INTEGRATION OF ACOUSTIC ABSORBING POROUS COMPONENTS IN VEHICLE ENVIRONMENT USING A NOVEL FINITE ELEMENT SOLVER

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Abstract

A new mixed finite element formulation very well adapted to analyze the propagation of elastic and acoustic waves in porous absorbing media is presented. The proposed new formulation is based on modified Biot's equations [1,2,3] written in terms of the skeleton displacement and the acoustic pressure in the interstitial fluid. It generalizes the previous formulation proposed by Professor Atalla & Co-authors[11], and has the great advantage over existing formulations [5 to 13]of automatically satisfying all boundary conditions without having to compute surface coupling integrals at porous sub-domain interfaces. When elastic forces in the skeleton are neglected, the formulation automatically degenerates to an equivalent fluid model taking into account inertial coupling with the skeleton. This generalized mixed formulation and associated equivalent fluid model has been implemented by STRACO (France) in RAYON-PEM Solver. It is shown in this paper, that the numerical results predicted with RAYON-PEM agrees very well with experimental results using impedance tube tests and vibration measurements on multi-layered plates [14].

1. INTRODUCTION

The production at minimum cost of safety and high quality vehicles characterised by very low environmental noise and outstanding interior comfort becomes a very challenging objective for car manufacturers and for transportation industry in general. Therefore customers are becoming very sensitive to interior comfort and governmental authorities are imposing more and more severe regulations to reduce the external noise. To optimise at low cost the performances of vehicles by reducing vibration and noise levels, it is necessary to develop computer aided engineering analysis tools capable of predicting the vehicle performances at an early stage of the design cycle. During the last two decades a lot of research efforts has been devoted to the development of experimental techniques dedicated to the measurement of the local absorbing characteristics of Poro-Elastic Materials (PEM) and associated constitutive laws[15,16,17]. In parallel several analytical [1 to 4] and numerical models [5 to 13] have been proposed to solve the classical linear system of Biot's equations which governs the propagation of acoustic and elastic waves in porous media [1,2,3].

Section 2 of the paper presents the system of modified Biot's equations written in terms of the skeleton displacement and of the acoustic pressure in the interstitial fluid. It is shown that this system has the advantage of explicitly involving the total stress tensor in the porous media and the inertial force induced by the interstitial acoustic wave. Section 3 and 4 derives the variational formulations associated to modified Biot's equations, which generalises the previous mixed formulation proposed by Professor Atalla and Co-authors [11]. It has the great advantage of automatically satisfying all boundary conditions without having to compute surface coupling integrals at sub-domain interfaces. Section 5 presents the numerical results predicted by the present formulation which has been implemented in STRACO's RAYON-PEM Solver. The numerical results compares very well with experimental results obtained using impedance tube and treated plates. The last section corresponds to the conclusion of the paper where the authors indicates the features of the new mixed formulation to solve vibro-acoustic problems encountered in transportation industries where the vehicle structure interacts with the passengers compartment through poro-elastic barriers.

2. MODIFIED BIOT'S EQUATIONS

Propagation of elastic and acoustic waves in porous media are governed by the system of Biot's equations [1], which could be written for time harmonic waves with a time dependence in *exp(-iωt))* :

$$\frac{\partial(\sigma^s_{kl})}{\partial x_l} = -\omega^2(\tilde{\rho}_{11}U^s_k + \tilde{\rho}_{12}U^f_k) \qquad (1.1)$$

$$\frac{\partial(-\phi\ p\delta_{kl})}{\partial x_l} = -\omega^2(\tilde{\rho}_{12}U^s_k + \tilde{\rho}_{22}U^f_k) \qquad (1.2)$$

where, ω is the circular frequency, U^s_k and U^f_k represents respectively the skeleton and fluid displacement components.

