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INTERACTION OF A SPHERICAL HIGH VOLTAGE PROBE WITH THE SPACE ENVIRONMENT: ELECTRON TRAPPING, CURRENT DRAIN AND COLLECTIVE PROCESSES

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ABSTRACT

The effects of electron trapping and plasma turbulence on the current-voltage relation for a positively charged high voltage spherical probe in a magnetized plasma are studied. Electrons can be bound in a force field consisting of an attractive radial electric field and a uniform magnetic field if they have insufficient energy to escape and cannot be captured by the sphere (by virtue of angular momentum conservation). Electrons which fall into such a potential well from infinity may become trapped if they lose energy as a result of wave-particle interactions associated with plasma collective effects. Analysis indicates that the structure of the sheath and the I-V relation for a high voltage probe configured as in the originally planned SPEAR I experiment (with a working plasma contactor) would be substantially influenced by the combined action of trapped electrons and wave-particle scattering processes.

1. INTRODUCTION

This work summarizes the results of a study conducted prior to the launch of SPEAR I (Space Power Experiment Aboard Rocket), for the purpose of aiding in the analysis of that experiment. A primary goal of the SPEAR I experiment was to obtain quantitative measures of the effects of an ambient magnetic field on electron collection by a high voltage spherical probe. The experiment, as originally planned, involved a series of charge-discharge cycles of a pair of spherical probes exposed to the space environment, and observation of the current-voltage relation during the discharge. The series included discharges in which one sphere was initially charged to a potential which ranged between 5 and 44 kV while the second was allowed to float, and discharges in which both were charged to different potentials. A plasma contactor was to have maintained the rocket body at nearly zero potential relative to the ambient plasma, but this did not function during the flight. Katz, et al. [1] have described the SPEAR I experiment in some detail and have analyzed the consequences of the contactor failure.

The aim of the present study is to clarify the role of plasma waves and instabilities in an axisymmetric high voltage electron collecting sheath in a magnetoplasma. This situation would have been realized in SPEAR I when only one sphere was charged. The failure of the plasma contactor destroyed the axisymmetry and complicated the comparison of observations with existing theory. The central question of SPEAR I, i.e., the nature and magnitude of magnetic field effects on electron collection must therefore be regarded as not yet fully resolved, although the work of Katz et al. [1] has provided valuable insights. Future experiments, including a proposed experiment in the SPEAR series will, hopefully, provide observations of the axisymmetric case, which is certainly more amenable to analysis.

In the course of formulating the model to be presented here it became clear that trapped electrons could play an important role in the structure and behavior of a high voltage axisymmetric sheath in a magnetoplasma when scattering due to collective processes is significant. Attempts to deal with this issue quickly became a major thrust of the effort. The electric field provides a potential well in which electrons can be bound if they have insufficient energy to escape and cannot be captured (by virtue of angular momentum conservation). Electrons entering such a well from infinity are not energetically trapped, but may nevertheless bounce within the well for a while before exiting, and may become trapped if they lose energy during this time as a result of some scattering process. The equilibrium trapped electron density within the sheath is determined as a balance between trapping rates, which depend on scattering rates, and trapped electron loss rates, which depend on both the scattering rate and the trapped electron density. Not surprisingly, when the number of trapped electrons becomes significant the effects of trapped electrons on the sheath structure and the current-voltage relation also become large.

2. ANALYSIS

Some previous studies of the current-voltage characteristics of symmetric sheaths are worthy of special mention before we proceed. Langmuir and Blodgett [2] derived expressions which may be used to compute the radius, R_{L-B}, and therefore also the collection current, of sheaths surrounding spherical and cylindrical charged objects, as a function of body potential Φ_0 and environmental parameters, in the absence of a magnetic field.

For a spherical probe of radius R_0 the Langmuir-Blodgett radius is approximately

$$R_{L-B} \simeq 1.13\times10^2 \ (1/j_0)^{2/7}(R_0 \ \Phi_0)^{3/7} \quad (1)$$

where j_0 is the charge flux density at the sphere for zero potential, and all quantities are in Gaussian units.

