



Interaction between a main-crack and a collinear micro-crack or two parallel micro-cracks

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ABSTRACT

The analysis of the interaction of a main crack with an array of microcracks, placed near the tip of the main crack, has been performed considering 2-D polycarbonate flat specimens. Analytical results, based on the elastic potential theory for the stress intensity factor K_I have been well compared with experimental results obtained through the application of the caustic method. Two different specimen configurations have been analysed involving one and two microcracks.

INTRODUCTION

The problem of the interaction of a main crack with an array of microcracks placed near its tip, of strong relevance in the prediction of the reliability of structural materials, can be experimentally analyzed by applying the caustic method on 2-D specimens. In fact, the method of caustics can be applied to the determination of the stress intensity factors at crack tips approaching other crack tips or boundaries of the specimen. Several problems of interaction of cracks with other cracks or boundaries have been reported [1,2]. In these problems two special cases can be distinguished: in the first case, the crack tip at which the stress intensity factor is determined does not lie very near another crack or boundary [3]. In the second case the crack tip lies very near another crack tip (or boundary) and the shape of the caustic around this crack tip is influenced by the other crack (or boundary).

On the other hand, the elastic interaction between both types of cracks can be analytically studied by applying a method based on the combination of the double layer potential technique and the Willis polynomial conservation theorem stating that the COD of a crack embedded into a polynomial stress field of degree N has the form (ellipse) \times (polynomial) [4]. Following this approach and

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expressing the displacement field as well as the stress field by an integral of the double layer potential type, the problem can be reduced to the one of finding vectorial functions-microcrack CODs $b(x)$ and the stress intensity factor K , at the macrocrack tip from the system obtained by solving the system of integral equations which express the boundary conditions on the crack faces. The microcrack CODs are represented in the form (ellipse)x(polynomial) where the first multiplier corresponds to the crack embedded into a uniform stress field and the second multiplier accounts for crack interactions. Thus, the system of singular integral equations is reduced to a system of linear algebraic equations which must be solved to obtain the polynomial coefficients.

POTENTIAL REPRESENTATION THEORY

The schematic representation of a main crack $(-l_o, l_o)$ which can interact with one microcrack is shown in Fig. 1.a and with two microcracks in Fig.1.b. In order to describe the elastic interaction between both cracks, plane stress conditions and Mode I are assumed. Moreover, the case where the microcrack is embedded into the stress field of the main crack tip is considered. Using the constant approximation the stress field within the microcrack line $(c-l, c+l)$ may be approximated by a constant equal to the value of the field at the microcrack centre $(x=c)$ and, following symmetry, $\sigma_w = \sigma_w(c)$ is the only stress component acting along the microcrack line.

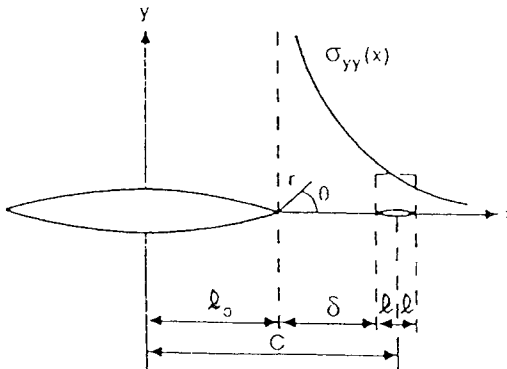


Fig.1.a: Main crack collinear with one microcrack.

Thus, the overall stress field in the vicinity of the main crack tip is given by the superposition:

$$\sigma(x) = \hat{\sigma}(x) + \sigma^l(x) \quad (1)$$

where:

$$\hat{\sigma}(x) = k_I^{eff} \frac{\phi[\theta(x)]}{\sqrt{2\pi r(x)}} \quad (2)$$

$$\sigma^l = T_x \int_{c-1}^{c+1} b(\xi) \cdot \Phi(\xi, x) d\xi \quad (3)$$

and $T\{u(x)\}$ is the stress operator which corresponds to the Hooke's law giving the transformation of the displacement field into the stress field:

$$T_{ij}\{u\} = \mu(u_{ij} + u_{ji}) + \lambda u_{kk} \delta_{ij} \quad (4)$$

Following the piecewise constant approximation the COD of the microcrack is elliptical and is given by:

