THE INTERACTION OF ELECTROMAGNETIC WAVES WITH SOME NATURAL SURFACES

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
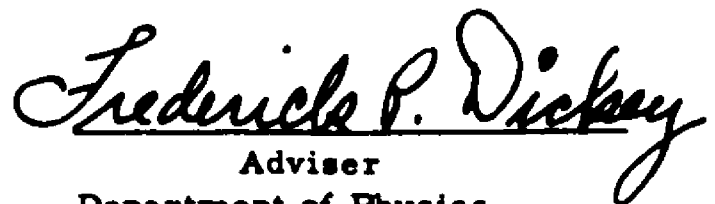School of The Ohio State University

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CHAPTER I

THE PROBLEM OF SCATTERING BY NON-UNIFORM SURFACES

A. Introduction

One of the major unsolved problems of electromagnetic theory is that of the interaction of electromagnetic radiation with non-uniform surfaces, particularly those natural surfaces that constitute the surface of the earth. Apart from its intrinsic interest, a general solution of this problem would have many applications, both scientific and technical. A complete solution for all frequencies and classes of surface is out of the question at the present time. However, there exist certain general relations between the properties of the surface and those of the radiation in interaction with it. There are also certain restricted wavelength intervals and classes of surface for which detailed solutions of the scattering problem may be obtained. It is the purpose of this study to derive some of these general relations, and to carry out the detailed solution for two special surfaces of practical interest in the microwave region.

B. The Taxonomic Problem and the Scattering Problem

There are two major difficulties that face the investigator of such problems. The first is that of solving the electromagnetic scattering problem for any particular, properly specified non-uniform surface. Unfortunately, present theories of scattering from

irregular surfaces are not sufficiently well developed to permit an exact solution of the scattering problem except in a few special cases. Thus approximate solutions must be used.

The second, and in some respects the more difficult, problem is that of describing the surfaces of interest, or at least of classifying them into categories that are significant for the scattering problem. These categories may be chosen intuitively, on the basis of the appearance of the surface; experimentally, on the basis of the observed characteristics of the interaction of the surface with radiation; or on theoretical grounds, on the basis of the kind of model used to represent the surface when formulating the scattering problem. It is clear that in any useful taxonomy, all three approaches should lead to the same grouping or association of terrains.

A good idea of the rather primitive state of the classification problem as it applies to the interaction of light waves with natural surfaces may be had from Penndorf's review[1] of Krinov's reflectivity data.[2] At microwave frequencies, the situation is similar.[35] Indeed even the relatively simple problem of describing the surface of the sea is only now,[3] after several decades of intensive research, approaching a state that is suitable for the solution of scattering problems.

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C. The Role of Models

As a consequence of these two difficulties, it has been customary to approach the problem of calculating the interaction of electromagnetic radiation with such surfaces by the use of models. That is, use has been made of idealized surfaces which are more or less amenable to theoretical treatment, and more or less representative of the category of natural surface under consideration. Not all surfaces can be represented in this way at the present time, but for those which can, the specification of a few basic parameters of the surface (which may be chosen on the basis of theoretical convenience or experimental evidence) will allow one to estimate the interaction of the surface with radiation.

In the past, most surface models have fallen into one of two classes, depending on which of these two principal difficulties their authors have wished to confront. In the first group[4,5,6] the models have been chosen so as to avoid the difficulties of the scattering problem, by representing the surface with assemblies of objects of known scattering properties (spheres, point scatterers, facets, etc.). The number, size, etc., of these scatterers have provided the parameters of the models, to be fitted by recourse to experiment. Insofar as it has been the aim of such models to predict the statistical properties of the scattered radiation, and to provide empirical formulas

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for its angular dependence, they have been quite successful. However, it is often rather difficult to relate the parameters of the model to directly observable properties of the surface.

The second group of models[7-16] have been aimed at providing detailed solutions of certain well defined scattering problems, usually of the boundary-value type, and are often of great theoretical interest. However, the lack of any real resemblance between the model surface and those encountered in practice have made it difficult to apply them directly.

D. General Approach

In contrast to such models, an effort has been made here to develop "complete" models which, while simple enough to permit the scattering problem to be solved with reasonable accuracy, still bear a close resemblance to the actual surface under consideration. Thus the parameters of the model, and consequently the characteristics of the scattered radiation, may be calculated from the surface properties alone. Conversely, once a model has been validated by scattering measurements, its parameters may be used to classify the surface in a rational and quantitative way.

In the next chapter (Chapter II) the scattering coefficients are defined for a general surface. The relation between them and several parameters of particular interest (the back-scattering coefficient,

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the absorption coefficient, the albedo) are derived. The scattering

coefficients for two "complete" model surfaces are then worked out

in detail. One of the models is applicable to slightly rough, con-

tinuous surfaces, and the other to surfaces covered with vegetation

in the form of long, thin lossy cylinders. In Chapter III the results

are applied to the interpretation of the radar return from terrain

(back-scattering).

In Chapter IV, the reciprocity properties of the scattering

coefficients are used to derive Kirchhoff's radiation law in a form

which takes account of the angular dependence of the emitted radi-

ation, and the polarization properties of the surface. The results

are then applied to the interpretation of measurements of the ap-

parent temperatures of some natural surfaces.

CHAPTER II
THE SCATTERING COEFFICIENTS

A. Introduction

An analysis of the interaction of electromagnetic waves with natural surfaces may most conveniently be carried out in terms of the differential scattering coefficients of the surface. It is the purpose of this chapter to derive these coefficients for two particular surface models. Before doing so, it is necessary to define them. Although many different definitions have been used in the past (differing chiefly in the manner in which normalization with respect to surface area and solid angle was carried out) the coefficient to be used here is a generalization of one frequently used in radar-return work.[17] Consider an infinite surface, forming, on the average, a plane, although over small regions it may have any complexity of structure (see Fig. 1). If radiation of intensity I_0(watts/meter^2) falls at an angle of incidence θ_0 and azimuth ϕ_0 on a given element of surface area S, and the intensity of the scattered radiation in the direction $\theta_s \phi_s$ at a distance R from S is I_s, then the scattering parameter $\gamma(\theta_0 \phi_0; \theta_s \phi_s)$ is defined by the equation

$$(1) \qquad I_0 \gamma(\theta_0 \phi_0; \theta_s \phi_s) S \cos \theta_0 = 4\pi R^2 I_s$$

or

(2) $\gamma(\theta_o\phi_o; \theta_s\phi_s) = (4\pi R^2 I_s)/(I_o S \cos \theta_o)$.

In this second expression, the numerator is the total power which would be scattered by S if it scattered isotropically into 4π steradians with the same intensity as it does in the scattering direction; the denominator is the power incident on S; and γ is their ratio.
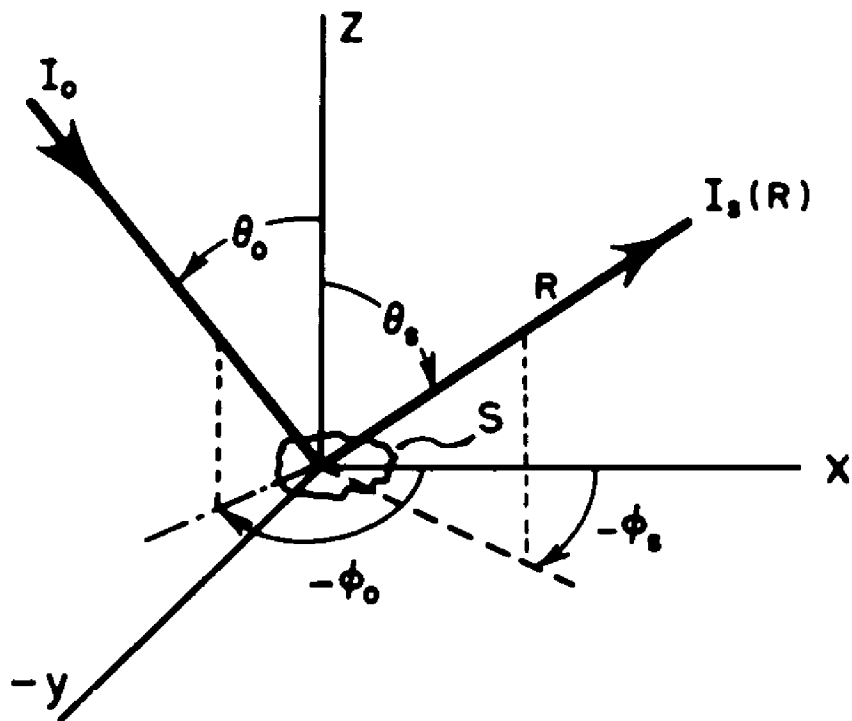
Fig. 1. Geometry of the scattering problem.

For the types of random surface of interest, it will be assumed that I_s is proportional to S, and inversely proportional to R^2 at sufficiently large distances from S; under these conditions γ becomes independent of R and S, and is a characteristic of the surface. It is clear that S, and thus the projected area of the

illuminating beam of radiation, must be much larger than any sig-
nificant structural features of the surface if γ is also to be inde-
pendent of the particular area at which the illumination is directed.

It is clear that for the general surface, γ should refer to,
and is a function of, the specific state of polarization of both the in-
cident and the scattered radiation. Any two orthogonal polarization
states may be used for this purpose. In this study, the conventions
and terminology of engineering usage have been chosen, although
they are somewhat ambiguous. These polarization states are indi-
cated via the notation:

	Incident Wave	Scattered Wave
γ_{vv}	Vertically Polarized	Vertically Polarized
γ_{hh}	Horizontally Polarized	Horizontally Polarized
γ_{vh}	Vertically Polarized	Horizontally Polarized
γ_{hv}	Horizontally Polarized	Vertically Polarized

The words "vertically polarized" refer to a wave with its
electric vector in the plane of incidence (parallel polarization); the
words "horizontally polarized" refer to a wave with its electric
vector in the plane of the surface (perpendicular polarization).

There are certain general relationships between these four
scattering coefficients that are consequences of the electromagnetic
reciprocity theorem.[20, 21] For example, consider first the situation

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in which the incident radiation is horizontally polarized, comes from the direction $\theta_0\phi_0$, has intensity I_0 at the surface, and has a horizontal component of scattered intensity I_s in the direction $\theta_s\phi_s$ at distance R from S. Then, from Eq. (2),

$$(3) \qquad \cos\theta_0\, Y_{hh}(\theta_0\phi_0;\ \theta_s\phi_s) = 4\pi R^2 I_s/(SI_0)\ .$$

Next consider the reverse situation in which horizontally polarized radiation is incident from the direction $\theta_s\phi_s$, has intensity I_0 at the surface, and produces scattered radiation with horizontal component of scattered intensity I_s' in the direction θ_0,ϕ_0 at distance R from S. Then from Eq. (2),

$$(4) \qquad \cos\theta_s\, Y_{hh}(\theta_s\phi_s;\ \theta_0\phi_0) = 4\pi R^2 I_s'/(SI_0)\ .$$

Now, from the reciprocity theorem, $I_s = I_s'$.[*] Thus, from (3) and (4) one obtains

$$(5a) \qquad \cos\theta_0\, Y_{hh}(\theta_0\phi_0;\ \theta_s\phi_s) = \cos\theta_s\, Y_{hh}(\theta_s\phi_s;\ \theta_0\phi_0)\ .$$

Similarly, for the other polarization states,

$$(5b) \qquad \cos\theta_0\, Y_{hv}(\theta_0\phi_0;\theta_s\phi_s) = \cos\theta_s\, Y_{vh}(\theta_s\phi_s;\theta_0\phi_0)$$

$$(5c) \qquad \cos\theta_0\, Y_{vh}(\theta_0\phi_0;\theta_s\phi_s) = \cos\theta_s\, Y_{hv}(\theta_s\phi_s;\theta_0\phi_0)$$

$$(5d) \qquad \cos\theta_0\, Y_{vv}(\theta_0\phi_0;\theta_s\phi_s) = \cos\theta_s\, Y_{vv}(\theta_s\phi_s;\theta_0\phi_0)\ .$$

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[*]See Appendix II.

In what follows, the coefficients will often be written in the form $Y(o, s)$ or $Y(s, o)$. Here (o) refers to the direction $\theta_o \phi_o$ and (s) refers to the direction $\theta_s \phi_s$; the first letter or pair of angles within the brackets refers to the direction <u>from which</u> the radiation is incident, and the second letter or pair of angles refers to the direction <u>into which</u> the radiation is scattered.

It is often convenient to account for specular reflection (as from a plane surface or a grating), by introducing Y's with the properties of delta-functions. This representation may easily be obtained by integrating Eq. (1) over scattering angles in the vicinity of the specular beam,

$$\frac{I_o \cos \theta_o S}{4\pi} \int Y(o, s)\, d\Omega_s = \int R^2 I_s\, d\Omega_s$$

where $d\Omega_s$ is the element of solid angle. Now the right hand side represents the total power in the specularly reflected beam, which may be written as $\rho(o, s) I_o S \cos \theta_o$, where $\rho(o, s)$, the reflection coefficient, is just the fraction of power in the incident beam that appears in the specularly reflected beam. Thus if $Y(o, s)$ has the form of a constant multiplied by $\delta(\theta_s - \theta_{sp}) \delta(\phi_s - \phi_{sp})$, where θ_{sp}, ϕ_{sp} define the specular direction, then

(6) $\qquad Y(o, s) = \dfrac{4\pi \rho(o, s)}{\sin \theta_{sp}}\, \delta(\theta_s - \theta_{sp}) \delta(\phi_s - \phi_{sp})$.

Appropriate subscripts must be supplied for the four polarization states.

As an example of the use of this representation, one may consider a perfectly flat surface, where $\theta_{sp} = \theta_o$ and $\phi_{sp} = \phi_o + \pi$. The reflection coefficients are $\rho_{hh} = |R_h|^2$; $\rho_{vv} = |R_v|^2$; $\rho_{vh} = \rho_{hv} = 0$; here R_h and R_v are the Fresnel coefficients (see Eq. (80)).

To conclude this section, certain commonly used parameters will be expressed in terms of the Y's. The first, the "radar cross-section per unit area of terrain", designated in the literature[21] by σ_o, is found from

$$\sigma_o(\theta_o) = \cos\theta_o \, Y(\theta_o\phi_o; \theta_o\phi_o)$$

and may refer to any of the four polarization combinations.

