Inter-Utilities Power-Exchange Coordination: A Market-Oriented Approach

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***Abstract*—A decentralized operation of the transmission grid for scheduling inter-utilities power exchanges is proposed. This approach is well suited for market-oriented environments; it achieves a market equilibrium while the operational independence of each interconnected utility is preserved. We employ *decomposition-coordination* techniques in combination with an *Interior-Point/Cutting-Plane* method in order to reduce the number of iterations of the decomposition algorithm. The paper includes test results on IEEE-based systems.**

***Index Terms*—Decomposition techniques, interior-point/cutting-plane methods, inter-utilities power exchanges.**

I. Introduction

RE-STRUCTURED electricity energy markets have attracted the attention of many researches in the last few years. Decentralization is one of the major topics within this area and it has been accomplished employing two well differentiated power grid operation entities: i) the Independent System Operator (ISO), and ii) the Power Exchange (PX). The former is in charge of the secure operation of the transmission system while the latter runs the energy market. However, little attention has been paid to the decentralization of the transmission grid itself, and few works can be found in the technical literature on this aspect. This paper proposes a decentralized and coordinated operation of the power grid.

Traditionally, tie-lines between different power systems were mainly motivated due to reliability reasons. The amount and price of exchanged power through tie-lines were assumed to be known and prices for those transactions were set according to some simple heuristic criteria; they were usually based on differences in system marginal prices. In general, those procedures do not guarantee an efficient power exchange operation policy. Nowadays, as the power industry moves into a more competitive environment, tie-lines play an important role in order to reduce operational costs of the power systems involved in the exchange. Rigorous procedures for defining the level of inter-utility power exchanges and pricing mechanisms are required.

A centralized operation of different interconnected system can result in an optimal operation policy; however disclosing utility data in order to achieve a market optimum may not be desirable or even allowed. Thus, it is advisable to develop trading tools to address the efficient operation of interconnected systems in an autonomous fashion while maintaining the advantages of a centralized approach.

One of the available tools to evaluate power exchanges among utilities is the Optimal Power Flow (OPF). As a result of the development of *spot pricing* theory for electricity markets [1], the OPF has emerged as an important tool for pricing of electrical energy. The use of an OPF is becoming increasingly more important in solving the problem of inter-utility power transactions in re-structured electricity markets.

Specifically, the problem of energy interchange has been previously studied from different points of view [2], [3]. In [4], the authors apply an Economic Dispatch to optimal scheduling of power exchanges. Fahd and Sheblé [5] propose a method in which the data required for interchange brokerage systems is the amount of power available for trading and the system incremental cost; the engine of this approach is a linearized OPF.

Ferrero and Shahidepour [6] propose a method for energy exchanges under deregulated environments; the basic idea is to replace the "buy curve" in each utility with an equivalent local generator that will compete with other generator in supplying the local load. They provide a comparison of their approach with the *equal lambda* criterion. Some other approaches take into account inter-temporal restrictions [7]–[9]. In [7], the authors solve the integrated power purchase and thermal scheduling problem using an Augmented Lagrangian and coordination method. Within this approach, *minimum purchase time* or *purchase level* constraints can be included. In [9], a heuristic technique to evaluate potential power exchanges among utilities is proposed, it considers factors such as generating units forced outages or network flow restrictions.

Other decomposition techniques have been applied to Distributed Optimal Power Flows (DOPF) problems. As shown in [10] and [11], an Augmented Lagrangian technique is used to solve a DOPF problem. On the other hand, a market for the coordination of Transmission Loading Relief (TLR) across multiple regions is studied in [12].

In this paper, the problem of determining the quantity and price of inter-utilities power exchanges is formulated as an optimization problem and solved in the following framework. The objective to be minimized is the overall production costs while preserving dispatch independence of the involved utilities. Moreover, inter-utility power exchanges are scheduled based on *spot prices*. Each utility solves a modified OPF which includes its own service area and some information on the tie-lines. All

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interconnected utilities interact with a coordination entity and the information exchange among them is minimal.

