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Intensity Distribution of the Finite Olver Beams through a Paraxial ABCD Optical System with an Aperture of Basis Annular Aperture

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Abstract

Based on the Collins diffraction formula and by means of the expansion of a hard aperture function into a finite sum of complex Gaussian functions, two analytical approaches of the Finite Olver beams (FOBs) passing through a paraxial ABCD optical system with a circular annular aperture or a rectangular one are developed in this paper. The propagation properties of the FOBs through an unapertured ABCD optical system or through this last with a circular (or rectangular) aperture or a circular (or rectangular) black screen are deduced, from the main results, as particular cases. Also, the characteristics of Finite ordinary Airy beam passing through the all considered optical systems are derived here that correspond to zeroth-order of the FOBs. According to the predicted formulas, computer simulation examples are given to deepen the understanding of the characteristics of the FOBs passing through some optical systems of annular aperture basis.

Keywords

Diffraction, Finite Olver Beams, Airy Beams, ABCD Optical System, Circular Annular Aperture, Rectangular Annular Aperture, Circular Black Screen, Rectangular Black Screen

1. Introduction

As is well known, the Finite Airy beam, as solution of paraxial wave equation, was firstly devoted in optical

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field by Siviloglou *et al.* [1] [2] in 2007, based on the Berry publication in the context of quantum mechanics which has introduced the Airy function for the first time as solution of the Schrödinger equation [3] in 1979. Airy beams constitute a complete and orthogonal family of non diffracting beams. To date, a number of papers have been developed to study the propagation and transformation of Airy beams [1]-[8]. Siviloglou Group's [6]-[8] was the first who had treated all theoretical and experimental properties of the Airy beams family, which allowed publishing more information on this beams class, that helped researchers to understand the behavior of this laser beams family. Recently, Belafhal *et al.* [9] have introduced for the first time the so-called "Finite Olver Beams" (FOBs) as theoretical generalization, with a beam order, of the ordinary Finite Airy beams. The authors have demonstrated that this novel beams class is a solution of Helmholtz equation and is a diffraction-free family [9]. In their principle reference, Belafhal *et al.* [9] have also devoted the phase masks for generating the beams in question in practice. For the purpose to understand and interpret the behavior of its propagation, the transformation of the FOBs by a paraxial aligned and misaligned optical system has investigated, respectively, by Hennani *et al.* [10] [11] in two above separate works.

Over the past twenty years, most of the optical properties of laser beams passing via annular aperture have been widely studied thanks to their theoretical and practical importance. Specifically, the focusing properties are well known when a beam is focused through various forms of apertures, whereby several theoretical studies on the propagation of laser beams through optical systems parallel to the optical axis have been made under a finite aperture condition [12]-[17] and especially the annular aperture that really exists in practice. The study of all transformations caused by the propagation of the laser beam through different forms of ABCD optical systems with annular apertures has assumed of great significance in the optics laser domain and its applications which allow describing the evaluation of a lot of laser beam characteristics [18]-[26].

On the other hand, the propagation of Finite Airy beams through an apertured paraxial optical system characterized by a transfer matrix ABCD has attracted much practical interest in optical applications. It is expected that the intensity diffracted by various forms of apertures changes, *i.e.* the intensity diffracted by a circular aperture differs to that diffracted by a rectangular one. This comparison will be the aim of this work. Therefore, the answer to this question will lead us to study the behavior of the FOBs propagating through an ABCD optical system with a circular annular aperture or with a rectangular one, using the generalized Huygens-Fresnel diffraction integral and the expansion of a hard aperture function into a finite sum of complex Gaussian functions.

The paper is organized as follows: In the coming Section, an analytical expression to describe the propagation of the FOBs through an ABCD optical system with a rectangular annular aperture will be established. In the third Section, we are going to investigate and analysis the propagation properties of the FOBs through a paraxial ABCD optical system with a circular annular aperture. So, some special cases will be treated and discussed in Section 4. In Section 5, the applications of the propagation properties formulas will be illustrated with detailed numerical calculations. Finally, a simple conclusion will be planned.

2. Approach Formula of the FOBs Passing through a Paraxial ABCD Optical System with a Rectangular Aperture

In the rectangular coordinates, the electrical field distribution of two-dimensional FOBs at the waist plane situated at propagation distance $z = 0$ is given by

$$U_1\left(x_1, y_1, z = 0\right) = O_n\left(\frac{x_1}{\omega_{0x}}\right)\exp\left\{a\frac{x_1}{\omega_{0x}}\right\}O_n\left(\frac{y_1}{\omega_{0y}}\right)\exp\left\{b\frac{y_1}{\omega_{0y}}\right\}, \tag{1}$$

where $O_n(.)$ is the Olver function of order n, ω_{0x} and ω_{0y} are the waist sizes of the incident beam in x and y directions, respectively. a and b are the exponential truncation factors in x and y directions, respectively. The transverse intensity distribution of the incident FOBs, for different parameters a, b and different beam orders n, is presented in **Figure 1**. The graphs of this figure show that the parameters a and b influence the apparition or not of the side-lobes in each direction, either x or y. This can be seen clearly around the orders $n = 0$ and 2. Consider now a FOBs illuminates a paraxial ABCD optical system with a rectangular annular aperture as shown in **Figure 2**. The rectangular annular aperture is of inner dimensional half width in x and y, b_x and b_y, respectively of outer dimensional half width in x and y-directions a_x and a_y, respectively (see **Figure 2**). Generally, the hard rectangular aperture can be expressed with a finite sum of complex Gaussian functions as [24] [27].
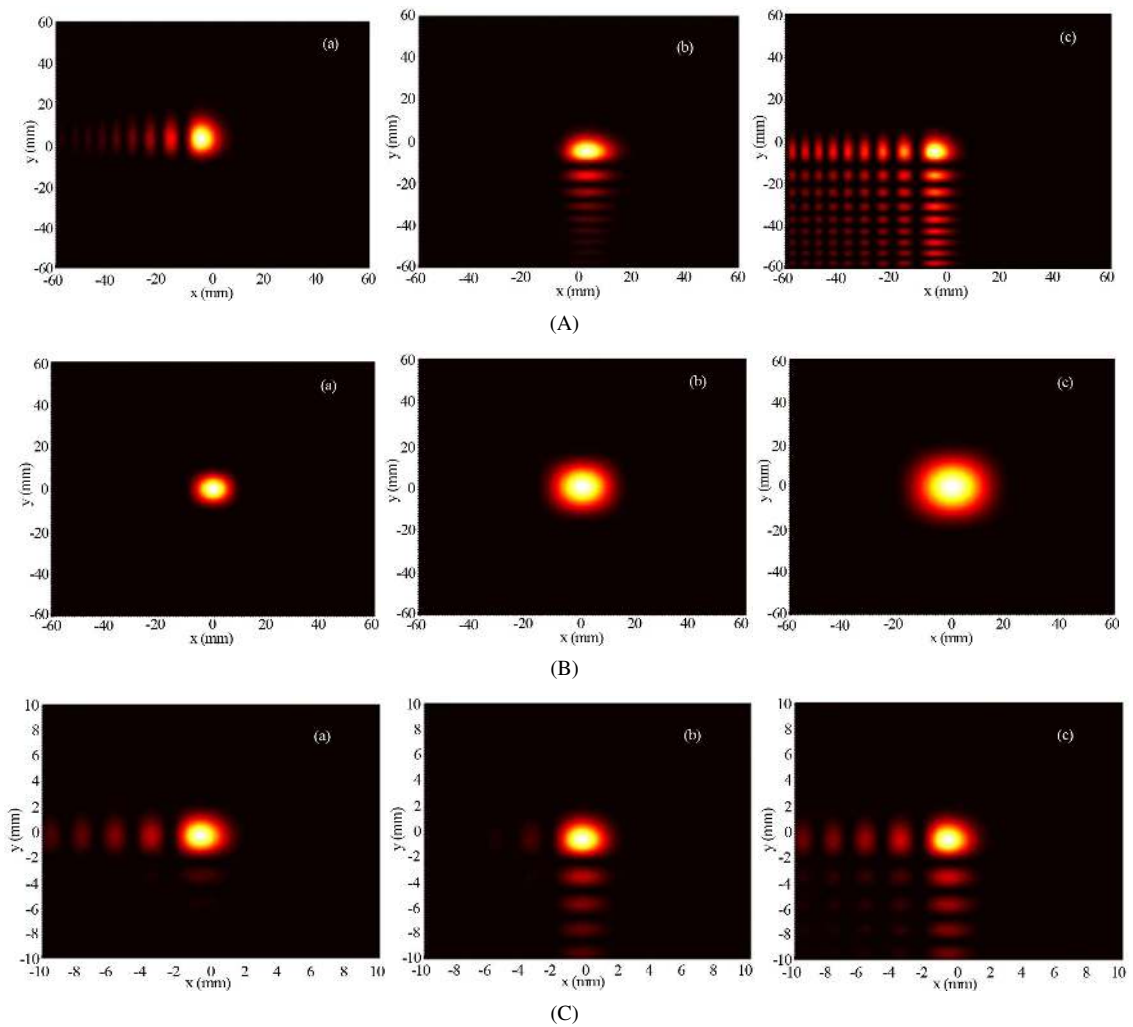
(A)

