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INTER-GROUP IMAGE REGISTRATION BY HIERARCHICAL GRAPH SHRINKAGE

Shihui Ying[1,2], **Guorong Wu**[1], **Shu Liao**[1], and **Dinggang Shen**[1,*]

[1]Department of Radiology and BRIC, University of North Carolina at Chapel Hill, NC 27599, USA

[2]Department of Mathematics, School of Science, Shanghai University, Shanghai 200444, China

Abstract

In this paper, we propose a novel inter-group image registration method to register different groups of images (e.g., young and elderly brains) simultaneously. Specifically, we use a hierarchical two-level graph to model the distribution of entire images on the manifold, with intra-graph representing the image distribution in each group and the inter-graph describing the relationship between two groups. Then the procedure of inter-group registration is formulated as a dynamic evolution of graph shrinkage. The advantage of our method is that the topology of entire image distribution is explored to guide the image registration. In this way, each image coordinates with its neighboring images on the manifold to deform towards the population center, by following the deformation pathway simultaneously optimized within the graph. Our proposed method has been also compared with other state-of-the-art inter-group registration methods, where our method achieves better registration results in terms of registration accuracy and robustness.

Index Terms

Inter-group image registration; graph shrinkage; topology preservation; diffeomorphism

1. INTRODUCTION

Recently, image analysis upon large population dataset becomes more and more popular in many neuroscience and clinical studies such as brain development, aging, and disease-induced abnormality [1, 2]. Catering for this requirement, many state-of-the-art groupwise registration methods have been proposed in last decade, which emerges as an unbiased approach to simultaneously align all images towards the population center [1–7].

Although the problem of groupwise registration has been widely investigated, few attentions have been paid for the registration problem between two or more image groups. For example, estimating the deformation from one group to another group is essential to characterize brain development or identify disease-related brain regions. Current methods usually first estimate the group-mean image of each group and then perform pairwise registration between two group-mean images [8]. However, there are several limitations of this kind of approach. *First*, the group-mean image of each group by simple averaging is usually very fuzzy, thus losing a lot of anatomical details compared with the individual subjects. Consequently, it is challenging to register two group-mean images both with fuzzy image contents. *Second*, although the recently proposed method [8] addressed this issue by using multi-channel registration method to encode not only intensity of group-mean image

*Corresponding author. dgshen@med.unc.edu.

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but also the intensity variances of all aligned subjects within the same group, the estimation of group-mean within each group and the inter-group registration are performed independently. Thus, the registration errors in the first step will be unavoidably propagated to the second step, which will affect the final registration performance between two groups. *Third*, current inter-group registration methods can only deal with two groups by registering all images in one group to the domain of group-mean image in the other group, regardless of the space anatomical differences between two group-mean images. However, it is common that some individual images in one group are much easier to register with the images in another group, due to their similar anatomical structures, which can be used as intermediate milestones to guide the estimation of deformation pathway from one group to another group.

In this paper, we propose to use the hierarchical graph to model the complex distribution of entire images which have already been partitioned into different groups based on clinical criterion (e.g., age and gender). Specifically, we first construct the intra-group graph for images in each group by adaptively setting threshold on all possible pairwise image distances. Then, the connections between two constructed intra-graphs are set up according to the distances between images in two graphs, resulting in the inter-group graphs. After obtaining the two-level graph with the intra-graph representing the image distribution in each group and the inter-graph describing the relationship between any two groups, we formulate the inter-group registration problem as the dynamic evolution of graph shrinkage, where each image (graph node) deforms along the graph edge until meets each other at the population center. Particularly, the advantages of our graph based inter-group registration method include: (1) registration error can be reduced by deforming each image only w.r.t. locally connected images in the graph, which have similar anatomical structures; (2) the topology of entire image distribution will not be changed during registration since each image consistently deforms along the graph edge. As we will demonstrate in the experiments, promising registration results have been achieved by our novel graph-based registration method, by comparison with other state-of-the-art counterpart methods.

2. METHODS

In this section, we present our graph-based inter-group registration method as follows.