This paper also aims at developing a computationally efficient implementation of the proposed decomposition coordination approach. A Lagrangian Relaxation (LR) decomposition technique is employed. Duality theory is used in order to decompose the problem along the boundary of the interconnected areas. We solve Optimal Power Flow problems for each area and coordinate the various area-OPFs in an outer loop through an iterative update of the Lagrange multipliers that are associated with certain constraints. Sub-gradient and Cutting-Plane based methods have widely been used to update the Lagrange multipliers which, in turn, is equivalent to optimize the dual function. In power-engineering, Interior Point methods have been successfully employed to solve linear and nonlinear problems and, more recently, they have been applied to solve nondifferentiable problems [13], [27]. A tailored *Interior-Point/Cutting-Plane* (IP/CP) method is used to deal with the resulting nondifferentiable problem. This approach avoids solution oscillation difficulties and speeds up the algorithm convergence.

This paper is organized as follows. In Section II, the proposed coordination procedure is presented. Notation is given in Section III. Section IV provides the problem's mathematical formulation. In Section V, a Lagrangian Relaxation technique is described. An *Interior-Point/Cutting-Plane* method to solve the dual problem is presented in Section VI. Case studies and test results are shown in Section VII. Conclusions close the paper.

II. Spot Market for Inter-Utilities Power Exchanges

In the proposed coordination procedure, we assume that there exists a Regional Scheduling Coordinator (RSC) who is in charge of the coordination of all power exchanges among utilities. Besides, each utility in the system controls its own operation scheduling so as to maximize the benefits from the tie-lines operation.

As shown by Schweppe *et al.* [1], optimal *spot prices* play a key role in the efficient dispatch of a power system; when each generator within a single system receives a price signal that is equal to the corresponding spot price, the system operates in such a way that the resulting operating point is optimum under an efficient economic point of view of that single system.

In a multiple (multi-utility or multi-country) integrated system setting, each integrated system "speaks" with its neighbor in terms of *spot prices* at their common borders; they buy or sell energy at the spot price of the specific instant and location [14]. The resulting operating point is the same as the one achieved under a fully centralized dispatch. When dispatching the utilities, the control center associated to each system must not discriminate between its own generators' power and power offered by the neighboring systems through tie-lines, except for economic reasons.

A multi-utility setting consisting of three coordinated areas is shown in Fig. 1. Each regional ISO operates its own power system and interacts with the RSC. When power exchanges are to be scheduled, the RSC starts an iterative procedure in which the utilities send tie-line power-flow information to the RSC.

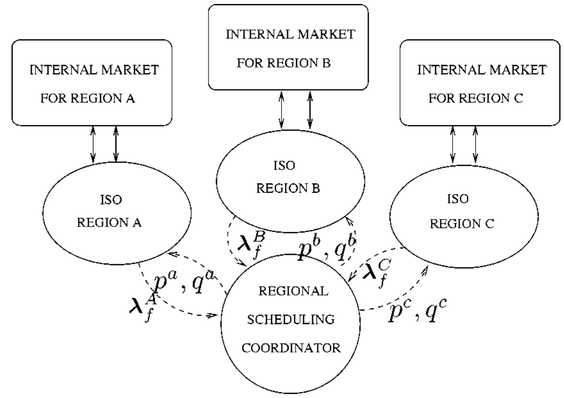


Fig. 1. Scheduling regional coordinator.

The RSC sends back spot price information for each power exchange involved within tie-lines. At each iteration, the control center associated with each utility re-schedules power exchanges through tie-lines; an Economic Dispatch or an OPF can be performed. Tie-lines are modeled as generators (or demands) from which they can buy (or sell) energy at the *spot price* as set by the RSC. This is repeated until a market equilibrium of the whole interconnected system is achieved.

