INTEGRATION OF SPATIAL INTERACTION MODELS: TOWARDS GENERAL THEORY OF SPATIAL INTERACTION

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Abstract The present study explores the possible extensions and improvements of spatial interaction modeling, and provides an overview of the generalized spatial interaction models including the new origin-destination pair effects.

It is demonstrated that a variety of spatial interaction models, given certain conditions, are actually identical: including the Poisson gravity model, the log-linear model, the logit model and the entropy-maximizing model.

The doubly constrained models and the saturated log-linear models are fitted to migration flows between prefectures in Japan in 1960 and 1985. The interaction effect and the *linkage coefficient* are identified. Then, improvements in the doubly constrained model are explored, adding the new explanatory variables relevant to the origin-destination pair (except for the distance between them).

Focusing on the similarity of these spatial interaction models allows the identification of the other origin-destination specific effects which are not accounted for in the conventional spatial interaction models. It is important to include the new variables corresponding to those effects in the spatial interaction model. Also, comparison with the multinomial logit model makes the interpretation of those effects in the behavioral context feasible.

Key words: spatial interaction model, entropy-maximizing model, Poisson gravity model, migration, Japan

1. Introduction

A theoretical extension of the classical gravity model based on social physics was provided by Wilson (1967), as Sugiura (1986) has shown. The importance of spatial interaction modeling as one of the sub-models of urban modeling, location modeling and location-allocation modeling has been recognized in the field of geography and urban planning (Yano, 1990). As a result of the continuing controversy regarding model misspecification, interpretation of distance-decay parameters, and calibrations (Baxter, 1983), various spatial interaction models have recently been developed (Ishikawa, 1988).

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At the same time, it has been found that the majority of these spatial interaction models are identical and have the same statistical properties (Baxter, 1982). Therefore, the spatial interaction models — the entropy-maximizing model (Wilson, 1967), the Poisson gravity models (Flowerdew and Aitkin, 1982), the log-linear model (Willekens, 1983a, b), the competing destinations model (Fotheringham, 1983) and the logit model (Ben-Akiva and Lerman, 1985) — are integrated using generalized linear modeling (Nelder and Wedderburn, 1972). These spatial interaction models are specified by Poisson regression, postulating that the flow is one of the occurrences of the random variable assumed to have a discrete probability distribution such as a Poisson distribution. Each spatial interaction model is distinguished by the included explanatory variables (Yano, 1991).

The present study explores the improvement in spatial interaction models, taking account of their similarity. A comparison between the doubly constrained model and the log-linear model allows the identification of other origin-destination specific effects (except the distance between them) not accounted for in conventional spatial interaction models. Moreover, in conceptualizing these effects and adding them to the doubly constrained model, it is possible to establish generalized spatial interaction models at an aggregate level. It is shown that origin-destination specific effects are related to the concept of the utility of the multinomial logit model, which is one of the discrete choice models that conceptually and operationally reflect the decision-making process.

These arguments will be illustrated with data on the numbers of migrants in 1960 and 1985 between pairs of the 46 prefectures in Japan. The results of fitting the doubly constrained model and the saturated log-linear model to the data are described. The relationships between origins and destinations, that is, the pair-specific effects, are identified, and their features are clarified by looking at the difference between the interaction effect of the log-linear model and the distribution function of the doubly constrained model. Improvements in generalized spatial interaction models are discussed, including the new explanatory variables relevant to the pair-specific effects.

2. The Integration of Spatial Interaction Models Using the Poisson Gravity Model

Most spatial interaction models developed recently are generalized linear models, *i.e.*, they regard the observed flow as one of the occurrences of a random variable assumed to have a discrete probability distribution. These spatial interaction models are specified by Poisson regression and differentiated by the explanatory variables included in them. One of the characteristics of Poisson regression is that an iteratively reweighted least squares procedure is needed to estimate the regression parameters. This can easily be done using a computer package such as GLIM (Generalized Linear Interactive Modeling) (Payne, 1987). In this section, the doubly constrained model and the log-linear model are respecified using the Poisson gravity model, and the possible extensions and improvements in spatial interaction modeling are explored by comparing the two models.

The Poisson gravity model

The characteristics of Poisson regression are clarified by contrasting them with Ordinary Least Squares regression. In Ordinary Least Squares regression the predicted value of the dependent variable is equal to a linear combination of the independent or explanatory variables, and the estimated mean of the random variable is assumed to have a continuous normal distribution. In short, the observed value of the dependent variable can be envisaged as a realization of the random variable. In Poisson regression the random variable is assumed to have a Poisson distribution. This characteristic is markedly different from that assumed by Ordinary Least Squares regression. That feature is appropriate in situations where the dependent variable consists of counts and is a non-negative integer. Moreover, in the Poisson gravity model, the total estimated frequency, that is, the total flow, is exactly equal to the total observed frequency (Fotheringham and Williams, 1983).

If there is a small constant probability that any individual in origin i will move to destination j, and that movements of individuals are independent, then if the population of i is large, the number of individuals recorded as moving from i to j will have a Poisson distribution. In Poisson regression the estimated value of the dependent variable is equal to the estimated mean value of the random variable and the variance (Lovett, 1984). According to a Poisson distribution, the probability of the flow between origin i and destination j, $p(t_{ij})$, is represented as follows:

$$p(t_{ij})=\frac{e^{-\theta_{ij}}\theta_{ij}^{t_{ij}}}{t_{ij}!}. \qquad (1)$$

It is assumed that the parameter θ_{ij} is logarithmically linked to a linear combination of the explanatory variables. Using the logarithms of the origin and destination populations, U_i and V_j, and distance, d_{ij}, as explanatory variables, the following equation is derived:

$$\theta_{ij}=\exp(\beta_0+\beta_1 \ln U_i+\beta_2 \ln V_j+\beta_3 \ln d_{ij}). \qquad (2)$$

The Poisson gravity model is able to overcome the disadvantages of the conventional gravity-type models (Fotheringham and Williams, 1983). However, it appears that the Poisson gravity model may be inadequate in migration flows, because most of the households involved in interurban migration have more than one member and each migrant is not independent (Flowerdew and Aitkin, 1982). Moreover, there is a problem regarding the inclusion of adequate explanatory variables into the model — a problem common to all spatial interaction models. Above and beyond the additional explanatory variables, other efforts have been made to refine and improve the spatial interaction model. Lovett *et al.* (1985) demonstrate that by incorporating sectoral and intervening opportunities variables, acceptable Poisson gravity models of the apprenticeship migration flows to Edinburgh between 1675 and 1799 may be identified. Flowerdew and Lovett (1988) show that substantial improvements have been made, through the use of the contiguity variable and the naval-base-interaction variable, in constrained models fitted to migration flows between labor market areas in Great Britain. In any case, the Poisson gravity models have the advantage that different or additional explanatory variables can be easily fitted.

The entropy-maximizing model

For the number of trips from origin i and destination j, T_{ij}, in the spatial interaction system with N origins and N destinations, the family of the entropy-maximizing models can be constructed using the combination of production-constraint and attraction-constraint given by equations (3) and (4). The four cases are known as unconstrained, production-constrained, attraction-constrained and doubly constrained models (Wilson, 1971).

$$O_i=\sum_j T_{ij},\ i, j=1, \dots N, \tag{3}$$
$$D_j=\sum_i T_{ij},\ i, j=1, \dots N. \tag{4}$$

The unconstrained model, *i.e.*, the model which has no constraint on either origin or destination totals, is given by equation (5). This model is identical to the conventional gravity model:

$$T_{ij}=K\ U_i\ V_j\ f_{ij}, \tag{5}$$

where U_i represents the origin emissiveness variable; V_j represents the destination attractiveness variable; f_{ij} represents the distribution function taken as the power function with distance; and K is the scale parameter, to ensure that the sum of the flows predicted by the model is equal to that of the actual ones.

In the production-constrained model, the total outflow, O_i, is assumed to be known. We use an attractiveness factor, V_j, to represent the influence of place j on choice of destinations. A single set of balancing factors, A_i, is calculated to ensure the constraint of the total outflows. The production-constrained model can be written:

$$T_{ij}=A_i\ O_i\ V_j^{\beta 2}\ f_{ij}, \tag{6}$$

with

$$A_i=1/\sum_j V_j^{\beta 2}\ f_{ij}. \tag{7}$$

The attraction-constrained model with total inflow, D_j, and an emissiveness factor of origin i, U_i, is

$$T_{ij}=B_j\ U_i^{\beta 1}\ D_j\ f_{ij}, \tag{8}$$

with

$$B_j=1/\sum_i U_i^{\beta 1}\ f_{ij}, \tag{9}$$

to ensure the constraint of the total inflows.

In the doubly constrained model, both outflow and inflow totals are known. Calculating the balancing factors, A_i and B_j, to ensure that two trip-end constraints of outflow and inflow totals are satisfied simultaneously, the following formulas are obtained:

$$T_{ij}=A_i\ O_i\ B_j\ D_j\ f_{ij}, \tag{10}$$
$$A_i=1/\sum_j B_j\ D_j\ f_{ij}, \tag{11}$$

$$B_j = 1/\sum_i A_i O_i f_{ij}. \tag{12}$$

The equivalence between the entropy-maximizing models and the Poisson gravity models applies where production- and/or attraction-constraints are employed (Flowerdew and Lovett, 1988). Fitting the production-constrained model is equivalent to fitting a factor (called a dummy variable) in the Poisson gravity model, where the factor has a different value for each origin. The attraction-constrained model can be fitted in an analogous manner, while a doubly constrained model is fitted by two factors simultaneously, one representing origins and one representing destinations. The unconstrained model is identical to the Poisson gravity model shown by equation (2), which is specified by the emissiveness variable of an origin, the attractiveness variable of a destination and the distance between them without factors.

The doubly constrained model with a power distance function then becomes the following gravity model taken as Poisson regression:

$$T_{ij} = \exp(\text{const.} + ORI(i) + DES(j) + \beta \ln d_{ij}), \tag{13}$$

where $ORI(i)$ and $DES(j)$ represent factors for origin i and destination j, respectively.

Equations (10) and (13) are identical, and in the Poisson gravity model an exponential value of a factor for origin i, $\exp(ORI(i))$, is proportionally equivalent to the product $A_i O_i$, and $\exp(DES(j))$ is proportionally equivalent to the product $B_j D_j$ in the doubly constrained model. The distance parameter estimated by the Poisson gravity model is identical to that estimated by the doubly constrained model with a power distance function (Yano, 1991).

The log-linear model

The log-linear model can be regarded as a special case of the Poisson gravity model with adequate factors for an origin and/or a destination.

The log-linear model is appropriate for data from categorical or qualitative variables. The main objective of this model is to specify and quantify the patterns of association between cross-classified variables. Log-linear models are useful in that 1) they decompose a data set into components or parameters, each of which represents the effects on the data of each variable and of combinations of variables in cross-classification; and 2) they describe these structures by a few parameters. The effect of a particular variable is referred to as the main effect, expressing the contribution to the data structure of the prevalence of the variable. The effect of composite variables is denoted as an interaction effect (Bishop *et al.*, 1975; Everitt, 1977).

In spatial interaction modeling, the origin-destination table is viewed as a cross-classification with R origins (rows) and C destinations (columns). The saturated log-linear model can be written in the following multiplicative form:

$$T_{ij} = w\, w_i^A\, w_j^B\, w_{ij}^{AB}, \tag{14}$$

where

$$w = [\Pi_{ij}\, T_{ij}]^{1/RC}, \tag{15}$$
$$w_i^A = (1/w)[\Pi_j\, T_{ij}]^{1/C}, \tag{16}$$

$$w_j^B=(1/w)[\Pi_i T_{ij}]^{1/R}, \tag{17}$$
$$w_{ij}^{AB}=T_{ij}/(w\ w_i^A\ w_j^B), \tag{18}$$

and the parameters satisfy the following constraints:

$$\Pi_i w_i^A=\Pi_j w_j^B=1.0, \tag{19}$$
$$\Pi_i w_{ij}^{AB}=\Pi_j w_{ij}^{AB}=1.0. \tag{20}$$

The overall mean effect is a size effect; it is the geometric mean of all T_{ij}-elements. The main effects denote the origin and destination effects, the former corresponding to the emissiveness of origin i, with the latter being the attractiveness of destination j. The interaction effect indicates an association between origin i and destination j; it captures the true spatial effect, and the distribution function is merely a proxy (Willekens, 1983a, b). If the interaction effect is absent in the origin-destination matrix, and hence w_{ij}^{AB}=1.0 for all i and j, the frequencies of flows may be determined by the overall mean effect and the main effects of the origin and the destination only.

The log-linear model can be specified by the Poisson gravity model as in equation (21). In the log-linear model, the overall mean, w, the main effects of row i and column j, w_i^A and w_j^B, and the interaction effect, w_{ij}^{AB}, correspond to the constant, the factors for origin i and destination j, *ORI*(i) and *DES*(j), and a factor for the pair of origin i and destination j, *ORI*(i).DES(j), in the Poisson gravity model.

$$T_{ij}=\exp(\text{const.}+ORI(i)+DES(j)+ORI(i).\ DES(j)). \tag{21}$$

In estimating the parameters of the log-linear model, there are two possible strategies, the 'centered effect' and the 'cornered effect' interpretations (Wrigley, 1985; Matsuda, 1988). The computer package GLIM uses the 'cornered effect' interpretation, assuming that the base cell is the cell i=1 and j=1. However, in the present study, the 'centered effect' interpretation will be adopted. Both methods can produce the same predicted frequencies; however, the parameter estimates of the log-linear models and the interpretation of these estimates differ according to the constraint systems used.

3. The Relationship between the Entropy-maximizing Model and the Log-linear Model

The strong relationship between the doubly constrained spatial interaction model and the multiplicative saturated log-linear model has been known (Willekens, 1983a,b). Comparison with the subscripts i and j, which denote origin i and destination j in these models, suggests that the parameters in the saturated log-linear model are composed of products (A_iO_i) and (B_jD_j). The interaction effect corresponds to the distribution function f_{ij}, which captures the true spatial effect exhibited by the observed flow matrix (Willekens, 1983a,b). In a doubly constrained model, the distribution function is the only proxy to the spatial effect.

The parameters included in both models can be calculated based on the maximum-likelihood estimation, known as the Poisson gravity model (Yano, 1991), and have similar statistical properties (Baxter, 1982). As Willekens (1983a) has shown using a numerical illustration, if the doubly constrained model is calibrated with the interaction effect rather than with the usual distribution function, then the predicted flows are identical to the observed ones. Also the balancing factors of the associated doubly constrained model are proportionally identical to the main effects. However, direct comparison of the distribution function f_{ij} and the interaction effect w_{ij}^{AB} is misleading, because the latter effect is scaled according to constraint (20). In order to identify the spatial effects or relationships between origin i and destination j included in the distribution function, the interaction effects of the log-linear model for the distribution function need to be determined. Although it is not usual to apply the log-linear model for continuous data, the matrix of the distribution function can be modeled by the log-linear model to derive the following relationship:

$$f_{ij}=d_{ij}^{\beta}=\tilde{w}\ \tilde{w}_i^A\ \tilde{w}_j^B\ \tilde{w}_{ij}^{AB}, \tag{22}$$

and

$$\tilde{w}=[\pi_{ij}\ d_{ij}^{\beta}]^{1/NN}, \tag{23}$$
$$\tilde{w}_i^A=(1/\tilde{w})[\Pi_j\ d_{ij}^{\beta}]^{1/N}, \tag{24}$$
$$\tilde{w}_j^B=(1/\tilde{w})[\Pi_i\ d_{ij}^{\beta}]^{1/N}, \tag{25}$$
$$\tilde{w}_{ij}^{AB}=d_{ij}^{\beta}/(\tilde{w}\ \tilde{w}_i^A\ \tilde{w}_j^B), \tag{26}$$

where the numbers of origins and destinations are N and N, respectively; the power distribution function f_{ij} $(=d_{ij}^{\beta})$ is specified by the doubly constrained model (10).

This interaction effect of the distribution function captures the relationship between origin i and destination j, accounted for in the doubly constrained model. Thus the doubly constrained model is one method of finding a matrix which satisfies the given marginal constraints and imposes the interaction effect of the distribution function onto the flow matrix. In this case, the main effects of origins and destinations on the distribution function are identical (as long as the matrix of the distances between them is symmetrical), to the geometric means of elements of a row and a column, respectively. Thus these effects represent the accessibilities of origins and destinations. The values have a tendency to be higher in the central area of the spatial system analyzed than in the periphery. This accessibility is defined by distances only (Pirie, 1979), unlike in Fotheringham's (1983) definition.

The relationship between the doubly constrained model and the saturated log-linear model may be seen in detail by applying a log-linear model to the predicted flows obtained by the doubly constrained model. The symbol "-" indicates the parameters modeled for the predicted flows, $\hat{T}_{ij}$'s:

$$\begin{aligned}\hat{T}_{ij}&=A_i\ O_i\ B_j\ D_j\ f_{ij},\\&=A_i\ O_i\ B_j\ D_j(\tilde{w}\ \tilde{w}_i^A\ \tilde{w}_j^B\ \tilde{w}_{ij}^{AB}),\\&=\bar{w}\ \bar{w}_i^A\ \bar{w}_j^B\ \bar{w}_{ij}^{AB}.\end{aligned} \tag{27}$$

As Willekens (1983a,b) indicates, it should be emphasized that the interaction effect of the predicted flow matrix, w_{ij}^{AB} with "-", is identical to that of the distribution function matrix, w_{ij}^{AB} with "~". This is why the entropy-maximizing model is called the biproportional adjustment model or the RAS method (Macgill, 1977). The biproportional adjustment model, in the case of the two-dimensional contingency table, consists of finding a matrix which satisfies the given marginal constraints and which is biproportional to the given matrix. This method is also called mostellerizing (Mosteller, 1968; Upton, 1978). The relationship between cells in the contingency matrix is an odd-rate which is independent of the scales of row totals and column totals, and is the interaction effect of that matrix. Accordingly, the doubly constrained model is a biproportional adjustment method for the OD (Origin-Destination) matrix, using the total outflow and total inflow of each place and the distribution function matrix.

