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Intelligent Control of an Autonomous Mobile Robot using Type-2 Fuzzy Logic

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Abstract— **We develop a tracking controller for the dynamic model of unicycle mobile robot by integrating a kinematic controller and a torque controller based on Fuzzy Logic Theory. Computer simulations are presented confirming the performance of the tracking controller and its application to different navigation problems.**

Index Terms—**Intelligent Control, Type-2 Fuzzy Logic, Mobile Robots.**

I. INTRODUCTION

Mobile robots are nonholonomic systems due to the constraints imposed on their kinematics. The equations describing the constraints cannot be integrated simbolically to obtain explicit relationships between robot positions in local and global coordinate's frames. Hence, control problems involve them have attracted attention in the control community in the last years [11].

Different methods have been applied to solve motion control problems. Kanayama et al. [10] propose a stable tracking control method for a nonholonomic vehicle using a Lyapunov function. Lee et al. [12] solved tracking control using backstepping and in [13] with saturation constraints. Furthermore, most reported designs rely on intelligent control approaches such as Fuzzy Logic Control [1][8][14][17][18][20] and Neural Networks [6][19].

However the majority of the publications mentioned above, has concentrated on kinematics models of mobile robots, which are controlled by the velocity input, while less attention has been paid to the control problems of nonholonomic dynamic systems, where forces and torques are the true inputs: Bloch and Drakunov [2] and Chwa [4], used a sliding mode control to the tracking control problem. Fierro and Lewis [5] propose a dynamical extension that makes possible the integration of kinematic and torque controller for a nonholonomic mobile robot. Fukao et al. [7], introduced an adaptive tracking controller for the dynamic model of mobile robot with unknown parameters using backstepping.

In this paper we present a tracking controller for the dynamic model of a unicycle mobile robot, using a control law such that the mobile robot velocities reach the given velocity inputs, and a fuzzy logic controller such that provided the required torques for the actual mobile robot. The rest of this paper is organized as follows. Sections II and III describe the formulation problem, which include: the kinematic and dynamic model of the unicycle mobile robot and introduces the tracking controller. Section IV illustrates the simulation results using the tracking controller. The section V gives the conclusions.

II. PROBLEM FORMULATION

A. The Mobile Robot

The model considered is a unicycle mobile robot (see Fig. 1), it consist of two driving wheels mounted on the same axis and a front free wheel [3].

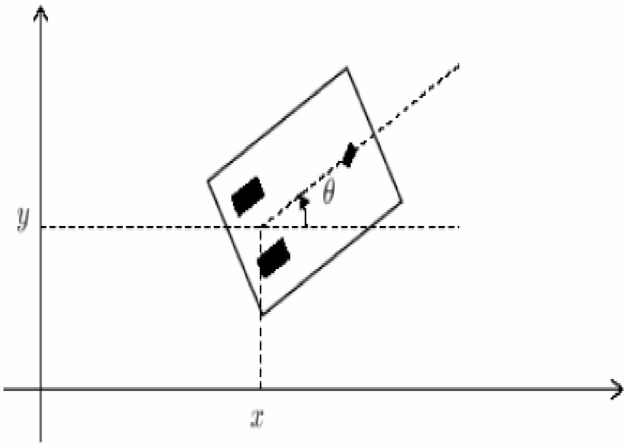

Fig. 1. Wheeled mobile robot.