$$b(\xi) = \frac{4l}{E} \frac{k_I^{eff}}{\sqrt{2\pi(l+\delta)}} e(\xi)n \quad (5)$$

where E is the Young's modulus, n is the unit vector in the y -direction and the ellipse $e(x)$ represents the opening with the extremities at the microcrack tips. When Eqs.(3) & (4) are substituted in Eqn.(3) and the stress operator is introduced under the integral sign the following equation for $\sigma(x)$ is obtained:

$$\sigma_{ij}^l(x) = \frac{4l}{E} \frac{k_I^{eff}}{\sqrt{2\pi(l+\delta)}} \int_{c-1}^{c+1} \left\{ \mu [\Phi_{2i,j}(\xi, x) + \Phi_{2j,i}(\xi, x)] + \lambda \Phi_{2k,k}(\xi, x) \delta_{ij} \right\} e(\xi) d\xi \quad (6)$$

Finally, after integration, the expression for $\sigma_{yy}(x)$ is obtained:

$$\sigma_{yy}^l(x) = \frac{k_I^{eff}}{\sqrt{2\pi(l+\delta)}} \left[\frac{1}{\sqrt{(1-l^2/(x-c)^2)}} - 1 \right] \quad (7)$$

Where the effective stress intensity factor is given by:

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$$k_l^{eff} = k_l^0 + \frac{1}{\sqrt{(\pi l_0)}} \int_{-l_0}^{l_0} \sqrt{\frac{l_0 + \xi}{l_0 - \xi}} \sigma'_{yy}(\xi) d\xi \quad (8)$$

This is the equation that needs to be plotted for experimental results comparison.

ELASTIC STRESS INTENSITY FACTORS DETERMINED BY CAUSTICS

The effect of the interaction between cracks and other defects on the fracture strength of structural materials has been recently described in a series of papers [5]. The same approach has been adopted in this research work to determine experimentally the influence of the interaction between one collinear micro-crack and one macro-crack or two parallel microcracks with one macrocrack.

Thin strips made of polycarbonate and containing either two collinear cracks or two parallel cracks with one macrocrack were subjected to axial tension. When these strips were observed using a coherent monochromatic light beam emitted from a *He-Ne* gas laser, the light reflected from the back surface of the strip was deviated as a consequence of the highly strained zone surrounding the crack-tip and formed a caustic.

In this section we will determine the equation of the caustic and the equation of its initial curve, when a caustic is formed on a screen after a light beam impinges on an elastic specimen of thickness d under plane stress conditions. The correspondence between a point $P(x,y)$ of the specimen and its image $R(X,Y)$ on the screen at a distance z_0 from the specimen is:

$$X = \lambda_m x + z_0 \frac{\Delta s}{\partial x}, \quad Y = \lambda_m y + z_0 \frac{\Delta s}{\partial y} \quad (9)$$

where λ_m is the magnification ratio of the optical arrangement and Δs is the increment of the optical path s of the rays of the light beam due to the specimen loading. By using elementary algebra, we can see that these conditions are satisfied if the Jacobian determinant vanishes:

$$J = \frac{\partial (X,Y)}{\partial (x,y)} = \begin{vmatrix} \partial X / \partial x & \partial X / \partial y \\ \partial Y / \partial x & \partial Y / \partial y \end{vmatrix} = 0 \quad (10)$$

Equation (10) define a curve on $z=f(x,y)$ called the initial curve, while the system of Eqs. (9) and (10) defines on the screen its corresponding caustic.

Now, if we consider the case of optically isotropic and mechanically isotropic materials, we have:

$$\Delta s = 4dc \operatorname{Re} \Phi(z) \quad (11)$$

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where c is an optical constant of material. Now if we define another constant:

$$C = 4z_0 d c / \lambda_{in} \quad (12)$$

it can be seen that the equation of initial curve of the caustic can be written:

$$|C\Phi''(z)| = 1 \quad (13)$$

where $\phi''(z)$ denotes the second derivative of the complex potential $\phi(z)$. This equation, which represents the initial curve of the caustic on the specimen, together with the following equation:

$$W = \lambda_{in} \left[z + \overline{C\Phi'(z)} \right] \quad (14)$$

which represents the equation of the caustic itself on the screen, have played an important role during the use of the method of caustic in a series of practical applications [6].

