that an anti-clockwise turn of $2\pi/n$, denoted as $turn^{(1)}$, in a path's direction results in path of the successor direction, e.g., $turn^{(1)}(N) = E$ for a four direction system. Since each direction symbol corresponds to an acceptance area rather than a specific path or set of paths, Frank (1992) defines two averaging rules that allow for the composition of directions that are one or more turns off from each other. The averaging rules determine which among a set of possibilities is selected as the result of the composition and this result is termed as being *Euclidean approximate* (Frank 1992). The Euclidean approximate results are sufficient if all that is required is a single-step inference result. If chain-reasoning, that is, a sequence of inferences with the result of one inference being the input to the next inference step, is required then it is necessary to use composition tables with disjunctions of base directional relations (Table 3.2) or tables with directional relations at different levels of resolution (see Papadias and Sellis (1994) for a definition of composition of high and low-resolution directional relations).

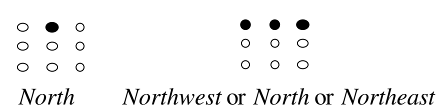


Figure 3.2 Iconic representation of directional relations.

The iconic representation (Figure 3.2) used in Table 3.2 should be interpreted as follows. Filled-in circles represent the directional relation or relations. Thus a filled-in top center circle represents *North*, while a filled-in top right circle represents *Northeast* and so on. If more than one circle is filled-in then that icon represents a disjunction of relations. This occurs, for instance, in the composition of *North* with *North* where the icon entry in the first row and column of the table has the top three circles filled-in and represents the relation *Northwest* or *North* or *Northeast*. If all circles are filled-in then that icon represents the universal relation, that is, the directional relation can be any one from the complete set of possible directional relations.

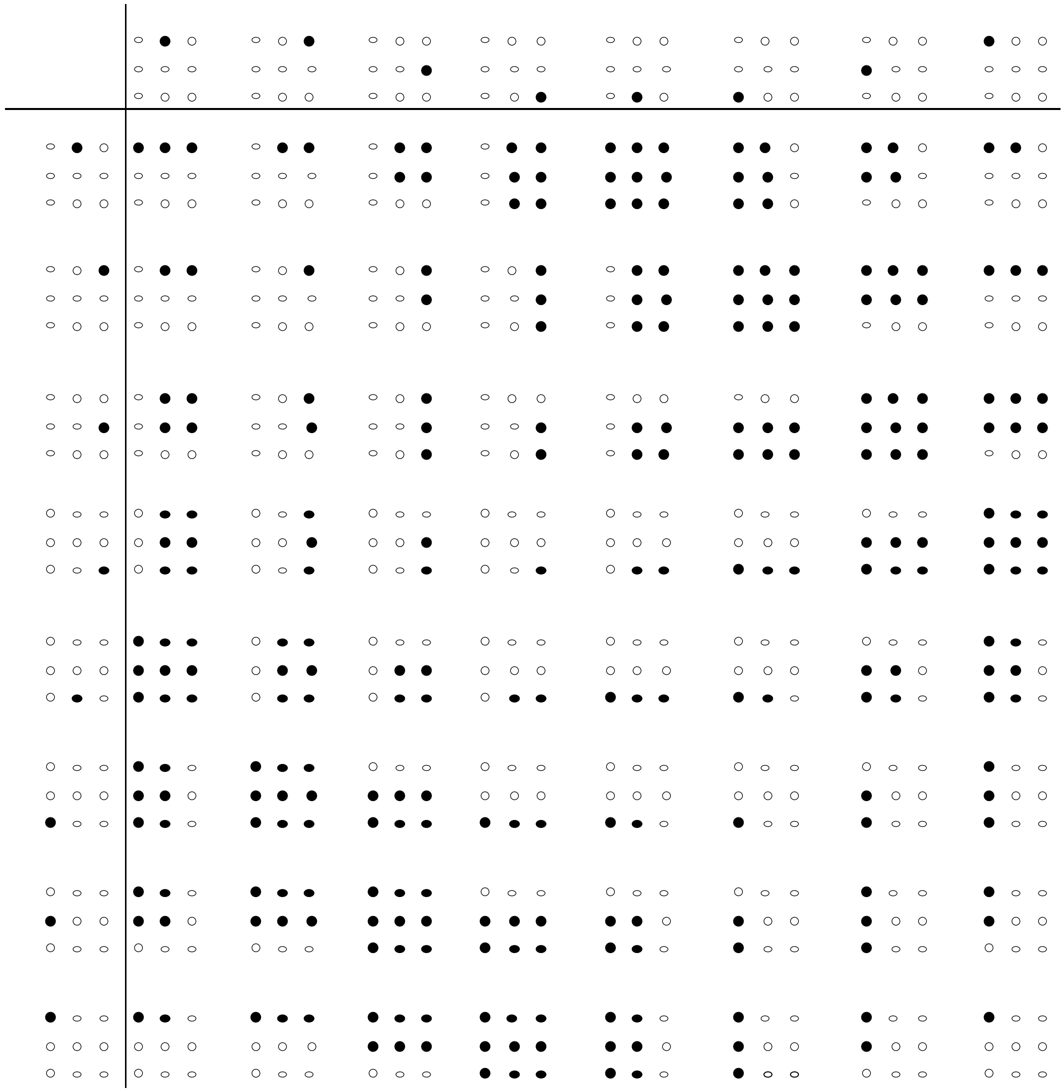


Table 3.2 Composition of directional relations between two regions.

The above table was derived using the projection-based system of directions for extended objects. The North-South and East-West extents of the objects are projected onto each axis and these projections recorded as interval relations between the two objects along each axis. The composition of interval relations is then used to determine the composition of the directional relations.

We use compositions that result in disjunctions of base relations in this thesis and therefore Table 3.2 for composition of directional relations.

3.4 Formalisms for Approximate Distances

Distances are quantitative values determined through measurements or calculated from known coordinates of the two objects in some reference system. People, however, frequently use approximations and qualitative notions such as near or far when reasoning about distances. Such qualitative distances are defined by a set of symbols, denoting qualitative measures such as *near* or *far*, and addition rules—sometimes in combinations with direction reasoning—such as *near* plus *far* is *far* (Frank 1992). Approximate distances have been mapped onto quantitative distances using fuzzy sets (Dutta 1989) or mutually exclusive distance intervals of increasing ratio (Hong 1994). Reasoning with approximate distances, however, is either necessarily *Euclidean approximate* or involves disjunctions of relations, except in the cases when the distances have the same or opposite orientations.

Frank (1992) and Hong (1994) derived the composition tables for approximate distances for the situations based on various definitions of distance systems. Composition in this case is addition of approximate distances. The composition table is derived by mapping the approximate distances onto a set of intervals that partition the real number line. The four approximate distances *vc*, *c*, *f*, and *F* correspond to the four half-open intervals [0, 1), [1, 4), [4, 13), and [13, ∞), respectively. Composition of approximate distances is done by performing an interval addition and mapping the mid-point of the resultant interval back onto an approximate distance.

Adding approximate distances using interval addition and subsequent mapping of the mid-point of the resultant interval is valid only for the addition of two distances. For example, the addition *vc* + *vc* + *vc* should result in *c* since the resultant interval is [0, 3). If interval arithmetic is not used for each addition, however, we get the result *vc* since the additions performed are:

$vc + vc + vc = (vc + vc) + vc = vc + vc = vc$

Interval addition also fails to take into account the fact that the two distances may be collinear and of opposite orientation. In such as case a distance subtraction rather than a distance addition must be performed. Hong's method for reasoning about qualitative distance and direction (Hong 1994) gives the composition table for qualitative distances, with direction information being disregarded, that contains disjunctions of qualitative distances as its entries (Table 3.3).

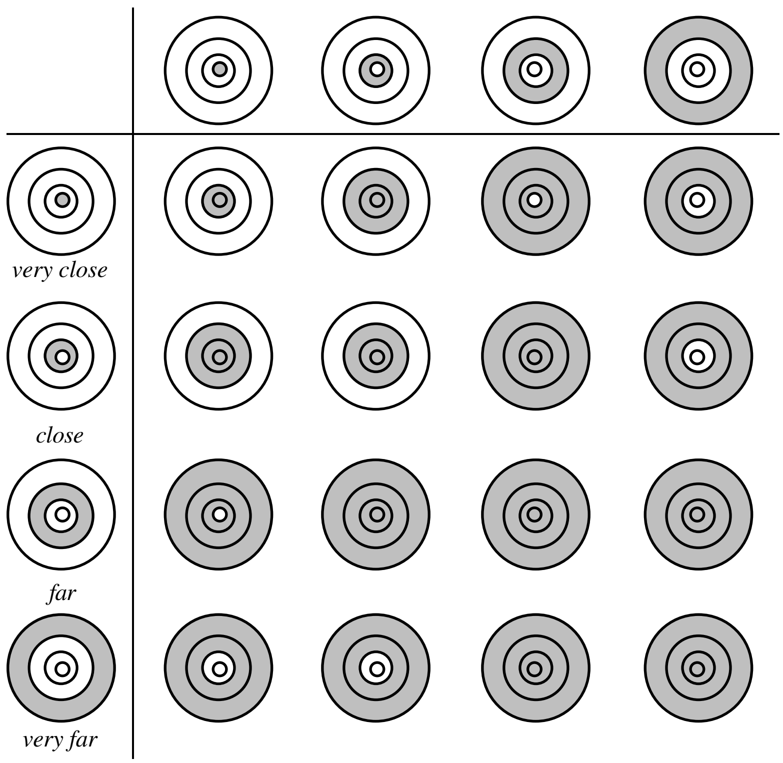


Table 3.3 Addition of approximate distances (adapted from Hong 1994).

Table 3.3 was derived using vector addition of distances and mapping the set of possible results onto qualitative distance symbols. In general, the result of adding two distances is essentially the result of adding two vectors (Figure 2.18). Vector addition is dependent on orientation, therefore, any relatively robust addition system for approximate distances must take orientation into account. For this reason we need an *integrated* qualitative distance and direction reasoning system (Hong, 1994).

3.5 Examples of Homogeneous Spatial Reasoning

The composition tables for spatial relations facilitate reasoning with incomplete spatial information by allowing the inference of new information. We illustrate the use of composition tables for assessing the consistency of complete and possibly incomplete spatial information about a scene. For a specified scene description the problem is determining if every given fact is consistent with inferred facts.

The example used in this section is for binary topological relations. A similar approach can be applied to reasoning about directions only (Hernandez, 1994; Frank, 1995) or approximate distances only (Frank, 1992). The composition table for binary topological relations enables the evaluation of topological consistency and inference of relations by expressing them as constraint satisfaction problems (Egenhofer and Sharma 1992; Smith and Park 1992; Egenhofer and Sharma 1993a). Each given topological relation between two objects *A* and *B*, is considered a constraint $t_i(A, B)$. Similarly the composition of two relations, $t_i(A, B)$ and $t_j(B, C)$, results in a set of constraints on objects *A* and *C*. The constraints set forth by all specified relations and possible compositions describe the topological consistency of a scene. That is, for any pair of objects *A* and *B*, the specified relation must exist in the set of relations inferred using all possible compositions.

Among n spatial object there are n^2 binary topological spatial relations. These can be represented by an $n \times n$ matrix called the *connectivity matrix*, in which each element t_{ij} represents the topological relation between the two objects *I* and *J* (Table 3.4).

	A	*B*	L	*N*
A	t_{11}	t_{12}	L	t_{1n}
B	t_{21}	t_{22}	L	t_{2n}
M	M	M	M	M
N	t_{n1}	t_{n2}	L	t_{nn}

Table 3.4 The connectivity matrix.

For example the topology of the scene in Figure 3.3 can be represented by the connectivity matrix in Table 3.5.

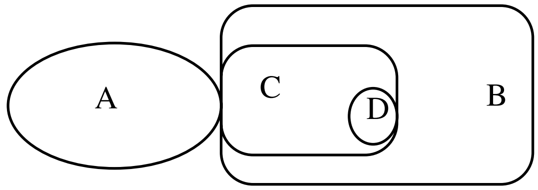


Figure 3.3 An example scene.

	A	*B*	*C*	*D*
A	*equal*	*meet*	*meet*	*disjoint*
B	*meet*	*equal*	*covers*	*contains*
C	*meet*	*coveredBy*	*equal*	*covers*
D	*disjoint*	*inside*	*coveredBy*	*equal*

Table 3.5 The connectivity matrix for the configuration in Figure 3.2.

The advantage of the connectivity matrix, over a graphical representation, is that it allows for the representation of incomplete information. Table 3.6 shows a connectivity matrix for a scene with unknown and incomplete information, denoted by "?" and "OR" respectively.

	A	*B*	*C*	*D*
A	*equal*	*?*	*meet*	*?*
B	*?*	*equal*	*covers OR contains*	*contains*
C	*meet*	*coveredBy*	*equal*	*covers*
D	*?*	*inside OR overlap*	*coveredBy*	*equal*

Table 3.6 An incomplete connectivity matrix.

The connectivity matrix has a certain structure that is determined by the properties converseness and the identity relation of topological relations. The topological relation between every object and itself is *equal*, and is the identity relation (Equation 3.4)

$$\forall_i t_{ii} = equal \tag{3.4}$$

Each of the eight possible topological relations, *t(A, B)*, between regions has a converse relation *t(B, A)* (Equation 3.5a-h).

- $disjoint\ (A, B) \Leftrightarrow disjoint\ (B, A)$ (3.5a)
- $meet\ (A, B) \Leftrightarrow meet\ (B, A)$ (3.5b)
- $equal\ (A, B) \Leftrightarrow equal\ (B, A)$ (3.5c)
- $overlap\ (A, B) \Leftrightarrow overlap\ (B, A)$ (3.5d)
- $inside\ (A, B) \Leftrightarrow contains\ (B, A)$ (3.5e)
- $contains\ (A, B) \Leftrightarrow inside\ (B, A)$ (3.5f)
- $covers\ (A, B) \Leftrightarrow coveredBy\ (B, A)$ (3.5g)
- $coveredBy\ (A, B) \Leftrightarrow covers\ (B, A)$ (3.5h)

These dependencies impose the following structure on the connectivity matrix:

$$\forall_{i,j}\ t_{ij} = converse\left(t_{ji}\right) \tag{3.6}$$

Two further relations, the universal relation U and the empty relation Ø, are introduced in order define additional constraints on the connectivity matrix. The universal relation is the disjunction of all possible topological relations (Equation 3.7) and holds true for the relation between any two objects.

$$\begin{aligned} U &= \cup\, t_i \\ &= disjoint \vee meet \vee equal \vee overlap \vee inside \vee coveredBy \vee covers \vee contains \end{aligned} \tag{3.7}$$

In terms of the connectivity matrix it implies that every t_{ij} must be a subset of the universal relation (Equation 3.8).

$$\forall_{i,j}\ t_{ij} \subseteq U \tag{3.8}$$

The empty relation is introduced to denote a consistency violation. If one or more elements of the connectivity matrix are empty then the set of topological relations for that scene is inconsistent. For consistent scene descriptions no relation may be empty (Equation 3.9).

$$\forall_{i,j}\ t_{ij} \neq \varnothing \tag{3.9}$$

The connectivity matrix can be transformed into a constraint network with the objects being the nodes and the relations between them forming the labeled edges of a directed graph (Montanari 1974; Mackworth 1977). A theorem by Montanari (1974) provides the basis for reducing the problem of evaluating the consistency of the whole network to the intersection of all binary compositions. It states that the network is path-consistent if the composition of all paths of length 2 is consistent and hence no other combinations of paths need be considered. For a network of relations, as in the case of a topological scene description, the theorem implies that the intersection of all pairs of compositions possible between every pair of objects, *I* and *J*, is sufficient to evaluate the consistency of the scene. For each pair the inferred or specified relation is consistent if and only if it is the intersection of all pairs of possible compositions (Equation 3.10).

$$\begin{aligned} t'_{ij} &= \left(t_{i1};t_{1j}\right) \cap \left(t_{i2};t_{2j}\right) \cap \ \ldots \ \cap \left(t_{in};t_{nj}\right) \\ &= \bigcap_{k=1}^{n} t_{ik};t_{kj} \end{aligned} \tag{3.10}$$

This consistency property may be used to derive a unique relation as the set intersection of multiple underdetermined compositions that link the same objects. For example, assume we are given four objects and the four relations *contains* (*A, B*), *meet* (*B, C*), *disjoint* (*A, D*), and *overlap* (*D, C*). The relation between *A* and *C* can be uniquely determined as the set intersection of the two compositions (*contains* ; *meet*) and (*disjoint* ; *overlap*), i.e.,

$$\begin{aligned}(contains; meet) \cap (disjoint; overlap) &= \{contains, covers, overlap\} \cap \\ &\quad \{disjoint, meet, inside, coveredBy, overlap\} \\ &= overlap\end{aligned}$$

If the set intersection of all compositions is empty then the topological constraints that describe the scene are inconsistent. Such a scene has no possible geometric interpretation, i.e., it cannot be realized in $\Re^2$. For example,

$$(inside; inside) \cap (contains; contains) = (inside \cap contains) = \varnothing$$

Evaluation of topological consistency is an extremely useful tool, particularly for spatial query processing where a contradiction in the specified spatial constraints can be detected. Detecting the contradiction saves processing time and computational resources since costly geometric computations can be avoided.

3.6 Limitations of Homogeneous Spatial Reasoning

While homogeneous spatial reasoning is useful it has certain limitations, especially when applied in inference of new spatial relations from a set of base spatial relations. Certain spatial relations, such as qualitative distances and directions, have a dependency that cannot be exploited using homogeneous spatial reasoning. In the case of qualitative distances, the directional relation between the two distances is vital in determining the result of their composition (Hong 1994) since the result of adding distances with the same orientation differs from that for distances of opposite orientation. Thus, reasoning about qualitative distances necessarily involves integrated spatial reasoning about qualitative distances and directions. Similarly, topological and directional relations have a dependency that can be exploited in spatial reasoning. In the case of topological relations, a contained object must necessarily have the same directional relations with other objects as that of the containing object. For instance, if *A* is *inside* *B* and *B* is *North* of *C* then *A* is also *North* of *C*. This inference is impossible using homogeneous spatial reasoning since the directional relation is implied by a topological relation. Such heterogeneous spatial reasoning is described in Chapter 4. In the case of directional relations, the definition of the relations imply a topological relation since directional relations for regions are based on the ordering of interval relations along two axes. Thus if region *A* is North of *B* then this fact implies that region *A* is *disjoint* from or *meets* region *B*. The topological relation between the two objects is implied by their directional relationship and this interdependence cannot be exploited by homogeneous spatial reasoning, which only deals with spatial relations of the same type.

3.7 Summary

The composition tables for individual types of spatial relations are derived based on the notion of transitivity of a particular relation. For example, in the case of point-set topological relations it is the transitivity of the subset relation that is used. Similarly the transitivity of the order relation is used in determining the composition of directional relations. These tables, however, are valid only for spatial relations of the same subtype. The following chapter introduces a method for determining the composition tables for combinations of different types of spatial relations, called heterogeneous spatial reasoning.

Chapter 4

Heterogeneous Spatial Reasoning

Heterogeneous spatial reasoning in the context of this thesis is the inference of spatial relations from specified information on spatial relations of different kinds between the objects of interest. An example of heterogeneous spatial reasoning is the inference of the directional relation between objects *A* and *C* from the topological relation between objects *A* and *B* and the directional relation between objects *B* and *C*.

The motivation for heterogeneous spatial reasoning comes from the analysis of homogeneous spatial reasoning presented in Chapter 3, which showed that there are situations that require combining spatial information, and utilizing the dependency among different types of spatial relations, for inferring new spatial information. As is evident from the existence of natural language terms for topological and directional relations, such as *disjoint* and *North*, and approximate distances, such as *near* and *far*, people distinguish between these kinds of spatial relations. Studies by cognitive psychologists and cognitive science researchers have shown, however, that people commonly use combined knowledge of various kinds of spatial relations when drawing inferences (Blades 1991; Mark 1993). For example, they may infer that Orono is *North* of Portland, Maine using the qualitative distance information, Orono is *near* Bangor and Bangor is *far* from Portland, and the directional information Bangor is *North* of Portland.

We are primarily concerned with reasoning using combined information about topological and directional spatial relations. This chapter introduces a formal framework for performing inferences using information about topology and direction. Since binary topological relations exist between objects with an extent we first outline our framework for defining directional relations between extended objects. We then show how the framework is applicable to deriving the composition tables for pairs of topological and directional relations, which is the basis of the inference mechanism. Next we demonstrate how the inference mechanism, using combined information, permits valid inferences that are not derivable from the topological and directional relation information individually.

4.1 Types of Heterogeneous Spatial Relation Compositions

There are three types of qualitative spatial relations: qualitative distances, topological relations, and directional relations. Three distinct pairings of these relations are possible, with each pairing having two possible orderings, and therefore, four distinct composition tables, giving a total of twelve composition tables (Figure 4.1).

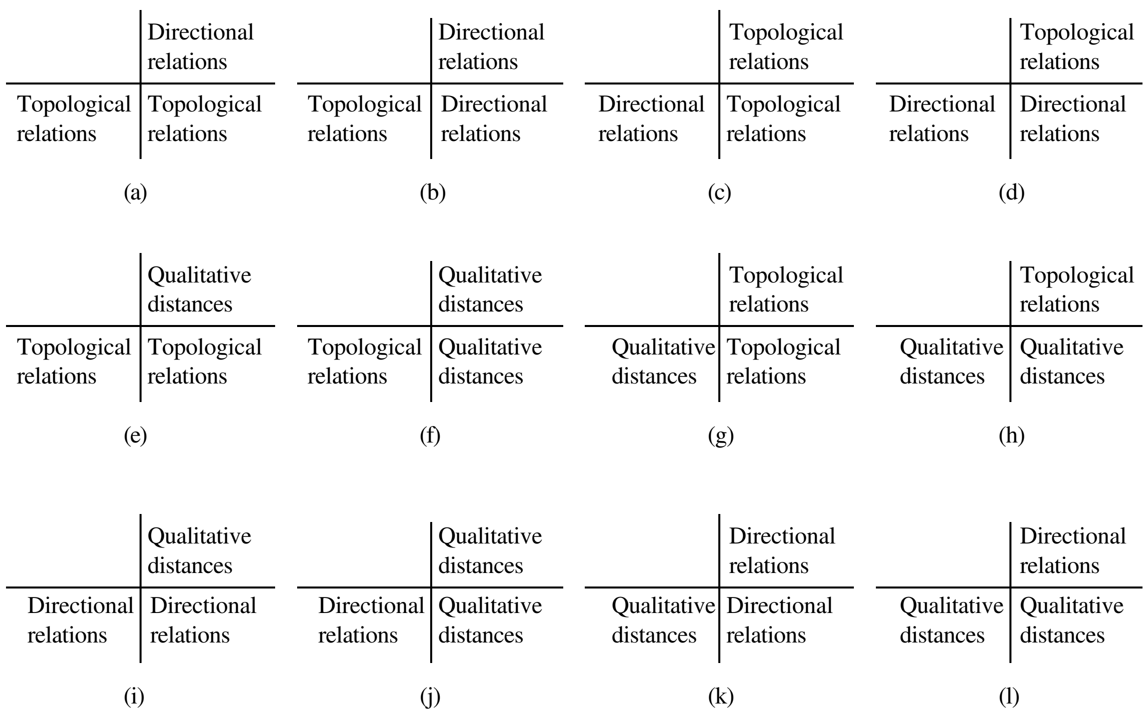


Figure 4.1 Heterogeneous spatial relation compositions.

The following sections describe the derivation of the composition tables for pairs of topological relations, directional relations, and qualitative distances. The method for deriving the composition tables for topological and directional relations is described in detail since these spatial relation types are the focus of this thesis. Section 4.5 gives the tables for topological and directional relations, while sections 4.6 and 4.7 give the tables for topological relations and qualitative distances and directional relations and qualitative distances, respectively.

4.2 Reasoning about Topology and Directions

Reasoning about topology and directions requires firstly, a definition and characterization of both kinds of spatial relations and secondly, a common representation scheme for them both, which would thereby facilitate heterogeneous spatial reasoning. We use a method based on point-set topology, the 9-intersection, to characterize topological spatial relations, and a projection-based system of directions for the definition of the directional relations between extended objects.

A topological relation between two regions, or extended objects, is invariant under certain continuous transformation, in particular rotation. A directional relation, however, is not. Thus a topological relation is independent of the ordering among the objects along any given axis, while a directional relation is dependent on, and in fact defined by, the ordering of objects along a specific axis. By knowing the directional relationship, the spatial information about the objects concerned is enhanced by information about their relative ordering. The topological relationship, on the other hand, contributes by providing information about the connectivity of the two objects.

It is this connectivity information that permits the inference of a directional relation in the following situation. Assume that the relation *North* between regions holds if and only if all points of region *A* are *North* of all points in region *B* and the regions have no points in common. Then given the three facts (1) *A North* of *B*, (2) *B meets C*, and (3) *C North* of *D* the directional relation between *A* and *D* follows from the ordering and connectivity information provided by three facts (Figure 4.2). The fact *B meets C* implies that objects *B* and *C* have some points in common, called *C´*. *C´* is part of both B and C. Hence from *A North* of *B* it follows that *A* is North of *C´* (since $C´ \subset B$) and from *C North* of *D* it follows that *C´* is *North* of *D* (since $C´ \subset C$). Since *North* implies an ordering and an ordering is transitive, we infer that *A* is *North* of *D*.

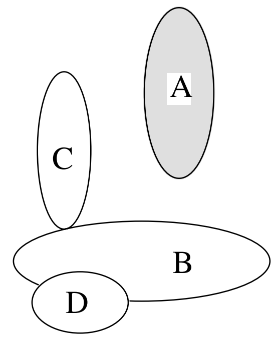


Figure 4.2 A situation where topology and direction reasoning is useful.

The definition of *North* used in the above example implied a topological relation, namely disjoint, between the regions. This definition is not the only possible one, for example one may say that the state of Utah is *North* of Arizona despite their sharing a border. The remainder of this section describes our method of defining directions between regions and its implications for combined topological and direction reasoning.