Parker and Murphy [3] studied the orbits of individual electrons in the vicinity of a charged object in a magnetic field, assuming a steady potential having an axis of symmetry oriented parallel to **B**. These authors concluded, on the basis of conservation of energy and angular momentum, that charged particles approaching the object from infinity along magnetic field lines could not be captured by the object if the displacement between the magnetic field line passing through the initial particle position (outside the sheath) and the symmetry axis exceeded a critical value, now called the Parker Murphy radius, R_{P-M}, which depended on B, the radius of the object, etc.

$$R_{P-M} = R_0 \left[1 + [8e\Phi_0/(m_j \Omega_j^2 R_0^2)]^{1/2} \right]^{1/2} \quad (2)$$

where m_j, Ω_j are the mass and cyclotron frequency of the charged particle.

Trapping is potentially important to sheath structure and I-V characteristics when a substantial portion of the sheath volume lies outside the Parker-Murphy radius. Then trapped electrons can accumulate in a substantial fraction of the total volume of the sheath, can contribute significantly to the total space charge in the sheath, and can constitute a significant reservoir of electrons which can be collected if they diffuse across the magnetic field as a result of collective electric field fluctuations. At low voltages, the Parker-Murphy radius for electrons can be larger than the Langmuir-Blodgett radius, in which case electron trapping should be an insignificant effect. For example, for a charged 10 cm radius sphere in a plasma with $n_e = 10^5$ cm^{-3}, $T_e = .1$ ev, and an ambient magnetic field of .3 Gauss, $R_{P-M} = R_{L-B}$ at about 300 volts, and $R_{P-M} > R_{L-B}$ for lower voltages (note that the Langmuir-Blodgett current is twice the Parker-Murphy limiting current when the two radii are equal). Thus we expect trapping effects to manifest themselves clearly when R_{P-M} is significantly smaller than R_{L-B}; this condition is well satisfied with sphere voltages of several kilovolts, as in the SPEAR experiments. Most previous attempts at controlled experimental studies of magnetic field effects in sheaths have been conducted at relatively low voltages (e.g., Szuszczewicz and Takacs [4]).

Linson [5] argued that plasma instabilities should be taken into account in models of the current-voltage characteristics of sheaths in magnetoplasmas. Without attempting a detailed analysis of collective effects in magnetosheaths, he constructed a phenomenological model for the I-V characteristic of a positively charged satellite which incorporated magnetic field, space charge, and turbulence effects in a simple way. Specifically, Linson solved a cylindrical Poisson equation for an azimuthally symmetric potential, under the assumption that potential gradients transverse to the magnetic field were much larger than gradients parallel to **B**, i.e.:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial\Phi}{\partial r} \right) + \frac{\partial\Phi^2}{\partial z^2} \simeq \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial\Phi}{\partial r} \right) \simeq 4\pi e n_e \quad (3)$$

The charge density en_e was assumed to be constant along a radius in what we have called the trapping plane, with a value determined by the condition $q \equiv (\omega_{pe}/\Omega_e)^2 \simeq 1$, where ω_{pe} is the plasma frequency in the sheath and Ω_e is the gyrofrequency. Thus $n_e = n_q \equiv B^2/4\pi m_e c^2$. Linson argued that collective effects would drive the charge density to this value. With the boundary conditions

$$\Phi(R_\perp) = \Phi'(R_\perp) = 0, \qquad \Phi(R_0) = \Phi_0 \quad (4)$$

the radial part of eq. (3) is an O.D.E. which may be treated as an eigenvalue equation for the transverse sheath radius $R_\perp$. Once $R_\perp$ is known, the current I collected by the object (electrons flowing along a cylindrical flux tube from $\pm\infty$) was found from the relation

$$I = I_0 \ (R_\perp/R_0)^2 \quad (5)$$

where R_0 is the radius of the probe, $I_0 = 2\pi R_0{}^2 e F_0$ is the current collected by the probe when $\Phi_0 = 0$, $F_0 = n_a v_a/(2\pi)^{1/2}$ is the ambient electron flux density, and n_a, v_a are the ambient electron density and electron thermal velocity. Linson does not explicitly mention trapped particles, but the notion seems to be implicit in his model.

Numerous other works (e.g., Borovsky, [6] and [7]; Chodura, [8]; Rubenstein and Laframboise, [9]; Sanmartin, [10]) treat various aspects of the sheath problem, i.e., space charge effects, magnetic field effects, and dynamic effects such as collective oscillations, but none of these works attempt to deal with all three at once. The interaction between magnetic field effects and collective electric field oscillations is of particular importance in affecting high voltage sheath structure, as we shall attempt to show, but a comprehensive quantitative analysis of this problem seems out of reach at present. The phenomenological approach still appears most viable, and the model which we construct below is essentially an extension of Linson's model.

