Integrating Dynamic Geometry Software, Deduction Systems, and Theorem Repositories

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August 10-12, 2006

Introduction

The axiomatic presentation of geometry fills the gap between formal logic and our spatial intuition.

The study of geometry is, and will always be, very important for a mathematical practitioner.

GeoThms framework provides an environment suitable for new ways of studying and teaching geometry at different levels and for storing geometrical knowledge: descriptions of construction; geometrical conjectures; geometrical proofs

Computers & Geometry

Computer technologies give new ways for studying geometry

Dynamic Geometry Software Visualise/Explore/Test Conjectures

Geometric Automated Theorem Proving synthetic proofs (human-readable) / algebraic proofs (efficiency).

Problems Repositories browse through the existing knowledge.

GeoThms integrates all these features bringing new forms in communicating mathematics.

GeoThms Framework

GeoThms integrates DGSs, ATPs, and a repository of constructive geometry theorems in one single tool.

Dynamic Geometry Software GCLC & Eukleides

Geometric Automated Theorem Proving GCLCprover (implements the area method).

Problems Repositories geoDB - geometric theorems, illustrations and proofs database.

GeoThms provides an environment suitable for new ways of studying and teaching geometry at different levels, and for storing geometrical knowledge: descriptions of construction; geometrical conjectures; geometrical proofs

Dynamic Geometry Software

Dynamic geometry software visualise geometric objects and link formal, axiomatic nature of geometry (most often — Euclidean) with its standard models (e.g., Cartesian model) and corresponding illustrations.
GCLC & Eukleides - two DGSs designed to be close to the traditional language of elementary Euclidean geometry.

- they provide support for primitive constructions based on ruler and compass
- transformations, labelling components of figures, interactive work, animations, etc.
- graphical user interface.

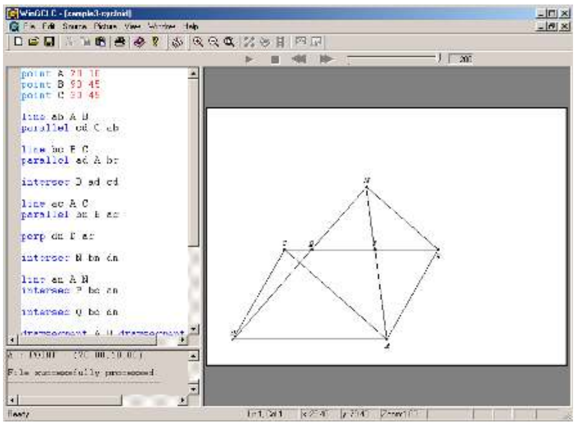
By using the set of primitive constructions, one can define more complex constructions.

A geometric construction is a sequence of specific, primitive construction steps. These primitive construction steps (also called *elementary constructions*) are based on using a *ruler* (or a *straightedge* and a *compass*, and they are:

- construction (with a *ruler*) of a line such that two given points belong to it;
- construction of a point which is an intersection of two lines (if such a point exists);
- construction (with a *compass*) of a circle such that its centre is one given point and such that the second given point belongs to it;
- construction of a segment connecting two points;
- construction of intersections between a given line and a given circle (if such points exist).

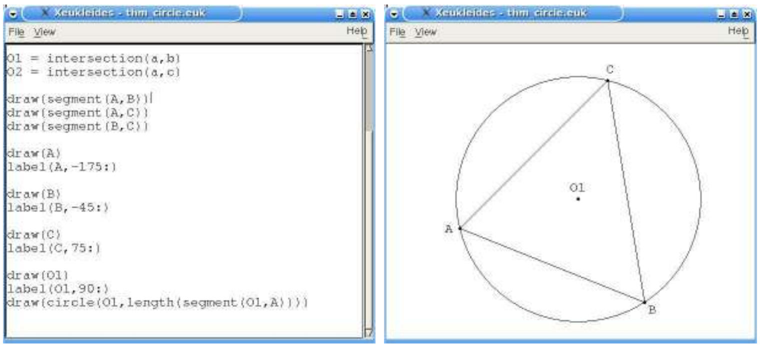
GCLC & Eukleides

GCLC[a] is a tool for teaching and studying mathematics, especially geometry and geometric constructions, and also for storing descriptions of mathematical figures and producing digital illustrations of high quality.