A direction between two points is an order relation along a primary axis regardless of whether a cone-shaped or projection-based direction system is used. Thus, if *North* corresponds to After and *South* to Before, then the statement *A* is *North* of *B* means that *A* is after *B* along the North-South axis. This does not mean that the points lie on the axis. Point *A* may lie anywhere within the acceptance region specified in the direction system that contains the North-South axis (Figure 4.3). A second important property of directions, namely converseness, follows from this notion of order. Since *A* after *B* implies *B* before *A* it is also true that *A* North of *B* implies *B* South of *A*, and vice versa. Thus a direction is an order relation and hence each direction has a converse direction.

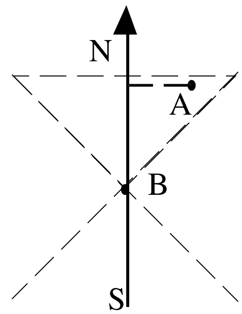


Figure 4.3 *A* North of *B* hence *A* is after *B* along the N-S axis.

These properties should also hold for directions between region objects. Let us consider various possible definitions for directions between simply connected, convex regions without holes. At this point we are considering reasoning along a single axis only and not reasoning about directions such as North and Northeast.

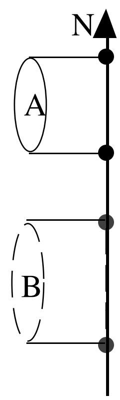


Figure 4.4 *A* completely North of *B*, hence interval *A* after interval *B* along the N-S axis.

Starting with the notion of order we define *North* as *A* is *North* of *B* if all points of *A* are *North* of all points of *B*. That is, for any pair of points (a, b) with $a \in A$ and $b \in B$, it is true that a is *North* of b. Since regions have an extent, the above definition means that there is an interval relationship between the two objects along the North-South axis (Figure 4.4). Thus any reasoning about directions, based on the above definitions, is interval-based reasoning. For example *North* ; *North* = *North* follows from the interval composition After ; After = After. This definition of direction between region objects preserves converseness. It also implies disjointness, e.g., *A North* of *B* implies *A disjoint*

B. However, disjointness is independent of order, therefore the fact *A North* of *B* enhances the information about the disjointness of *A* and *B* in that it specifies that *A* is after *B* on the North-South axis. It is this ordering, combined with connectivity, which enables topology-direction reasoning in the situation shown in Figure 4.2.

There are two observations of note in the above situation. First, the direction reasoning is along a single axis, North-South in this case, and hence the relative order of *A* and *B* and *C* and *D* is known. Second, the topological relation specifies that the objects have some point or points in common along the North-South axis. Since all points of *A* are After all points *B* along the North-South axis, the points of *A* are also After all the points that *B* has in common with *C*. Now since these points are in object *C* and all points of object *C* are After all points of *D*, it follows from the transitivity of After that all points of *A* are After all points of *D* along the North-South axis. This implies that *A* is *North* of *D*.

The definition based on order excludes cases where regions have a border point in common along the reference axis. If we relax the definition to include such cases then the interval relation between the objects' extents is a Meet/MetBy relation. This definition also implies an ordering along the reference axis and converseness. Hence interval based reasoning is applicable and permits the inference *North* ; *North* = *North*. In fact since all Allen relations except Equal, During, and Contains specify an order along the axis and have a converse relation they could all be used in defining the notion of direction between region objects along a particular axis (Figure 4.5). For example, if we allow the objects to *overlap* then the composition of intervals OverlappedBy ; OverlappedBy = After or MetBy or OverlappedBy, all three of which are included in the definition of *North*, therefore, the inference *North* ; *North* = *North* is still valid.

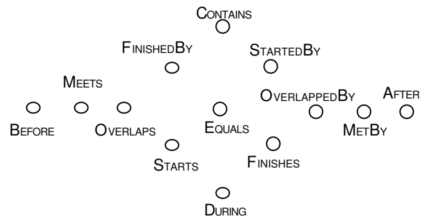


Figure 4.5 Iconic representation of Allen's interval relations.

Now let us consider those intervals that imply that a portion of one interval is before or after all of the second interval. Starting with the strict definition that all points of *A* are *North* of all points of *B* we consider the interval relation After and its composition table (Table 4.1). The result of the composition is After and hence the closure property is satisfied.

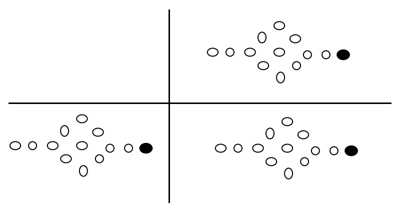


Table 4.1 Composition for the interval relation subset {After}.

Relaxing the definition of Northness to include portions of one object being *North* of all of the second object allows additional interval relations, MetBy, OverlappedBy, or StartedBy to be included in the definition of *North*. These interval relation are successive neighbors of each other as apparent from the conceptual neighborhood graph in Figure 4.5. Examining the composition tables for the subsets of intervals {After, MetBy} (Table 4.2), {After, MetBy, OverlappedBy} (Table 4.3), {After, MetBy, OverlappedBy, StartedBy} (Table 4.4) we note that the closure property is satisfied in all these cases, i.e., the result of the composition of any pair of interval relations is a set of elements belonging to the original subset.

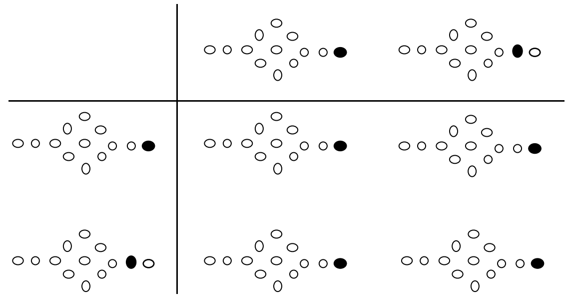


Table 4.2 Composition for the interval relation subset {After, MetBy}.

Extending the set with the interval relation OverlappedBy gives a composition table (Table 4.3) whose the entries are all elements of the subset of interval relations used to define the directional relation *North*.

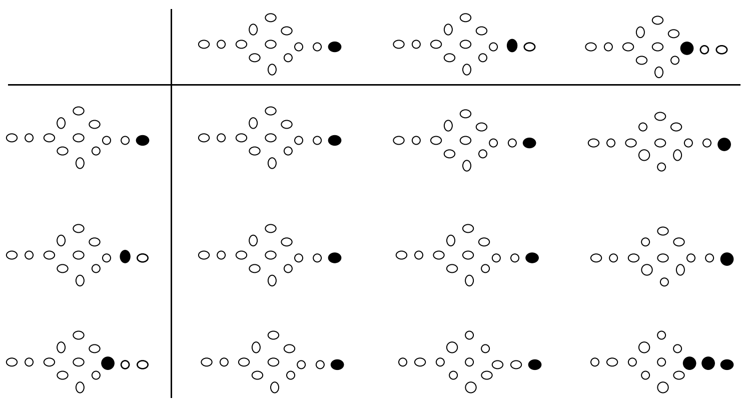


Table 4.3 Composition for the interval relation subset {After, MetBy, OverlappedBy}.

Further relaxing the definition of *North* and thereby adding StartedBy to the subset of interval relations results in Table 4.4.

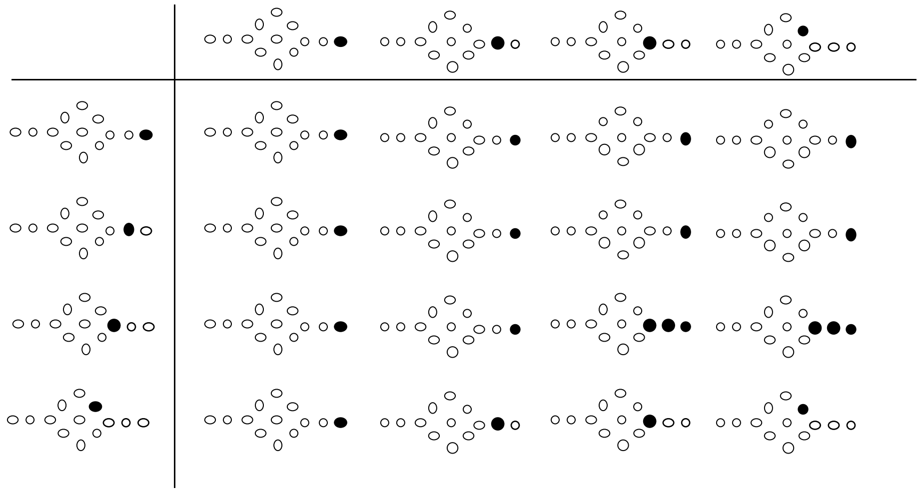


Table 4.4 Composition for the interval relation subset {After, MetBy, OverlappedBy, StartedBy}.

The intervals {After, MetBy, OverlappedBy, StartedBy} all imply that some part of *A* is *North* of all of *B*. From the conceptual neighborhood graph we see that OverlappedBy has two neighbors, namely StartedBy and Finishes, and the intuition underlying the conceptual neighborhood concept suggests that either or both neighbors should be included in the relaxed definition of Northness. The interval relation Finishes differs from the others since it implies that all of *A* is *North* of some part of *B*. Table 4.5 gives the composition table of the expanded subset of intervals.

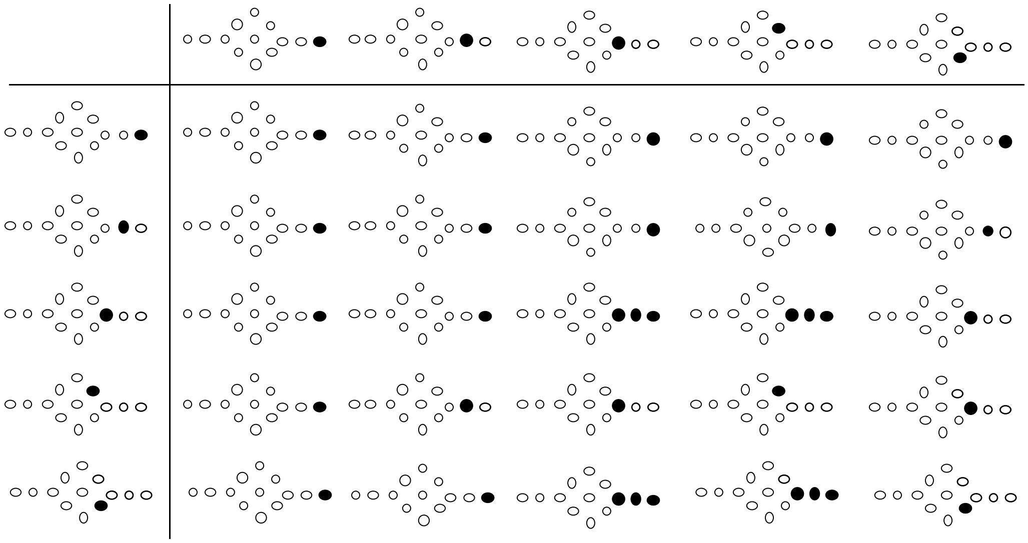
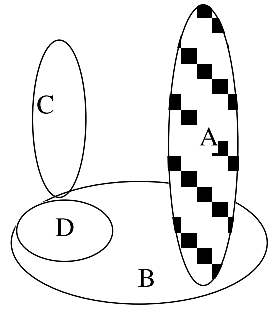


Table 4.5 Composition for the interval relation subset {After, MetBy, OverlappedBy, StartedBy, Finishes}.

The results of the composition are from the set {Finishes, StartedBy, OverlappedBy, MetBy, After} which defines Northness. Thus even if we relax the definition of *North* to permit the inclusions *B* is in the Northern or Southern part of *A*, the inference *North* ; *North* = *North* is valid. This inference holds regardless of the shape or relative size of the objects.

While the relaxation in the definition of Northness is immaterial for pure direction reasoning, it has implications for combined topology and direction reasoning. Recall that the effect of directions was to order the participating objects along a particular axis and the effect of the topological relation was to determine whether or not the two projected intervals could have any points in common. If we relax the definition of direction to include *overlap* then the overlap information obtained from the topological relation provides no additional constraints. As a result the inference *A North* of *D* from the facts *A North* of *B*, *B meets C*, and *C North D* is invalid because there can be some part of *A South* of *D* (Figure 4.6).

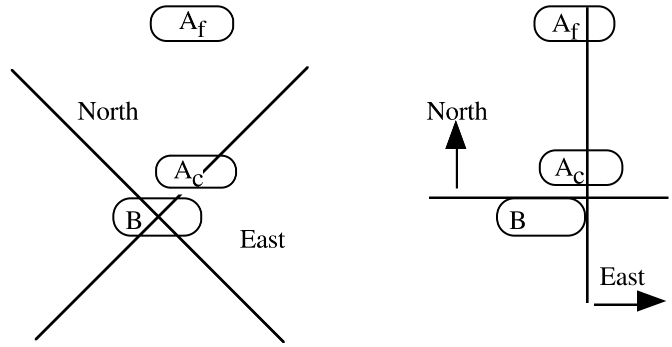


A North B, B *meets* C, C North of D.

Figure 4.6 Situation where a relaxed definition of direction limits the inferences possible, because one cannot infer *A North* of *D* as there can be some part of *A South* of *D*

The above discussion of the definition of directional relations based on order assumed a primary axis for the direction. In the case of two points the line between them defines the primary axis of the direction. This is not the case for regions. Since regions have an extent, two regions can have a North-South as well as an East-West relationship. So we need to assess which direction relations can exist between two regions for 4 and 8 direction systems.

First we examine the possible directions relations in a four direction system based on a cone-shaped model and on projections. For a cone-shaped model the region of acceptance changes with distance from the point of origin and hence the directional relation between regions can change with distance, because we consider the extent of the primary object when determining the relationship (Figure 4.7a). When *A* is close to *B* it is both *North* and *East* of it, but *A* is solely *North* of *B* when it is at a farther distance. For a projection-based system the directional relation is independent of the distance between the objects (Figure 4.7b).



(a) (b)

Figure 4.7 Effect of distance on the directional relation between regions.

Since our preferred definition is such that direction is independent of distance between objects we use the projection-based method for both four and eight direction systems. In the four direction system we are concerned with intervals along the N-S and E-W axes formed by the Minimum Bounding Rectangles (MBRs) of the objects and hence directions between regions can be defined in terms of directions between MBRs. Thus the direct consequences of defining directions between regions in terms of intervals are (1) the projection-based system of directions must be used, and (2) the directions between regions are determined by the relationships between their MBRs.

The possible relations between MBRs that can correspond to a directional relationship are in turn determined by the interval relations used to define the directional relationships. We will consider this effect by incrementally relaxing an initially strict definition of direction.

We start by allowing only the intervals Before or After in defining a directional relation, that is, all of *A* must be *North*, *South*, *East*, or *West* of all of *B*. This means that the MBRs of the two objects are necessarily disjoint, because the interval relationship along either axis is from the set {Before, After} (Figure 4.8). If the MBRs were not *disjoint* then the intervals along at least one axis would have points in common and hence that directional relation would be undefined. The directional relation is said to be

SameEastWesterly or SameNorthSoutherly if the interval relation along the E-W or N-S axis is from the set SDI = {Meets, Overlaps, FinishedBy, Contains, Starts, Equals, StartedBy, During, Finishes, OverlappedBy, MetBy}. Thus the possible directional relations in terms of pairs of intervals relations are:

{Before, Before} ⇒ *A* is *West* and *South* of *B* (Figure 4.8a)

{Before, Y} Y ∈ SDI ⇒ *A* is *West* and *SameNorthSoutherly* of *B* (Figure 4.8b)

{Before, After}⇒ *A* is *West* and *North* of *B* (Figure 4.8c)

{After, Before} ⇒ *A* is *East* and *South* of *B* (Figure 4.8d)

{After, Y} Y ∈ SDI ⇒ *A* is *East* and *SameNorthSoutherly* of *B* (Figure 4.8e)

{After, After}⇒ *A* is *East* and *North* of *B* (Figure 4.8f)

{X, Before} X ∈ SDI ⇒ *A* is *South* and *SameEastWesterly* of *B* (Figure 4.8g)

{X, After} X ∈ SDI ⇒ *A* is *North* and *SameEastWesterly* of *B* (Figure 4.8h)

{X, Y} X ∈ SDI, Y ∈ SDI ⇒ No direction relation between *A* and *B* can be specified and they have points in common.

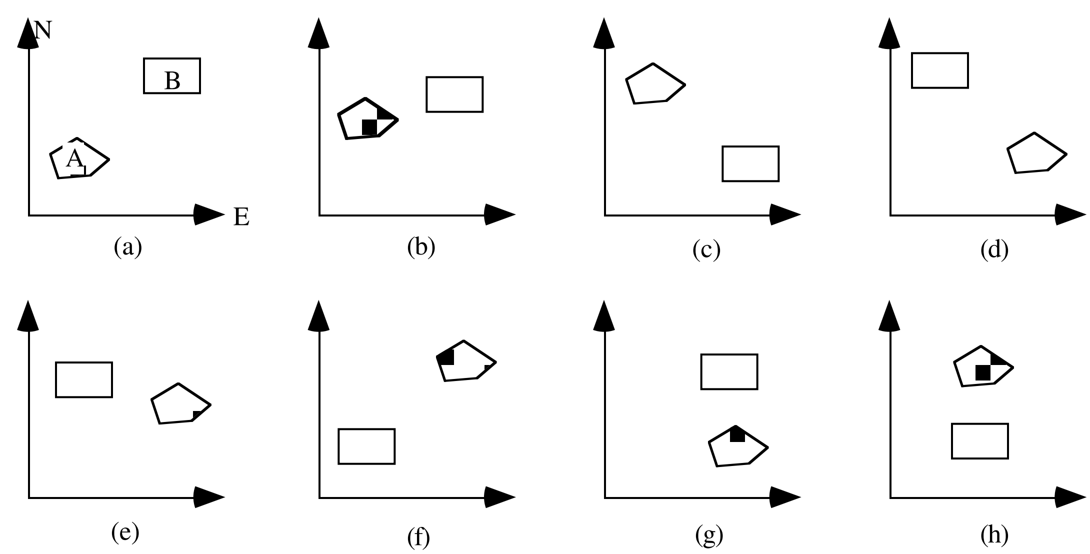


Figure 4.8 Possible directions between regions for a strict definition of direction.

If the definition of direction is relaxed to include the situation where regions have a border point in common along either axis then the possible directional relations are defined in terms of pairs of elements from the sets {Before, Meets, MetBy, After} and SDI = {Overlaps, FinishedBy, Contains, Starts, Equals, StartedBy, During, Finishes, OverlappedBy}. They are:

{Before or Meets, Before or Meets} => *A* is *West* and *South* of *B* (Figure 4.9a)

{Before or Meets, Y} Y ∈ SDI => *A* is *West* and *SameNorthSoutherly* of *B* (Figure 4.9b)

{Before or Meets, After or MetBy}=> *A* is *West* and *North* of *B* (Figure 4.9c)

{After or MetBy, Before or Meets} => *A* is *East* and *South* of *B* (Figure 4.9d)

{After or MetBy, Y} Y ∈ SDI => *A* is *East* and *SameNorthSoutherly* of *B* (Figure 4.9e)

{After or MetBy, After or MetBy}=> *A* is *East* and *North* of *B* (Figure 4.9f)

{X, Before or Meets} X ∈ SDI => *A* is *South* and *SameEastWesterly* of *B* (Figure 4.9g)

{X, After or MetBy} X ∈ SDI => *A* is *North* and *SameEastWesterly* of *B* (Figure 4.9h)

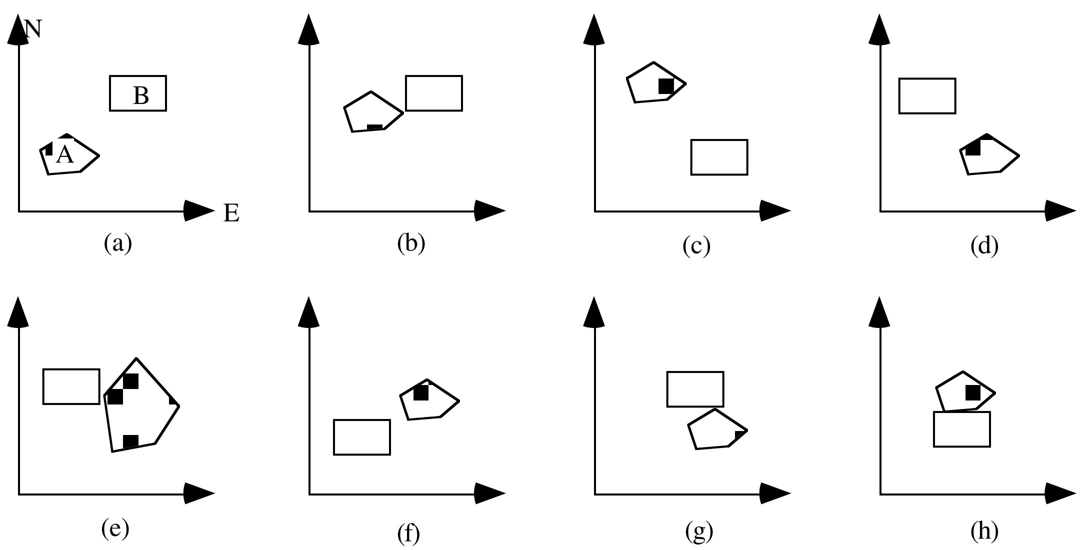


Figure 4.9 Possible directions between regions whose interval relations are disjoint or meet.

The consequence, for topological reasoning, of restricting the interval relations to the subset {Before, Meets, MetBy, After} is that the bounding rectangles between the objects can only be disjoint from or touching each other. The implications for topological relations is that objects may only be disjoint or may meet along either or both the direction axes. Therefore, the interval-based definition of directional relations also restricts the topological relations between objects.

Having decided upon a characterization of directional relations for region objects which has implications for topological reasoning, we need a common representation for both types of spatial relations. The following section outlines the mapping of topological and directional relations onto pairs of interval relations.

4.3 Interval-based Representation of Topological and Directional Relations

Since directional relations between regions are defined only when the regions are disjoint or meet, the interval relation subset {Before, Meets, MetBy, After} is used to characterize directional relations. Therefore, each of the directions *North*, *Northeast*, *East*, *Southeast*, *South*, *Southwest*, *West* and *Northwest* corresponds to mutually exclusive subsets of the 169 possible pairs of interval relations. Figure 4.10a shows the mappings for *North* and Figure 4.10b shows the mappings for the directions *Northeast*, *Southeast*, *Southwest*, and *Northwest*.

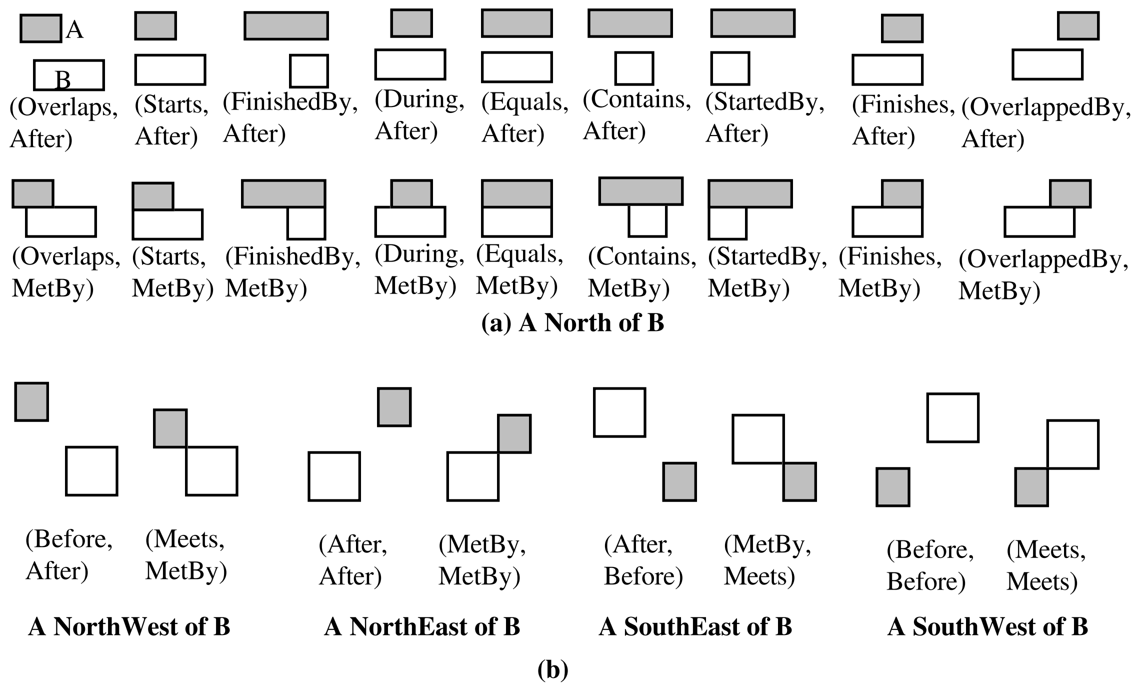


Figure 4.10 Mapping directions onto pairs of interval relations.

The mappings for directional relations are such that one and only one subset of the 169 possible pairs of inetrval relations corresponds to one directional relation. For topological relations, however, the subsets are not mutually exclusive, primarily because the ordering between the objects is unimportant. Hence a particular interval relation pair can correspond to more than one binary topological relation. For example, the pair (Equals, Equals) can occur if the regions *A* and *B* are *disjoint*, *meet*, *overlap*, *equal*, or if region *A covers* B or *A* is *coveredBy B*. Figure 4.11 shows the possible pairs of interval relations for the topological relation *A covers B*. Each of the eight topological relations between regions is mapped onto some set of pairs of interval relations.

Once both directional and topological relations have been mapped onto pairs of interval relations the composition of pairs of topological and directional relations can be determined from the composition of corresponding pairs of interval relations.