Now by substituting the equation of the initial curve in the expression for $\phi(z)$, we find that this curve is a circle of radius

$$r = r_0 = \left[\frac{3}{8(2\pi)^{1/2}} |CK| \right]^{2/5} = 0.4677 |CK|^{2/5} \quad (15)$$

This equation reveals that the initial curve of the caustic on the specimen depends only on the absolute value of the stress intensity factor K and on the overall constant C of the experimental arrangement. Regarding the absolute value $|K|$ of the complex stress intensity factor K , it can be obtained by measuring the maximum diameter D_i of the caustic. In fact, under this conditions, the following relationship holds:

$$|K| = 6.6843 r_0^{5/2} / |C| = 0.3735 (D_i / \lambda_{in})^{5/2} / |C| \quad (16)$$

In this paper, pictures of the interactions at different load level are reported. The sequence follows three steps: no interaction, interaction, and crack propagation. The first set of picture (fig.3) regards one microcrack collinear with the main crack, while the second set (fig.4) regards two microcracks parallel and not collinear to the main crack.

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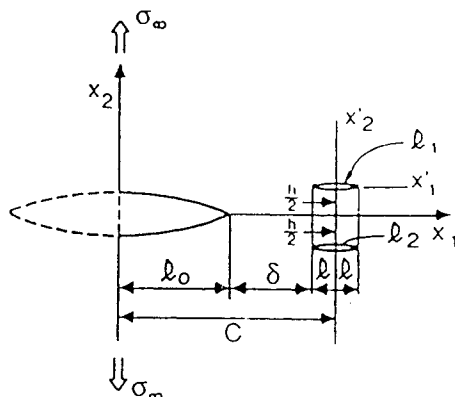


Fig.1.b: Configuration with two microcracks parallel to the main crack line.

PLOTTED RESULTS & COMPARISON

- **Microcrack collinear to the maincrack:** Reminding that a piecewise constant approximation has been used to approximate the stress field within the microcrack line, the analytical solution obtained gives a low estimate for K_I^{eff} since a straight line parallel to x-axis on the centre to microcrack gives the average value.

Experimental analysis conducted up to $d/l = 1.33$ reflects this difference with the analytical solution (see fig. 2); the value of this gap for $d/l = 1.33$ is 0.41, i.e. 33% of the analytical value. However, the more the microcrack moves away the less this gap is significant.

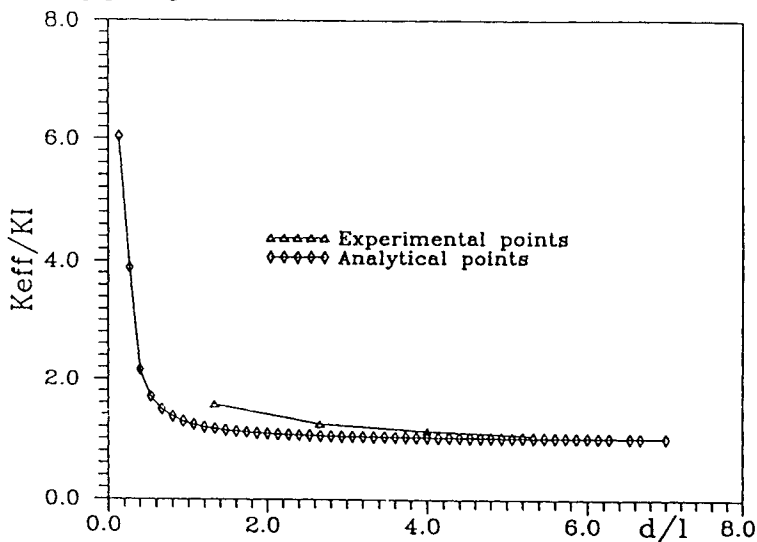


Fig. 2: K_{eff}/K_I ratio versus d/l (see fig.1.a) for analytical & experimental analysis

