Intensity-based modal analysis of partially coherent beams with Hermite–Gaussian modes

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Many partially coherent beams are made up of a superposition of mutually uncorrelated Hermite–Gaussian modes. We prove that knowledge of the transverse intensity profile of such a beam is sufficient for evaluating the weights of the modes in an exact way. Simulations indicate that the proposed method resists noise well. © 1998 Optical Society of America

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The output beam of a stable-cavity laser oscillating in many transverse modes can be written as an incoherent superposition of Hermite–Gaussian (HG) beams.[1,2] As is well known,[3] this superposition gives rise to partially coherent beams. The same type of representation holds true, according to Wolf's modal theory of coherence,[4] for several partially coherent beams of interest.[3,5–8] For all cases, the problem consists of determining the weights of the underlying modes, starting from experimentally determined quantities. This problem has been tackled in various ways, including coherence measurements,[9,10] M^2-factor analysis,[11] best-fitting procedures,[12,13] and matrix-inversion methods.[14]

In this Letter we prove that the problem can be solved in a fairly simple way, starting from intensity measurements and making use of Fourier-transform techniques. According to numerical simulations the present method also exhibits good performance in the presence of noisy data.

Let us consider a superposition of independently oscillating HG modes. We limit ourselves to the one-dimensional case. Extension to the (rectangular) two-dimensional case is straightforward.

The expression for the disturbance of the nth HG mode at its waist plane, say, the plane $z = 0$, is[2]

$$G_n(x;\, v_0) = \left(\frac{2}{\pi v_0{}^2}\right)^{1/4} \frac{1}{\sqrt{2^n n!}}\, H_n\!\left(\frac{x\sqrt{2}}{v_0}\right)\exp\!\left(-\frac{x^2}{v_0{}^2}\right), \tag{1}$$

where H_n is the Hermite polynomial[15] of order n and v_0 is the spot size. The functions in Eq. (1) are normalized in the sense that

$$\int_{-\infty}^{+\infty} G_n{}^2(x;\, v_0)\mathrm{d}x = 1\,, \qquad n = 0,\, 1,\, 2,\, \ldots . \tag{2}$$

Since the modes are independent of one another, their time-averaged intensities are additive, and the transverse intensity profile of the beam at its waist plane is

$$I(x) = \sum_{n=0}^{\infty} c_n G_n{}^2(x;\, v_0)\,, \tag{3}$$

where c_n is a positive coefficient representing the power content of the nth mode. The problem of interest to us is to evaluate the c_n coefficients when $I(x)$ is known. In doing so, we assume that the spot size of the modes is known.

The difficulty with this evaluation stems from the fact that, although the G_n constitute an orthogonal set, their squares obviously do not. Consequently, the usual scalar-product rule that we would adopt for evaluating the coefficients of a series expansion into orthogonal functions cannot be applied. We can even wonder whether a unique solution exists for the values of c_n. Surprisingly, we have found that when we pass to the Fourier-transform domain the coefficients can be evaluated by the scalar product rule; this may sound contradictory because the scalar product is conserved under Fourier transformation. As a matter of fact, the property that we exploit here is slightly subtler. Briefly, it turns out that the Fourier transforms of the $G_n{}^2$, although they are not orthogonal on the whole p axis (p is a spatial-frequency variable), are indeed orthogonal on the half-axis $p \geq 0$ with respect to the variable p^2.

Our derivation starts from the following Fourier-transform relation:

$$\mathcal{F}\{G_n{}^2(x;\, v_0)\}(p) = \Psi_n(\pi^2 v_0{}^2 p^2)\,, \tag{4}$$

where $\mathcal{F}$ denotes the Fourier-transform operator with respect to x and the functions

$$\Psi_n(t) = L_n(t)\exp\!\left(-\frac{t}{2}\right) \tag{5}$$

are introduced, where L_n is the nth-order Laguerre polynomial.[15] Equation (4) can be easily derived from formula (22.13.20) of Ref. 15.

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It is well known that the Ψ_n functions are orthogonal on the half-axis $t \geq 0$, i.e.,

$$\int_0^\infty \Psi_n(t)\Psi_m(t)\mathrm{d}t = \delta_{n,m}\,, \tag{6}$$

where $\delta_{n,m}$ is the Kronecker symbol.

On Fourier transforming both sides of Eq. (3) and using Eq. (4), we obtain

$$\tilde{I}(p) = \sum_{n=0}^{\infty} c_n \Psi_n(\pi^2 v_0{}^2 p^2)\,, \tag{7}$$

where the tilde stands for the Fourier transform. Then, by exploiting the orthogonality of the Ψ_n functions [see Eq. (6)], we find the following expression for the expansion coefficient c_n:

$$c_n = 2\pi^2 v_0{}^2 \int_0^\infty \tilde{I}(p)\Psi_n(\pi^2 v_0{}^2 p^2) p\mathrm{d}p\,. \tag{8}$$

This amazingly simple equation completely solves the problem of finding the powers that are contributed to the beam by the various modes. Furthermore, thanks to the completeness of the set of Ψ_n functions, Eq. (8) proves that c_n values are indeed unique.