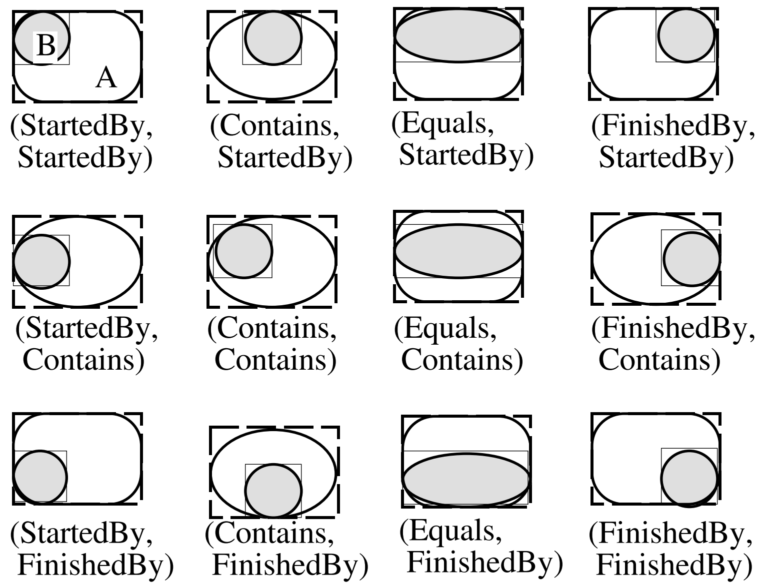


Figure 4.11 Interval relations pairs possible for the topological relation *covers*.

4.4 Composition of Pairs of Interval Relations

Composition of pairs of interval relations are performed using Allen's composition tables for interval relations. The first interval of the first pair is composed with the first interval of the second pair, and similarly for the second interval of each pair. Each composition results in a set of intervals. The composition of the two pairs of intervals is given by the Cartesian product of the results of the composition of corresponding elements of the pairs. For example, consider the composition of interval pairs P = (Before, MetBy) and Q = (Meets, After). The composition of P ; Q is given by the Cartesian product (×) of the individual compositions Before ; Meets and MetBy ; After (Equation 4.1).

$$\begin{aligned} P;Q &= (Before, MetBy);(Meets, After) \\ &= (Before; Meets) \times (MetBy; After) \\ &= \{Before\} \times \{After\} = (Before, After) \end{aligned} \tag{4.1}$$

For topological and directional relations, however, each relation maps onto a set of interval relation pairs. The composition of pairs of interval relations can then be used to determine the composition of combinations topological and directional relations, or for determining the topological relation implied by the composition of directional relations. For example, the direction *Northwest* implies that any one of the interval pairs from among the sets {(Before, After), (Before, MetBy), (Meets, After), (Meets, MetBy)} is possible, while the topological relation *contains* implies the set {(Contains, Contains)}. Therefore, determining the topological relations implied by the composition *Northwest* ; *contains* involves the composition of the elements formed by the Cartesian product of the above two sets of pairs of interval relations (Equation 4.2).

$$\begin{aligned}
&Northwest;contains = \\
&\{(Before, After),(Before, MetBy),(Meets, After),(Meets, MetBy)\} \\
&;\{(Contains, Contains)\} \\
&= \{(Before, After);(Contains, Contains)\} \cup \{(Before, MetBy);(Contains, Contains)\} \\
&\cup \{(Meets, After);(Contains, Contains)\} \cup \{(Meets, MetBy);(Contains, Contains)\} \\
&= \{(Before, After)\} \cup \{(Before, MetBy)\} \cup \{(Meets, After)\} \cup \{(Meets, MetBy)\} \\
&= \{(Before, After),(Before, MetBy),(Meets, After),(Meets, MetBy)\} \\
&= disjoint \vee meet
\end{aligned} \tag{4.2}$$

The composition tables for combinations of the eight topological and eight directional relations between regions are derived using the above method and are given in the following section.

4.5 Composition Tables for Topological and Directional Relation Pairs

Topological and directional relations map onto sets of interval relations. Compositions of these sets of interval relation pairs is accomplished using Allen's tables for the composition of individual interval relations. Therefore, an integrated mechanism exists with which pairs of topological and directional relations can be combined. Composition of the corresponding sets of interval relation pairs determines the set of interval relation pairs that correspond to the composition of a topological with a directional relation or directional with a topological relation. The resultant set of pairs of interval relations is then mapped back onto sets of topological relations and directional relations, thereby giving the results of the composition of topological and directional relations. This process is carried out systematically and exhaustively for each topological-directional relation and directional-topological relation pair and results in a set of composition tables.

The composition tables are given below. Table 4.6 describes the composition of topological with directional relations giving directional relations as the result.

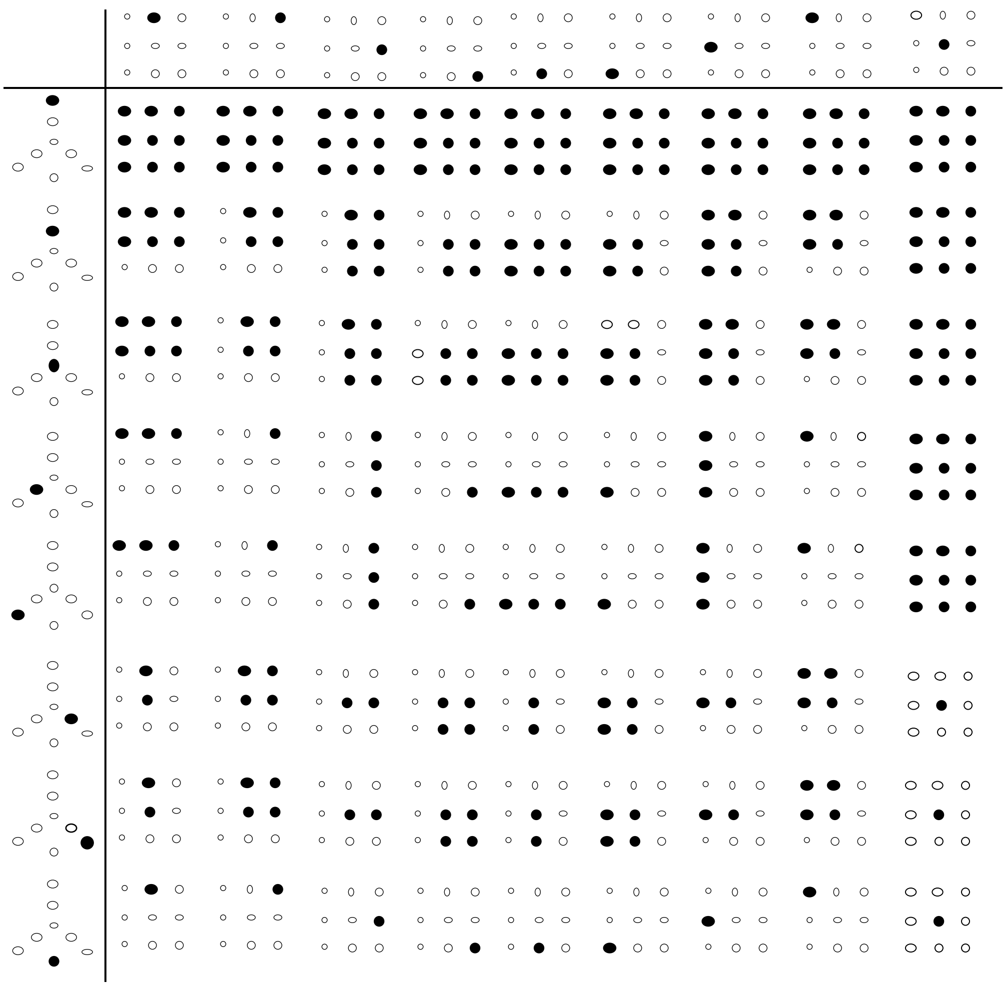


Table 4.6 Topological relation ; directional relation ⇒ directional relations.

An inspection of the above table indicates that the strongest inference is made when the topological relation between the first pair of objects is one of containment, i.e., any one of *equal*, *inside*, or *coveredBy*. This occurs because the contained object must necessarily have the same directional relations as the containing object. A similar observation can be made in the next table, which is for the composition of topological and directional relations giving topological relations as the result (Table 4.7).

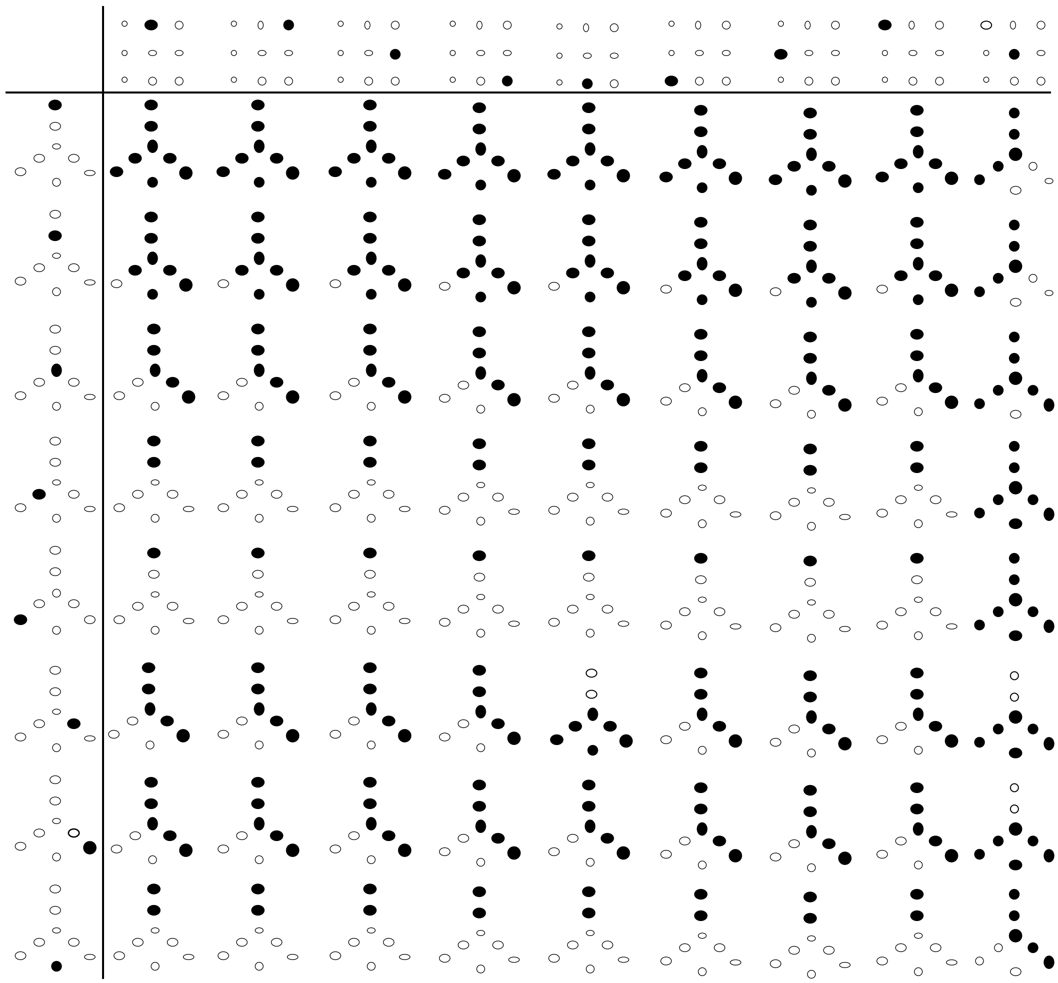


Table 4.7 Topological relation ; directional relation ⇒ topological relations.

In the case of Table 4.7 we note that the topological relations *disjoint* or *meet*, implied by the definition of directional relations, are the ones that are inferred whenever containment is the topological relationship participating in the composition.

Table 4.8 describes the composition of directional and topological relations giving directional relations as the result.

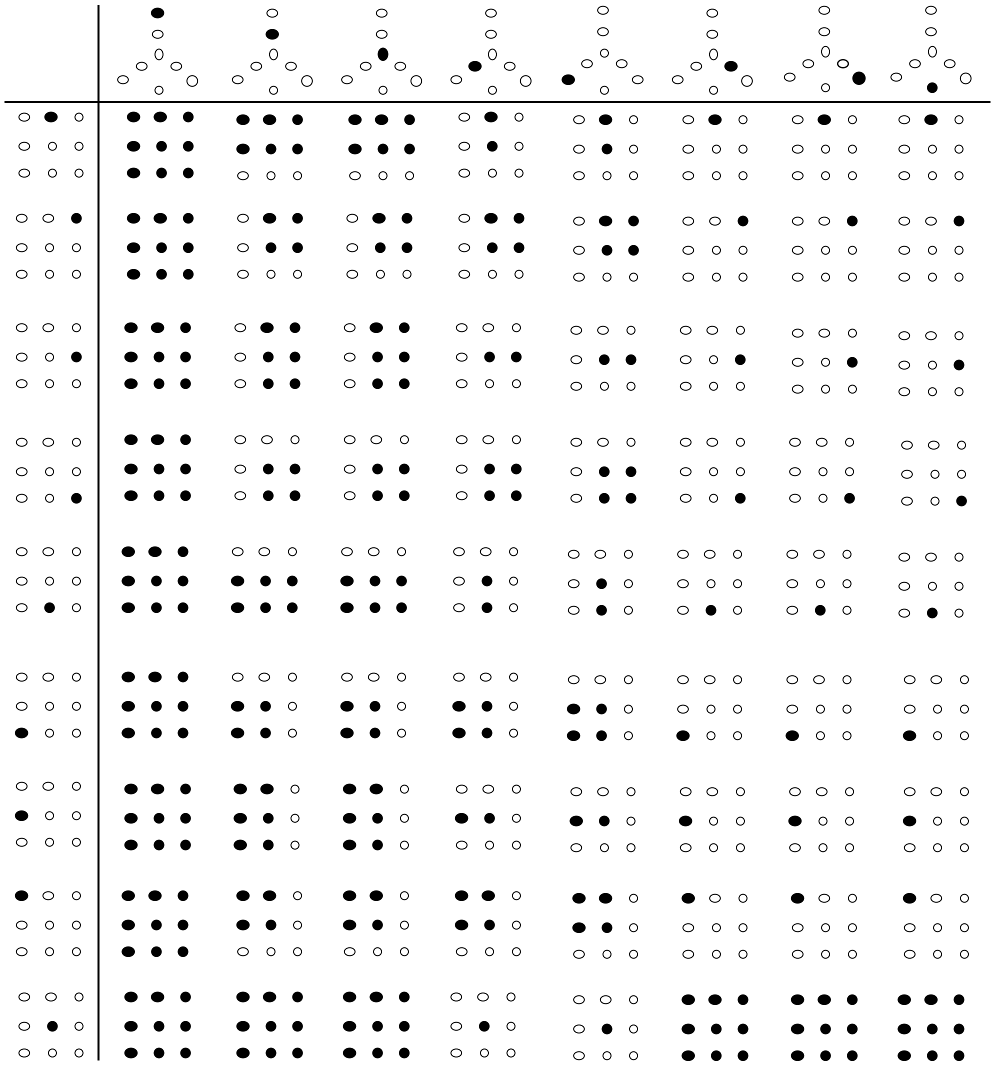


Table 4.8 Directional relation ; topological relation ⇒ directional relations.

Finally, Table 4.9 defines the composition of directional and topological relations giving topological relations as the result.

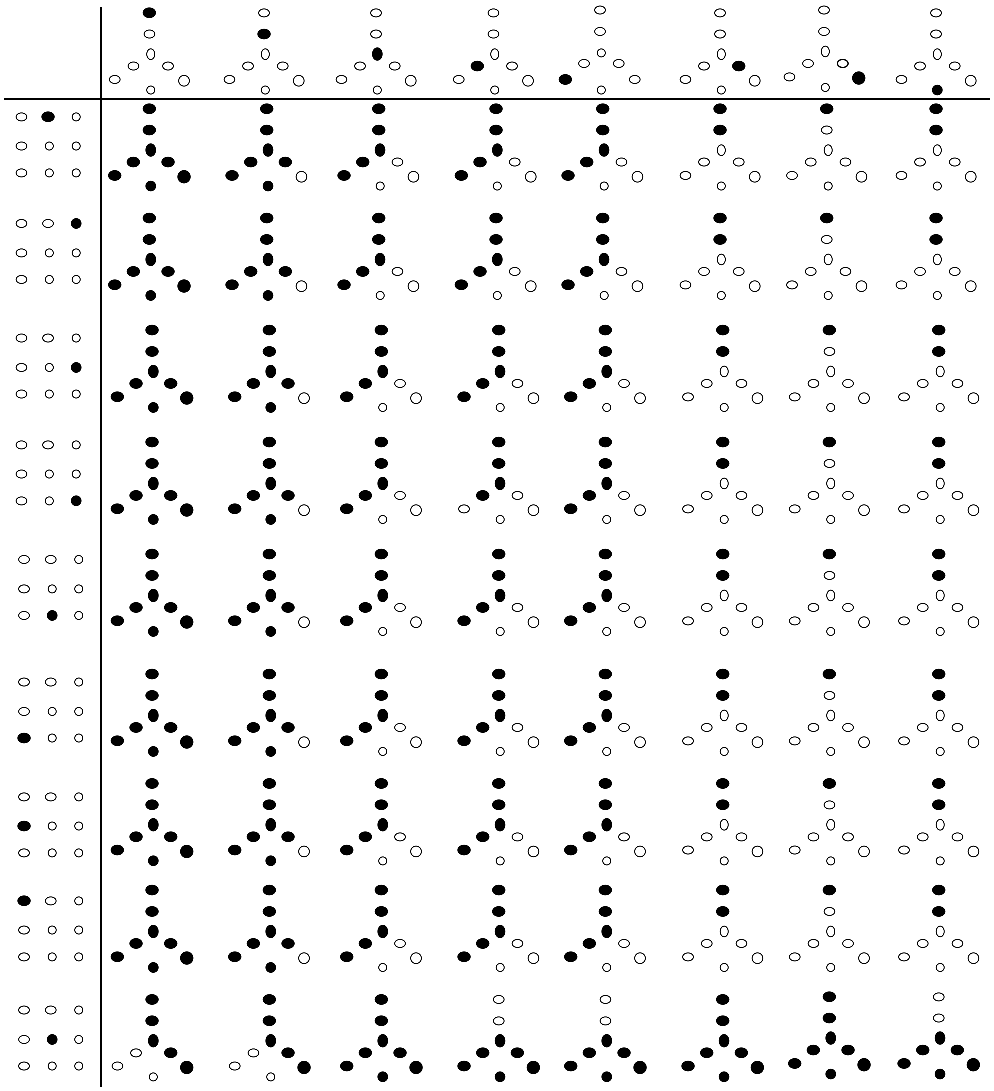


Table 4.9 Directional relation ; topological relation ⇒ topological relations.

Tables 4.6-4.9 show that heterogeneous spatial reasoning about topology and direction is most effective when there is a containment relationship between one pair of objects participating in the composition. This is a valid observation for single-step inferences. The following section demonstrates, however, that heterogeneous spatial reasoning using pairs of interval relations is useful for multi-step compositions even when there is no containment relationship among the participating objects.

4.6 Reasoning About Topology and Distance

Reasoning about topology and distance requires a definition of distance relations between extended objects that is compatible with the 9-intersection formalism for binary topological spatial relations. While distances and qualitative distances between point objects have been defined (Section 2.2.3) the same has not been done for objects with a linear or areal extent.

Defining distances between regions is a complex task since the size of each object plays an important role in determining the possible distances. Some possibilities for defining distances between regions are: taking the distance between the centroids of the two regions; taking the shortest distance between the two regions; or taking the furthest distance between the two regions. Each of these definitions has potential for inconsistencies and contradictions particularly if one or both regions contain other regions and the distance relation between the containers is inherited by the contained regions.

The continental United States, as a region, contains 48 states. Similarly Mexico is a region that can be considered as one unit or as a container of smaller regions, namely its states. If the shortest distance between two regions is used to determine the qualitative distance relationship then one would state that Mexico is *very close* to the United States. While this is correct for Mexico and the U.S. as regions, this same qualitative distance relation does not hold between Maine, contained in the U.S., and Mexico. Similarly the facts Mexico is *very close* to the U.S. and the U.S. is *very close* to Canada result in the possibly incorrect inference that Mexico is *very close* or *close* to Canada.

In view of the above potential inconsistencies any definition of qualitative distance relations among regions must take into account the sizes of the regions. This consideration adds much complexity to the definition of qualitative distance relations. Since the primary purpose of this thesis is a systematic study of types of spatial reasoning for topological and directional relations, and qualitative distances are included for the sake of completeness only, we assume a very simple notion of distance between regions. We consider only two possible qualitative distance relations, *SamePlace* and *Not SamePlace*, i.e., two regions have some interior points in common or have no interior points in common, respectively. The advantage of this simplified definition is that the relation *SamePlace* corresponds to the topological relations {*overlap* ∨ *inside* ∨ *coveredBy* ∨ *contains* ∨ *covers* ∨ *equal*} and the relation *Not SamePlace* corresponds to {*disjoint* ∨ *meet*}. This correspondence enables the derivation of the composition tables given in this section.

Tables 4.10 through 4.13 give the results of composing topological relations and qualitative distances. The entries in each table were determined by substituting the sets of topological relations {*disjoint* ∨ *meet*}, for the qualitative distance *SamePlace*, and {*overlap* ∨ *inside* ∨ *coveredBy* ∨ *contains* ∨ *covers* ∨ *equal*}, for *Not SamePlace*. and subsequently performing the appropriate compositions between topological relations.

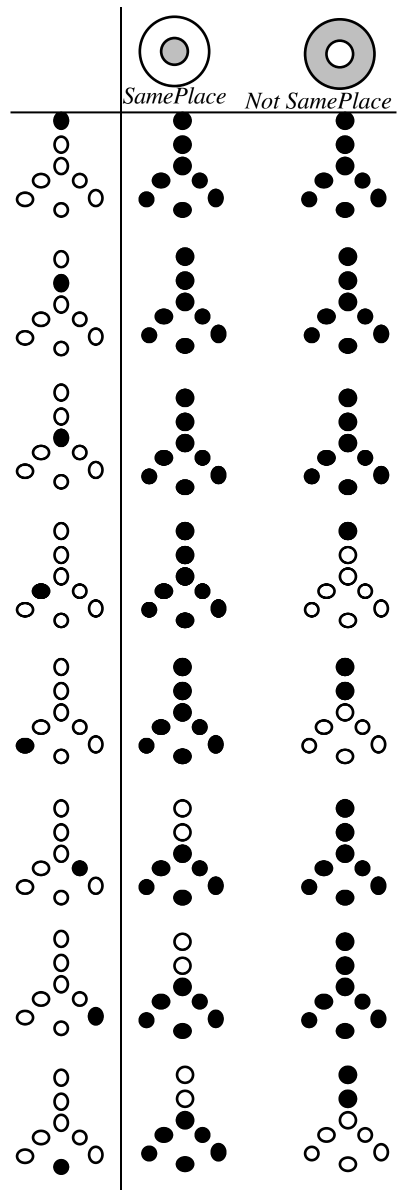


Table 4.10 Topological relation ; qualitative distance ⇒ topological relations.

The entry for *coveredBy* ; *Not SamePlace* in Table 4.10, for example, was derived performing the composition *coveredBy* ; $\{disjoint \vee meet\}$ which results in the set $\{disjoint \vee meet\}$ (Equation 4.3).

$$\begin{aligned} coveredBy;\{disjoint \vee meet\} &= \{coveredBy;disjoint\} \cup \{coveredBy;meet\} \\ &= \{disjoint\} \cup \{disjoint \vee meet\} \\ &= \{disjoint \vee meet\} \end{aligned} \tag{4.3}$$

Table 4.11 gives the qualitative distances that can be inferred from the composition of a topological relation with a qualitative distance.

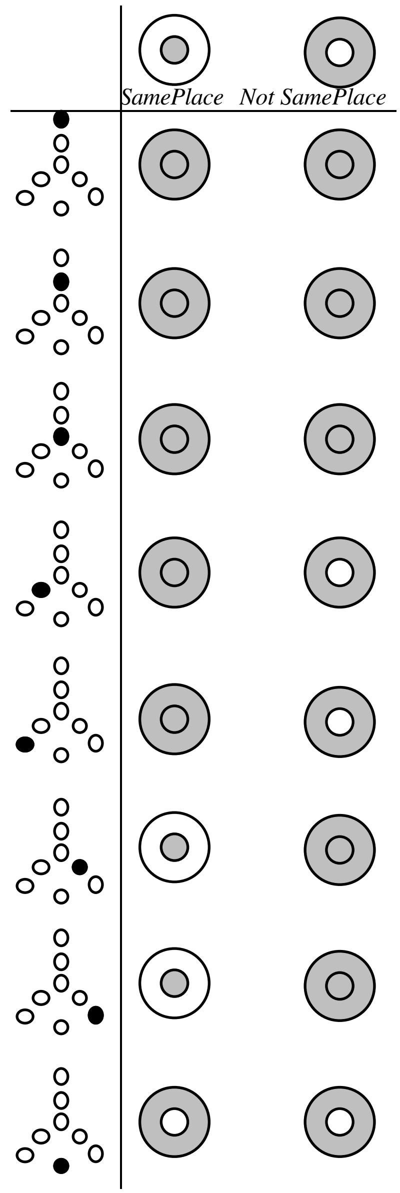


Table 4.11 Topological relation ; qualitative distance ⇒ qualitative distances.

Using the same method as for Table 4.10 the entry for *coveredBy* ; *Not SamePlace* in Table 4.11 is the qualitative distance Not *SamePlace*, since it is equivalent to {*disjoint* ∨ *meet*}. The same approach was used for Tables 4.12 and 4.13.