I-V Model Development

We begin by reducing the Poisson equation to one dimension in order to obtain an easily solvable eigenvalue equation for $R_\perp$, but instead of simply dropping the $\partial^2\Phi/\partial z^2$ term in the Laplacian, which leads to unrealistic potential contours near the sphere (makes the sphere look like a very long cylinder), we assume that the equipotential surfaces are ellipsoids with eccentricity which varies with distance from the sphere. Specifically, we assume $\Phi(r,z) = \Phi(\eta)$, where η is defined by the equation

$$r^2 + (C - \eta^2 D)z^2 = \eta^2$$

and C and D are constants. Substituting $\Phi(\eta)$ into Poisson's equation leads to an equation for $\Phi(r, z=0)$ of the form

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\Phi}{\partial r}\right) + \left(\frac{C - r^2D}{r}\right)\frac{\partial\Phi}{\partial r} = 4\pi e n_e \quad (6)$$

The constants C and D may be chosen so that the equipotential surfaces are ellipsoidal at the sheath edge and spherical at $r = R_0$. Since the current collection rate essentially depends on the cross sectional area of the sheath in the plane perpendicular to the magnetic field, the length of the sheath axis parallel to the magnetic field, a_z, is not a critical parameter, and we may take some liberty in approximating this value. For example, if assume that $a_z = R_{L-B}$ then

$$C = (R_\perp/R_{L-B})^2 + R_\perp^2\,[1 - (R_\perp/R_{L-B})^2]/(R_\perp^2 - R_0^2)$$

$$D = [1 - (R_\perp/R_{L-B})^2]/(R_\perp^2 - R_0^2)$$

We take a larger step away from Linson's model in our treatment of the charge density in Poisson's equation. In the absence of particle trapping the electron density in the sheath is mostly controlled by the local sheath potential, because the local flow velocity is then determined primarily by $\Phi(r)$ and flux must be conserved. Plasma instabilities have a greater effect on the charge density when trapping is important, because the dependence of the density of trapped electrons, n_{eT}, on $\Phi(r)$ is less direct. Turbulent scattering is needed to get electrons into the trapping region, as discussed in section 1, and also controls the rate at which trapped electrons diffuse toward the sphere and are lost. The equilibrium density of trapped electrons is determined as a balance between inflow and outflow. It is not obvious, however, that $q = 1$ defines the balance point; we have insufficient knowledge of instabilities in magnetized sheaths to make such a claim with confidence. The value of n_e which makes $q = 1$ may be sensible as an upper limit on density, but turbulent scattering is unlikely to increase n_e if the nonturbulent dynamics tends to make $n_e < n_q$.

There is another area of concern: if we assert that $n_e = n_q$ we lose contact with particle orbit dynamics, and we have no way of knowing whether the velocities required to maintain $q = 1$ (via flux conservation) are physically reasonable. With or without turbulence, the possible range of cross field flow velocities is still governed by the available energy, which is a function of the potential distribution.

In an effort to resolve these concerns we shall try to get closer to the orbit dynamics by making the cross field flow velocity the fundamental quantity in our model and determining the electron density from this, via a flux conservation condition. We introduce turbulence phenomenologically by making the trapped particle diffusion rate, which is a function of the turbulence level, a free parameter in the model. We will not attempt to infer the correct turbulence level a priori, but stay within physically reasonable limits (cross field flow $\leq$ Bohm rate, for example). We shall show the effects of turbulent scattering and of the condition $q \leq 1$ by computing I-V characteristics for several levels of scattering with and without this condition.

Electron Density in the Sheath

The sheath electrons exhibit three distinct types of behavior: direct capture, in which electrons are able to travel from the sheath edge to the sphere, either ballisticly or via drift orbits, without changing their energy or angular momentum; trapping, in which electrons enter the sheath, become trapped, and diffuse across field lines until they are captured; and transient flow, in which electrons essentially flow through the sheath and escape, possibly after a few bounces in the potential well. Direct capture is the primary behavior mode for particles approaching the sheath along field lines within the Parker-Murphy radius. Outside R_{P-M}, either of the other modes is possible, while direct capture is not.