(B)

(C)

Figure 1. Transverse intensity distribution of the incident FOBs for different parameters a and b. (A): ($n = 0$) (a) $a = 0.1, b = 1$; (b) $a = 1, b = 0.1$; (c) $a = b = 0$ and $\omega_{0x} = \omega_{0y} = 5$ mm; (B): ($n = 1$) (a) $\omega_{0x} = \omega_{0y} = 5$ mm; (b) $\omega_{0x} = \omega_{0y} = 8$ mm; (c) $\omega_{0x} = \omega_{0y} = 10$ mm; (C): ($n = 2$) (a) $a = 0.1, b = 1$; (b) $a = 1, b = 0.1$; (c) $a = b = 0$ and $\omega_{0x} = \omega_{0y} = 5$ mm.
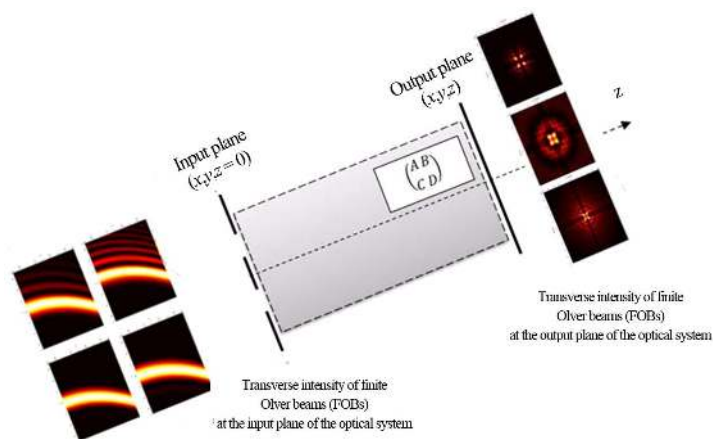

Figure 2. Schematic diagram of the propagation of the FOBs through an ABCD optical system with a rectangular or a circular annular aperture.

$$
\begin{cases}
A_{p_1}(x,y) = \sum_{h_1=1}^{N} A_{h_1} \exp\left\{-\frac{B_{h_1}}{a_x^2}x^2\right\} \sum_{h_2=1}^{N} A_{h_2} \exp\left\{-\frac{B_{h_2}}{a_y^2}y^2\right\}, \\
A_{p_2}(x,y) = \sum_{g_1=1}^{N} A_{g_1} \exp\left\{-\frac{B_{g_1}}{b_x^2}x^2\right\} \sum_{g_2=1}^{N} A_{g_2} \exp\left\{-\frac{B_{g_2}}{b_y^2}y^2\right\},
\end{cases}
\tag{2}
$$

where $A_{h,g_{1,2}}$ and $B_{h,g_{1,2}}$ denote the expansion and Gaussian coefficients, respectively.

Within the framework of the paraxial approximation, the propagation of a laser beam passing through a paraxial ABCD optical system over a diffraction surface Σ, at source plane $(z=0)$, is treated by means the Collins integral formula [28] and the output electric field distribution in the receiver plane placed at propagation distance z is given as

$$
U_2(x_2,y_2,z) = C_0 \iint_{\Sigma} U_1(x_1,y_1,z=0)\exp\left\{\frac{ik}{2B}\left(A(x_1^2+y_1^2)-2(x_1x_2+y_1y_2)\right)\right\}\mathrm{d}x_1\mathrm{d}y_1,
\tag{3}
$$

with

$$
C_0 = \left(\frac{ik}{2B}\right)\exp\{-ikz\}\exp\left\{-\frac{ik}{2B}D(x_2^2+y_2^2)\right\},
\tag{4}
$$

where $k=2\pi/\lambda$ is the wave number, A, B, C and D are the elements of the transfer matrix and λ being the wavelength.

If the diffraction surface Σ is a rectangular annular aperture, Equation (3) returns to

$$
\begin{aligned}
U_2(x_2,y_2,z) = {}& C_0 \int_{-a_x}^{a_x}\int_{-a_y}^{a_y} A_{p_1}(x_1,y_1)U_1(x_1,y_1,z=0)\exp\left\{\frac{ik}{2B}\left(A(x_1^2+y_1^2)-2(x_1x_2+y_1y_2)\right)\right\}\mathrm{d}x_1\mathrm{d}y_1 \\
& - C_0 \int_{-b_x}^{b_x}\int_{-b_y}^{b_y} A_{p_2}(x_1,y_1)U_1(x_1,y_1,z=0)\exp\left\{\frac{ik}{2B}\left(A(x_1^2+y_1^2)-2(x_1x_2+y_1y_2)\right)\right\}\mathrm{d}x_1\mathrm{d}y_1.
\end{aligned}
\tag{5}
$$

Substituting Equation (1) and Equation (2) into Equation (5), we find

$$
\begin{aligned}
U_2(x_2,y_2,z) = {}& C_0 \sum_{h_1=1}^{N} A_{h_1} \int_{-\infty}^{+\infty} O_n\left(\frac{x_1}{\omega_{0x}}\right)\exp\left\{-\left(\frac{B_{h_1}}{a_x^2}+\frac{ikA}{2B}\right)x_1^2+\left(\frac{a}{\omega_{0x}}+\frac{ik}{B}x_2\right)x_1\right\}\mathrm{d}x_1 \\
& \times \sum_{h_2=1}^{N} A_{h_2} \int_{-\infty}^{+\infty} O_n\left(\frac{y_1}{\omega_{0y}}\right)\exp\left\{-\left(\frac{B_{h_2}}{a_y^2}+\frac{ikA}{2B}\right)y_1^2+\left(\frac{b}{\omega_{0y}}+\frac{ik}{B}y_2\right)y_1\right\}\mathrm{d}y_1 \\
& - C_0 \sum_{g_1=1}^{N} A_{g_1} \int_{-\infty}^{+\infty} O_n\left(\frac{x_1}{\omega_{0x}}\right)\exp\left\{-\left(\frac{B_{g_1}}{b_x^2}+\frac{ikA}{2B}\right)x_1^2+\left(\frac{a}{\omega_{0x}}+\frac{ik}{B}x_2\right)x_1\right\}\mathrm{d}x_1 \\
& \times \sum_{g_2=1}^{N} A_{g_2} \int_{-\infty}^{+\infty} O_n\left(\frac{y_1}{\omega_{0y}}\right)\exp\left\{-\left(\frac{B_{g_2}}{b_y^2}+\frac{ikA}{2B}\right)y_1^2+\left(\frac{b}{\omega_{0y}}+\frac{ik}{B}y_2\right)y_1\right\}\mathrm{d}y_1.
\end{aligned}
\tag{6}
$$