Eukleides[b] is an Euclidean geometry drawing language (with localised versions).

- `eukleides` is a compiler for typesetting geometric figures within a (La)TeX document.
- `xeukleides` is a GUI front-end for creating interactive geometric figures.



[a] Predrag Janičić, www.matf.bg.ac.yu/~janicic/gclc/
[b] Christian Obrecht; EukleidesPT (Pedro Quaresma) gentzen.mat.uc.pt/~EukleidesPT/

ATP in Geometry

Automated theorem proving in geometry has two major lines of research:

algebraic proof style Algebraic proof style methods are based on reducing geometry properties to algebraic properties expressed in terms of Cartesian coordinates. These methods are usually very efficient, but the proofs they produce do not reflect the geometry nature of the problem and they give only a yes/no conclusion.

synthetic proof style Synthetic methods attempt to automate traditional geometry proof methods that produce human-readable proofs.

GCLCprover

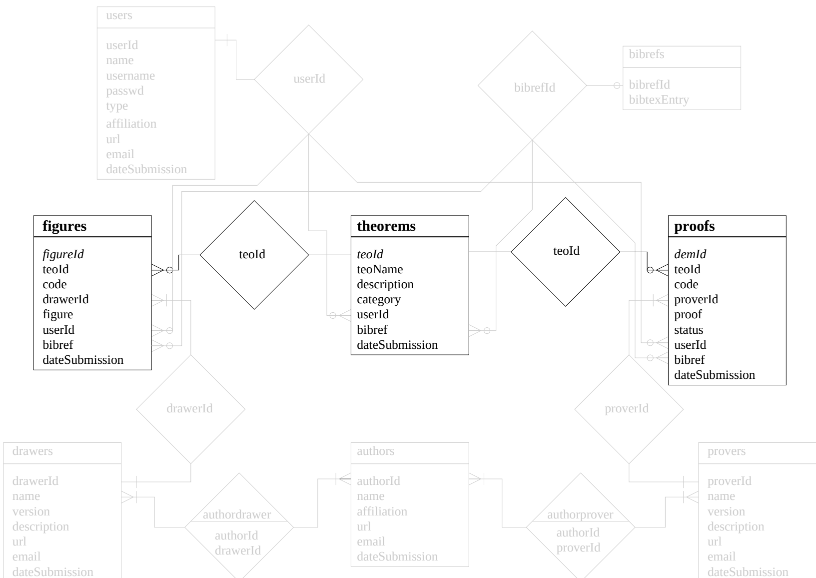
GCLCprover - synthetic geometric ATP (area method)

- implements the area method
- simple and tight integration with GCLC and Eukleides
- human-readable proofs
- very efficient for many conjectures

GCLCprover was implemented in C++ (as GCLC) and is very efficient.
The prover can prove many complex geometric problems in milliseconds.

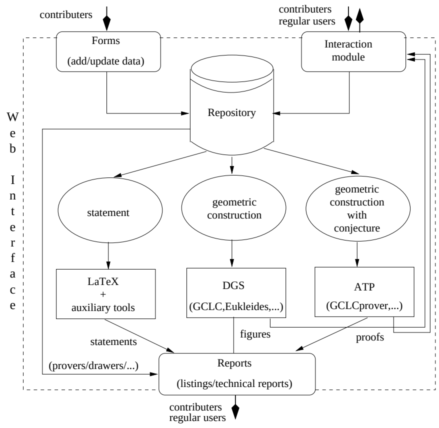
Theorem	elimination steps	geometric steps	algebraic steps	time (sec)
Ceva	3	6	23	0.001
Gauss line	14	51	234	0.029
Midpoint	8	19	45	0.002
Thales	6	18	34	0.001
Menelaus	5	9	39	0.002
Pappus' Hexagon	24	65	269	0.040
Areas of Parallelograms	62	152	582	0.190

