Intelligent control of HVAC systems.
Part I: Modeling and synthesis

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Abstract: *This is the first part of a work on intelligent type control of Heating, Ventilating and Air-Conditioning (HVAC) systems. The study is performed from the perspective of giving a unitary control method to ensure high energy efficiency and air quality improving. To illustrate the proposed HVAC control technique, in this first part it is considered as benchmark problem a single thermal space HVAC system. The construction of the mathematical model is performed only with a view to obtain a framework of HVAC intelligent control validation by numerical simulations. The latter will be reported in a second part of the study.*

Key Words: *HVAC, HVAC mathematical model, HVAC intelligent control, fuzzy logic control, neural network control, PID control, self-tuning PID-type fuzzy adaptive control*

1. INTRODUCTION

The energy performance of buildings has become nowadays very important because increasingly more buildings owners are becoming more receptive to costs. The context and the reasons are multiple: limited traditional resources, global warming, pollution etc. It seems incredible, but in the 1990s it was estimated that the energy consumption by **H**eating, **V**entilating, and **A**ir **C**onditioning (HVAC) equipment in industrial and commercial buildings accounted for around 50% of the world energy consumption [1], [2]. From this point of view, HVAC systems are among the most challenging plants in process control. It is noteworthy that in the 2010s this consumption decreased to about 20-40%, but accounts around 33% of the global CO_2 emissions [3]. Consequently, it is expected that future intelligent buildings should be provided, for high energy efficiency and comfort, with increasingly sophisticated control systems. Like in the project referred to [4], project which aims to improve air quality in hospital operation rooms (ORs) [5] (Figs. 1, 2), the HVAC systems can address specific objectives involving fluid dynamics, such as impinging jet ventilation strategy [6]-[8]. In other cases, such as a metro system or in the skyscrapers with hundred floors, it is necessary to have a complete air conditioning system, which could keep the temperature, humidity or pressure within acceptable ranges.

The most HVAC systems are typically set to operate at designed conditions defined by thermal loads. Without a robust control law, the system will become unstable, because the actual thermal loads are time-varying and consequently the HVAC system would overheat or

overcool spaces. Depending on application, the HVAC systems are designed and built as either *self-contained unit packages* or as *central systems*. The self-contained HVAC unit converts a primary energy (electricity or gas) by providing heating or cooling towards the conditioned space. Such systems are *air conditioning units* for rooms, *air-to-air heat pumps*, *rooftop HVAC systems*. With central systems, the primary conversion from fuel of electricity into a form of thermal energy occurs in a central location, and then this energy will be distributed to a particular building (school, office, industrial building, house etc.). Thus, the central systems combine central supply and end use zone systems. A first example is the central hot and/or chilled water distributed to multiple fan systems. The fan systems use water-to-air heat exchangers called coils to provide hot and/or cold air for the controlled spaces. End-use subsystems can be fan systems or terminal units. Another example concerns a central chiller and boiler for the conversion of primary energy, as well as a central fan system to delivery hot and/or cold air. Fig. 3 shows the thermodinamic cooling cycle in an air-conditioning system.
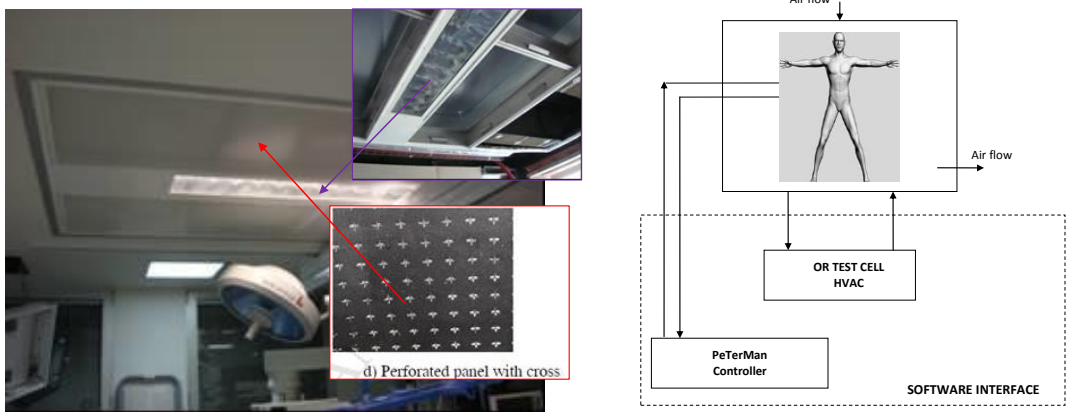

Fig. 1 – a) Air diffuser for OR with the suggestion of introducing perforated panels with innovative orifice geometry; b) software interface connecting two hardware parts – thermal manikin PeTerMan and experimental model of OR (test cell)
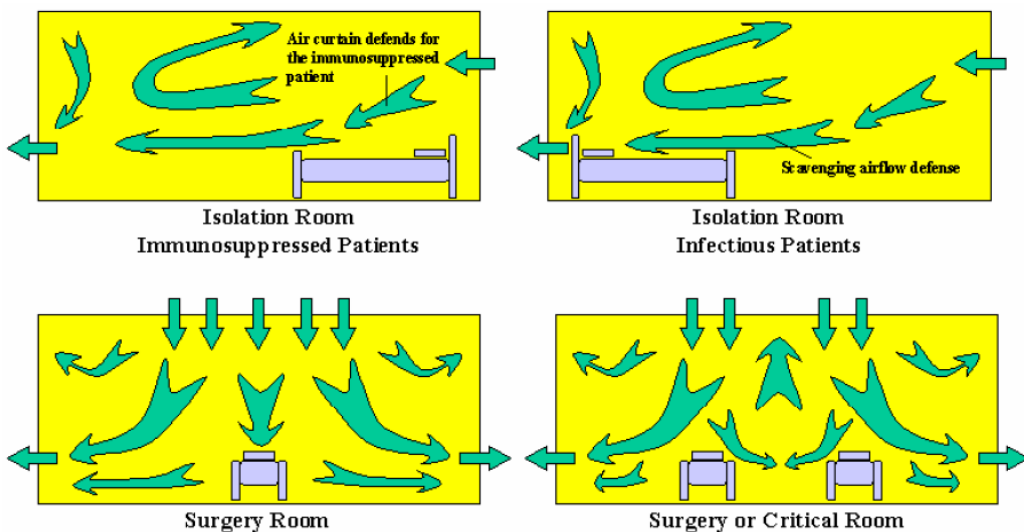