The albedo of a surface usually refers to the fraction of incident radiation of random polarization rescattered by the surface for a band of frequencies in the optical region. Here the albedo A is defined as the fraction of the power incident on the surface from the direction $\theta_o\phi_o$ (at a specific polarization and frequency) that is rescattered. With this definition the albedo is

$$A(\theta_o, \phi_o) = \int \frac{I_s R^2 \, d\Omega_s}{S I_o \cos\theta_o}$$

so that, from Eq. (1), taking account of the polarization properties of the surface, the albedo $A_v(\theta_0\phi_0)$ for vertically polarized incident radiation is

$$(7a) \qquad A_v(\theta_0\phi_0) = (4\pi)^{-1} \int \left[\gamma_{vv}(o, s) + \gamma_{vh}(o, s)\right] d\Omega_s.$$

There is a corresponding expression for horizontally polarized incident radiation

$$(7b) \qquad A_h(\theta_0\phi_0) = (4\pi)^{-1} \int \left[\gamma_{hh}(o, s) + \gamma_{hv}(o, s)\right] d\Omega_s.$$

Another surface parameter of interest is the absorption coefficient (a), defined here as the fraction of power (of a given polarization and frequency) incident on the surface from the direction $\theta_0\phi_0$ that is absorbed by the surface. Clearly, for the types of surfaces considered here, which are assumed to be thick enough so that no energy is transmitted through the body they define, the absorption coefficient is just unity minus the albedo,

$$(8a) \qquad a_h(\theta_0\phi_0) = 1 - A_h(\theta_0\phi_0)$$

$$(8b) \qquad a_v(\theta_0\phi_0) = 1 - A_v(\theta_0\phi_0) .$$

B. The Slightly Rough Surface

The first surface model for which the scattering coefficients will be calculated in detail is one that may be called the "slightly

rough surface." This is a practically plane surface in which the slope is everywhere much less than unity, and for which the maximum height above the mean plane surface is much less than a wavelength. The surface may be described formally by giving its elevation $z = f(xy)$ at every point in the x-y plane (see Fig. 2).
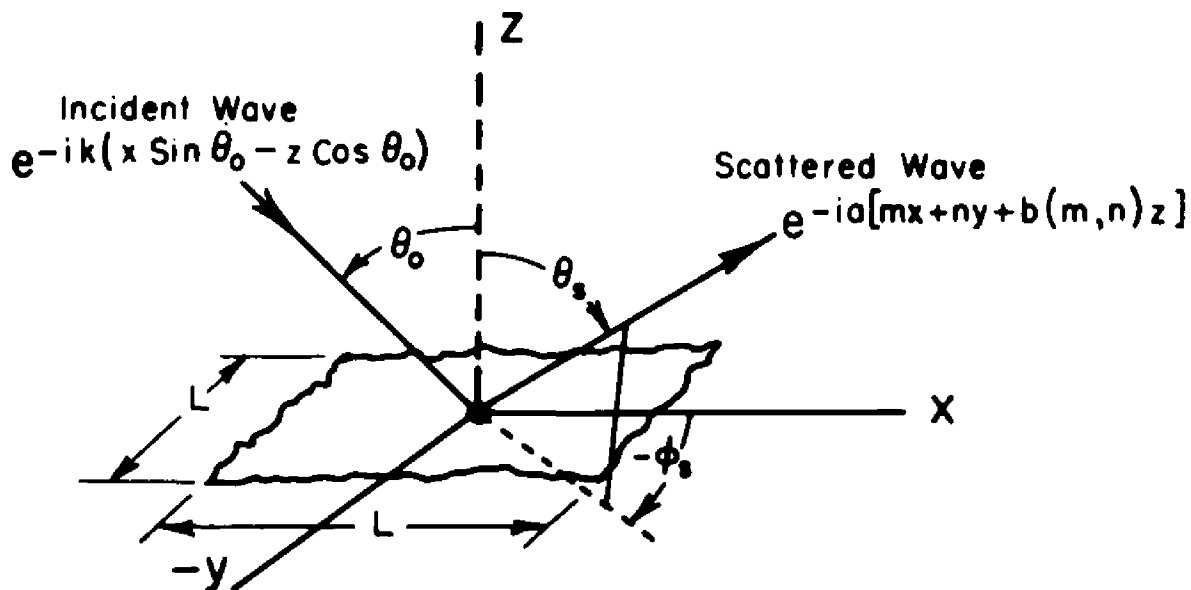
Fig. 2. Scattering from a slightly rough surface.

Now if attention is confined to a large square of side L, the altitude may be expanded in a Fourier series

$$(9) \qquad z = f(x, y) = \sum_{-\infty}^{\infty} \sum_{-\infty}^{\infty} P(m, n) \exp\left[-ia(mx+ny)\right] ,$$

$$a = 2\pi/L \qquad m, n \text{ integers}$$

where, because f is real, $P(m,n) = P^*(-m,-n)$. The interest here is not in regular surfaces of a periodic type,[12] but rather in randomly rough surfaces; the Fourier decomposition permits a straightforward

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definition of what is meant by randomness. According to S. O. Rice,[16] the surface is considered a random one if an average, designated by $<>$, over an infinite set of such elements of side L leads to the following properties of the coefficients;

(10) $<P(m,n)> = 0$

which implies the randomness of the individual coefficients;

(11) $<P(m,n)P^*(m,n)> = (\pi/L)^2 W(p,q)$

$p = 2\pi m/L \qquad q = 2\pi n/L$

which implies that the average exists, and defines the spectral density function $W(p,q)$ for the surface; and finally

(12) $<P(m,n)P(u,v)> = 0 \quad m \neq -u \qquad n \neq -v$,

which implies the independence of the individual coefficients. It is also convenient to define the mean square surface roughness,

(13) $\overline{z^2} = <f^2(x,y)>$

it being assumed that the mean surface elevation is zero, i.e., $<f(x,y)> = 0$. The correlation function $\rho(\xi,\eta)$ for the surface is defined by

(14) $\overline{z^2}\rho(\xi,\eta) = <f(x+\xi, y+\eta) f(xy)>$,

i.e., $\rho(o,o) = 1$. The functions $\overline{z^2}\rho(\xi,\eta)$ and $W(pq)$ are the Fourier transforms of each other.

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The scattering from such a surface is calculated by a perturbation method introduced by Rayleigh,[18] and extended to the electromagnetic case in detail by S. O. Rice[16] and others.[15] It is based upon an expansion of the scattered field into a series of plane waves $E(m,n;z)e^{i\omega t}$, where, in the notation of Rice, which will be followed as far as possible in what follows,

$$E(m,n;z) = \exp\left[-ia(mx+ny) - ib(m,n)z\right]$$

$$(15) \qquad b(m,n) = \begin{cases} \left[K^2 - a^2m^2 - a^2n^2\right]^{\frac{1}{2}} & m^2+n^2 < K^2/a^2 \\ -i\left[a^2m^2 + a^2n^2 - K^2\right]^{\frac{1}{2}} & m^2+n^2 > K^2/a^2 \end{cases}$$

In the simplest case, in which the surface is a perfect conductor, and the incident wave is horizontally polarized, the total electric fields in the region above the surface may be written

$$E_x = \sum_{-\infty}^{\infty} A_{mn} E(m,n;\ z)$$

$$(16) \qquad E_y = 2i \sin(Kz\cos\theta_0)\exp(-iKx\sin\theta_0) + \sum B_{mn} E(m,n;z)$$

$$E_z = \sum C_{mn} E(m,n;z)$$

where the first term in E_y is just the sum of the incident and specularly reflected waves for a perfectly flat surface, and A_{mn}, B_{mn}, C_{mn} are the amplitudes of the scattered waves (relative to an incident wave of unit amplitude). The horizontal component of the

scattered field E_h^s in the direction specified by the integers m, n, is then

(17) $E_h^s(m, n) = (-A_{mn} \sin \phi_s + B_{mn} \cos \phi_s) \, E(m, n; z)$;

the corresponding vertical component of the scattered field is

(18) $E_v^s(m, n) = (-A_{mn} \cos \theta_s \cos \phi_s - B_{mn} \cos \theta_s \sin \phi_s$

$+ C_{mn} \sin \theta_s) \, E(m, n; z)$.

To evaluate the scattering cross-sections for the surface it is convenient to average over a small range of solid angles. By comparing equivalent expressions for a plane wave specified by the integers (m, n) or the scattering angles $(\theta_s \phi_s)$, one finds that $am = K \sin \theta_s \cos \phi_s$; $an = K \sin \theta_s \sin \phi_s$. Consequently the differential unit of solid angle may be found by evaluating the Jacobian of the transformation, to give

(19) $d\Omega_s = \sin \theta_s \, d\theta_s \, d\phi_s = \left[\cos \theta_s\right]^{-1} d(am/K) \, d(an/K)$,

where the differentials $d(am/K)$ are small in the limit $L \rightarrow \infty$. Thus the power P^s scattered by the surface per unit solid angle in the direction $\theta_s \phi_s$ is, for horizontal polarization,

(20) $P_h^s = \dfrac{z_o}{2} \left|E_h^s(m, n)\right|^2 \dfrac{\Delta m \Delta n}{d\Omega_s} L^2 \cos \theta_s$

$= \dfrac{z_o}{2} (K/a)^2 \cos^2\theta_s \left|-A_{mn} \sin \phi_s + B_{mn} \cos \phi_s\right|^2 L^2$

where z_0 is the impedance of free space and where the summation of the A_{mn} and B_{mn} over a small range $\Delta m \Delta n$ of m and n has been approximated by their mean values multiplied by the intervals. Keeping in mind the definition of γ and P_h^s, and the fact that the incident wave had unit field strength, (or intensity $z_0/2$), one finds

$$(21) \qquad \gamma_{hh}(o, s) = \frac{4\pi P_s}{(z_0/2) \cos\theta_0 L^2}$$

or

$$(22) \qquad \gamma_{hh}(\theta_0, o; \theta_s \phi_s) = 4\pi \left(\frac{\cos^2\theta_s}{\cos\theta_0} \right) (K/a)^2 \left| B_{mn} \cos\phi_s - A_{mn} \sin\phi_s \right|^2$$

and similarly

$$(23) \qquad \gamma_{hv}(\theta_0, o; \theta_s \phi_s) = 4\pi \left(\frac{\cos^2\theta_s}{\cos\theta_0} \right) (K/a)^2 \left| \begin{array}{c} -A_{mn} \cos\theta_s \cos\phi_s \\ -B_{mn} \cos\theta_s \sin\phi_s \\ +C_{mn} \sin\theta_s \end{array} \right|^2$$

where, of course, the values of m and n are those appropriate to the scattering direction $\theta_s \phi_s$.

Now for surfaces such that $|f| \ll \lambda$ and $|\partial f/\partial x|$; $|\partial f/\partial y| \ll 1$, i.e., for surfaces which have everywhere an elevation much less than a wavelength, and a slope much less than unity, one can obtain the coefficients $A_{mn} B_{mn} C_{mn}$ by a perturbation calculation. For a surface of arbitrary conductivity and dielectric constant, and for a horizontally polarized incident wave, the coefficients have been worked out by Rice.[16] To the first order in (f/λ) they are given by

$$A_{mn} = 2iU(a^2 mn) \, P(m-\nu, n)(Dd)^{-1}$$

(24)
$$B_{mn} = 2iU(-a^2 m^2 - bc) \, P(m-\nu, n)(Dd)^{-1}$$

$$C_{mn} = 2iU(anc) \, P(m-\nu, n)(Dd)^{-1}$$

where

$$U = (\sigma^2 - \tau^2) \left[1 + (\tau \cos \theta_0')(\sigma \cos \theta_0)^{-1} \right]^{-1}$$

$$a\nu = K \sin \theta_0$$

$$K = 2\pi/\lambda$$

λ = free space wavelength

$$\sigma = iK$$

$$\tau = \sigma(\epsilon_1 - i\epsilon_2)^{\frac{1}{2}}$$

ϵ_1 = relative dielectric constant of surface material

$$\epsilon_2 = g/(\omega \epsilon_0)$$

g = conductivity of surface material

ϵ_0 = permittivity of free space

$$\cos \theta_0' = \left[1 - (\sigma/\tau)^2 \sin^2 \theta_0 \right]^{\frac{1}{2}}$$

$$d = b + c$$

$$ib = \left[\sigma^2 + a^2(m^2 + n^2) \right]^{\frac{1}{2}}$$

$$ic = \left[\tau^2 + a^2(m^2 + n^2) \right]^{\frac{1}{2}}$$

$$D = a^2 m^2 + a^2 n^2 + bc$$

and the $P(m-\nu, n)$ are to be found from Eq. (9). When all the algebraic reductions are carried out, one finds that for a slightly rough surface with arbitrary complex dielectric constant $\epsilon' = \epsilon_1 - i\epsilon_2$, the

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scattering parameters for horizontal incidence are given by

$$\gamma_{hh}(\theta_0 0; \theta_s \phi_s) = \frac{4}{\pi} \overline{z^2} K^4 \cos \theta_0 \cos^2\theta_s \, T_{hh} \, I$$

(25)

$$\gamma_{hv}(\theta_0 0; \theta_s \phi_s) = \frac{4}{\pi} \overline{z^2} K^4 \cos \theta_0 \cos^2\theta_s \, T_{hv} \, I$$

where

$$T_{hh} = \left| \frac{(\epsilon' - 1) \cos \phi_s}{\left[\cos \theta_0 + \sqrt{\epsilon' - \sin^2\theta_0}\right]\left[\cos \theta_s + \sqrt{\epsilon' - \sin^2\theta_s}\right]} \right|^2$$

$$T_{hv} = \left| \frac{-\sin\phi_s \sqrt{\epsilon' - \sin^2\theta_s}\,(\epsilon' - 1)}{\left[\cos \theta_0 + \sqrt{\epsilon' - \sin^2\theta_0}\right]\left[\epsilon'\cos \theta_s + \sqrt{\epsilon' - \sin^2\theta_s}\right]} \right|^2$$

$$I = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{ig(\xi,\eta)} \rho(\xi,\eta) \, d\xi \, d\eta$$

$$g(\xi \eta) = K\xi(\sin \theta_s \cos \phi_s - \sin \theta_0) + K\eta \sin\theta_s \sin \phi_s \, .$$

In deriving the expression I, use was made of the fact that the correlation function $\overline{z^2}\rho(\xi\eta)$ is the Fourier transform of the power spectral density $W(p,q)$ for the surface.

The corresponding results for vertically polarized incident radiation may be found in an entirely similar way by utilizing the corresponding coefficients (ABC) for that case. Since these were not given by Rice, they are worked out in the Appendix. They lead to the formulas

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$$Y_{vv}(\theta_0 0; \theta_s \phi_s) = \frac{4}{\pi} \overline{z^2} K^4 \cos\theta_0 \cos^2\theta_s \; T_{vv} \; I$$

(26)

$$Y_{vh}(\theta_0, 0; \theta_s \phi_s) = \frac{4}{\pi} \overline{z^2} K^4 \cos\theta_0 \cos^2\theta_s \; T_{vh} \; I$$

where

$$T_{vv} = \left| \frac{(\epsilon'-1)\left[\epsilon'\sin\theta_0\sin\theta_s - \cos\phi_s \sqrt{\epsilon'-\sin^2\theta_0}\sqrt{\epsilon'-\sin^2\theta_s}\right]}{\left[\epsilon'\cos\theta_0 + \sqrt{\epsilon'-\sin^2\theta_0}\right]\left[\epsilon'\cos\theta_s + \sqrt{\epsilon'-\sin^2\theta_s}\right]} \right|^2$$

$$T_{vh} = \left| \frac{-(\epsilon'-1)\sin\phi_s\sqrt{\epsilon'-\sin^2\theta_0}}{\left[\epsilon'\cos\theta_0 + \sqrt{\epsilon'-\sin^2\theta_0}\right]\left[\cos\theta_s + \sqrt{\epsilon'-\sin^2\theta_s}\right]} \right|^2 .$$

If the surface is isotropic, i.e., if the correlation function $\rho(\xi,\eta)$ has the property $\rho(\xi,\eta) = \rho\left(\sqrt{\xi^2+\eta^2}\right)$, then one may introduce polar coordinates

$$\xi = r \cos\psi \qquad \eta = r \sin\psi$$

and, by integrating over the polar angle, obtain

(27) $$I = 2\pi \int_0^\infty r\, \rho(r)\, J_0\left[Kr(\sin^2\theta_0 + \sin^2\theta_s - 2\sin\theta_0\sin\theta_s \cos\phi_s)^{\frac{1}{2}}\right] dr.$$

The scattering coefficients have an interesting property. They show that in the approximation to which they are correct, the scattering from a slightly rough surface is the product of two factors, one of which (I) involves only the surface structure, but is independent of the material. The other (T_{hh} etc.) depends on the complex dielectric constant but is independent of the structure of the surface.

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The results obtained above will be used in Chapter III to calculate the back scattering of microwaves from asphalt and concrete surfaces. It may be of interest to note here these results can also be used to calculate the "diffusion" of light waves by very highly polished glass or metal surfaces. Maréchal[36] has shown, in a much less detailed calculation, that the total diffuse scattering from such a surface is proportional to the mean square surface roughness $(\overline{z^2})$, as is also obviously predicted by Eqs. (26) and (27). These equations should also predict the dependence of the "diffused" light on polarization, dielectric constant, scattering and incidence angles, etc.