The relationship among the main effect, w_i^A with "-", total outflow, O_i, and the balancing factor, A_i, is as follows:

$$\bar{w}_i^A \approx A_i \, O_i \, \tilde{w}_i^A, \tag{28}$$

$$A_i \approx \bar{w}_i^A / (O_i \, \tilde{w}_i^A) = 1/\tilde{w}_i^A. \tag{29}$$

The main effect of origin i in the log-linear model for the predicted flow matrix, w_i^A with "-", is proportional to the total outflow emanating from that origin, *i.e.*, O_i. The emissiveness of origin i in the doubly constrained model is represented in the multiplication of the balancing factor of origin i, A_i, the total outflow of origin i, and the main effect of that origin on the distribution function, w_i^A with "~". Several interpretations of balancing factors have been provided (Yano, 1989). Based on a comparison with the log-linear model, balancing factors are interpreted as the inverses of the main effects of the distribution function, which are equivalent to the accessibilities of origins or destinations. Accordingly, balancing factors play the role of eliminating the influence of the spatial pattern of origins or destinations, within the distribution function, from the emissiveness or attractiveness in the doubly constrained model. In other words, balancing factors can be regarded as terms adjusting the accessibilities of origins or destinations, which correspond to the main effects of the distribution function. These results prove the validity of Wilson's (1970) interpretation of balancing factors (Yano, 1992b).

The relationship between an origin and a destination, removing the effects of accessibilities of that origin and that destination from the distribution function, is correctly used in the conventional doubly constrained model. This relationship shows that the distances in the accessible area are overestimated, while those in the inaccessible area are underestimated. Consequently, in the doubly constrained model, the predicted flows in the accessible area are inevitably overestimated, while those in the inaccessible area are underestimated (Yano, 1992b).

Several attempts have been made recently to adopt the historical flow matrix instead of the distribution function (Snickars and Weibull, 1977; Scholten and Wissen, 1985), and these attempts demonstrate the superiority of the use of the historical flow matrix. In this case, the adopted relationship between an origin and a destination is not the value of the historical flow but the interaction effect of the historical flow matrix, calculating the balancing factors not corresponding to the inverses of the accessibilities of an origin and a destination. This indicates that, although the values of flows fluctuate greatly, the relationships between places do not change over time (Murauskas *et al.*, 1986). Therefore, the doubly constrained model incorporating either the historical flow matrix or the interaction effect contained in that matrix may advance the use of the spatial interaction model for forecasting.

These studies indicate that the comparison between the interaction effects of the observed flow matrix and of the specified distribution function allows the identification of the unspecified relationships of origin-destination pairs in the doubly constrained model. The specification of those relationships and their addition as new explanatory variables to the doubly constrained model make the improvement of spatial interaction modeling feasible. Adding further variables referring to either the origin or the destination alone, however, makes no sense since these effects are adjusted by the balancing factors (Flowerdew and Lovett, 1988). Accordingly, the appropriate form of pair-specific components must be added in the doubly constrained model.

4. Towards the Improvement of Spatial Interaction Models

Although spatial interaction models have long been used to analyze and forecast spatial flow patterns, there is little agreement as to model misspecification, interpretation of distance-decay parameters, and calibrations (Baxter, 1983). The misspecification problem is well known as the 'map pattern problem' (Sugiura, 1986; Ishikawa, 1988), which originated in the following controversial statement: "The estimated effects of geographic distance are likely to be biased in gravity models by the spatial structure (*i.e.*, the particular configuration of origins and destinations) under investigation, and by the existence of spatial autocorrelation between mass variables" (Curry, 1972; Curry *et al.*, 1975; Sheppard *et al.*, 1976; Cliff *et al.*, 1974, 1975 and 1976; Johnston, 1973, 1975 and 1976). The controversy surrounding this problem suggests that spatial interaction models for data on flows between sets of origins and destinations may often be misspecified because of a failure to account for the influence of spatial structure and other factors that affect flows.

To address this controversy, spatial variation of distance-decay parameters of origin-specific constrained models (Itoh, 1986) and the relationship between spatial structure and spatial interaction (Griffith and Jones, 1980) have been empirically investigated. Fotheringham (1983) demonstrates that the spatial variation in origin-specific distance-decay parameters harmonizes with the spatial pattern in accessibility, which measures population potential of a destination with respect to all other destinations. Further, he shows that including that term in the production-constrained gravity model eliminates spatial variation of distance-decay parameters. This model is termed a competing destinations model, based on a behavioral interpretation when destination choice is hierarchical. The competing destinations model attempts to identify a spatial structure variable that should be incorporated in conventional spatial interaction models, and shows the importance not only of the usual mass and distance effects but also of the elements of accessibility and competitiveness in flow (Fotheringham and O'Kelly, 1989; Ishikawa, 1988; Sugiura, 1988).

Accessibility, H_{ij}, defined by Fotheringham (1983), is specified as follows:

$$H_{ij}=\sum_k M_k\, d_{jk}^{\sigma} \neq i, j, \tag{30}$$

where M_k represents the mass of place k, d_{jk} represents the distance between k and j, and σ is a parameter estimated independently or exogenously. H_{ij} measures the accessibility of destination j to all other alternative destinations as perceived by migrants in origin i — that is, its spatial structure. The value of the accessibility increases within the area where the other destinations are clustered close together around destination j, and decreases within the area where they lie separately around that destination. Thus the competing destinations model is given by

$$T_{ij}=A_i\, O_i\, B_j\, D_j\, d_{ij}^{\beta}\, H_{ij}^{\alpha}. \tag{31}$$

In the competing destinations model, $\alpha<0$ when competitive forces are dominant in the spatial interaction system, $\alpha>0$ when agglomerative forces are present, and $\alpha=0$ when the two forces are in equilibrium. Equation (31), then, is identical to the doubly constrained model. Consequently, Fotheringham (1983) demonstrates that including the accessibility of the destination in the constrained models may avoid the misspecification caused by the effect of the spatial structure — that is, the 'map pattern problem'.

This accessibility can be regarded as one of the origin-destination pair-specific effects not accounted for in the conventional spatial interaction models. It is one of the factors making up the relationship between an origin and a destination (other than the distance), which contributes to producing the difference between the interaction effects of the observed flow matrix and the distribution function.

Another pair-specific effect must also be considered. Baxter and Ewing (1986) explore the possibility of improvement of spatial interaction models (with the exception of Fotheringham's competing destinations model), adding the following terms for measuring origin-destination relationships: 1) the nearest opportunity effect which represents whether destination j is the nearest destination from origin i, and 2) the nearness/cost interaction which takes into account the distances to destinations up to some critical threshold.

Recently, Fik and Mulligan (1990) developed the competing central places model based on Fotheringham's competing destinations model. This model incorporates the new explanatory variables defined in the next section — the effects of competing and intervening central places within the hierarchical spatial system. The effects of competing central places can be thought of as a jointly competing origins-destinations accessibility, given that both the order of the origin and the distance between i and j play an explicit role in delimiting the range of competing flows. The effects of hierarchical intervening opportunities are defined by opportunities between i and j, for the order of destinations and for intervening opportunities at distances equal to or less than the distance from origin i to destination j. Certainly the conventional spatial interaction models have omitted a vital variable for measuring the hierarchical spatial structure effects. However, Fik and Mulligan (1990) use a multiple regression technique: the conventional log-normal gravity model, not the generalized linear model. Hence comparison of results with those obtained using the entropy-maximizing model is quite unsatisfactory.

It seems reasonable to suppose that the spatial pattern of the other places (excluding places making up that origin-destination pair) and the hierarchical structure of places are incorporated in the conventional spatial interaction models. The former effects can be seen as an extension or a generalization of sociologist Stouffer's (1960) intervening opportunities and competing migrants formulations. The intervening opportunities are defined by the number of opportunities, such as total number of in-migrants, within the area between origin i and destination j. The competing migrants are defined by the number of opportunities from a destination-based standpoint: for example, the opportunities at a distance equal to or less than the distance from a destination to an origin. However, Stouffer uses the intervening opportunities instead of the distance between origin i and destination j. Porter (1964) constructs the generalized spatial interaction model by incorporating the effects of intervening opportunities into the classical log-normal gravity model, and explores the validity of the model by applying it to telephone calls, migration and highway traffic data in Nebraska. Consequently, he notes that it is important to conceptualize the ideas of the effects while taking into account the decision-

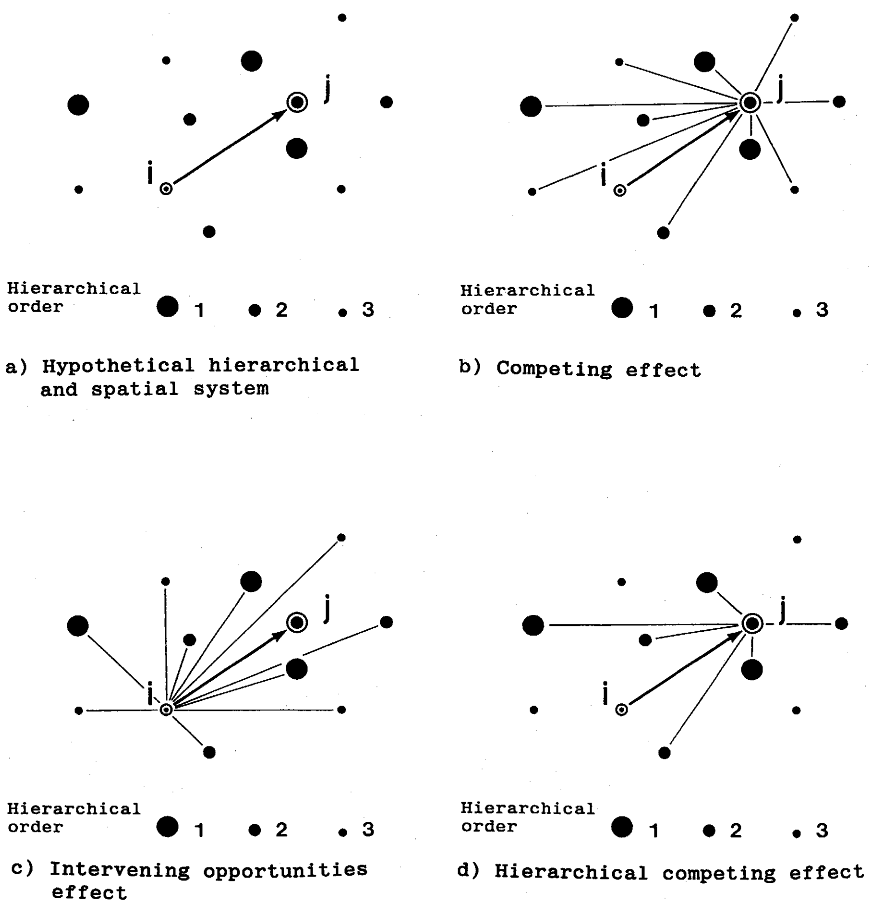


Fig. 1 Illustration of origin-destination pair-specific effects

making process of the mover.

In a study of interurban migration in Great Britain, Flowerdew and Lovett (1988) demonstrate that substantial improvements have been made through the incorporation of the contiguity variable and the naval-base-interaction variable into the Poisson constrained gravity model. These additional variables are based on the kind of spatial interaction, and detected through the analysis of residuals. Therefore, this study provides a poor representation of the conceptualization of the decision-making process.

The above arguments indicate that the variables of the origin-destination pair-specific effects added to the spatial interaction models are the competing effect, the intervening opportunities effect and the hierarchical effect. In the present study a generalized spatial interaction model will be constructed, incorporating these effects into the doubly constrained model. In the hypothetical spatial interaction system presented in Fig. 1-a, these effects are illustrated diagrammatically.

The competing effect (Fig. 1-b)

The competing effect represents the spatial relationships of other destinations for some origin-destination pair from a destination-based standpoint. This effect is measured by the accessibility of destination j to all other alternative destinations as perceived by migrants of origin i. There are two kinds of accessibility. The first is a distance measure of accessibility as in equation (32), and the other is a gravity measure as in equation (33) (Pirie, 1979):

$$H_{ij}=\sum_k d_{jk}^{\sigma},\ k\neq i, j, \tag{32}$$
$$H_{ij}=\sum_k M_k\, d_{jk}^{\sigma},\ k\neq i, j, \tag{33}$$

where M_k represents the mass variable of place k. Although the other alternative destinations — except for origin i and destination j — may be circumscribed by the area relating to the position of the origin-destination pair (Fik and Mulligan, 1990), the effects of the relevant destinations on accessibility vary with the value of the distance parameter, σ, in equations (32) and (33). Thus, as the value of σ increases, the effects on accessibility of alternative destinations far from destination j decrease, and can be eliminated. Therefore, it is not necessary to define the area that is affected by accessibility. By changing the distance parameter, that area can be delimited. In the present study, those cases in which the values of σ are -1.0, -1.5 and -2.0 are examined. It may be noted that equation (33) is identical to the accessibility in Fotheringham's competing destinations model.

The intervening opportunities effect (Fig. 1-c)

The intervening opportunities effect represents the spatial relationships of the other destinations for some origin-destination pair from an origin-based standpoint. This effect is measured by the accessibility of origin i to all other alternative destinations except for origin i and destination j, as perceived by migrants of origin i, as is the competing effect:

$$H_{ij}=\sum_k d_{ik}^{\sigma},\ k\neq i, j, \tag{34}$$
$$H_{ij}=\sum_k M_k\, d_{ik}^{\sigma},\ k\neq i, j. \tag{35}$$

Although it is possible to delimit the area which includes the relevant origin (Stouffer, 1960), as in the case of the competing effect, this particular area is not delimited in the present study.

The hierarchical effect (Fig. 1-d)

The hierarchical effect represents the hierarchical relation of the origins or the destinations and is incorporated in the competing and the intervening opportunities effects (Fik and Mulligan, 1990). For example, the hierarchical competing effect is specified by the accessibility of the destination to the other alternative destinations which are of the same or a higher order. The accessibility is calculated by equation (32) or (33). The hierarchical orders of each place are specified exogenously. The hierarchical intervening opportunities effect is measured as well, with the distance to destination j replaced with the distance from origin i:

$$H_{ij}=\sum_k d_{jk}^{\sigma},\ k\neq i, j \text{ and } k\in\{S\,;\, z_k\geq z_j\}, \tag{36}$$
$$H_{ij}=\sum_k M_k\, d_{jk}^{\sigma},\ k\neq i, j \text{ and } k\in\{S\,;\, z_k\geq z_j\}. \tag{37}$$

In specifying the hierarchical effect, two restrictions are used: 1) for those destinations of the same or a higher order than destination j ($k\in\{S;\ z_k\geq z_j\}$), and 2) for those destinations of the same or a higher order than origin i ($k\in\{S;\ z_k\geq z_i\}$), where z_k represents the order of place k.

The effects discussed in this section are summarized in Table 1. They are not independent but interrelated: the accessibility based on a gravity measure which includes the population of each destination is inevitably incorporated into the hierarchical effect. Therefore, because strong correlations exist between these effects, in the present study, explanatory variables measuring these effects are incorporated separately in the generalized spatial interaction models.

Table 1. New origin-destination pair-specific effects

EFFECT	ACCESSIBILITY	HIERARCHICAL EFFECT	MODEL
Competing	A) Distance	a) None	CAa1·2·3
		b) $z_k\geq z_i$	CAb1·2·3
		c) $z_k\geq z_j$	CAc1·2·3
	B) Gravity	a) None	CBa1·2·3
		b) $z_k\geq z_i$	CBb1·2·3
		c) $z_k\geq z_j$	CBc1·2·3
Intervening opportunities	A) Distance	a) None	IAa1·2·3
		b) $z_k\geq z_i$	IAb1·2·3
		c) $z_k\geq z_j$	IAc1·2·3
	B) Gravity	a) None	IBa1·2·3
		b) $z_k\geq z_i$	IBb1·2·3
		c) $z_k\geq z_j$	IBc1·2·3

Note: The numbers which identify the models correspond to the distance-decay parameters in calculating the accessibilities of equations (32) to (37): 1 is when $\sigma=-1.0$, 2 is when $\sigma=-1.5$ and 3 is when $\sigma=-2.0$.

5. Calibration Results for the Inter-prefectural Migration Flows in Japan in 1960 and 1985

The generalized spatial interaction models are fitted to the 1960 and 1985 migration flows between 46 prefectures in Japan (all except Okinawa Prefecture) based on the data from residence registration (Fig. 2). The mass of each prefecture is measured by population data available from the 1960 and 1985 census data. The distances between them are measured by the great circle distances between the cities in which prefectures' head offices are located, while the intra-prefectural distance, d_{ii}, is approximated by equation (38), according to Batty (1976, p. 248):

$$d_{ii}=r_i/\sqrt{2}, \tag{38}$$

where r_i is the radius of the roughly circular zone with the same area as prefecture i.

According to Ishikawa's (1978) study on inter-prefectural migration in postwar Japan, there was a turning point in internal migration in the latter half of the 1960s. Until 1970, migration was characterized by an increase in population flows from nonmetropolitan areas to some leading areas (Wakabayashi, 1987). The new trend is represented by an increase in population flows from metropolitan areas to nonmetropolitan areas, and between metropolitan areas (Kuroda, 1979). Thus, the 1960 and 1985 migration flows took place before and after the turning point, respectively.