The basic motivation of using an iterative spot market for inter-utilities power-exchange is the possibility of increasing efficiency through iterations while keeping self-dispatch operation among ISOs. A key aspect of the proposed approach is that the amount of information exchange among utilities and the RSC is minimal. As opposed to other iterative schemes [15], at each iteration there is no need to perform a fully centralized dispatch. Decentralized self-dispatch as applied to the unit-commitment problem has been recently proposed in [16] where generators iteratively submit bids in response to a trial sequence of market prices. Similar frameworks could also be suitable for other problems such as the Transmission Loading Relief (TLR) problem across multiple regions [12].

III. Problem's Mathematical Formulation

The notation is organized as follows:

Indices

i, j	Index of buses.
f	Index of frontiers buses.
a	Index for the number in areas; $a = 1, \ldots, \mathcal{A}$.

Sets

$\mathcal{A}$	Set of interconnected areas.
$\mathcal{F}$	Set of frontier buses.
Ω_n	Set of buses connected to bus n.

Functions

f_a	Objective function of area a.
h_a	Power flow equations of area a.
g_a	Operational limits of area a.

The approach presented in this paper can be formulated as an Optimal Power Flow problem that involves multiple areas. The equations that appear in this problem are similar to those of a single OPF problem, except for some equations that couple variables from different areas. These equations are the active and reactive power balance at the frontier (boundary) buses; linear coupling constraints are introduced to force the variables on both sides of the boundary to be the same. For the power flow at the tie-lines, these equations are formulated as follows:

- Active power balance at frontier buses

$$\sum_{j\in\Omega_f} p_{fj} = 0, \qquad \forall f \in \mathcal{F}. \tag{1}$$

- Reactive power balance at frontier buses

$$\sum_{j\in\Omega_f} q_{fj} = 0, \qquad \forall f \in \mathcal{F}. \tag{2}$$

where p_{fj} and q_{fj} are the active and reactive power flow form bus f to j respectively. The Lagrange multipliers associated to constraints (1), (2) are the *spot prices* (marginal cost) of the power resources at the frontier buses; thus, they give a sound basis for pricing power exchanges. The power exchanges among the interconnected utilities are scheduled based on these Lagrange multipliers. These spot prices are the required information to coordinate the utilities involved in the energy transactions.

An optimization problem that involves a system of interconnected utilities can now be formulated. Let $\mathcal{A}$ be the set of the interconnected areas (utilities subsystems). Let l be the total number of constraints that couple variables from different areas, and n_a the number of coupling constraints of area a. If we assume that the objective function is separable with respect to the areas, which is the case when modeling operational costs, then the problem can be decoupled. The multi-area OPF problem is formulated as follows:

In compact form, the multi-area Optimal Power Flow problem can be re-written as

$$\begin{aligned} \text{minimize} \quad & \sum_{a=1}^{\mathcal{A}} f_a(\boldsymbol{x}_a) \\ \text{subject to} \quad & \\ & \boldsymbol{h}_a(\boldsymbol{x}_a) = 0 \qquad a = 1, \ldots, \mathcal{A} \\ & \boldsymbol{g}_a(\boldsymbol{x}_a) \le 0 \qquad a = 1, \ldots, \mathcal{A} \\ & \sum_{a=1}^{\mathcal{A}} \boldsymbol{C}_a \boldsymbol{x}_a = 0. \end{aligned} \tag{3}$$

where vector $\boldsymbol{x}_a$ contains variables from area a, and matrix $\boldsymbol{C}_a \in \mathcal{R}^{l\times n_a}$ selects the components of $\boldsymbol{x}_a$ that are involved in the coupling constraints. The equation $\sum_{a=1}^{\mathcal{A}} \boldsymbol{C}_a\boldsymbol{x}_a = 0$ forces variables of neighboring systems to be equal. The elements of $\boldsymbol{C}_a$ are either 0 or 1.

IV. Decompostion-Coordination Techniques

In the power-engineering community, decomposition techniques have widely been applied to large-scale Unit Commitment problems where direct approaches fail to find a solution in a reasonable amount of time. Decomposition techniques as applied to solve problem (3) are oriented to preserve operational independence of power systems. Drawbacks of available decomposition-based methods, such as Lagrangian Relaxation [17] or Augmented Lagrangian [10], are slow convergence or need of fine tuning of parameters.