C. The Surface of Long, Thin Lossy Cylinders

The second surface model for which the scattering coefficients will be calculated in detail is one which, it is hoped, can represent terrain surfaces covered with vegetation in the form of long, thin lossy cylinders. It is assumed that the cylinders are placed at random over the surface, and are far enough apart (a few wavelengths, see Reference 35) so that the single scattering approximation may be used. However, in order to account for the attenuation of the incident radiation as it penetrates the scattering layer, it is supposed that the incident field is equal to the free space field multiplied by an exponential attenuation factor. The reason for this

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assumption is to bring the single scattering approximation into at least qualitative agreement with the physical facts of the situation. It is clear that the incident wave must be attenuated as it penetrates a surface of infinitely long, lossy cylinders. The single scattering approximation (in the strict sense, where the incident field is taken equal to the free space field in the absence of scatterers), entirely fails to account for this attenuation. Moreover, the scattered radiation in that approximation is a function of the length of the cylinders, which is also unreasonable physically.

The assumption of an exponentially attenuated incident wave is equivalent to assigning an imaginary part to the propagation constant, that is to say, it is an approximation in the same spirit as the conventional "multiple scattering" approximation.[19, 35] The latter also assigns to the medium an effective propagation constant which may be calculated in terms of the number of particles per unit volume of the medium and the forward scattering amplitude. The relation between the two approximations is discussed at the end of this chapter.

The scattering from a single cylinder, due to the incident field just described, will first be calculated. The cylinder is taken with one end, its upper end, at the origin of coordinates (see Fig. 3), and with its axis specified by the unit vector $\vec{n}$ and the angles θ, ϕ.
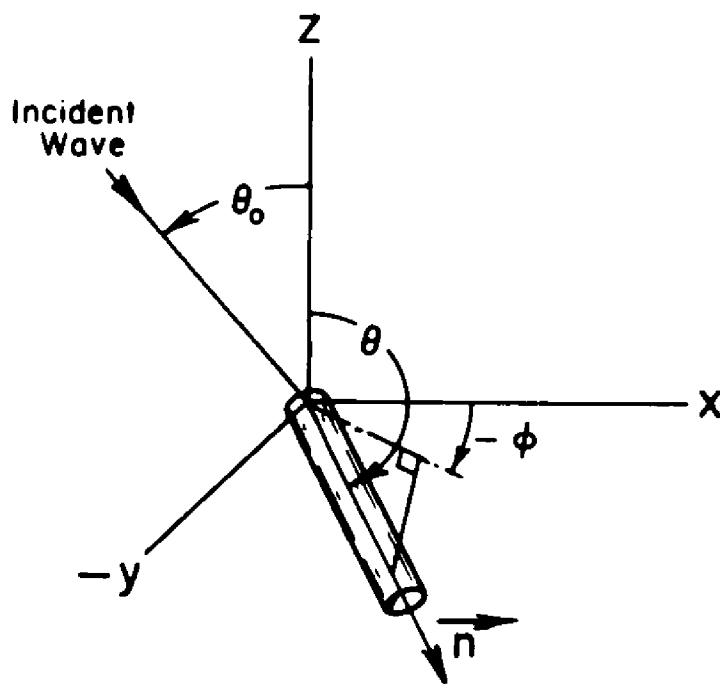
Fig. 3. Scattering by a cylinder.

It is supposed that the cylinder extends downwards from the plane $z = 0$, i.e., $\pi/2 < \theta < \pi$. The assumed incident fields for vertical and horizontal polarization are then

(28a) $\qquad \vec{E}_h^i = -\vec{j}\, E_0\, \psi$

(28b) $\qquad \vec{E}_v^i = (\vec{i}\, \cos\theta_0 + \vec{k}\, \sin\theta_0)\, E_0\, \psi$

(28c) $\qquad \psi = \exp\left[-iK(x\, \sin\theta_0 - z\, \cos\theta_0) + \alpha\, z\right] \quad z < 0$

where $\vec{i}\ \vec{j}\ \vec{k}$ are unit vectors along the three coordinate axes; E_0 is the amplitude of the incident field at the plane $z = 0$; and α is the assumed attenuation rate of the incident field strength. Now if $\vec{E}_t$, $\vec{H}_t$ are the total electric and magnetic fields in the presence of the

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scatterer, $\vec{E}^i \vec{H}^i$ are the assumed incident fields in the absence of the scatterer, and $\vec{E}_s \vec{H}_s$ are the scattered fields, defined by

$$\vec{E}_s = \vec{E}_t - \vec{E}^i$$

$$\vec{H}_s = \vec{H}_t - \vec{H}^i$$

then, within the scatterer

$$\nabla \times \vec{H}_t = (g + i\omega\epsilon_0\epsilon_1)\vec{E}_t$$

where g and ϵ_1 are the conductivity and relative dielectric constant of the cylinder material.

The incident field satisfies

$$\nabla \times \vec{H}^i = i\omega\epsilon_0 \vec{E}^i \quad .$$

Subtracting, it is found that the scattered field satisfies

$$(29) \qquad \nabla \times \vec{H}_s = i\omega\epsilon_0\vec{E}_s + \left[g + i\omega\epsilon_0(\epsilon_1 - 1)\right]\vec{E}_t \ .$$

Thus the scattered field is generated by the effective current density

$$(30) \qquad \vec{J}_s = \left[g + i\omega\epsilon_0(\epsilon_1 - 1)\right]\vec{E}_t = i\omega\epsilon_0(\epsilon' - 1)\vec{E}_t \quad .$$

It is still necessary to calculate E_t, which will only be done approximately. For this purpose, the incident field is resolved into two parts, one parallel to the cylinder axis and the other perpendicular

to it. It is assumed that for a scatterer much less than a wavelength in diameter, the parallel component of the electric field inside is equal to the parallel component of the incident field (continuity of tangential electric field). The perpendicular component of the electric field within the scatterer is taken equal to (t) times the perpendicular component of the incident field (static approximation). For a non-conducting dielectric circular cylinder, for example, $t = 2/(1+\epsilon_1)$. However for cylinders of arbitrary cross-section and conductivity, t is a tensor function of the angle between the perpendicular component of the incident field and some axis which specifies the orientation of the cylinder cross-section. For example, if the cylinder were of elliptical cross-section, both the magnitude and the direction of the internal field are functions of the angle between the external field and the major axis of the ellipse. Thus, strictly speaking, t has a simple interpretation only for circular cylinders; in the general case it may be regarded as a parameter having some effective value. With these assumptions, the total field within the cylinder is

$$\vec{E}_t = (\vec{E}^i \cdot \vec{n})\vec{n} + t\left[\vec{E}^i - (\vec{E}^i \cdot \vec{n})\vec{n}\right]$$

(31)

$$\vec{n} = \vec{i}\sin\theta\cos\phi + \vec{j}\sin\theta\sin\phi + \vec{k}\cos\theta$$

from which the current may be found by using Eq. (30). The

scattered fields due to such a current distribution may be found by standard methods (Reference 20, p. 89) to be, at a great distance R from the scatterer,

$$E_v^s = \frac{-i\omega\mu}{4\pi R} \ e^{-iKR} \int (\vec{J}_s \cdot \vec{i}_{\theta_s}) \ e^{iK\vec{\rho} \cdot \vec{R}_1} dV$$

(32)

$$E_h^s = \frac{-i\omega\mu}{4\pi R} \ e^{-iKR} \int (\vec{J}_s \cdot \vec{i}_{\phi_s}) \ e^{iK\vec{\rho} \cdot \vec{R}_1} dV$$

where μ is the permeability of free space, $\vec{\rho}$ is the vector which specifies the position of dV, $\vec{R}_1$ is a unit vector in the scattering direction

(33) $\vec{R}_1 = \vec{i} \sin\theta_s \cos\phi_s + \vec{j} \sin\theta_s \sin\phi_s + \vec{k} \cos\theta_s$

and $\vec{i}_{\phi_s}$, $\vec{i}_{\theta_s}$ are unit vectors in the "horizontal" and "vertical" polarization directions, respectively:

(34) $\vec{i}_{\theta_s} = -\vec{i} \cos\theta_s \cos\phi_s - \vec{j} \cos\theta_s \sin\phi_s + \vec{k} \sin\theta_s$

(35) $\vec{i}_{\phi_s} = -\vec{i} \sin\phi_s + \vec{j} \cos\phi_s$.

For a thin cylinder of cross-sectional area A, the integration over the cylinder volume may be replaced by an integration over a length coordinate s along the cylinder, with $dV \approx A \ ds$ and $\vec{\rho} \approx s\vec{n}$. With this final approximation the integrals of Eq. (32) may be evaluated. To save space only the case where both incident and scattered fields are horizontally polarized will be carried out in detail. When

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Eqs. (28a), (28c), (30), (31), (33) and (35) are substituted into

Eq. (32b), the expression for the scattered field becomes

$$(36) \qquad E_{hh}^s = \frac{E_0 A K^2(\epsilon'-1)}{4\pi R} e^{-iKR}\left[t \cos\phi_s + (1-t)\sin^2\theta \sin\phi \sin(\phi-\phi_s)\right]$$

$$\cdot \int_0^\infty e^{SM}\, ds$$

where

$$M = \alpha \cos\theta - iK(\sin\theta \sin\theta_0 \cos\phi - \cos\theta\cos\theta_0)$$

$$+ iK(\sin\theta\sin\theta_s \cos(\phi-\phi_s) + \cos\theta\cos\theta_s)\ .$$

The integral is just $M^{-1}\exp(SM)$ which has the value M^{-1} at the

lower limit and the value zero at the upper limit, si e $\cos\theta$ is

always negative and α is positive. Thus

$$(37) \qquad \left|\frac{E_{hh}^s}{E_0}\right|^2 = \frac{A^2 K^4\left|\epsilon'-1\right|^2}{(4\pi R)^2\left|M\right|^2}\left[(1-t)\sin^2\theta\sin\phi\ \sin(\phi_s-\phi)+t\cos\phi_s\right]^2 .$$

It is now necessary to average over the distribution of cylinder

orientations to obtain the average power scattered by a single cylinder.

It will be assumed that the cylinder axes are oriented at random.

However, the probability that the axis of a given cylinder lies in the

element of solid angle $d\Omega$ in the direction $\vec{n}$, will be assumed to be

proportional to $\cos^2\theta\, d\Omega$. Thus the average value (designated by

$\overline{[\quad]}$) of any function F of θ,ϕ will be defined by

$$(38) \qquad \widetilde{F(\theta,\phi)} = \frac{\int F(\theta,\phi)\cos^2\theta\, d\Omega}{\int \cos^2\theta\, d\Omega} \qquad .$$

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This choice of averaging function maintains symmetry about the vertical axis, gives a slight preference to the vertical direction, and makes certain integrals easier to evaluate. Nevertheless it has not been possible to carry out the average of Eq. (37) correctly. Instead the numerator and denominator have been averaged separately, and their ratio taken, i.e., it has been assumed that the following equation

$$(39) \qquad \frac{\overline{\left[t \cos \phi_s + (1-t) \sin^2\theta \sin\phi \sin(\phi-\phi_s) \right]^2}}{|M|^2}$$

$$= \frac{\overline{\left[t \cos \phi_s + (1-t) \sin^2\theta \sin\phi \sin(\phi-\phi_s) \right]^2}}{\overline{|M|^2}}$$

is true at least approximately. If there are N cylinders per unit area of surface then the scattering coefficients, from Eq. (1), are given by

$$(40) \qquad \gamma = 4\pi R^2 \, N \, \sec\theta_o \, \overline{|E^s/E_o|^2}.$$

Substituting (38) into (40) and making use of (39), the coefficients are

$$(41a) \qquad \gamma_{hh}(o, s) = B\left[(3-2\sin^2\phi_s) + t(8-10\sin^2\phi_s) + t^2(24-23\sin^2\phi_s) \right]$$

$$(41b) \qquad \gamma_{hv}(o, s) = B\left[(1+2\sin^2\theta_s + 2\cos^2\theta_s \sin^2\phi_s \right.$$
$$+t(-2-4\sin^2\theta_s + 10\cos^2\theta_s \sin^2\phi_s)$$
$$\left. +t^2(1+2\sin^2\theta_s + 23\cos^2\theta_s \sin^2\phi_s) \right]$$

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(41c) $\gamma_{vh}(o,s) = B\left[(1+2\sin^2\theta_0 + 2\cos^2\theta_0 \sin^2\phi_s)\right.$

$$+t(-2-4\sin^2\theta_0 + 10\cos^2\theta_0 \sin^2\phi_s)$$

$$\left.+t^2(1+2\sin^2\theta_0 + 23\cos^2\theta_0 \sin^2\phi_s)\right]$$

(41d) $\gamma_{vv}(o,s) = B\left[(1-t)^2(3-2\cos^2\theta_s \cos^2\theta_0 \sin^2\phi_s)\right.$

$$\left.+(\sin\theta_s \sin\theta_0 - \cos\theta_s \cos\theta_0 \cos\phi_s)Q\right]$$

where

$$(42)\quad B = \frac{(AN)(AK^2)\left[(\epsilon_1-1)^2 + \epsilon_2^2\right]}{28\pi\cos\theta_0\left[3/5(\alpha/K)^2 + 3(\cos\theta_0 + \cos\theta_s)^2 + (\sin^2\theta_0 + \sin^2\theta_s - 2\sin\theta_0 \sin\theta_s \cos\phi_s)\right]}$$

$$Q = 12(1-t)^2\sin\theta_s\sin\theta_0 + 14t(1-t)(3\sin\theta_0\sin\theta_s - \cos\theta_0\cos\theta_s\cos\phi_s)$$

$$+ 35t^2(\sin\theta_0\sin\theta_s - \cos\theta_0\cos\theta_s \cos\phi_s)\ .$$

The attenuation factor may, together with t, be regarded as parameters of the theory. However, that part of the attenuation due to absorption (i.e., due to the imaginary component $i\epsilon_2$ of the dielectric constant of the cylinders) may be calculated from the model. To do so, one may start with an expression for the average power per unit volume dissipated in the cylinder material,

$$(43)\quad \frac{dP}{dV} = (g/2)\ \vec{E}_t \cdot \vec{E}_t^{*}\qquad .$$

Thus the total power dP absorbed in a vertical increment dz, or volume $A(\cos\theta)^{-1}$ dz of the cylinder material, is

$$(44)\quad dP = \frac{dP}{dV} A\, dz(\cos\theta)^{-1}\ .$$

But the average incident power P at that level associated with a single cylinder is just the power density in the incident beam multiplied by the effective area $N^{-1} \cos \theta_0$,

$$(45) \qquad P = (z_0/2) \, \vec{E}_i \cdot \vec{E}_i^* \, (N^{-1} \cos \theta_0) \quad .$$

Thus, averaging over the cylinder distribution and recalling that α was defined as the attenuation rate for the field strength, there results

$$(46) \qquad 2\alpha = \frac{1}{P} \frac{dP}{dz} = (g/z_0) \left(\frac{AN}{\cos \theta_0} \right) \left(\frac{\overbrace{E_t \cdot E_t^*}}{E_i \cdot E_i^* \cos \theta} \right) \quad .$$

Upon carrying out the average according to (38), this gives

$$(47a) \qquad \alpha_h/K = (3/16) \, AN\epsilon_2 \, \sec \theta_0 \, (1 + 3t^2) \quad ,$$

for horizontally polarized incident radiation, and

$$(47b) \qquad \alpha_v/K = (3/16) AN\epsilon_2 \, \sec \theta_0 \left[(1+3t^2) + \sin^2 \theta_0 (1-t^2) \right]$$

for vertically polarized incident radiation.