The mass density coefficients appearing in equations (1.1) and (1.2) are given by:

$$\tilde{\rho}_{11} = (1-\phi)\rho_s - (1-\tau)\phi\rho_f - \phi\, b/j\omega \quad (2.1)$$

$$\tilde{\rho}_{22} = \tau\phi\rho_f - \phi\, b/j\omega \quad (2.2)$$

$$\tilde{\rho}_{12} = (1-\tau)\phi\rho_f + \phi\, b/j\omega \quad (2.3)$$

where ϕ is the porosity, τ is the turtuosity, ρ_s and ρ_f are the mass densities of the skeleton material and of the interstitial fluid. The coefficient b represents the viscous coupling between solid and fluid phases given by [1],

$$b = \phi\sigma\sqrt{1-4j\omega\eta\rho_f\left(\frac{\tau}{\Lambda\phi\sigma}\right)^2} \quad (3)$$

where σ is the flow resistivity, η is the viscosity of the interstitial fluid and Λ is the viscous characteristic length of the media. σ^s_{kl} and $\sigma^f_{kl} = -\phi\, p\delta_{kl}$ represent the components of the stress tensors respectively in the skeleton and in the interstitial fluid where p is the pressure.
The stress tensor components are related to the strain tensor components by the constitutive laws of the porous media,

$$\sigma^s_{kl} = (A\,div(U^s) + Q\,div(U^f))\delta_{kl} + G\left(\frac{\partial U^s_k}{\partial x_l} + \frac{\partial U^s_l}{\partial x_k}\right) \quad (4.1)$$

$$-\phi\, p = Q\,div(U^s) + R\,div(U^f) \quad (4.2)$$

where, G is the skeleton shear modulus, Q is the coupling modulus, A and R are the first Lamé coefficient and the bulk modulus of the porous media. Coefficient A, Q and R are related to the bulk modulus K_s of the skeleton material, the bulk modulus K_b of the skeleton in vacuum and to the bulk modulus K_f of the interstitial fluid by the following formulas [1]:

$$A = \frac{(1-\phi)((1-\phi)K_s - K_b) + \phi K_s K_b / K_f}{1-\phi - K_b/K_s + \phi K_s/K_f} - \frac{2G}{3} \quad (4.3)$$

$$Q = \frac{((1-\phi)K_s - K_b)\phi}{1-\phi - K_b/K_s + \phi K_s/K_f} \quad (4.4)$$

$$R = \frac{K_s\phi^2}{1-\phi - K_b/K_s + \phi K_s/K_f} \quad (4.5)$$

The relative displacement vector $(U^f - U^s)$ is given from equation (1.2) by,

$$U^f - U^s = \frac{1}{\omega^2\tilde{\rho}_f}grad(\phi\, p) - \tilde{\beta}\, U^s \quad (5)$$

where $\tilde{\beta} = (1 + \tilde{\rho}_{12}/\tilde{\rho}_{22})$ *is* the inertial coupling factor.
Modified Biot's equations are derived by substituting constitutive laws (4.1) and (4.2) into equation (1.1) and into the divergence of equation (5). This leads to the following system of Modified Biot's equations:

$$\tilde{\rho}_s\omega^2 U^s + div(\tilde{\sigma}^s_{kl}(U^s)) - \tilde{\alpha}\phi\, p\delta_{kl}) + \tilde{\beta} grad(\phi p) = 0 \quad (6.1)$$

$$div\left(\frac{1}{\omega^2\tilde{\rho}_f}grad(\phi p) - \tilde{\beta}\, U^s\right) + \frac{\phi p}{R} + \tilde{\alpha}\, div(U^s) = 0 \quad (6.2)$$

where $\tilde{\alpha} = (1 + Q/R)$ is the stiffness coupling factor between the skeleton and the fluid.
The equivalent masses $\tilde{\rho}_f$ and $\tilde{\rho}_s$ appearing in modified Biot's equations (6.1) and (6.2) are given by:

$$\tilde{\rho}_f = \phi\rho_e \quad (7.1)$$

$$\tilde{\rho}_s = (1-\phi)\rho_s + \phi\rho_f\left(1 - \frac{\rho_f}{\rho_e}\right) \quad (7.2)$$

where $\rho_e = \rho_f\tau - b/j\omega$, is the effective mass of the interstitial fluid which includes the viscous coupling factor b with the skeleton.