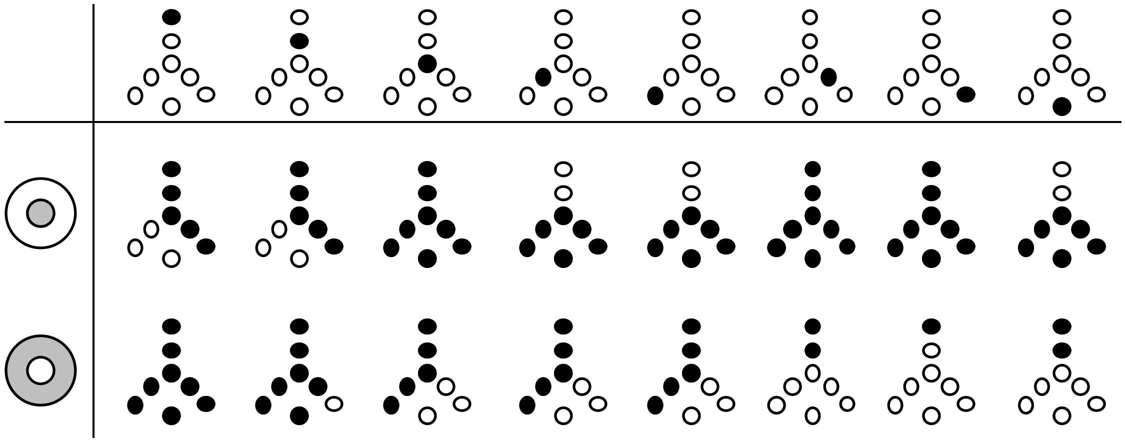


Table 4.12 Qualitative distance ; topological relation ⇒ topological relations.

Table 4.12 gives the topological relations that can be inferred from the composition of a qualitative distance with a topological relation.

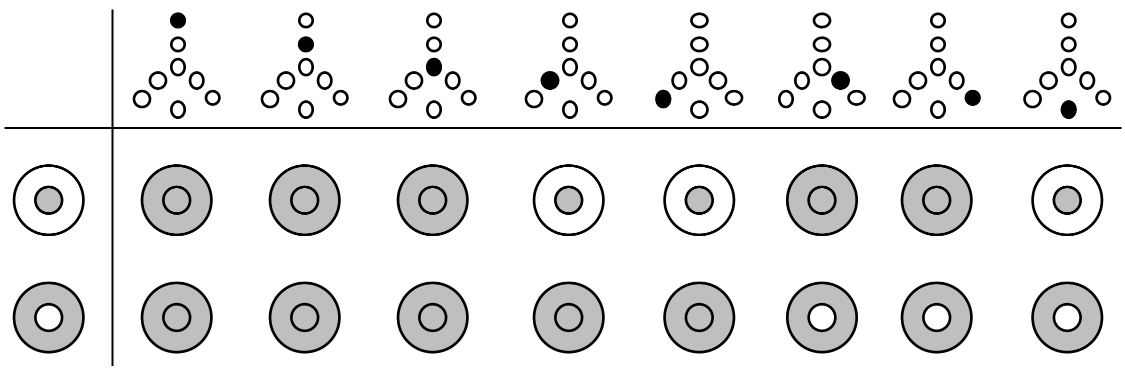


Table 4.13 Qualitative distance ; topological relation ⇒ qualitative distances.

Table 4.13 gives the qualitative distances that can be inferred from the composition of a qualitative distance with a topological relation.

4.7 Reasoning About Distance and Direction

This section gives the composition tables for qualitative distances and directions. The composition tables are derived as follows. First, the two qualitative distances, *SamePlace* and *Not SamePlace*, are mapped onto topological relations. Second, topology and distance reasoning is used to determine the relevant compositions. Third, the relations resulting from the second step are mapped onto qualitative distance and directional relations, thereby giving the desired composition tables 4.14 through 4.17.

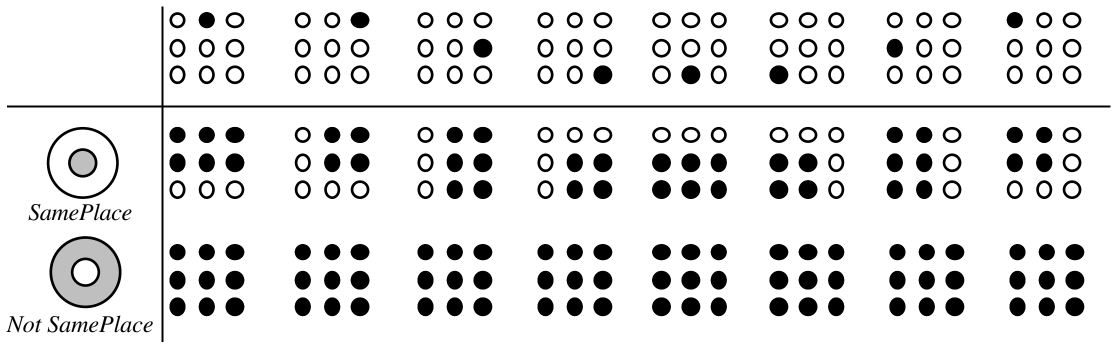


Table 4.14 Qualitative distance ; directional relation ⇒ directional relations.

Table 4.14 gives the directional relations that can be inferred from the composition of a qualitative distance with a directional relation.

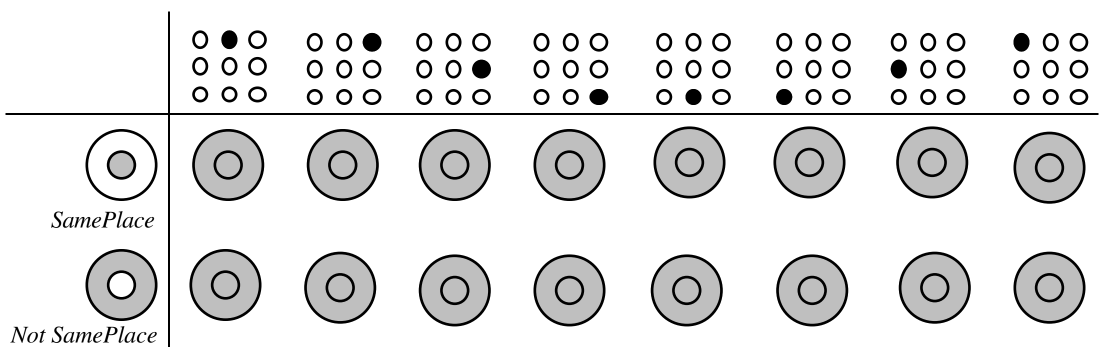


Table 4.15 Qualitative distance ; directional relation ⇒ qualitative distances.

Table 4.15 gives the qualitative distances that can be inferred from the composition of a qualitative distance with a directional relation.

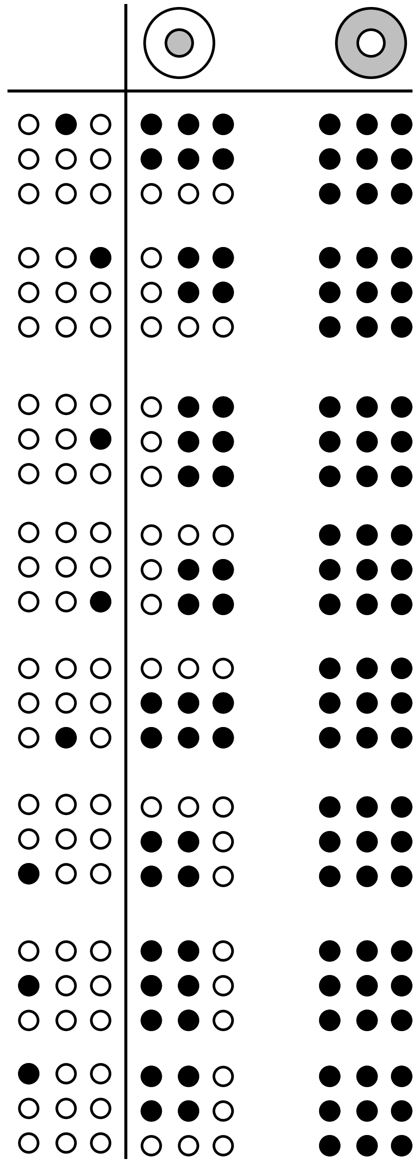


Table 4.16 Directional relation ; qualitative distance ⇒ directional relations.

Table 4.16 gives the directional relations that can be inferred from the composition of a directional relation with a qualitative distance.

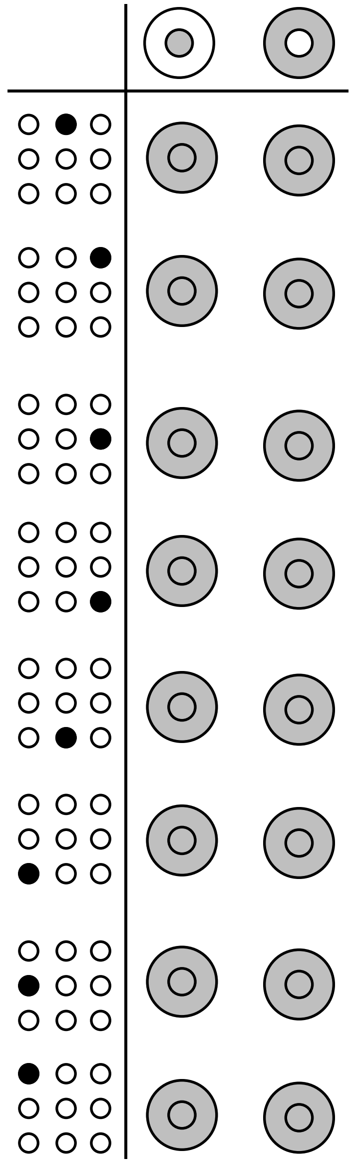


Table 4.17 Directional relation ; qualitative distance ⇒ qualitative distances.

Table 4.17 gives the qualitative distances that can be inferred from the composition of a directional relation with a qualitative distance.

4.8 Using the Composition of Interval Pairs for Multi-step Inferences

Composition of interval pairs is also useful for determining the composition of sequences of directional and topological relations. Consider the situation illustrated in Figure 4.2 where the directional relation between objects A and D is obtained by the three relation composition *A North B* ; *B meet C* ; *C North D*. We simplify the following presentation by assuming that only the North-South relationship of *A* and *D* is of concern. Thus only intervals along the North-South axis are of importance in the reasoning process.

The multi-step composition using intervals along the North-South axis is done as follows. The directional relation *North* implies the interval relation set {After, MetBy}, while the topological relation *meet* implies the interval relation set {Meets, Overlaps, Starts, FinishedBy, Contains, Equals, During, Finishes, StartedBy, OverlappedBy, MetBy}. Their compositions are:

$$
\begin{aligned}
North;meet;North &= \{After, MetBy\}; \\
&\quad \left\{\begin{matrix} Meets, Overlaps, Starts, FinishedBy, Contains, Equals, \\ During, Finishes, StartedBy, OverlappedBy, MetBy \end{matrix}\right\}; \\
&\quad \{After, MetBy\} \\
&= \left\{\begin{matrix} After, MetBy, OverlappedBy, StartedBy, \\ FinishedBy, Contains, Equals, Finishes \end{matrix}\right\}; \{After, MetBy\} \\
&= \{After, MetBy\} = North
\end{aligned}
$$

While the mapping onto interval relations allows for the multi-step composition of any sequence of directional and topological relations we are particularly interested in compositions of the form d_i ; t_j ; d_i where $d_i \in$ {North, South, East, West} and t_j is a binary topological relation.

Sistla *et al*. (1994) stated the inference rule

A x D :: *A x B, B overlaps C, C x D,*

where *x* denotes any one of the orientation relationships {*left_of*, *right_of*, *front_of*, *behind*}. They provide no formal justification for the rule, its derivation, and validity. Our interval-based representation of spatial relations provides a formal definition and derivation for the above inference rule and thereby determines its validity. We determined that the inference rule holds for all topological relations, with the exception of *disjoint*, between two regions. These inference rules were derived by constructing eight 13x13 composition tables, one for each topological relation.

For example, the composition table for the topological relation *meet* gives the results of the composition I_i ; I_{meet} ; I_j where I_i and I_j are interval relations and I_{meet} is the set of interval relations along a particular axis that are mapped onto the relation *meet*. Based on these composition tables the derived inference rules are:

- $A\ d_i\ B$, B *meets* C, $C\ d_i\ D \Rightarrow A\ d_i\ D$
- $A\ d_i\ B$, B *overlaps* C, $C\ d_i\ D \Rightarrow A\ d_i\ D$
- $A\ d_i\ B$, B *equal* C, $C\ d_i\ D \Rightarrow A\ d_i\ D$
- $A\ d_i\ B$, B *contains* C, $C\ d_i\ D \Rightarrow A\ d_i\ D$
- $A\ d_i\ B$, B *covers* C, $C\ d_i\ D \Rightarrow A\ d_i\ D$
- $A\ d_i\ B$, B *inside* C, $C\ d_i\ D \Rightarrow A\ d_i\ D$
- $A\ d_i\ B$, B *coveredBy* C, $C\ d_i\ D \Rightarrow A\ d_i\ D$

where d_i is one of the directions North, South, East, or West.

Thus an interval-based representation of spatial relations is useful for determining the composition of combinations of different types of spatial relations since it provides a canonical representation of these relations.

4.9 Summary

This chapter defined heterogeneous spatial reasoning and presented a formalism for performing inferences over heterogeneous spatial relations, in particular topological and directional relations. Examining the composition tables for such heterogeneous reasoning we note that it is most useful whenever the topological relationships between the objects concerned is one of containment. This is because the contained object always has the same directional relationships with other objects as the containing object.

The composition tables given in this chapter determine the result of composing pairs of spatial relations of two different types to give inferred relations of either type. It is possible to envision yet another type of composition table. One which gives spatial relations of a third type inferred from the composition of a pair of spatial relations of two different types. An example of such a composition table would be one that gives the topological relations inferred from the composition of a qualitative distance and a directional relation. Such heterogeneous composition tables are not considered in this work.

If, however, a mapping can be established from say qualitative distance onto directions and vice versa, then the above heterogeneous composition becomes a case of composing two qualitative distances or two directions and inferring topological relations. This is mixed spatial reasoning, a variation on heterogeneous spatial reasoning, whereby spatial relations of one type, e.g. topological, are inferred from the composition of spatial relations of a different type, such as directional relations. The following chapter describes a mechanism for mixed spatial reasoning.

Chapter 5

Mixed Spatial Reasoning

Mixed spatial reasoning, in the context of this thesis, is the inference of spatial relations of one type from the composition of two spatial relations of a different type. An example of mixed spatial reasoning is the inference of topological relations, between objects *A* and *C*, from the composition of the directional relation between objects *A* and *B* and the directional relation between objects *B* and *C*.

This chapter introduces a formal framework for performing such inferences about topology, direction, and qualitative distance information. It starts with a definition of the types of mixed spatial relation compositions that can be performed. The subsequent sections describe the composition of: topological relations giving directional relations or qualitative distances (Section 5.2); directional relations giving topological relations or qualitative distances (Section 5.3); and qualitative distances giving topological relations or directional relations (Section 5.4).

5.1 Types of Mixed Spatial Relation Compositions

For each of the three types of spatial relations, the composition of a pair of relations results in spatial relations of the other two types. Thus a total of six distinct composition tables must be derived (Figure 5.1).

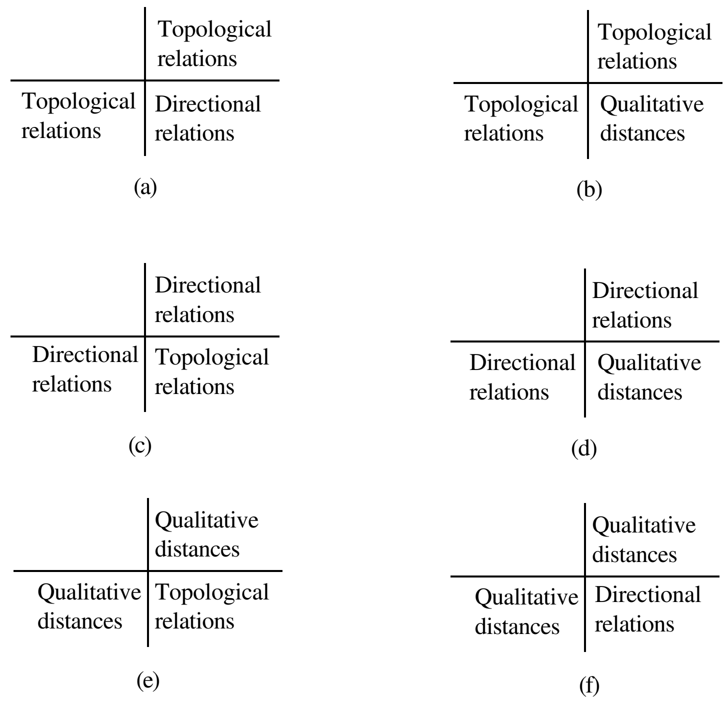


Figure 5.1 Mixed spatial relation compositions.

The following sections give the composition tables for topological relations, directional relations, and qualitative distances.

5.2 Composition of Topological Relations

The composition table for topological relations, Table 3.1 in Section 3.2, can be used to derive the tables for inferring directional relations and qualitative distances from the composition of topological relations. The topological relations *disjoint* or *meet* map onto the qualitative distance *Not SamePlace* or any one of the eight directional relations. Similarly, the topological relations other than *disjoint* or *meet* all map onto the qualitative distance *SamePlace* or onto the directional relation, *OverlapNoDirection*.

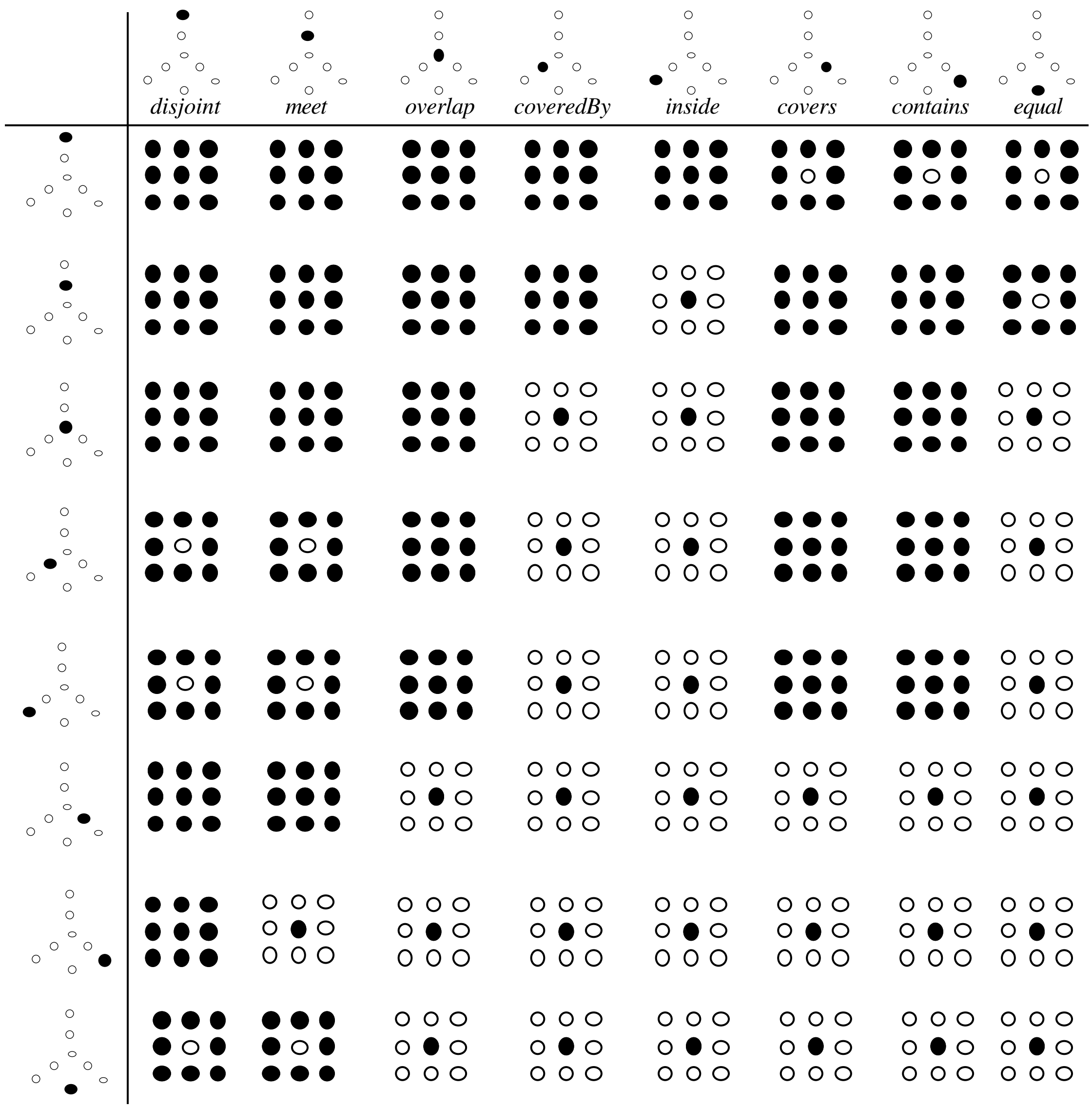


Table 5.1 Topological relation ; topological relation ⇒ directional relations.

Table 5.1 gives the directional relations implied by the composition of topological relations. It is evident that the only information provided by this table is that the composition of topological relations implies only that a directional relation exists or that it does not exist. No specific directional relation, or set of possible directional relations, can be determined because topological relations are independent of the ordering among objects, whereas directional relations depend on the ordering of objects along an axis.

[Table 5.2](#) gives the qualitative distances implied by the composition of topological relations. It was derived by mapping the topological relations *disjoint* or *meet* onto the qualitative distance relation *Not SamePlace*, and the remaining topological relations onto the qualitative distance relation *SamePlace*.

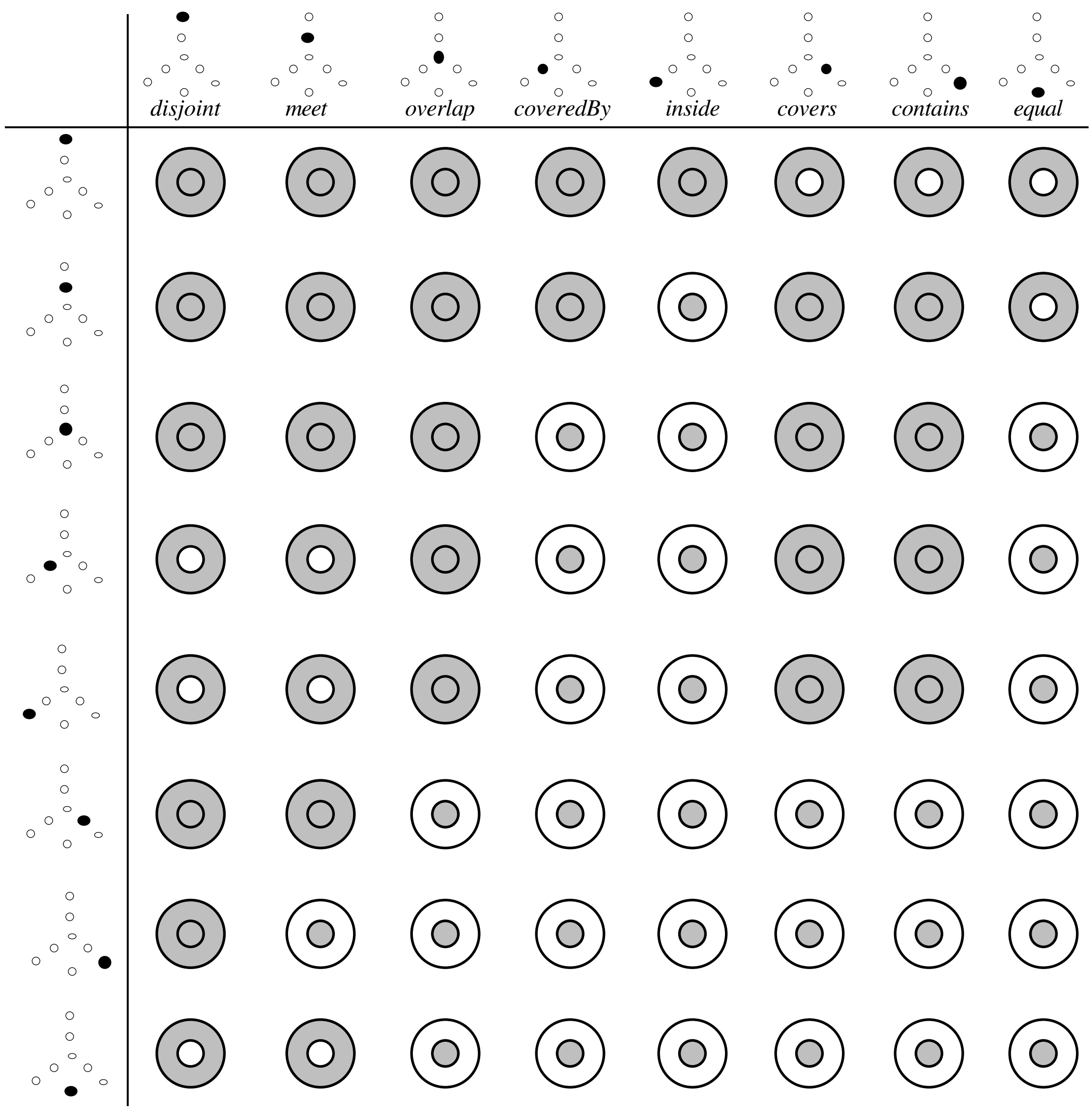


Table 5.2 Topological relation ; topological relation ⇒ qualitative distances.

Table 5.2 is identical to table 5.1 and can be obtained from it by substituting *SamePlace* wherever *OverlapSameDirection* occurs and *Not SamePlace* wherever *OverlapSameDirection* does not occur.