Because the preceding calculations have the appearance of a typical single-scattering approximation, it might be supposed that one could dispense with the empirical attenuation factor α, and calculate an effective propagation constant for the "medium" of the scatterers using standard methods.[37] This is not the case, because

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of the hypothesis, supported (at certain frequencies) by experimental evidence to be given in Chapter IIID, that the incident radiation is essentially extinguished in a depth much less than the length of the cylinders. The consequence is that only N, the number of cylinders per unit area of surface, has any significance for the model, whereas the standard calculation expresses the propagation constant in terms of the number of particles per unit volume. However, by comparing the results of the standard calculation with the one just given, a value may be inferred for the effective number of cylinders per unit volume, by considering the cylinders to be broken up into effective lengths which act independently. Not surprisingly, this effective length turns out to be of the same magnitude as $1/\alpha$, the penetration depth. To show this formally, suppose that the incident wave is not the expression (28) containing (α), but rather

(48) $\psi_o = \exp\left[-iK_m(x \sin \theta_0 - z \cos \theta_0)\right]$

where K_m is the desired (complex) propagation constant for the medium. If this is substituted into Eq. (35) and the forward scattering ($\phi_s = 0$; $\theta_s = \pi - \theta_0$) evaluated, one gets

$$\frac{E_{hh}^s(\theta_0, \pi; \pi-\theta_0, o)}{E_o} = \frac{(\epsilon'-1) AK^2 \, e^{-iKR}}{4\pi R(K_m-K)(\cos \theta \cos \theta_0 - \sin \theta \sin \theta_0 \cos \phi)}$$

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where it is assumed that radiation takes place into free space. The average over cylinder orientations must now be carried out according to Eq. (38). For the simplest case of normal incidence this leads to the following value for $\tilde{S}(o)$, the average forward scattering amplitude

$$\tilde{S}(o) \cong iKR \, e^{iKR} \left(\frac{E_{hh}^*}{E_o} \right)$$

$$= \frac{3}{8} (1+3t) \left[\frac{iAK^3(\epsilon'-1)}{4\pi(K_m-K)} \right] \quad .$$

From the forward scattering theorem (Reference 37, p. 33) one has, formally,

$$K_m/K = 1 - 2\pi i n K^{-3} \, \tilde{S}(o)$$

where n is the number of scatterers per unit volume. Thus, combining the last two equations,

(49) $\qquad (K_m-K)^2 = \frac{3}{16} n\,AK(\epsilon'-1)(1+3t) \quad \theta_o = 0 \quad .$

Now K_m may be written in the form $K_m = K + \Delta K - iK_2$, where ΔK is the correction to the real part of the propagation constant (i.e., $1 + (\Delta K/K)$ is the refractive index) and K_2 is the imaginary part of K_m.

To compare this result with Eq. (47) it is necessary to choose the parameters so that the assumptions of each derivation are

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satisfied. The forward scattering method required $|K_m - K| \ll K$; the calculation involving α, which ignored the contribution of scattering to the extinction of the incident radiation, should be valid if $\alpha / K \ll 1$ and $\epsilon_2 \gg \epsilon_1 - 1$. Under these conditions, (49) has the solution

$$\Delta K = K_2 = \left[\frac{3}{32} \, n A K \, \epsilon_2 (1 + 3t) \right]^{\frac{1}{2}} \qquad \begin{array}{c} \epsilon_2 \gg \epsilon_1 - 1 \\ \theta_0 = 0 \end{array}$$

whereas from (47),

$$\alpha / K = \frac{3}{8} \, A N \, \epsilon_2 (1 + 3t) \ .$$

By comparing the expressions (28) and (48) for the incident wave, it is clear that at normal incidence one should have $\alpha = K_2$, which requires that

$$n = 2 N K_2 \ .$$

The effective length of the cylinders is thus $(1/2K_2)$, as might have been expected.

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CHAPTER III

THE BACK-SCATTERING COEFFICIENTS, WITH APPLICATIONS TO THE RADAR RETURN FROM TERRAIN

A. Introduction

The simplest special case of the formulas derived in the preceding chapter is that of back-scattering. Since this corresponds to predicting the radar return from terrain, some recently published experimental data[17] will allow a check on the formulas just derived. That is, it will permit one to determine whether the models are adequate to represent the more complex surfaces encountered in practice. To conform to engineering notation, in this section only the angle between the incoming propagation vector and the plane of the surface, called the "grazing angle" or "incidence angle," will be designated by θ. Thus $\theta + \theta_0 = \pi/2$. The back scattering coefficient $\gamma(\theta_0\phi_0; \theta_0\phi_0)$ will be designated simply by γ.

B. Models with Undetermined Parameters

Before discussing the models just mentioned, it may be of interest to describe briefly some of the more useful models with undetermined parameters, since these have often been used in radar design. In his study of radar ground return,[4,21] R. E. Clapp proposed three simple models with which airborne measurements might be compared. His first model corresponded to a surface obeying

Lambert's law

(50) $Y = c \sin \theta$ $\sigma_o = c \sin^2 \theta$

where c is an undetermined parameter. Although such a law may have a certain validity at optical wavelengths, it was rejected by Clapp on the grounds that it did not fit his measurements, and that no easily visualized mechanism could account for scattered radiation that was independent of the direction of the incident radiation. Clapp's second model was composed of a single layer of spheres, N per unit area, each with a scattering cross-section A_K, giving

(51) $Y = NA_K \cosec \theta$; $\sigma_o = NA_K$.

Clapp rejected this model also, partly because he considered a single layer of spheres to be a poor representation of a three dimensional surface. However, the model might apply to certain types of vegetation, such as a field of oats in head, where the heads of grain represent essentially a single layer of rather large scatterers. An example of measured back-scattering in agreement with this model is given in Fig. 4. Here, and throughout this section, the ordinate, labelled "γ in db", represents $10 \log Y$, and the experimental measurements of Y are due to R. C. Taylor.[17] The calculated curve in Fig. 4 is from Eq. (51) with the parameter (NA_K) chosen arbitrarily.
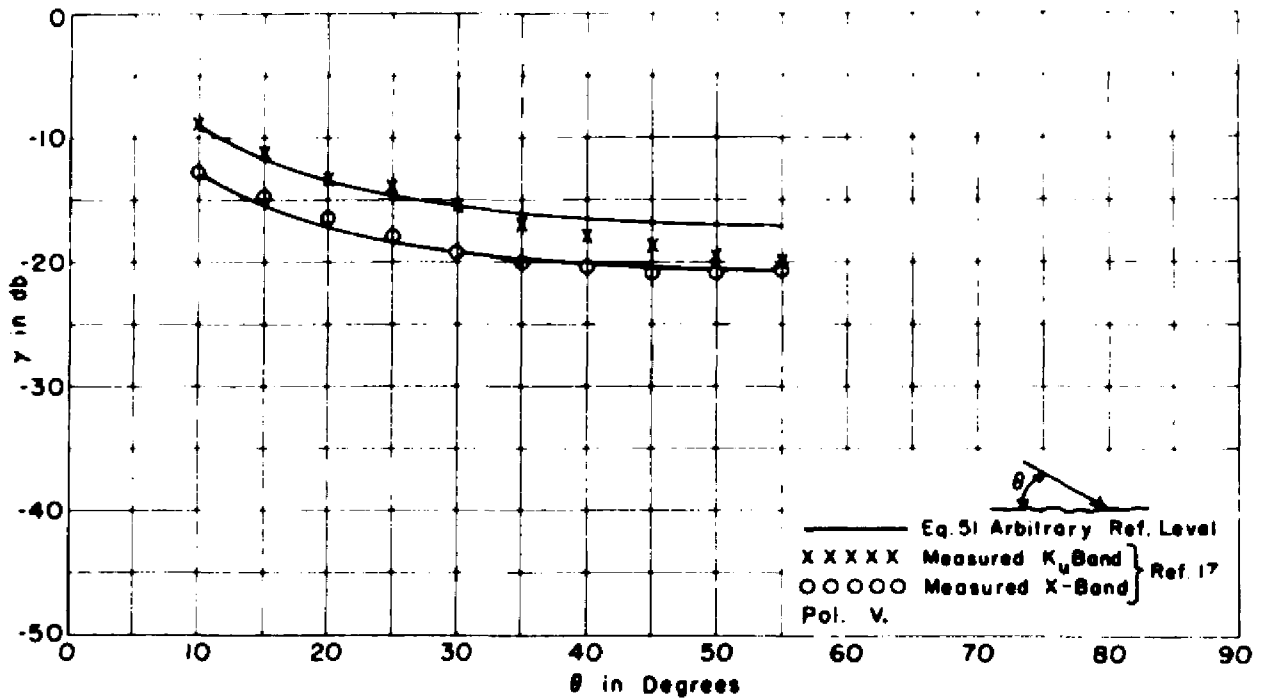
Fig. 4. Back scattering from a vegetation covered
surface (15" grass in head) illustrating Eq.(51).

In the third model proposed by Clapp, the terrain surface was

represented by many layers of spheres, each of which absorbed a

fraction (1-δ) of the energy falling on it, and reradiated the rest

isotropically. Then

(52) $Y = \delta$ $\sigma_0 = \delta \sin \theta$.

This model was in good agreement with Clapp's measurements and

with many subsequent measurements of the radar return from vege-

tation covered surfaces. An excellent illustration of data fitted by

this model may be found in Fig. 5. Here the experimental data

follow almost exactly the horizontal straight line predicted by
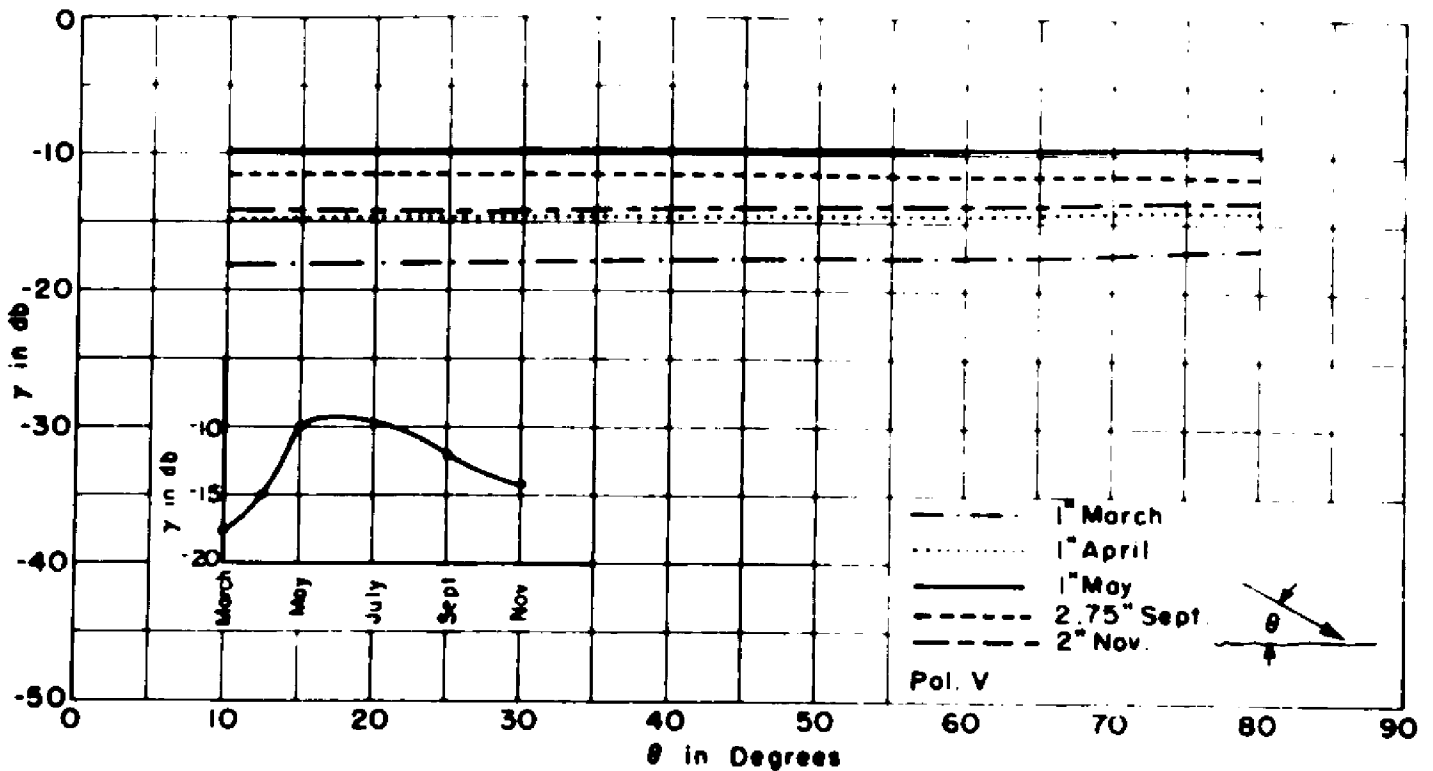
Fig. 5. Back scattering from a vegetation covered
surface (short grass) illustrating Eq. (52).

Eq. (52). In connection with this model it may be mentioned that

a paper by W.S. Ament[8] has sometimes been quoted as showing

that no physical surface can have $\sigma_0 = \delta \sin\theta$, as predicted by

Eq. (52). Insofar as Ament's proofs are correct, however, they

show only that a surface cannot reradiate uniformly in all direc-

tions, and cannot have $\gamma \rightarrow$ constant as $\theta \rightarrow 0$; in no case do the

proofs apply to the practical terrain return problem.

A somewhat more elaborate terrain model, which represents

the terrain by arrays of facets, has been proposed by Katz and

Spetner.[5] It has been used by its authors to fit data taken with air-

borne radars.

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C. The Slightly Rough Surface

The radar return from slightly rough surfaces, such as might be represented in practice by asphalt or concrete roadways, can be calculated from the results of Chapter IID. For back scattering, Eqs. (26) and (28) become

$$
(53) \qquad Y = 8 \sin^3 \theta \, T \, \overline{z^2} \, K^4 \int_0^\infty \rho(r) \, J_0(2Kr \cos \theta) \, r \, dr
$$

where, for incident and back-scattered radiation both horizontally polarized,

$$
(54a) \qquad T_h = \left| \frac{\epsilon_1 - i\epsilon_2 - 1}{\left[\sin \theta + (\epsilon_1 - i\epsilon_2 - \cos^2 \theta)^{\frac{1}{2}} \right]^2} \right|^2
$$

and for incident and back-scattered radiation both vertically polarized,

$$
(54b) \qquad T_v = \left| \frac{(\epsilon_1 - i\epsilon_2 - 1)\left[(\epsilon_1 - i\epsilon_2)(1 + \cos^2 \theta) - \cos^2 \theta \right]}{\left[(\epsilon_1 - i\epsilon_2) \sin \theta + (\epsilon_1 - i\epsilon_2 - \cos^2 \theta)^{\frac{1}{2}} \right]^2} \right|^2 \quad .
$$

In order to compare the predictions of these equations with the measurements of Taylor[17] on a series of asphalt and concrete surfaces, it is necessary to know $\overline{z^2}$, $\rho(r)$, ϵ_1 and ϵ_2. The parameters $\overline{z^2}$ and $\rho(r)$ were estimated by making a plaster cast of a representative area of each surface. These casts were then sliced into sections to expose typical surface profiles. The surface height was measured every 0.25 mm (in some cases every 0.1 mm) along the

profile using a comparator. A linear correction term was applied to the raw data to ensure zero average height and zero average slope, and the mean square roughness and correlation function computed by conventional methods. The results are shown in Figures 6 and 7, each point representing an average of some five or six sections of a given plaster-cast. Similar results for runway surfaces have been given in References 22 and 23.

It was not possible to measure the complex dielectric constant of these surface materials directly. Instead, representative samples of asphalt and concrete from other sources were measured using standard waveguide methods,[24] and were found to have the following values of complex dielectric constant:

$$\epsilon_1 - i\epsilon_2 = 6.5 - i1.5 \text{ Concrete X-band } (\lambda = 3.0 \text{ cm})$$

$$5.5 - i0.5 \text{ Concrete } K_a\text{-band } (\lambda = 0.86 \text{ cm})$$

$$4.3 - i0.1 \text{ Asphalt X-band}$$

$$2.5 - i0.6 \text{ Asphalt } K_a\text{-band.}$$

No great significance should be placed on these values, since the measurements are not only rather inaccurate (especially for the asphalt surface) but the true value of ϵ' depends on the state, composition, etc., of the particular sample.