Modified Biot's equations (6.1) and (6.2) have the great advantage of involving explicitly the total stress tensor, $\sigma^{tot}_{kl} = \tilde{\sigma}^s_{kl} - \tilde{\alpha}\phi\, p\delta_{kl}$
where,

$$\tilde{\sigma}^s_{kl} = \left(K_b - \frac{2G}{3}\right)div(U^s)\delta_{kl} + G\left(\frac{\partial U^s_k}{\partial x_l} + \frac{\partial U^s_l}{\partial x_k}\right)$$

is the stress tensor of the skeleton with vacuum inside.

3. MIXED VARIATIONAL FORMULATION OF MODIFIED BIOT'S EQUATIONS

The mixed variational formulation associated to modified Biot's equations is simply derived by multiplying equation (6.1) by a virtual displacement vector V^s and equation (6.2) by a virtual pressure ϕq, and by integrating over the domain Ω occupied by the porous media,

$$Z(U^s,V^s) + A(p,q) - \hat{C}(p,V^s) - \hat{C}(q,U^s) = \tilde{C}_S(T,V^s) + C_S(q,W_n - U^s_n) \quad (8)$$

for admissible $V^s(\Omega)$ and $q(\Omega)$ satisfying prescribed boundary conditions.

$$Z(U^s,V^s) = K(U^s,V^s) - \omega^2 M(U^s,V^s) \quad (9)$$

is the mechanical impedance operator of the skeleton where K and M are the stiffness and mass operators given by,

$$K(U^s,V^s) = \int_\Omega (\tilde{\sigma}^s_{kl}(U^s)\tilde{\varepsilon}^s_{kl}(V^s d\Omega \quad (9.1)$$

$$M(U^s,V^s) = \int_\Omega \tilde{\rho}_s(U^s,V^S)d\Omega \quad (9.2)$$

$A(p,q) = H(p,q)-Q(p,q)/\omega^2$ is the acoustic admittance operator of the interstitial fluid where H and Q are the inertial and stiffness operators given by,

$$H(p,q)=\int_{\Omega}\frac{1}{\tilde{\rho}_f}((grad(\phi p),grad(\phi q))d\Omega \quad (10.1)$$

$$Q(p,q)=\int_{\Omega}\frac{\phi^2 pq}{R}d\Omega \quad (10.2)$$

$$\hat{C}(p,V^s)=$$
$$\int_{\Omega}\left\{\tilde{\alpha}\phi p\,div(V^s)+(\tilde{\beta}\,V^s,grad(\phi\ p))\right\}d\Omega \quad (11)$$

is the volume coupling operator.
The coupling operator given by equation (11) is composed by the sum of the volume stiffness coupling operator proportional to $\tilde{\alpha}$ and of the volume inertial coupling operator proportional to $\tilde{\beta}$.

$$\tilde{C}_S(T,V^s)=\int_S(T,V^s)dS \quad (12)$$

is the surface loading operator, where $T_k=\sigma_{kl}^{tot}n_l$ represents the component on the coordinate axis x_k of the total surface stress vector T. Finally,

$$C_S(q,W_n-U_n^s)=\int_S q(W_n-U_n^s)dS \quad (13)$$

is the surface cinematic coupling operator, where n is the outgoing unitary vector normal to the boundary S of the porous domain Ω.
As shown in figure 1, for practical applications the porous media is attached on a part S_1 of its boundary to an impervious master structure and is coupled on another part S_0 to an acoustic cavity.