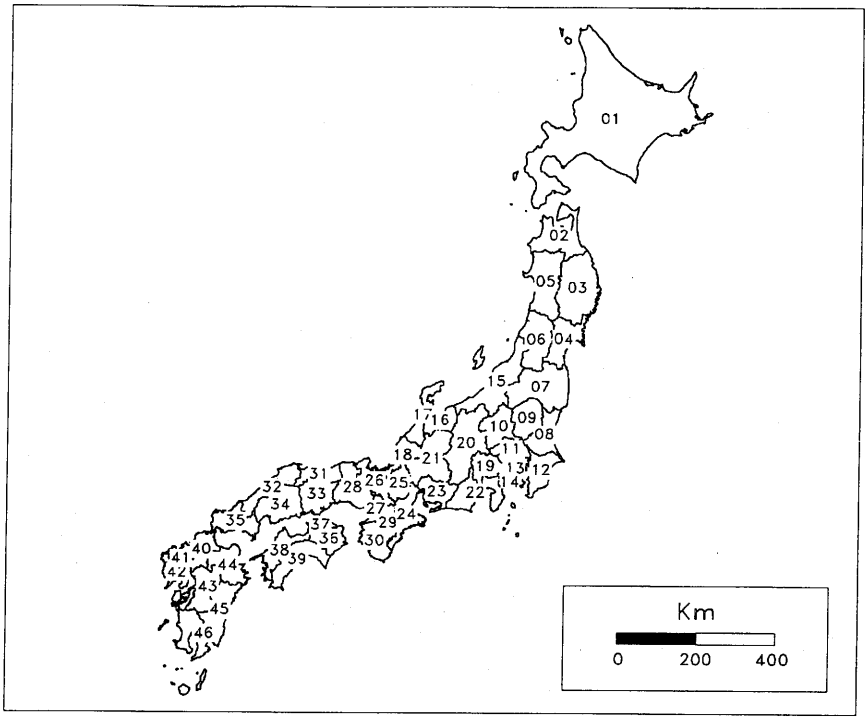
In this section, the results of fitting the entropy-maximizing models and the log-linear models to the data are described. Comparison of the doubly constrained model and the log-linear model allows the introduction of unspecified origin-destination relationships into conventional spatial interaction models. The generalized spatial interaction model, incorporating the accessibilities defined in the previous chapter, is also examined.

The parameter estimates of entropy-maximizing models can be calibrated as Poisson gravity models using the GLIM package, one of the statistical packages for generalized linear modeling (Aitkin *et al.*, 1989). In GLIM, goodness-of-fit for the Poisson gravity model can be assessed from several different perspectives. The overall difference between the observed values of the dependent variable and those predicted by the model is measured by the value of the scaled deviance, which is distributed in a manner similar to chi-square. In a Poisson regression model it is given by the following equation:

$$D=2[\{\Sigma_i\, y_i \ln(y_i/\theta_i)\}-\{\Sigma_i(y_i-\theta_i)\}], \tag{39}$$

where y_i is the observed value of the dependent variable in case i and θ_i is the predicted value of the dependent variable in case i. The scaled deviance is large when the correspondence between the two sets of values is poor and small when the correspondence is good. It has a value of zero if the model fits perfectly, while the upper limit depends on the nature of the data.

It is possible to evaluate the adequacy of a model by comparing the calculated deviance with the critical value of chi-square for the appropriate significance level and



1 Hokkaido	11 Saitama	21 Gifu	31 Tottori	41 Saga
2 Aomori	12 Chiba	22 Shizuoka	32 Shimane	42 Nagasaki
3 Iwate	13 Tokyo	23 Aichi	33 Okayama	43 Kumamoto
4 Miyagi	14 Kanagawa	24 Mie	34 Hiroshima	44 Oita
5 Akita	15 Niigata	25 Shiga	35 Yamaguchi	45 Miyazaki
6 Yamagata	16 Toyama	26 Kyoto	36 Tokushima	46 Kagoshima
7 Fukushima	17 Ishikawa	27 Osaka	37 Kagawa	
8 Ibaraki	18 Fukui	28 Hyogo	38 Ehime	
9 Tochigi	19 Yamanashi	29 Nara	39 Kochi	
10 Gumma	20 Nagano	30 Wakayama	40 Fukuoka	

Fig. 2 Forty-six prefectures of Japan under consideration

the number of degrees of freedom (which is equal to the number of observations minus the number of fitted parameters). A model can be considered adequate at the specified significance level if its deviance is less than the critical value. However, because the scaled deviance is dependent on the scale of the sample, it is usually compared with the deviance of the null model which is obtained by using the overall mean as an estimate. For example, when the stage is reached at which variables are added incrementally, the significance of individual variables can be examined by comparing the reduction in deviance with the critical value of chi-square corresponding to the change in the number of degrees of freedom.

Another significant aspect of the parameters is that they can be assessed through the division of each estimate by its standard error. As a rule of thumb, if the resulting value is greater than 2.0, then the parameter is significant; if it is less than 1.0, then the parameter is insignificant and should perhaps be removed from the fitted model (Lovett, 1984).

In Poisson regression the residuals of each case are not simply equal to the differences between the observed and fitted values, but are given by the following equation:

$$R_i = (y_i - \theta_i)/\sqrt{\theta_i}. \tag{40}$$

As a general rule, any residuals larger than ± 2.0 are abnormal values and are worthy of further investigation (Lovett, 1984).

Results of fitting entropy-maximizing models

The results of fitting the various types of constrained Poisson gravity models to the data in 1960 and 1985 are summarized in Table 2. The scaled deviances are smallest for the doubly constrained model, in the following order: the production-constrained, the attraction-constrained, and the unconstrained models for both years. The superiority of the production-constrained model over the attraction-constrained model indicates that the attractiveness of destinations is more important than the emissiveness of origins. However, the scaled deviances obtained suggest that these entropy-maximizing models are a long way from being satisfactory. The doubly constrained model fitted to the 1960 data resulted in a scaled deviance of 2,506,720 and degrees of freedom equal to 2,024. The calculated chi-square value should be less than 2,024.9 for the model fitted to avoid rejection at the 0.05 significance level. Yanagawa (1986) states that the chi-square value is dependent on the scale of the sample. Therefore, assessing only this perspective will produce misleading interpretations. On the other hand, since the deviance of the null model is 27,177,386 in 1960 and 28,611,598 in 1985, it is clear that the goodness-of-fit of all models improves rapidly. The unconstrained models account for about 80 percent of the null models, and the doubly constrained models for approximately 90 percent.

The values of distance parameters are from -1.471 to -1.195; the effects of the distance-decay of the constrained models are larger than those of the unconstrained models. Scanning the parameters of populations of an origin and a destination—measuring the emissiveness and attractiveness of prefectures—the values of the parameters of the population of a destination fitting the unconstrained and the production-constrained models to the data in 1960 are greater than 1.0, while those fitting the attraction-constrained model to the data in 1960 and the models to the data in 1985 are less than 1.0. The decrease in the importance of the attractiveness might reflect the change in the tendency to migrate in postwar Japan. For example, in 1960, compared with the population size of the prefecture, the attractiveness of the destination is overestimated; for 1985 the effect of attractiveness declines relatively.

The coefficients of the dummy variables in the constrained models are calculated; the dummy variables of origins are incor orated in the production-constrained and the doubly constrained models, while the dummy variables of destinations are incorporated in the attraction-constrained and the doubly constrained models. The coefficients of these dummy variables correspond to the products of the balancing factors and total outflows or inflows in the entropy-maximizing models, as mentioned in the previous section; the natural logarithm of A_iO_i (or B_jD_j) is equivalent to the parameter of the dummy variable of origin i (or of destination j). We should note that these values are affected by the spatial structure. The emissiveness of origins and the attractiveness of

Table 2. Results of entropy-maximizing models fitted to the migration flows in 1960 and 1985

Constrained Poisson gravity model	Variables included	1960 Parameter	S.E.	Scaled deviance	D.F.	1985 Parameter	S.E.	Scaled deviance	D.F.
Null	Const.	7.891	0.000	27,177,386	2,115	8.011	0.000	28,611,598	2,115
Unconstrained	Const.	−13.260	0.012	4,715,137	2,112	−6.515	0.011	5,655,750	2,112
	$\ln d_{ij}$	−1.195	0.000			−1.211	0.000		
	$\ln P_i$	0.769	0.001			0.682	0.001		
	$\ln P_j$	1.107	0.001			0.724	0.001		
Production-constrained	Const.	−0.860	0.011	2,716,197	2,068	5.951	0.010	3,307,708	2,068
	$\ln d_{ij}$	−1.440	0.000			−1.425	0.000		
	$\ln P j$	1.280	0.001			0.829	0.001		
Attraction-constrained	Const.	5.629	0.011	3,741,085	2,068	6.909	0.010	3,676,868	2,068
	$\ln d_{ij}$	−1.335	0.000			−1.403	0.000		
	$\ln P_i$	0.806	0.001			0.753	0.001		
Doubly constrained	Const.	19.310	0.003	2,506,720	2,024	19.270	0.003	2,836,010	2,024
	$\ln d_{ij}$	−1.471	0.001			−1.413	0.000		
Including the contiguity dummy variable, C_{ij}	Const.	18.170	0.006	2,457,116	2,023	17.700	0.005	2,713,422	2,023
	$\ln d_{ij}$	−1.313	0.001			−1.197	0.001		
	C_{ij}	0.408	0.002			0.562	0.002		

destinations are weighted by the inverse of the accessibility included in the distribution function. Therefore, these coefficients of the dummy variables of origins and destinations are interpreted as combined variables made up of the spatial structure and the emissiveness of origins or the attractiveness of destinations.

In the GLIM package, these coefficients of the dummy variables, called factors, are relative values constraining the first category of each dummy variable to zero. There is a tendency for values of the coefficients to be high in prefectures whose outflows or inflows are large, or which are located within an inaccessible area, as Table 3 indicates. This is a reflection of the fact that the values are measured by the multiplication of the total outflows or the total inflows of prefectures and the inverse of accessibilities which represent the spatial structure. For example, the values of Tokyo, Osaka and Aichi, whose total outflows and total inflows are high, and the values of Hokkaido and Fukuoka, which are located in inaccessible areas, are relatively high.

The residuals of the flows are measured by the relative residual defined by equation (40) and the absolute residual defined by the difference between the observed value and the predicted one. Table 4 shows that as variance and range of residuals of any model decrease, goodness-of-fit, measured by the deviance, is high, for both 1960 and 1985 (see also Table 2). Briefly, the residuals of each flow are as follows. Large positive residuals — where the values of predicted flows exceed those of the observed flows — occur for intra-prefectural flows and flows between metropolitan areas and nonmetropolitan areas with relatively long distances. Flows within metropolitan areas, on the other hand, produce large negative residuals. As is shown in Fig. 3, systematic tendencies are indicated: the intra-prefectural flows are overestimated, and the flows within metropolitan areas where the prefectures with large emissiveness and attractiveness are near each other are underestimated. Thus, further explanatory variables relevant to these residuals should be incorporated in the spatial interaction models.

Next, the doubly constrained models including the contiguity variable are examined. The dummy variable of the contiguity, C_{ij}, has a value of 1 if prefectures i and j are contiguous, and of 0 otherwise. The parameter estimates of the contiguity variables are biased toward a less positive direction (see Table 2). However, the addition of the contiguity variable does not greatly improve goodness-of-fit (see Table 4). That is, in general, contiguity has a positive effect on intra-prefectural flows, but leads to further overestimation within metropolitan areas. Accordingly, negative residuals in the metropolitan area are fostered, and little improvement is made as a whole.

Results of fitting log-linear models

The log-linear models were fitted to the same data sets. The results — overall mean effects and the main effects of origins and destinations for the 1960 and 1985 data — are presented in Table 5. The comparison of two different years makes the meaning of these effects clear. The value of the overall mean effect rises from 271.7 to 377.3. This increment is caused by a change in the total migrants. The important point to note is that the main effects of origins and destinations vary greatly between 1960 and 1985. Basically, the origin effects represent the relative emissiveness of the origins and the destination effects represent the relative attractiveness of the destinations, and these

Table 3. Dummy variables of origins and destinations of the doubly constrained models fitted to the migration flows in 1960 and 1985

	Prefecture	1960		Exponential		1985		Exponential	
		ORI (*i*)	*DES* (*j*)	*ORI* (*i*)	*DES* (*j*)	*ORI* (*i*)	*DES* (*j*)	*ORI* (*i*)	*DES* (*j*)
1	Hokkaido	0.000	0.000	1.000	1.000	0.000	0.000	1.000	1.000
2	Aomori	−1.798	−2.118	0.166	0.120	−1.883	−1.964	0.152	0.140
3	Iwate	−1.766	−2.084	0.171	0.124	−1.941	−1.905	0.144	0.149
4	Miyagi	−1.914	−2.167	0.147	0.115	−1.970	−1.674	0.139	0.187
5	Akita	−1.876	−2.393	0.153	0.091	−2.316	−2.311	0.099	0.099
6	Yamagata	−2.313	−2.642	0.099	0.071	−2.858	−2.667	0.057	0.069
7	Fukushima	−1.669	−1.935	0.188	0.144	−2.212	−1.944	0.109	0.143
8	Ibaraki	−2.490	−1.924	0.083	0.146	−2.334	−1.719	0.097	0.179
9	Tochigi	−2.813	−2.388	0.060	0.092	−2.841	−2.265	0.058	0.104
10	Gumma	−2.742	−2.267	0.064	0.104	−2.779	−2.253	0.062	0.105
11	Saitama	−3.595	−1.839	0.027	0.159	−2.304	−1.525	0.100	0.218
12	Chiba	−3.132	−1.576	0.044	0.207	−2.082	−1.322	0.125	0.267
13	Tokyo	−1.655	0.100	0.191	1.106	−1.447	−0.772	0.235	0.462
14	Kanagawa	−2.833	−0.947	0.059	0.388	−1.869	−1.095	0.154	0.335
15	Niigata	−1.647	−1.819	0.193	0.162	−2.010	−1.791	0.134	0.167
16	Toyama	−2,923	−2.714	0.054	0.066	−3.095	−2.795	0.045	0.061
17	Ishikawa	−2.967	−2.667	0.051	0.069	−2.840	−2.553	0.058	0.078
18	Fukui	−3.229	−2.956	0.040	0.052	−3.369	−3.094	0.034	0.045
19	Yamanashi	−3.328	−2.853	0.036	0.058	−3.369	−2.822	0.034	0.059
20	Nagano	−2.021	−1.745	0.133	0.175	−2.222	−1.787	0.108	0.167
21	Gifu	−2.934	−2.155	0.053	0.116	−2.818	−2.481	0.060	0.084
22	Shizuoka	−2.025	−1.257	0.132	0.285	−1.950	−1.474	0.142	0.229
23	Aichi	−1.819	−0.751	0.162	0.472	−1.508	−1.136	0.221	0.321
24	Mie	−2.784	−2.211	0.062	0.110	−2.719	−2.366	0.066	0.094
25	Shiga	−4.037	−3.363	0.018	0.035	−3.754	−3.364	0.023	0.035
26	Kyoto	−2.633	−1.929	0.072	0.145	−2.382	−2.134	0.092	0.118
27	Osaka	−1.964	−0.702	0.140	0.496	−1.530	−1.340	0.217	0.262
28	Hyogo	−1.996	−1.052	0.136	0.349	−1.682	−1.463	0.186	0.232
29	Nara	−4.124	−3.386	0.016	0.034	−3.428	−3.049	0.032	0.047
30	Wakayama	−3.200	−2.733	0.041	0.065	−3.228	−3.133	0.040	0.044
31	Tottori	−2.990	−3.083	0.050	0.046	−3.268	−3.102	0.038	0.045
32	Shimane	−2.280	−2.705	0.102	0.067	−2.731	−2.606	0.065	0.074
33	Okayama	−2.114	−2.169	0.121	0.114	−2.305	−2.138	0.100	0.118
34	Hiroshima	−1.534	−1.865	0.216	0.155	−1.554	−1.436	0.211	0.238
35	Yamaguchi	−1.621	−2.299	0.198	0.100	−2.149	−2.123	0.117	0.120
36	Tokushima	−2.878	−3.036	0.056	0.048	−3.093	−2.962	0.045	0.052
37	Kagawa	−2.828	−3.083	0.059	0.046	−3.022	−2.835	0.049	0.059
38	Ehime	−1.845	−2.493	0.158	0.083	−2.380	−2.307	0.093	0.100
39	Kochi	−2.227	−2.676	0.108	0.069	−2.770	−2.668	0.063	0.069
40	Fukuoka	−0.715	−1.662	0.489	0.190	−1.205	−1.160	0.300	0.313
41	Saga	−2.158	−3.467	0.116	0.031	−3.098	−3.155	0.045	0.043
42	Nagasaki	−1.243	−2.544	0.289	0.079	−2.050	−2.143	0.129	0.117
43	Kumamoto	−1.254	−2.513	0.285	0.081	−1.979	−1.965	0.138	0.140
44	Oita	−1.762	−2.667	0.172	0.069	−2.358	−2.316	0.095	0.099
45	Miyazaki	−1.534	−2.589	0.216	0.075	−2.089	−2.091	0.124	0.124
46	Kagoshima	−0.879	−2.229	0.415	0.180	−1.646	−1.614	0.193	0.199