The contributions of T_v and T_h to the overall return have been calculated for these particular dielectric constants and are shown in Figs. 8 and 9.
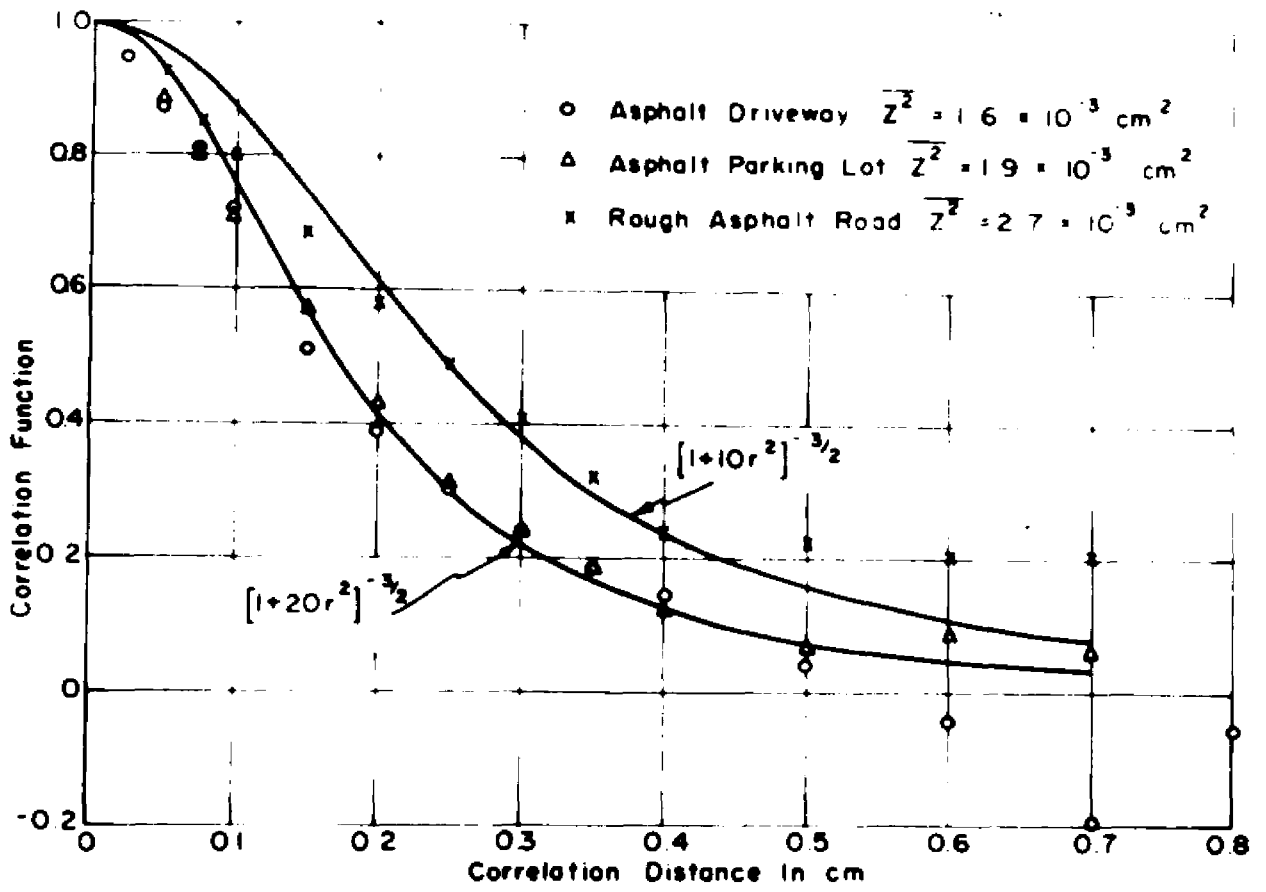
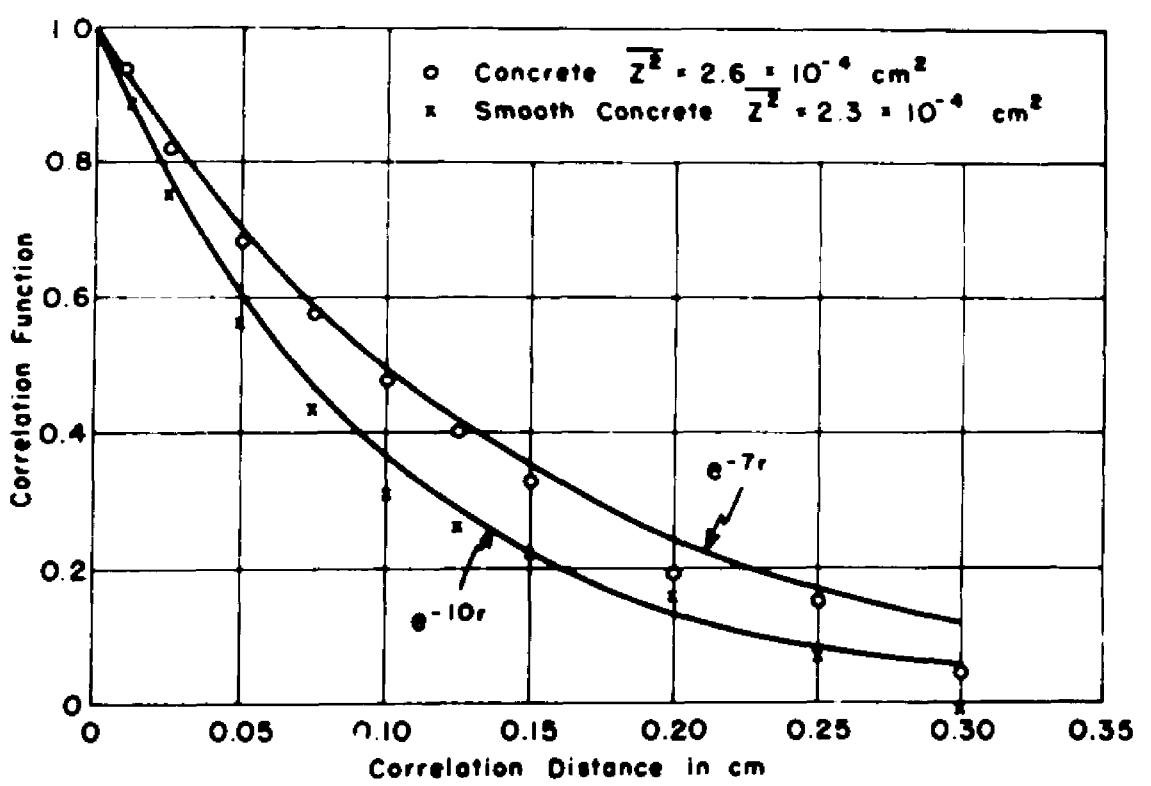Fig. 6. Correlation functions for 3 asphalt surfaces.

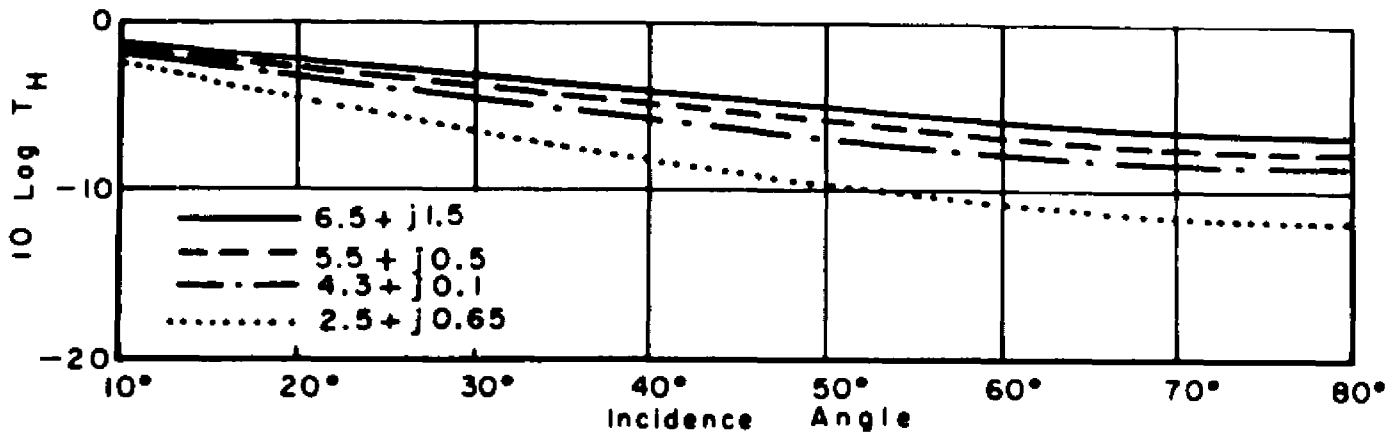Fig. 7. Correlation functions for 2 concrete surfaces.

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Fig. 8. Effect of dielectric constant on return from a slightly rough surface (horizontal polarization).
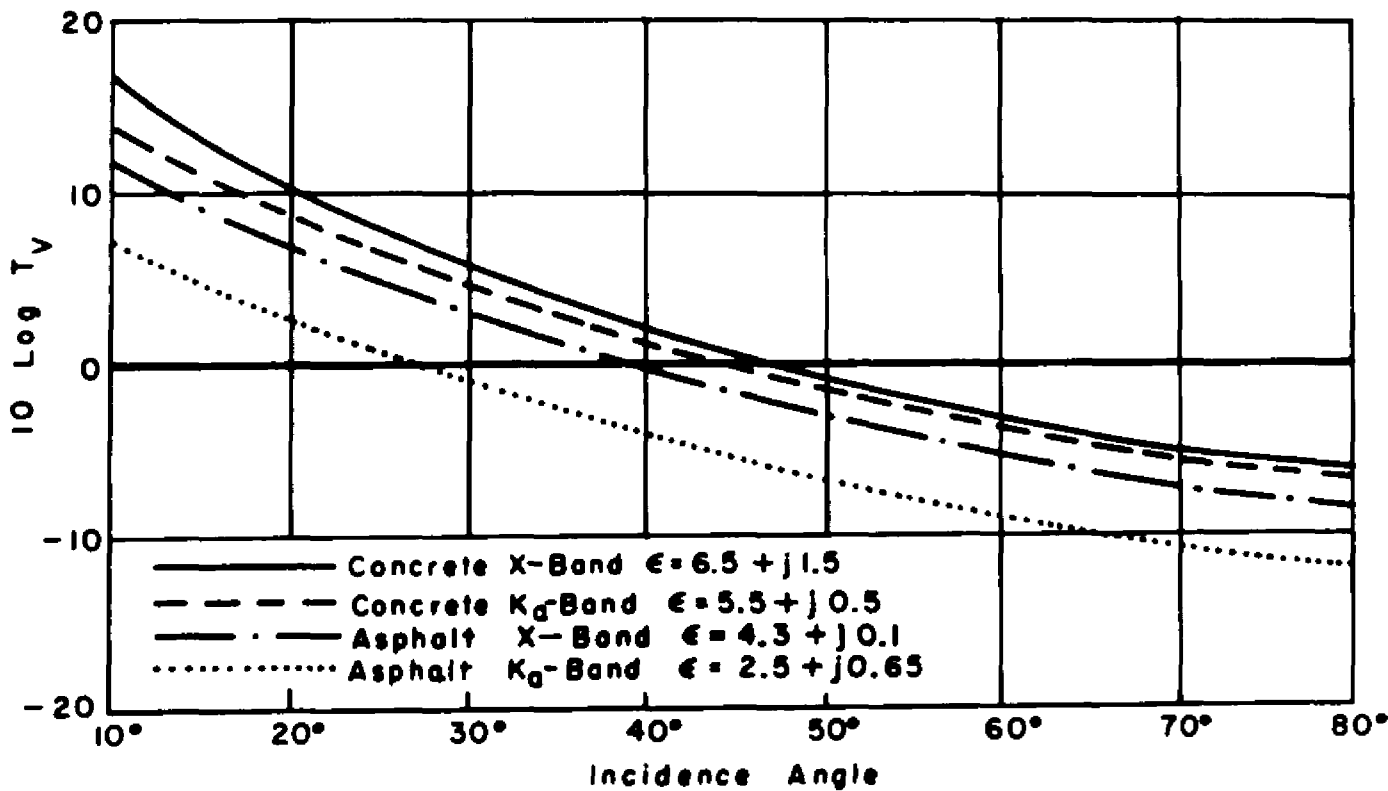

Fig. 9. Effect of dielectric constant on return from a slightly rough surface (vertical polarization).

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The extent to which the perturbation theory, as given by Eq. (53), can actually predict the back-scattering from real terrain surfaces may be judged from Figs. 10 to 19. In these figures the ordinate is 10 log Y and the abscissa is incidence angle. In each case the measured points are due to Taylor,[17] and the calculated curves are based on the measured surface properties only. It is clear from these figures that the dependence of the back-scattered power on surface roughness, polarization, frequency and angle of incidence are very well represented by the theory. The differences between the measured and calculated return are due partly to the failure of the surface to satisfy the theoretical assumptions (e.g., slopes of order unity, inhomogeneous dielectric) and partly to the crudeness of the estimates of dielectric constants and correlation functions. In particular, the fact that the correlation functions were calculated only for correlation distances of less than a few millimeters, means that the tail of the actual correlation function may be considerably different from the assumed curves of Figs. 6 and 7. The calculated return is most sensitive to the shape of the tail at the longer wavelengths and at incidence angles towards normal. Since the discrepancy between theory and measurement is greatest under just these conditions, it is probable that more accurate values for $\rho(r)$ would give better agreement. Nevertheless,

it is felt that for all practical purposes, the model may be used to calculate the return from such surfaces to within a few db. Conversely, the success of the model implies that an accurate prediction of the return from such surfaces requires specification of the surface roughness, correlation function, and complex dielectric constant.

D. Surfaces Covered with Vegetation in the form of Long Thin Lossy Cylinders

For surfaces covered with certain types of vegetation, such as grass, flags, etc., for which it may be assumed that the individual blades or stems scatter like long thin lossy cylinders distributed at random over the surface, the scattered return signal may be found from Eqs. (41). For back scattering, these reduce to

$$(55a) \qquad Y_{hh} = \frac{A^2 NK^2 \left[(\epsilon_1 - 1)^2 + \epsilon_2^2 \right] \left[3 + 8t + 16t^2 \right]}{28\pi \sin\theta \left[3(\alpha_h/K)^2 + 4(1 + 2\sin^2\theta) \right]} \qquad ,$$

$$(55b) \qquad Y_{vv} = \frac{A^2 NK^2 \left[(\epsilon_1 - 1)^2 + \epsilon_2^2 \right] \left[3 + 8t + 16t^2 + \cos^2\theta(12 + 4t - 16t^2) \right]}{28\pi \sin\theta \left[3(\alpha_v/K)^2 + 4(1 + 2\sin^2\theta) \right]} \qquad ,$$

$$(55c) \qquad Y_x = \frac{A^2 NK^2 \left[(\epsilon_1 - 1)^2 + \epsilon_2^2 \right] (1-t)^2 (1 + 2\cos^2\theta)}{28\pi \sin\theta \left[3(\alpha/K)^2 + 4(1 + 2\sin^2\theta) \right]} \qquad ,$$

$$(56a) \qquad \alpha_h/K = (3/16) AN \epsilon_2 \csc\theta (1 + 3t^2) \ ,$$

$$(56b) \qquad \alpha_v/K = (3/16) AN \epsilon_2 \csc\theta \left[1 + 3t^2 + \cos^2\theta(1 - t^2) \right]$$

where Y_x is the cross polarized return (Y_{hv} or Y_{vh}). The value of α appropriate for the incident radiation must be used.