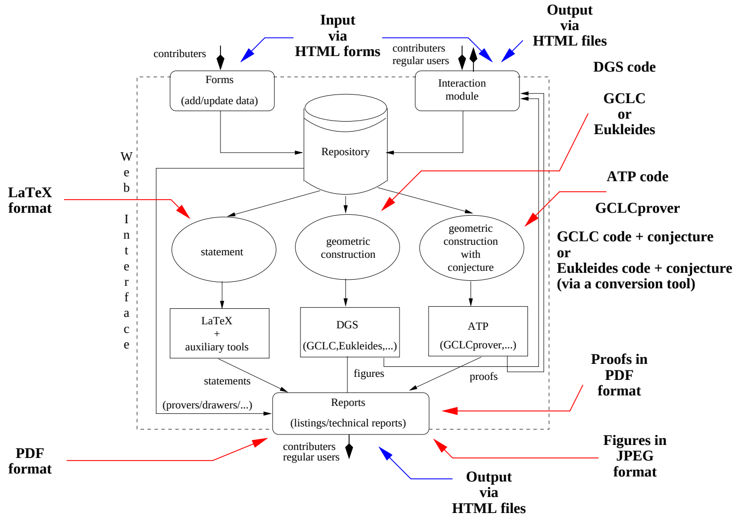
GeoDB - ERD



GeoThms

GeoThms[a], is a framework that links dynamic geometry software (GCLC, Eukleides), geometry theorem provers (GCLCprover), and a repository of geometry problems (geoDB).





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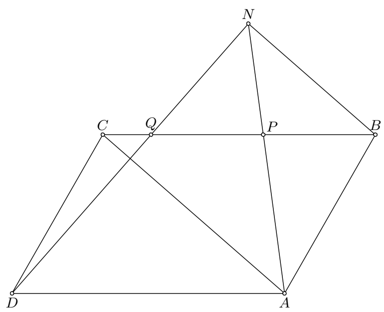
[a]GeoThms is accessible from `http://hilbert.mat.uc.pt/~geothms`

GeoThms - by Example

Theorem 1 (Gramy P143[a]) *Given a parallelogram $ABCD$, a point N, obtained by the intersection of a line parallel to AC passing through B, and a line perpendicular to AC passing through D, then the point P, which is given by the intersection of AN and BC, is the midpoint of QB, where Q is the intersection of BC and DN.*



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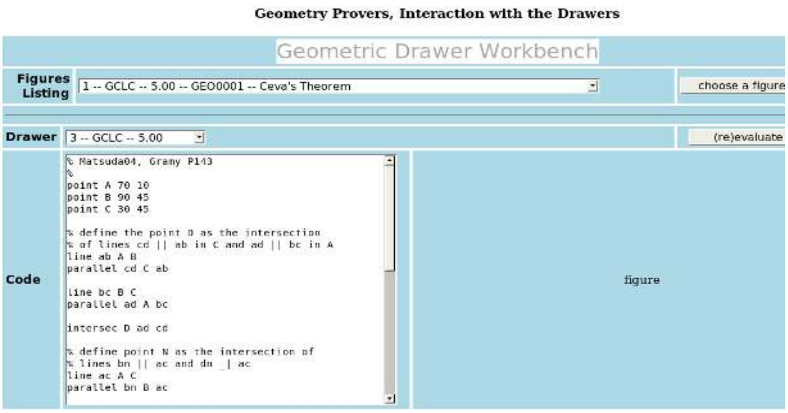
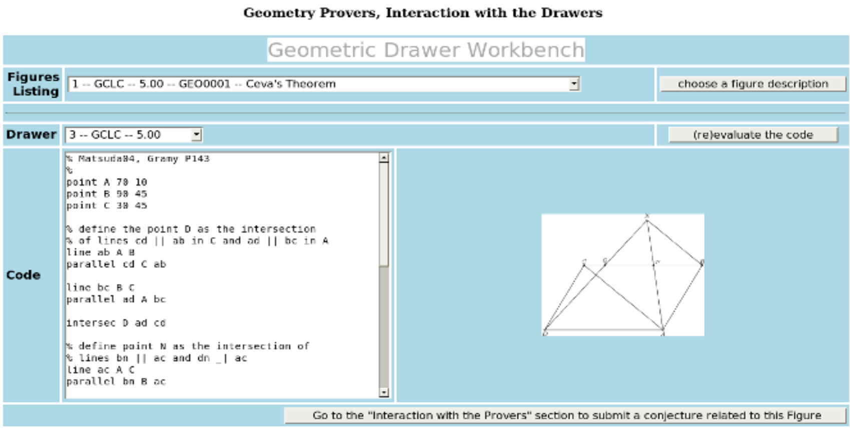
[a]P143 of "Gramy: A Geometry Theorem Prover Capable of Construction" by Matsuda and Vanlehn.