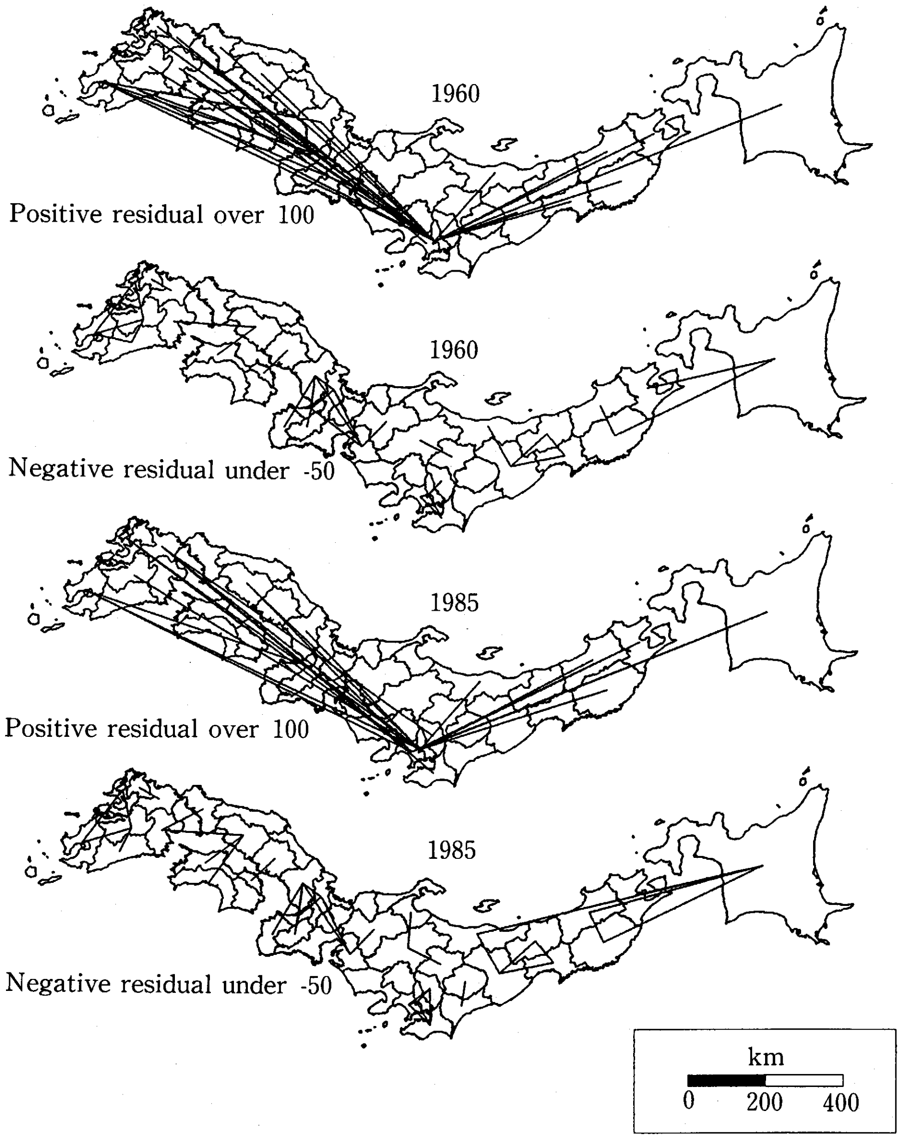


Fig. 3 Spatial pattern of relative residuals of the doubly constrained models fitted to the migration flows in 1960 and 1985

effects correspond to the total outflows and the total inflows. The spatial patterns of the origin and destination effects are harmonious, because those prefectures with large outflows also have large inflows. For 1960 the destination effects of Tokyo, Osaka, Aichi, Kanagawa and Hyogo are relatively high, exceeding their origin effects. The destination effects of Tokyo and Osaka for 1985 decline, and the origin and destination effects are in balance. The destination effects of Chiba and Saitama, which are part of the Tokyo metropolitan area, experience a relative rise, presumably because of increas-

Table 4. Summary of residuals of entropy-maximizing models fitted to the migration flows in 1960 and 1985

Constrained Poisson gravity model		1960 Observed flows	Predicted flows	Relative residuals	Absolute residuals	1985 Observed flows	Predicted flows	Relative residuals	Absolute residuals
Unconstrained	SUM	5,652,659.0	5,652,826.0	−8,797.2	−166.8	6,376,468.0	6,376,659.0	−6,568.2	−191.3
	MEAN	2,671.4	2,671.5	−4.2	−0.1	3,013.5	3,013.5	−3.1	−0.1
	S.D.	18,226.5	17,956.2	58.7	8,674.6	17,388.1	14,892.1	74.1	10,387.9
	MAX	631,344.0	671,142.4	1,546.8	235,105.9	435,652.0	380,267.2	2,501.8	311,011.4
	MIN	1.0	76.1	−329.6	−140,909.0	2.0	86.6	−288.7	−124,219.0
	RANGE	631,343.0	671,066.3	1,876.4	376,014.9	435,650.0	380,180.6	2,790.5	435,230.4
Production-constrained	SUM	5,652,659.0	5,652,697.0	−3,353.3	−38.0	6,376,468.0	6,376,538.0	204.6	−70.0
	MEAN	2,671.4	2,671.4	−1.6	0.0	3,013.5	3,013.5	0.1	0.0
	S.D.	18,226.5	18,902.5	40.3	5,248.9	17,388.1	15,857.8	45.7	6,777.8
	MAX	631,344.0	714,195.2	328.5	77,668.5	435,652.0	416,975.8	401.3	142,836.9
	MIN	1.0	19.0	−201.4	−82,851.2	2.0	32.5	−223.0	−88,466.6
	RANGE	631,343.0	714,176.2	529.9	160,519.7	435,650.0	416,943.3	624.3	231,303.5
Attraction-constrained	SUM	5,652,659.0	5,652,659.0	−4,122.6	0.0	6,376,468.0	6,376,468.0	555.6	−0.3
	MEAN	2,671.4	2,671.4	−1.9	0.0	3,013.5	3,013.5	0.3	0.0
	S.D.	18,226.5	17,879.4	49.9	6,449.6	17,388.1	15,495.0	50.7	7,228.3
	MAX	631,344.0	651,127.3	547.6	128,729.6	435,652.0	399,951.7	610.5	163,743.5
	MIN	1.0	35.3	−301.2	−121,920.0	2.0	41.4	−278.0	−117,399.0
	RANGE	631,343.0	651,092.0	848.8	250,649.6	435,650.0	399,910.3	888.5	281,142.5
Doubly constrained	SUM	5,652,659.0	5,652,727.0	−2,491.5	−67.5	6,376,468.0	6,376,503.0	−2,035.8	−35.0
	MEAN	2,671.4	2,671.4	−1.2	0.0	3,013.5	3,013.5	−1.0	0.0
	S.D.	18,226.5	18,876.1	40.1	4,586.0	17,388.1	16,329.7	41.4	5,661.8
	MAX	631,344.0	700,488.9	360.6	73,071.9	435,652.0	409,442.2	402.1	81,965.4
	MIN	1.0	9.4	−193.5	−69,144.9	2.0	27.0	−193.8	−70,182.3
	RANGE	631,343.0	700,479.5	554.1	142,216.8	435,650.0	409,415.2	595.9	152,147.7
Doubly Constrained	SUM	5,652,659.0	5,652,711.0	−3,375.1	−52.3	6,376,468.0	6,376,489.0	−3,335.8	−20.6
	MEAN	2,671.4	2,671.4	−1.6	0.0	3,013.5	3,013.5	−1.6	0.0
Including the	S.D.	18,226.5	18,845.1	39.1	4,684.0	17,388.1	16,370.1	39.7	5,884.2
contiguity dummy	MAX	631,344.0	696,106.1	325.0	71,971.3	435,652.0	396,338.4	388.8	80,024.2
variable, C_{ij}	MIN	1.0	11.5	−199.5	−66,668.1	2.0	29.2	−203.9	−80,131.8
	RANGE	631,343.0	696,094.6	524.5	138,639.4	435,650.0	396,309.2	592.7	160,156.0

ing out-migration from Tokyo within the metropolitan area.

The interaction effects, which represent the pure relationships of the origin-destination pairs, are calculated in a matrix form such as a distribution function. The matrix of the interaction effects is asymmetrically different from the distance distribution matrix, though approximately symmetrical itself. It is difficult to understand the entirety of the relationships of the origin-destination pairs at once; in this case the number of pairs is 2,116. In order to derive the relationships, a multidimensional scaling method is applied to the matrix of interaction effects, measuring the similarities between origins and destinations. SSA-II, which produces a two-dimensional configuration of objects from an asymmetrical distance matrix, is used (Guttman, 1968).

Figure 4 shows the 1960 and 1985 maps of the interaction effects of internal migration flows in Japan. On these maps, the prefectures, arranged in a series from the Tohoku region to the Kyushu region, are configured as a circle running counterclockwise. This configuration is a reflection of the real geographical pattern of the prefectures. The prefectures at the periphery of Japan (the Tohoku and Kyushu regions) are close together, while those in the middle (the Chubu region) are dispersed. The configuration by SSA-II, based on a row (or origin) solution, is similar to one based on a column (or destination) solution. It is worth noting that the relationships between prefectures are nearly symmetrical, and these relationships have not changed over time. To measure the degree of resemblance between configurations in 1960 and 1985, Tobler's (1983) bidimensional regression analysis is employed (Sugiura, 1989). The correspondence between the two spacings is extremely high (R^2=0.971). From these results it may be concluded that the variation of the volumes of migration flows, the asymmetry characterized by the dominance of flows from the nonmetropolitan areas to the metropolitan areas, and the difference between the flow patterns of migrants in 1960 and those in 1985 are caused not by the change over time in the relationships between prefectures, but by the change in the total outflows from origins and the total inflows to destinations. Therefore, it can be shown that the relationships of the origin-destination pairs are stable over time.

The maps based on the interaction effects of the distribution functions are similar to the maps drawn from the observed flow matrix (see Fig. 5). The correspondence rate between the two configurations is quite high. The coefficients of determination of bidimensional regression, R^2, are 0.871 for 1960 and 0.835 for 1985. The relationships of the origin-destination pairs included in the observed migration are closely related to those pairs included in the distribution function. It is suggested that the origin-destination relationships of the migration flows correspond to the relationships measured by the distances between origins and destinations, excluding the effects of the configuration of the prefectures — that is, the accessibility. The doubly constrained models use the relative distribution function with the accessibilities of each prefecture removed as the origin-destination relationships. The values of the distribution function have a tendency to be higher in the central area of the spatial system concerned than at the periphery. Thus, the effects of the distance-deterrence of prefectures in the central area on the other prefectures are high, while those at the periphery are low. In the constrained models, however, these values are standardized. As the results of SSA-II indicate, therefore, the

Table 5. Main effects of origins and destinations of the log-linear models fitted to the migration flows in 1960 and 1985

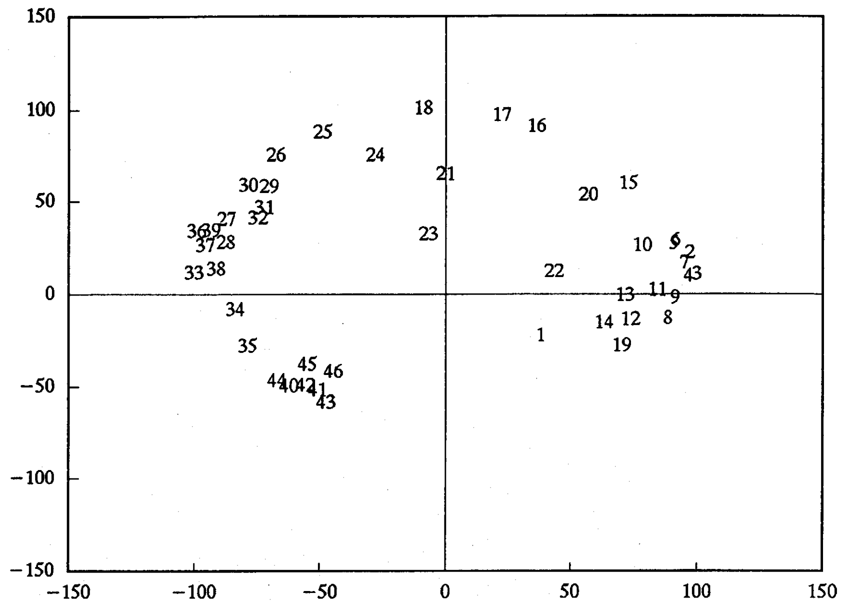
1) 1960

	Prefecture	Main effects of an origin w_i^A	Main effects of a dentination w_j^B	Population P_i	Intra-prefectural flow T_{ii}	Total outflow O_i	Total inflow D_j
1	Hokkaido	2.8761	2.8070	5,039,206	258,209	326,296	311,987
2	Aomori	0.6256	0.4912	1,426,606	29,614	60,978	48,298
3	Iwate	0.5232	0.3801	1,448,517	29,300	62,266	47,551
4	Miyagi	0.8316	0.7293	1,743,195	35,424	84,700	65,450
5	Akita	0.6133	0.3742	1,335,580	22,206	57,451	37,187
6	Yamagata	0.4918	0.3877	1,320,664	21,648	58,395	41,194
7	Fukushima	0.8020	0.6114	2,051,137	46,443	111,322	74,831
8	Ibaraki	0.5978	0.9031	2,047,024	31,204	84,300	67,288
9	Tochigi	0.4225	0.4482	1,513,624	24,396	62,723	44,600
10	Gumma	0.4634	0.5215	1,578,476	26,674	67,216	50,655
11	Saitama	1.0909	2.3734	2,430,871	41,881	108,109	142,154
12	Chiba	1.1735	2.4768	2,306,010	45,375	113,646	130,819
13	Tokyo	15.9535	34.4182	9.683,802	631,344	1,020,108	1,230,821
14	Kanagawa	3.1710	7.5827	3,443,176	112,246	216,909	310,244
15	Niigata	1.1629	0.9686	2,442,037	42,158	105,763	73,040
16	Toyama	0.6206	0.5209	1,032,614	16,499	37,002	29,333
17	Ishikawa	0.5349	0.6015	973,418	15,556	34,853	30,015
18	Fukui	0.4391	0.4119	752,696	11,745	28,242	22,230
19	Yamanashi	0.2521	0.3613	782,062	14,087	36,895	26,197
20	Nagano	0.8119	0.8425	1,981,433	47,237	97,422	75,129
21	Gifu	1.0574	1.5483	1,638,399	32,258	74,724	72,764
22	Shizuoka	1.9635	2.9085	2,756,271	72,142	134,357	130,246
23	Aichi	3.7145	8.3161	4,206,313	166,935	256,293	324,900
24	Mie	0.9041	1.2137	1,485,054	28,217	65,591	57,919
25	Shiga	0.5679	0.9555	842,695	12,940	36,750	34,695
26	Kyoto	1.8987	2.4137	1,993,403	63,279	120,565	114,790
27	Osaka	5.8776	12.8024	5,504,746	283,692	432,482	583,477
28	Hyogo	3.2085	5.4339	3,906,487	131,451	238,080	272,439
29	Nara	0.6139	0.7053	781,058	12,770	35,899	30,391
30	Wakayama	0.4733	0.6058	1,002,191	19,527	44,359	38,847
31	Tottori	0.4179	0.3074	599,135	10,894	29,520	21,614
32	Shimane	0.5720	0.4058	888,886	16,774	44,550	31,102
33	Okayama	0.9746	0.9561	1,670,454	39,407	81,578	69,260
34	Hiroshima	1.4243	1.5721	2,184,043	62,973	116,007	109,692
35	Yamaguchi	1.4724	1.0987	1,602,207	41,768	92,289	76,332
36	Tokushima	0.4583	0.3186	847,274	15,118	40,669	26,892
37	Kagawa	0.5332	0.4100	918,867	16,622	45,959	33,894
38	Ehime	1.1231	0.6958	1,500,687	36,449	84,238	61,237
39	Kochi	0.5828	0.4154	854,595	25,457	50,892	37,701
40	Fukuoka	3.8060	2.4527	4,006,679	160,395	291,867	261,151
41	Saga	0.6480	0.2545	942,874	20,013	63,409	42,565
42	Nagasaki	1.4723	0.6930	1,760,421	51,538	117,321	86,312
43	Kumamoto	1.3319	0.7509	1,856,192	44,963	106,957	77,392
44	Oita	0.8833	0.6094	1,239,655	28,098	69,552	51,956
45	Miyazaki	1.0009	0.6078	1,134,590	30,279	71,152	54,813
46	Kagoshima	1.4587	0.8211	1,963,104	54,408	133,003	91,255
	Total	Overall mean effect $w=271.6946$		93,418,428	2,981,613	5,652,659	5,652,659