2.1. Overview of Our Graph-based Registration Method

Here, we consider all images $\boldsymbol{I} = \{I_i | \, i=1,\ldots,N\}$ sitting on the high-dimensional manifold. Then we construct a two-level graph to model the distribution of all images from different groups (Section 2.2). Given the graph, we further regard the registration of each individual image to the population center as a dynamic procedure of time variable t ($0 < t < \infty$). Thus, we use $I_i(t)$ to represent the warped image I_i at time t with $I_i(0) = I_i$. To model the distribution of images, we introduce a dynamic graph $\mathcal{G}(t) = \{I(t), E, V(t)\}$ on the image manifold, where $I(t) = \{I_i(t) | i=1,\ldots,N\}$ denotes the deformed graph nodes at time t and $E = \{e_{ij} | i,j = 1,\cdots,N\}$ denotes the graph edge, respectively. $e_{ij} = 1$ if $I_i(t)$ and $I_j(t)$ are connected, and 0 otherwise. Since we do not allow self-loop in the graph, we further define $e_{ii} = 0$ for all $i=1,\cdots,N$. We also define a weighted adjacency matrix $V(t) = (\exp(v_{ij}(t)))_{N\times N}$ to represent the geodesic distance between two images, where $v_{ij}(t)$ is the velocity vector of the geodesic from $I_i(t)$ to $I_j(t)$ at $I_i(t)$, and 'exp' denotes exponential mapping [9]. The energy function in our graph-base registration method is defined as follows:

$$F(t)=\sum_{i,j=1}^{N} e_{ij}\|v_{ij}(t)\|^2. \quad (1)$$

The principle behind Eq. (1) is demonstrated in Fig. 1. The topology of their distribution is described by the graph, where the graph edges denote the local connectivity between graph nodes. Specifically, the velocity vector $v_{ij}(t)$ is associated with each graph edge, where the integration along $v_{ij}(t)$ forms the geodesic distance from $I_i(t)$ to $I_j(t)$. Thus, the minimization of $F(t)$ can be regarded as a dynamic graph shrinking procedure, which deforms each image from $I_i(t)$ to $I_i(t+\Delta t)$ with the decreased overall geodesic distance, while keeping the topology of the entire graph. As time t increases, all $I_i(t)$ s are supposed to meet at the population center, with the properly determined velocity vector $v_{ij}(t)$ and the time increment Δt (Section 2.3).

2.2. Graph Construction

Since the final goal of our method is to register all images from different groups to the common space, it is straightforward to construct the two-level graph for all images from different groups, where the intra-graph represents the distribution of images in the same group and the inter-graph encodes the relationship between intra-graphs. The idea of constructing the two-level graph is displayed in Fig. 2 where we use three groups as example (solid lines and dashed lines represent the edges of the intra-graphs and the inter-group graph, respectively). Next, we will explain the method for constructing intra- and inter-graph.

Intra-group Graph—Assume that the whole dataset $\boldsymbol{I}$ has M groups. For each group, we apply the following steps to construct the intra-group graph: (1) for each image I_i in group S_m (m=1,…,M, and $\cup_{m=1}^{M} S_m = \boldsymbol{I}$), we calculate the distance w.r.t. all other images I_j in $\boldsymbol{I}$, where we use the geodesic distance estimated from Log-Demons method [10] as the measurement. Note, if I_i and I_j belongs to two different groups, we set the distance to infinity. (2) We adaptively determine the threshold h which is the smallest degree to make every node in the intra-graph of S_m having at least one connection. (3) We construct the intra-group by removing the connections with its geodesic distance obtained in Step (1) which is larger than h, i.e., $e_{ij}^{m}=1$ if the geodesic distance is smaller than h. Otherwise $e_{ij}^{m}=0$.