In the following, we discuss an LR technique for the numerical solution to problem (3). The LR technique dualizes a small subset of constraints—the *coupling constraints*—by adding them to the objective function of an optimization problem. By using duality theory, the original problem is reformulated in the space of the dual variables associated with the dualized constraints. As a result of using a Lagrangian duality, the problem is transformed into a nondifferentiable problem. A specialized algorithm for nondifferentiable optimization is used to solve the transformed problem.

Within the problem we are solving, the *coupling constraints* are the variables involved in the active and reactive power balances at the frontier buses. In this application, the number of coupling constraints is much smaller than the total number of constraints. If constraints (1), (2) are relaxed, problem (3) can be decomposed into $|\mathcal{A}|$ independent subproblems. Hence, the independent solution of the OPF of each subproblem can be performed.

Let us consider the primal problem in compact form as presented in (3). The *coupling constraints*—in compact form—can be appended to the objective function to form a partial Lagrangian, *i.e.,*

$$\mathcal{L}(\boldsymbol{x}, \boldsymbol{\lambda}_f) = \sum_{a\in\mathcal{A}} f_a(\boldsymbol{x}_a) + \boldsymbol{\lambda}_f^T \sum_{a\in\mathcal{A}} \boldsymbol{C}_a\boldsymbol{x}_a \tag{4}$$

where $\boldsymbol{\lambda}_f \equiv \text{column}[\boldsymbol{\lambda}_{pf}, \boldsymbol{\lambda}_{qf}]$, and $\boldsymbol{\lambda}_{pf}$ and $\boldsymbol{\lambda}_{qf}$ are the vectors of Lagrange multipliers associated to the active and reactive power balance at the frontier buses, respectively. The *Dual Problem* (DP) of (3) is given by

$$\text{maximize}_{\lambda_f} \ \phi(\boldsymbol{\lambda}_f) \tag{5}$$

where $\phi(\boldsymbol{\lambda}_f)$ is the dual function. This is a concave function [18]. Despite the small dimension of its variable space, the transformed problem is usually nondifferentiable [19]. In our case, the primal problem is nonconvex and the dual function is nondifferentiable. This function is defined as

$$\begin{aligned} \phi(\boldsymbol{\lambda}_f) \equiv \text{minimize}_x \quad & \mathcal{L}(\boldsymbol{x}, \boldsymbol{\lambda}_f) \\ \text{subject to} \quad & \\ & \boldsymbol{h}_a(\boldsymbol{x}_a) = 0 \qquad a = 1, \ldots, \mathcal{A} \\ & \boldsymbol{g}_a(\boldsymbol{x}_a) \le 0 \qquad a = 1, \ldots, \mathcal{A}. \end{aligned} \tag{6}$$

The dual function $\phi(\boldsymbol{\lambda}_f)$ has the interesting property that it can be decomposed per interconnected area. Instead of solving Problem (3), the *Dual Problem* (5) can be solved. As the dual function $\phi(\boldsymbol{\lambda}_f)$ is not explicitly known, an iterative procedure is implemented to compute it; updating the Lagrange multiplier vector $\boldsymbol{\lambda}_f$ is equivalent to maximizing the dual function.

At the optimal solution, the vector $\boldsymbol{\lambda}_f^*$ has a physical-economical interpretation [1]: this is the *spot price* associated to the power exchanges at the different tie-lines.

Given $\boldsymbol{\lambda}_f^k$, problem (6) is decomposed into one subproblem per interconnected system; thus each utility solves a modified OPF problem that has the following structure:

$$\begin{aligned} \text{minimize} \quad & f_a(\boldsymbol{x}_a) + \boldsymbol{\lambda}_{fa}^{k^T} \boldsymbol{C}_a \boldsymbol{x}_a \\ \text{subject to} \quad & \\ & \boldsymbol{h}_a(\boldsymbol{x}_a) = 0 \\ & \boldsymbol{g}_a(\boldsymbol{x}_a) \leq 0. \end{aligned} \tag{7}$$

It is worth noticing that each problem can solve its own version of problem (7); it has the same structure than a standard OPF except for the linear term in the objective function that models the interaction with neighboring utilities.