We have referred to the case in which the intensity is known across the waist plane, which is easily generalized. Indeed, any beam constituted by an incoherent superposition of HG modes is endowed with well-known shape-invariance properties.[6] To a constant factor accounting for energy conservation, the intensity distribution at any plane $z \neq 0$ is a magnified version of the one specified by Eq. (3), and the same magnification occurs for the spot sizes of the modes. Accordingly, we could rephrase our analysis for a plane $z \neq 0$, substituting the value of the spot size of the propagated modes, v_z, for v_0. The spot size of the modes as well as the waist position can be determined in various ways. For example, if the beam is generated by a laser, knowledge of the cavity parameters allows us to specify the mode characteristics. On the other hand, if the laser cavity is unknown or if the beam is generated by a synthetic source, the mode parameters can be evaluated starting from measurements of the intensity profile at distinct cross sections.

Let us now discuss how Eq. (8) can be exploited in practice. Suppose that the intensity profile $I(x)$ has been measured. Using standard sampling criteria, we can evaluate the Fourier transform $\tilde{I}(p)$ by use of fast Fourier-transform and interpolation techniques. We can then use a quadrature rule to compute the values of c_n through Eq. (8). An important question is how resistant such a procedure is to noise. Without going into detailed analysis, we present here the results of a few simulations from which some hints about the performance of the method can be gained.

We simulated a flat-top intensity distribution by means of a flattened Gaussian profile[16] of order $N = 20$. Noise was added to the intensity value of each sample as a random number with a uniform distribution in the interval $(-\varepsilon I_0, \varepsilon I_0)$, where I_0 is the maximum value of the intensity profile and ε is a positive quantity. Possible negative values of the corrupted intensity were set to zero.

In Fig. 1(a) we show intensity profiles for $\varepsilon = 0, 0.05, 0.1$. In Fig. 1(b) the c_n values obtained from Eq. (8) for these three cases are shown. For $\varepsilon = 0$ they are joined by a dashed curve and coincide with the theoretical values that were derived in the study reported in Ref. 14. For $\varepsilon \neq 0$ it should be noted that noise breaks down the even character of the intensity profile, which leads to possibly complex values for the coefficients, whereas the true values are real and positive. For this reason we show in Fig. 1(b) the real part of the values of c_n; the imaginary part can be ascribed to only noise. Some negative, and hence meaningless, values are exhibited, but this occurs only for those values of n for which the true c_n is near zero. Accordingly, the performance of the method with noisy data seems satisfactory.

In summary, a simple method of evaluating the modal content of a partially coherent beam obtained as a incoherent superposition of HG beams has been presented. The method requires the knowledge of the propagation characteristics of the modes and uses the intensity distribution of the beam at an arbitrarily chosen transverse plane to yield the set of the mode weights. In turn, this method permits the evaluation of the cross-spectral density function of the field and then leads to a complete characterization of the beam.[3] Although transverse modes emitted from most stable-cavity resonators are well described by HG

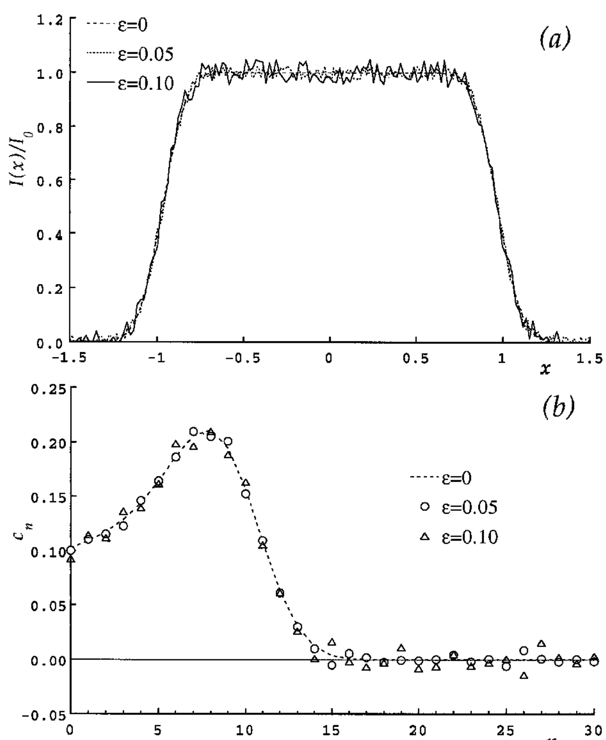

Fig. 1. (a) Intensity flat-top distributions in the presence of additive noise. (b) Real part of the expansion coefficients. $N = 20$.

beams, sometimes the beams of interest can be of the Laguerre–Gauss family. The possibility of extending the present method to Laguerre–Gauss modes remains to be seen.

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