Describe the Construction

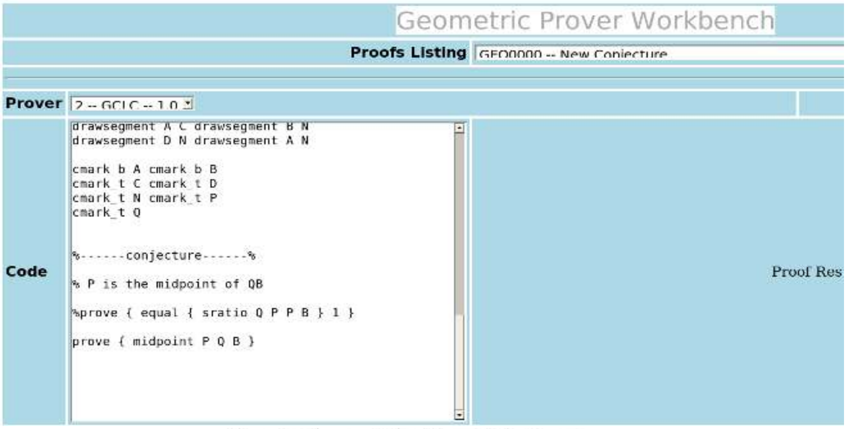
We begin by specifying the construction in the DGSs language.

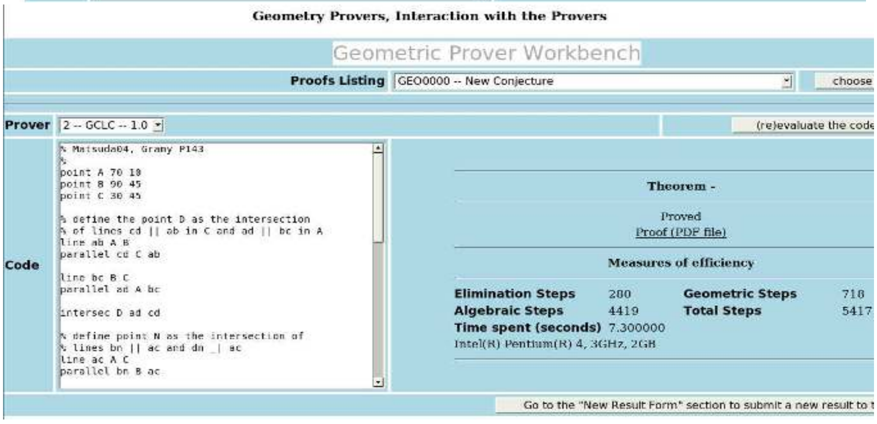




Testing the Conjecture

Having described the construction of the figure, now we have to add the conjecture, P is the midpoint of QB.





All the commands used in the construction of the figure are internally (within the prover) transformed into primitive constructions of the area method.