Fig. 2 – Airflow configurations for critical areas in hospital [5]

In this paper we are above all interested in *HVAC control synthesis*. Generally speaking, each HVAC system has its own control strategy. In this paper we propose a strategy that we consider most appropriate, given the history of the field and our own experience. A considerable amount of literature has been published on HVAC control synthesis. The development of these technological systems − whose importance has significantly grown since the oil crisis of the 1970's − closely follows the history of automatic control over the last six decades. An excellent survey on HVAC control systems can be found in [10]. Very simplified, the HVAC control methods can be classified into conventional/classical techniques, also called *hard control methods* [10] (Proportional-Integral-Derivative-PID control, Linear Quadratic Regulator-LQR, adaptive control, Lyapunov-based nonlinear control etc.) and nonconventional artificial intelligence based techniques, also called *soft control methods* [10] (fuzzy logic control, neural network control, agent-based intelligent control systems etc.). It should be noted that overlapping of these categories is inevitable.
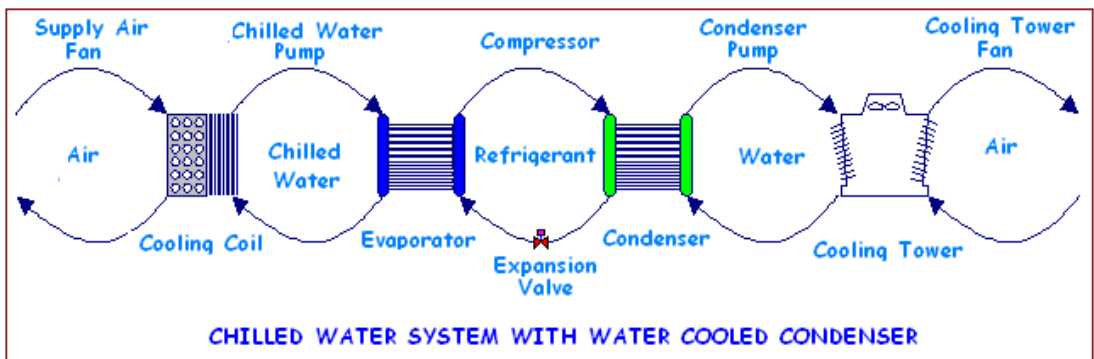

Fig. 3 – The schematic view of chilled water air-conditioning system with water-cooled condenser [9]

The conventional methods have their background in automatic control theory. Basically, any classical control law synthesis starts from a mathematical model of the system, usually called *state model*. In the *online* control process, the inputs and the feedbacks from the previous system state are used by the control algorithm to optimize the control of the system in the next time step. But the classical control laws have been synthesized/designed based on mathematical models, and their weakness lies right here: due to inherent modeling limits, burdened with measurement noises and incomplete observations of the state, the control of complex processes becomes a really difficult task. Therefore, the classical control, based on mathematical models, remains vulnerable to inaccurate and noisy inputs or feedbacks, even after a history of over six decades. It seems strange that over 95% of the control algorithms belong to prehistoric PID control [11]. It should be said however that this surprising outcome can be also made in connection with the well known schism, invoked more than once [12], between theoreticians and practitioners in the field.

2. A PHYSICAL MODEL OF HVAC SYSTEM

To illustrate our HVAC control technique, let us consider the single-zone/space HVAC physical model described in [2], often used as benchmark problem in the field (78 citations on Google currently; e.g., [13], [14]). Fig. 4 refers to the system operating on the cooling mode (air-conditioning). In all its details, the system components include thermal space, heating/cooling coil, humidifier/dehumidifier, mixing box, air filter, supply and return fans, filters, dampers, and ductwork.

Operations performed in the system will be reflected in mathematical modeling (section 3). In this system, fresh air enters and mixes with 75% of the return air (position 5) at the flow mixer (position 1), and remaining air is exhausted. The purposes of comfort anf hygiene are considered in this system-to-fresh-air volumetric flow-rate ratio. Then, mixed air passes through the heat exchanger components and finally by supply fan enters the thermal space as supply air (position 2), to offset (compensate) the sensible (actual heat) and latent (humidity) heat thermal loads acting upon the system; specifically, by changing of thermal load, the system controller simultaneously varies volumetric flow rate of air and water, so that the desired setpoints in temperature and relative humidity are maintained. Finally, the air in the thermal space is drawn through a fan (position 4), 75% of this air gets recirculated and the rest is exhausted from the system.

Some remarks from *Wikipedia* are given below for a quick understanding of the concepts. An *air conditioner* is designed to change the air temperature and humidity within an area (used for cooling and sometimes heating, depending on the air properties at a given time). The cooling is typically done using a simple refrigeration cycle, but sometimes *evaporation* is used, commonly for comfort cooling in buildings and motor vehicles. In construction, a complete system of heating, ventilation and air conditioning is referred to as "HVAC".

Ventilating is the process of "changing" or replacing air in any space to provide high indoor *air quality* (i.e. to control temperature, replenish oxygen, or remove moisture, odors, smoke, heat, dust, airborne bacteria, and carbon dioxide). Ventilation is used to remove unpleasant smells and excessive moisture, introduce outside air, to keep interior building air circulating, and to prevent stagnation of the interior air.