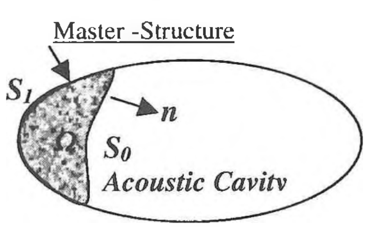

Figure 1: Porous Component attached to the Master-Structure and coupled with an acoustic cavity

The master structure S_1 communicates to the porous media a total displacement vector $W=U^s$. Reciprocally the porous component applies to the master structure a surface loading T. At the interface S_0 with the acoustic cavity the normal component W_n of the total acoustic displacement and the normal components U_n^s and U_n^f of the skeleton and of the fluid are related by the equation,

$$W_n-U_n^s=\phi(U_n^f-U_n^s) \quad (14)$$

For impervious surfaces the relative displacement is null, and for perforated surfaces there exist a relative displacement between the skeleton and the fluid. At the interface S_0 the acoustic cavity applies to the porous component a surface pressure loading $T_k = -pn_k$, and reciprocally the porous media communicates to the cavity a relative displacement $W_n-U_n^s$.
In summary the master structure communicates to the porous component a cinematic displacement and the acoustic cavity apply on it an acoustic surface loading. The porous component reacts by applying a surface loading on the master structure, and by communicating a relative displacement flux to the cavity.

In the configuration of figure 1, equation (8) could be rewritten in the following form,

$$Z(U,V)+A(p,q)-\overline{C}(p,V)-\overline{C}(q,U)=\tilde{C}_1(T,V)+C_0(q,W_n) \quad (15)$$

where,

$$\overline{C}(p,V)=C_0(p,V)-\hat{C}(p,V) \quad (16)$$

In order to avoid the assemblage of the complex porous component to the acoustic cavity, the continuity of the pressure at the interface S_0 could be relaxed and imposed in a weak form by a Lagrange multiplier X_n:

$$C_0(X_n,(p-p^c))=\int_{S_0}(p-p^c)X_n dS_0=0 \quad (17)$$

where p^c is the pressure applied by the cavity. This technique allows the use of incompatible finite element meshes at the interfaces between the porous component and the surrounding environment.
Finally, the addition of equations (15) and (17) leads to the mixed variational equation,

$$Z(U,V)+A(p,q)-\overline{C}(p,V)-\overline{C}(q,U)$$
$$-C_0(q,W_n)-C_0(p,X_n)=\tilde{C}_1(T,V)-C_0(p^c,X_n) \quad (18)$$

The right hand side of equation (18) is exactly proportional to the total energy exchanged between the porous media and its surrounding environment. The mechanical energy absorbed by the porous component from the Master-Structure corresponds to the first term, and the acoustic energy absorbed from the acoustic cavity corresponds to the second term.

4. EQUIVALENT FLUID- MODEL

For very soft porous media, elastic internal forces in the skeleton could be neglected. In this particular case the system of modified Biot's equations reduces to :

$$\tilde{\rho}_s\omega^2U^s+div(-\tilde{\alpha}\phi\ p\delta_{kl})+\tilde{\beta}grad(\phi p)=0 \quad (19.1a)$$

$$div\left(\frac{1}{\omega^2\tilde{\rho}_f} grad(\phi p) - \tilde{\beta}\, U^s\right) + \frac{\phi p}{R} + \tilde{\alpha}\, div(U^s) = 0 \qquad (19.2)$$

Multiplying equation (19.2) by the trial function ϕq and integrating over the porous media domain Ω leads to the following variational equation,

$$A(p,q) - \omega^2 M(U^s, V^s) = C_S(q, W_n - (1-\phi\tilde{\alpha})U_n^s) \qquad (20)$$

where the displacement vector V^s satisfies the equation,

$$\tilde{\rho}_s \omega^2 V^s + div(-\tilde{\alpha}\phi\, q\delta_{kl}) + \tilde{\beta} grad(\phi q) = 0 \qquad (19.1b)$$

which is equivalent to equation (19.1a) where the pressure p is replaced by the trial pressure q.
The coupling operator appearing in the right side of equation (20) is given by the integral,

$$C_S(q, W_n - (1-\phi\tilde{\alpha})U_n^s) = \int_S q(W_n - (1-\phi\tilde{\alpha})U_n^s) dS \qquad (21)$$