By using the integral representation of Olver function [9]

$$
O_n(x) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\exp\left(a(iu)^\gamma+iux\right)\mathrm{d}u,
\tag{7}
$$

with $\begin{cases} \gamma = n+3, \\ |a| = \dfrac{1}{n+3}, \end{cases}$

recalling the integral formula [29]

$$
\int_{-\infty}^{+\infty}\exp\left(-p^2x^2\pm qx\right)\mathrm{d}x = \frac{\sqrt{\pi}}{p}\exp\left\{\frac{q^2}{4p^2}\right\},
\tag{8}
$$

$$R_e \left\{ p^2 \right\} > 0,$$

by means of the integral form [9]

$$\frac{1}{2\pi} \int_{-\infty}^{+\infty} \exp\left[a(it)^{n+3} - \frac{1}{2} st^2 + itx \right] dt = \exp\left\{ \frac{s^3}{12} + \frac{sx}{2} \right\} O_n \left[x + \frac{s^2}{4} \right], \tag{9}$$

and after some calculations, the output electrical field of a FOBs passing through an ABCD optical system which is limited by a rectangular annular aperture is found as follows

$$
\begin{aligned}
U_2(x_2, y_2, z) = & C' \sum_{h_1=1}^{N} A_{h_1} \left(\frac{1}{P_{h_1}} \right)^{1/2} \exp\left\{ \frac{q_x^2}{4P_{h1}} + \frac{1}{96\omega_{0x}^6 P_{h_1}^3} + \frac{q_x}{8\omega_{0x}^3 P_{h_1}^2} \right\} O_n \left(\frac{q_x}{2\omega_{0x} P_{h_1}} + \frac{1}{16\omega_{0x}^4 P_{h_1}^2} \right) \\
& \times \sum_{h_2=1}^{N} A_{h_2} \left(\frac{1}{P_{h_2}} \right)^{1/2} \exp\left\{ \frac{q_y^2}{4P_{h_2}} + \frac{1}{96\omega_{0y}^6 P_{h_2}^3} + \frac{q_y}{8\omega_{0y}^3 P_{h_2}^2} \right\} O_n \left(\frac{q_y}{2\omega_{0y} P_{h_2}} + \frac{1}{16\omega_{0y}^4 P_{h_2}^2} \right) \\
& - C' \sum_{g_1=1}^{N} A_{g_1} \left(\frac{1}{P_{g_1}} \right)^{1/2} \exp\left\{ \frac{q_x^2}{4P_{g1}} + \frac{1}{96\omega_{0x}^6 P_{g_1}^3} + \frac{q_x}{8\omega_{0x}^3 P_{g_1}^2} \right\} O_n \left(\frac{q_x}{2\omega_{0x} P_{g_1}} + \frac{1}{16\omega_{0x}^4 P_{g_1}^2} \right) \\
& \times \sum_{g_2=1}^{N} A_{g_2} \left(\frac{1}{P_{g_2}} \right)^{1/2} \exp\left\{ \frac{q_y^2}{4P_{g_2}} + \frac{1}{96\omega_{0y}^6 P_{g_2}^3} + \frac{q_y}{8\omega_{0y}^3 P_{g_2}^2} \right\} O_n \left(\frac{q_y}{2\omega_{0y} P_{g_2}} + \frac{1}{16\omega_{0y}^4 P_{g_2}^2} \right),
\end{aligned} \tag{10}
$$

where

$$C' = \frac{1}{4\pi} C_0, \tag{11}$$

$$
\begin{cases}
P_{h_1} = \dfrac{B_{h_1}}{a_x^2} + \dfrac{ikA}{2B}, \\[2mm]
P_{h_2} = \dfrac{B_{h_2}}{a_y^2} + \dfrac{ikA}{2B}, \\[2mm]
P_{g_1} = \dfrac{B_{g_1}}{b_x^2} + \dfrac{ikA}{2B}, \\[2mm]
P_{g_2} = \dfrac{B_{g_2}}{b_y^2} + \dfrac{ikA}{2B},
\end{cases} \tag{12}
$$

and

$$
\begin{cases}
q_x = \dfrac{a}{\omega_{0x}} + \dfrac{ik}{B} x_2, \\[2mm]
q_y = \dfrac{b}{\omega_{0y}} + \dfrac{ik}{B} y_2.
\end{cases} \tag{13}
$$

Equation (10) is the main first result of this work. It characterizes the transformation of a FOBs by an ABCD optical system with a rectangular annular aperture. In the following paragraph, we will focus to the propagation of FOBs through a circular annular aperture.

3. Propagation Equation of FOBs through a Circular Annular Apertured Paraxial ABCD Optical System

Let us consider a circular annular aperture located at the waist plane of $z = 0$ followed by an ABCD optical

system illuminated by a 2-D-FOBs as shown in **Figure 2**. The hard circular annular aperture is expanded into a finite sum of complex Gaussian functions in two dimensions as [5]

$$T(x, y) = \sum_{h=1}^{N} A_h \left[\exp\left\{ -\frac{B_h}{R^2}(x^2 + y^2) \right\} - \exp\left\{ -\frac{B_h}{r^2}(x^2 + y^2) \right\} \right], \tag{14}$$

where A_h and B_h denote the expansion and Gaussian coefficients respectively, which can obtained by optimization-computation directly [27]. R and r are the outer and inner radii of the annular aperture, respectively. The propagation of the FOBs passing through a paraxial ABCD optical system with circular annular aperture is characterized by the Collins integral of the form [28]

$$U_2(x_2, y_2, z) = C_0 \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} T(x_1, y_1) U_1(x_1, y_1, z = 0) \exp\left\{ -\frac{ik}{2B}(A(x_1^2 + y_1^2) - 2(x_1 x_2 + y_1 y_2)) \right\} dx_1 dy_1. \tag{15}$$

The parameters A, B, C and D are as described in Section 2. Substituting Equation (1) and Equation (14) into Equation (15), we can obtain

$$U_2(x_2, y_2, z) = C_0 \sum_{h=1}^{N} A_h \left(\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} O_n\left(\frac{x_1}{\omega_{0x}}\right) O_n\left(\frac{y_1}{\omega_{0y}}\right) \exp\left\{ -\frac{B_h}{R^2}(x_1^2 + y_1^2) \right\} \exp\left\{ a\frac{x_1}{\omega_{0x}} \right\} \right.$$

$$\times \exp\left\{ b\frac{y_1}{\omega_{0y}} \right\} \exp\left\{ -\frac{ik}{2B}(A(x_1^2 + y_1^2) - 2(x_1 x_2 + y_1 y_2)) \right\} dx_1 dy_1$$

$$\tag{16}$$

$$- \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} O_n\left(\frac{x_1}{\omega_{0x}}\right) O_n\left(\frac{y_1}{\omega_{0y}}\right) \exp\left\{ -\frac{B_h}{r^2}(x_1^2 + y_1^2) \right\} \exp\left\{ a\frac{x_1}{\omega_{0x}} \right\}$$

$$\left. \times \exp\left\{ b\frac{y_1}{\omega_{0y}} \right\} \exp\left\{ -\frac{ik}{2B}(A(x_1^2 + y_1^2) - 2(x_1 x_2 + y_1 y_2)) \right\} dx_1 dy_1 \right).$$

By using the integral representation of Olver functions given by Equation (7), recalling the integral formulae as given in Equation (8) and Equation (9), and after some calculations, we find the output optical field of a Finite Olver beam passing through an ABCD optical system with a circular annular aperture as follows

$$U_2(x_2, y_2, z) = C' \sum_{h=1}^{N} A_h \left[\frac{1}{P_R} \exp\left\{ \frac{q_x^2 + q_y^2}{4P_R} \right\} \exp\left\{ \frac{1}{96\omega_{0x}^6 P_R^3} + \frac{q_x}{8\omega_{0x}^3 P_R^2} \right\} \exp\left\{ \frac{1}{96\omega_{0y}^6 P_R^3} + \frac{q_y}{8\omega_{0y}^3 P_R^2} \right\} \right.$$

$$\times O_n\left(\frac{q_x}{2\omega_{0x} P_R} + \frac{1}{16\omega_{0x}^4 P_R^2} \right) O_n\left(\frac{q_y}{2\omega_{0y} P_R} + \frac{1}{16\omega_{0y}^4 P_R^2} \right) - \frac{1}{P_r} \exp\left\{ \frac{q_x^2 + q_y^2}{4P_r} \right\} \exp\left\{ \frac{1}{96\omega_{0x}^6 P_r^3} + \frac{q_x}{8\omega_{0x}^3 P_r^2} \right\} \tag{17}$$

$$\left. \times \exp\left\{ \frac{1}{96\omega_{0y}^6 P_r^3} + \frac{q_y}{8\omega_{0y}^3 P_r^2} \right\} O_n\left(\frac{q_x}{2\omega_{0x} P_r} + \frac{1}{16\omega_{0x}^4 P_r^2} \right) O_n\left(\frac{q_y}{2\omega_{0y} P_r} + \frac{1}{16\omega_{0y}^4 P_r^2} \right) \right],$$

where

$$\begin{cases} P_R = \dfrac{B_h}{R^2} + \dfrac{ikA}{2B} \\ P_r = \dfrac{B_h}{r^2} + \dfrac{ikA}{2B} \end{cases}, \tag{18}$$

and

$$\begin{cases} q_x = \dfrac{a}{\omega_{0x}} + \dfrac{ik}{B} x_2 \\ q_y = \dfrac{b}{\omega_{0y}} + \dfrac{ik}{B} y_2 \end{cases}. \tag{19}$$

Equation (17) is considered as the main second result of this work. It characterizes the transformation of a FOBs by an ABCD optical system with a circular annular aperture. In the following paragraph, we will deduce the special cases describing the form of rectangular and circular apertures.