A *fan* is a machine used to create flow within a fluid, typically a gas such as air. Fans produce air flows with high volume and low pressure (however higher than ambient pressure), as opposed to *compressors* which produce high pressures at a comparatively low volume.
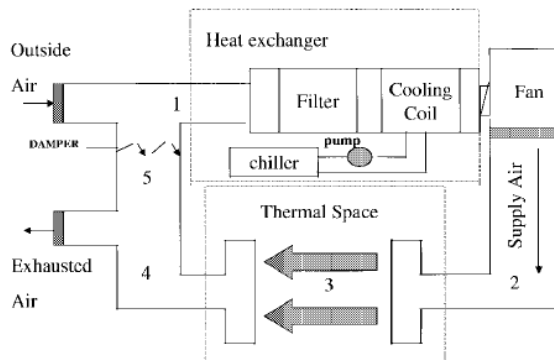

Fig. 4 – HVAC system operating on the cooling mode, as air-conditioning system – schematic view [2]

3. MATHEMATICAL MODEL

In this Section, the construction of the mathematical model is performed with only a view to obtain a framework of HVAC fuzzy logic controller validation by numerical simulations. The controller design will be described in the next Section. The constitutive assumptions of the mathematical model are the following [2]: 1) ideal gas behavior; 2) perfect mixing; 3) constant pressure process; 4) negligible wall and thermal storage 5) negligible thermal losses between components; 6) negligible infiltration and exfiltration effects; and 7) negligible

transient effects in the flow splitter and mixer. We introduce the following notations for system's parameters, constants and variables: ρ − air mass density [kg/m^3]; h_w − enthalpy of liquid water [J/kg]; Δh_w − variation of enthalpy of water vapor [J/kg]; h_{wv} − enthalpy of water vapor [J/kg]; W_o − humidity ratio of outdoor air; W_2 − humidity ratio of supply air; $W_3(t)$ − humidity ratio of thermal space; V_{he} − volume of heat exchanger [m^3]; c_p − specific heat of air [J/(kg^{o}C)]; $T_o(t)$ − temperature of outdoor air [^{o}C]; $T_2(t)$ − temperature of supply air [^{o}C]; $T_3(t)$ − temperature of thermal space [^{o}C]; V_3 − volume of thermal space [m^3]; M − humidity (moisture) load [kg/s]; Q − sensible heat load [W]; q_a − volumetric flow rate of air [m^3/s]; q_w − flow rate of chilled/heated water [m^3/s]. We so specify that the air with temperature $T_o(t)$ and flow rate $q_a(t)$ passes through the heat exchanger where an amount of heat is exchanged with the air. Since we have the assumption of perfect mixing, the air temperature within and exiting the heat exchanger is $T_2(t)$, which represents the supply air temperature. Obviously, the air and the heat exchanger capacitance must be taken into account, hence the resulting temperature has a *transient response*. After being cooling or heating in the heat exchanger, the air at temperature $T_2(t)$ passes into the thermal space with the help of fan and the air temperature in the thermal space is $T_3(t)$. Consistenly with the made assumptions, the effect of variations in instantaneous airspeed pressure zones is neglected. There is no air leakage except exhaust valve areas.

The air flow in thermal space is homogeneous. The heat load Q in the thermal space and the heat input Q_{he} in the heat exchanger (positive for heating and negative for cooling) are considered. Thermal losses between components are neglected and, thus, temperatures in the locations 4 and 5 (Fig. 3) are equal to the temperature of the air exiting the thermal space. Given that the infiltration and exfiltration effects are neglected, the flow rates at locations 2-3 are equal to $q_a(t)$.

The mathematical modeling is based on the energy conservation principle. For simplicity, we neglect a moment the effects of humidity. Consider the basic relationship

$$Q\Delta t = mc_p\Delta T \qquad (1)$$

relating the amount of heat $Q\Delta t$ absorbed by a mass m having specific heat c_p to achieve a temperature change ΔT in the time interval Δt. Physically, the temperature variation implies a transient regime and so, given the air mixture assumed in Section 3, the dynamics of temperatures in the two key locations 2-3 are configured like this:

$$\rho V_{he} c_p \frac{dT_2}{dt} = \rho c_p q_a \left(0.25T_o + 0.75T_3 - T_2\right) + Q_{he}$$

$$\rho V_3 c_p \frac{dT_3}{dt} = \rho q_a c_p \left(T_2 - T_3\right) + Q \qquad (2)$$

Next, add a simple dynamic model of air humidity and a completion of thermal loads. Thus, the differential equations describing the dynamic behavior of the HVAC system in Fig. 3 can be written as follows:

$$\dot{T}_3 = \frac{q_a}{V_3}(T_2 - T_3) - \frac{h_{wv}q_a}{c_p V_3}(W_2 - W_3) + \frac{1}{\rho c_p V_3}(Q - h_{wv}M)$$

$$\dot{W}_3 = \frac{q_a}{V_3}(W_2 - W_3) + \frac{M}{\rho V_3} \tag{3}$$

$$\dot{T}_2 = \frac{q_a(0.25T_o + 0.75T_3 - T_2)}{V_{he}} - \frac{q_a h_w}{c_p V_{he}}\left((0.25W_o + 0.75W_3) - W_2\right) - \frac{\rho \Delta h_w q_w}{\rho c_p V_{he}}$$

Indeed, in the right side of the first equation of (4), we observe a thermal load term generated by the enthalpy of the water vapors, namely $-\rho h_{wv}q_a(W_2 - W_3)$; then, we have a second thermal load term $-h_{wv}M$, in which the moisture mass is directly involved; and, finally, a third thermal load term defined by the sensible heat load Q (simply, the *sensible heat* is in connection with the amount of heat required to increase or decrease the temperature of an object or space, without changing its state of aggregation; *latent heat* is the amount needed to change its aggregation state).

All these terms are linearly concatenated based on the principle of superposition of effects. Now, it is easy to see how the other two equations are similarly obtained. As for the signs of the thermal load terms, plus or minus, these signs indicate the influence on state variables $T_2(t), T_3(t), W_3(t)$. For instance, the negative term $-h_{wv}M$ contributes to the decreasing of the temperature $T_3(t)$, and so on. It should be added that the actuators dynamics have been neglected.