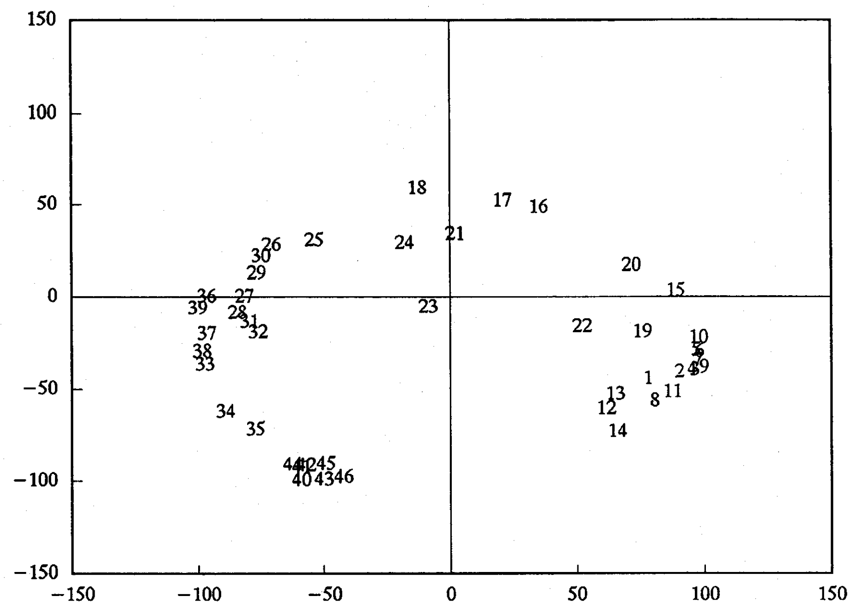
Table 5. Continued

2) 1985

	Prefecture	Main effects of an origin w_i^A	Main effects of a dentination w_j^B	Population P_i	Intra-prefectural flow T_{ii}	Total outflow O_i	Total inflow D_j
1	Hokkaido	2.5187	2.0139	5,679,439	326,465	416,801	389,901
2	Aomori	0.6975	0.5393	1,524,448	32,045	74,856	64,364
3	Iwate	0.4469	0.3724	1,433,611	34,298	70,213	62,231
4	Miyagi	0.9936	1.0526	2,176,295	53,635	111,780	112,075
5	Akita	0.3734	0.3170	1,254,032	21,359	48,524	41,594
6	Yamagata	0.3156	0.2772	1,261,662	20,464	44,468	40,183
7	Fukushima	0.6579	0.6333	2,080,304	37,298	80,841	75,966
8	Ibaraki	1.2150	1.4081	2,725,005	54,222	110,323	118,384
9	Tochigi	0.6704	0.8151	1,866,066	30,012	66,555	69,925
10	Gumma	0.5603	0.6524	1,921,259	35,786	68,636	70,191
11	Saitama	3.6521	4.5284	5,863,678	131,567	291,044	330,982
12	Chiba	4.0148	4.9456	5,148,163	114,702	274,343	306,226
13	Tokyo	13.8233	17.3902	11,829,363	435,652	912,842	914,462
14	Kanagawa	6.7241	8.7330	7,431,974	230,199	458,431	507,601
15	Niigata	0.8772	0.8138	2,478,470	43,976	88,464	80,115
16	Toyama	0.4930	0.4577	1,118,369	15,526	34,717	33,111
17	Ishikawa	0.6724	0.6437	1,152,325	20,718	44,833	43,220
18	Fukui	0.3593	0.3655	817,633	11,052	25,957	24,980
19	Yamanashi	0.3180	0.4027	832,832	17,965	36,232	38,263
20	Nagano	0.7873	0.8979	2,136,927	51,101	88,248	89,853
21	Gifu	1.0316	1.0267	2,028,536	37,625	76,715	75,803
22	Shizuoka	1.9207	2.1999	3,574,692	72,696	144,059	146,730
23	Aichi	4.4356	5.1141	6,455,172	188,868	314,606	319,927
24	Mie	0.9361	1.0268	1,747,311	32,097	67,835	70,097
25	Shiga	0.7675	0.8766	1,155,844	19,091	44,520	50,359
26	Kyoto	2.2586	2.3304	2,586,574	71,744	142,887	138,565
27	Osaka	7.4084	7.1171	8,668,095	279,363	504,056	484,058
28	Hyogo	3.5728	3.5162	5,278,050	144,470	268,560	266,163
29	Nara	0.8818	0.8944	1,304,866	22,932	58,073	66,290
30	Wakayama	0.4440	0.3696	1,087,206	17,353	39,680	34,932
31	Tottori	0.3198	0.3004	616,024	10,175	25,185	23,852
32	Shimane	0.3493	0.3494	794,629	16,846	36,190	34,116
33	Okayama	1.1719	1.1255	1,916,906	35,012	79,875	77,494
34	Hiroshima	1.8658	1.9224	2,819,200	95,114	171,285	167,940
35	Yamaguchi	1.0291	0.9487	1,601,627	36,445	81,067	75,361
36	Tokushima	0.3767	0.3376	834,889	17,934	35,869	33,328
37	Kagawa	0.5687	0.5898	1,022,569	19,078	46,118	45,721
38	Ehime	0.8151	0.7063	1,529,983	35,785	69,698	65,593
39	Kochi	0.4057	0.3915	839,784	19,960	37,819	35,470
40	Fukuoka	2.9711	2.8403	4,719,259	189,871	313,928	309,203
41	Saga	0.3750	0.3256	880,013	16,528	42,738	38,596
42	Nagasaki	0.8866	0.7920	1,593,968	40,898	89,604	80,660
43	Kumamoto	0.8626	0.8464	1,837,747	46,231	92,829	89,198
44	Oita	0.6492	0.5936	1,250,214	28,364	60,521	57,405
45	Miyazaki	0.7096	0.6350	1,175,543	36,795	70,549	65,691
46	Kagoshima	0.9833	0.9410	1,819,270	63,545	114,094	110,289
	Total	Overall mean effect $w=377.3389$		119,869,826	3,312,862	6,376,468	6,376,468

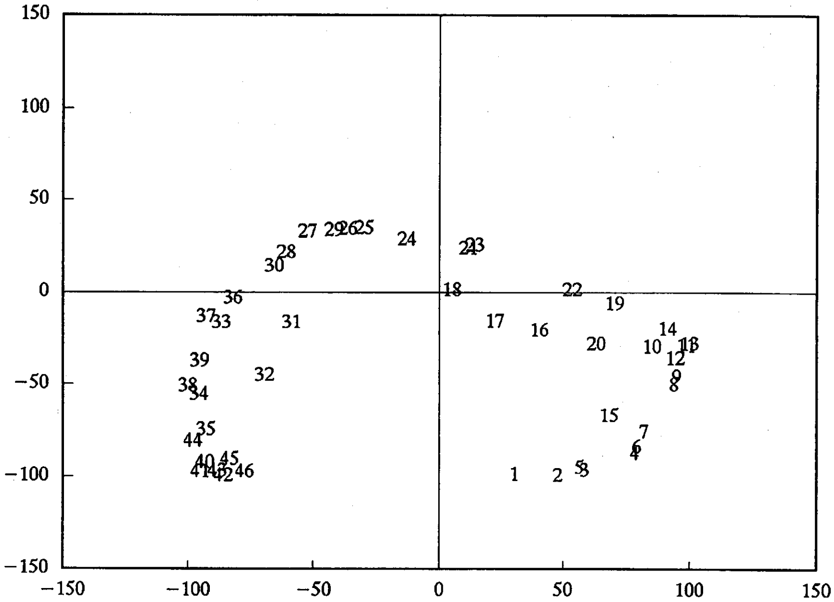


Notes: Coefficient of alienation = 0.180
Numbers correspond to prefecture code numbers in Figure 2.

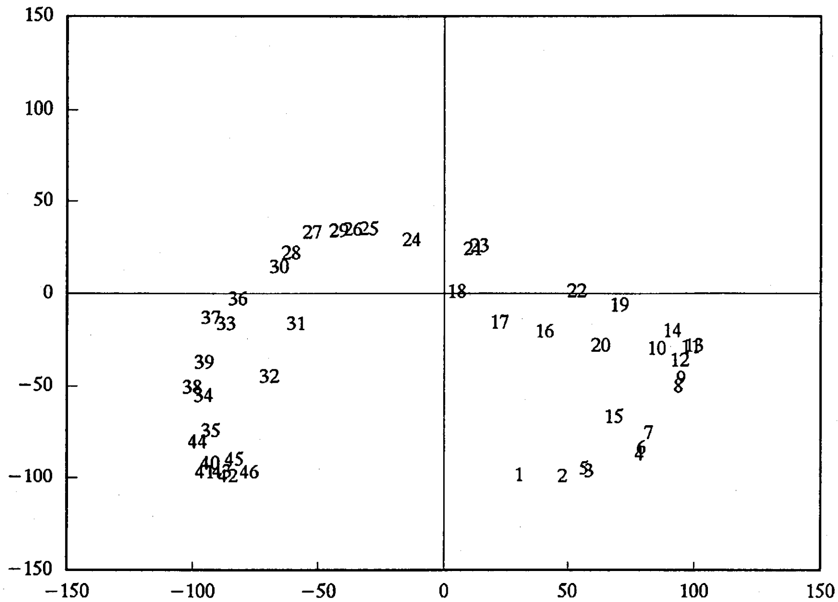


Notes: Coefficient of alienation = 0.147
Numbers correspond to prefecture code numbers in Figure 2.

Fig. 4 Two-dimensional configurations of interaction effects of the observed migration flows in 1960 (top) and 1985 (above), recovered by SSA-II (column solution)



Notes: **Coefficient of alienation = 0.038**
Numbers correspond to prefecture code numbers in Figure 2.



Notes: **Coefficient of alienation = 0.038**
Numbers correspond to prefecture code numbers in Figure 2.

Fig. 5 Two-dimensional configurations of interaction effects of the matrices of the distribution function in 1960 (top) and 1985 (above), recovered by SSA-II (column solution) effects of the distance-deterrence of prefectures in the central area, such as the Chubu region, are underestimated, and these prefectures are dispersed in Fig. 4. Those effects at the periphery, such as the Tohoku and Kyushu regions, are overestimated, and these prefectures are close together in Fig. 4. It can be shown that in the constrained models the effects of the configuration of prefectures in the whole system concerned — that is — accessibilities, are immediately removed (Yano, 1992b). But these relationships are based only on the distances between origins and destinations; the configuration of the other origins and/or destinations are omitted completely. For example, the competing effect, which is defined as the accessibility of the destination to all other alternative destinations, is independent of the effect of distance (Fotheringham, 1983). Therefore, it is reasonable to add the competing effect to conventional constrained models.

Linkage coefficients

What is the pure relationship between an origin and a destination? In this section, I shall discuss the relationship between them, removing the distance effect, which is captured by an index hereinafter referred to as the linkage coefficient. The linkage coefficient is measured by dividing the interaction effect of the observed flow matrix by that of the distribution function, as in equation (41):

$$L_{ij} = w_{ij}^{AB} / \tilde{w}_{ij}^{AB}, \tag{41}$$

subject to, $\Pi_i\ w_{ij}^{AB} = \Pi_j\ w_{ij}^{AB} = 1.0$; $\Pi_i\ \tilde{w}_{ij}^{AB} = \Pi_j\ \tilde{w}_{ij}^{AB} = 1.0$.

In other words, the coefficient suggests the relationship between an origin and a destination overlooked in the doubly constrained model. The linkage coefficient is an origin-destination pair-specific variable, not referring to either the origin or the destination alone. Adding this linkage coefficient to the doubly constrained model can make the model fit perfectly.

Conceptually, the coefficient is composed of systematic and unique factors. Although it is impossible to distinguish these factors exactly, systematic factors are specified as new variables which can be incorporated in the constrained model. These variables take into account the competing effect, the intervening opportunities effect and the hierarchical effect in the present study. Unique factors are regarded as random or unique ones.

Figure 6 shows the spatial patterns of the linkage coefficients in 1960 and 1985. The values of linkage coefficients of greater than 3.0 are drawn as lines. Comparison of the linkage coefficients in 1960 and 1985 indicates similar tendencies, though for 1960 there are many more pairs whose values are more than 3.0 than for 1985. The spatial patterns indicate the existence of several clusters of prefectures: the linkage between the Kyushu region and Osaka, Aichi, and Gifu, and the clusters of the Tohoku, Chugoku, Shikoku and Hokuriku regions. In order to extract these clusters objectively, factor analysis based on the standardized cross-product matrix (Yano, 1985) is applied to matrices of the linkage coefficients. These matrices are approximately symmetrical, reflecting the observed flow matrices. In this case, factor analyses of both the R-mode (destinations pattern) and the Q-mode (origins pattern) can be employed. Both analyses produced similar results. Accordingly, the results of the Q-mode factor analysis, based on the similarities of out-migrant patterns, are examined. The origin-destination pairs with

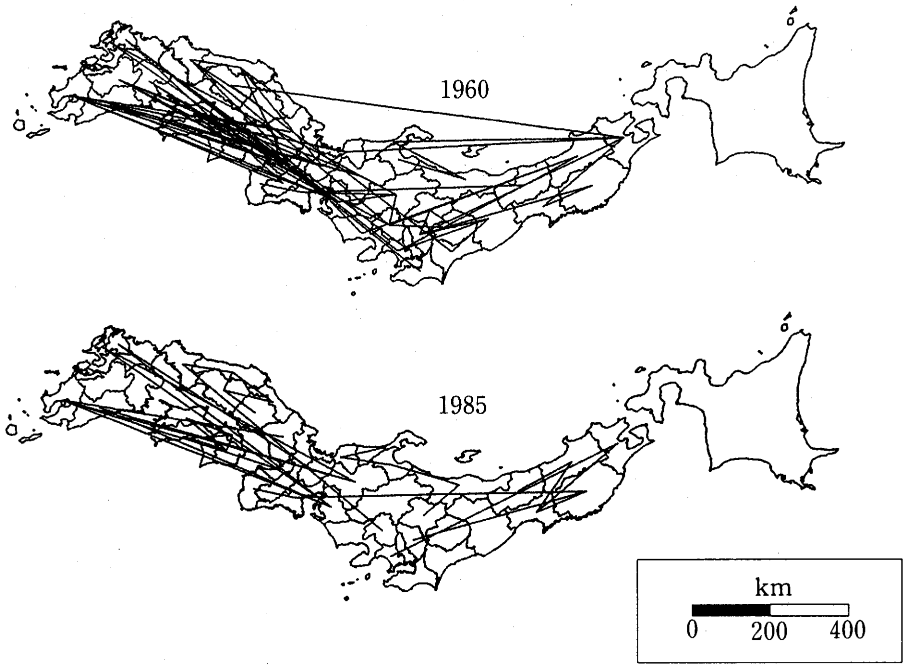


Fig. 6 Spatial pattern of the linkage coefficients over 3.0

high values of linkage coefficients have prominent factor loadings and factor scores for some factors corresponding to the regional clusters (Fig. 7).

The results for 1960 and 1985 are similar, because the interaction effects are stable over time. For 1960, factor analysis has produced six factors which represent the regional linkages and clusters: the linkage between the leading metropolitan areas and the Kyushu region (Factor 1), the linkage between the periphery of the Tokyo metropolitan area and the Tohoku region (Factor 2), the linkage between Gifu-Aichi and the Kyushu region (Factor 3), the cluster in the Japan Sea coast region (Factor 4), the Chugoku region (Factor 5) and the Shikoku region (Factor 6). These linkages and clusters represent the origin-destination pair relationships, except for the distance effect, which affect migration flows. These extracted factors support the conventional interpretation of internal migration in Japan. For example, the linkages between the leading metropolitan areas and the Kyushu and Tohoku regions are based on the relationship between the demand for and supply of labor (Aoki, 1979). The clusters on the Japan Sea coast and in the Chugoku and Shikoku regions are regional clusters having resulted from traffic conditions after the early seventeenth century. Although a multiple regression analysis of inter-prefectural migration (Ishikawa, 1978) has revealed that the predicted flows between the contiguous prefectures are underestimated, this effect is also one of the origin-destination pair-specific relations.

Conversely, the linkage coefficients between prefectures within a metropolitan area are low, and the pair-specific effects are ess than the distance effects. This seems contradictory, given that the intra-metropolitan migrations are heaviest. However,