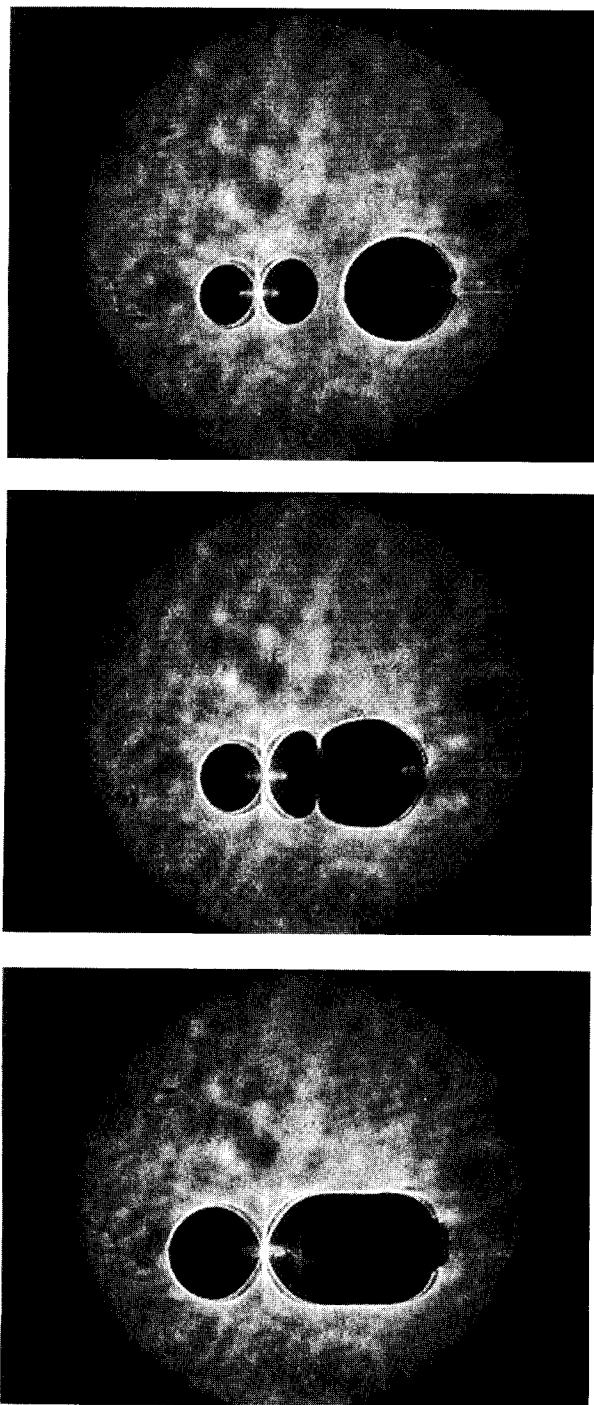


Fig. 3: Experimental caustics evolution of one collinear microcrack.