5.3 Composition of Directional Relations

In this thesis directional relations are characterized by pairs of interval relations. The composition of pairs of interval relations also determines topological and qualitative distance relations. For example, the direction *Northwest* implies that any one of the interval pairs from among the set {(Before, After), (Before, MetBy), (Meets, After), (Meets, MetBy)} is possible. Therefore, determining the topological relation implied by the composition *Northwest* ; *Northwest* involves the compositions of the elements formed by the Cartesian product of the above set with itself (Equation 5.1).

$$
\begin{aligned}
&\mathit{Northwest; Northwest} = \\
&\quad \{(\mathit{Before, After}),(\mathit{Before, MetBy}),(\mathit{Meets, After}),(\mathit{Meets, MetBy})\}; \\
&\quad \{(\mathit{Before, After}),(\mathit{Before, MetBy}),(\mathit{Meets, After}),(\mathit{Meets, MetBy})\} \\
&= \{(\mathit{Before, After});(\mathit{Before, After})\} \cup \{(\mathit{Before, After});(\mathit{Before, MetBy})\} \\
&\cup \{(\mathit{Before, After});(\mathit{Meets, After})\} \cup \{(\mathit{Before, After});(\mathit{Meets, MetBy})\} \\
&\mathrm{M} \\
&\cup \{(\mathit{Meets, MetBy});(\mathit{Before, After})\} \cup \{(\mathit{Meets, MetBy});(\mathit{Before, MetBy})\} \\
&\cup \{(\mathit{Meets, MetBy});(\mathit{Meets, After})\} \cup \{(\mathit{Meets, MetBy});(\mathit{Meets, MetBy})\} \\
&= (\mathit{Before, After}) \\
&= \mathit{disjoint}
\end{aligned} \tag{5.1}
$$

Table 5.3 gives the topological relations implied by the composition of directional relations.

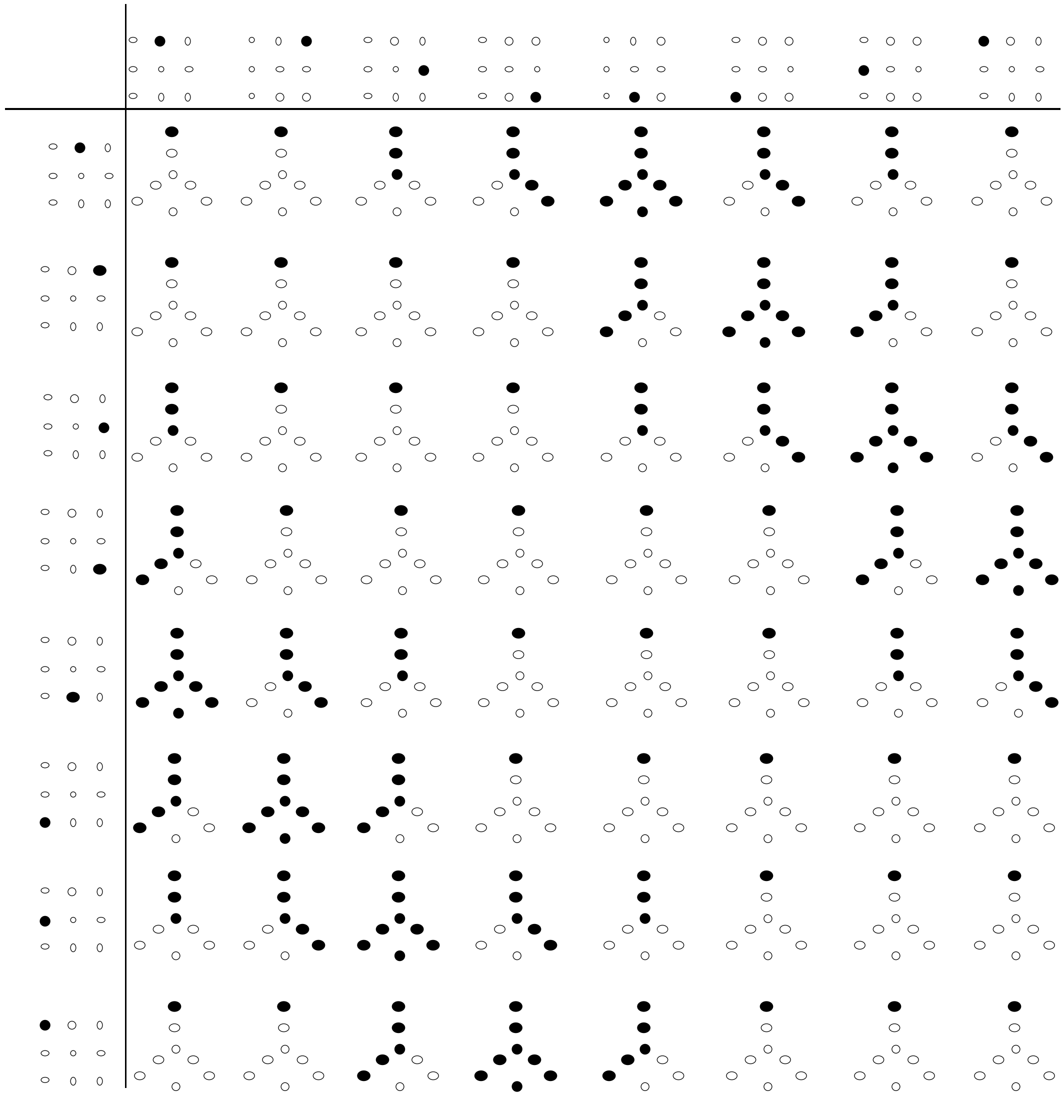


Table 5.3 Topological relations inferred from the composition of directional relations.

From Table 5.3 one may observe that the relation *disjoint* is inferred whenever the pair of composed directional relations has a direction in common. Thus, *disjoint* is the inferred relation when composing *North* with *North*, *Northwest*, or *Northeast* and similarly when composing *Northwest* with *North*, *Northwest*, *Northeast*, *West*, or *Southwest*.

Mapping the topological relations in Table 5.3 onto the qualitative distance relations *SamePlace*, *Not SamePlace*, and *SamePlace* or *Not SamePlace* gives Table 5.4.

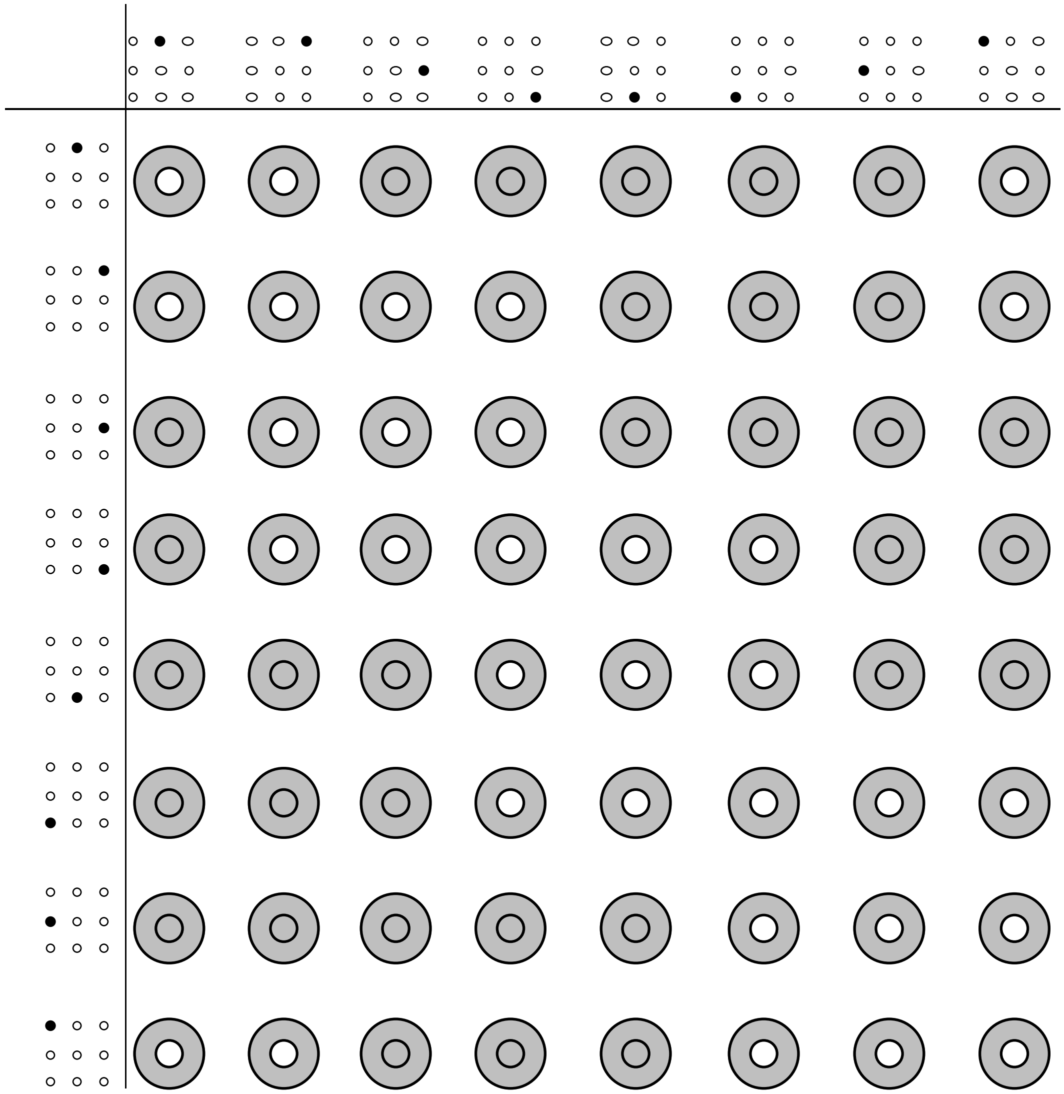


Table 5.4 Qualitative distances inferred from the composition of directional relations.

Table 5.4 is identical to table 5.3 since the topological relations map onto qualitative distances. The following section presents the composition tables for inferring topological and directional relations from the composition of qualitative distances.

5.4 Composition of Qualitative Distances

The simple definition of qualitative distances between regions, *SamePlace* and *Not SamePlace*, essentially means that no direction information can be inferred from their composition (Table 5.5).

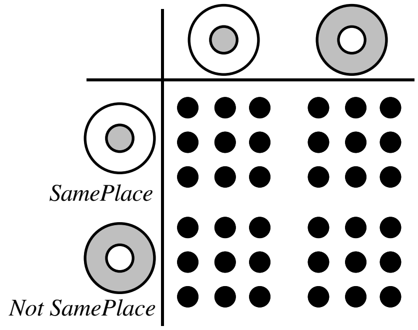


Table 5.5 Directional relations inferred from the composition of qualitative distances.

In the case of topological relations, however, some information can be obtained since the qualitative distances map onto topological relations. *Not SamePlace* maps onto the relations *disjoint* or *meet*, whereas *SamePlace* maps onto the remainder of the set of eight binary topological relations. Table 5.6 was derived using these mappings.

The entry for *SamePlace* ; *Not SamePlace*, for example, was derived from the composition of the two sets of topological relations (Equation 5.2).

$$
\begin{aligned}
&SamePlace; Not\ SamePlace = \\
&\quad \{overlap, inside, coveredBy, contains, covers, equal\}; \{disjoint, meet\} \\
&= \{disjoint, meet, overlap, covers, contains\} \cup \{disjoint, meet\} \\
&\cup \{meet, overlap, covers, contains\} \cup \{disjoint\} \cup \{meet\} \\
&\cup \{overlap, covers, contains\} \\
&= \{disjoint, meet, overlap, covers, contains\}
\end{aligned} \tag{5.2}
$$

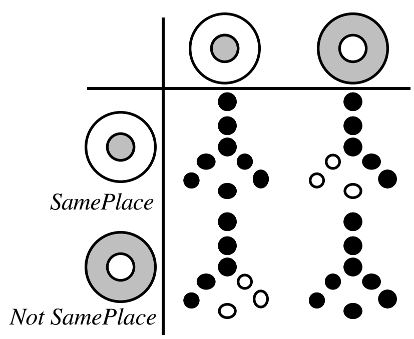


Table 5.6 Topological relations inferred from the composition of qualitative distances.

The difference between tables 5.5 and 5.6 arises from the fact that topological relations do not record the ordering among objects and hence do not imply a directional relation. The qualitative distances map onto topological relations and therefore the composition of qualitative distances provides no direction information.

5.5 Summary

This chapter presented the composition tables that allow for the inference of spatial relations of one type from the composition of spatial relations of a different type. While the tables seem to indicate that mixed spatial reasoning often provides little or no additional information, this is always the case. The following chapter demonstrates that mixed spatial reasoning is crucial for combined reasoning about topological and directional relations and makes combined spatial reasoning equivalent to integrated spatial reasoning.

Chapter 6

Integrated Spatial Reasoning

Integrated spatial reasoning in the context of this thesis is the inference of spatial relations from completely specified information on relations of different kinds for each object pair of interest. For example, integrated spatial reasoning is involved when inferring the topological and directional relation between objects *A* and *C* given the topological and directional relations between objects *A* and *B*, as well as the same types of relations between *B* and *C*. The term *integrated* is used to denote the fact that different kinds of spatial relations are used together when specifying the spatial relation between objects such as "*A* is North of and *disjoint* from *B*." For the remainder of this thesis the term integrated spatial relations refers to the conjunction of topological and directional spatial relations.

Information on both topological and directional relations may enhance our knowledge about the locational relationship between objects. For example, knowing that *A* is *North* of *B* in addition to the fact *A disjoint B* limits the subset of space in which *A* can lie relative to *B*. Similarly knowing that *A meets B* in addition to the fact *A* is *North* of *B* also constrains the possible location of *A* relative to *B* to a greater degree than the single fact *A North* of *B*. This situation, where both topological and directional relations between all pairs of objects are known, differs from situations where heterogeneous spatial reasoning is used. In the case of heterogeneous spatial reasoning the available information consists of a spatial relation of one type, such as topological, between one pair of objects and a spatial relation of a different type, say directional, between the other pair of objects. When information about all types of spatial relations between both pairs of objects is available then that information may be integrated when used for spatial reasoning. This chapter describes a framework for spatial reasoning by integrating information about different spatial relations between pairs of objects.

Integrating spatial information allows inferences such as A is North of and *disjoint* from C given the facts that (1) A is *North* of and *meets* B and (2) B is *North* of and *meets* C. The above inference is impossible using directional or topological relations individually. Thus integrated spatial reasoning may provide additional capabilities for any automated spatial reasoning system. In the above case, however, utilizing the fact that topological relations can be inferred from the composition of directional relations allows for the same inference from the same facts using combined spatial reasoning. We evaluate the capabilities of integrated spatial reasoning and compare them with combined spatial reasoning. This comparison forms the test of our hypothesis that combined spatial reasoning, about topological and directional relations, is equivalent to integrated spatial reasoning and both provide the same set of inferred spatial relations.

In order to perform inferences about integrated spatial relations, however, we first need a definition of these spatial relations and a representation scheme. Since we are concerned with only topological and directional relations, we use interval relations as the representation and method for defining integrated topological and directional spatial relations. As a consequence, the composition of pairs of interval relations is used to determine the composition of the integrated topological and directional spatial relations.

The following sections give an example demonstrating the utility of integrated spatial reasoning, followed by a description of the spatial relations themselves and a mechanism for determining their composition tables. Finally, we compare the results of using integrated spatial reasoning, when all relevant information is available, with the results of using combined spatial reasoning.

6.1 Integrated Topological and Directional Spatial Relations

Directional relations determine the relative orientation between objects whereas topological relations do not. Based on the definition of directional relations used in this thesis the topological relation between objects that have a particular directional relation may only be *disjoint* or *meet*. Thus taken together the two types of relations may constrain the spatial relationship between objects further than the individual relations do, and provide the enhanced capability to reason about conjunctions of spatial relations. A prerequisite for such integrated reasoning, however, is the definition of integrated spatial relations. The following paragraphs define the integrated topological and directional relations used in this study.

The directional relations of interest is the set {*North, Northeast, East, Southeast, South, Southwest, West, Northwest*}. From among the various definitions possible for directional relations (Section 4.2) between region objects we choose one that allows objects to be *disjoint* or *meet* along the reference axis, North-South or East-West. This definition implies a topological relationship, thereby facilitating integrated reasoning. The implication of the definition of directional relations is that the only possible topological relations between object that have one of the above eight directional relationships are *disjoint* and *meet*. Therefore, the integrated topological and directional relations must at least be a Cartesian product of the sets {*disjoint*, *meet*} and {*North, Northeast, East, Southeast, South, Southwest, West, Northwest*}, giving sixteen relations. These are: *DisjointNorth, DisjointNortheast, DisjointEast, DisjointSoutheast, DisjointSouth, DisjointSouthwest, DisjointWest, DisjointNorthwest, MeetNorth, MeetNortheast, MeetEast, MeetSoutheast, MeetSouth, MeetSouthwest, MeetWest, and MeetNorthwest*. The above set of sixteen relations, however, does not provide a complete coverage since objects may *overlap* or have a containment relationship. For such situations we introduce the broadly defined relationship *OverlapNoDirection*. This relation specifies that the objects share a common region and includes the possibility that one object is contained in or equal to the other. Thus the sixteen relations plus *OverlapNoDirection* provide a complete coverage and an exhaustive set of integrated topological and directional relations between regions.

The following section describes how the sixteen integrated relations are mapped onto pairs of interval relations between objects. The pairs of interval relations are the ones used in determining the composition of integrated topological and directional relations.

6.2 Mapping Integrated Relations onto Interval Pairs

The mapping of integrated relations onto interval pairs is done by taking the intersection of the individual mappings of topological and directional relations onto interval pairs. The topological relations of interest are *disjoint* and *meet* only due to the definition of directional relations between regions. These topological relations map onto interval pairs formed by the Cartesian product of the set {Before, Meets, MetBy, After} with itself. In the case of directional relations the interval relations Before and Meets correspond to West or South depending on the axis. Similarly, the interval relations After and MetBy correspond to East or North depending on the axis. For pairs of interval relations the order of the elements in the pair is important. Figure 6.1 shows the correspondence between direction information and interval pairs.

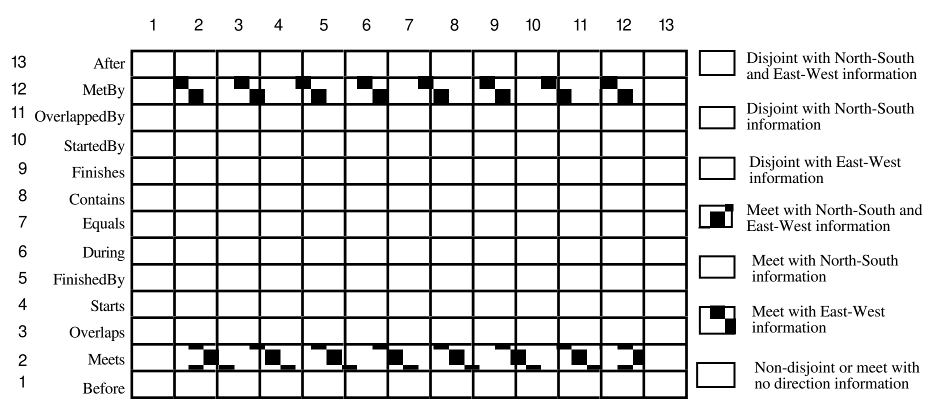


Figure 6.1 Directions and corresponding interval pairs.

The numbers along each edge represent the interval relations ranging from Before to After. The top right corner, therefore, corresponds to the interval pair (After, After) and the bottom left corner to the pair (Before, Before). Now After implies East along the first axis, horizontal in Figure 6.1, and North along the second axis, which is the vertical axis in Figure 6.1. Therefore the top right corner corresponds to *Northeast* and the bottom left to *Southwest*. The topological relation information implied by the interval relations is *disjoint*, *meet*, or neither In the figure, the cross-hatched patterns correspond to relations where the objects may touch and the horizontal and vertical hatched patterns indicate disjointedness of objects. The interval relation After implies *disjoint* regardless of the axis. If After is interval relation along the vertical axis then it implies the directional relation North while the East-West relationship is implied by the interval relation along the horizontal axis. If, however, the interval relation along the horizontal axis is from the set {Overlaps, Starts, FinishedBy, During, Equals, Contains, Finishes, StartedBy, OverlappedBy} then interval relation After along the vertical implies that one region is North of the other. Using this correspondence the relation *DisjointNorth* maps on to the set of interval pairs, {(Overlaps, After), (Starts, After), (StartedBy, After), (Contains, After), (Equals, After), (Finishes, After), (FinishedBy, After), (During, After), (OverlappedBy, After)}. Similarly, the relation *MeetNorth* is obtained by substituting the interval relation Meets for After in the above set.

6.3 Composition of Integrated Topological and Directional Relations

The integrated topological and directional relations are represented iconically using a cyclic pattern of circles corresponding to the eight directions, two topological relations, and overlapping objects (Figure 6.2).

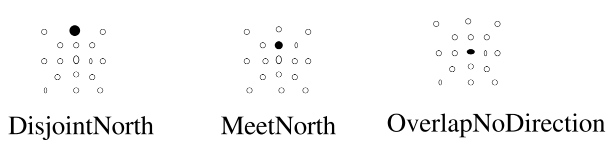


Figure 6.2 Iconic representation of integrated relations.

The outer circles correspond to the relations *DisjointNorth* through *DisjointNorthwest*, the inner circles to the relations *MeetNorth* through *MeetNorthwest*, and the center to *OverlapNoDirection*. The same iconic representation can be used to indicate a disjunction of relations. Figure 6.3 shows the representation of the relations Not South and (DisjointNortheast or MeetNortheast).

Not South

DisjointNortheast
or
MeetNortheast

Figure 6.3 Iconic representation of disjunctions of integrated relations.

Each of the integrated spatial relations, such as *DisjointNorth* or *MeetNorth*, map onto a set of interval relation pairs. For example, *DisjointNorth* maps onto the set {(Overlaps, After), (OverlappedBy, After), (Starts, After), (StartedBy, After), (Finishes, After), (FinishedBy, After), (Contains, After), (During, After), (Equal, After)} and *MeetNorth* maps on to {(Overlaps, MetBy), (OverlappedBy, MetBy), (Starts, MetBy), (StartedBy, MetBy), (Finishes, MetBy), (FinishedBy, MetBy), (Contains, MetBy), (During, MetBy), (Equal, MetBy)}. The composition of integrated spatial relations is then obtained by the same procedure used for heterogeneous topological and directional relation compositions. The composition of the two sets of pairs of interval relations is performed using Allen's transitivity table for interval relations and the resulting set of interval pairs is mapped back onto an integrated spatial relation.

For example, the result of the composition *DisjointNorth* ; *MeetNorth* is determined as follows:

$$
\begin{aligned}
DisjointNorth;MeetNorth &= \{(Overlaps, After) \ldots (OverlappedBy, After)\}; \\
&\quad \{(Overlaps, MetBy) \ldots (OverlappedBy, MetBy)\} \\
&= \left\{ \begin{matrix} (Overlaps, After);(Overlaps, MetBy) \\ \cup \ldots \cup (OverlappedBy, After);(OverlappedBy, MetBy) \end{matrix} \right\} \\
&= \{(Overlaps, After) \ldots (OverlappedBy, After)\} \\
&= DisjointNorth
\end{aligned}
$$

The ellipses used above convey that the set of pairs of interval relations is such that interval relation along the second axis is always After or MetBy while the interval relation along the first axis goes from Overlaps through to OverlappedBy thereby forming the nine pairs of interval relations corresponding to *DisjointNorth* or *MeetNorth*.

The following four tables give the results of the composition of integrated topological and directional relations. For each table the topological relation between objects pairs is fixed and the eight rows and columns correspond to the eight directions clockwise from *North* to *Northwest*. Table 6.1 is the composition table for the case when the topological relation between both pairs of objects (*A, B*) and (*B, C*) is *disjoint*.

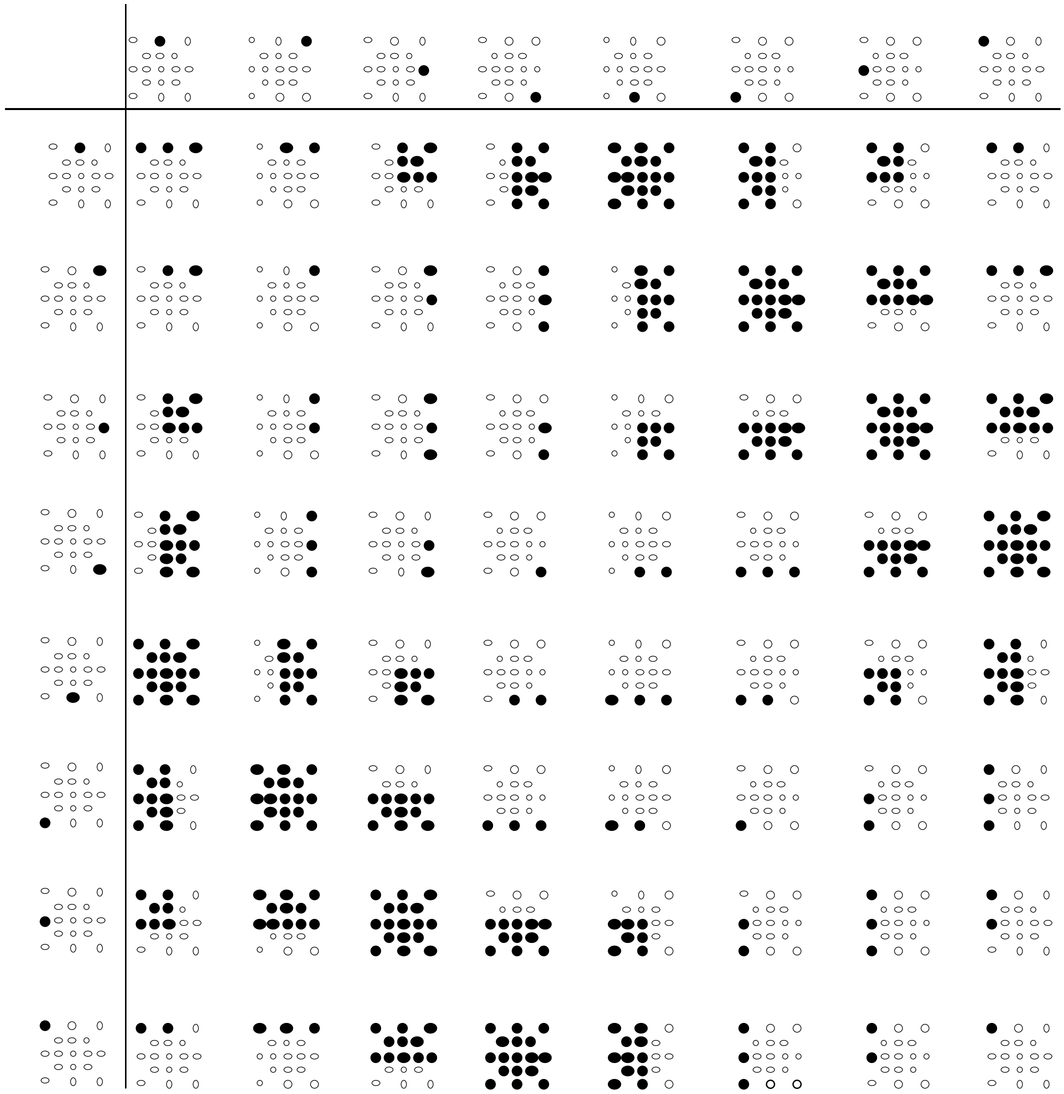


Table 6.1 Direction composition table with A *disjoint* B and B *disjoint* C.