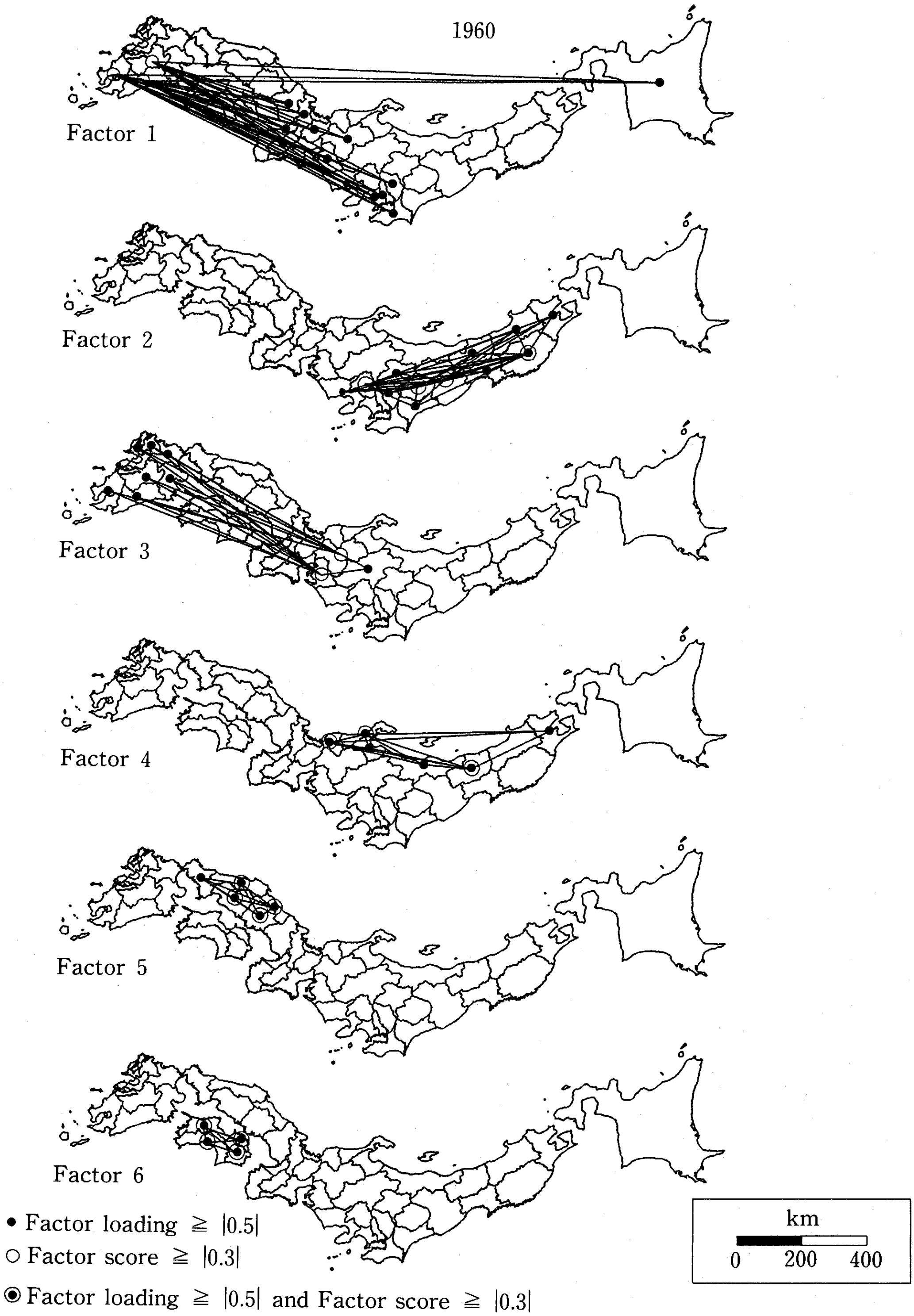


Fig. 7 Results of factor analysis of the linkage coefficients in 1960 and 1985

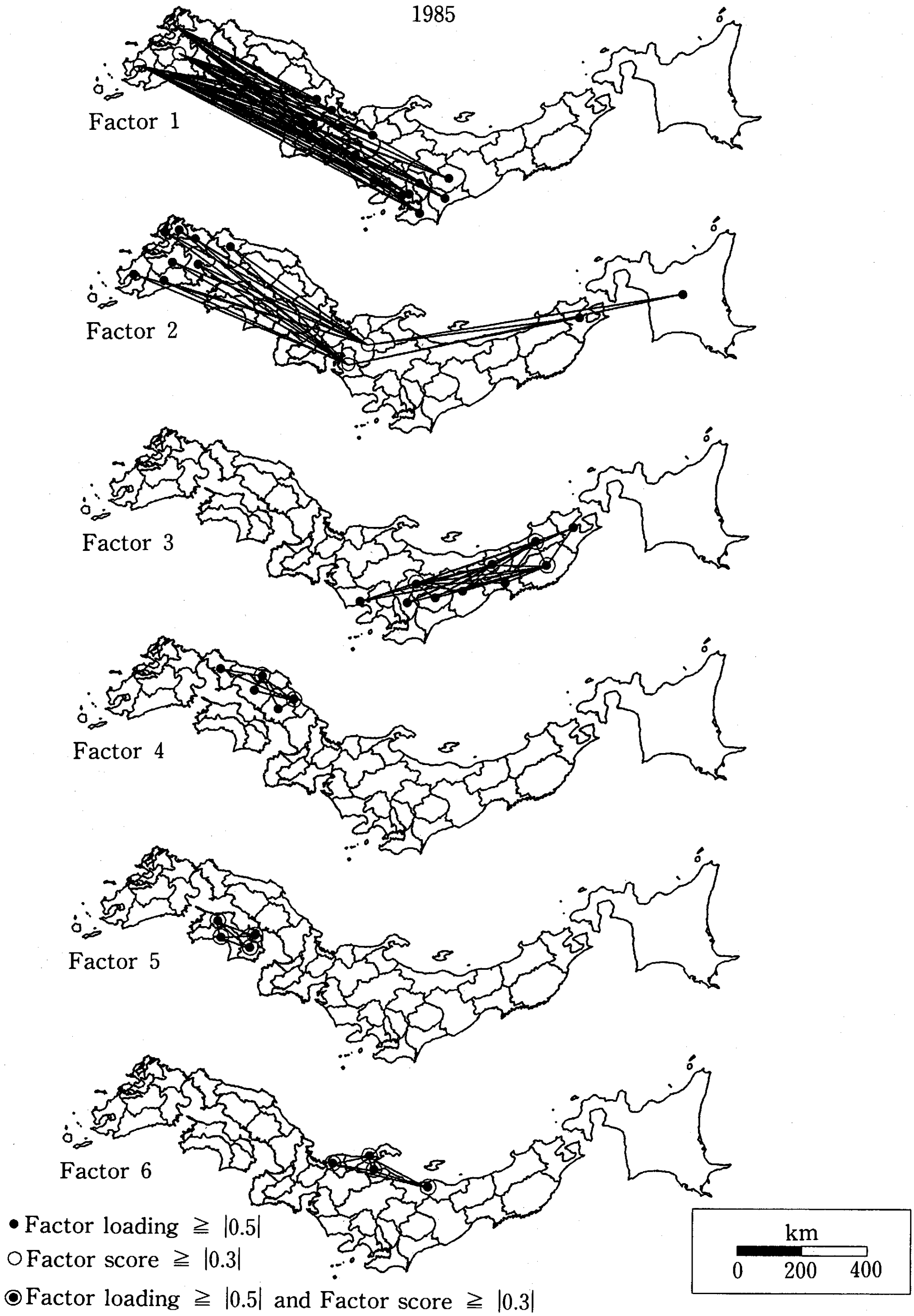


Fig. 7 continued

heavy migration flows within the metropolitan areas are caused by the great emissiveness and attractiveness there, so that pair-specific effects are poor. In examining the residuals of the doubly constrained model, the largest negative residuals are found in flows within the metropolitan areas. The overestimation of predicted flows indicates that the relationships between origins and destinations within those areas are not strong in comparison with the distances.

These results do not necessarily coincide with those derived from factor analysis of the migration flows in the periods 1960-1965 and 1966-1970 (Saino and Higashi, 1978). Their analysis is applied to the values of migration flows directly. Then they extract the broader areas: East Japan, West Japan, and the regions of Osaka, Chukyo and Kyushu. In the present study, the origin-destination pair-specific relationships, with the exception of the effects of the scales of the origins and destinations and the distances between them, are examined. The pairs with high values for the linkage coefficient have a stronger relationship than the one that the distance effects produce. This indicates the existence of new relationships not specified in the conventional spatial interaction models.

Results of generalized spatial interaction models

This section presents the results of fitting generalized spatial interaction models incorporating the competing effect, the intervening opportunities effect and the hierarchical effect as pair-specific effects (except the distance effect). These new effects are defined by two kinds of accessibility — a distance measure and a gravity measure, as in equations (32) to (37). The mass variables of place k, M_k, are measured by the population in each year. The distance-decay parameters in the accessibility formulation, σ, is set equal to -1.0, -1.5 and -2.0, and each case is calculated.

In the hierarchical effect, 46 prefectures are classified into three hierarchical orders by population, as is shown in Fig. 8. The highest order is composed of Tokyo and Osaka; the second-order includes Hokkaido, Kanagawa, Aichi and Hyogo, including Saitama and Chiba in 1985; and the third is composed of the remaining prefectures.

Table 6 summarizes the results of calibrating the doubly constrained models incorporating the above effects. In the case of the 1960 data, the model including the hierarchical competing effect had the best goodness-of-fit, using only destinations of the same order or higher than destination j in computing the accessibility, Model CAc1. It produced a deviance value of 1,651,827 — about 34.1 percent less than that of the doubly constrained model. The value of the distance-decay parameter is -1.4990, near that of the conventional constrained model. The value of the parameter of the hierarchical competing effect is positive at 11.20. This effect represents the accessibility of the destination for the other alternative destinations of the same or a higher order. This positive parameter estimate indicates that the agglomeration force is dominant in migration flows in Japan. This result is reasonable. It accurately corresponds to the strong tendency to migrate in 1960, characterized by a predominance of population flows from nonmetropolitan areas to some of the leading metropolitan areas and among the metropolitan areas.

As regards migration in 1985, the models adding the intervening opportunities or competing effects excluding the hierarchical effects, based on the distance measure of

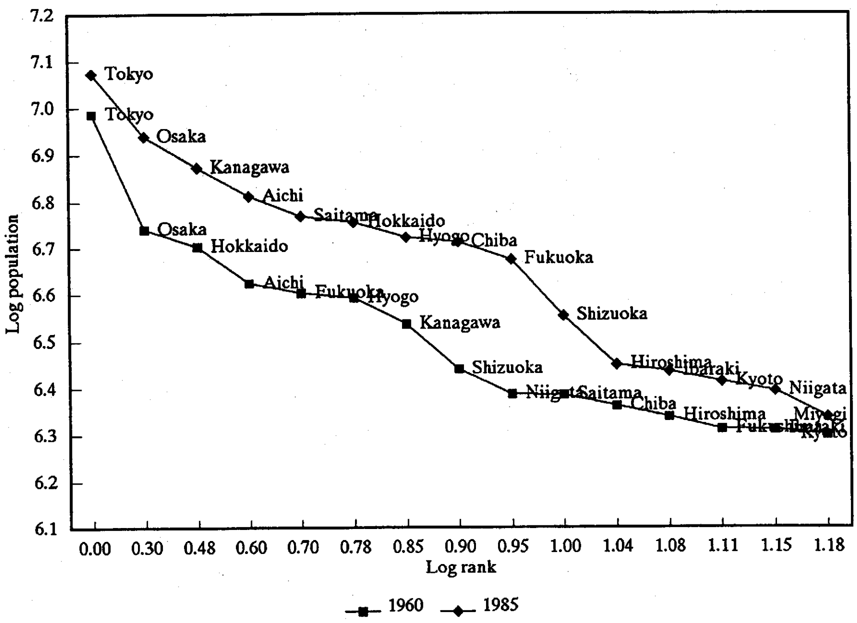


Fig. 8 Classification of fifteen largest prefectures based on rank-size distributions in Japan in 1960 and 1985

accessibility, Models IAa1 and CAa1, produced the best goodness-of-fit. The deviances of these models are 1,786,917 and 1,801,910. Reductions in deviance of approximately 37 percent from the doubly constrained model including no further explanatory variables, were produced. Both these effects have positive parameter estimates. For migration in 1985, the intervening opportunities and the competing effects coexist. The positive force of the intervening opportunities effect implies that some migration flow (when the other destinations around that origin are close together) is greater than that predicted by the conventional model. The positive competing force implies that when the other destinations around that destination are close together, that flow is greater as well. These forces appear when the migration flows between prefectures are close together, and are within the accessible areas measured by distance. Thus migration in Japan in 1985 is characterized by heavy flows within some leading metropolitan areas and among them.

Let us examine further the relationships between the above additional pair-specific effects and the linkage coefficients. The linkage coefficients are scaled such that $\Pi_i L_{ij} = \Pi_j L_{ij} = 1.0$. The additional pair-specific effects are not scaled. Then the log-linear model is applied to the matrix of the pair-specific effects, and a comparison between the calculated interaction effects and the linkage coefficients is established. The interaction effects of the accessibility of the hierarchical competing effect with $\sigma = -1.0$ in 1960, Model CAc1, whose addition to the doubly constrained model produced the best goodness-of-fit, are examined in detail. The accessibility itself may be affected by the scales of the origins and destinations and the spatial structure. The effects of these scales

Table 6. Results of generalized spatial interaction models fitted to the migration flows in 1960 and 1985

1) 1960

Model	Scaled deviance	Const.	$\ln d_{ij}$	$\ln H_{ij}$	Reduction* in deviance
CAc1	1,651,827	52.52	−1.499	11.2000	34.1%
CBc1	1,747,503	−74.96	−1.490	8.3040	30.3%
IAa1	1,818,183	40.90	−1.480	7.3460	27.5%
CAa1	1,846,028	40.43	−1.476	7.1930	26.4%
IAb1	1,922,286	45.93	−1.473	9.0110	23.3%
IBb1	1,985,654	−57.50	−1.471	6.7590	20.8%
IBa3	2,059,930	14.26	−1.517	1.0620	17.8%
CBb1	2,122,892	−25.32	−1.529	3.9510	15.3%
IAa3	2,179,394	27.79	−1.727	1.0410	13.1%
CAa3	2,181,830	27.20	−1.715	0.9651	13.0%
Doubly constrained	2,506,720	19.31	−1.471	------	0.0%

2) 1985

Model	Scaled deviance	Const.	$\ln d_{ij}$	$\ln H_{ij}$	Reduction* in deviance
IAa1	1,786,917	42.32	−1.432	7.8260	37.0%
CAa1	1,801,910	42.08	−1.432	7.7460	36.5%
CAc1	1,827,937	51.83	−1.438	10.9800	35.5%
IAb1	1,947,008	49.08	−1.441	10.0500	31.3%
CAc1	1,973,313	−49.55	−1.438	5.9420	30.4%
CAb1	2,005,179	43.46	−1.465	8.1110	29.3%
IBb1	2,070,079	−43.71	−1.439	5.4390	27.0%
IAc1	2,142,194	40.95	−1.459	7.2690	24.5%
IBa1	2,158,291	−13.85	−1.425	2.8150	23.9%
CBa1	2,223,876	−11.50	−1.425	2.6160	21.6%
Doubly constrained	2,836,010	19.27	−1.413	------	0.0%

*: Percentage of reduction in deviance unexplained by the doubly constrained model.

are made explicit in the main effects of rows and columns derived from the log-linear model. In the case of the hierarchical competing effect, the main effects of the destinations have a spatial variation, while those of the origins are remarkably constant over space. The destinations in the central area of Japan have high main effects, whereas those in the peripheral area have low ones. However, these effects do not make sense in the doubly constrained model, because the balancing factors, especially the destinations, adjust them. Therefore, it is the interaction effects that substantially affect the goodness-of-fit. Comparison between the interaction effects of the hierarchical competing accessibility and the linkage coefficients allows the validity of incorporating that effect to be evaluated.

The linkage coefficient is composed of pair-specific relations which can be specified by the variables or effects mentioned above, and by those which cannot be generalized and are peculiar to the migration flows. The hierarchical competing accessibility is

Table 7. Relationships between the linkage coefficients and the hierarchical competing accessibility in 1960

Origin	Destination	Linkage coefficient	Interaction effect of CAcl	Observed flows	Doubly constrained model			Doubly constrained model adding Cacl		
					Predicted flows	Relative residuals	Absolute residuals	Predicted flows	Relative restduals	Absolute residuals
1 25 Shiga	25 Shiga	5.1488	1.0436	12,940	1,289.4	324.5	11,650.6	5,778.8	94.2	7,161.2
2 26 Kyoto	26 Kyoto	3.2881	1.0436	63,279	19,904.1	307.4	43,374.9	36,341.5	141.3	26,937.5
3 29 Nara	29 Nara	5.1655	1.0372	11,668	1,232.0	297.3	10,436.0	3,755.3	129.1	7,912.7
4 21 Gifu	21 Gifu	6.7833	1.0318	32,258	6,369.7	324.4	25,888.3	13,204.1	165.8	19,053.9
5 37 Kagawa	37 Kagawa	5.7503	1.0311	16,622	9,987.3	66.4	6,634.7	15,174.0	11.8	1,448.0
6 26 Kyoto	13 Tokyo	0.6777	1.0310	5,476	3,192.0	40.4	2,284.0	6,181.4	−9.0	−705.4
7 25 Shiga	13 Tokyo	0.7815	1.0307	1,974	820.6	40.3	1,153.4	2,389.4	−8.5	−415.4
8 30 Wakayama	30 Wakayama	11.6634	1.0306	19,527	4,975.0	206.3	14,552.0	7,676.0	135.3	11,851.0
9 10 Gumma	10 Gumma	17.3707	1.0301	26,674	10,057.1	165.7	16,616.9	14,442.0	101.8	12,232.0
10 33 Okayama	33 Okayama	7.9986	1.0300	39,407	19,181.3	146.0	20,225.7	26,153.2	82.0	13,253.8
11 9 Tochigi	9 Tochigi	14.1020	1.0298	21,262	8,223.9	143.8	13,038.1	12,721.1	75.7	8,540.9
12 24 Mie	24 Mie	5.3463	1.0296	28,217	10,953.5	165.0	17,263.5	15,180.3	105.8	13,036.7
13 36 Tokushima	36 Tokushima	13.8169	1.0295	15,118	5,567.4	128.0	9,550.6	7,997.8	79.6	7,120.2
14 19 Yamanashi	19 Yamanashi	19.1514	1.0288	12,872	4,028.8	139.3	8,843.2	6,012.9	88.5	6,859.1
15 18 Fukui	18 Fukui	9.6096	1.0280	11,745	4,223.6	115.7	7,521.4	5,611.2	81.9	6,133.8
16 17 Ishikawa	17 Ishikawa	6.1230	1.0279	15,556	7,331.1	96.1	8,224.9	9,825.1	57.8	5,730.9
17 20 Nagano	20 Nagano	18.6230	1.0277	47,237	19,931.2	193.4	27,305.8	25,401.5	137.0	21,835.5
18 16 Toyama	16 Toyama	6.2519	1.0277	16,499	7,214.2	109.3	9,284.8	9,943.1	65.7	6,555.9
19 25 Shiga	40 Fukuoka	0.8718	1.0276	160	80.9	8.8	79.1	246.4	−5.5	−86.4
20 6 Yamagata	6 Yamagata	9.9192	1.0276	21,648	8,018.7	152.2	13,629.3	17,266.4	33.3	4,381.6
21 26 Kyoto	40 Fukuoka	1.8101	1.0272	1,148	339.8	43.8	808.2	684.5	17.7	463.5
22 26 Kyoto	6 Yamagata	1.0325	1.0269	82	117.3	−3.3	−35.3	238.4	−10.1	−156.4
23 25 Shiga	6 Yamagata	1.4453	1.0267	35	29.4	1.0	5.6	90.0	−5.8	−55.0
24 26 Kyoto	4 Miyagi	0.8418	1.0267	124	170.4	−3.6	−46.4	295.8	−10.0	−171.8
25 38 Ehime	38 Ehime	4.6807	1.0267	36,449	21,443.2	102.5	15,005.8	30,970.9	31.1	5,478.1
26 25 Shiga	4 Miyagi	0.6236	1.0265	28	42.7	−2.3	−14.7	111.6	−7.9	−83.6
27 26 Kyoto	11 Saitama	0.5295	1.0264	299	470.0	−7.9	−171.0	689.5	−14.9	−390.5
28 7 Fukushima	7 Fukushima	12.1287	1.0264	46,443	23,214.4	152.5	23,228.6	35,641.5	57.2	10,801.5
29 8 Ibaraki	8 Ibaraki	5.1982	1.0262	30,155	18,841.2	82.4	11,313.8	25,164.4	31.5	4,990.6
30 26 Kyoto	12 Chiba	0.7474	1.0261	446	530.4	−3.7	−84.4	781.1	−12.0	−335.1