4. Particular Cases

4.1. Particular Cases Concerning the Incident Beam

4.1.1. Propagation of a Finite Airy Beam through a Rectangular

In this case, the propagation of this beams family passing through a paraxial ABCD optical system with rectangular annular aperture is treated. This case is obtained when $n = 0$ and by the use of Equation (10), the output field is given by

$$
\begin{aligned}
U_2(x_2, y_2, z) = &\ C' \sum_{h_1=1}^{N} A_{h_1} \left(\frac{1}{P_{h_1}}\right)^{1/2} \exp\left\{\frac{q_x^2}{4P_{h1}} + \frac{1}{96\omega_{0x}^6 P_{h_1}^3} + \frac{q_x}{8\omega_{0x}^3 P_{h_1}^2}\right\} Ai\left(\frac{q_x}{2\omega_{0x} P_{h_1}} + \frac{1}{16\omega_{0x}^4 P_{h_1}^2}\right) \\
&\times \sum_{h_2=1}^{N} A_{h_2} \left(\frac{1}{P_{h_2}}\right)^{1/2} \exp\left\{\frac{q_y^2}{4P_{h_2}} + \frac{1}{96\omega_{0y}^6 P_{h_2}^3} + \frac{q_y}{8\omega_{0y}^3 P_{h_2}^2}\right\} Ai\left(\frac{q_y}{2\omega_{0y} P_{h_2}} + \frac{1}{16\omega_{0y}^4 P_{h_2}^2}\right) \\
&- C' \sum_{g_1=1}^{N} A_{g_1} \left(\frac{1}{P_{g_1}}\right)^{1/2} \exp\left\{\frac{q_x^2}{4P_{g1}} + \frac{1}{96\omega_{0x}^6 P_{g_1}^3} + \frac{q_x}{8\omega_{0x}^3 P_{g_1}^2}\right\} Ai\left(\frac{q_x}{2\omega_{0x} P_{g_1}} + \frac{1}{16\omega_{0x}^4 P_{g_1}^2}\right) \\
&\times \sum_{g_2=1}^{N} A_{g_2} \left(\frac{1}{P_{g_2}}\right)^{1/2} \exp\left\{\frac{q_y^2}{4P_{g_2}} + \frac{1}{96\omega_{0y}^6 P_{g_2}^3} + \frac{q_y}{8\omega_{0y}^3 P_{g_2}^2}\right\} Ai\left(\frac{q_y}{2\omega_{0y} P_{g_2}} + \frac{1}{16\omega_{0y}^4 P_{g_2}^2}\right),
\end{aligned}
\tag{20}
$$

where
$$
\begin{cases}
P_{h_1} = \dfrac{B_{h_1}}{a_x^2} + \dfrac{ikA}{2B} \\[4pt]
P_{h_2} = \dfrac{B_{h_2}}{a_y^2} + \dfrac{ikA}{2B} \\[4pt]
P_{g_1} = \dfrac{B_{g_1}}{b_x^2} + \dfrac{ikA}{2B} \\[4pt]
P_{g_2} = \dfrac{B_{g_2}}{b_y^2} + \dfrac{ikA}{2B}
\end{cases}
\text{and}
\begin{cases}
q_x = \dfrac{a}{\omega_{0x}} + \dfrac{ik}{B} x_2 \\[6pt]
q_y = \dfrac{b}{\omega_{0y}} + \dfrac{ik}{B} y_2
\end{cases}.
$$

Equation (20) gives the propagation of a Finite ordinary Airy beam through an ABCD optical system apertured by a rectangular annular aperture. It is in agreement with the main result established by Equation (15) of the investigation developed by Ez-zariy *et al.* [24] on the transformation of the two-dimensional finite Airy beams by an ABCD optical system truncated by a rectangular annular aperture.

4.1.2. Propagation of Finite Airy Beam through a Circular Annular Aperture

The propagation of a Finite ordinary Airy beam passing through a paraxial ABCD optical system with circular annular aperture is examined as a special case when $n = 0$

$$
\begin{aligned}
U_2(x_2, y_2, z) = &\ C' \sum_{h=1}^{N} A_h \left[\frac{1}{P_R} \exp\left\{\frac{q_x^2 + q_y^2}{4P_R}\right\} \exp\left\{\frac{1}{96\omega_{0x}^6 P_R^3} + \frac{q_x}{8\omega_{0x}^3 P_R^2}\right\} \exp\left\{\frac{1}{96\omega_{0y}^6 P_R^3} + \frac{q_y}{8\omega_{0y}^3 P_R^2}\right\} \right. \\
&\times Ai\left(\frac{q_x}{2\omega_{0x} P_R} + \frac{1}{16\omega_{0x}^4 P_R^2}\right) Ai\left(\frac{q_y}{2\omega_{0y} P_R} + \frac{1}{16\omega_{0y}^4 P_R^2}\right) - \frac{1}{P_r} \exp\left\{\frac{q_x^2 + q_y^2}{4P_r}\right\} \exp\left\{\frac{1}{96\omega_{0x}^6 P_r^3} + \frac{q_x}{8\omega_{0x}^3 P_r^2}\right\} \\
&\left. \times \exp\left\{\frac{1}{96\omega_{0y}^6 P_r^3} + \frac{q_y}{8\omega_{0y}^3 P_r^2}\right\} Ai\left(\frac{q_x}{2\omega_{0x} P_r} + \frac{1}{16\omega_{0x}^4 P_r^2}\right) Ai\left(\frac{q_y}{2\omega_{0y} P_r} + \frac{1}{16\omega_{0y}^4 P_r^2}\right) \right],
\end{aligned}
\tag{21}
$$

$$\text{where } \begin{cases} P_R = \dfrac{B_h}{R^2} + \dfrac{ikA}{2B} \\ P_r = \dfrac{B_h}{r^2} + \dfrac{ikA}{2B} \end{cases}, \text{ and } \begin{cases} q_x = \dfrac{a}{\omega_{0x}} + \dfrac{ik}{B}x_2 \\ q_y = \dfrac{b}{\omega_{0y}} + \dfrac{ik}{B}y_2 \end{cases}.$$

Equation (21) is obtained from Equation (17) and gives the output electrical field of a Finite Airy beam through an ABCD optical system truncated by a circular annular aperture. This equation is similar to Equation (9) regarded as the principle finding obtained by Wen *et al.* [5] about a Finite Airy beam traveling a paraxial ABCD optical system with circular annular aperture.

4.2. Special Cases Concerning the Rectangular Annular Aperture

4.2.1. FOBs through an ABCD Optical System with a Rectangular Aperture

In this particular case, we treat the transformation of a FOBs passing through a paraxial ABCD optical system with rectangular aperture, by choosing $b_x = b_y \to 0$. The form of Equation (10) reduces to

$$U_2(x_2, y_2, z) = C' \sum_{h_1=1}^{N} A_{h_1} \left(\frac{1}{P_{h_1}}\right)^{1/2} \exp\left\{\frac{q_x^2}{4P_{h1}} + \frac{1}{96\omega_{0x}^6 P_{h_1}^3} + \frac{q_x}{8\omega_{0x}^3 P_{h_1}^2}\right\} O_n\left(\frac{q_x}{2\omega_{0x} P_{h_1}} + \frac{1}{16\omega_{0x}^4 P_{h_1}^2}\right)$$

$$\times \sum_{h_2=1}^{N} A_{h_2} \left(\frac{1}{P_{h_2}}\right)^{1/2} \exp\left\{\frac{q_y^2}{4P_{h_2}} + \frac{1}{96\omega_{0y}^6 P_{h_2}^3} + \frac{q_y}{8\omega_{0y}^3 P_{h_2}^2}\right\} O_n\left(\frac{q_y}{2\omega_{0y} P_{h_2}} + \frac{1}{16\omega_{0y}^4 P_{h_2}^2}\right), \tag{22}$$

with

$$\begin{cases} P_{h_1} = \dfrac{B_{h_1}}{a_x^2} + \dfrac{ikA}{2B} \\ P_{h_2} = \dfrac{B_{h_2}}{a_y^2} + \dfrac{ikA}{2B}, \\ P_{g_1} = P_{g_2} \to \infty \end{cases}$$

and

$$\begin{cases} q_x = \dfrac{a}{\omega_{0x}} + \dfrac{ik}{B}x_2 \\ q_y = \dfrac{b}{\omega_{0y}} + \dfrac{ik}{B}y_2 \end{cases}.$$