The Proof - Area Method

(1) $$\frac{\overrightarrow{QP}}{\overrightarrow{PB}} = 1$$, by the statement

(2) $$\left(-1 \cdot \frac{\overrightarrow{PQ}}{\overrightarrow{PB}}\right) = 1$$, by geometric simplifications

(3) $$\left(-1 \cdot \frac{S_{PDF^3_{dn}}}{S_{PDBF^3_{dn}}}\right) = 1$$, by Lemma 37 , second case — points P, B, and C are collinear (point Q eliminated)

(4) $$\left(-1 \cdot \frac{S_{DF^3_{dn}P}}{\left(S_{DBP} + S_{BF^3_{dn}P}\right)}\right) = 1$$, by geometric simplifications

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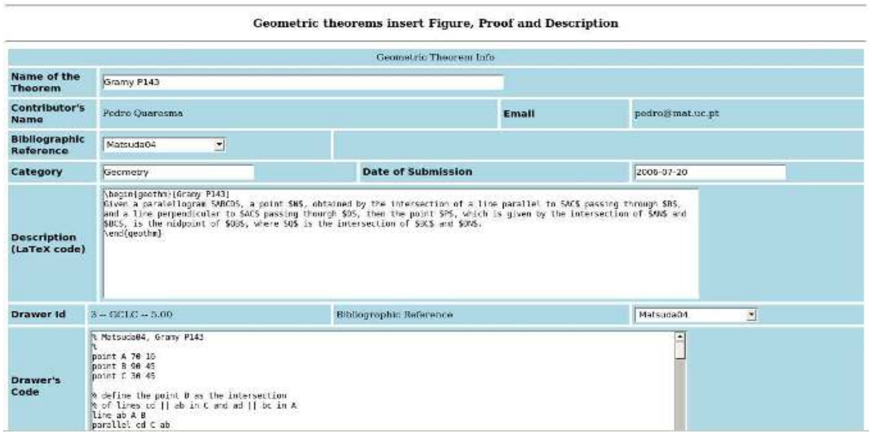
(5) $$\frac{\left(-1 \cdot S_{DF^3_{dn}P}\right)}{\left(S_{DBP} + S_{BF^3_{dn}P}\right)} = 1$$, by algebraic simplifications

(6) $$\frac{\left(-1 \cdot \frac{\left(\left(S_{BAN} \cdot S_{DF^3_{dn}C}\right) + \left(-1 \cdot \left(S_{CAN} \cdot S_{DF^3_{dn}B}\right)\right)\right)}{S_{BACN}}\right)}{\left(S_{DBP} + S_{BF^3_{dn}P}\right)} = 1$$, by Lemma 30 (point P eliminated)

(7) $$\frac{\left(\left(-1 \cdot \left(S_{BAN} \cdot S_{DF^3_{dn}C}\right)\right) + \left(S_{CAN} \cdot S_{DF^3_{dn}B}\right)\right)}{\left(\left(S_{BACN} \cdot S_{DBP}\right) + \left(S_{BACN} \cdot S_{BF^3_{dn}P}\right)\right)} = 1$$, by algebraic simplifications

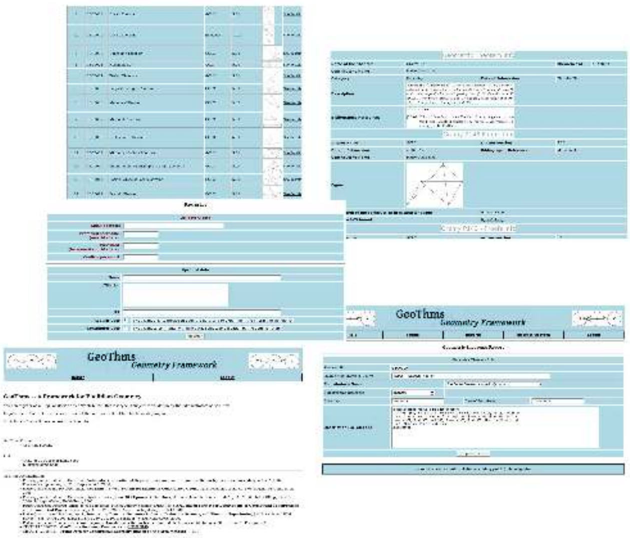
Adding a New Theorem to the Database

The user (with the status of contributer) can select the "Forms" section in order to add a statement for the new result and the corresponding figure and proof.