The bulk flow velocities associated with each of these behavior types are different. We determine the electron density in the sheath by solving an equation of the form

$$\frac{\partial}{\partial r}\left(r\, n_{ej}\, V_{rj}\right) \simeq 2P_jF_0 \quad (7)$$

for the trapped and direct capture electrons, and a simple flux conservation condition for transient electrons. In eq. (7) F_0 is the ambient flux density defined following eq. (5). Eq. (7) is obtained from the usual continuity equation by integrating along the magnetic field within the sheath, at a fixed radius r, and suppressing a geometric factor of order unity. The quantities n_{ej} and V_{rj} are averaged one-dimensional representations of the electron density and radial bulk flow speed for species j. P_j is the 'collection fraction' for species j. For example, $P_C(r)$ is the fraction of electrons incident at radius r which are captured directly. P_C vanishes for $r > R_{P-M}$ and is taken to be 1 for $r < R_{P-M}$ (we smooth the transition near $r = R_{P-M}$).

In order to estimate P_T, the collection fraction for electron trapping, we note that in the absence of scattering all the electrons at any point r within the sheath have kinetic energy $e\Phi(r) + k_BT_{ai}$, where k_BT_{ai} is the initial thermal energy of the i^{th} electron, regardless of the path they followed to reach r. In the case of a multi-kilovolt body in the relatively cold ionospheric plasma k_BT_{ai} is negligible, so that the electrons occupy a thin spherical shell of radius $v_\Phi = [2e\Phi(r)/m_e]^{1/2}$ and thickness $\sim v_a \ll v_\Phi$ in velocity space (see fig. 1). Electric field fluctuations may be generated in a number of different ways: this shell distribution may be unstable; pre-sheath instabilities may modulate electron flow into the sheath; or instabilities of the type discussed by Linson may operate. In any case, the result of such fluctuations is likely to be velocity space diffusion which would move electrons down the very steep gradients on the inner and outer surfaces of the shell. Electrons moving inward are energetically trapped if they lose more energy than their initial thermal energy

($k_B T_a$ ~ .1 eV in the ionosphere), and the amount of scattering required to achieve this seems trivial considering the large amount of energy available to drive turbulence in a multi-kilovolt sheath. Electrons moving outward escape the sheath, carrying with them the energy lost by the trapped particles. Energy conservation suggests that the flux into the sheath outside the Parker-Murphy radius should be roughly evenly divided between trapped and transient particles, as long as the energy fluctuations experienced by the electrons as a result of the turbulence are much larger than $k_B T_a$. Since this last condition is so mild, we take P_T ~ 1/2 for electrons entering the sheath outside the Parker-Murphy radius, and let P_T go smoothly to zero for $r < R_{P-M}$, as P_C rises toward 1.

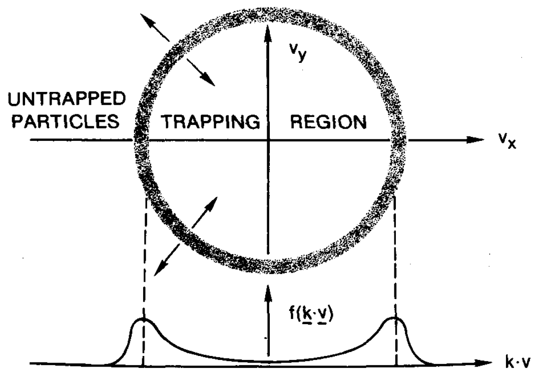

Fig. 1: Schematic representation of the electron distribution function in velocity space.

We note in passing that the amount of charge required to fill the sphere in velocity space which we have designated as the trapping region, and thus prevent further trapping by eliminating the slope on the inner edge of the shell, is orders of magnitude greater than the charge on the probe, for reasonable values of the parameters. Thus space charge effects must be the limiting factor on the total number of trapped electrons if sufficient turbulence is present, which implies that trapped electrons must affect the structure of the sheath.

Given P_C and P_T, the collection fraction for the transient particles, P_{Trans}, is calculated from the relationship

$$P_C(r) + P_T(r) + P_{Trans}(r) = 1 \qquad (8)$$

which simply says that all electrons hitting the sheath exhibit one of the three behavior modes discussed above.