4.2.2. FOBs through an ABCD Optical System with a Rectangular Black Screen

This case is obtained when $a_x = a_y \to \infty$. In these conditions, the parameters established in Equation (12) and Equation (13) could be changed to

$$\text{where } \begin{cases} P_{h_1} = P_{h_2} = \dfrac{ikA}{2B} = P \\ P_{g_1} = \dfrac{B_{g_1}}{b_x^2} + \dfrac{ikA}{2B} \\ P_{g_2} = \dfrac{B_{g_2}}{b_y^2} + \dfrac{ikA}{2B} \end{cases}, \text{ and } \begin{cases} q_x = \dfrac{a}{\omega_{0x}} + \dfrac{ik}{B}x_2 \\ q_y = \dfrac{b}{\omega_{0y}} + \dfrac{ik}{B}y_2 \end{cases}.$$

Under the above conditions, the rectangular annular aperture reduces to a rectangular black screen and the transformation of FOBs passing through a paraxial ABCD optical system with rectangular black screen is characterized by the following field

$$U_2(x_2, y_2, z) = \frac{C'}{P} \exp\left\{\frac{q_x^2 + q_y^2}{4P} + \frac{q_x}{8\omega_{0x}^3 P^2} + \frac{q_y}{8\omega_{0y}^3 P^2} + \frac{1}{96\omega_{0x}^6 P^3} + \frac{1}{96\omega_{0x}^6 P^3}\right\}$$

$$\times O_n\left(\frac{q_x}{2\omega_{0x}P} + \frac{1}{16\omega_{0x}^4 P^2}\right) O_n\left(\frac{q_y}{2\omega_{0y}P} + \frac{1}{16\omega_{0y}^4 P^2}\right)$$

$$-C'\sum_{g_1=1}^{N} A_{g_1}\left(\frac{1}{P_{g_1}}\right)^{1/2} \exp\left\{\frac{q_x^2}{4P_{g_1}} + \frac{1}{96\omega_{0x}^6 P_{g_1}^3} + \frac{q_x}{8\omega_{0x}^3 P_{g_1}^2}\right\} O_n\left(\frac{q_x}{2\omega_{0x}P_{g_1}} + \frac{1}{16\omega_{0x}^4 P_{g_1}^2}\right)$$

$$\times \sum_{g_2=1}^{N} A_{g_2}\left(\frac{1}{P_{g_2}}\right)^{1/2} \exp\left\{\frac{q_y^2}{4P_{g_2}} + \frac{1}{96\omega_{0y}^6 P_{g_2}^3} + \frac{q_y}{8\omega_{0y}^3 P_{g_2}^2}\right\} O_n\left(\frac{q_y}{2\omega_{0y}P_{g_2}} + \frac{1}{16\omega_{0y}^4 P_{g_2}^2}\right).$$

(23)

When the Finite Olver beam order equal to zero, this beams family becomes a Finite Airy beam and in this situation one obtains a formula similar to that elaborated by Equation (18) of Ref. [24].

4.2.3. FOBs through an Unapertured ABCD Optical System

An unapertured ABCD optical system conducted under the condition that $a_x = a_y \to \infty$ and $b_x = b_y \to 0$. In this case, we have

$$U_2(x_2, y_2, z) = \frac{C'}{P_1} \exp\left\{\frac{q_x^2 + q_y^2}{4P} + \frac{q_x}{8\omega_{0x}^3 P^2} + \frac{q_y}{8\omega_{0y}^3 P^2} + \frac{1}{96\omega_{0x}^6 P^3} + \frac{1}{96\omega_{0x}^6 P^3}\right\}$$

$$\times O_n\left(\frac{q_x}{2\omega_{0x}P} + \frac{1}{16\omega_{0x}^4 P^2}\right) O_n\left(\frac{q_y}{2\omega_{0y}P} + \frac{1}{16\omega_{0y}^4 P^2}\right)$$

(24)

where $\begin{cases} P_{h_1} = P_{h_2} = \dfrac{ikA}{2B} = P \\ P_{g_1} = P_{g_2} \to \infty \end{cases}$, and $\begin{cases} q_x = \dfrac{a}{\omega_{0x}} + \dfrac{ik}{B}x_2 \\ q_y = \dfrac{b}{\omega_{0y}} + \dfrac{ik}{B}y_2 \end{cases}$.

This result can be compared with that given in Ref. [10].

4.3. Special Cases Concerning the Circular Annular Aperture

4.3.1. FOBs through an ABCD Optical System with Circular Aperture

In this case, we treat the transformation of a Finite Olver beam passing through a paraxial ABCD optical system coupled with a circular aperture. This case is obtained when $r \to 0$ and the form of Equation (15) reduces to

$$U_2(x_2, y_2, z) = C'\sum_{h=1}^{N} A_h\left[\frac{1}{P_R}\exp\left\{\frac{q_x^2 + q_y^2}{4P_R}\right\}\exp\left\{\frac{1}{96\omega_{0x}^6 P_R^3} + \frac{q_x}{8\omega_{0x}^3 P_R^2}\right\}\exp\left\{\frac{1}{96\omega_{0y}^6 P_R^3} + \frac{q_y}{8\omega_{0y}^3 P_R^2}\right\}\right.$$

$$\left.\times O_n\left(\frac{q_x}{2\omega_{0x}P_R} + \frac{1}{16\omega_{0x}^4 P_R^2}\right) O_n\left(\frac{q_y}{2\omega_{0y}P_R} + \frac{1}{16\omega_{0y}^4 P_R^2}\right)\right],$$

(25)

with $\begin{cases} P_R = \dfrac{B_h}{R^2} + \dfrac{ikA}{2B} \\ P_r \to \infty \end{cases}$, and $\begin{cases} q_x = \dfrac{a}{\omega_{0x}} + \dfrac{ik}{B}x_2 \\ q_y = \dfrac{b}{\omega_{0y}} + \dfrac{ik}{B}y_2 \end{cases}$.

This analytical expression characterizes the propagation of a Finite Olver beam through a circular aperture followed by an ABCD optical system.

4.3.2. FOBs through an ABCD Optical System with Circular Black Screen

Under this condition $R \to \infty$, and by the use of Equation (17), the transformation of the FOBs by a paraxial

ABCD optical system with circular black screen is given by

$$
\begin{aligned}
U_2\left(x_2, y_2, z\right) =& \frac{C'}{P}\exp\left\{\frac{q_x^2+q_y^2}{4P}+\frac{q_x}{8\omega_{0x}^3 P^2}+\frac{q_y}{8\omega_{0y}^3 P^2}+\frac{1}{96\omega_{0x}^6 P^3}+\frac{1}{96\omega_{0y}^6 P^3}\right\} \\
&\times O_n\left(\frac{q_x}{2\omega_{0x}P}+\frac{1}{16\omega_{0x}^4 P^2}\right)O_n\left(\frac{q_y}{2\omega_{0y}P}+\frac{1}{16\omega_{0y}^4 P^2}\right) \\
&-C'\sum_{h=1}^{N}\left[\frac{A_h}{P_r}\exp\left\{\frac{q_x^2+q_y^2}{4P_r}\right\}\exp\left\{\frac{1}{96\omega_{0x}^6 P_r^3}+\frac{q_x}{8\omega_{0x}^3 P_r^2}\right\}\exp\left\{\frac{1}{96\omega_{0y}^6 P_r^3}+\frac{q_y}{8\omega_{0y}^3 P_r^2}\right\}\right. \\
&\left.\times O_n\left(\frac{q_x}{2\omega_{0x}P_r}+\frac{1}{16\omega_{0x}^4 P_r^2}\right)O_n\left(\frac{q_y}{2\omega_{0y}P_r}+\frac{1}{16\omega_{0y}^4 P_r^2}\right)\right],
\end{aligned}
\tag{26}
$$

with $\begin{cases}P_R=\dfrac{ikA}{2B}=P\\ P_r=\dfrac{B_h}{r^2}+\dfrac{ikA}{2B}\end{cases}$, and $\begin{cases}q_x=\dfrac{a}{\omega_{0x}}+\dfrac{ik}{B}x_2\\ q_y=\dfrac{b}{\omega_{0y}}+\dfrac{ik}{B}y_2\end{cases}.$