The motion can be described with equation (1) of movement in a plane [5]:

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$$\dot{q} = \begin{vmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{vmatrix} \begin{vmatrix} v \\ w \end{vmatrix}$$

$$M(q)\dot{v} + V(q,\dot{q})v + G(q) = \tau \qquad (1)$$

Where $q = [x, y, \theta]^T$ is the vector of generalized coordinates which describes the robot position, *(x,y)* are the cartesian coordinates, which denote the mobile center of mass and θ is the angle between the heading direction and the *x*-axis (which is taken counterclockwise form); $v = [v, w]^T$ is the vector of velocities, *v* and *w* are the linear and angular velocities respectively; $\tau \in R^r$ is the input vector, $M(q) \in R^{nxn}$ is a symmetric and positive-definite inertia matrix, $V(q,\dot{q}) \in R^{nxn}$ is the centripetal and Coriolis matrix, $G(q) \in R^n$ is the gravitational vector. Equation (1.a) represents the kinematics or steering system of a mobile robot. Notice that the no-slip condition imposed a non-holonomic constraint described by (2), that it means that the mobile robot can only move in the direction normal to the axis of the driving wheels.

$$\dot{y}\cos\theta - \dot{x}\sin\theta = 0 \qquad (2)$$

B. *Tracking Controller of Mobile Robot*

Our control objective is established as follows: Given a desired trajectory $q_d(t)$ and orientation of mobile robot we must design a controller that apply adequate torque τ such that the measured positions *q(t)* achieve the desired reference $q_d(t)$ represented as (3):

$$\lim_{t\to\infty} \left\| q_d(t) - q(t) \right\| = 0 \qquad (3)$$

To reach the control objective, we are based in the procedure of [5], we deriving a *τ(t)* of a specific $v_c(t)$ that controls the steering system (1.a) using a Fuzzy Logic Controller (FLC). A general structure of tracking control system is presented in the Fig. 2.

III. CONTROL OF THE KINEMATIC MODEL

We are based on the procedure proposed by Kanayama et al. [10] and Nelson et al. [15] to solve the tracking problem for the kinematic model, this is denoted as $v_c(t)$. Suppose the desired trajectory q_d satisfies (4):

$$\dot{q}_d = \begin{vmatrix} \cos\theta_d & 0 \\ \sin\theta_d & 0 \\ 0 & 1 \end{vmatrix} \begin{vmatrix} v_d \\ w_d \end{vmatrix} \qquad (4)$$

Using the robot local frame (the moving coordinate system x-y in figure 1), the error coordinates can be defined as (5):

$$e = T_e(q_d - q), \begin{vmatrix} e_x \\ e_y \\ e_\theta \end{vmatrix} = \begin{vmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} x_d - x \\ y_d - y \\ \theta_d - \theta \end{vmatrix} \qquad (5)$$

And the auxiliary velocity control input that achieves tracking for (1.a) is given by (6):

$$v_c = f_c(e, v_d), \begin{vmatrix} v_c \\ w_c \end{vmatrix} = \begin{vmatrix} v_d + \cos e_\theta + k_1 e_x \\ w_d + v_d k_2 e_y + v_d k_3 \sin e_\theta \end{vmatrix} \qquad (6)$$

Where k_1, k_2 and k_3 are positive constants.

IV. FUZZY LOGIC CONTROLLER

The purpose of the Fuzzy Logic Controller (FLC) is to find a control input *τ* such that the current velocity vector *v* to reach the velocity vector v_c this is denoted as (7):

$$\lim_{t\to\infty} \left\| v_c - v \right\| = 0 \qquad (7)$$

As is shown in Fig. 2, basically the FLC have 2 inputs variables corresponding the velocity errors obtained of (7) (denoted as *ev* and *ew*: linear and angular velocity errors respectively), and 2 outputs variables, the driving and rotational input torques τ (denoted by *F* and *N* respectively). The membership functions (MF)[9] are defined by 1 triangular and 2 trapezoidal functions for each variable involved due to the fact are easy to implement computationally.

Fig. 3 and Fig. 4 depicts the MFs in which N, C, P represent the fuzzy sets [9] (Negative, Zero and Positive respectively) associated to each input and output variable, where the universe of discourse is normalized into [-1,1] range.