In deriving the above formulas, it was assumed that the cylinder axes were distributed according to Eq. (38). This slight preference for the vertical direction is not necessarily characteristic of actual vegetation. Thus the angular dependence, and more especially the polarization dependence, of the calculated return may not represent the measured dependence in all respects. Nevertheless, it is felt that this model may be used to explain some of the more prominent features of the radar return from certain types of vegetation covered surfaces.

The justification for using this particular model may be found in certain data taken by Taylor[17] which show that the return from grass, at K_a-band, is significantly greater than the return from the bare ground beneath. At the same time, the return is essentially independent of the length of the grass (after a certain minimum of vegetation cover has appeared). These two observations suggest that the incident radiation must be almost completely extinguished before reaching the ground underlying the vegetation. Thus the major assumption of the model of Chapter IIC is satisfied (at least at K_a-band; the evidence at X-band is not so clear cut). Furthermore, Fig. 5 shows that there is also a marked seasonal dependence

of the return, with a large increase in the spring despite the fact

that constant mowing kept the grass at essentially constant depth.

This suggests that the water content and the number of blades per

unit area may have a decided influence on the return, since both

these parameters are known to increase during the spring growth,

and to diminish slowly during the later summer, fall and winter.

In order to determine whether, in fact, Eqs. (55) and (56) can ac-

count for the return from grass covered surfaces, it is only necessary

to determine the parameters A, N, ϵ_1, ϵ_2 for the surfaces in ques-

tion. It has not been possible to do this in detail, especially since

some of the measurements were made before the theory was devel-

oped. However, some rough measurements for the ordinary lawn

grass under consideration show that $A \simeq 10^{-2}$ cm^2 and that the

product AN lies in the range $0.10 > AN > 0.01$. The complex di-

electric constant of a grass blade might also be measured, for ex-

ample by cavity methods.[26] For the qualitative use of Eqs. (55)

and (56), however, the effective dielectric constant of the blades,

and its relation to their water content, has been estimated by the

following procedure. It is assumed that for vegetation with a water

content of $100f$ % by weight, the dielectric constant is given by

(57) $\epsilon_1 - i\epsilon_2 = 2.5(1-f) + f\epsilon_w$

where the first term is supposed to represent the contribution of the solid matter, and ϵ_w is the dielectric constant of water, given by the usual relaxation formula[24]

$$(58) \qquad \epsilon_w = 5 + \frac{75}{1 + i(1.85/\lambda)} \; ; \; (\lambda \text{ in cm}).$$

Such an estimate is admittedly rather crude, especially since it is known[25,26] that absorbed or chemically bound water may not satisfy Eq. (58). Nevertheless, it is believed that the use of Eq. (58) will serve to illustrate the effect of season, frequency, etc., on the return. For this purpose, Figs. 20 to 27 have been prepared, based on Eqs. (55) to (58), with $A = 1.3 \times 10^{-2}$ cm^2, and the other parameters as specified on each figure. The value of t has been taken as $2/(1 + \epsilon_1)$. In each case, representative measurements by Taylor are included for comparison. Figures 20 to 23 show the effect of increasing the moisture content from 30%, representative of dried grass, to 90%, representative of very moist green grass.[32] It may be seen from these figures that such a change of moisture content is adequate to account for the observed seasonal dependence of the return. The general level of the return is also accounted for at both wavelengths (X- and K_a-bands) although the details of the angular dependence are not. Figures 24-27 show the dependence of the return on the parameter AN. Here the general level of the

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observed return corresponds to choosing $AN \simeq 0.03$, which is in reasonable agreement with the observed value of the parameter for grass covered surfaces. Figures 28 and 29 show the calculated polarization dependence of the return for several values of AN and f, together with some of Taylor's measurements. The measured polarization dependence is not always in good agreement with the predictions of the model, due partly to the fact that the calculated results are somewhat sensitive to the assumed choice of distribution for the orientation of the cylinder axes. However, the tendency for the drier and sparser vegetation to exhibit a greater vertically polarized return as the angle of incidence moves away from the normal is reproduced by the model. Finally, Fig. 30 shows the measured frequency dependence of a wide variety of vegetation surfaces, together with the predictions of the model just described. The line labelled "Dielectric Constant Independent of Wavelength" shows what would be obtained if the wavelength dependence of the dielectric constant were ignored. It is clear from this figure that the wavelength dependence of the dielectric constant, even though it may not be exactly as given in Eq. (58), must still play an important role in determining the overall wavelength dependence of the return.

The cross-polarized return from such surfaces has been reported by Campbell,[35] and is about 5 db below the horizontally polarized return. This is also in qualitative agreement with the prediction of Eq. (55).

Although the detailed agreement between the calculated and the measured return is not so good as it was for the slightly rough surfaces, it is found that the general level of the return, the frequency dependence, the seasonal dependence, and the statistical properties[4,21] of the return can be accounted for qualitatively. This suggests that the model may be of use in predicting other features (e.g., albedo, bistatic scattering) of the interaction of electromagnetic radiation with vegetation surfaces, at least as to order of magnitude. Conversely, to the extent that the model is successful, it shows that the number of cylinders per unit area, their cross-sectional area, and their dielectric constant must be specified to describe the surface properly.

E. Other Theoretical Models

In addition to the simple models of Section IIIB, and the "complete" models of Sections IIIC and IIID, there is a third class of models for scattering from non-uniform surfaces. In this class, interest is concentrated more on the theoretical details of the scattering problem than on maintaining any particular

resemblance between the model surface and some actual terrain. For example, Twersky[14] has considered a surface of perfectly conducting hemispherical bosses distributed at random over a perfectly conducting plane; Ament[8] has considered a surface of perfectly conducting half-planes or wires; and Carlson and Heins[16] have considered the scattering from a regular array of plates. The bibliography (References 6 - 16) includes a representative sample of American, British and Russian literature on such models.

Fig. 10. Calculated and measured return from a slightly
rough asphalt surface (vertical polarization).

Fig. 11. Calculated and measured return from a slightly
rough asphalt surface (horizontal polarization).

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Fig. 12. Calculated and measured return from a slightly
rough asphalt surface (vertical polarization).

Fig. 13. Calculated and measured return from a slightly
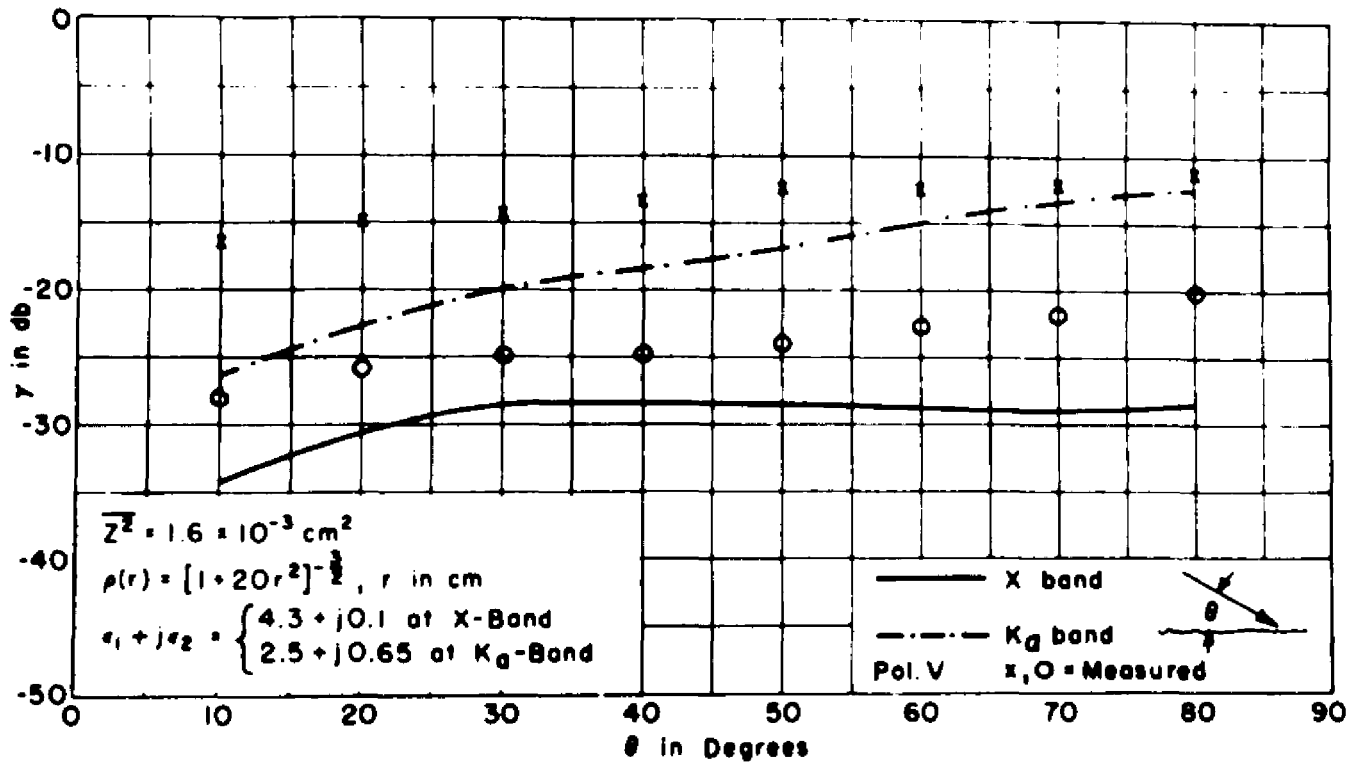rough asphalt surface (horizontal polarization).