The Lagrangian Relaxation algorithm can be summarized as follows:

Step 0) *Initialize the multiplier vector*. Start with a previous value of the $\boldsymbol{\lambda}_f$ vector; alternatively, an average of the marginal cost of the system can be used.

Step 1) *Solve the decomposed primal problems*. Once the Lagrange multipliers associated to the coupling constraints are fixed, the problem decomposes into subproblems. Within the proposed approach, each utility is responsible for the operation of its system and, hence, it solves its own OPF.

Step 2) *Update the Multiplier Vector*. This step is equivalent to maximizing the dual function. There exits a number of possible strategies that can be applied to Problem (5). They are discussed in Section V.

Step 3) *Check for ϵ-optimality*. If some convergence criteria are satisfied, stop. Else, return to *Step 1*.

This LR algorithm fits properly within the model proposed in Section II.

V. Lagrange Multipliers Updating

The proposed *decomposition-coordination* approach relies on an efficient update of the Lagrange multipliers; this is equivalent to maximizing the dual function (6). Several methods have been proposed in the technical literature to deal with the non-differentiable dual function.

A. Sub-Gradient Method

The sub-gradient method is an extension of the steepest ascent method and it is easy to implement. However, no information about the accuracy of the solution is obtained [20]. The sub-gradient method simply update the current Lagrange multiplier $\boldsymbol{\lambda}^k$ in the direction of the sub-gradient. At each iteration k, a sub-gradient is readily available as $\boldsymbol{\xi}^k = \sum_{a=1}^{\mathcal{A}} \boldsymbol{C}_a \boldsymbol{x}_a$. The updated Lagrange multiplier is obtained from

$$\boldsymbol{\lambda}^{k+1} = \boldsymbol{\lambda}^k + \beta^k \frac{\boldsymbol{\xi}^k}{\left\| \boldsymbol{\xi}^k \right\|}, \tag{8}$$

where β^k satisfies the following conditions:

$$\lim_{k \to \infty} \beta^k \to 0, \sum_{k=1}^{\infty} \beta^k \to \infty. \tag{9}$$

At each iteration, β^k can be chosen as $\beta^k = 1/(\alpha_1 + \alpha_2 k)$ where α_1 and α_2 are positive constants that satisfy (9). Performance of this method as applied to a multi-area DC OPF is reported in [17]. The major drawback of this method is the need of a fine tuning of parameters α_1, α_2. A similar situation happens when applying an Augmented Lagrangian technique; however, better convergence performance is obtained [10].

B. Cutting-Plane Methods

The cutting-plane algorithm [21] is a well known method to optimize nondifferentiable problems. The basic idea underlying cutting-plane-based algorithms is to build iteratively an outer approximation of the dual function. These methods solve to optimality the relaxed dual function problem. The polyhedral approximation of the dual function is defined by

$$\mathcal{S}_{cp} \equiv \left\{ (z, \boldsymbol{\lambda}) \colon z \leq \phi\left(\boldsymbol{\lambda}^{(k)}\right) - \left(\boldsymbol{\lambda} - \boldsymbol{\lambda}^{(k)}\right)^T \boldsymbol{\xi}^k, \right. \\ \left. \boldsymbol{\lambda} \geq \boldsymbol{\lambda}_{\min}, \boldsymbol{\lambda} \leq \boldsymbol{\lambda}_{\max}, \, \forall k \in K \right\} \tag{10}$$

where k is the iteration counter and K is the set of iterations. Cutting-plane methods try to find an optimal point of the set defined by $\mathcal{S}_{cp}$. A stabilized version of this algorithm is the Bundle method where a quadratic term is added to the objective function to improve convergence.