4.3.3. FOBs through an Unapertured ABCD Optical System

An unapertured ABCD optical system is conducted under the condition that $R\to\infty$ and $r\to 0$. Under these conditions, the output electrical field of the FOBs through an ABCD optical system is given by

$$
\begin{aligned}
U_2\left(x_2, y_2, z\right) =& \frac{C'}{P}\exp\left\{\frac{q_x^2+q_y^2}{4P}+\frac{q_x}{8\omega_{0x}^3 P^2}+\frac{q_y}{8\omega_{0y}^3 P^2}+\frac{1}{96\omega_{0x}^6 P^3}+\frac{1}{96\omega_{0y}^6 P^3}\right\} \\
&\times O_n\left(\frac{q_x}{2\omega_{0x}P}+\frac{1}{16\omega_{0x}^4 P^2}\right)O_n\left(\frac{q_y}{2\omega_{0y}P}+\frac{1}{16\omega_{0y}^4 P^2}\right)\Big],
\end{aligned}
\tag{27}
$$

with $\begin{cases}P_R=\dfrac{ikA}{2B}=P\\ P_r\to\infty\end{cases}$, and $\begin{cases}q_x=\dfrac{a}{\omega_{0x}}+\dfrac{ik}{B}x_2\\ q_y=\dfrac{b}{\omega_{0y}}+\dfrac{ik}{B}y_2\end{cases}.$

This result can be compared with that given in Equation (24) and also with the principle finding of Ref. [10].

5. Numerical Simulation Examples

5.1. Rectangular Annular Aperture

In order to investigate the effects of the rectangular (or circular) annular aperture on propagation properties of the FOBs through an ABCD optical system, some numerical simulation examples are performed in this paper using the above analytical results elaborated in Equation (10) and Equation (17) concerning the rectangular annular aperture and the circular one, respectively.

The normalized intensity distribution of the FOBs through an annular aperture followed by a free space is calculated numerically at receiver plane situated at propagation distance z. The parameters chosen in the simulations, are $\lambda=632.8\,\text{nm}$, $a=0.01\,\text{mm}$ and $b=0.01\,\text{mm}$. The transfer matrix corresponding to the free space is established by

$$
\begin{pmatrix} A & B \\ C & D \end{pmatrix}=\begin{pmatrix} 1 & z \\ 0 & 1 \end{pmatrix},
\tag{28}
$$

where z denotes the distance between the input and output plans of the optical system.

In **Figure 3**, we display the normalized intensity distribution of the zeroth-order FOBs passing through an ABCD optical system with rectangular aperture. This study is investigated for making a comparison of three-dimensional behavior for different inner dimensional half width in x and y directions which are illustrated in the following way: **Figure 3(A)**: $a_x = a_y = 10 \, \text{mm}$ is fixed and the parameter $b_x = b_y$ is varied as: (a): $b_x = b_y = 0.01 \, \text{mm}$, (b): $b_x = b_y = 2 \, \text{mm}$, (c): $b_x = b_y = 8 \, \text{mm}$ and (d) $b_x = b_y = 9 \, \text{mm}$ In **Figure 3(B)**, one fixes the outer parameters of the used aperture in x and y direction as $b_x = b_y = 2 \, \text{mm}$, then we change the value of the outer half widths as: (a): $a_x = a_y = 3 \, \text{mm}$, (b): $a_x = a_y = 5 \, \text{mm}$, (c): $a_x = a_y = 8 \, \text{mm}$ and (d) $a_x = a_y = 10 \, \text{mm}$ For different propagation distances z in **Figure 3(C)**, the outer and inner half widths are fixed at $a_x = a_y = 10 \, \text{mm}$ and $b_x = b_y = 4 \, \text{mm}$, and with change of z distance propagation as (a): $z = 200 \, \text{mm}$, (b): $z = 500 \, \text{mm}$, (c): $z = 1000 \, \text{mm}$ and (d): $z = 2000 \, \text{mm}$. For the same study, we display in **Figure 4** the normalized transverse intensity distribution of the FOBs in second order (n = 2). The calculations parameters are: $\lambda = 1632.8 \, \text{nm}$ and $\omega_{0x} = \omega_{0y} = 0.8 \, \text{mm}$.

5.2. Circular Annular Aperture

In order to study the propagation characteristics of FOBs for various orders ($n = 0, 1, 2, \cdots$) passing through an annular aperture followed by a free space expressed by the transfer matrix elements given in Equation (27), we will make some numerical calculations for the approximate analytical expression given by Equation (17), which are presented in **Figure 5** and **Figure 6** obtained by the choice of the same parameters given in the previous pa-
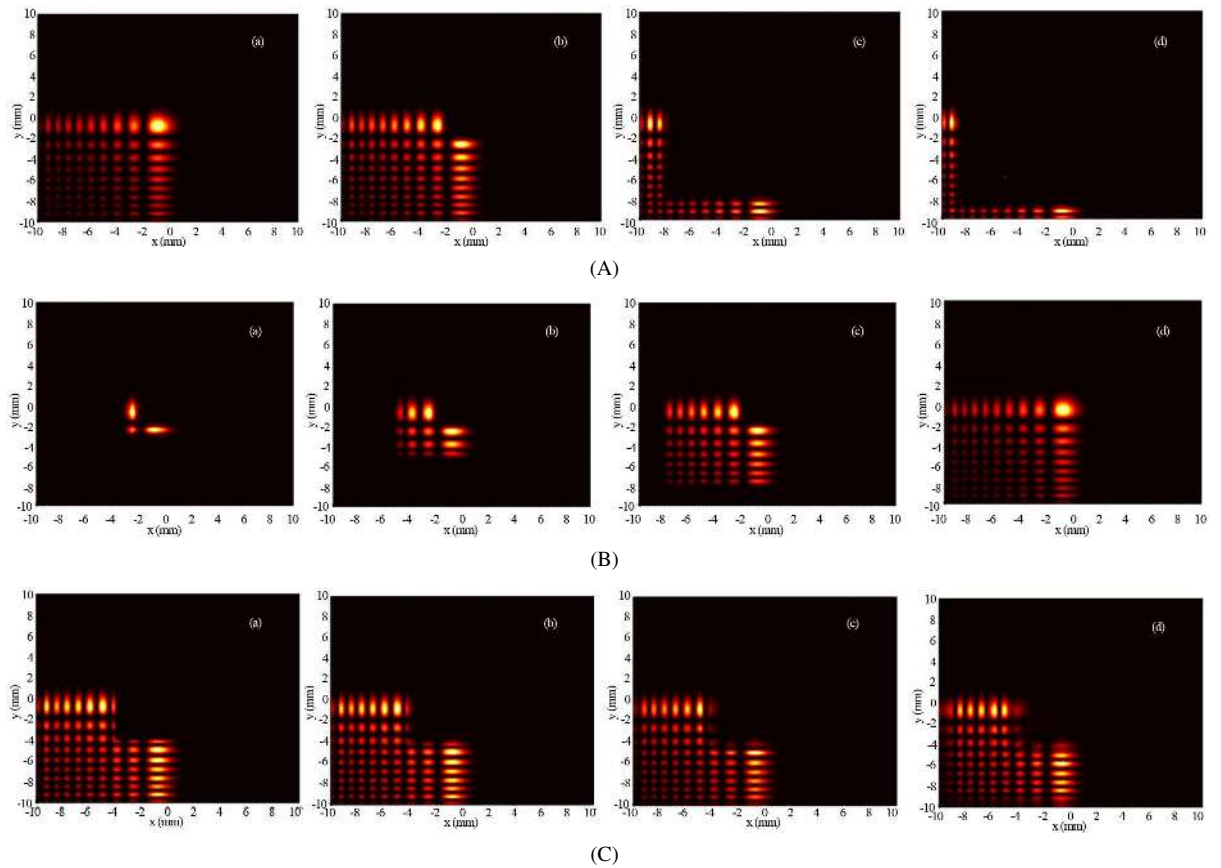

(A)

(B)

(C)