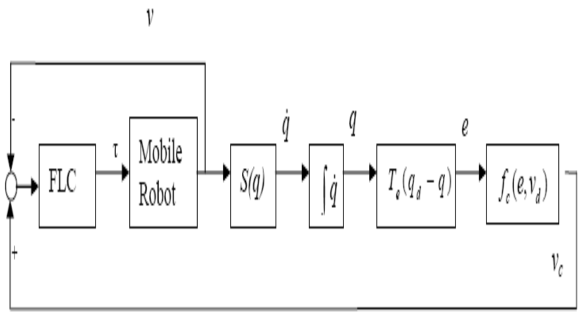

Fig. 2. Tracking control structure

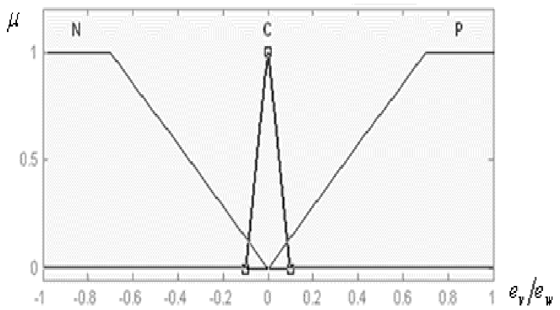

Fig. 3. Membership function of the input variables e_v and e_w

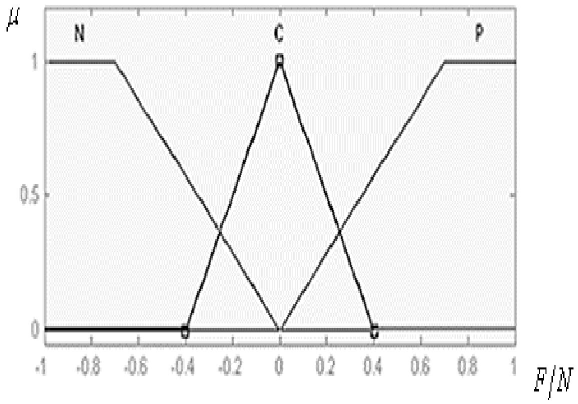

Fig. 4. Membership functions of the output variables F and N.

The rule set of FLC contain 9 rules which governing the input-output relationship of the FLC and this adopts the Mamdani-style inference engine [16], and we use the center of gravity method to realize defuzzification procedure. In Table I, we present the rule set whose format is established as follows:

Rule *i*: If e_v is G_1 and e_w is G_2 then F is G_3 and N is G_4

Where $G_1..G_4$ are the fuzzy set associated to each variable and *i*= 1 ... 9.

TABLE 1
FUZZY RULE SET

e_v / e_w	N	C	P
N	*N/N*	*N/C*	*N/P*
C	*C/N*	*C/C*	*C/P*
P	*P/N*	*P/C*	*P/P*

In Table I, N means NEGATIVE, P means POSITIVE and C means ZERO.

V. SIMULATION RESULTS

Simulations have been done in Matlab® to test the tracking controller of the mobile robot defined in (1). We consider the initial position $q(0) = (0, 0, 0)$ and initial velocity $v(0) = (0,0)$. From Fig. 5 to Fig. 8 we show the results of the simulation for the case 1. Position and orientation errors are depicted in the Fig. 5 and Fig. 6 respectively, as can be observed the errors are sufficient close to zero, the trajectory tracked (see Fig. 7) is very close to the desired, and the velocity errors shown in Fig. 8 decrease to zero, achieving the control objective in less than 1 second of the whole simulation. We show in Fig. 9 the Simulink block diagram to test the controller. We also show in Fig. 10 the tracking errors in the three variables. Finally, we show in Fig. 11 the evolution of the genetic algorithm that was used to find the optimal parameters for the fuzzy controller.