Inter-group Graph—To build the connection between two groups, we exhaustively calculate the geodesic distance of each possible pair of graph node I_i in one intra-group graph w.r.t graph node I_j in another intra-group graph and select k pairs with p-smallest distance (p= 30 in our paper). Then the inter-group edge $e_{ij}^{inter}=1$ if the distance between I_i and I_j (I_i and I_j belong to different groups) is top p smallest. Otherwise $e_{ij}^{inter}=0$. Eventually, all graph edges e_{ij} (both e_{ij}^{m} and e_{ij}^{inter}) in $\mathcal{G}(t)$ (Section 2.1) can be calculated by:

$$e_{ij}=\begin{cases} e_{ij}^{m} & I_i \in S_m \text{ and } I_j \in S_m \\ e_{ij}^{inter} & \text{otherwise} \end{cases} \quad (2)$$

2.3 Graph Shrinkage

As we formulate the problem of groupwise registration as the dynamic shrinkage of graph, it is critical to determine the deformation of each image $I_i(t)$ at time t, which can reduce the energy function $F(t)$ in Eq. (1). Based on the local connectivity of each node $I_i(t)$ in the graph, it is reasonable to move $I_i(t)$ along the average direction according to its connected nodes. Since the velocity vector sits on the tangent space of $I_i(t)$ on the manifold, it can be efficiently calculated by linear averaging as:

$$\hat{v}_i(t)=\frac{1}{N_i}\sum_{j=1}^{N}e_{ij}v_{ij}(t). \quad (3)$$

where $N_i=\sum_{j=1}^{N}e_{ij}$ is the number of connections for $I_i(t)$. Next, we will prove that such velocities make our objective function $F(t)$ monotone decreasing. First, the increment of the objective function $F(t)$ from time t to $t+\Delta t$ is:

$$\begin{aligned}\Delta F(t;\Delta t)&=F(t+\Delta t)-F(t)=\textstyle\sum_{i,j=1}^{N}e_{ij}\left(\|v_{ij}(t+\Delta t)\|^2-\|v_{ij}(t)\|^2\right)\\&\approx -4(\textstyle\sum_{j=1}^{N}N_i\|\hat{v}_i(t)\|^2)\cdot\Delta t+2(\textstyle\sum_{j=1}^{N}(N_i+1)\|\hat{v}_i(t)\|^2)\cdot\Delta t^2,\end{aligned} \quad (4)$$

where we discard the high-order terms $o(\Delta t^2)$. Note, $v_{ij}(t+\Delta t)$ can be approximated by using the BCH formula [9] on the geodesic $\exp(v_{ij}(t+\Delta t))$, where $\exp(v_{ij}(t+\Delta t)) = \exp(-\hat{v_i}(t)\cdot\Delta t)\circ\exp(v_{ij}(t))\circ\exp(\hat{v_j}(t)\cdot\Delta t)$. Then, the derivative of $F(t)$ is given as

$$F'(t)=\lim_{\Delta t\to 0}\frac{\Delta F(t;\Delta t)}{\Delta t}=-4\left(\sum_{i=1}^{N}N_i\|\hat{v}_i(t)\|^2\right). \quad (5)$$

Obviously, $F'(t)$ is always negative, which leads to the objective function $F(t)$ strictly and monotonously decreasing when t increases to infinity. As time t goes to infinity, all the graph nodes shrink to the population center with the degree of $F'(t)$ tended to 0.

To implement this minimization procedure, we propose a discrete descent method as follows. Suppose each image has been deformed from $I_i(t_0)$ to $I_i(t^k)$, where $\{t^k\}$ is the discretization of time t ($k = 0,1,2, \cdots$, $t^0 = 0$ and $t^k \to \infty$ as $k \to \infty$). Let $I_i^k=I_i(t^k)$ be the warped image at time t^k and $\hat{v}_i^k=\hat{v}_i(t^k)$ (by Eq. (3)) be the velocity vector. Given these velocity vectors, the optimal step size Δt^k is determined as follows. According to the convergent condition of the Taylor series when we apply BCH formula to calculate $v_{ij}(t+\Delta t)$ in Eq. (4), Δt^k should be small enough, i.e., $\Delta t^k\cdot\|\hat{v}_i^k\|<1$ for all i=1, ⋯,N. Notice that the increment $\Delta F(t^k; \Delta t)$ can be considered as a positive definite quadratic function of Δt. Then, to make our method quickly converge, Δt^k is selected to encourage the increment $\Delta F(t^k; \Delta t)$ decreased as large as possible. Thus, it is straightforward to determine the optimal value of Δt^k by

$$\Delta t^k=\min\left\{\frac{1}{\|\hat{v}_i^k\|},\frac{\sum_{j=1}^{N}N_i\|\hat{v}_i^k\|^2}{\sum_{j=1}^{N}(N_i+1)\|\hat{v}_i^k\|^2}\right\} \quad (6)$$

Eventually, the deformation field from I_i^0 to population canter can be obtained by concatenating the deformation segments as $\exp(\Delta t^k\hat{v}_i^k)\circ\cdots\circ\exp(\Delta t^0\hat{v}_i^0)$.