Table 6.2 is the composition table for the case when the topological relation between the first pair of objects (*A, B*) is *disjoint* and the second pair of objects (*B, C*) *meet*. That is, the table gives the composition of relations such as *DisjointNorth* with *MeetEast*.

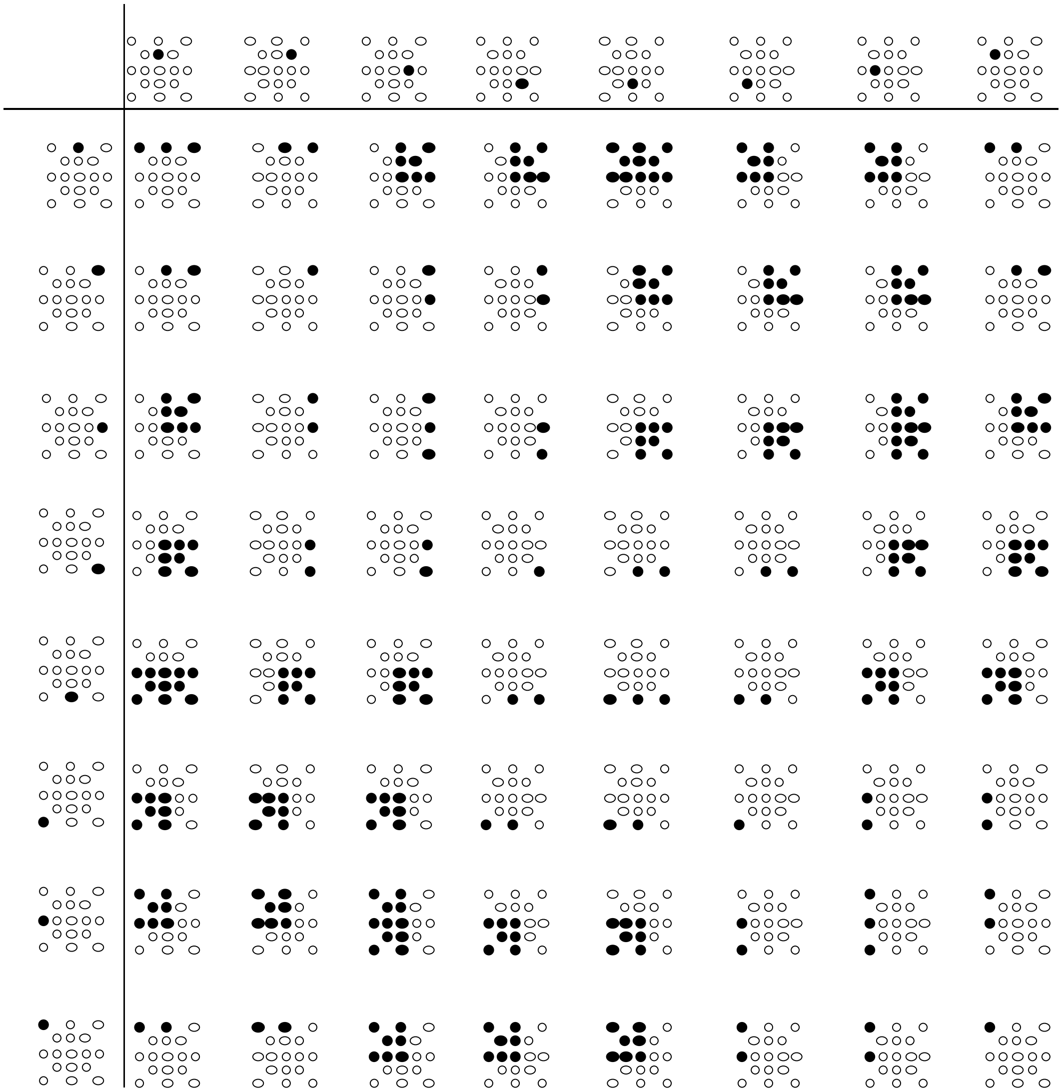


Table 6.2 Direction composition table with A *disjoint* B and B *meet* C.

Table 6.3 is the composition table for the case when the topological relation between the first pair of objects (*A, B*) is *meet* and the second pair of objects (*B, C*) is *disjoint*, that is, the table gives the composition of relations such as *MeetNorth* with *DisjointEast*.

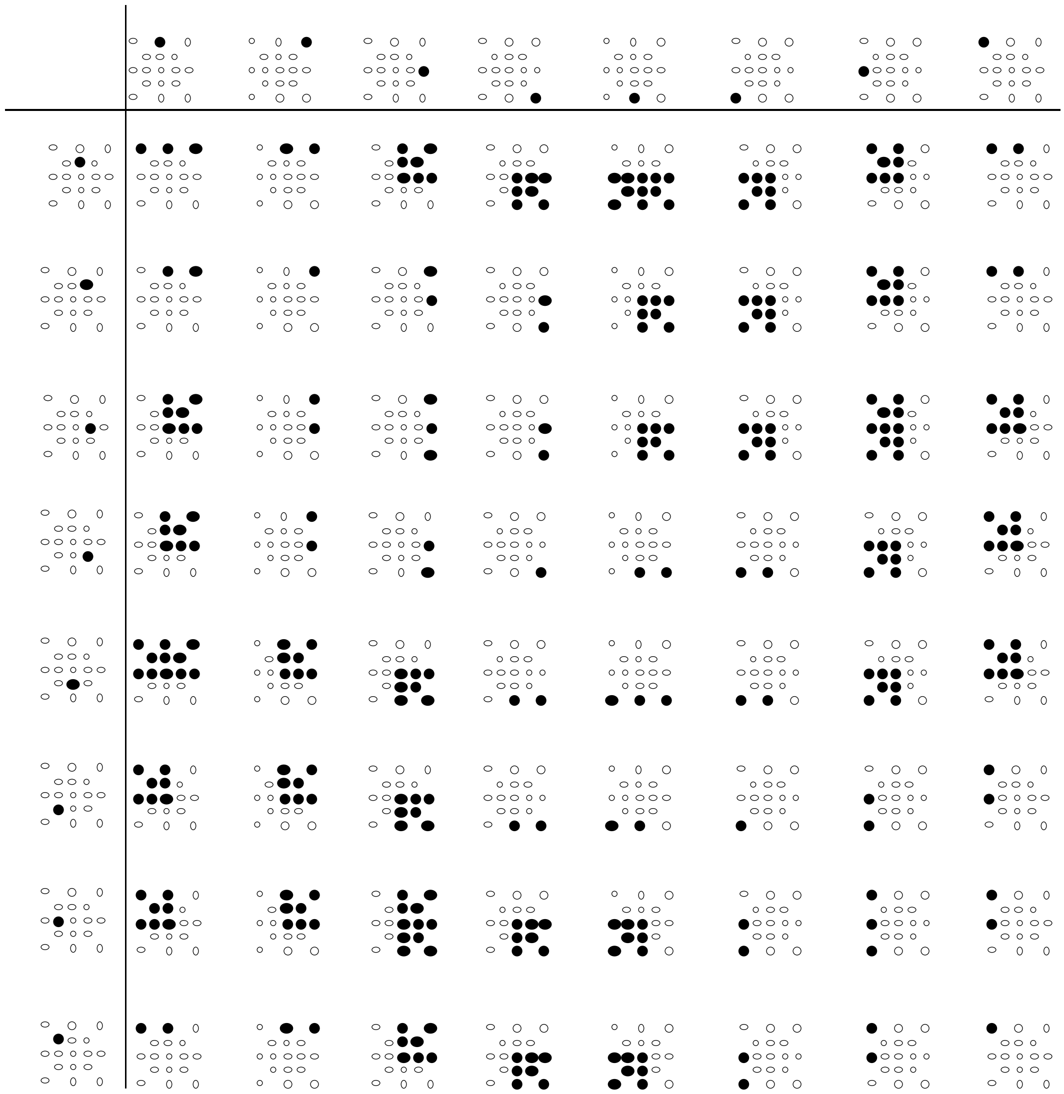


Table 6.3 Direction composition table with A *meet* B and B *disjoint* C.

Table 6.4 is the composition table for the case when the topological relation between both pairs of objects (*A, B*) and (*B, C*) is *meet*, that is, the table gives the composition of relations such as *MeetNorth* with *MeetEast*.

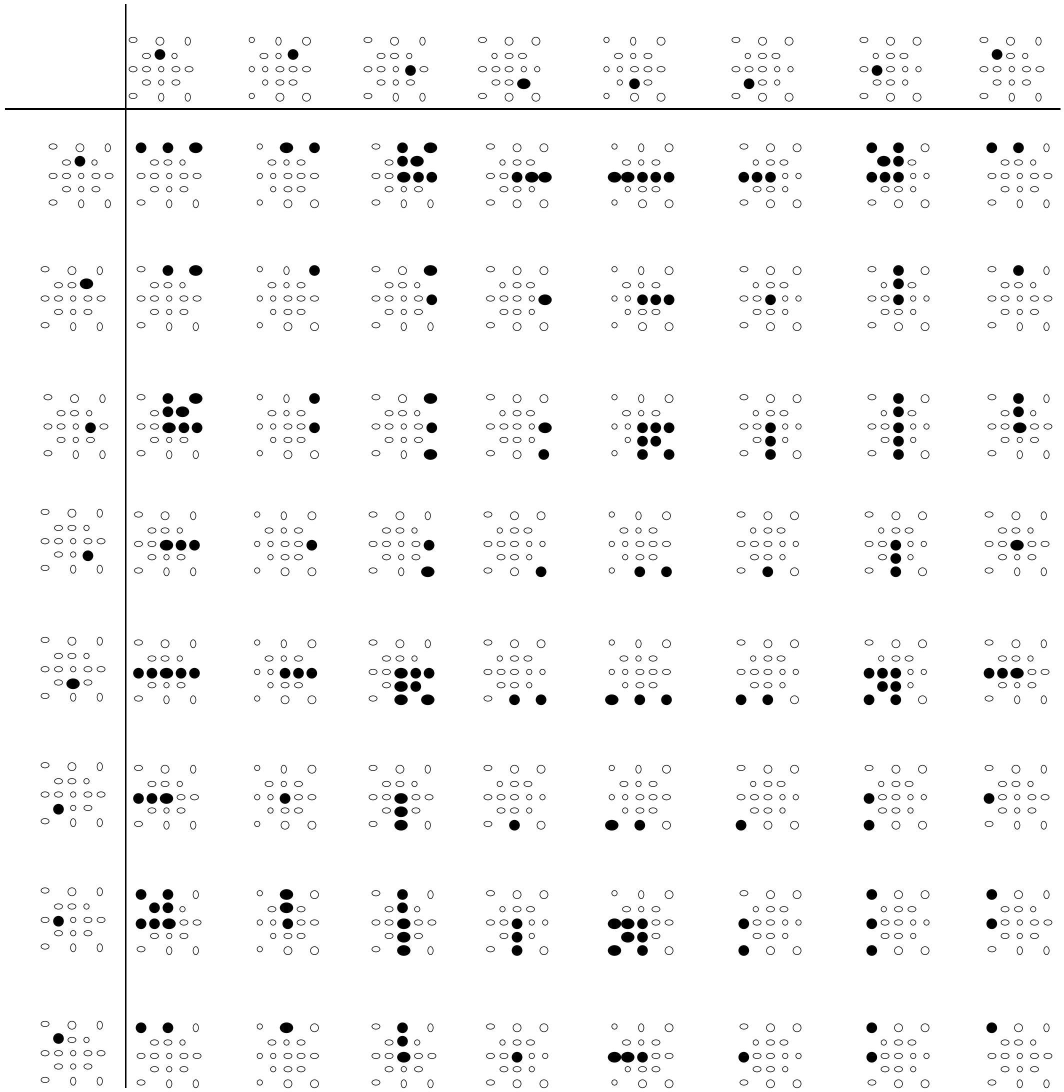


Table 6.4 Direction composition table with A *meet* B and B *meet* C.

Examining tables 6.1 through 6.4 we see that the diagonal [n, n], and off-diagonal, [n, n-1] and [n-1, n], entries all contain the topological relation *disjoint* only. For the directional relations *Northeast*, *Northwest*, *Southeast*, and *Southwest* the composition result contains only *disjoint* when the direction difference between the composed relations is two or less. Entries in the diagonal, and one- and two-off the diagonal, all contain only *disjoint* and not *meet*. An explanation of this pattern is provided through a comparison of integrated spatial reasoning with combined spatial reasoning in the following section.

6.4 Comparison of Integrated with Combined Spatial Reasoning

The only topological relations that participate in both types of spatial reasoning, i.e., combined and integrated, are the relations *disjoint* and *meet*. Therefore, we consider the reasoning capabilities of the two approaches in the situations where the topological relation involved is *disjoint* or *meet*.

Combined spatial reasoning is the coupling of homogeneous and heterogeneous and mixed spatial reasoning (Equation 6.1), whereas integrated spatial reasoning deals with the compositions of tuples of spatial relations.

$$\text{Combined} = \text{Homogeneous} \wedge \text{Heterogeneous} \wedge \text{Mixed} \tag{6.1}$$

For topological and directional relations, combined spatial reasoning, is formally specified by Equations 6.2a and 6.2b,

$$\{t_i \;;_{\text{t}}\, t_j\} \wedge \{t_i \;;_{\text{t}}\, d_j\} \wedge \{d_i \;;_{\text{t}}\, t_j\} \wedge \{d_i \;;_{\text{t}}\, d_j\} \Rightarrow t_k \tag{6.2a}$$

and

$$\{t_i \;;_{\text{d}}\, t_j\} \wedge \{t_i \;;_{\text{d}}\, d_j\} \wedge \{d_i \;;_{\text{d}}\, t_j\} \wedge \{d_i \;;_{\text{d}}\, d_j\} \Rightarrow d_k \tag{6.2b}$$

and integrated spatial reasoning by Equation 6.3.

$$[t_i, d_i] \;;\; [t_j, d_j] \Rightarrow \{[t_k, d_k]\} \tag{6.3}$$

The composition operators, however, differ for the two types combined and integrated spatial reasoning. For instance, combined spatial reasoning about topology and direction requires two composition operators "$;_{\text{t}}$" and "$;_{\text{d}}$". The first one returns topological relations and the second returns directional relations, given two spatial relations of either type (Equations 6.2a and 6.2b) whereas integrated spatial reasoning requires a single operator (Equation 6.3).

One form, combinations of spatial relations, is mapped onto the other, integrated spatial relations using the operations *group* and *ungroup*. Given the set $\{t_i, d_j\}$ of spatial relations between two objects, the operator *group* creates a tuple $[t_i, d_j]$, which is the integrated spatial relation between the two objects. The inverse operator, *ungroup*, creates the set from the tuple. *Group* and *ungroup* are required since the composition of integrated spatial relations is defined only on a tuple of spatial relations.

From the fact that the tuple denoting an integrated spatial relation is obtained by *grouping* a set of spatial relations of different types we note that Equation 6.3 is a compact expression of the compositions defined by Equations 6.2a and 6.2b. Hence the sets of spatial relations obtained using either inference mechanism, on combinations of spatial relations, should be equal (Figure 6.4).

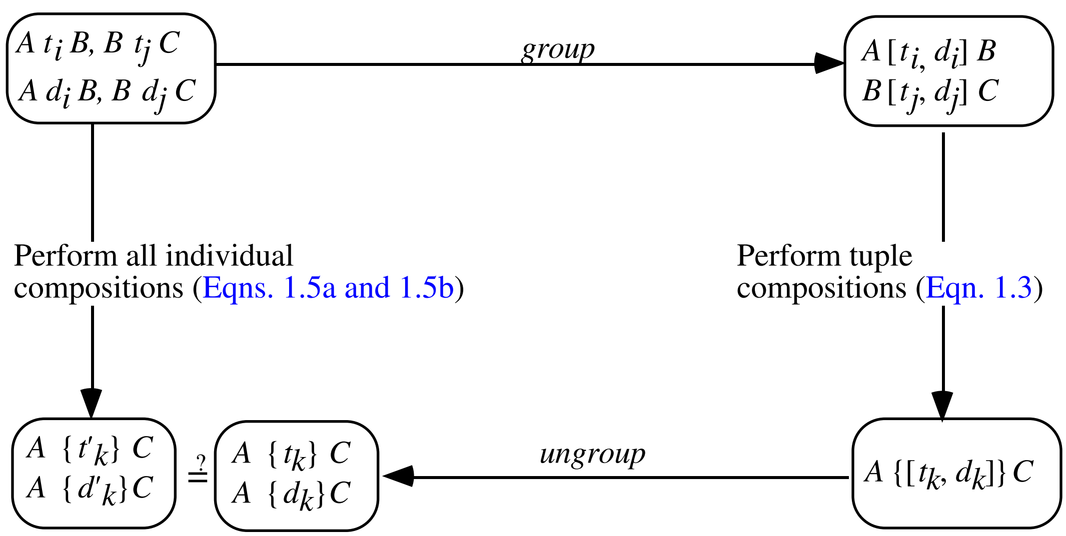


Figure 6.4 Composition of combinations of spatial relations.

This equality of sets of inferred results is indeed valid in the following situation. Given spatial relations (1) *A North B*, (2) *A meet B*, (3) *B East C*, (iv) *B meet C*. The integrated spatial relations in this instance are *A MeetNorth B* and *B MeetEast C* and the composition to be performed is *MeetNorth* ; *MeetEast*. Similarly, for this instance the combinations of spatial relations and their compositions are: (1) *North* $;_d$ *East*, (2) *North* $;_t$

East, (3) *meet* $;_t$ *meet*, (4) *meet* $;_d$ *meet*, (5) *North* $;_t$ *meet*, (6) *North* $;_d$ *meet*, (7) *meet* $;_t$ *East*, and (8) *meet* $;_t$ *East*. Using either integrated or combined spatial reasoning the inferred spatial relations are (1) *A* {*disjoint, meet, overlap, contains, covers*} *C*, and (2) *A* {*North*, *Northeast*, *East*} *C*.

Given that both methods for spatial reasoning give the same results in the above instance the question of interest is whether the two are equivalent in all instances. In order to determine equivalence we need the four composition tables (Tables 6.5 - 6.8 below) for combined spatial reasoning that correspond to the four composition tables for integrated spatial reasoning (Tables 6.1 - 6.4).

Tables 6.5 through 6.8 are derived using the composition tables for homogeneous, heterogeneous, and mixed spatial reasoning about topological and directional relations given in Chapters 3-5.

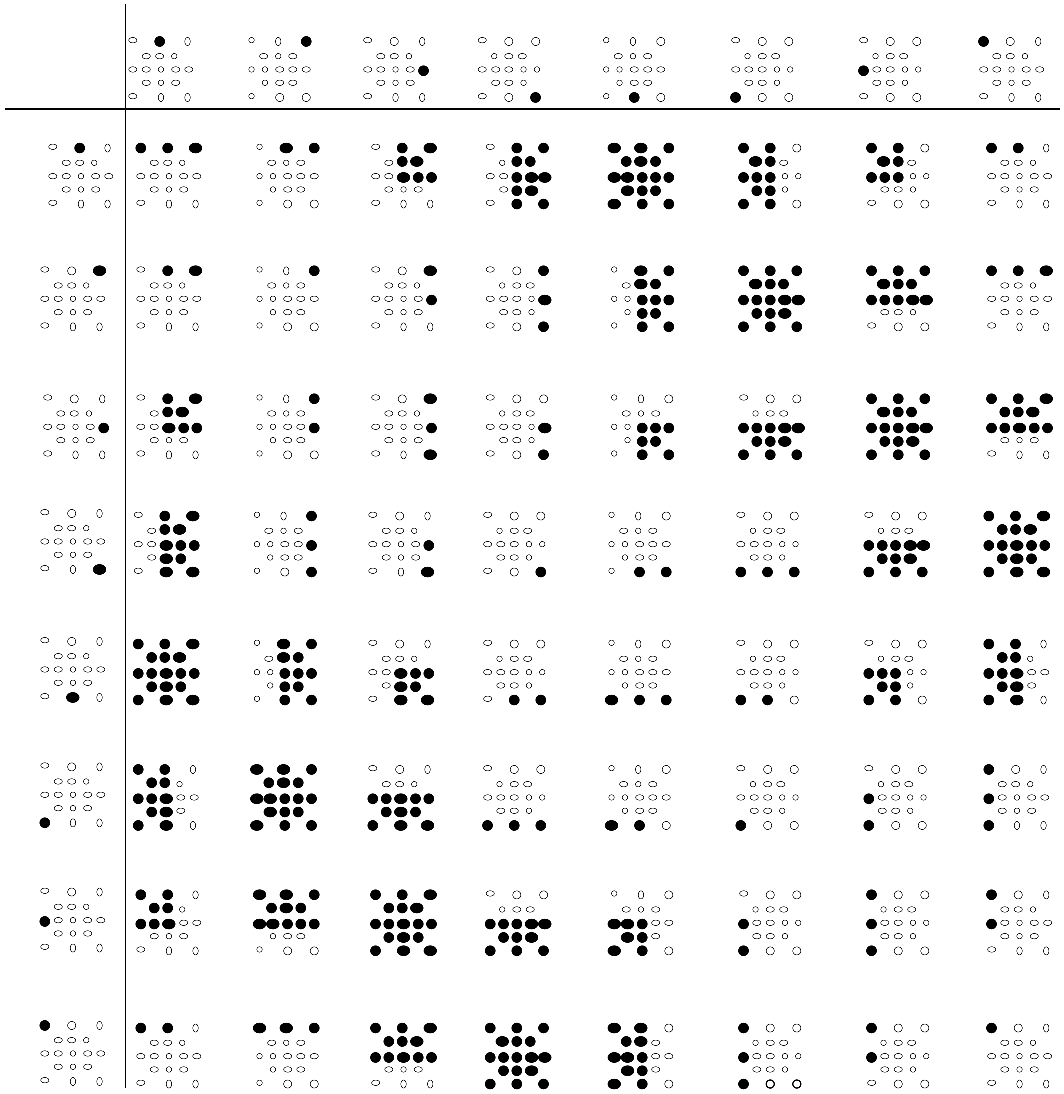


Table 6.5 Combined spatial reasoning with *A disjoint B* and *B disjoint C*.

Table 6.6 is the combined reasoning composition table for the situations where the first pair of objects are *disjoint* and the second pair *meet*.

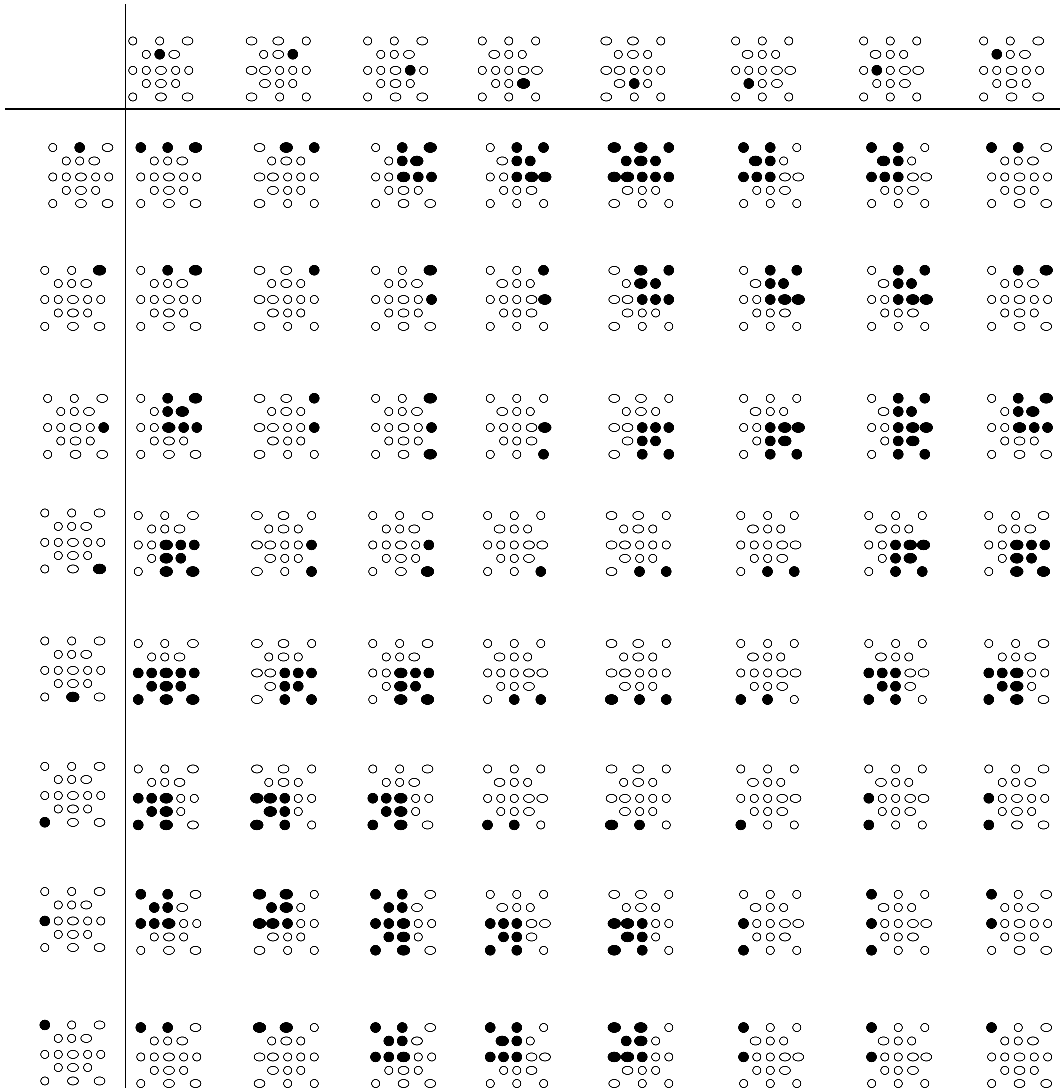


Table 6.6 Combined spatial reasoning with *A disjoint B* and *B meet C*.