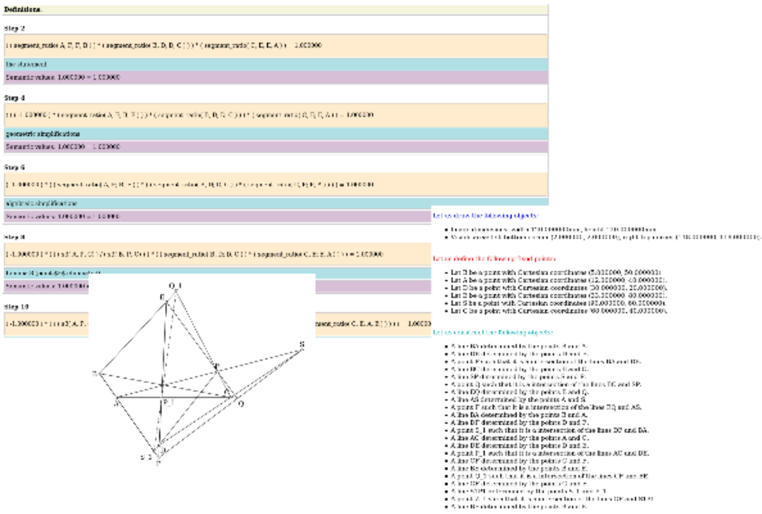
GeoThms - Browsing

The user has many other options for browsing the database.



Recent work

XML and SVG support.



- geometrical constructions stored in strictly structured files; easy to parse, process, and convert into different forms and formats
- input/output tasks will be supported by generic, external tools and different geometry tools will communicate easily
- growing corpora of geometrical constructions will be unified and accessible to users of different geometry tools
- easier communication and exchange of material with the rest of mathematical and computer science community
- there is a wide and growing support for XML
- different sorts of presentation (text form, LaTeX form, HTML) easily enabled
- strict content validation of documents with respect to given restrictions.

Related Systems

Tool	DGS	ATP	Readable Proofs	Web Interface	Repository of Problems	Verification of Constructions
GeoThms	√	√	√	√	√	√
Geometry Explorer	√	√	√			
MMP/Geometer	√	√	√			
GEX (old version)	√	√	√			
GEX (new version)	√	√		√		
GEOTHER	√	√			√	
Cinderella	√					
Discover	√	√				
geometriagon	√			√	√	
GeoView	√	√				
GeoGebra	√					
Theorema		√	√			

Table 1: Comparison between tools that combine DGS, ATP and repository of geometry theorems

Conclusions

GeoThms:

- DGSs (GCLC and Eukleides)
- ATP (GCLCprover)
- Database - GeoDB

All accessible through a Web interface. GeoThms system is, as far as we know, **the only system that integrates DGSs, ATPs, and a database of geometric problems in a Web interface**.
This framework provides:

- an environment suitable for new ways of studying and teaching geometry at different levels.
- an environment for storing mathematical knowledge (in explicit, declarative way) — about geometrical constructions, proofs, and illustrations.

We hope that GeoThms would contribute to a modern mathematical education.

Future Work

We hope that with the support from interested parties GeoThms can grow and became a widely used repository. We would try to make GeoThms a major Internet resource for geometrical constructions.

We will also work on the following tasks:

- To implement a e-Learning module for the study of Euclidean geometry at high-school and university level.
- To implement a module for proof visualisation and for moving through the generated proofs
- To improve the search mechanism
- To further develop the XML based interchange format (and the corresponding XML suite) that can link most of the current geometrical software.
- To implement/develop additional proving methods, primarily synthetic ones (e.g. angle method).
- To link additional geometry programs and additional theorem provers to our framework and to further develop the Web interface.