3. EXPERIMENTS

In this section, our proposed method is evaluated by performing inter-group registration on the IXI database[1]. The proposed method is also compared with the conventional groupwise registration method [5] where the deformations for all images are jointly estimated towards

[1] http://biomedic.doc.ic.ac.uk/brain-development/index.php?n=Main.HomePage

the single population center, and also our previously developed pairwise inter-group registration method [8]. 30 brain MR images (from 20 to 54 years old) are used in the following experiment, each with 83 manually delineated ROIs. The image size and voxel spacing are 256 × 256 × 198 and $1\times1\times1mm^3$, respectively. Based on the age information, we divide it into two groups (below and above 30y). Some typical images from the IXI database are shown in Fig. 3.

The linear alignment among different images is served as the preprocessing step. Then, we perform the groupwise registration with the conventional groupwise registration method, pairwise inter-group registration method, and our proposed graph-based inter-group registration method. The group-mean images (i.e. simple averaging of all warped images) by three methods are shown in Fig. 4. It is clear that the group-mean image obtained by our method is sharper than those obtained by the conventional groupwise registration method and pairwise inter-group method.

To quantitatively evaluate the registration accuracy, we use the average of all pairwise Dice ratios for 83 ROIs, in which the Dice ratio used to measure the overlap degree between ROI *A* and ROI *B* is defined by

$$\text{Dice}(A, B)=2 \times \frac{|A \cap B|}{|A|+|B|}, \quad (7)$$

where $|\cdot|$ means the volume of the particular ROI. The average Dice ratio of our method is 68.49%, where we achieve 4.03% and 1.48% more performance improvement than the conventional groupwise registration method and the pairwise inter-group registration method, respectively.

To demonstrate the advantage of topology preserving, we project 30 warped images at different shrinkage stages onto the 3D space by PCA. For clarity, we only show 4 images (dots) in the first group and 5 images (boxes) in the second group in Fig. 5. The lines are used to represent the graph edges. It is clear that the graph is synchronously shrinking to the population center with the topological structure well preserved, which brings the improvement in registration accuracy. As shown in Fig. 6, the Dice ratios of two typical ROIs (Hippocampus and Postcentral Gyrus) consistently increase (red curves) in our method with progress of registration, while the Dice ratios by the conventional groupwise registration method (blue curves) even decrease in the middle of groupwise registration. Also, the evolution curves by our method is eventually above those (green lines) by the pairwise inter-group registration method.

4. CONCLUSION

In this paper, we have developed a novel graph-based intergroup registration method by using a hierarchal graph to model the entire image distribution. Then, the procedure of groupwise registration is formulated as the dynamic shrinkage procedure of the graph on the manifold, which brings the advantage of preserving the topology of the image distribution during the groupwise registration. Our proposed method has been evaluated on the IXI data set, where our method achieves the best registration results in comparison with other two state-of-the-art methods.

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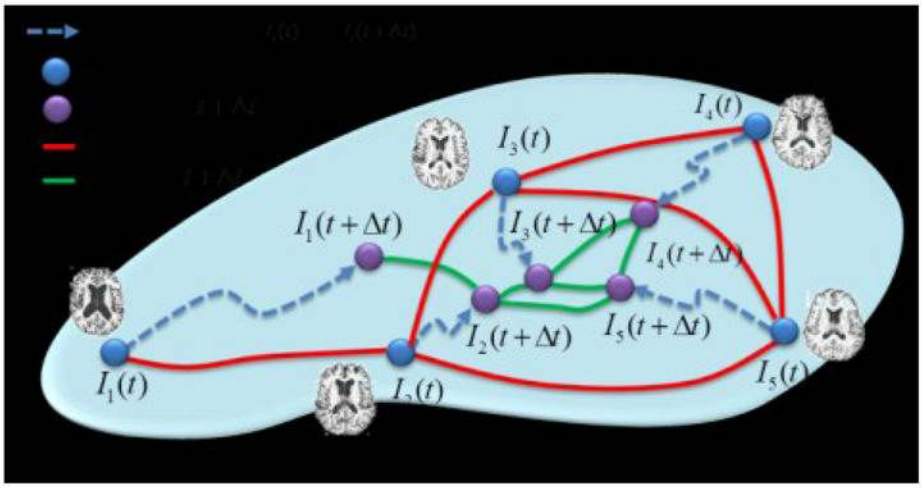