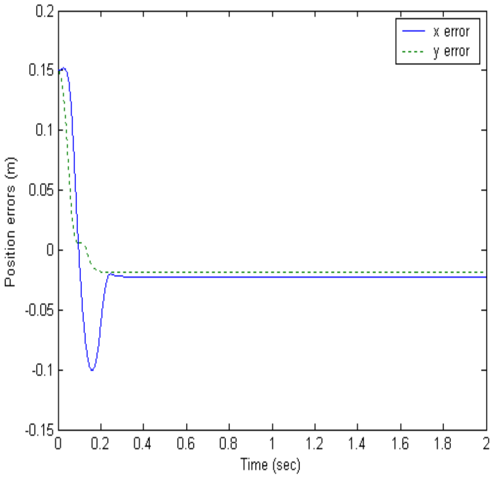

Fig. 5. Positions error with respect to the reference values. Solid: error in x, dotted: error in y.

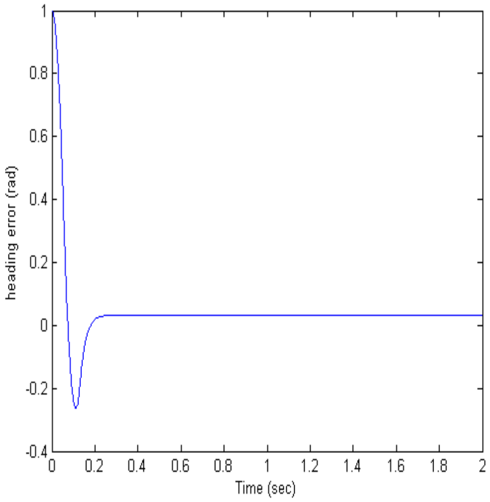

Fig. 6. Orientation error with respect to the reference values.

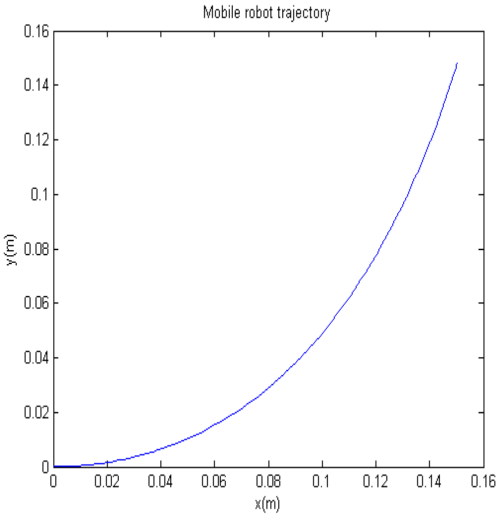

Fig. 7. Mobile Robot Trajectory.