It is very easy to demonstrate by using equations (4.4) and (4.5) that,

$$1 - \phi\tilde{\alpha} = \frac{K_b}{K_s} \qquad (22)$$

for soft porous material $K_b << K_s$, so the second surface integral relative to the term $(1-\phi\tilde{\alpha})$ can be neglected. Finally by neglecting this second surface integral and by substituting U^s and V^s from equations (19.1a) and (19.1b) the variational equation (20) could be written in the following form,

$$A(p,q) + \tilde{H}(p,q)/\omega^2 = C_S(q, W_n) \qquad (23)$$

where,

$$\tilde{H}(p,q) = \int_\Omega \left\{ (grad\tilde{\alpha}\phi p - \tilde{\beta} grad\phi p)(grad\tilde{\alpha}\phi q - \tilde{\beta} grad\phi q) / \tilde{\rho}_s \right\} d\Omega \qquad (24)$$

The variational equation (23) expressed in term of the acoustic pressure corresponds to the fluid equivalent model of the porous media, where the inertia of the skeleton is taken into account by the additional term given by equation (24).

5. FINITE ELEMENT RESULTS

Discretisation by the Finite Element Method (FEM) of equations (18) and (23) allows the computation of the mixed impedance matrix of the porous component Ω by eliminating all internal degrees of freedom (dof) of the porous component except those dof's attached to the master structure and to the acoustic cavity. *This has the great advantage of not increasing the size of the global vehicle model and of allowing suppliers to compute separately the impedance matrices of their components and deliver these matrices to vehicle manufacturers.* FEM results predicted by RAYON-PEM Solver developed by STRACO (France) in cooperation with the University of Sherbrooke (Canada) are presented. Figure 2.a shows an excellent agreement between simulation and measurement for the case of homogeneous limp wool sample, protected by a thin perforated screen and placed at the end termination of an impedance tube. Figure 2.b compares with very good agreement the real and imaginary parts of the surface impedance predicted by the proposed model and impedance tube measurements for a multi-layers simple including a septum [4].

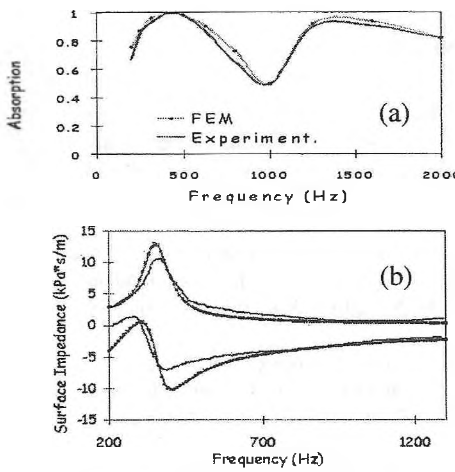

Figure 2: Impedance Tube Simulations
(a) homogeneous simple
(b) multi-layer simple with septum

Finally Figure 3 shows the results obtained for a system made up from two clamped rectangular plates separated by an unbounded foam. One plate is excited by a shaker and the normal quadratic velocity is measured on the facing plate. Good agreements are again achieved using both poro-elastic and equivalent fluid finite element models.