C. Interior-Point/Cutting-Plane Method

More recently, Interior-Point methods have been applied to solving nondifferentiable problems [19], [22], [23]. They have successfully been applied to power-engineering [13], [27] and multi-regional planning problems [24].

It is not always necessary to find the optimum of the relaxed outer approximation defined by (10); sometimes, it may be advantageous to find *central points* on the dual function space. At each iteration, the IP/CP algorithm computes an approximate center, the *analytic center*, of a current set defined by the sub-gradients generated in previous iterations. This set is called the localization set and is defined by

$$\mathcal{S}_{ip} \equiv \left\{ (z, \boldsymbol{\lambda}) \colon z \geq \theta^k, \, z \leq \phi\left(\boldsymbol{\lambda}^{(k)}\right) - \left(\boldsymbol{\lambda} - \boldsymbol{\lambda}^{(k)}\right)^T \boldsymbol{\xi}^k, \right. \\ \left. \boldsymbol{\lambda} \geq \boldsymbol{\lambda}_{\min}, \boldsymbol{\lambda} \leq \boldsymbol{\lambda}_{\max}, \, \forall k \in K \right\} \tag{11}$$

where θ^k is the best recorded lower bound of the dual function at iteration k.

Taking an interior point of $\mathcal{S}_{ip}$, the sub-gradient associated with this point cuts the localization set in two parts; the part that lies below the cut defines a new smaller localization set. The cuts generated with this procedure are deep, and they achieve a large reduction in the localization set.

The *analytic center* is defined as the unique maximizer of the products of the slacks to each of inequalities (11) or, equivalently, the sum of their logarithms. Mathematically, the *analytic center* problem can be formulated as follows:

$$\begin{aligned} \text{maximize} \quad & \sum_i \log s_i \\ \text{subject to} \quad & \\ & \boldsymbol{A}\boldsymbol{x} + \boldsymbol{s} = \boldsymbol{c} \\ & \boldsymbol{s} \geq 0. \end{aligned} \tag{12}$$

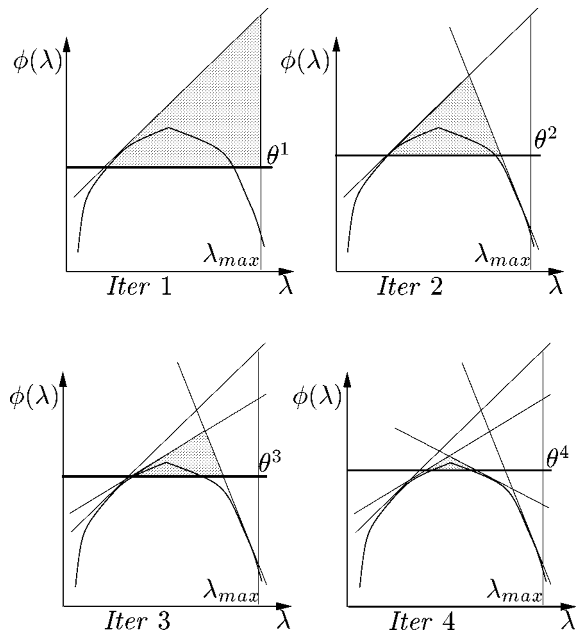


Fig. 2. Interior point cutting plane method evolution.

where

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{I} & -\boldsymbol{\xi}^k \\ -1 & 0 \\ 0 & -\boldsymbol{I} \\ 0 & \boldsymbol{I} \end{bmatrix}, \qquad \boldsymbol{x} = \begin{bmatrix} z \\ \boldsymbol{\lambda} \end{bmatrix},$$

$$\boldsymbol{s} = \begin{bmatrix} s_z \\ \boldsymbol{s}_\lambda \end{bmatrix}, \qquad \boldsymbol{c} = \begin{bmatrix} \phi(\boldsymbol{\lambda}^k) - \boldsymbol{\lambda}^{T(k)}\boldsymbol{\xi}^k \\ -\theta^k \\ -\boldsymbol{\lambda}_{\min} \\ \boldsymbol{\lambda}_{\max} \end{bmatrix}. \tag{13}$$

and $\boldsymbol{I}$ is the indentity vector of appropriate dimensions. In a typical iteration, new sub-gradients are added after the *analytic center* of the current localization set has been computed. At each iteration, the localization set shrinks and thus provides and increasingly accurate approximation of the optimal solution. The *analytic center* is computed using a primal–dual interior-point method.