Figure 3. Computer simulated transverse intensity distribution of zeroth-order of FOBs passing through a rectangular annular aperture, with: (A) $a_x = a_y = 10 \, \text{mm}$ and (a): $b_x = b_y = 0.01 \, \text{mm}$; (b): $b_x = b_y = 2 \, \text{mm}$; (c): $b_x = b_y = 8 \, \text{mm}$ and (d): $b_x = b_y = 9 \, \text{mm}$ at $z = 0.5 \, \text{m}$; (B) $b_x = b_y = 2 \, \text{mm}$ and (a): $a_x = a_y = 3 \, \text{mm}$; (b): $a_x = a_y = 5 \, \text{mm}$; (c): $a_x = a_y = 8 \, \text{mm}$ and (d): $a_x = a_y = 10 \, \text{mm}$ at $z = 0.5 \, \text{m}$; (C) $a_x = a_y = 10 \, \text{mm}$, $b_x = b_y = 2 \, \text{mm}$ and different z (a) $z = 0.2 \, \text{m}$; (b): $z = 0.5 \, \text{m}$; (c): $z = 1 \, \text{m}$ and (d): $z = 2 \, \text{m}$.
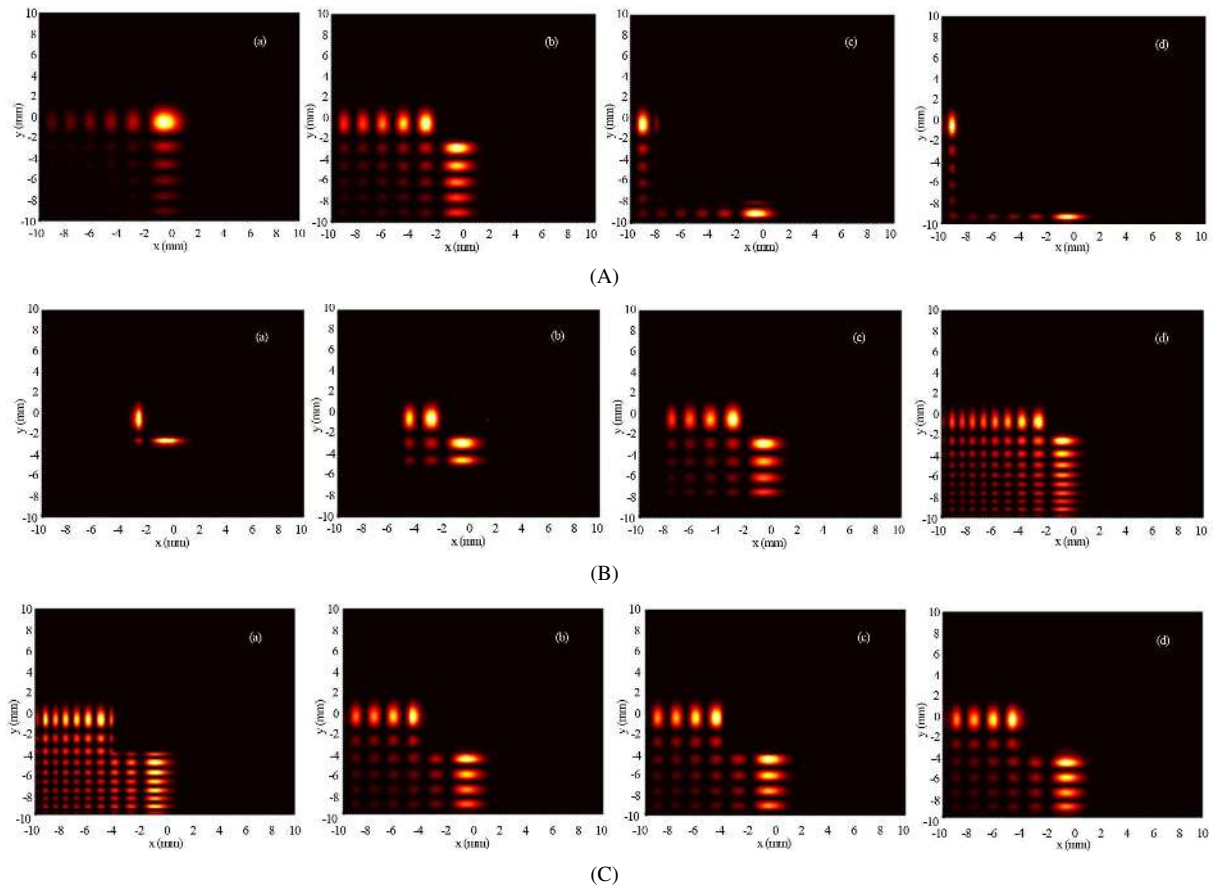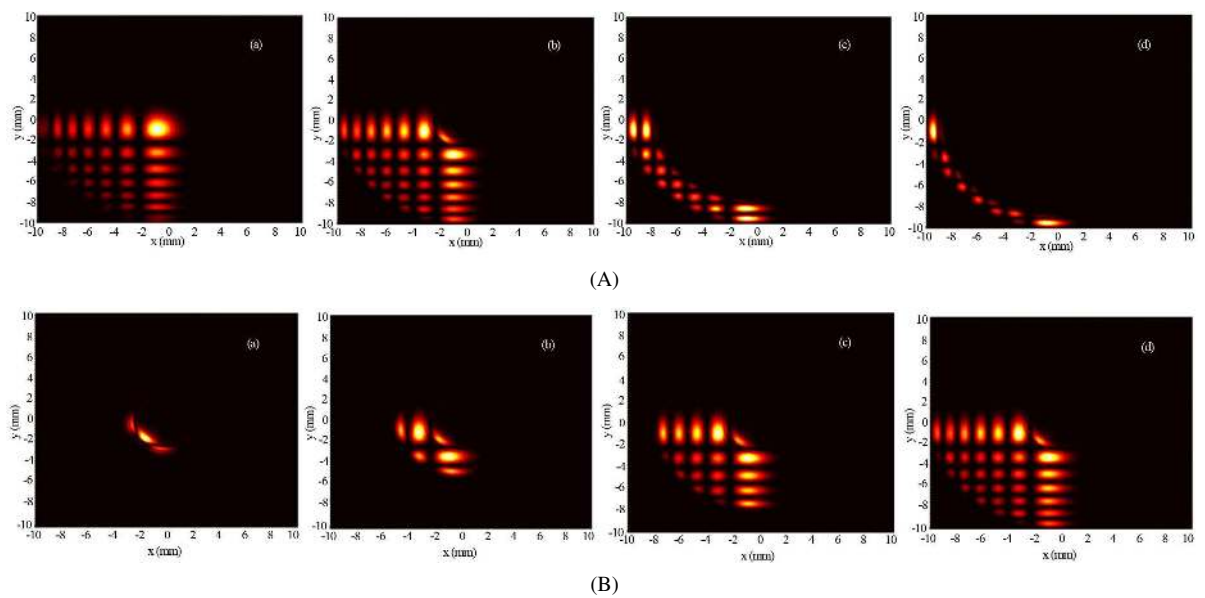
(A)

(B)

(C)

Figure 4. Computer simulated of transverse intensity distribution of FOBs for the second order ($n = 2$) passing through a rectangular aperture, with: (A) $a_x = a_y = 10\,\text{mm}$ and (a): $b_x = b_y = 0.01\,\text{mm}$; (b): $b_x = b_y = 2\,\text{mm}$; (c): $b_x = b_y = 8\,\text{mm}$ and (d): $b_x = b_y = 9\,\text{mm}$ at $z = 0.5\text{m}$; (B) $b_x = b_y = 2\,\text{mm}$ and (a): $a_x = a_y = 3\,\text{mms}$; (b): $a_x = a_y = 5\,\text{mm}$; (c): $a_x = a_y = 8\,\text{mm}$ and (d): $a_x = a_y = 10\,\text{mm}$ at $z = 0.5\,\text{m}$; (C) $a_x = a_y = 10\,\text{mm}$, $b_x = b_y = 2\,\text{mm}$ and different z (a) $z = 0.2\,\text{m}$; (b): $z = 0.5\,\text{m}$; (c): $z = 1\,\text{m}$ and (d): $z = 2\,\text{m}$.
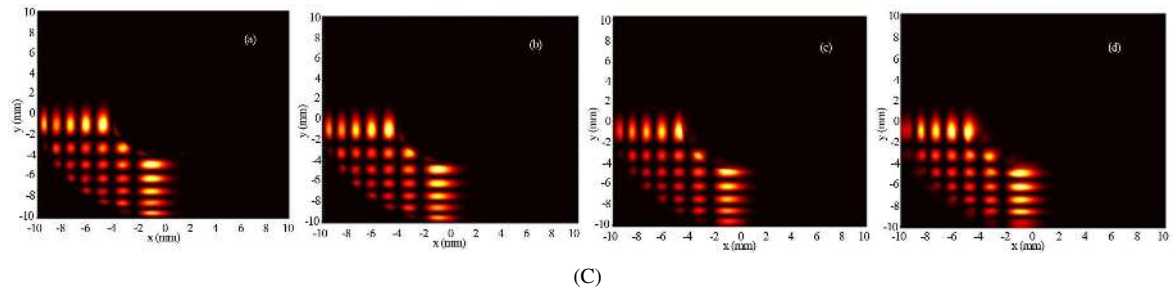