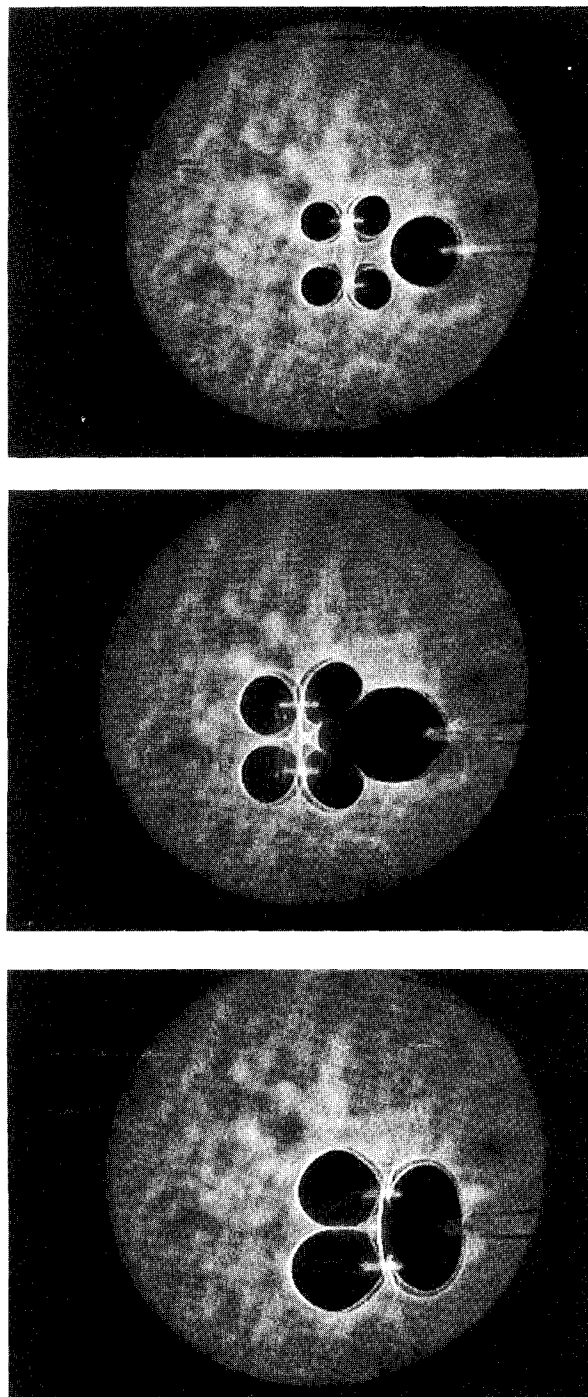


Fig. 4: Experimental caustics evolution of two parallel microcracks.

-Two microcracks parallel to the maincrack. The configuration considered is shown in Fig. 1.b ; the problem, as the one before, will be studied in the piecewise constant approximation. Two different effects of crack interaction may occur depending on the relative values of the geometrical parameters; one when $c \gg l_0$ and the microcracks “amplify” the stress concentration ($K_I^{eff} > K_I^0$), the other when $c \equiv l_0$ and the microcracks “shield” the macrocrack tip ($K_I^{eff} < K_I^0$). In the intermediate range of c/l_0 these two effects compete. Tacking into account the fact that crack propagation direction, according to Von Mises criterion for isotropic media, is around 60° we expect higher values for that microcrack position. In fact, as plotted in Fig. 5 for $d/l = 1.33$ we obtained a higher ratio K_I^{eff}/K_I^0 for $h = 4$ (corresponding to $\theta = 75^\circ$) than for $h = 8$ ($\theta = 82^\circ$).

On the other hand for $d/l = 5.33$ we obtained a lower ratio K_I^{eff}/K_I^0 for $h = 4$ ($\theta = 26^\circ$) than for $h = 8$ ($\theta = 45^\circ$).

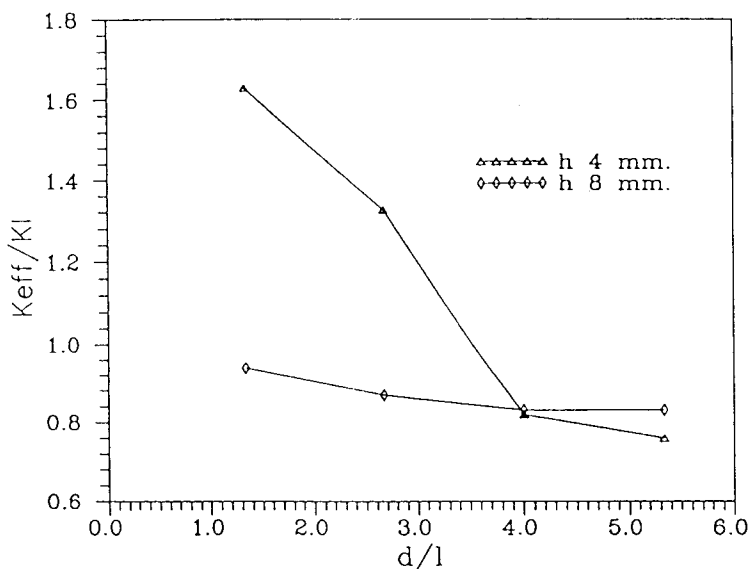


Fig.5: K_{eff}/K_I ratio versus d/l (see fig.1.b) for experimental analysis.

CONCLUSIONS

Experimental interaction between a main crack and one or two microcrack for plane stress polycarbonate specimens has been analysed with caustic method. Theory of elastic interaction based upon Willis polynomial conservation theorem has been used, just for one microcrack collinear to the main crack, as instrument to compare results, that fit approximately well. In the case of two



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microcracks parallel but not collinear to the main crack it has been demonstrated that we have higher stress intensity factors when the microcracks position approaches Von Mises propagation directions. Pictures reporting interactions at different load levels, are also reported.

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