Table 6.7 is the combined reasoning composition table for the situations where the first pair of objects *meet* and the second pair are *disjoint* .

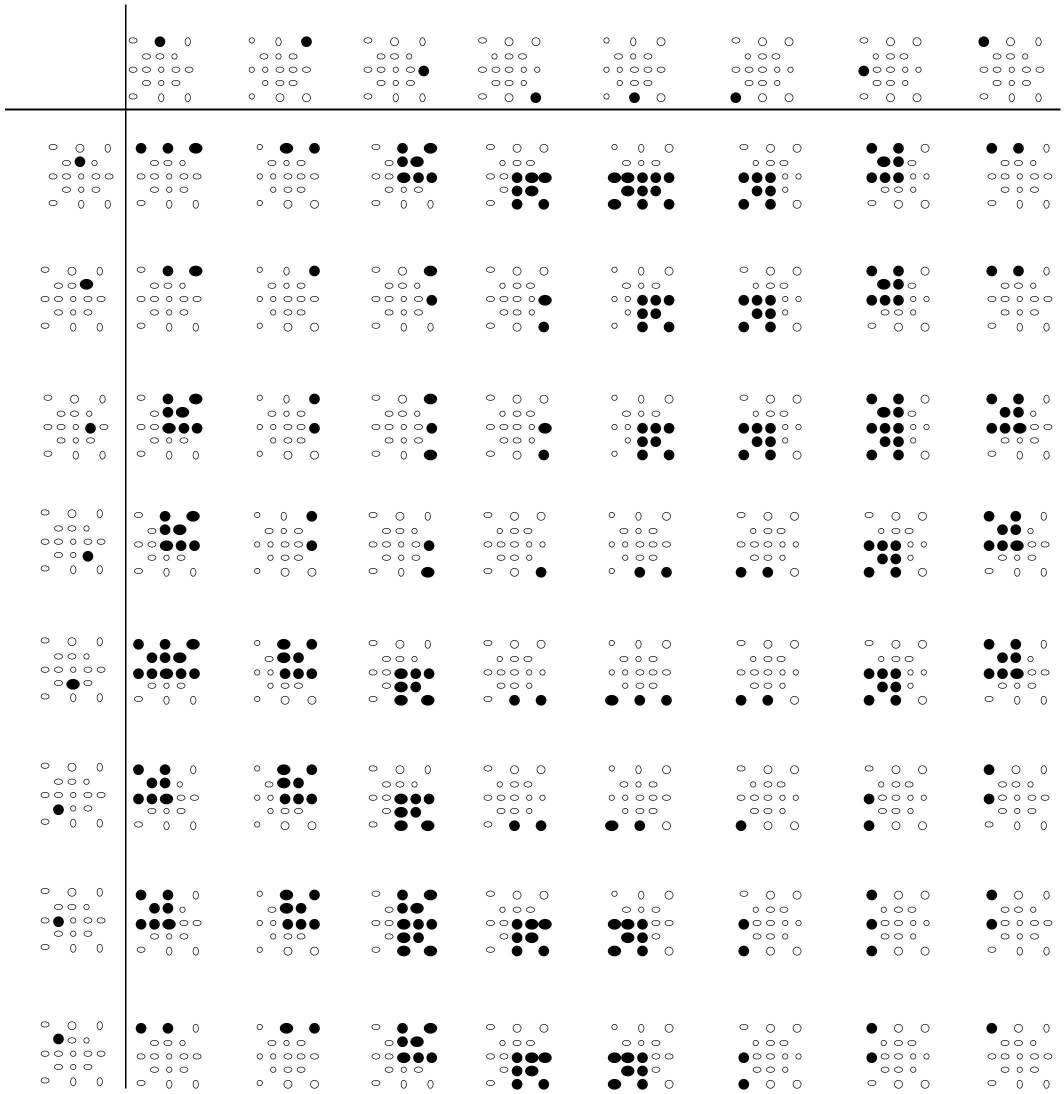


Table 6.7 Combined spatial reasoning with *A meet B* and *B disjoint C*.

Table 6.8 is the combined reasoning composition table for the situations where the topological relation between both pair of objects is *meet*.

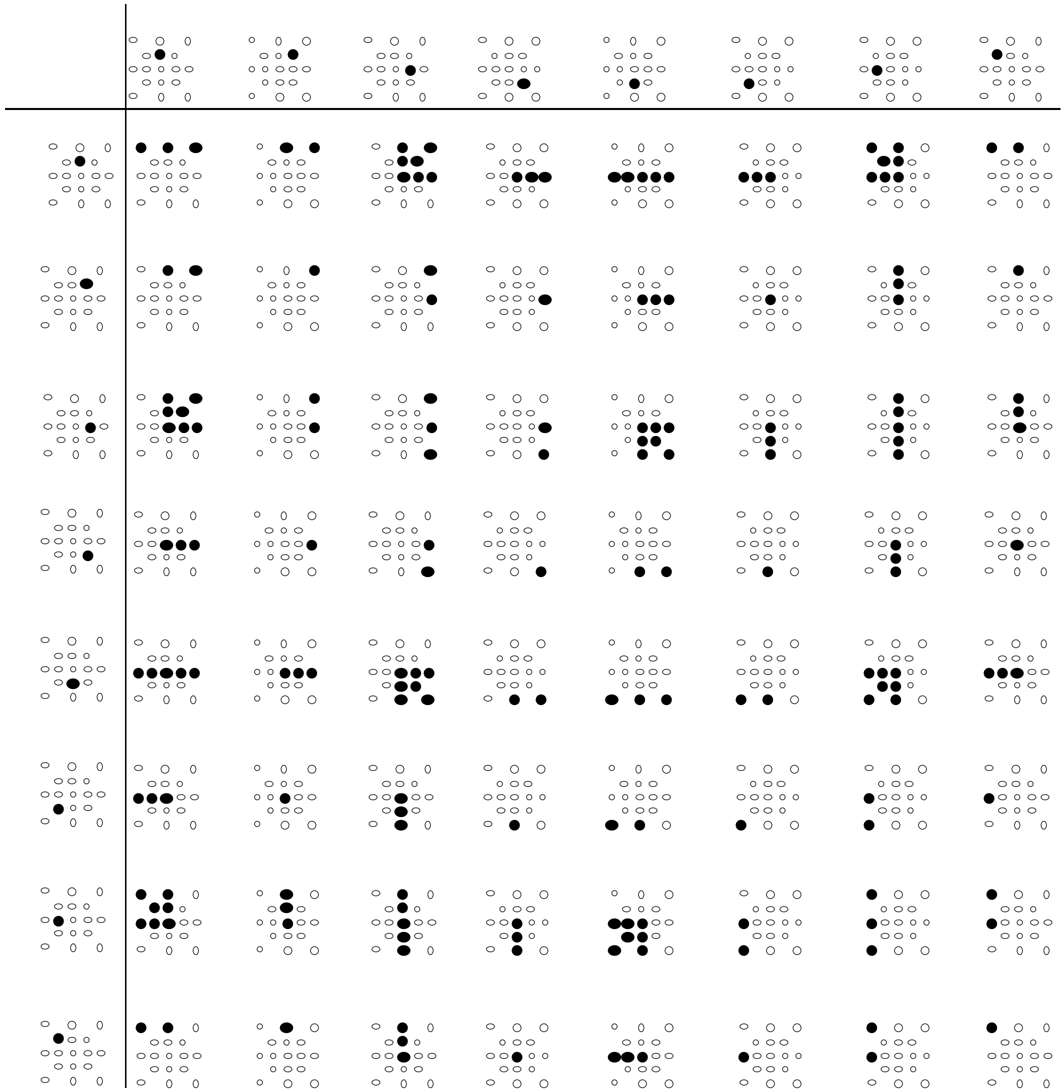


Table 6.8 Combined spatial reasoning with *A meet B* and *B meet C*.

Proposition : Integrated spatial reasoning and combined spatial reasoning result in the same set of inferred relations.

Proof: Tables 6.1 through 6.4 and tables 6.5 through 6.8 are identical. For topological and directional spatial relations, therefore, both integrated and combined spatial reasoning give the same set of inferred relations.

Since the set of inferred spatial relations is the same regardless of the spatial reasoning mechanism that is used we conclude that our hypothesis is valid and that integrated spatial reasoning is equivalent to combined spatial reasoning.

The same approach, comparing composition tables for combined and integrated spatial reasoning, is valid when qualitative distances are involved.

6.5 Discussion

The tables which give the composition for integrated and combined spatial reasoning have some patterns from which the following conclusions can be drawn:

- Integrated spatial reasoning gives the same results as combined spatial reasoning if mixed spatial reasoning is used for inferring topological relations from the composition of directional relations.

- If mixed spatial reasoning is not used then inferring a topological relation from the composition of directional relations can be done only by mapping the resultant directional relations onto topological relations. By definition a directional relation holds whenever the two objects are *disjoint* or *meet* each other. Using homogeneous or heterogeneous reasoning, therefore, the topological relationships *meet* and *disjoint* are in the inferred relation set. In the case of integrated reasoning, however, the topological relation *meet* is excluded from the set inferred from the composition of the same topological and directional relations. This is because the pairs of interval relations, resulting from the composition, contain information about directional and topological relations.

- The composition of directions that are conceptual neighbors does not ever result in the topological relation *meet* (Table 4.6). Therefore only *disjoint* is in the inferred set of spatial relations whenever the composition of topological and directional relations involves directions that are conceptual neighbors. This creates the pattern observed among the diagonal and off-diagonal entries in Tables 6.1 through 6.4.

6.7 Summary

In summary, we may conclude that integrated spatial reasoning is equivalent to combined spatial reasoning. Since a single table-lookup is more efficient than multiple lookups followed by set intersections, integrated spatial reasoning is the preferred choice whenever complete information is available about the qualitative spatial relationships among the objects. By complete information we mean information about the qualitative spatial relations of each type between each pair of objects. In other instances the homogeneous and heterogeneous methods of reasoning about topological and directional relationships should be used for inferring spatial relations. The following chapter proposes a framework and implementation of a comprehensive qualitative spatial reasoner that is capable of effectively utilizing the mechanisms for homogeneous, heterogeneous, mixed, and integrated spatial reasoning described thus far.

Chapter 7

A Framework for Qualitative Spatial Reasoning in GIS

The paradigm of "complete and exact spatial information" underlies the use of any current commercial Geographic Information System (GIS). Complete and exact refer to spatial data modeled in a Cartesian coordinate space with all coordinate information available for all spatial entities whose geometry is represented exactly by the coordinates. Therefore, current GISs are very good at integrating and analyzing *quantitative* spatial information.

The emphasis on quantitative is a result of the underlying assumptions regarding databases and information systems. These assumptions are that databases should contain consistent, accurate, and non-redundant data and that information systems must provide a definitive answer to any query (Egenhofer and Mark 1995).

We introduce a framework that pursues a radically different approach to handling geographic data. Rather than forcing a user to translate all spatial concepts into a quantitative framework, users can reason about *qualitative spatial information* within a purely qualitative environment. While quantitative models use absolute values, *qualitative models* deal with magnitudes. The advantage of qualitative reasoning models is that they can separate numerical analyses from the determination of magnitudes or events, which may be assessed differently depending on the context. In qualitative reasoning a situation is characterized by variables that can only take a small, predetermined number of values and the inference rules that use these values *in lieu* of numerical quantities that approximate them (de Kleer and Brown 1984).

7.1 Qualitative Spatial Reasoning in GIS

For a qualitative spatial reasoner, it is important to find representations that support partial and imprecise information. This qualitative spatial reasoning framework presented here is based on the representation of *explicit spatial relations*. It allows users to record spatial-relation information independent of the actual geometry of the spatial objects. Examples of explicit spatial relations are *cardinal directions* such as north or north-east; *approximate distances* such as near and far; and *topological relations* such as inside, disjoint, or overlap. Unlike Euclidean-based geographic databases and GISs, which provide for a single way of determining such relations by quantitative calculations, the qualitative spatial reasoner can infer information at different conceptual levels. In geographic applications entities are defined both in terms of their attributes and the complex spatial relations, such as proximity and connectivity, between them. This spatial structure is of primary interest in geographic databases and hence must be represented or modeled in some form. The two approaches are (1) to define it explicitly in a relational form or (2) to construct it using rules in a deductive database. We combine these approaches by defining relations as first-class objects, which have an associated spatial reasoning system; therefore, such a spatial reasoner has three basic approaches to determining the result of a geographic database query:

- use explicitly stored, qualitative information if it meets the requirements;
- infer the result with qualitative reasoning formalisms; and
- compute a qualitative spatial relation by transforming the query into a quantitative Euclidean coordinate space, in which the problem gets solved by applying algorithms

on some model such as a raster or vector representation, and map the quantitative result back onto a qualitative value.

Qualitative spatial inferences are performed using composition tables that define the set of possible spatial relations that can exist between two objects when their spatial relationships with a common third object are known. Homogeneous composition tables are required when the known spatial relations are all of the same type, for example, topological; heterogeneous composition tables when the relations are of different types, for example the topological relation between *A* and *B* and the directional relation between *B* and *C*; while integrated composition tables are required when two or more spatial relations between each pair of objects is known, for example both the topological and directional relations for the object pairs (*A*, *B*) and (*B*, *C*).

7.1.1 Explicitly Stored Relations

In the most simple situation, the spatial relation for which a user asks is explicitly stored in the database; therefore, a particular kind of spatial relation can be retrieved immediately and query processing becomes a simple table look up. For example, if the cardinal direction between Bangor and Orono is recorded, then there is no need to compute it.

7.1.2 Qualitatively Inferred Relations

The behavior of spatial relations is captured in relation algebras. These algebras formalize particular properties of relations that are crucial when deriving information.

- A relation r is symmetric if $A\ r\ B$ implies $B\ r\ A$.
- Two relations r_1 and r_2 are converse if $A\ r_1\ B$ implies $B\ r_2\ A$.

The most powerful property is the *composition*. It infers the relation r_1 between two objects A and C from the knowledge of the relations $A\ r_2\ B$ and $B\ r_3\ C$.

Frequently, the inferred relation is imprecise and represented by a disjunction of several possible relations. An important property of composition is that it distributes over disjunctions, i.e.,

$$r_i \,;\, (r_j \vee r_k) = (r_i \,;\, r_j) \vee (r_i \,;\, r_k) \tag{7.1}$$

The transitivity of a relation is a special case of the composition in which $r_1 = r_2 = r_3$ such that $A\ r\ B$ and $B\ r\ C$ implies $A\ r\ C$.

A constraint network (Guesgen and Hertzberg 1993) is used for representing the relations and for evaluating consistency or inferring relations. The nodes in the network represent individual objects, while each arc is labeled by the possible relations between the two objects at its nodes. Consistency is maintained by computing all consequences whenever a new relation is added to the network (Hernández 1993). The consequent relations are determined by computing the transitive closure of the relations using the appropriate transitivity table.

Such qualitative inferences allow a query processor to derive answers even if the particular spatial or temporal relation has not been recorded explicitly. For example, if a system stores that "Orono is *North* of Bangor" and a user asks the query for the cardinal direction between Bangor and Orono, given the knowledge that *North* and *South* are converse relations, the query processor can infer that Bangor is *South* of Orono.

7.1.3 Quantitatively Calculated Relations

Quantitative evaluation of spatial relations is the most commonly used method in commercial GISs. The geometry of the objects is represented in terms of a set of coordinates or pixels in some Cartesian reference grid. Computational geometry algorithms such as nearest neighbor search, point in polygon tests, and line-line intersections are used to compute spatial relations like *inside*, *within* X miles, or *north* of.

It is important to note that these spatial computations are performed on a model and a necessarily finite precision representation of the reality of interest. The results of the computations are either the values of the deisred spatial relations or they are mapped onto qualitative spatial relations between the real world entities. The underlying assumption here is that the model and its representation are an accurate and valid encapsulation of the nature and properties of the geographic entities.

7.2 Object-Oriented Design for a Qualitative Spatial Reasoner

The conceptual design of our qualitative spatial reasoner differs from conventional GISs and spatial reasoning systems by treating relations as explicit *objects*, rather than labeled *links* between spatial objects. This looser framework permits the system to be used as a test bed for qualitative reasoning, which can be expanded to accommodate additional types of (spatial) relations. It leads to an object-oriented implementation in which objects as well as relations have operations and respond to messages.

Figure 7.1 shows the object hierarchy developed for the reasoner. There are two first-order classes in this hierarchy, the **Relation** and the **Relative**. The **Relative** is the abstract class for objects involved in **Relations**. It provides methods for getting access to the **Relations** in which a **Relative** is involved, getting its name, and for dissolving **Relations** when the **Relative** is deleted. All **Relatives** require a name, which is used as an identifier.

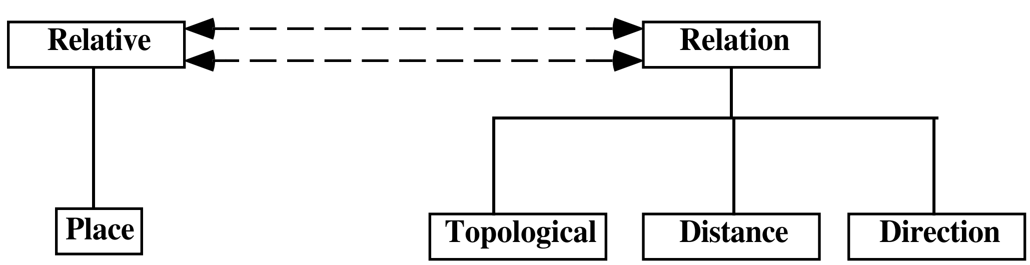


Figure 7.1 Object hierarchy for a qualitative geographic reasoner.

A **Place** is a **Relative** with a location and may participate in a spatial relation. For example, the **Place** Bangor is *South* of Orono. The location was defined as a 3-dimensional Cartesian coordinate. Much of the reasoning over **Places** assumes a representative coordinate for objects that have, at least, a two-dimensional extent. This ambiguous representation is very similar to human reasoning over hierarchically ordered spaces (Frank 1992), but can produce erroneous results on occasion. For instance, when you are in Reno Nevada, California is to the *Northwest*, *West*, *Southwest*, *South* and *Southeast*, but the result calculated from the representative points would be *South* or even *Southeast*.

The **Relation** class is an abstract class that binds two **Relatives** in a spatial relation. It provides basic utilities for relation management such as retrieving the **Relatives** involved, and creating and destroying the **Relation**. Each of the types of relations described in the previous section is a subclass of **Relation**. It has its methods for accessing an object's value, e.g., the value *overlaps* in a **Topological Relation** (Figure 7.2). Again it should be noted that the **Relation** has a value appropriate to its type, and limited reasoning can be accomplished without reference to the values stored by the **Relatives** involved in the **Relation**.

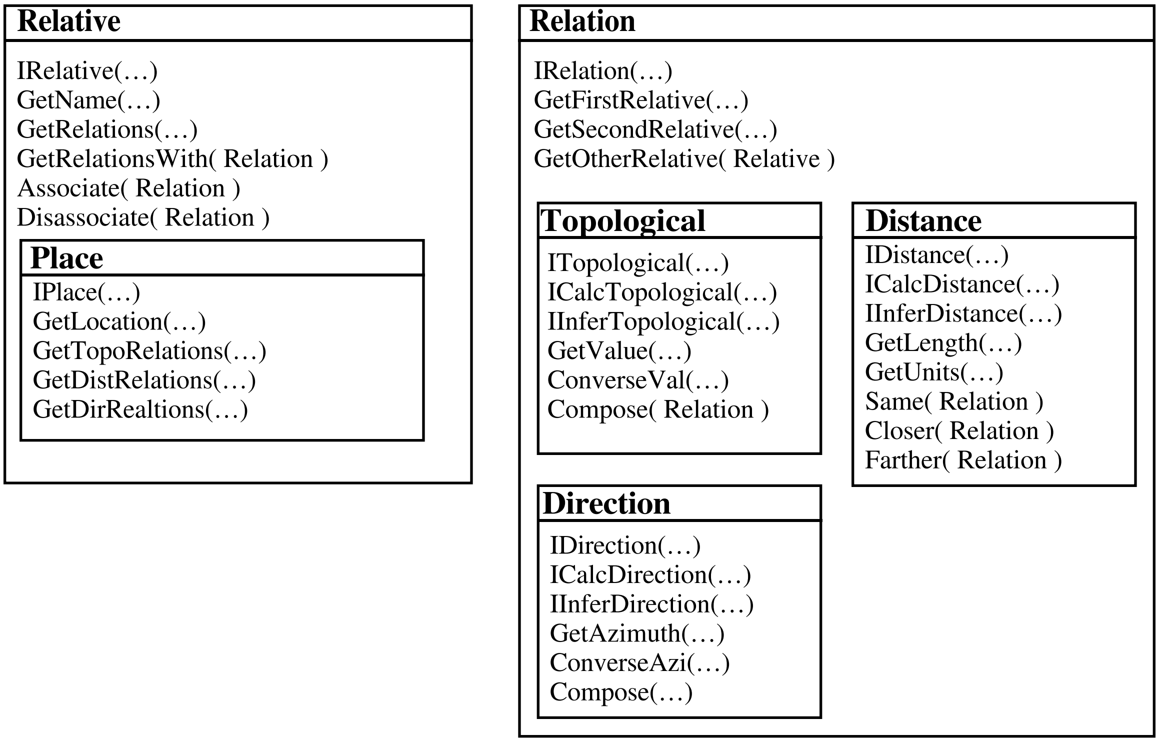


Figure 7.2 Methods for **Relative** and **Relation** classes and their subclasses.

Each class provides three separate initializers for explicit, calculated, and inferred creation.

- Explicit creation is used when a fact is known about two **Relatives**, for example Edinburgh *is north of* London. Neither of the relatives referenced in the relation need to have exact coordinates to be part of an explicit definition. This is similar to receiving a news report that there was rioting in South Central Los Angeles.

- Calculated creation depends on both **Relatives** being completely defined quantitatively. This is the way most GISs currently treat topology definition and, therefore, complete and accurate geometric definition is assumed on the part of users of the system.

- Inferred creation is used when a **Relation** is needed between two **Relatives**, but the relationship is not known explicitly and the **Relatives** are not quantitatively defined. It is possible that such a request will fail, meaning that no unambiguous relation could be derived at this time.

7.3 The Spatial Reasoning Subsystems

The relation classes and the query handling system interface with a spatial reasoning subsystem that performs homogeneous, heterogeneous, and integrated spatial reasoning and combinations thereof. When a relation class requires an inferred creation of an object it directly requests the isolated reasoning subsystem to perform the necessary inference process. Heterogeneous and integrated reasoning are invoked by the query handler or by the spatial reasoning subsystem if it determines the need for them.

The query handler identifies the types of spatial constraints, such as topological or directional, specified in the query and depending on the combinations of constraints requests either type of reasoning. For example, if a query is "Find scenes such that region *A* is *West* of region *B*, region B *meets* region *C* and is *West* of it, and *C* is *disjoint* from and *West* of region *D*" then both isolated and integrated spatial reasoning may be used to determine the unspecified relations before passing the scene description on to a scene matching subsystem. The spatial reasoner itself can evaluate which of the three types of reasoning or their combinations can be used based on the relationships among objects of interest and related objects that have been stored in the database. It does so by constructing a labeled directed multigraph with objects as nodes and relationships as edges, i.e., a constraint network.

The option of storing, inferring, or computing spatial relationships can lead to inconsistencies within the database. Three types of conflicts are possible: (1) between the stored and computed value, (2) between the inferred and computed value, and (3) between an inferred value and a subsequent update due to a value being expressly specified or recomputed. At present we do not deal with resolving such conflicts. The computed value is always assumed to be the most accurate or correct one and replaces any stored value. Future work will explore mechanisms for resolving such conflicts and for integrating qualitative and quantitative methods for spatial data handling.

The working prototype of the spatial reasoning subsystems for combined topological and direction reasoning, written in C, exists on a DEC Alpha workstation and the shell archive of the source files can be downloaded using the appropriate link in the web page http://www.ncgia.maine.edu/jayant.html. A web-based user interface to the inference mechanism is being constructed and will be available through the same link. The interface will permit a user to test the inference mechanism by specifying a set of spatial constraints and having the system evaluate its consistency.

The prototype uses initialized tables to store the composition tables for topological and directional relations. The scene description or set of spatial constraints is represented by a set of constraint networks and evaluation of consistency or determining unspecified spatial relations is done by constraint propagation. The constraint propagation algorithm used is modified version of Mackworth's (1977) path consistency algorithm which maintains a queue of all paths of length two that must be considered. If any constraints associated with a path are modified due to a path consistency check then all paths likely to be affected by the change are appended to the queue. Constraint propagation ends when the queue is empty or if an inconsistency is found.