(A)

(B)

(C)

Figure 5. Computer simulated transverse intensity distribution of zeroth-order of FOBs passing through a circular annular aperture: (A) $R = 10\,\text{mm}$ as: (a): $r = 0.01\,\text{mm}$; (b): $r = 2\,\text{mm}$; (c): $r = 8\,\text{mm}$ and (d): $r = 9\,\text{mm}$; (B) $r = 2\,\text{mm}$ as: (a): $R = 3\,\text{mm}$; (b): $R = 5\,\text{mm}$; (c): $R = 8\,\text{mm}$, and (d): $R = 10\,\text{mm}$; (C) $R = 10\,\text{mm}$ and $r = 2\,\text{mm}$ for different z (a) $z = 0.2\,\text{m}$; (b): $z = 0.5\,\text{m}$; (c): $z = 1\,\text{m}$ and (d): $z = 2\,\text{m}$.
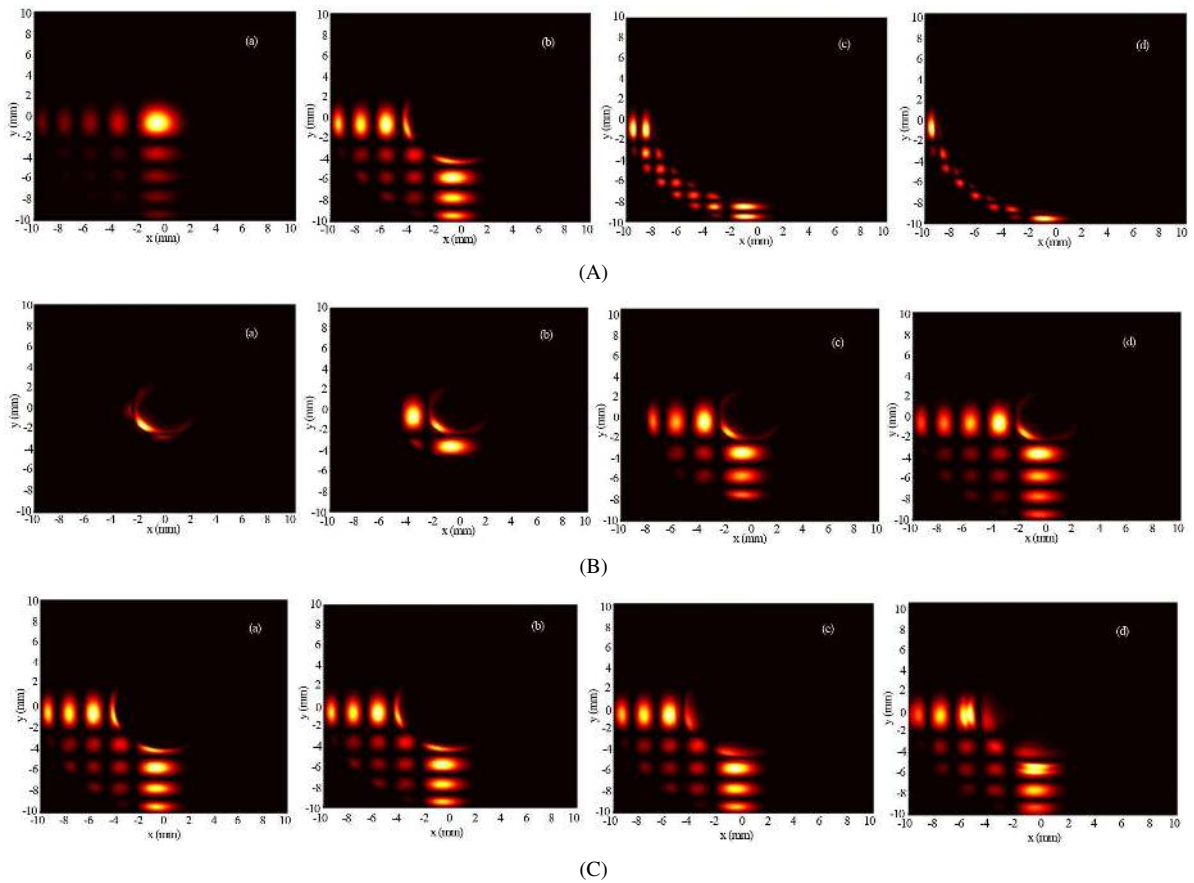

(A)

(B)

(C)

Figure 6. Computer simulated transverse intensity distribution of FOBs for the second order ($n = 2$) passing through a circular annular aperture, and (A) $R = 10\,\text{mm}$ as: (a): $r = 0.01\,\text{mm}$; (b): $r = 2\,\text{mm}$; (c): $r = 8\,\text{mm}$ and (d): $r = 9\,\text{mm}$; (B) $r = 2\,\text{mm}$ as: (a): $R = 3\,\text{mm}$; (b): $R = 5\,\text{mm}$; (c): $R = 8\,\text{mm}$, and (d): $R = 10\,\text{mm}$; (C) $R = 10\,\text{mm}$ and $r = 2\,\text{mm}$ as: (a) $z = 0.2\,\text{m}$; (b): $z = 0.5\,\text{m}$; (c): $z = 1\,\text{m}$ and (d): $z = 2\,\text{m}$.

ragraph as $z = 500\,\text{mm}$, $\lambda = 632.8\,\text{nm}$ and $\omega_{0x} = \omega_{0y} = 0.8\,\text{mm}$ with different values of the outer-radius R and the inner-radius r characterizing the form of the annular aperture. For making a comparison in the behavior of the three-dimensional intensity distribution at different propagation distance z of the Finite Olver beams passing through an ABCD optical system with annular aperture shown below in the figures. The representation of the FOBs of zero and first orders as **Figure 5(A)**: The outer radius is fixed at $R = 10\,\text{mm}$ is fixed and the

inner radius r is varied as (a): $r = 0.01\,\text{mm}$, (b): $r = 2\,\text{mm}$, (c): $r = 8\,\text{mm}$ and (d) $r = 9\,\text{mm}$ In **Figure 5(B)**, one fixes the inner radius of the used aperture at $r = 2\,\text{mm}$. Then, we change the value of the outer radius R: (a): $R = 3\,\text{mm}$, (b): $R = 5\,\text{mm}$, (c): $R = 8\,\text{mm}$ and (d) $R = 10\,\text{mm}$ For different propagation distances z in **Figure 5(C)** the other parameters are chosen as $R = 10\,\text{mm}$ and $R = 4\,\text{mm}$, and consequently changing the distance z as (a): $z = 200\,\text{mm}$, (b): $z = 500\,\text{mm}$, (c): $z = 1000\,\text{mm}$ and (d): $z = 2000\,\text{mm}$. For the same study, we display in **Figure 6** the normalized transverse intensity distribution of FOBs of second order ($n = 2$).

These figures show that there is a change in profile on the intensity distribution, which allows us to understand the transformation of the propagation of this Finite Olver beams family by changing the annular aperture size or the propagation distance in free space.

6. Conclusion

In this work, the approximate analytical representations of the Finite Olver beams passing through a paraxial ABCD optical system with rectangular annular and circular one are obtained. The method used in this paper allows us to understand the mechanism of the propagation of the Finite Olver beams by changing the aperture parameters as its size is perfected to the very small size of apertures where the real diffraction experiment cannot be performed easily. The normalized intensity distribution of this beams family in free space is graphically illustrated with numerical examples for analyzing the behavior of this beams class. Moreover, our method used in this work can be applied to other non-diffracting laser beams.

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