4. THE PROBLEM OF HVAC SYSTEM SYNTHESIS

There are two objectives of a quality HVAC system: thermal comfort represented herein by the state variables of the thermal space $T_3(t), W_3(t)$ and energy savings. Most conventional HVAC systems are based on a single rotational speed of the fan/compressor. A system with variable speed control can control the heating/cooling capacity by changing the rotational speed of the fan/compressor for load matching and thermal comfort; therefore it must be complemented with a good control algorithm, to maintain comfort under any thermal loads. In other words, the rotational speed of the fan, which in our case is proportional to the volumetric flow rate of air q_a, must be conceived as a control variable; similarly must be considered the flow rate of chilled/heated water q_w. This control strategy characterize the HVAC system as a *variable-air-volume system* (VAV) that results in the lowest energy consumption.
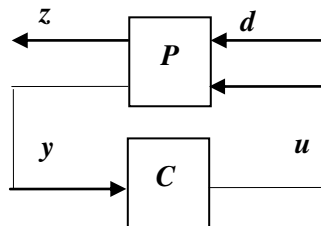

Fig. 5 − Feedback control system: standard block-diagram

In fact, the system (3) is the mathematical model of a feedback control system (Fig. 5). In the figure, P represents the physical system to be controlled, C and d the mathematical models of the controller and the external disturbance at the output of the plant, respectively. z represents a regulated/quality output. Further on, usual notations concern the *state vector* x characterizing the plant P, *the measurement output vector* y, and the *control vector* u. In our problem the two quantities z and y coincide. In this context, we will rewrite the system (3) by introducing the variables in usual state form

$$
\begin{aligned}
&x_1 := T_3, x_2 := W_3, x_3 := T_2 \\
&y_1 := T_3, y_2 := W_3 \\
&u_1 := q_a, u_2 := q_w \\
&d_1 := Q, d_2 := M
\end{aligned}
\tag{4}
$$

This means that the effort to build a controller is based on measuring the two states characterizing thermal space. With simplified notations for parameters and constants

$$
\alpha_1 = \frac{1}{V_3}, \alpha_2 = \frac{h_{wv}}{c_p V_3}, \alpha_3 = \frac{1}{\rho c_p V_3}, \alpha_4 = \frac{1}{\rho V_3}, \beta_1 = \frac{1}{V_{he}}, \beta_2 = \frac{\Delta h_w}{c_p V_{he}}, \beta_3 = \frac{h_w}{c_p V_{he}}
\tag{5}
$$

one obtains

$$
\begin{aligned}
\dot{x}_1 &= \alpha_1 (x_3 - x_1) u_1 - \alpha_2 (W_2 - x_2) u_1 + \alpha_3 (d_1 - h_{wv} d_2) \\
\dot{x}_2 &= \alpha_1 (W_2 - x_2) u_1 + \alpha_4 d_2 \\
\dot{x}_3 &= \beta_1 (x_1 - x_3) u_1 + 0.25 \beta_1 (T_o - x_1) u_1 - \beta_3 ((0.25 W_o + 0.75 x_2) - W_2) u_1 - \beta_2 u_2 \\
&\quad y_1 = x_1, y_2 = x_2
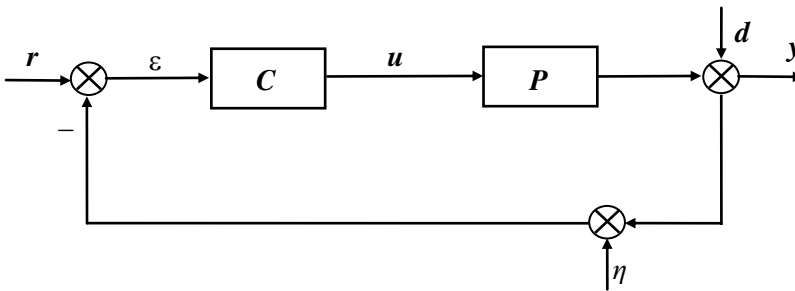\end{aligned}
\tag{6}
$$

Fig. 6 − Basic configuration of a stabilization system

For the sake of compliance, we mention that the feedback systems are characterized by two basic paradigms: the *regulator*, or *stabilization system*, and the *tracking system* [15], [16]. The HVAC system is essentially a regulator: the controller goal is that to maintain the thermal space temperature x_1 and humidity ratio x_2 at a certain set point $r = \begin{pmatrix} x_1^e & x_2^e \end{pmatrix}^T$, i. e., to counteract the error signals $\varepsilon = \begin{pmatrix} x_1(t) - x_1^e & x_2(t) - x_2^e \end{pmatrix}^T$ (Fig. 6) (by a^T is denoted the transpose to a vector/matrix a). The HVAC control synthesis problem can be defined as follows

Suppose that variations occur in the ambient temperature T_o, the humidity ambient ratio W_o, the humidity ratio of supply air W_2 and the thermal loads Q, M from a certain equilibrium point $\left(T_o^e, W_o^e, W_2^e, Q^e, M^e\right)$ which is in connection with a certain state equilibrium $\left(x_1^e, x_2^e, x_3^e\right)$. Provide an output feedback control law $\boldsymbol{u}(t) := \boldsymbol{u}\left(\boldsymbol{y}(t)\right)$ that brings the state $\left(x_1(t), x_2(t)\right)$ to the equilibrium point $\left(x_1^e, x_2^e\right)$. More specifically, the regulated (measured, herein) output $\boldsymbol{y}(t) = \left(x_1(t) \quad x_2(t)\right)^{\mathrm{T}}$ is required to approach the equilibrium point $\left(x_1^e, x_2^e\right)$.