2087	11	Saitama	12	Chiba	0.2817	0.8914	2,681	4,069.4	−21.8	−1,388.4	3,295.1	−10.7	−614.1
2088	25	Shiga	29	Nara	0.1758	0.8912	197	740.5	−20.0	−543.5	374.0	−9.2	−177.0
2089	43	Kumamoto	42	Nagasaki	0.4249	0.8906	2,772	8,835.2	−64.5	−6,063.2	2,511.1	5.2	260.9
2090	16	Toyama	17	Ishikawa	2.0869	0.8893	2,256	2,745.6	−9.3	−489.6	736.4	56.0	1,519.6
2091	3	Iwate	5	Akita	0.7637	0.8891	987	4,983.3	−56.6	−3,996.3	1,469.1	−12.6	−482.1
2092	12	Chiba	13	Tokyo	0.1671	0.8870	45,375	65,284.5	−77.9	−19,909.5	41,972.2	16.6	3,402.8
2093	28	Hyogo	27	Osaka	0.2810	0.8868	45,057	108,926.2	−193.5	−63,869.2	51,376.4	−27.9	−6,319.4
2094	11	Saitama	14	Kanagawa	0.1555	0.8862	4,700	8,730.9	−43.1	−4,030.9	5,985.0	−16.6	−1,285.0
2095	17	Ishikawa	16	Toyama	2.1674	0.8856	1,749	2,506.3	−15.1	−757.3	634.6	44.2	1,114.4
2096	43	Kumamoto	46	Kagoshima	0.3536	0.8846	2,817	11,198.7	−79.2	−8,381.7	3,134.6	−5.7	−317.6
2097	33	Okayama	37	Kagawa	1.1294	0.8846	1,639	5,441.5	−51.5	−3,802.5	1,371.7	7.2	267.3
2098	5	Akita	3	Iwate	0.7134	0.8836	1,098	6,079.8	−63.9	−4,981.8	1,784.4	−16.2	−686.4
2099	8	Ibaraki	9	Tochigi	1.6262	0.8826	1,701	4,896.9	−45.7	−3,195.9	1,279.6	11.8	421.4
2100	12	Chiba	14	Kanagawa	0.1557	0.8826	6,038	14,170.5	−68.3	−8,132.5	6,198.9	−2.0	−160.9
2101	42	Nagasaki	41	Saga	2.1577	0.8801	6,171	4,223.1	29.8	1,937.9	925.7	172.4	5,245.3
2102	37	Kagawa	33	Okayama	1.2337	0.8800	2,284	6,641.6	−53.5	−4,357.6	1,668.0	15.1	616.0
2103	7	Fukushima	6	Yamagata	0.5953	0.8781	1,149	8,685.1	−80.9	−7,536.1	2,834.8	−31.7	−1,685.8
2104	14	Kanagawa	12	Chiba	0.1285	0.8771	4,398	10,187.1	−57.4	−5,789.1	4,051.9	5.4	346.1
2105	41	Saga	42	Nagasaki	2.2390	0.8757	7,676	4,264.4	52.2	3,411.6	1,572.3	153.9	6,103.7
2106	6	Yamagata	7	Fukushima	0.6526	0.8750	1,218	9,240.3	−83.5	−8,022.3	2,733.9	−29.0	−1,515.9
2107	29	Nara	27	Osaka	0.2862	0.8732	12,770	19,957.0	−50.9	−7,187.0	11,179.1	15.0	1,590.9
2108	14	Kanagawa	13	Tokyo	0.0660	0.8683	54,620	109,420.4	−165.7	−54,800.4	48,919.4	25.8	5,700.6
2109	21	Gifu	23	Aichi	0.5731	0.8679	21,939	38,339.1	−83.8	−16,400.1	13,854.8	68.7	8,084.2
2110	9	Tochigi	8	Ibaraki	1.3854	0.8650	2,064	5,637.9	−47.6	−3,573.9	1,168.6	26.2	895.4
2111	4	Miyagi	6	Yamagata	0.5844	0.8507	1,701	8,640.4	−74.7	−6,939.4	1,975.6	−6.2	−274.6
2112	6	Yamagata	4	Miyagi	0.5773	0.8378	1,869	9,315.3	−77.2	−7,446.3	1,773.8	2.3	95.3
2113	11	Saitama	13	Tokyo	0.1060	0.8264	41,881	75,110.2	−121.2	−33,229.2	33,216.7	47.5	8,664.3
2114	41	Saga	40	Fukuoka	0.5251	0.8211	13,395	25,730.5	−76.9	−12,335.5	6,037.4	94.7	7,357.6
2115	26	Kyoto	25	Shiga	0.1726	0.6927	4,298	15,535.3	−90.2	−11,237.3	489.8	172.1	3,808.2
2116	25	Shiga	26	Kyoto	0.3108	0.6899	5,850	16,009.7	−80.3	−10,159.7	446.9	255.6	5,403.1
		STATISTIC		SUM	2819.2	2116.8	5,652,659	5,652,727.0	−2,491.5	−67.5	5,652,695.0	−4,628.7	−35.6
				MEAN	1.3323	1.0004	2,671	2,671.4	−1.2	0.0	2,671.4	−2.2	0.0
				S.D.	1.4250	0.0259	18,227	18,876.1	40.1	4,586.0	20,141.8	33.4	3,752.1
				MAX	19.1514	1.0436	631,344	700,488.9	360.6	73,071.9	759,214.7	348.7	45,200.1
				MIN	0.0660	0.6899	1	9.4	−193.5	−69,144.9	16.7	−146.8	−127,870.7
				RANGE	19.0853	0.3538	631,343	700,479.6	554.1	142,216.9	759,197.9	495.5	173,070.8

interpreted as one of the former. Since the interaction effects of the hierarchical competing accessibility and the linkage coefficients are scaled in a similar way, values of more than 1.0 indicate strong linkages between origins and destinations, while values of less than 1.0 indicate weak linkages. The relationships between the linkage coefficients and the interaction effects of the hierarchical competing accessibility, including the results of the doubly constrained model and one incorporating the hierarchical competing accessibility, are shown in Table 7. The pairs of the thirty largest and the thirty smallest values of the interaction effects of the hierarchical competing accessibility, Model CAc1, are listed in Table 7, ranked according to the sizes of these values. The interaction effects of the hierarchical competing accessibility have a small range of 0.3538 and a standardized deviance of 0.0259, while the linkage coefficients have a large range of 19.0853 and a standardized deviance of 1.4250. The correlation coefficient between them is 0.08445. However, several correspondences between them are detected. The origin-destination pairs whose origins are prefectures within the Kinki region, the most accessible region in Japan, and intra-prefectural pairs have high interaction effects, whereas contiguous origin-destination pairs have low effects. Pairs identified by factor analysis, as well as intra-prefectural pairs, have high values of linkage coefficients, whereas the pairs within the metropolitan areas and those of contiguous prefectures have low values. Similarly, the characteristics of linkage coefficients can be identified by the tendency of the interaction effects of the hierarchical competing accessibility. Hierarchical competing effects are specified objectively or exogenously, whereas the linkage coefficients are extracted from the observed migration flows, including the peculiar relationships. Although the correlation coefficient shows no relationship between them, pairs with extremely high or low values (see Table 7) have a close relationship. As a result, adding the hierarchical competing effects to the doubly constrained model produces a reduction in deviance from 2,506,720 to 1,651,827, and in the range and variance of residuals.

Ishikawa's (1987) analysis of the Japanese migration flows in 1960 and 1980, using Fotheringham's origin-specific production-constrained competing destinations model, suggested that system-wide calibrations do not necessarily produce an average image of origin-specific calibrations. This result indicates that the origin-destination pairs for which incorporating the above effects is effective coexist with those for which it has no effect.

It may be concluded that the addition of the new origin-destination pair-specific effects may greatly improve the goodness-of-fit of the constrained models. However, it is not affirmed that the above-defined effects correspond to the linkage coefficients. In the present study, the linkage coefficients are conceptually classified into systematic factors and unique factors. The latter factors are not captured by the origin-destination pair-specific effects. Since these peculiar relationships are dependent on the spatial interaction concerned and the spatial structure of origins and destinations, they should be considered as error terms. The present study revealed that the pair-specific effects to be included in spatial interaction models could be identified as linkage coefficients. The results contribute to a better understanding and interpretation of spatial interaction modeling.

In the next section, a behavioral interpretation of these pair-specific effects will be presented using discrete choice modeling.

6. Behavioral Interpretation of the Doubly Constrained Model

Although the above-discussed spatial interaction models are concerned with an analysis at an aggregate level or on a meso-scale, they can also be specified using the multinomial logit model which is one of the discrete choice models at a disaggregate level or on a micro-scale (Anas, 1983). In this section, the doubly constrained model is redefined as the multinomial logit model, and an attempt is made to develop a behavioral interpretation of the origin-destination pair-specific effects.

The multinomial logit model attempts to describe, explain and predict the discrete choice behavior of an individual, based on the utility maximization theory (Ben-Akiva and Lerman, 1985; Wrigley, 1985). Consider a population of individual decision-makers (h=1, ..., H), who have homogeneous preferences with an additive stochastic term. Each decision-maker faces a choice among discrete alternatives (j=1, ..., J), the set of which is called the choice set. Provided that the utility of each alternative is assumed to be a linear function of the utility attributes, the utility function is specified by equation (42). The utility function of the multinomial logit model is composed of 1) the indices of relevant characteristics of the alternatives, and 2) the indices of relevant characteristics of the decision-makers. The former is further classified into alternative specific variables, alternative specific dummy variables and generic variables (Morisugi, 1984).

$$U_j^h = a_{oj} + \textstyle\sum_k a_k X_{jk}^h + e_j^h, \tag{42}$$

where $\boldsymbol{\alpha}=[\alpha_{oI}, ..., \alpha_{oJ}, \alpha_1, ..., \alpha_K]$ are the utility coefficients common to all decision-makers in the population; X_{ik}^h represents the value of the k-th attribute for alternative j and decision-maker h; and e_j^h is the vector of stochastic utility which is distributed over the population. The coefficients α_{oj} are alternative-specific constants which measure the unspecified part of utility for each alternative, and which are the average probabilities over the estimation sample for each alternative.

The utility-maximizing choice model is derived from the probability that decision-maker h chooses alternative i:

$$P_i^h = \text{Prob.}[U_i^h > U_j^h\,;\, j \neq i]. \tag{43}$$

Provided that each stochastic utility, e_i^h, is independently and identically distributed over the population and for each decision-maker according to the Weibull distribution, the multinomial logit model is derived as follows (Ben-Akiva and Lerman, 1985):

$$P_i^h = \frac{\exp(\beta_{oi} + \sum_k \beta_k X_{ik}^h)}{\sum_j \exp(\beta_{oj} + \sum_k \beta_k X_{jk}^h)}, \tag{44}$$

where the utility coefficients β cannot be identified, but the scaled coefficients $\boldsymbol{\beta}$= $[\beta_{oI}, ..., \beta_{oJ}, \beta_1, ..., \beta_K]$ are uniquely estimable with the exception of one of the alternative specific constants, say, β_{oJ}. The differences $\beta_{oj}-\beta_{oJ}$ are uniquely estimable for each j (Anas, 1983).

Using the multinomial logit model on disaggregate data, the doubly constrained model can be specified (Anas, 1983). Suppose now that the migration system is described by the OD matrix with N origins and N destinations, and that the number of total migrants in this system is T (= $\sum_{ij}t_{ij}$). In this case, the doubly constrained model can be regarded as the discrete choice model of joint origin-destination choice derived from stochastic utility maximization. In other words, the doubly constrained model is identical to the multinomial logit model, which defines the origin-destination pairs on the OD matrix as the alternatives, and the distance between them as the alternative attribute.

The utility for decision-maker h of choosing an origin-destination pair (i, j) is as follows:

$$U_{ij}^{h}=\beta_{oi}+\beta_{dj}+\beta\ d_{ij}^{h}+e_{ij}^{h}, \tag{45}$$

where e_{ij}^{h} is the random utility, β_{oi} is the origin-specific constant, β_{dj} is the destination-specific constant, d_{ij}^{h} is distance between origin i and destination j as the generic variable for the origin-destination pairs and β is its parameter estimate. From the comparison between the doubly constrained model and the multinomial logit model, it is clear that the multiplications of the balancing factors of origins and destinations, A_i and B_j, and the total outflow and the total inflow, O_i and D_j — that is, A_iO_i and B_jD_j, respectively — correspond to the origin-specific and the destination-specific constants (Anas, 1983; Yano, 1992a). The distance between an origin and a destination in a distribution function corresponds to one of the alternative attributes of the origin-destination pair (i, j) perceived by the decision-maker h, *i.e.*, the generic variables. For the identity of both models, the assumption that all migrants evaluate the distance between an origin and a destination uniformly as the alternative attribute is needed. Therefore, the generic variable of the distance d_{ij}^{h} is regarded as the mean for the distances over which the decision-makers will choose any specific origin i and any specific destination j:

$$d_{ij}^{h}=\sum_h \delta_{ij}^{h}\ d_{ij}^{h}/\sum_h \delta_{ij}^{h}, \tag{46}$$

where δ_{ij}^{h}=1 if migrant h chooses the origin-destination pair (i, j), and δ_{ij}^{h}=0 if he does not.

The doubly constrained model is identical to the multinomial logit model in equation (44), in which only one generic variable is present, *i.e.*, the distance between an origin and a destination. The formulation is

$$p_{ij}=\frac{\exp(\beta_{oi}^{A}+\beta_{dj}^{A}+\sum_k \beta_k^{A}\ X_{ijk})}{\sum_{mn}\exp(\beta_{om}^{A}+\beta_{dn}^{A}+\sum_k \beta_k^{A}\ X_{mnk})}. \tag{47}$$

Consequently, the multinomial logit model can predict the same parameter estimates calibrated by the doubly constrained model. The generic utility coefficient in the multinomial logit model is the distance-decay parameter in the doubly constrained model. As Yano (1992a) has shown using a numerical illustration, the origin-specific and the destination-specific constants correspond to the balancing factors of origins and destinations in the doubly constrained model (Anas, 1983):

$$A_i = \exp(\beta^A_{oi})/O_i, \tag{48}$$

$$B_j = \exp(\beta^A_{dj})/D_j. \tag{49}$$

In spatial interaction modeling, the distance-decay parameter has conventionally been interpreted as the effect of the friction factor of distance. However, in the utility maximization theory, on which the multinomial logit model is based, the distance between origin i and destination j is interpreted as one of the alternative attributes specifying the utility based on which migrants choose the origin-destination pair (i, j). The difference between the utility for destination 1, which a migrant in origin i considers, and that for destination 2 is $\beta(d_{i1}-d_{i2})$, where β is a negative value, *ceteris paribus*. Accordingly, the more the difference increases positively — that is, $d_{i1}<d_{i2}$ — the more the choice probability of destination 1 increases.

Based on the distance between them, it is possible to give a behavioral interpretation to the interaction effects derived from the log-linear model. In the case where migrants face a choice among origin-destination pairs, the interaction effect corresponds to the linear combination of the generic variables, excluding the individual attributes and the alternative specific constants.

The alternative specific constants are interpreted as the inherent utility of that alternative unspecified by the alternative specific variables, or the average probability of the random utility, similar to the intercept term in regression analysis (Train, 1986). In the multinomial logit model, the alternative specific constants are relative values. The coefficient β_{oi} is included in the utility function when a migrant chooses origin i, and the coefficient β_{dj} is included when destination j is chosen. Thus in the context of spatial interaction modeling these alternative specific constants correspond to the emissiveness of origins and the attractiveness of destinations, respectively. Therefore, the exponential of the linear combination of generic variables in the multinomial logit model is identical to the interaction effect of the log-linear model, and the distribution function in the doubly constrained model is the part of the utility explained by the generic variables, composed of only the distance between an origin and a destination. Thus we see that the interaction effects are interpreted as the differences of utilities among the alternatives, which are commonly recognized by all migrants when they face the choice of origin-destination pairs.

7. Conclusion and Further Research

In the present study, I have demonstrated that a variety of spatial interaction models developed recently are identical and possess the same mathematical and statistical characteristics. Specifically, the Poisson gravity model, the log-linear model and the logit model, expanded in the 1980s, are of exactly the same type as Wilson's entropy-maximizing model, which provided the theoretical justification for conventional gravity models. These models have developed from different contexts, and present apparently diverse model structures. However, statistical analyses with these models show that they are identical. When certain conditions are met, they produce the same parameter estimates.

First, it is demonstrated that Wilson's entropy-maximizing models are the same as the Poisson gravity models when dummy variables of origins and destinations are incorporated appropriately. This allows the addition of new variables in addition to the mass variables of origins and destinations and the distance between them. Next, the close relationship between the entropy-maximizing model and the log-linear model is shown. In other words, the main effects correspond to the emissiveness of origin i and the attractiveness of destination j. The interaction effect corresponds to the distribution function. In the doubly constrained model, no further variables can be incorporated referring to either the origin or the destination alone, because these variable are adjusted by balancing factors or dummy variables of origins and destinations. Thus, it is the variables relevant to the origin-destination pairs that can be included in the doubly constrained model. The interaction effect of the saturated log-linear model represents the relationship between the origin and the destination, which yields the perfect fit in the doubly constrained model. Therefore, the difference between the interaction effects of the distance function and the observed flow matrix has not been taken into account in conventional spatial interaction models.