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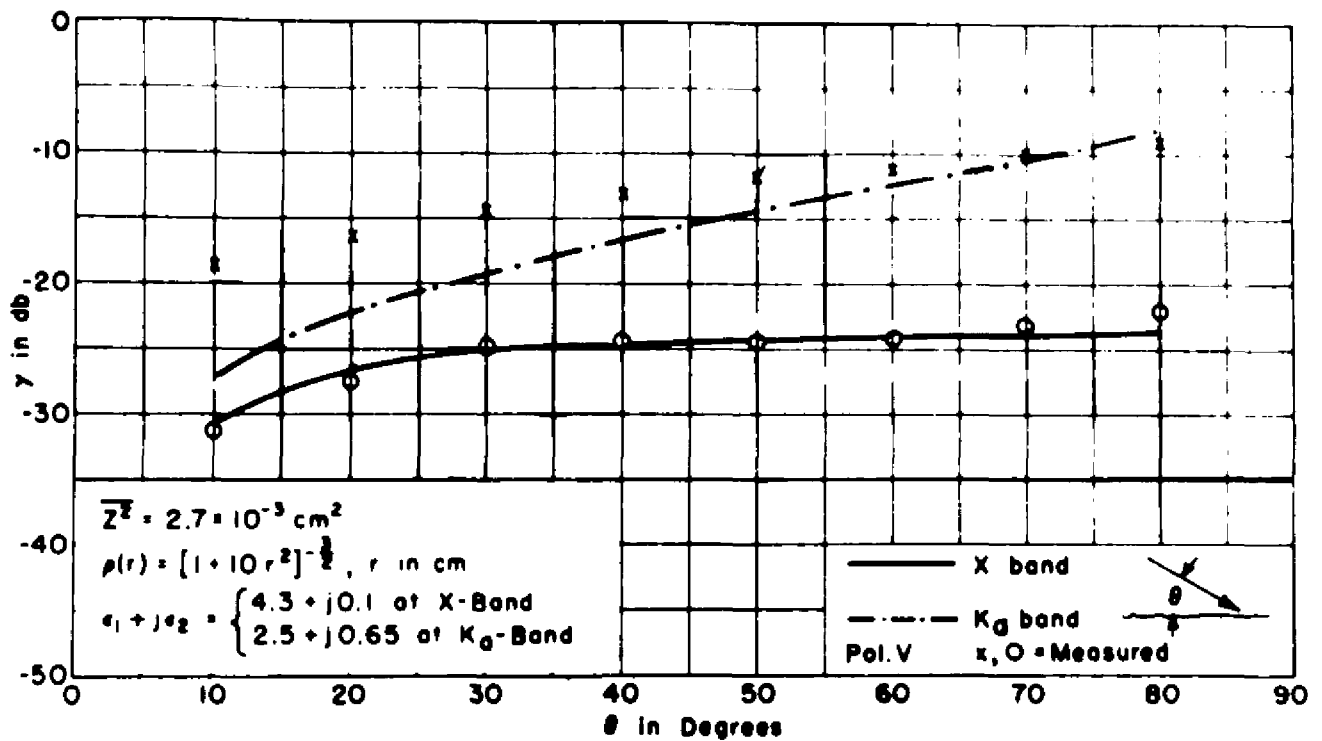
Fig. 14. Calculated and measured return from a slightly
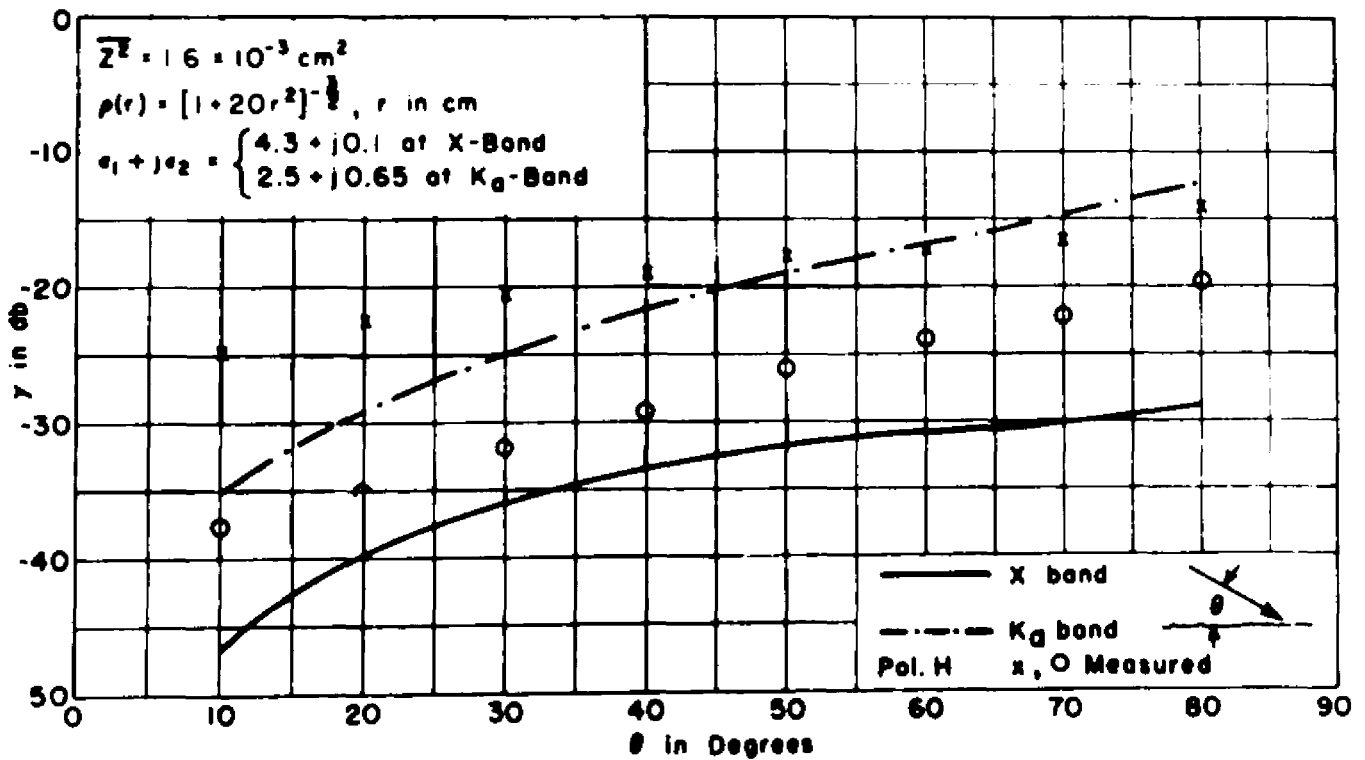rough asphalt surface (vertical polarization).
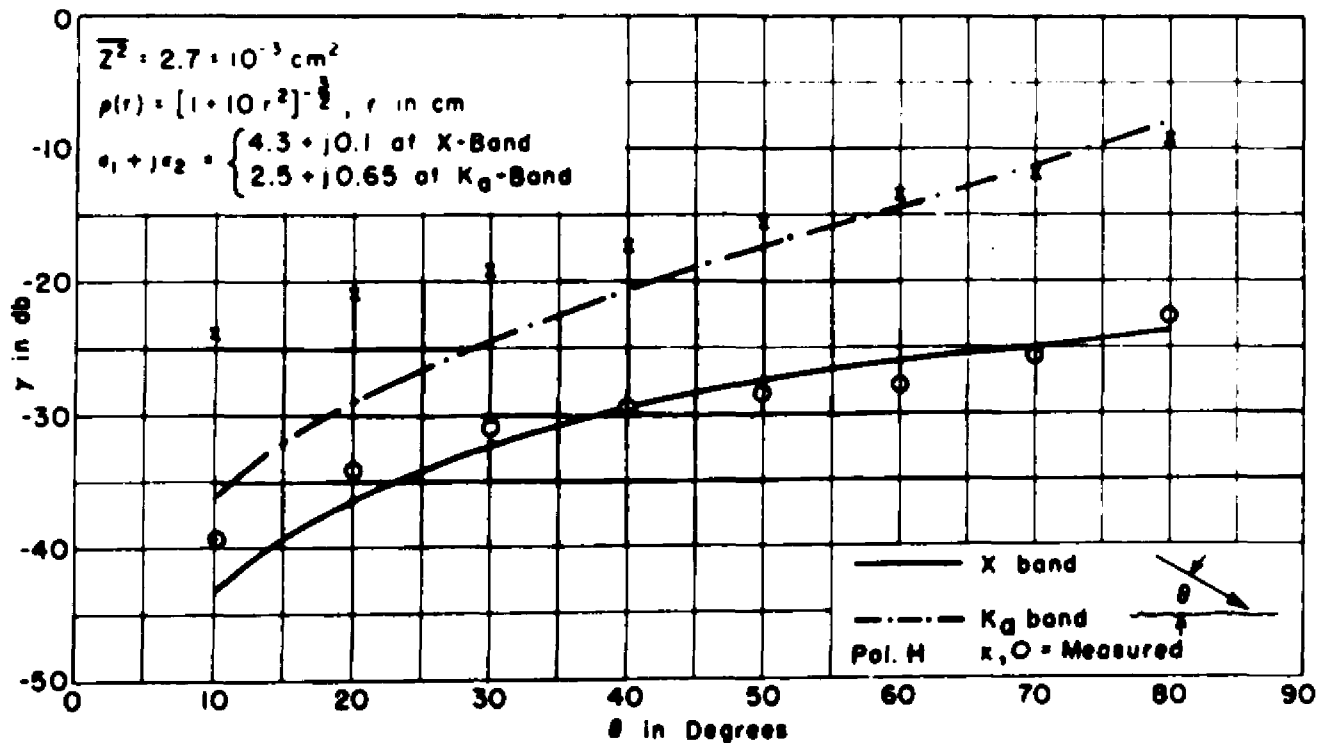
Fig. 15. Calculated and measured return from a slightly
rough asphalt surface (horizontal polarization).
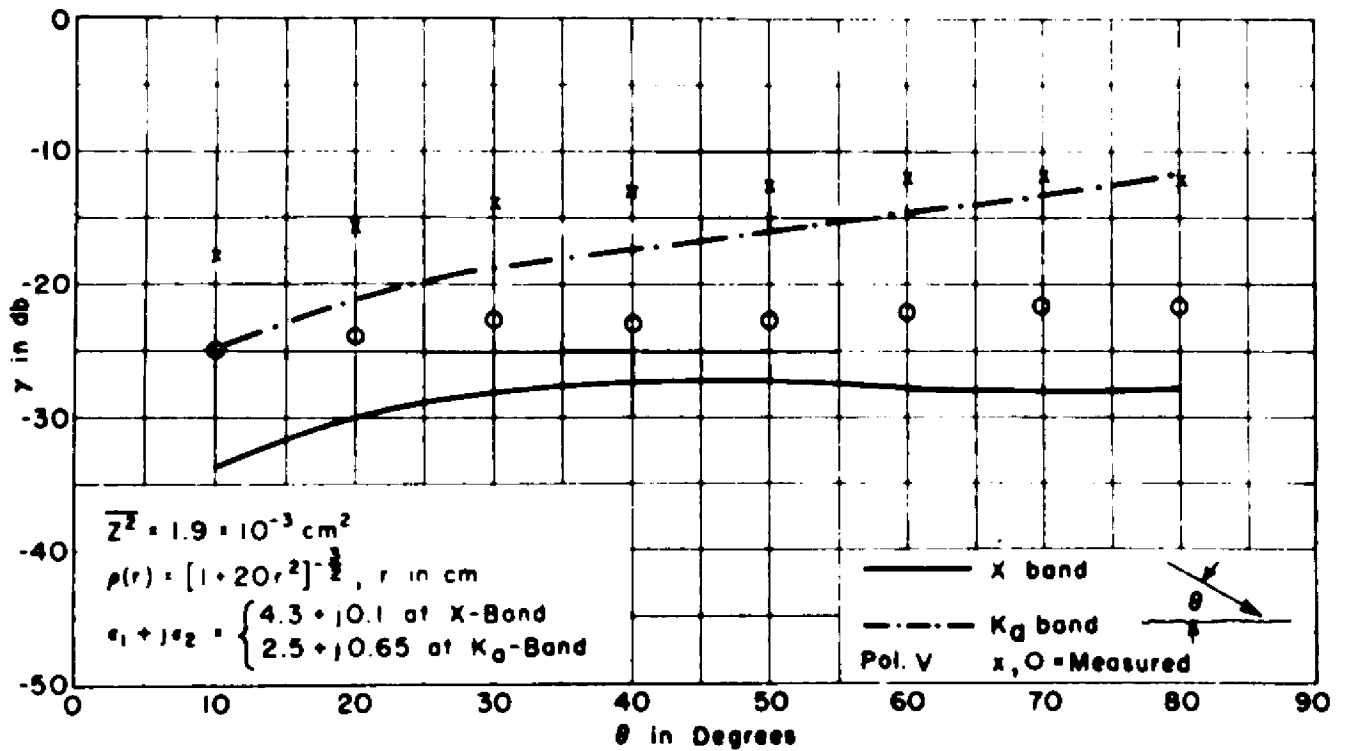
52
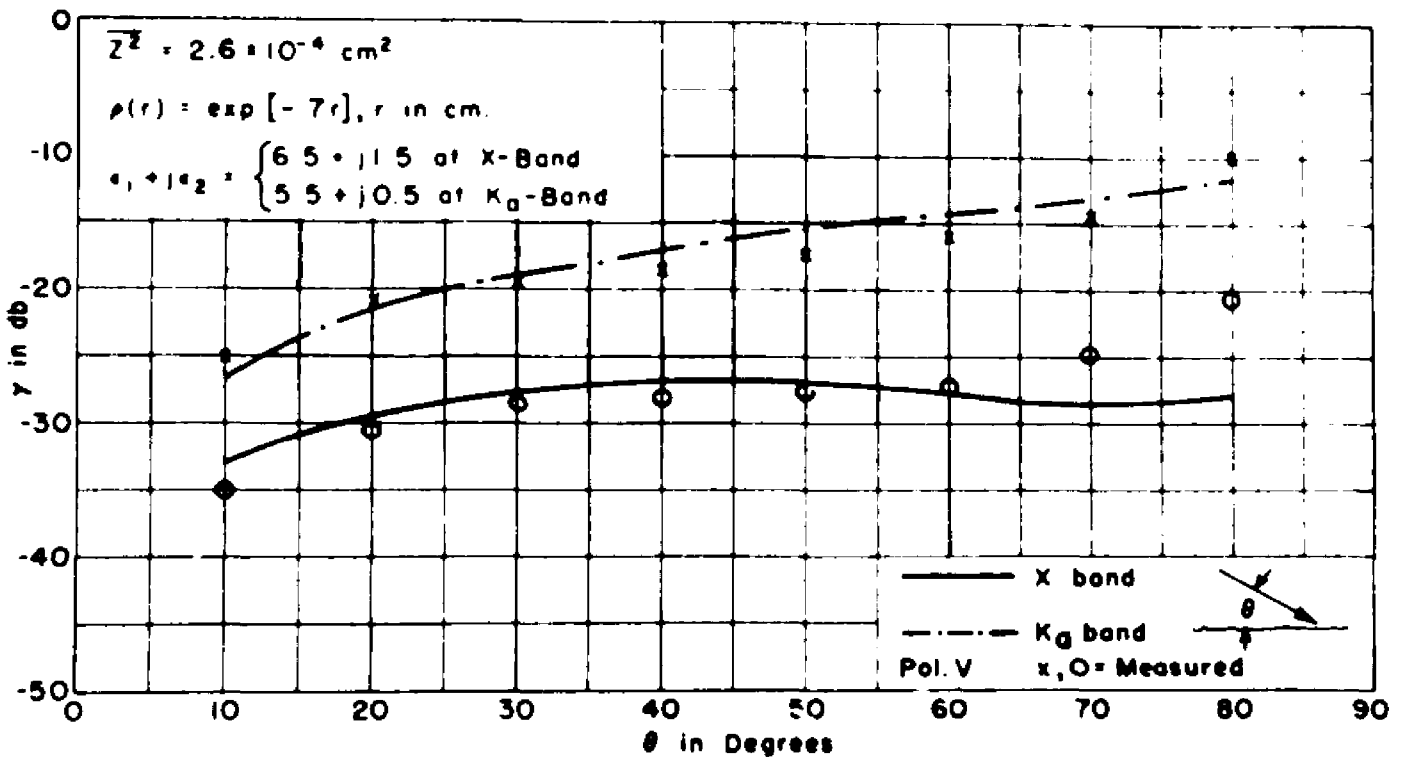
Fig. 16. Calculated and measured return from a slightly
rough concrete surface (vertical polarization).
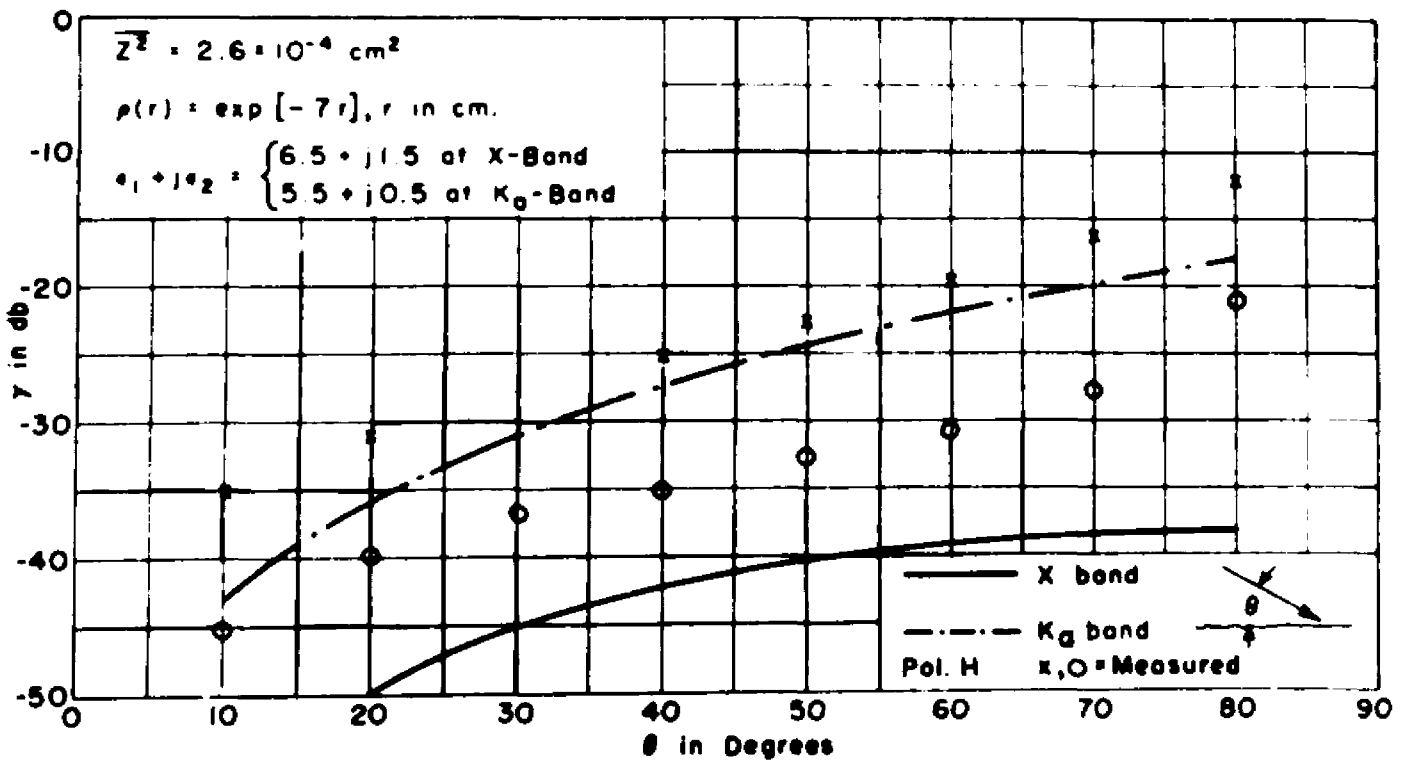
Fig. 17. Calculated and measured return from a slightly
rough concrete surface (horizontal polarization).