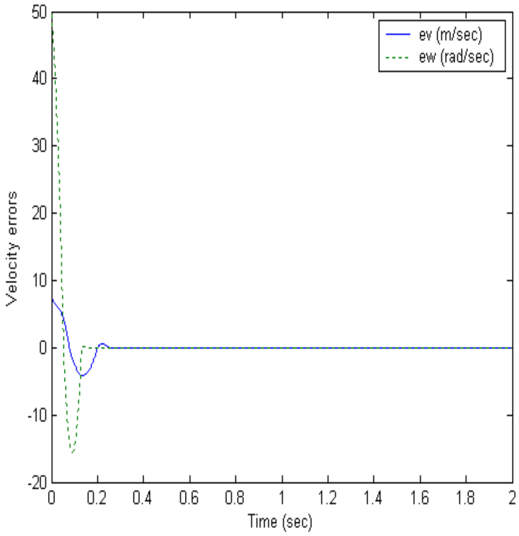

Fig. 8. Velocity errors: Solid: error in e_v, dotted: error in e_w

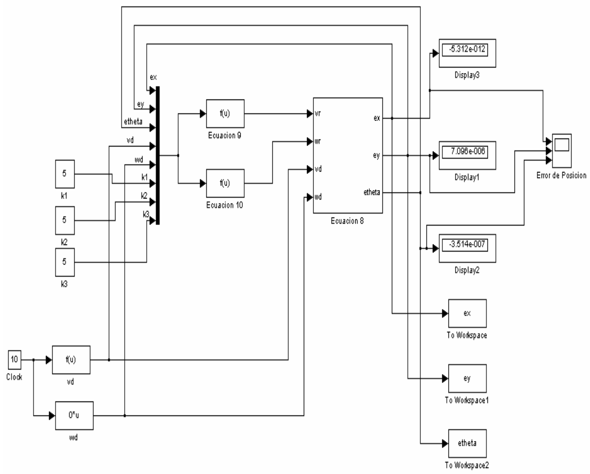

Fig. 9 Simulink block diagram of the controller.

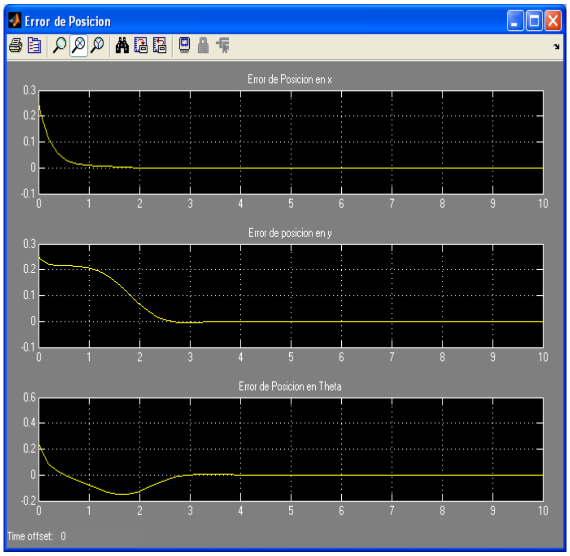

Fig. 10 Tracking errors in the three variables.

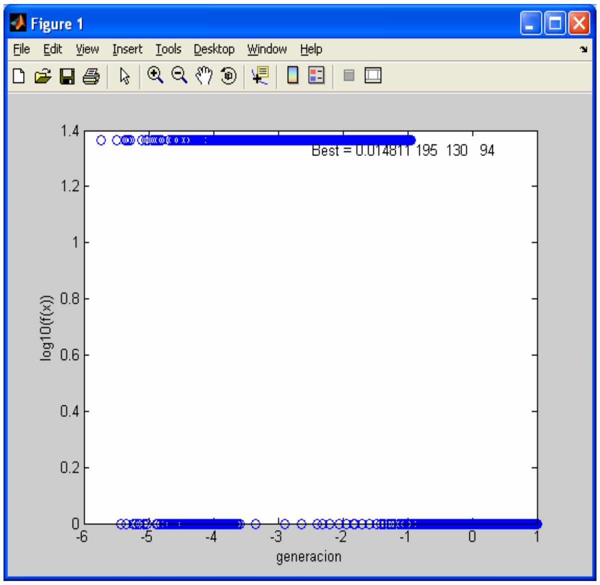

Fig. 11 Evolution of GA for finding optimal Controller.

In Table II we show simulation results for 25 experiments with different conditions for the gains of the fuzzy controller. We can also appreciate from this table that different reference velocities and positions were considered.

TABLE II
SIMULATION RESULTS FOR DIFFERENT EXPERIMENTS WITH THE FUZZY CONTROLLER.