It is known in the literature and in the practice of the field that the difficulty of solving the problem is especially amplified in the presence of the thermal load changes (disturbances d) [2] and of inherent measurement noise η (Fig. 6). To specify, we write the system [7] as model in *variations* by introducing the equilibrium point $\left(\boldsymbol{x}^e, \boldsymbol{u}^e, \boldsymbol{d}^e\right)$

$$\delta x_1 = x_1 - x_1^e, \delta x_2 = x_2 - x_2^e, \delta x_3 = x_3 - x_3^e$$
$$\delta u_1 = u_1 - u_1^e, \delta u_2 = u_2 - u_2^e, \delta d_1 = d_1 - d_1^e, \delta d_2 = d_2 - d_2^e \tag{7}$$

Substitute (7) in (6) and consider the definition of the *equilibrium point* as that solution of the system (6) that cancels the right-sides in the first three equations (6)

$$0 = u_1^e \alpha_1 \left(x_3^e - x_1^e\right) - u_1^e \alpha_2 \left(W_2^e - x_2^e\right) + \alpha_3 \left(d_1^e - h_{wv} d_2^e\right)$$
$$0 = u_1^e \alpha_1 \left(W_2^e - x_2^e\right) + \alpha_4 d_2^e \tag{8}$$
$$0 = u_1^e \beta_1 \left(x_1^e - x_3^e\right) + 0.25 u_1^e \beta_1 \left(T_o^e - x_1^e\right) - u_1^e \beta_3 \left(\left(0.25 W_o^e + 0.75 x_2^e\right) - W_2^e\right) - u_2^e \beta_2$$

A *bilinear system* in variations is obtained in matrix-form

$$\delta \dot{x} = A_0 \delta x + \sum_{i=1}^{3} \delta x_i B_i \delta_u + B_0 \delta_u + E \delta_d, \, y = C \delta x \tag{9}$$

where

$$A_0 = \begin{bmatrix} -\alpha_1 u_1^0 & \alpha_2 u_1^0 & \alpha_1 u_1^0 \\ 0 & -\alpha_1 u_1^0 & 0 \\ 0.75 \beta_1 u_1^0 & -0.75 \beta_3 u_1^0 & -\beta_1 u_1^0 \end{bmatrix}$$

$$B_0 = \begin{bmatrix} \alpha_1 \left(x_3^0 - x_1^0\right) - \alpha_2 \left(W_s - x_2^0\right) & 0 \\ \alpha_1 \left(W_s - x_2^0\right) & 0 \\ 0 & -\beta_2 \end{bmatrix} E = \begin{bmatrix} \alpha_3 & -h_{fg} \alpha_3 \\ 0 & \alpha_4 \\ 0 & 0 \end{bmatrix} \tag{9'}$$

$$B_1 = \begin{bmatrix} -\alpha_1 & 0 \\ 0 & 0 \\ 0.75 \beta_1 & 0 \end{bmatrix}, B_2 = \begin{bmatrix} \alpha_2 & 0 \\ -\alpha_1 & 0 \\ -0.75 \beta_3 & 0 \end{bmatrix}, B_3 = \begin{bmatrix} \alpha_1 & 0 \\ 0 & 0 \\ -\beta_1 & 0 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

A principle of *parsimony*, taken and used in systems identification [17], can be also invoked in mathematical modeling. This philosophical principle says that entities should not be multiplied needlessly; the simpler of two competing theories is to be preferred. The mathematical model of the system, as it was obtained in (6), proved to be a parsimonious one, that is neither too simple nor too complicated, so ultimately credible [2], [13], [14], [18]. Indeed, the model is bilinear, so positioned as mathematical complexity between linearity and nonlinearity. In terms of validation of an intelligent control strategy (based on fuzzy logic, neural, networks) the model is so representative. However, it should be added that an intelligent control strategy is largely free of mathematic model, as such its validation in process, *on-line*, depends on preliminary validation on model only to a small extent.

We said in Section 2 that we will use the numerical data provided in [2] as reference data for the proposed herein synthesis method. In order to transcribe the data from [2] in SI units were used information from sites [19] and [20]. These numerical values resulted as follows:

the nominal operating conditions, including the thermal loads:

$T_o^e = 29.44°\text{C}$; $W_o^e = 0.018$; $W_2^e = 0.007$; $d_1^e = 84960\,\text{W}$; $d_2^e = 0.02092\,\text{kg/s}$;

$u_1^e = 8.023\,\text{m}^3/\text{s}$; $u_2^e = 0.365\,\text{m}^3/\text{s}$

constructive and functional HVAC system data base:

$\rho = 1.19\,\text{kg/m}^3$; $c_p = 1005\,\text{J}/(\text{kg}°\text{C})$; $V_{he} = 1.719\,\text{m}^3$; $V_3 = 1655.115\,\text{m}^3$; $h_{wv} = 2431700\,\text{J/kg}$ at

(see [19]); $h_w = 53450\,\text{J/kg}$ at $x_3^e = 12.77\,°\text{C}$ (see [19]).

The analytical term $-\rho \Delta h_w q_w / (\rho c_p V_{he})$ in the system (3) is a generalization of the corresponding analytical-numerical term $-6000\text{gpm} / (\rho c_p V_{he})$ in the original model given in [2], in which the coefficient 6000 is physically dimensional. Data of [2] were converted herein in SI units. Substituting these data in the system (8) and choosing $\Delta h_w = 10000\,\text{J/kg}$, an equilibrium state $x_1^e = 21.4855\,°\text{C}, x_2^e = 0.0091725, x_3^e = 12.631\,°\text{C}$ very close to that given in [2], $x_1^e = 21.66\,°\text{C}\,(71\,°\text{F})$, $x_2^e = 0.0092$, $x_3^e = 12.77\,°\text{C}\,(55\,°\text{F})$, was obtained.

In developing the mathematical model (6), the actuator dynamics were neglected. The above discussion on the equilibrium points is not affected by this simplification. Therefore, the control signals can be implemented using a simple dynamic model of order one

$$G(s) = z(s)/u(s) = k/(1+\tau s) \tag{10}$$

5. ARTIFICIAL INTELLIGENCE BASED CONTROL OF HVAC SYSTEM

As already has been stated, the methods of the artificial intelligence in the solution of the control problems are based in principle only on the input-output data of the process. Therefore, herein *the mathematical models* (6) or (9) *will serve only as illustration of applying an artificial intelligence based control strategy. In the case of physical process, the mathematical model is naturally substituted by the physical system.*