The doubly constrained models and the saturated log-linear models are fitted to migration flows between prefectures in Japan in 1960 and 1985. The interaction effects and the linkage coefficients, which represent the difference between the interaction effect and the distribution function, are identified. Moreover, the factor analysis of the linkage coefficient matrix provides several regional patterns which have a closer relationship than that measured by distance.

Improvements in the doubly constrained model are explored, adding the new explanatory variables relevant to the origin-destination pair with the exception of distance between them. The conventional doubly constrained model ignores the competing effect, the intervening opportunities effect and the hierarchical effect. In the present study, these effects are defined in terms of accessibility, and the extended doubly constrained models incorporating them are tested using the Poisson gravity model. The validity of the generalized spatial interaction models is discussed.

Finally, the behavioral interpretation of these relationships of origin-destination pairs is given by comparing the multinomial logit model, which is based not on macro-information theory for aggregate data but on micro-behavioral postulates for disaggregate data. The doubly constrained model is identical to the multinomial logit model of migrant's joint origin-destination choice, consistent with stochastic utility maximization. Therefore, the distance between an origin and a destination is regarded as one of the alternative specific variables making up the utility function, in addition to the effects included in the generalized spatial interaction models. The interaction effects of the saturated log-linear model measure the specified part of utility for each alternative or origin- destination pair. In the present study, in contrast with the conventional gravity model, the distance between places is interpreted as one of the origin-destination pair-specific relationships, and the existence of the other relationships is assumed. However, it may be appropriate that to regard interaction effect derived from the log-linear model as the true spatial relationship or the pure relationship between an origin and a destination perceived by migrants, as Willekens (1983a) indicates.

Table 8 represents the relationship in the present study of the spatial interaction models: the doubly constrained model, the Poisson gravity model, the log-linear model and the multinomial logit model. The various spatial interaction models developed after the entropy-maximizing model derived by Wilson (1967) postulate that the flow is regarded as a random variable, assumed to have a discrete probability distribution such as a Poisson distribution. However, these spatial interaction models have been looked upon as different, because the explanatory variables included in each model are various and interpretations of these variables and hypotheses of the behavioral content are different. The fact that these spatial interaction models are identical made improvement and advancement of spatial interaction modeling possible. Propounding a view of the similarity of these spatial interaction models allowed the identification of the other origin-destination pair-specific effects which are not accounted for in conventional spatial interaction models. It is important to include the new variables corresponding to those effects in the spatial interaction model. Also, comparison with the multinomial logit model allowed the interpretation of those effects in a behavioral context.

Table 8. Integration of spatial interaction models

	Doubly constrained model $T_{ij}=A_i\ O_i\ B_j\ D_j\ f_{ij}$	Poisson gravity model $T_{ij}=\exp(\text{const.}+ORI(i)+DES(j)+\beta\ln d_{ij}$
Constant	None	exp(const.)
Emissiveness of an origin	$A_i\ O_j$: Total outflow of an origin multiplied by balancing factor	$\exp(ORI(i))$: Dummy variable of an origin
Attractiveness of a destination	$B_j\ D_j$: Total inflow of a destination multiplied by balancing factor	$\exp(DES(j))$: Dummy variable of a destination
Relationship between places	f_{ij}: Distance-decay function	$\exp(\beta\ln d_{ij})$: Distance between places

	Log-linear model $T_{ij}=w\ w_i^A\ w_j^B\ w_{ij}^{AB}$	Multinomial logit model $P_{ij}=\frac{\exp(\beta_{oi}^A+\beta_{dj}^A+\sum_k\beta_k^A X_{ijk})}{\sum_{mn}\exp(\beta_{om}^A+\beta_{dn}^A+\sum_k\beta_k^A X_{mnk})}$
Constant	w: Overall mean effect	$1/\sum_{mn}\exp(\beta_{om}^A+\beta_{dn}^A+\sum_k\beta_k^A X_{mnk})$
Emissiveness of an origin	w_i^A: Main effect of an origin	$\exp(\beta_{oi}^A)$: Origin-specific constant
Attractiveness of a destination	w_j^B: Main effect of a destination	$\exp(\beta_{dj}^A)$: Destination-specific constant
Relationship between places	w_{ij}^{AB}: First-order interaction effect	$\exp(\sum_k\beta_k^A X_{ijk})$: Generic variable

In the present study, only the doubly constrained model of the family of entropy-maximization models was considered. However, it is possible to extend the comparison of these models to the unconstrained, the production-constrained and the attraction-constrained models. In the doubly constrained model, the improvement of goodness-of-fit is realized by the inclusion of adequate origin-destination pair-specific effects. In the other constrained models, the adequate origin-specific and/or destination-specific effects, relating to the emissiveness of an origin and the attractiveness of a destination, must be further identified and incorporated in those models. These new explanatory variables are closely related not only to the kinds of flows but also to the spatial structure under investigation. Further advances in spatial interaction modeling could be carried out by conceptualizing these effects in addition to specifying the mass variables of origins and destinations and the distance between them, and by including these effects in the Poisson gravity type spatial interaction models.

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References Cited

Aitkin, M., Anderson, D., Francis, B. and Hinde, J. (1989): *Statistical Modelling in GLIM*. Oxford University Press, Oxford, 374p.

Anas, A. (1983): Discrete choice theory, information and multinomial logit and gravity models. *Transportation Research*, **17B**, 13-23.

Aoki, E. (1979): *Shushoku ido* (Migration caused by getting jobs). In Itho, T., Naito, H. and Yamaguchi, F. (eds.) "*Jinko ido no Chi'iki Kozo: Nippon no Chi'iki Kozo (The Regional Structure of Migration: Japanese Regional Structure)*". Taimeido, Tokyo, 105-115.*

Batty, M. (1976): *Urban Modelling: Algorithms, Calibrations, Predictions.* Cambridge

University Press, Cambridge, 381p.
Baxter, M. (1982): Similarities in methods of estimating spatial interaction models. *Geographical Analysis*, **14**, 267-272.
—— (1983): Estimation and inference in spatial interaction models. *Progress in Human Geography*, **7**, 40-59.
Baxter, M. and Ewing, G. O. (1986): A framework for the exploratory development of spatial interaction models: a recreation travel example. *Journal of Leisure Research*, **18**, 320-336.
Ben-Akiva, M. and Lerman, S. (1985): *Discrete Choice Analysis: Theory and Application to Predict Travel Demand.* MIT Press, Cambridge, MA, 390p.
Bishop, Y. M. M., Finberg, S. E. and Holland, P. W. (1975): *Discrete Multivariate Analysis: Theory and Practice.* MIT Press, Cambridge, MA, 557p.
Cliff, A. D., Martin, R. L. and Ord, J. K. (1974): Evaluating the friction of distance parameter in gravity models. *Regional Studies*, 8, 281-286.
—— (1975): Map pattern and friction of distance parameters: reply to comments by R. J. Johnston, and by L. Curry, D. A. Griffith and E. S. Sheppard. *Regional Studies*, **9**, 285-288.
—— (1976): A reply to the final comment. *Regional Studies*, **10**, 341-342.
Curry, L. (1972): A spatial analysis of gravity flows. *Regional Studies*, **6**, 131-147.
——, Griffith, D. A. and Sheppard, E. S. (1975): Those gravity parameters again. *Regional Studies*, **9**, 289-296.
Everitt, B. S. (1977): *The Analysis Contingency Table.* Chapman and Hall, London, 128p.
Fik, T. J. and Mulligan, G. F. (1990): Spatial flows and competing central places: towards a general theory of hierarchical interaction. *Environment and Planning A*, **22**, 527-549.
Flowerdew, R. and Aitkin, M. (1982): A method of fitting the gravity model based on the Poisson distribution. *Journal of Regional Science*, **22**, 191-202.
—— and Lovett, A. (1988): Fitting constrained Poisson regression models to interurban migration flow. *Geographical Analysis*, **20**, 297-307.
Fotheringham, A. S. (1983): A new set of spatial-interaction models: the theory of competing destinations. *Environment and Planning A*, **15**, 15-36.
—— and O'Kelly, M. E. (1989): *Spatial Interaction Models: Formulations and Applications.* Kluwer, Dordrecht, 221p.
—— and Williams, P. A. (1983): Further discussion on the Poisson model. *Geographical Analysis*, **15**, 343-347.
Griffith, D. A. and Jones, K. G. (1980): Explorations into the relationship between spatial structure and spatial interaction. *Environment and Planning A*, **12**, 187-201.
Guttman, L. (1968): A general nonmetric technique for finding the smallest coordinate space for a configuration of points. *Psychometrika*, **33**, 496-506.
Ishikawa, Y. (1978): *Sengo niokeru kokunai jinko ido* (Internal migration in postwar Japan). *Geographical Review of Japan*, **51**, 433-450.**
—— (1987): An empirical study of the competing destinations model using Japanese interaction data. *Environment and Planning A*, **19**, 1359-1373.
—— (1988): *Kukanteki Sogosayo Moderu (Development of Spatial Interaction Modeling:*

An Overview). Chijin Shobo, Kyoto, 254p.*
Itoh, S. (1986): An analysis of the distance parameter of spatial interaction model — a case of the greater Tokyo metropolitan area —. *Geographical Review of Japan*, **59**(Ser. B), 103-118.
Johnston, R. J. (1973): On friction of distance and regression coefficients. *Area*, **5**, 187-191.
—— (1974): Map pattern and friction of distance parameters: a comment. *Regional Studies*, **9**, 281-283.
—— (1976): On regression coefficients in comparative studies of the 'friction of distance'. *Tijdschrift voor Economische en Sociale Geografie*, **67**, 15-28.
Kuroda, T. (1979): *Nippon Jinko no Tenkan Kozo (Transitional Structure of Japanese Population)*. Kokon Shoin, Tokyo, 262p.*
Lovett, A. A. (1984): *Poisson Regression Using the GLIM Package*. Computer Package Guide, No.5, Department of Geography, University of Lancaster, Lancaster, 30p.
——, Whyte, I. D. and Whyte, K. A. (1985): Poisson regression analysis and migration field: the example of the apprenticeship records of Edinburgh in the seventeenth and eighteenth centuries. *Transactions, Institute of British Geographers, N.S.*, **10**, 317-332.
Macgill, S. M. (1977): Theoretical properties of biproportional matrix adjustments. *Environment and Planning A*, **9**, 687-701.
Matsuda, N. (1988): *Shitsuteki Joho no Tahenryo Kaiseki (Multivariate Analysis for Categorical Information)*. Asakura Shoten, Tokyo, 214p.*
Morisugi, H. (1984): *Hi-shukei kodo moderu no suitei to kentei* (Estimation and statistical test of disaggregate behavioral model). In Doboku Gakkai Doboku Keikakugaku (ed.) "*Hi-shukei Kodo Moderu no Riron to Jissen (Theory and Practice in the Disaggregate Behavioral Modeling)*". Doboku Gakkai (Japan Society of Civil Engineers Membership), Tokyo, 25-66.*
Mosteller, F. (1968): Association and estimation in contingency tables. *Journal of the American Statistical Association*, **63**, 1-28.
Murauskas, G. T., Green, M. B. and Bone, R. M. (1986): Modeling changes in the in-migration patterns of northern Saskatchewan communities: a log-linear approach. *Cahier de Géographie du Qubec*, **30**, 53-67.
Nelder, J. A. and Wedderburn, R. W. M. (1972): Generalised linear models. *Journal of the Royal Statistical Society*, **A135**, 370-384.
Payne, C. D. (ed.) (1987): *The GLIM System Release 3.77 Manual-edition 2*. Numerical Algorithms Group Limited, Oxford. 183p.
Pirie, G. H. (1979): Measuring accessibility: a review and proposal. *Environment and Planning A*, **11**, 299-312.
Porter, H. (1964): *Application of Intercity Intervening Opportunity Models to Telephone, Migration, and Highway Traffic Data*. Department of Geography, Northwestern University, Ph.D. dissertation, Evanston, 164p.
Saino, T. and Higashi, K. (1978): *Wagakuni niokeru todofukenkan jinko ido no kozo to sono henka* (Structure and its change of inter-prefectural migration in Japan). *Geographical Review of Japan*, **51**, 864-875.**
Scholten, H. J. and van Wissen, L. (1985): A comparison of the loglinear interaction

model with other spatial interaction models. In Nijkamp, P., Leitner, H. and Wrigley, N. (eds.) "*Measuring the Unmeasurable.*" Martinus Nijhoff Publishers, Dordrecht, 141-176.

Sheppard, E. S., Griffith, D. A. and Curry, L. (1972): A final comment on misspecification and autocorrelation in those gravity parameters. *Regional Studies*, **10**, 337-339.

Snickars, F. and Weibull, J. E. (1977): A minimum information principle: theory and practice. *Regional Science and Urban Economics*, **7**, 137-168.

Stouffer, S. A. (1960): Intervening opportunities and competing migrants. *Journal of Regional Science*, **2**, 279-287.

Sugiura, Y. (1986): *Kukanteki sogosayo moderu no kin'nen no tenkai: juryoku moderu kara entoropi saidaika moderu made* (Recent development of spatial interaction models: from gravity model to entropy-maximizing model). In Nogami, M. and Sugiura, Y. (eds.) "*Pasokon niyoru Suri Chirigaku Enshu (Exercise in Mathematical Geography Using Personal Computers)*". Kokon Shoin, Tokyo, 137-185.*

—— (1988): *Chakuchi sentakugata kukanteki sogosayo moderu niyoru chizu patan mondai no kokufuku no kanosei nitsuite* (On possibility of a solution of the map pattern problem by using a spatial interaction model of destinations choice). In Terasaka, A. (ed.) "*Kodo-johoka Shakai niokeru Chi'iki Kozo no Henyo (Changes of Regional Structure in Information-oriented Society).*" The Report of Grant-in-aid by Japanese Ministry of Education, Science and Culture, Tokyo, 141-155.*

—— (1989): *Ten bunpu patan bunseki-gijutsu to FORTRAN puroguramu* (Point pattern analysis and its FORTRAN programs). *Comprehensive Urban Studies*, **Special Edition**, 5-23.**

Tobler, W. R. (1983): *Bidimensional Regression: a Computer Program*. Discussion Paper, No.6, Department of Geography, University of California, Santa Barbara, Santa Barbara, 72p.

Train, K (1986): *Qualitative Choice Analysis: Theory, Econometrics, and an Application to Automobile Demand*. MIT Press, Cambridge, MA, 252p.

Upton, G. J. G. (1978): *The Analysis of Cross-tabulated Data.* John Wiley, New York, 148p.

Wakabayashi, K. (1987): *Jinko ido to chi'iki seisaku* (Migration and regional policy). In Hasumi, O., Yamamoto, E. and Takahashi, A. (eds.) "*Nippon no Shakai 2: Shakai Mondai to Kokyo Seisaku (Japanese Society 2: Social Problems and Public Policy)*". Tokyo Daigaku Shuppankai, Tokyo, 51-82.*

Willekens, F. J. (1983a): Log-linear modelling of spatial interaction. *Papers of the Regional Science Association*, **52**, 187-205.

—— (1983b): Specification and calibration of spatial interaction models: a contingency-table perspective and an application to intra-urban migration in Rotterdam. *Tijdschrift voor Economische en Sociale Geografie*, **74**, 239-252.

Wilson, A. G. (1967): A statistical theory of spatial distribution models. *Transportation Research*, **1**, 221-227.

—— (1970): *Entropy in Urban and Regional Modelling*. Pion, London, 166p.

—— (1971): A family of spatial interaction models, and associated developments. *Environment and Planning*, **3**, 1-32.

Wrigley, N. (1985): *Categorical Data Analysis for Geographers and Environmental Scientists.* Longman, New York, 392p.

Yanagawa, A. (1986): *Risan Tahenryo Deta no Kaiseki (Analysis of Discrete Multivariate Data).* Kyoritsu Shuppan, Tokyo, 215p.*

Yano, K. (1985): *Chiri gyoretsu eno chokusetsu inshibunsekiho no tekiyo ni kansuru ichi kosatsu* (A note on direct factor analysis of geographic matrices: a case of binary data matrices). *Geographical Review of Japan*, **58**(Ser. A), 516-535.**

—— (1989): *Entoropi saidaikagata kukanteki sogosayo moderu no kinko inshi ni kansuru ichi kosatsu* (A note on interpretation of the balancing factors of spatial interaction model). *Notes on Theoretical Geography,* **No. 6**, 17-34.*

—— (1990): *Igirisu o chushintoshita toshi moderu kenkyu no doko* (Developments of urban modelling research in Britain: a citation analysis). *The Human Geography*, **42**, 118-145.**

—— (1991): *Ippan senkei moderu niyoru kukanteki sogosayo moderu no togo* (The integration of spatial interaction models using generalized linear modeling). *Geographical Review of Japan*, **64**(Ser. A), 367-387.**

—— (1992a): *Risanteki sentaku moderu toshitemita kukanteki sogosayo no moderuka nitsuite: entoropi saidaika moderu kara tako rojitto moderu e* (A note on a family of spatial interaction models as a discrete choice model: from entropy-maximizing models to multinomial logit models). *Notes on Theoretical Geography*, **No. 8**, 55-75.*

—— (1992b): On the relationships between origins and destinations of the doubly constrained spatial interaction model. *Geographical Analysis*, to be submitted.

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