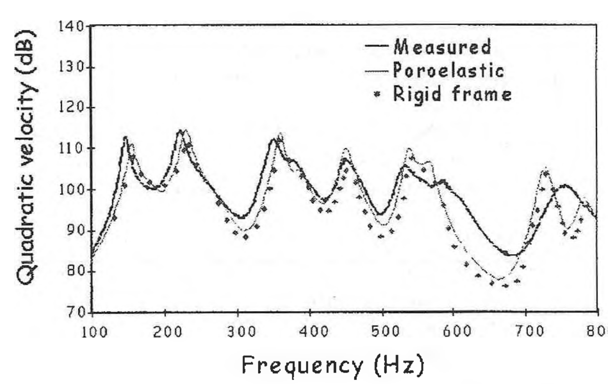

Figure 3: Finite Element Results for two rectangular plates coupled with foam

6. CONCLUSION AND PERSPECTIVES

The proposed mixed formulation based on modified Biot's equations has the advantage over existing formulations of automatically satisfying all interior and exterior boundary conditions at sub-domain interfaces. In addition it reduces at minimum the number of degrees of freedoms and considerably simplify the modelling effort by allowing the direct computation of the impedance matrix added by a complex porous component to the vibro-acoustic impedance matrix of the bare vehicle body. This new approach also simplifies the co-operative work between vehicle suppliers and manufacturers. The resulting RAYON-PEM Solver developed by STRACO and commercially available in the environment of I-DEAS Vibro-acoustics software constitutes a very promising and powerful analysis tool, which could be advantageously used to optimize the acoustic treatment of industrial vehicles.

7. REFERENCES

[1] J.F. ALLARD, "*Propagation of sound in porous media, Modelling of sound absorption,* Charpman" & Hall, North Way Andover, Hampshire, SP105BE, England , (1993).

[2] M.A BIOT, *Theory of propagation of elastic waves in a fluid-saturated porous solid. I. Low frequency range*, Journal of the Acoustical Society of America 28, 168-178 (1956);

[3] M.A BIOT, *Theory of propagation of elastic waves in a fluid-saturated porous solid. II. Higher frequency range*, Journal of the Acoustical Society of America 28, 179-191 (1956);

[4] P.M Morse, "Vibration and Sound", New York, McGraw-Hill, republished by the Acoustical Society of America (1981).

[5] J.P. COYETTE, H. BLOEMHOF, *A generalized procedure for modelling multi-layer insulation systems*, Proceeding of ISMA19, Katholic Universiteit Leuven, Leuven Belgium, (1996);

[6] V. EASWAREN, W. LAURIKS and J.P. COYETTE, "*Displacement-Based Finite Element Method for Guded Wave Propagation Problems: Application to Poroelastic Media*", Journal of Acoustical Society of America, 100, 2989-3002, 1996.

[7] Y.J KANG and J.S BOLTON, "*Finite Element Modeling of Isotropic Elastic Porous Materials Coupled with Acoustical Finite Elements*" Journal of the Acoustical Society of America 98, 635-643, (1995).

[8] J.S BOLTON, N.M SHIAU, Y.J KANG, *Sound transmission through multi-panel structures lined with elastic porous materials,* Journal of Sound and Vibration 191(3), 317-347, (1996)

[9] R. PANNETON, N. ATALLA, *Numerical prediction of sound transmission through finite multi-layer systems with poro-elastic materials,* Journal of Acoustical Society of America 100(1), 346-354 (1996);

[10] R. PANNETON, N. ·ATALLA, *An efficient sheme for solving the three-dimensional pro-elasticity problem in acoustics,* Journal of Acoustical Society of America 101(6), 3287-3298 (1997);

[11] N. Atalla, R. Panneton and P. Debergue, " *A mixed pressure-displacement formulation for poroelastic materials* " J. Acoust. Soc. Amer., 104(3), 1444-1452, (1998).

[12] P. GORENSON, *A 3D Symmetric finite element formulation of the Biot Equations for a fluid saturated linear elastic porous medium,* international Journal for Numerical Methods in Engineering, 41, 167-192, (1998)

[13] P.GORENSON, *Numerical modelling of dynamics of light porous materials,* PHD, Lund University (1998)

[14] B. BROUARD & al. *Measurement and prediction of the surface impedance of a resonant sound absorbing structure*, Acta Acoustica, Vol. 2, 301-306, (1994).

[15]A. F SYBERT, D.F ROSS, *Experimental determination of acoustic properties using a two microphones random excitation technique,* Journal of Acoustical Society of America, 61, 1362-1370, (1977).

[16] D.A. BIES, "Acoustical Properties of Porous Materials",Noise and Vibration Control, Capter 10, Edition L.L. BERANEK (1988).

[17] M.E. DELANEY and E.N BAZLEY, "Acoustical characteristics of fibrous absorbent materials", National Physical Laboratory, Aerodynamics Division Report AC37, (1969).