In this paper we consider a neuro-fuzzy strategy [22], [23] for the HVAC system control. It has two components: a) a neuro-control and b) a fuzzy logic control supervising the neuro-control to counteract the saturation. To generate the two control signals − the volumetric flow rate of air and the flow rate of chilled/heated water −, an *elementary*

perceptron scheme [24], [25] is considered sufficiently efficient (Fig. 7). The elementary perceptron is a unilayered neural network with a single neuron. For simplicity, we use a generic notation u for the two control signals u_1 and u_2. In the figure, $\nu = [\nu_1 \ \nu_2]^{\mathrm{T}}$ is the weighting vector of the neural network, which is "trained" online by the *gradient descent learning method* to reduce the cost J
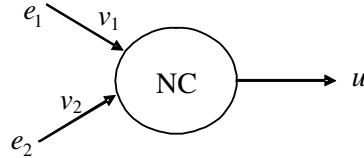

Fig. 7 − Perceptron type neuro-control

$$J = \frac{1}{2n}\sum_{i=1}^{n}\left(q_1 e_1^2(i) + e_2^2(i) + q_2 u_{nc}^2(i)\right) := \frac{1}{2n}\sum_{i=1}^{n} J(i) \tag{11}$$

q_1, q_2 are weights on the cost and $u := u_{nc}$ is the provided neuro-control signal

$$u := u_{nc} = \nu_1 e_1 + \nu_2 e_2 \tag{12}$$

From the system's view point, the input is u_{nc} and the output is $e = (e_1, e_2)$. From neuro-control training viewpoint, the system performance is assessed by the *cost function*, a criterion supposing a trade-off between the first input e_1 − the tracking error, the second input component e_2 − the rate of change of tracking error, and the control u_{nc}. In fact, the procedure (12) generates two neuro-control signals based on the sets of errors

$$e_{1t} := x_{1ref} - x_1, e_{2t} = \dot{e}_{1t}$$
$$e_{1h} := x_{2ref} - x_2, e_{2h} = \dot{e}_{1h} \tag{13}$$

where x_{1ref} and x_{2ref} are reference inputs (commands); in the case of the tracking systems, these are time variable signals. Consequently, the update is given by the expression

$$\mathbf{v}(n+1) = \mathbf{v}(n) + \Delta\mathbf{v}(n)$$

$$\Delta\mathbf{v}(n) := -\mathrm{diag}(\delta_1, \delta_2)\frac{\partial J}{\partial \mathbf{v}(n)} = -\mathrm{diag}(\delta_1, \delta_2)\sum_{i=n-N}^{n}\left(\frac{\partial J(i)}{\partial e(i)}\frac{\partial e(i)}{\partial u(i)} + \frac{\partial J(i)}{\partial u(i)}\right)\frac{\partial u(i)}{\partial \mathbf{v}(i)} \tag{14}$$

In the relation (14), the matrix $\mathrm{diag}(\delta_1, \delta_2)$ introduces the learning scale vector, $\Delta\mathbf{v}(n)$ is the weight vector update and *N* marks a back memory (of *N* time steps). The derivatives in (14) require only input-output information about the system. $\partial e(i)/\partial u(i)$ is online approximated by the relationship

$$\partial e(i)/\partial u(i) \approx (e(i) - e(i-1))/(u(i) - u(i-1)) \tag{15}$$

To counteract the risk of the neuro-control saturation and to achieve the enhancement of the learning system, a fuzzy supervised neuro-control (FSNC) was developed in [22], [23]. It should be emphasized that for fast systems, such as the hydraulic servomechanisms, with time constants in value of tenths of a second, the procedure has given good results. This time

we consider the application of the idea in the case of a slow system, such as the HVAC system. This means that the neuro-control switches to a Mamdani type fuzzy logic control whenever the just described neuro-control is saturated. In this way, the two components mutually complete a strategy with valences of optimality (neuro-control) and operational safety (fuzzy control). The mathematical foundations of the approximations with neural networks were laid in the paper of Cybenko [26]. Similarly, a new topic − fuzzy theory −, is born with the work of Zadeh [27]. The ideas of fuzzy set and fuzzy control are introduced with the aim to control the systems that are structurally difficult to model. Then, the fuzzy control has been extensively studied and applied, in the Mamdani linguistic variant, begining with the reference paper [28].
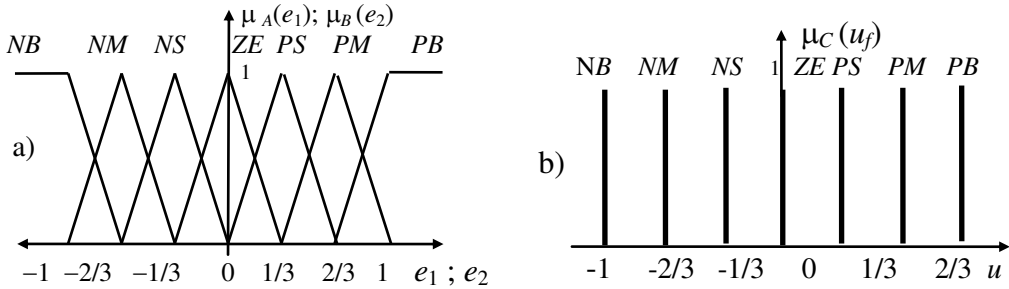

Fig. 8 −Membership functions for: a) scaled input variables y_1, y_2 and b) scalled fuzzy control u_f

The three well-known components of the fuzzy control − *fuzzyfier, fuzzy reasoning, and defuzzyfier* − will be succinctly described below. The *fuzzyfier* component converts the crisp input signals

$$e_{1k}, e_{2k}, k = 1, 2, \ldots \tag{16}$$

into their relevant fuzzy variables (or, equivalently, membership functions, MFs) associated to the following set of linguistic terms: "zero" (ZE), "positive or negative small" (PS, NS), "positive or negative medium" (PM, NM), "positive or negative big" (PB, NB) (for the sake of simplicity, the most natural and unbiased membership functions are chosen, of triangular and singleton type, Fig. 8). All MFs are defined on the common interval $[-1, 1]$. This means that previously have been defined some scale factors (SFs) $k_{e_1}, k_{e_2}, k_{u_f}$ for all variables e_1, e_2, u_f, respectively. The selection of suitable values for thes SFs involves apriori knowledge about the process, but also can be done through *trial and error* to achieve the best possible control performance. In principle, there is no well-defined method for good setting of SF's for FLC's. Special attention should be paid to compute on-line the effective SF of the control, α (see below). Thus the relationships between the SFs and the input and output variables of the self-tuning FLC are as follows

$$e_{1N} = k_{e_1} e_1, \quad e_{2N} = k_{e_2} e_2, \quad u_{fN} = \left(\alpha k_{u_f} \right) u \tag{17}$$