The transient electrons are largely cospatial with the trapped electrons, and their density is much less than n_{eT}, because they spend much less time in the sheath. We approximate their density as follows:

$$n_{e\,Trans} \sim P_{Trans}\, F_0 \sqrt{k_B T_a/[2e\Phi(r)/m_e + k_B T_a]} \qquad (9)$$

and find the total charge density from

$$n_e = n_{eC} + n_{eT} + n_{e\,Trans} \qquad (10)$$

Eq. (16) underestimates $n_{e\,Trans}$, but the error is not significant because $n_{e\,Trans}$ makes a negligible contribution to the total charge density in the sheath when trapping is important. In the no-turbulence /no-trapping limit our results reduce to a value close to the Parker-Murphy limiting current, as one would expect.

Since some electrons must escape from the sheath, we can no longer use the simple relationship in eq. (5) to compute the collected current. Instead, we have

$$I = I_0 \int_0^{R_\perp} (2r/R_0^2)\, [P_C(r) + P_T(r)]\, dr \qquad (11)$$

Velocity Formulas

The average radial velocity of direct capture electrons is the dielectric drift velocity

$$V_{rC} = (c/B\Omega_e)\, v_z \partial E/\partial z = -\, v_z\, (c/B\Omega_e)\, \partial^2\Phi/\partial r \partial z$$

as long as this expression gives a result less than v_z. If this is not the case we assume equipartition of the kinetic energy and take $V_{rC} \simeq -\, v_\Phi/3^{1/2}$. We can approximate

$$\partial\Phi/\partial z \sim \pm\, (a_r/a_z)\, \partial\Phi/\partial r \quad \text{and} \quad v_z \simeq \pm\, v_\Phi/3^{1/2},$$

where a_r and a_z are the axes of our elliptical potential contours; hence, in the drift regime,

$$V_{rC} \simeq -\, (a_r/a_z)\, (v_\Phi/3^{1/2})\, (e/m\Omega^2)\, \partial^2\Phi/\partial r^2 \qquad (12)$$

For trapped electrons we postulate that

$$V_{rT} \simeq -\, \beta_S\, v_\Phi \quad \text{when } r > R_{P-M} \qquad (13)$$

where β_S is a constant which parameterizes the rate of cross field diffusion due to turbulence, and that

$$V_{rT} \to -\, v_\Phi \quad \text{as} \quad r \to R_0 \quad \text{when} \quad r < R_{P-M}. \qquad (14)$$

We have experimented with a number of variations of these velocity formulas, and find that the results are extremely insensitive to changes in coefficients, smoothing prescriptions, etc. In fact it will be seen in the next section that varying the value of β_S through two orders of magnitude results in only a factor of 4 variation in the current collected by the probe.

Eq's (6) through (14) may be combined with the boundary conditions

$$\Phi(R_\perp) = \Phi'(R_\perp) = 0, \qquad \Phi(R_0) = \Phi_0$$

$$n_{e\,Trans} = n_a \quad \text{and} \quad n_{eC} = n_{eT} = 0 \quad \text{at} \quad r = R_\perp$$

and solved for $R_\perp$ and for the radial profiles of potential, charge density, etc., for a specified value of probe potential. The electron current can be computed from these profiles and (11).

Model results

Fig. 2 shows current-voltage characteristics for a variety of values of β_S, for a probe with a radius of 10 cm. Ambient plasma density and temperature are 10^5 cm^{-3} and .1 eV, respectively, and the ambient magnetic field is .3 Gauss. As noted above, the I-V curves are surprisingly insensitive to β_S.

Consistent with our philosophy of making the radial trapped particle diffusion rate a free parameter, we have placed no constraints on the value of q defined earlier. We find that q is always less than 1 for the larger values of β_S in figure 2, but q values do exceed unity at some radii for smaller values of β_S.

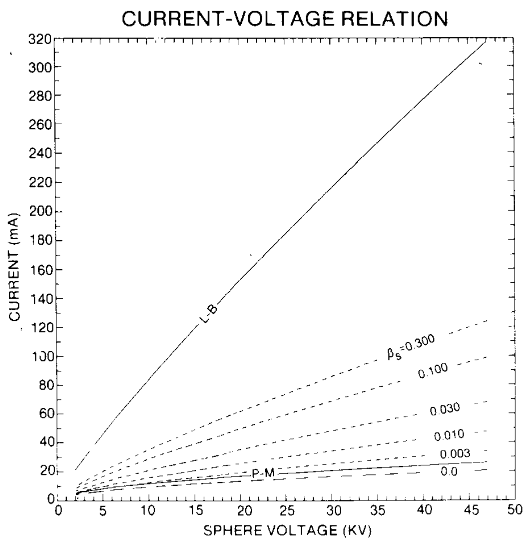

Fig. 2: Current voltage relations based on the model outlined in the text. The value of 'q' is not limited in these calculations.