This *Interior-Point/Cutting-Plane* method avoids solution oscillation difficulties and speeds up the algorithm convergence. Four iterations of the method in a one dimensional case are shown in Fig. 2; the lower bound $\boldsymbol{\lambda}_{\min}$ has been set to zero.

VI. Numerical Results

In order to simulate the performance of the proposed *coordination-decomposition* approach, a prototype implementation has been developed under the MATLAB [25] environment. The algorithm has been tested using two (small- and medium-size) IEEE-based power systems. The IEEE-RTS96 [26] builds on a basis of three IEEE-RTS24, each corresponding to one area. Figures for this power system are 72 buses, 96 generators, 119 lines and 5 tie-lines. Similarly, the IEEE-118×3 testing system is set up on a basis of three IEEE-118, each corresponding to one area, and contains 354 buses, 162 generators, 558 lines and 6 tie-lines. Both test systems have three well differentiated areas.

TABLE I
Operational Cost (US$ p.u.)

SCENARIOS	*IEEE-RTS96*	*IEEE-118×3*
Scenario A	2.9583	2.9462
Scenario B	2.9952	2.9891
Scenario C	2.9616	2.9510

A. Testing Scenarios

Several *scenarios* are run in order to illustrate the performance of the centralized and decentralized approaches. Within all simulations, the objective is to minimize the total production cost of the overall power system.

Scenario A: A centralized operation of the power grid is assumed; all interconnected areas are considered as a single system. Generators' outputs and inter-utilities power exchanges are scheduled as a result of a centralized dispatch in which the total production cost is minimized.

Scenario B: The interconnected power systems are dispatched by regional ISOs but power exchanges among utilities are coordinated based on heuristics criteria. The amount of power to be exchanged through tie-lines is previously agreed and the prices for that transactions is set to the mean value of the marginal prices at both sides of the tie-lines.

Scenario C: Each power system is operated by a regional ISO and inter-utilities power exchanges are scheduled under the intervention of the Regional Scheduling Coordinator.

Operational costs[1] for the two test systems and for the three *scenarios* described above are displayed in Table I. In *Scenario A*, the global optimum of the whole system is achieved. Within *Scenario B*, a heuristic rule based on previous knowledge of the spot prices at the borders has been implemented; the operational cost increases with respect to *Scenario A*. The decentralized but coordinated approach, *Scenario C*, converges to essentially the same results as those obtained with a centralized approach. For the tested *scenarios*, up to 1.5% of the operational costs can be saved. The main engine for inter-utilities power exchanges, in all *scenarios*, is a noncontingency Optimal Power Flow.

Within *Scenario C*, a quasioptimal solution is obtained as there are power flows mismatches at tie-lines; the convergence criterion for those mismatches has been set to 0.03 p.u. This criterion is appropriate in this case since more accurate results, in terms of power flows mismatches at tie-lines, has little effect (less than 0.0016 p.u.) in the variation of total operational cost. From a computational point of view, it should be noted that if a highly accurate solution is required, say, 10^{-6} p.u. power mismatch tolerance, the number of iterations rapidly increase, degrading the efficiency of the approach.