The *fuzzy reasoning* is defined by a fixed set of control rules (or, rules base, RB), normally derived from experts' knowledge. For instance, in the tracking control, the construction of a rules base embodies the idea of a (direct) proportion between the error signal e_1 and the required fuzzy control u_f [22], [23], [29]. In this case, of a regulator type

controller, the approach in the construction of the rules base is different. We specify that there is a booming literature of the field, but especially we highlight the papers [30]-[32]. Observe that there is no consensus in the literature on the terminology used in describing various types of fuzzy controllers. A fuzzy logic controller (FLC) is called *adaptive* if any one of its tunable parameters (SFs, MFs, and rules base) changes when the controller is being used, otherwise it is a *nonadaptive* or *conventional* FLC. An adaptive FLC that tunes an already working controller by modifying either its MFs or SFs or both of them is called a *self-tuning* FLC. If a FLC is tuned by automatically changing its RB, then it is called a *self-organizing* FLC [33].

e_2

u_f	NB	NM	NS	ZE	PS	PM	PB	
NB	NB	NB	NB	NM	NS	NS	ZE	
NM	NB	NM	NM	NM	NS	ZE	PS	
NS	NB	NM	NS	NS	ZE	PS	PM	
ZE	NB	NM	NS	ZE	PS	PM	PB	
PS	NM	NS	ZE	PS	PS	PM	PB	
PM	NS	ZE	PS	PM	PM	PM	PB	
PB	ZE	PS	PS	PM	PB	PB	PB	a)

(e_1 labels the rows)

e_2

α	NB	NM	NS	ZE	PS	PM	PB	
NB	VB	VB	VB	B	SB	S	ZE	
NM	VB	VB	B	B	MB	S	VS	
NS	VB	MB	B	VB	VS	S	VS	
ZE	S	SB	MB	ZE	MB	SB	S	
PS	VS	S	VS	VB	B	MB	VB	
PM	VS	S	MB	B	B	VB	VB	
PB	ZE	S	SB	B	VB	VB	VB	b)

(e_1 labels the rows)

Fig. 9 − a) Rules base for computation of u_f ; b) rules base for computation of updating factor α [30]

A *conventional FLC of Proportional Derivative (PD)-type* will be first described. This totals a number of $n = 7 \times 7$ IF..., THEN... rules, that is the number of the elements of the Cartesian product $A \times B$, $A = B$: = {NB; NM; NS; ZE; PS; PM; PB}. These sets are associated with the sets of linguistic terms chosen to define the membership functions for the fuzzy variables e_1 and, respectively, e_2. The structure of the n rules is shown in Fig. 9 a). Let T be the discrete sampling time. Consider the two scaled (normalized) input crisp variables e_{1Nk} and e_{2Nk}, at each time step $t_k = kT$ (k = 1, 2,...). Taking into account the two ordinates corresponding in the figure to each of the two crisp variables, a number of M $\leq 2^2$ combinations of two ordinates must be investigated. Having in mind these combinations, a number of M if..., then... rules will operate in the form

$$\text{if } e_{1Nk} \text{ is } A_i \text{ and } e_{2Nk} \text{ is } B_i, \text{then } u_{fNk} \text{ is } C_i, \ i = 1, 2, ..., M \qquad (18)$$

(A_i, B_i, C_i are linguistic terms belonging to the sets *A*, *B*, *C*, and $A = B = C$, see Fig. 9a). Note that RB in Fig. 9a) characterizes the requirements involving a two-dimensional phase plane,

so that the conventional FLC drives the system into the so-called *sliding mode* [34]. A *self-tuning* FLC is obtained by online updating the control gain α. This serves to counteract the controller overshoot and to improve the overall control performance. RB for α depends on each process, and of each "rough" RB operating, in this case of the one given in Fig. 9a). RB shown in Fig. 9b) is used in [30], with applications which gave good results on theoretical mathematical models. The new associated linguistic terms are "very small" (VS), "small" (S), "small big" (SB), "medium big" (MB), "big" (B), "very big" (VB).

It is worthy to note that the authors of the paper [30] present in [31] a new application, this time on HVAC systems and another RB for control gain α was obtained.

		e_2				
	k_P	**NB**	**NS**	**ZE**	**PS**	**PB**
	NB	PVB	PVB	PVB	PB	PM
	NS	PVB	PVB	PB	PB	PM
e_1	**ZE**	PB	PB	PM	PS	PS
	PS	PM	PS	PS	PS	PS
	PB	PS	PS	ZE	ZE	ZE

a)

		e_2				
	k_D	**NB**	**NS**	**ZE**	**PS**	**PB**
	NB	ZE	ZE	PS	PS	PB
	NS	ZE	ZE	ZE	ZE	PS
e_1	**ZE**	ZE	ZE	ZE	PS	PB
	PS	PS	PS	PS	PB	Z
	PB	ZE	ZE	ZE	PS	PB

b)

		e_2				
	k_I	**NB**	**NS**	**ZE**	**PS**	**PB**
	NB	PVB	PB	PM	PM	PM
	NS	PVB	PB	PB	PM	PS
e_1	**ZE**	PM	PS	ZE	ZE	ZE
	PS	PM	PM	PS	ZE	ZE
	PB	PS	ZE	ZE	ZE	ZE

c)

Fig. 10 − Rules base for self-tuning a PID-type fuzzy adaptive control: a) k_P ; b) k_I ; c) k_D

Other authors [32] consider as a starting point in the synthesis of a self-tuning FLC the well-known relationship which defines a Proportional-Integral-Derivative (PID) control

$$u(k) = k_P e(k) + k_I \sum_{i=1}^{k} e(i) + k_D \left[e(k) - e(k-1) \right] \tag{19}$$