No.	Controlador	Escala	Velocidades Iniciales		e					Constantes		
			vd	wd	ex	ey	etheta	ev	ew	k1	k2	k3
1	4	[-100 100]	0.25 – 0.25cos(2πt/5)	0	-0.435	2.36	2.79	0.0139	4.4	5	5	5
2	4	[-100 100]	0.25 – 0.25cos(2πt/5)	0	-0.339	-0.0997	-0.5	-14	-0.0773	45	70	45
3	4	[-100 100]	0.25 – 0.25cos(2πt/5)	0	-0.339	-0.0996	-0.5	-32.9	-0.0311	100	160	110
4	4	[-100 100]	0.1	0	-0.323	-0.106	-4.491	0.232	0.4	5	5	5
5	4	[-100 100]	0.1	0	-0.323	-0.106	-0.491	0.232	0.4	45	70	45
6	4	[-100 100]	0.1	0	-0.329	-0.104	-0.498	-30	0.328	100	160	110
7	4	[-100 100]	0.15 – 0.15cos(2πt/5)	0	-0.339	-0.0997	-0.5	-0.6	0.0859	5	5	5
8	4	[-100 100]	0.15 – 0.15cos(2πt/5)	0	-0.314	-0.112	-0.494	-10	0.627	45	70	45
9	4	[-100 100]	0.15 – 0.15cos(2πt/5)	0	-0.314	-0.112	-0.494	-10	0.627	100	160	110
10	4	[-100 100]	0.5	0	1.24	1.16	1.25	5.33	5.32	5	5	5
11	4	[-100 100]	0.5	0	-0.316	-0.108	-0.484	-10.5	-11.8	45	70	45
12	4	[-100 100]	0.5	0	-0.336	-0.101	-0.449	-31.5	-33.6	100	160	110
13	5	[-100 100]	0.25 – 0.25cos(2πt/5)	0	-0.339	-0.0995	-0.5	-0.818	-0.000181	5	5	5
14	5	[-100 100]	0.25 – 0.25cos(2πt/5)	0	-0.339	-0.0996	-0.5	-14.2	0.0325	45	70	45
15	5	[-100 100]	0.25 – 0.25cos(2πt/5)	0	-0.339	-0.0995	-0.5	-33	0.00623	100	160	110
16	5	[-100 100]	0.1	0	-0.333	-0.102	-0.496	-0.185	0.185	5	5	5
17	5	[-100 100]	0.1	0	1.5	-0.425	1.81	40.9	8.57	45	70	45
18	5	[-100 100]	0.1	0	-0.338	-0.1	-0.499	-32.1	-6.57	100	160	110
19	5	[-100 100]	0.5	0	-0.285	-0.119	-0.466	1.21	-1.29E-14	5	5	5
20	5	[-100 100]	0.5	0	-0.3	-0.113	-0.472	-9	-11	45	70	45
21	5	[-100 100]	0.5	0	-0.318	-0.107	-0.485	-28	-31.9	100	160	110
13	1	[-100 100]	0.25 – 0.25cos(2πt/5)	0	-0.319	-0.106	-0.482	0.311	1.01	5	5	5
14	1	[-100 100]	0.25 – 0.25cos(2πt/5)	0	-0.327	-0.104	-0.491	-12.9	0.694	45	70	45
15	1	[-100 100]	0.25 – 0.25cos(2πt/5)	0	-0.339	-0.0997	-0.5	-32.8	0.0839	100	160	110
16	1	[-100 100]	0.1	0	-0.241	-0.127	-0.419	2	1.84	5	5	5
17	1	[-100 100]	0.1	0	-0.312	-0.109	-0.483	-11.8	-1.87	45	70	45
18	1	[-100 100]	0.1	0	-0.321	-0.108	-0.494	-29.3	-6.32	100	160	110
19	1	[-100 100]	0.5	0	0.0567	-0.17	-0.226	6	2.76	5	5	5
20	1	[-100 100]	0.5	0	-0.309	-0.113	-0.487	-11.4	-13.6	45	70	45
21	1	[-100 100]	0.5	0	-0.29	-0.118	-0.473	-25	-32.5	100	160	110

VI. CONCLUSIONS

We described the development of a tracking controller integrating a fuzzy logic controller for a unicycle mobile robot with known dynamics, which can be applied for both, point stabilization and trajectory tracking. Computer simulation results confirm that the controller can achieve our objective. As future work, several extensions can be made to the control structure of Fig. 2, such as to increase the tracking accuracy and the performance level.

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