Fig. 1.
Overview of our graph-based registration method by graph shrinking.

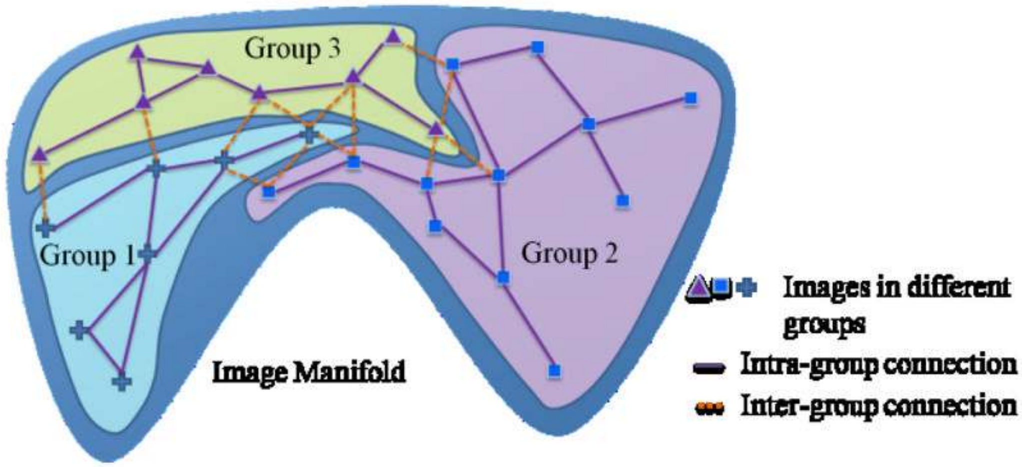

Fig. 2.
Illumination of two-level graph for entire images with three groups.

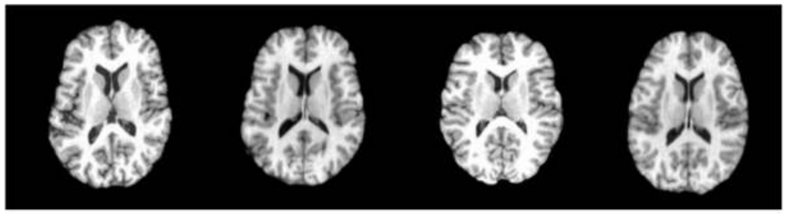

Fig. 3.
Four typical images from the IXI database.

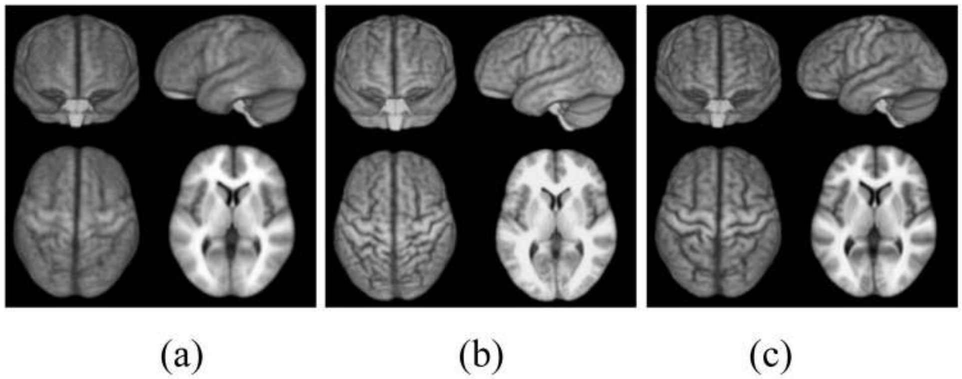

Fig. 4.
Group-mean images by three methods. (a) conventional groupwise registration [5], (b) pairwise inter-group method [8], and (c) our method.