The coefficients k_P, k_I, k_D are usually tuned according to certain criteria of classical synthesis [35], [36], [32]. RB shown in Fig. 10 is used in [32].
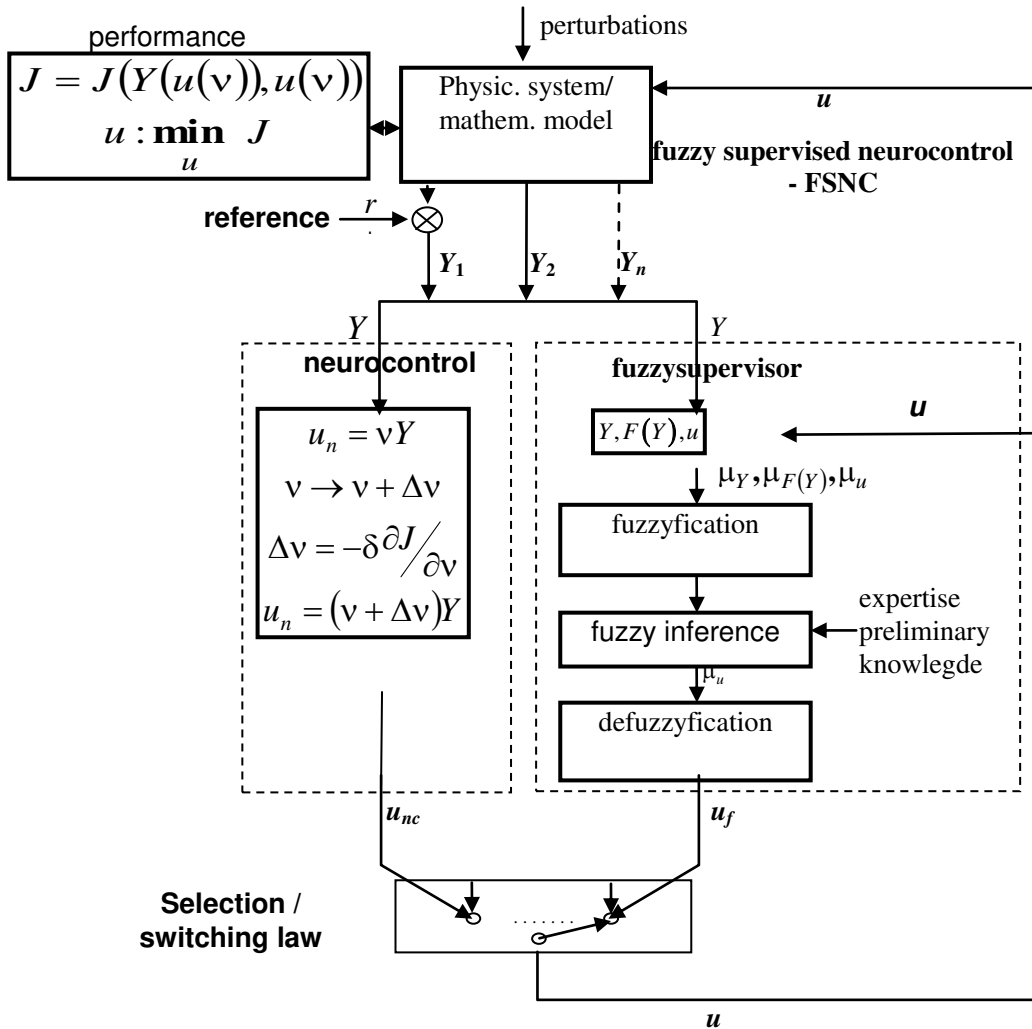
Fig. 11 − Sketch of the fuzzy supervised neural-control (FSNC)

The *defuzzyfier* concerns just the transforming of these if …, then rules into a mathematical formula giving the output control variable u_f. In terms of fuzzy logic, each rule of the form (18) defines a fuzzy set $A_i \times B_i \times C_i$ in the input-output Cartesian product space R^3, whose membership function can be defined in the manner

$$\mu_{u_i} = \min[\mu_{A_i}(e_{1k}), \mu_{B_i}(e_{2k}), \mu_{C_i}(u)], i = 1, \dots, M, (k = 1, 2, \dots) \tag{20}$$

For simplicity, the singleton-type membership function $\mu_C(u)$ of control variable has been preferred; in this case, $\mu_{C_i}(u)$ will be replaced by u_i^0, the singleton abscissa. Therefore, using 1) the singleton fuzzyfier for u_f, 2) the center-average type defuzzyfier, and 3) the min inference, the *M* if…, then… rules can be transformed, at each time step $k\tau$, into a formula giving the crisp control u_f [37], [38]

$$u_f = \sum_{i=1}^{M} \mu_{u_i} u_i^0 / \sum_{i=1}^{M} \mu_{u_i} \tag{21}$$

The FSNC operates as fuzzy logic control u_f in the case when neuro-control u_n saturated. In the case of fuzzy control operating, the fuzzy neuro-control u_n is concomitantly updated in the context of the real acting fuzzy control u_f. To obtain the rigor and accuracy of regulated process tracking, fuzzy logic control switches on neuro-control whenever readjusted neuro-control u_n is not saturated. At time t_s, when the switching from fuzzy logic control to neuro-control occurs, the readjusted weighting vector v_r will be derived by considering a scale factor u_f/u_n [38]

$$v_{1r} = \left(u_f - v_2 y_2\right)u_f \Big/ \left(u_{nc} y_1\right), \, v_{2r} = v_2 u_f / u_{nc}$$

The aforementioned FSNC was brought to the proof in various numerical simulations reported in [22], [23], [25], [38], and also in laboratory tests [29], [39]. A sketch of the FSNC is shown in Fig. 11.

5. CONCLUDING REMARKS

The mathematical model described in Section 3 is one of the most commonly HVAC models referenced in literature. Therefore, it will be used, in a second part to this paper, as a benchmark for the numerical simulation study of a special control strategy called FSNC (Fyzzy Supervised Neuro-Control). It must be said that, although independent of a mathematical model of the controlled system, the intelligent control requires a careful evaluation by numerical simulations, especially with regard to the adoption of the component "rules base". We mention also that the results of this study will serve as a starting point in the development of the project [4].

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