B. Lagrange Multiplier Updating Performance

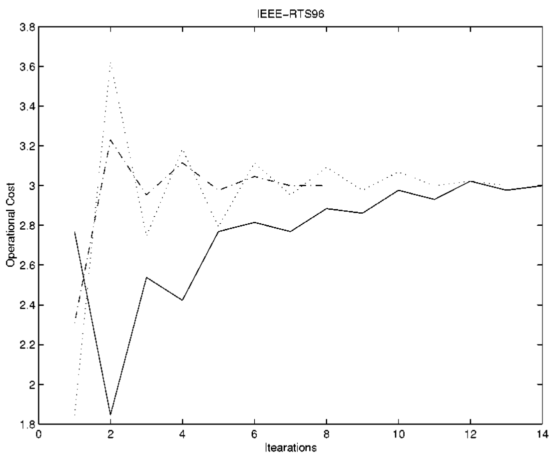
Several cases have been run under different load conditions. The number of iterations and number of flops of the implemented methods are shown in Table II. It should be noted that the major computational effort is consumed in the solution of area-OPFs at each iteration. In all cases, the algorithms have

[1]For comparison purposes, the operational cost has been normalized to the operational cost of a single system for each case.

TABLE II
LAGRANGE MULTIPLIER UPDATE COMPARISON

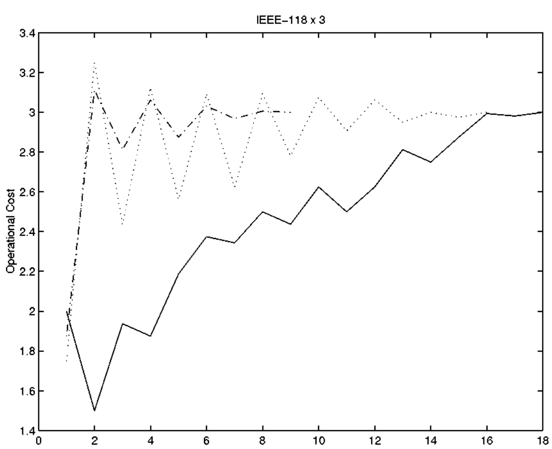
METHOD	ITERATIONS		FLOPS (10^8)	
	RTS-96	118×3	RTS-96	118×3
SG	14	18	1,426	4,839
CP	13	16	1,265	4,335
IP/CP	8	9	0,682	2,469

SG: Sub-Gradient, CP: Cutting-Plane, IP/CP: Interior-Point/Cutting-Plane



SG(−), CP(···), IP/CP(-·-)

Fig. 3. Operational cost evolution (US$ pu). IEEE-RTS96.



SG(−), CP(···), IP/CP(-·-)

Fig. 4. Operational cost evolution (US$ pu). IEEE-118×3.

converged and achieved the same solution. However, it has been observed that as the ratio "Total Power Generated/Power Through Tie-lines" increases, a higher number of iterations is required.

The evolution for of the objective function for the IEEE-RTS96 and IEEE-188×3 cases is shown in Figs. 3 and 4, respectively. The starting point for $\boldsymbol{\lambda}_f$ has been selected as the mean value of the spot prices at frontiers buses. Extensive parameter tuning have been carried out to achieve a good performance with the sub-gradient method. As expected, the sub-gradient is quite sensitive to parameters changes and also to the starting point. When a good starting point is available for $\boldsymbol{\lambda}_f$, the sub-gradient method has proved to be competitive. The Cutting-plane method has proved to be oscillating, although no Bundle variant has been implemented. The *Interior-Point/Cutting-Plane* gets close to the optimal solution in the very first iterations and it requires less number of iterations than any of the other methods. Moreover, within this method there is no need of parameter tuning. Although results presented in this section can not be generalized for larger systems, the proposed algorithm has proved to be reliable and robust for the test systems.

VII. CONCLUSION

A market-oriented approach has been proposed to deal with inter-utilities power exchanges. The approach is based on a decentralized operation of the transmission grid.

In the framework of *decomposition-coordination* strategies, a Lagrangian Relaxation technique is presented; different methods are explored in order to get an efficient algorithm. As opposed to traditional methods to deal with the dual function, such as sub-gradient or cutting-plane-based methods, an *Interior-Point/Cutting-Plane* method has been employed. This method avoids solution oscillation difficulties and speeds up the algorithm convergence; moreover, there is no need of tuning parameters. The overall *decomposition-coordination* approach performance achieves robustness and great reliability.

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