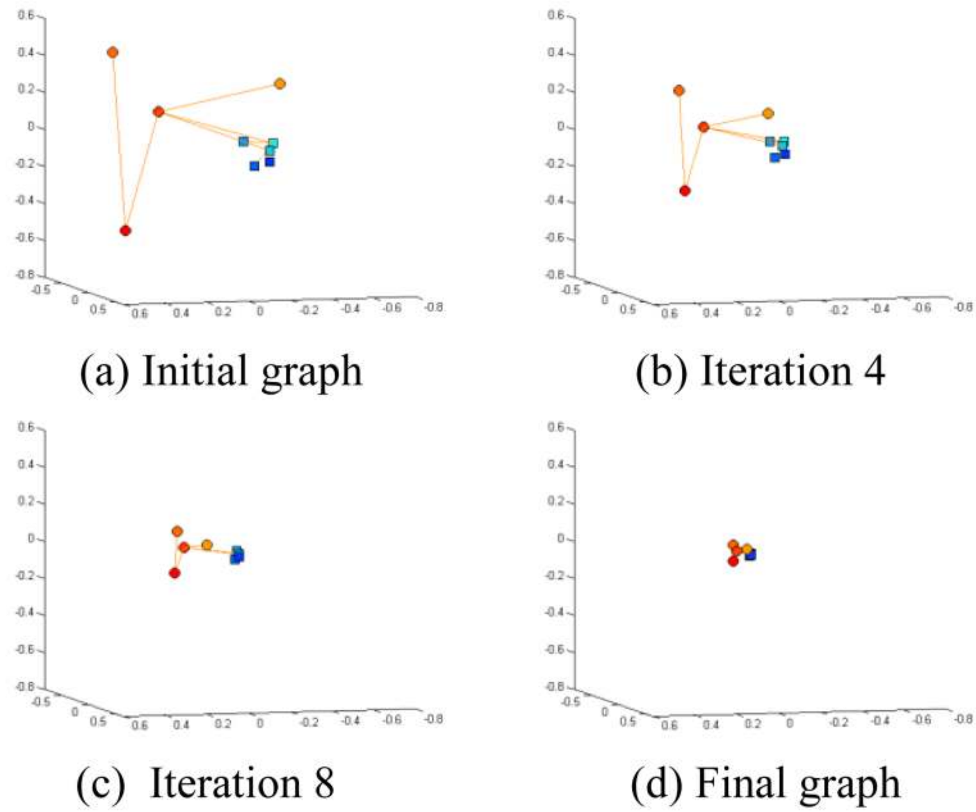

Fig. 5.
Evolution of the graph of 9 selected images in the projected space.

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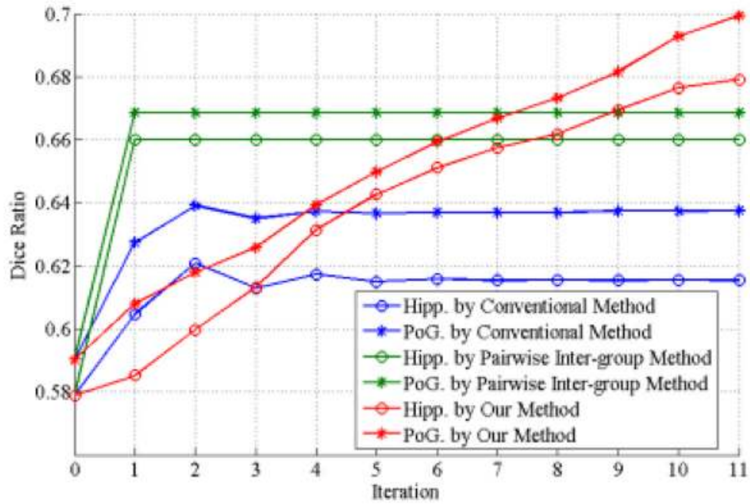

Fig. 6.

Dice ratios of two ROIs during the registration by three methods with respect to different iterations, respectively.