The prototype implementation demonstrates that (i) combined spatial reasoning is more powerful than homogeneous spatial reasoning, and (ii) when both topological and directional relations are specified then integrated and combined spatial reasoning are equivalent. The implementation, however, does not demonstrate how the spatial inference mechanism could be used for spatial query processing tasks such as minimization of a set of spatial constraints.

The following section describes the results of two tests run on the prototype system for demonstrating how the query process responds to a request for information. The examples show the use of retrieval of stored and inference of unknown spatial relationships only.

7.4 Test Results

The two examples in this section demonstrate the use of homogeneous, heterogeneous, and integrated spatial reasoning in answering a query regarding the spatial relationships between objects from a database that contains less than complete spatial information. For the purposes of these examples we are concerned only with qualitative spatial information, i.e., the spatial relationships among the objects.

The first example illustrates the use of homogeneous spatial reasoning and shows how it is used to evaluate the consistency of topological constraints specified in a query. The procedure for evaluating consistency described in Chapter 3 (Section 3.5) applies also to spatial query processing. It enables the detection of inconsistent queries over topological spatial relations without having to perform expensive geometric computations over the actual data. For example, the following query formulated in a spatial SQL dialect,

```
SELECT   lake, name
FROM     state, county, lake
WHERE    state.geometry CONTAINS county.geometry and
         county.geometry CONTAINS lake.geometry and
         (state.geometry DISJOINT lake.geometry or
          state.geometry MEET lake.geometry)
```

can be immediately rejected since the constraints in the `WHERE` clause are inconsistent. Table 7.1 shows the connectivity matrix for the above query using the fact that each object is equal to itself and the universal relation in place of unspecified constraints.

	state	county	lake
state	*equal*	*contains*	*disjoint OR meet*
county	*U*	*equal*	*contains*
lake	*U*	*U*	*equal*

Table 7.1 The connectivity matrix for the spatial SQL query.

Evaluating the consistency of the description shows that the relation $t'_{state,lake}$ is empty.

$$\left(equal;\left(disjoint\ \ or\ \ meet\right)\right)\cap\left(contains;contains\right)$$

$$\cap\left(\left(disjoint\ \ or\ \ meet\right);equal\right)$$

$$=\left\{disjoint,meet\right\}\cap\left\{contains\right\}\cap\left\{disjoint,meet\right\}$$

$$=\varnothing$$

By evaluating queries for topological consistency, expensive computations and time delays can be avoided for ill-formulated constraints.

The second example illustrates the use of heterogeneous and integrated spatial reasoning. Heterogeneous spatial reasoning is useful when the topological relationship between one or more pairs of objects is a containment relationship. Integrated spatial reasoning is used when both the topological and directional relations between the objects are known.

Consider the scene containing four distinct objects *A*, *B*, *C*, and *D* having twenty-four binary topological and directional relations among them. Of these twenty-four relations only six are specified and stored in the database. These are {*A inside B* ; *B disjoint C*, *B disjoint D*, *C meet D*, *B North C*, *C North D*} and can be represented by the constraint graph shown in Figure 7.3. The following queries are asked and in the given order: (i) What is the topological relation between *B* and *A*?, (ii) What are the topological and directional relations between *A* and *C*?, (iii) What is the directional relational between *B* and *D*?, and (iv) What are the topological and directional relations between *A* and *D*?.

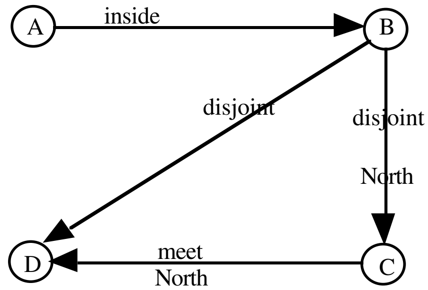


Figure 7.3 The constraint graph for the spatial relations {*A inside B* ; *B disjoint C*, *B disjoint D*, *C meet D*, *B* North *C*, *C* North *D*}.

• Query 1. What is the topological relation between *B* and *A*?

The topological relation *A inside B* is stored in the database. Hence this relation is retrieved and the converse relation *B contains A* is returned as the response to the query.

• Query 2. What are the topological and directional relations between *A* and *C*?

From the constraint graph (Figure 7.3) we see that the composition *A inside B* ; *B disjoint C* permits the inference of the topological relation *A disjoint C*. There is no path from *A* to *C* that is labeled by directional relations for each edge. Hence the only recourse is the heterogeneous composition *A inside B* ; *B North C* which gives the inferred directional relation *A North C*. The database is updated with these inferred relations and the constraint graph is now as shown in Figure 7.4.

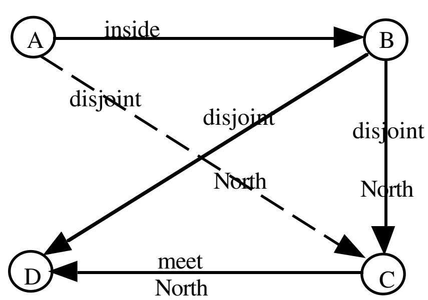


Figure 7.4 The constraint graph after query 2.

• Query 3. What is the directional relational between *B* and *D*?

No directional relation between *B* and *D* is stored in the database but those between objects *B* and *C* and *C* and *D* are stored. These relations are used in the composition *B North C* ; *C North D* to infer the relation *B North D* which is then stored in the database giving the constraint graph shown in Figure 7.5.

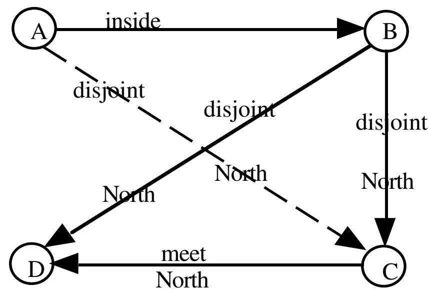


Figure 7.5 The constraint graph after query 3.

• Query 4. What are the topological and directional relations between *A* and *D*?

From the constraint graph (Figure 7.5) we can see that the stored relations *C meet D* and *C North D* and the inferred and subsequently stored relations *A disjoint C* and *A North C* form a path that can be used to infer the spatial relations between *A* and *D*. Since both the topological and directional relations are known, the integrated composition *A DisjointNorth C* ; *C MeetNorth D* is used to infer *A DisjointNorth D*. This inferred relation implies the topological relation *A disjoint D* and the directional relation *A North D*. After updating the database the constraint graph is as shown in Figure 7.6.

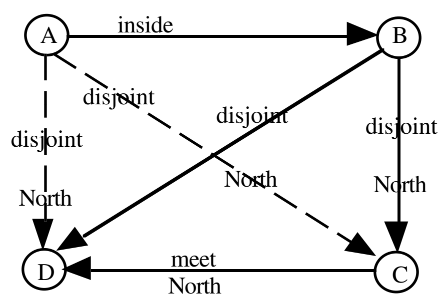


Figure 7.6 Constraint graph after query 4.

Of the twenty-four topological and directional spatial relationships that exist for the objects in the scene shown in Figure 7.7, pure homogeneous spatial reasoning requires at least seven of them to be specified in order to infer the complete set of relations. Twelve relations are inferred using the converseness property of topological and directional relations. Five others can be inferred from the base set of seven specified relations which are {*A inside B* ; *B disjoint C*, *B disjoint D*, *C meet D*, *A North C*, *B North C*, *C North D*}. Of these seven, the directional relation *A North C* can be inferred using the heterogeneous composition *A inside B* ; *B North C*.

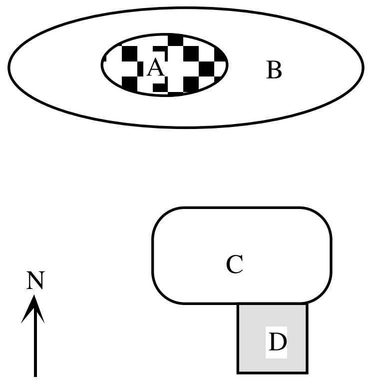


Figure 7.7 A situation where heterogeneous spatial reasoning is useful.

Therefore a qualitative spatial reasoner can effectively exploit the added reasoning power provided by integrated and heterogeneous compositions whenever the situation warrants it. The applicability of heterogeneous or integrated compositions is determined using the constraint graph. The labeled edges specify which relationships are known. Hence integrated compositions are used when, for example, both topological and directional relations are given. Heterogeneous compositions are used when a containment relationship exists between two objects or when one of the topological or directional relations between the objects is not known. Determining which composition or which sequences of composition to use, given a constraint network graph, is not a trivial problem. We propose to investigate and evaluate efficient algorithms for handling this problem which has been much studied within the context of Artificial Intelligence research on constraint-satisfaction (Guesgen and Hertzberg 1992; Tsang 1993; Meyer 1995).

7.5 Summary

We have described a framework for implementing a comprehensive qualitative spatial reasoning system. The system consists of subsystems that can perform inferences on individual types of spatial relations, such as topological relations, on combinations of different types of relations, and on integrated relations formed as conjunctions of different spatial relations such as topological and directional relations. The spatial reasoner does not replace but rather complements an existing database and information system which uses quantitative data and computes spatial relationships based on this data. The query handler determines whether a query must be satisfied by a stored, inferred, or computed relationship and takes the appropriate steps to answer the query. Updates, however, pose a special problem since the possibility exists of conflicts from inconsistencies among stored, inferred, and computed relationships. Further work is required in order to devise strategies for resolving the various conflicts that can occur. We also show how and when the qualitative spatial reasoner could utilize combined knowledge of different types of spatial relations in order to reduce the number of stored relations in the database.

Chapter 8

Conclusions and Future Work

This thesis dealt with computational methods that exploit qualitative spatial information for making inferences about objects in a geographic database. The motivation was to enhance Geographic Information Systems with intelligent mechanisms to deal with complex spatial concepts and providing facilities for the representation of qualitative spatial information and making inferences.

The qualitative spatial information we dealt with consisted of a system of qualitative directions, such as {*North*, *Northeast*, *East*, *Southeast*, *South*, *Southwest*, *West*}, and binary topological relations, such as {*Disjoint, Meet, Overlap, CoveredBy, Inside, Covers, Contains, Equal*}. We defined the semantics of these spatial relations for homogeneously two-dimensional simply connected objects. The definition of the spatial relations also consists of composition tables that define the result of composing two spatial relations of the same type to obtain a set of spatial relations of the same type. The composition tables form the basis of the inference mechanism used throughout this thesis. Spatial inference is the process of utilizing known spatial information and rules to deduce new spatial information. Thus given a pair of spatial relations over a common object one can infer the unspecified spatial relation or relations possible between the two objects related to the common object by using the composition tables.

We investigated the inference of qualitative spatial information from stored base facts, that is, finding those spatial relations that are implied by a particular configuration from a set of objects and a set of spatial constraints relating these objects. In particular we examined the various types of inferencing over combined or integrated spatial relations and developed the appropriate composition tables for topological and directional relations.

8.1 Major Results

The primary contributions of this thesis are:

- A clear definition of the types of spatial inferences that can be performed in a comprehensive spatial reasoning system. The four types are (1) homogeneous spatial reasoning about relations of the same type, (2) heterogeneous spatial reasoning about pairs of relations of different types such as topological and directional relations, (3) mixed spatial reasoning about spatial relations of one type from the composition of pairs of spatial relations of a different type, and (4) integrated spatial reasoning about conjunctions of topological and directional relations considered as integrated relations such as the relation DisjointNorth denoting the spatial relations *disjoint* and *North*.

- A systematic consideration of each element of a comprehensive framework for spatial reasoning in geographic information systems. We identify the missing elements and develop the formalisms, namely composition tables for heterogeneous and integrated spatial reasoning.

- Comparing the result of using a combination of homogeneous and heterogeneous inferences with that of using an integrated inference mechanism. This comparison is useful in determining which method should be used when all the necessary information is available. Integrated spatial reasoning provides smaller sets of inferred relations when the topological relation between object pairs is either *disjoint* or *meet*. Heterogeneous reasoning is useful when one object contains another since all its directional relations also hold for the contained object.

- Identifying that all types of spatial reasoning, and their combinations, provide the same set of inferred relations when the directional relation between objects are not along the same axis. One such example is when the composition involves the directions *North* and *East* or *South* and *West*.

- The design of a framework for comprehensive qualitative spatial reasoning that allows spatial relations to be stored in a database. This facilitates the integration of quantitative and qualitative spatial data handling mechanisms.

- Identification of the possible sources of data value conflicts when spatial relationships are stored or inferred in addition to being computed from geometric information in the database. The three main sources are: errors in the geometric information, propagation of computational errors resulting in incorrect determination of qualitative spatial relationships, and errors in stated qualitative spatial relationships added to the database thereby resulting in contradictions among the stored facts.

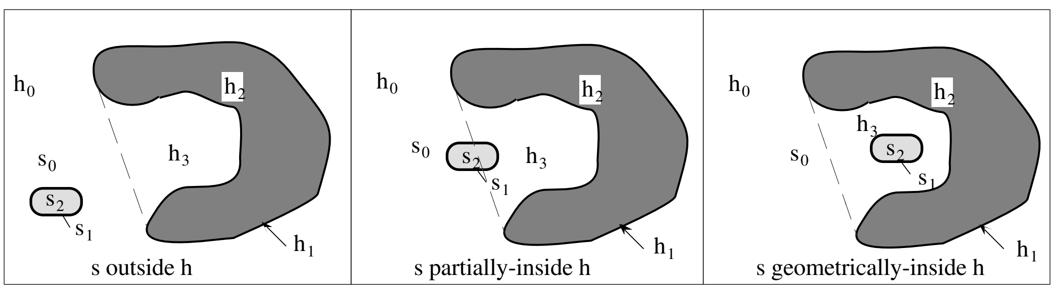
8.2 Directions for Future Work

Beyond the particular results this thesis provides a starting point for much further work in the area of integrated spatial reasoning. In this section we identify new research questions, ranging from conceptual to implementations issues, that are complementary to the results presented in this thesis. This research would enable the construction of a comprehensive qualitative spatial reasoning system capable of handling large data sets consisting of both quantitative and qualitative spatial information.

8.2.1 Extending the 9-intersection for Concave Regions

The 9-intersection method for characterizing binary topological spatial relationships is valid for convex regions. An extension of the 9-intersection applies to regions with holes (Egenhofer, Clementini, and Di Felice 1994b). A further generalization of the intersection-based formalism is required, however, for dealing with regions of arbitrary shape (Abdelmoty 1995). Each region is decomposed into a number of components and the topological relationship between two regions is characterized by the values of the set intersections of the components of the regions. The set of intersections can be represented as a matrix and if every object has exactly the three components, interior, boundary, and exterior then this representation is equivalent to the 9-intersection formalism.

The generality of this method lies in the fact that an object can have any number of components. The components may be virtual in that they have no physical boundary that delineates their extent. The advantage of this flexibility is that spatial relationships such as *geometrically-inside* and *partially-inside*, as defined by Cui *et al.*(1993), can be defined using an intersection matrix. This definition is useful for distinguishing qualitative spatial relationships such as a ship being *outside*, *partially-inside*, or *geometrically-inside* a harbor (Figure 8.1). These distinctions cannot be made using the 9-intersection formalism.



	h_0	h_1	h_2	h_3
s_0	1	1	1	1
s_1	1	0	0	0
s_2	1	0	0	0

	h_0	h_1	h_2	h_3
s_0	1	1	1	1
s_1	1	0	0	1
s_2	1	0	0	1

	h_0	h_1	h_2	h_3
s_0	1	1	1	1
s_1	0	0	0	1
s_2	0	0	0	1

Figure 8.1 Component intersection method for distinguishing between *outside*, *partially-inside*, and *geometrically-inside*.

Composition of topological relations, characterized by the above method, is derived using the two rules (Equations 3.1 and 3.2) specified for the 9-intersection. Thus the above method can potentially replace the 9-intersection. Additional work is required, however, before such a step can be taken.

Consider the case illustrated in Figure 8.1. The distinction between outside, partially-inside, and geometrically-inside depends on the division of the exterior of the object h into two components, h_0 and h_3. In order for a symbolic reasoner to distinguish between an exterior that has been partitioned into virtual components and one that has not, there must be an additional rule that specifies when a partition is made. For the above instance, an appropriate rule could be that the exteriors of all concave objects are partitioned into components. One component being the exterior of the convex hull of the object and the rest being the portions of the exterior enclosed by the boundary of the convex hull and the boundary of the object, such as component h_3 above. Thus, while Abdelmoty and colleagues proposed method is indeed a generalization of the 9-intersection, it requires a clear specification of how an object should be partitioned into components and what these components represent.

8.2.2 Integrating Qualitative Distances and other Spatial Relations

This thesis assumed a very simple model for qualitative distances since the focus of this work was investigating the power of interval-based reasoning. Interval-based methods, however, are inappropriate for representing and reasoning about qualitative distances. Integrated topological and distance relations, such as *DisjointFar* or *DisjointNear*, cannot be characterized by interval relations, therefore an alternative representation must be devised.

The same representation scheme should also be suitable for characterizing directions and qualitative distances between regions. The key problem, however, is defining distance relations between extended objects since they may vary depending on the sizes of the objects concerned. If the sizes of the regions are disregarded and the distance between two regions is defined as the distance between their centroids, nearest points, or furthest points then inconsistencies may arise between inferred and computed distance relations. For example, if the shortest distance between two regions defines their qualitative distance relationship, then the facts Mexico is very *close* to the U.S. and Canada is *very close* to the U.S. imply that Mexico is *very close* or *close* to Canada. Computing the shortest distance between the two regions, however, may result in the qualitative distance *far*, depending on the definition of the qualitative distances *very close*, *close*, and *far*.

Once a suitable method for characterizing distances between extended objects has been devised, we need a means of characterizing integrated topological and distance relations and integrated distance and direction relations between objects. Having the appropriate representation for the three types of qualitative spatial relations will enable the construction of a truly comprehensive spatial reasoning system.

8.2.3 Integrating Qualitative and Quantitative Spatial Information

- Identifying the specific problems in integrating quantitative and qualitative spatial information with particular emphasis on maintaining and checking consistency.

The basic purpose of qualitative approaches to spatial data handling is to enhance the ability of an information system to deal with less than complete information. Problems can occur, however, when the qualitative information derived by inference either contradicts or differs slightly from the qualitative information derived by computation on quantitative information. The sources and causes of inconsistency include (1) lower resolution or imprecision in the specified qualitative information as compared to the quantitative information; (2) inaccuracies in the quantitative information such as the position of the objects; and (3) mismatches between the model for spatial relations used the information system and the model used by the information gatherer or specifier.

The first source of inconsistency, lower resolution information, can lead to conflicts between inferred and computed spatial relation information because the computational process results in a single possibility, while the inference process results in a set of possibilities. Resolving the conflict is fairly simple since the computed information is expected to be a subset of the inferred information.

Computed information may not always be a subset of inferred information particularly if the quantitative values used are near the "landmark," i.e., border of a range, values for a qualitative spatial relation. For example, assume that intervals of increasing size are used to define qualitative distances. The intervals are [0, 100), [100, 1000), and [1000, 10000) miles for the qualitative distances *near*, *far*, and *very far* respectively. Now assume for points *A, B*, and *C* the distance between A and B is very close to but just above 0 miles and that between *B* and *C* is very close but less than 100 miles and the points are collinear. It is possible that using qualitative inference the distance between *A* and *C* is inferred to fall in the range *near* but due to computational errors is computed to be in the range *far*. Such conflicts are quite unlikely to occur but nevertheless must be identified and resolved when they do occur.

The third source of inconsistency, difference in mental models, is particularly problematic. An example of such a difference is when an information system uses a projection-based system of directions, while a user thinks of directions in terms of a cone-shaped system. The composition tables for the two systems differ and so will the inference results. Based on the unexpected results users may believe the system to be unreliable. There is no clear solution to this problem. Further research on mental models with particular emphasis on determining if different classes of users have distinct models or if there is a substantial commonalty in their mental models will help considerably in finding effective solutions. An information system should also be reconfigurable or customizable such that the underlying qualitative spatial data and relation model is selectable and modifiable by a domain expert.

8.3.4 Directional Relations Between Contained Objects

- A definition of directional relations between contained objects, that is, relations such as Maine is in the Northeastern part of the United States, and their use in direction reasoning.

The directional relations may be defined by partitioning the containing object. One possibility would be to subdivide the object A into four subobjects with the centroid being the meeting point. Then a contained object B can be classified as being in the Northeastern, Southeastern, Southwestern, or Northwestern part of A if one of the four subdivisions of A completely contains B. If B spans subdivisions then the projection based definition of *North*, *South*, *East*, and *West* can be used to classify B as being in the Northern or whichever part of A. A second possibility would be to use a projection-based segmentation with a neutral zone around the centroid of the container object A. This definition will facilitate reasoning since the interval-pair composition mechanism used in this work will be directly applicable.

8.3.5 Directions Between Overlapping Objects

- Reasoning about direction relations between overlapping objects.

This is useful in cases such as when object A overlaps with the Northern part of B and C overlaps with the Southern part of B. If the definition of Northern and Southern is such that there is no overlap between the parts then one can infer that A is *North* of B in the above situation.

Segmentation of both objects and the definition of Northern, Northeastern and such like parts of each object is one possible approach to the problem. The overlap relationship can then be specified with greater detail as in *A* overlaps the Northern part of *B*. Interval-based reasoning is applicable here too, because the parts of each object are assumed to be non-overlapping.

8.3.6 Hierarchical Spatial Reasoning

- A mechanism for hierarchical spatial reasoning. More specifically a means for qualitative reasoning about spatial objects represented at the same or different levels of detail or a containment hierarchy.

Reasoning about objects at the same and different levels of detail may be necessary for instance when information about the directional relations between cities in a country such as India is stored as follows. The country is divided into states, each having a capital city. The directional relations among cities within each state are stored as are the directional relations among the capital cities of all states. Thus the capital cities form the link between the non-capital cities of the various states. Reasoning about the directional relation between two non-capital cities in two different states is done as follows. The directional relation between each city and the capital city of each state is known as is the directional relation between the two capital cities. Thus a sequence of compositions of the three known directional relations may permit the inference of the directional relational between the two cities. For example, Calcutta the capital of West Bengal if Southeast of Lucknow, the capital of Uttar Pradesh. Diamond Harbor in West Bengal is South of Calcutta. Dehra Dun in Uttar Pradesh is Northeast of Lucknow. Given these facts the directional relation Dehra Dun Northwest of Diamond Harbor can be inferred using hierarchical spatial reasoning.

Reasoning about objects in a containment hierarchy occurs, for example, when inferring the directional relation between Athens, Greece and Paris, France based on directional relations between cities in Europe versus inferring the directional relationship between the two cities based on that of the two countries in which the cities exist.

Constraint processing and constraint satisfaction form the basis of the reasoning methods described above. Therefore special attention must be paid to constraint processing techniques for spatial information.

8.3.7 Heuristics for Constraint Satisfaction

- Developing heuristics for improving the efficiency of constraint satisfaction mechanisms used for inferring relations and evaluating consistency.

In particular, we need heuristics for ensuring locality of inference such that inferences concerning cities in the state of Maine do not consider relationships between cities in Oregon for example.

Constraint reasoning is done using a constraint network graph. Inferences or consistency checks are done by first finding a sequence of edges that connect the reference and primary objects between whom the spatial relationship must be determined. Numerous paths may exist. Consistency requirements dictate that the inferred relations belong to the intersection of the sets of relations inferred using each one of the possible paths. This may not be required or desirable, however, if it can be assumed that local consistency implies global consistency. In such cases we need heuristics that identify and use paths consisting of objects within a certain neighborhood of the objects of interest. If the objects of interest are not "close" then many intermediate objects must be found. The intermediate objects on the selected paths should be such that they are "closer" to the objects of interest than intermediate objects on discarded paths. The definition of close need not be based on a metric value. One possible definition is that each pair of adjacent edges, and hence a composition, should be chosen such that the cardinality of the set of inferred relations is less than that for any other path. In particular if a composition at some intermediate stage in a path results in a universal relation then that path should be discarded. Another thumb rule, specific to topological spatial relations, could be to seek paths such that the compositions involve containment or *overlap* relationships rather than *disjoint* or *meet* since compositions involving the latter typically result in a larger set of possibilities. Determining suitable paths or constructing constraint graphs that reflect locality is made easier if indexes exist on the spatial objects and relations in the database.

8.3.8 Indexing Schemes for Spatial Relations

- Evaluation of indexing schemes for stored spatial relations that allow retrieval of relation objects based on the spatial relationship or on the geographic entities involved in the relationship.

Indexes on the geographic or spatial entities are useful when constructing a constraint graph that is not concerned with the complete database but rather a subset of objects that lie within some neighborhood of interest. This is useful for restricting constraint processing to objects and relations within a local area of the database, as in the case of reasoning about cities in Maine while disregarding relations among cities in California. If objects are clustered by spatial location then all objects on a certain set of disk pages can be retrieved and form the nodes of the constraint graph. Next the index on stored spatial relations can be used to label the edges in the graph.

The indexing mechanisms will differ for the spatial entities of various dimension or shape and for the spatial relations. While spatial entities can be clustered based on location, determining criteria for clustering of spatial relations is a complex problem. Another problem with indexing spatial relations is deciding whether relations should be indexed individually, as in the case of topological relations, or whether they should be indexed jointly since they usually used in conjunction, as in the case of qualitative distance and orientation relations. Thirdly, the indexing mechanisms may be different based on the type of spatial relation involved such as topological, directional, or qualitative distance.

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Jayant Sharma was born in Calcutta, India on April 22, 1962. He attended high school at the St. Xavier's Collegiate School in Calcutta. He received his Bachelor of Engineering degree in December, 1985 from the Birla Institute for Technology and Science, Pilani, India

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In September 1991 he was enrolled for graduate study in computer science at the University of Maine and served as a Research Assistant at the Dept. of Computer Science and the National Center for Geographic Information and Analysis. He received the Master of Science degree in computer science from the University of Maine, Orono in August 1993.

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