Recalling Linson's claim that instabilities would enhance cross field diffusion when $q \geq 1$, we recalculated the I-V curves after modifying our expression for the radial flow speed of trapped electrons in such a way that V_{rT} increases as needed to maintain $q \leq 1$, while allowing eq's (13) and (14) to operate wherever $q < 1$. The modified I-V curves are shown in fig 3. Note that the effect of the q limit is to cause the curves for $\beta_S \leq .03$ to overlap. If we postulate that small-to-moderate rates of radial diffusion are most probable, we would conclude that the most likely I-V characteristic would lie within or just above this group of curves, i.e., between the curves for $\beta_S = 0.03$ and $\beta_S = 0.1$ in fig. 2.

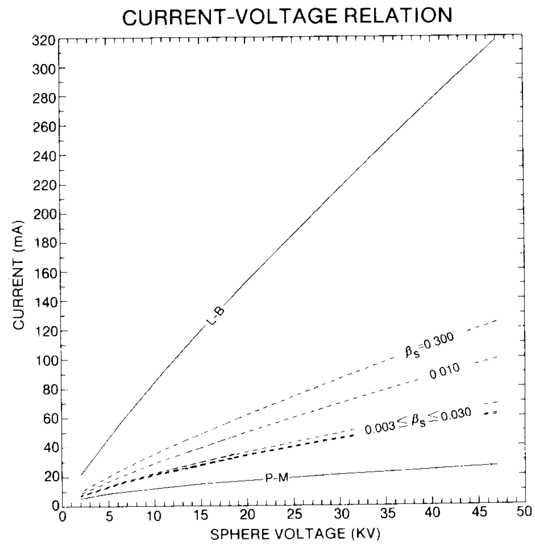

Fig. 3: Current voltage relations similar to those in Fig. 2, except that the value of 'q' is limited in these calculations by the assumption that radial diffusion is enhanced when $q < 1$.

3. SUMMARY AND DISCUSSION

We have presented the results of model calculations intended to provide insight into the consequences of electron trapping and collective electric field fluctuations in an axisymmetric high voltage magnetized sheath such as would have been encountered in the SPEAR I experiment had the plasma contactor operated as planned. Considering the phenomenological nature of the model, the results should be regarded as qualitative rather than quantitative, but it would nevertheless be useful to test them experimentally.

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REFERENCES

[1] Katz, I., G.A. Jongeward, V.A. Davis, M.J. Mandel, R.A. Kuharski, J.R. Lilley, Jr., W.J. Raitt, D.L. Cooke, R.B. Torbert, G. Larson, and D. Rau, Structure of the bipolar plasma sheath generated by SPEAR I, J. Geophys. Res., 94, 1450, 1989.

[2] Langmuir, I., and K. Blodgett, Currents limited by space charge between concentric spheres, Phys. Rev, 24, 49, 1924.

[3] Parker, L.E., and B.L. Murphy, Potential buildup on an electron-emitting satellite, J. Geophys. Res., 72, 1631, 1967.

[4] Szuszczewicz, E.P., and P.Z. Takacs, Magnetosheath effects on cylindrical Langmuir probes, Phys. Fluids, 22, 2424, 1979.

[5] Linson, L.M., Current-voltage characteristics of an electron-emitting satellite in the ionosphere, J. Geophys. Res., 74, 2368, 1969.

[6] Borovsky, J.E., The theory of Langmuir probes in strong electrostatic potential structures, Phys. Fluids, 29, 718, 1986.

[7] Borovsky, J.E., The dynamic sheath: objects coupling to plasmas on electron-plasma-frequency timescales, Phys. Fluids, 31, 1074, 1988.

[8] Chodura, R., Plasma-wall transition in an oblique magnetic field, Phys. Fluids, 25, 1628, 1982.

[9] Rubenstein, J., and J.G. Laframboise, Theory of a spherical probe in a collisionless magnetoplasma, Phys. Fluids, 25, 1174, 1982.

[10] Sanmartin, J.R., Theory of a probe in a strong magnetic field, Phys. Fluids, 13, 103, 1970.