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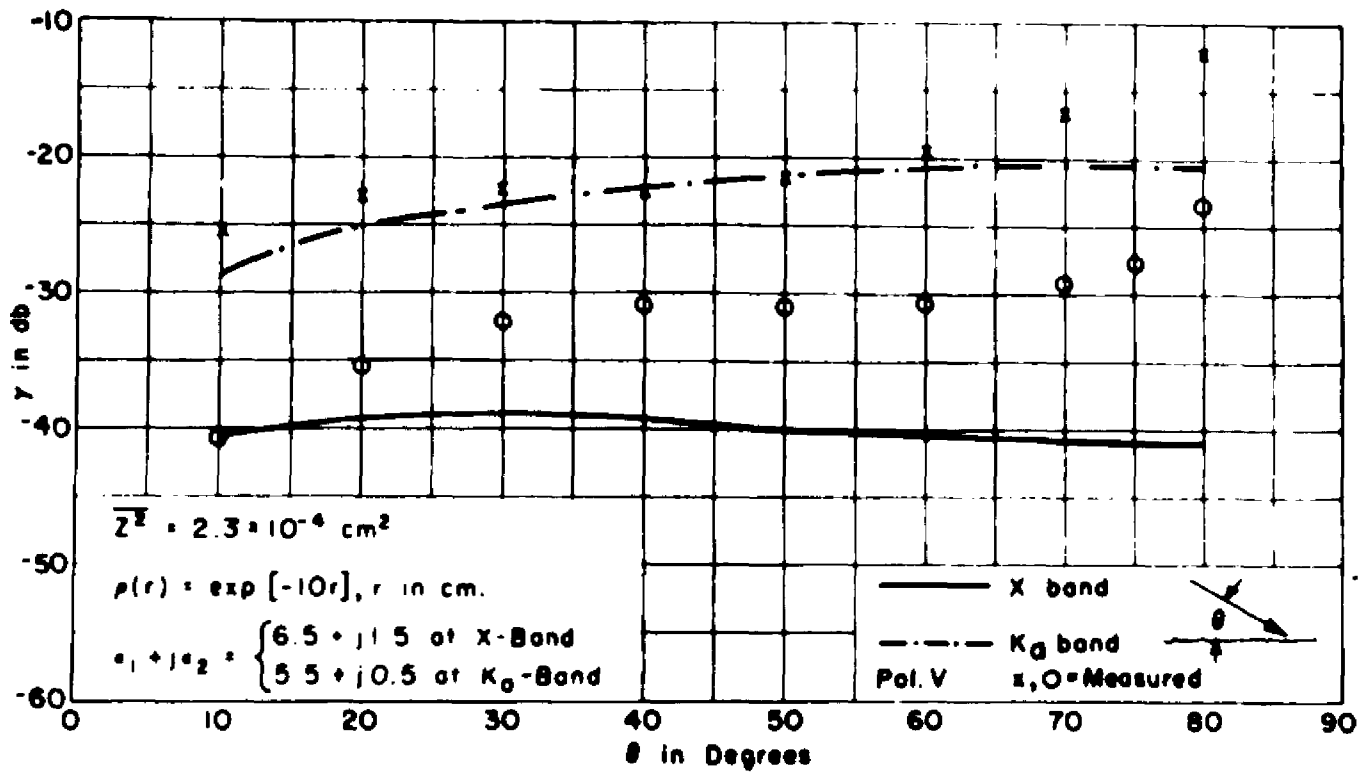
Fig. 18. Calculated and measured return from a slightly rough concrete surface (vertical polarization).
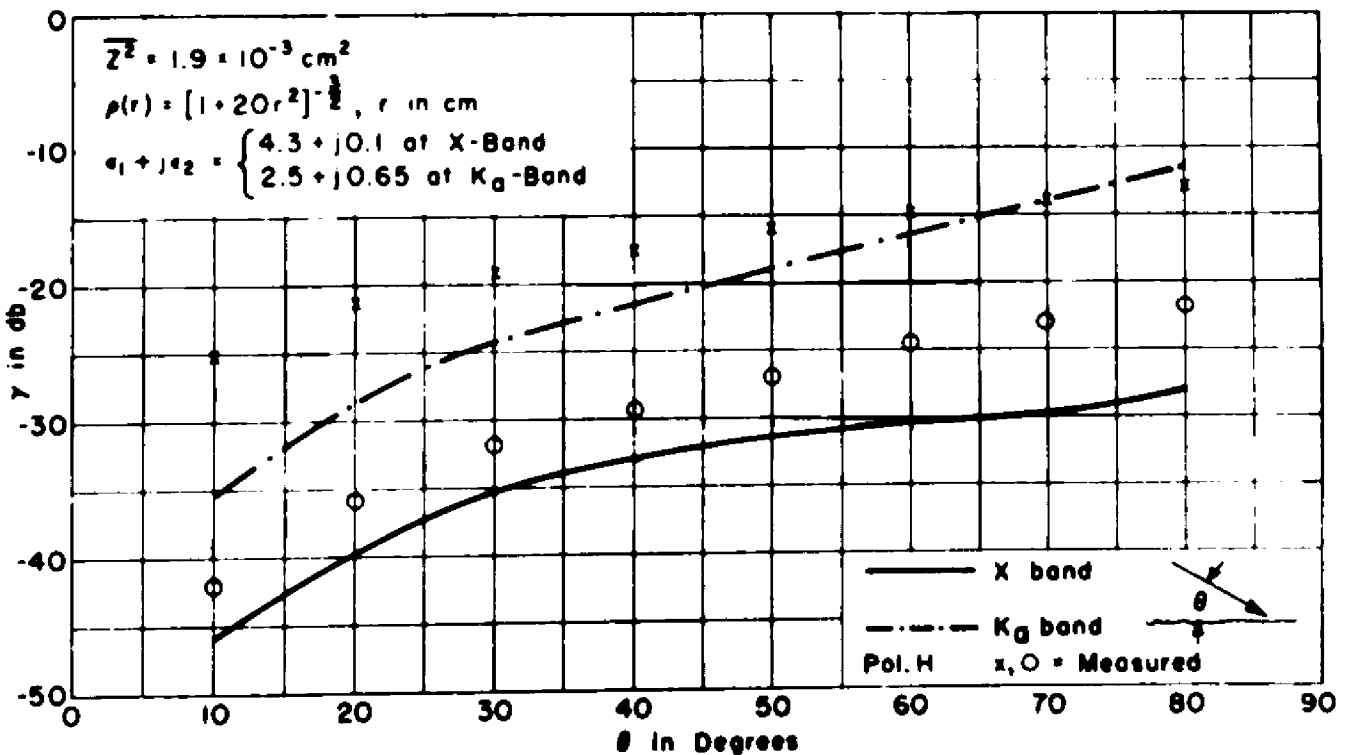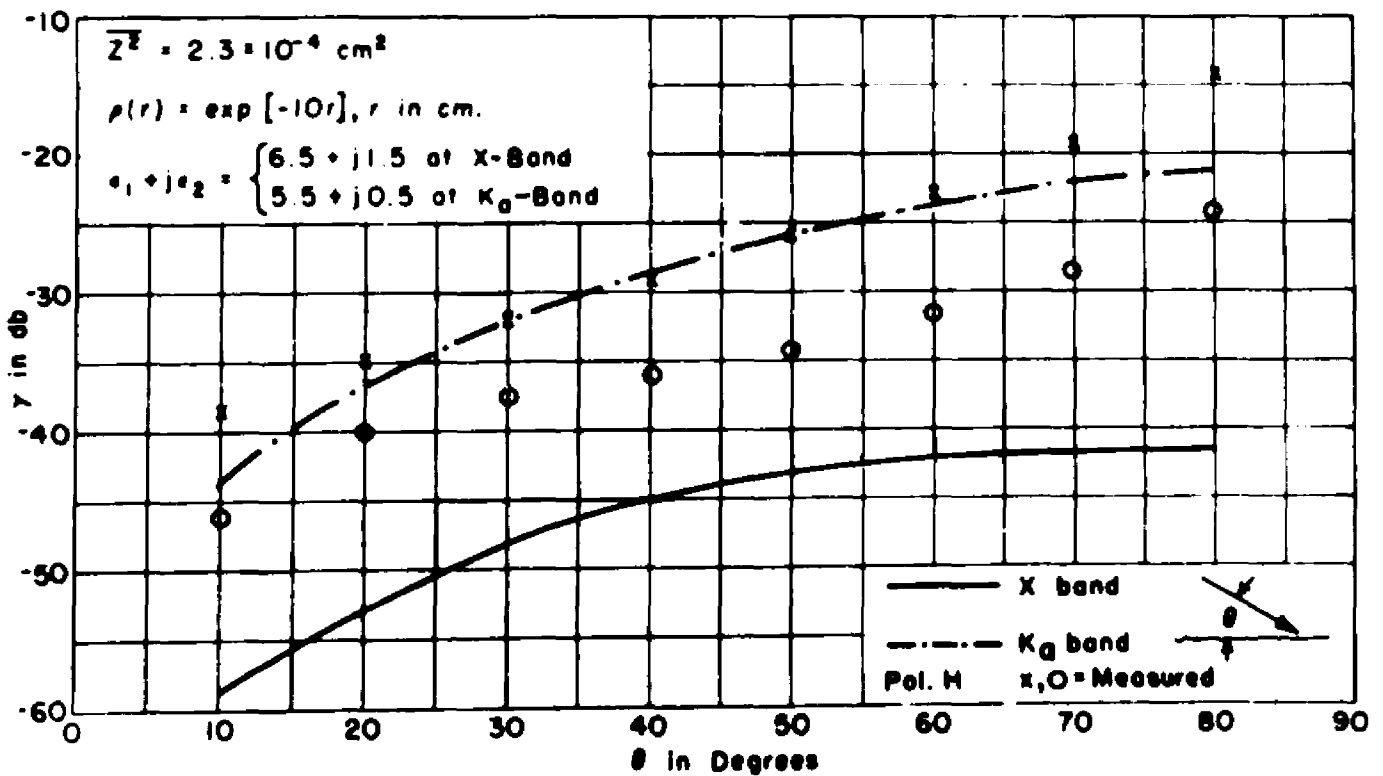

Fig. 19. Calculated and measured return from a slightly rough concrete surface (horizontal polarization).
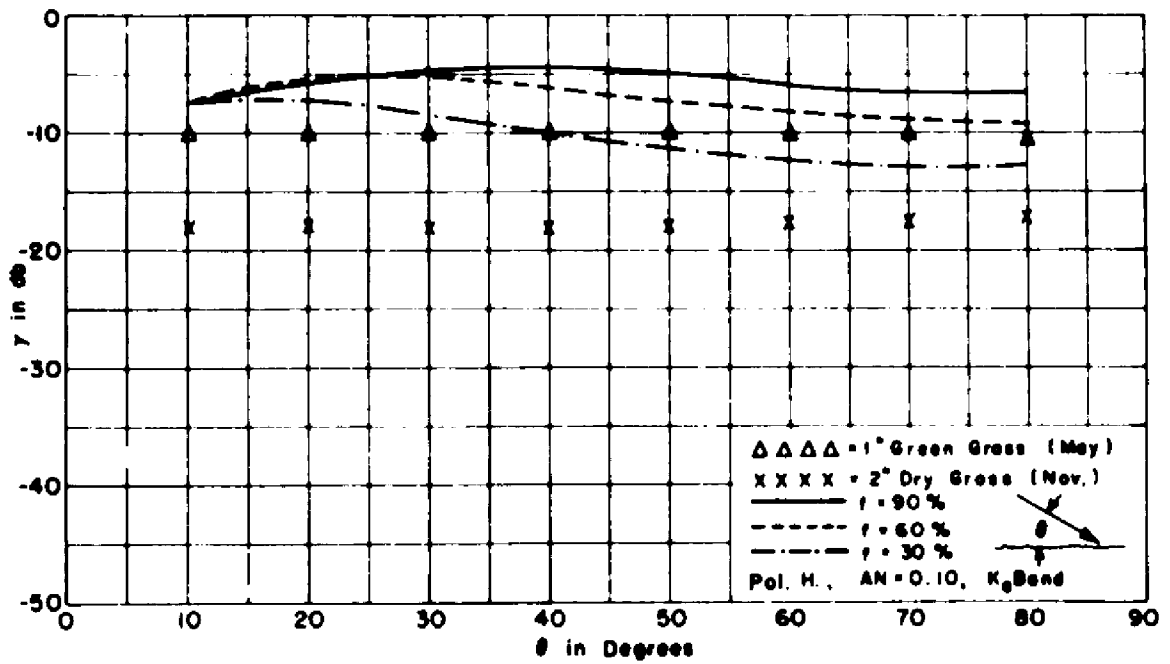
Fig. 20. Calculated and measured return from vegetation at
8.6 mm wavelength. Dependence on water content.
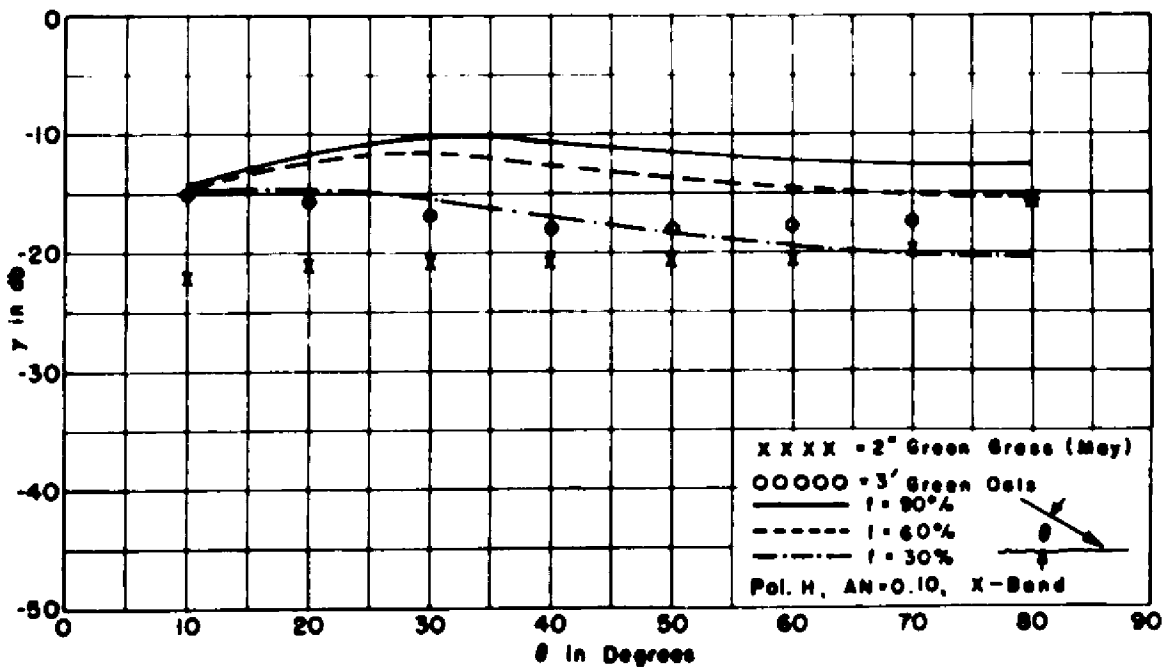
Fig. 21. Calculated and measured return from vegetation at
3 cm wavelength. Dependence on water content.
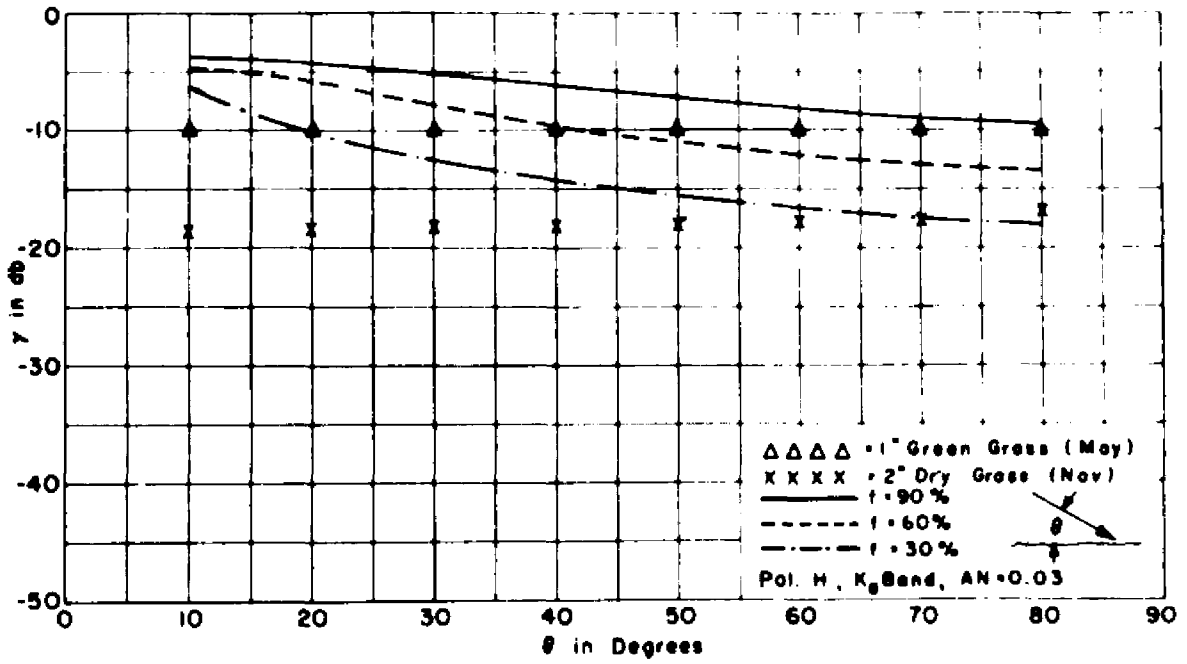
Fig. 22. Calculated and measured return from vegetation at
8.6 mm wavelength. Dependence on water content.
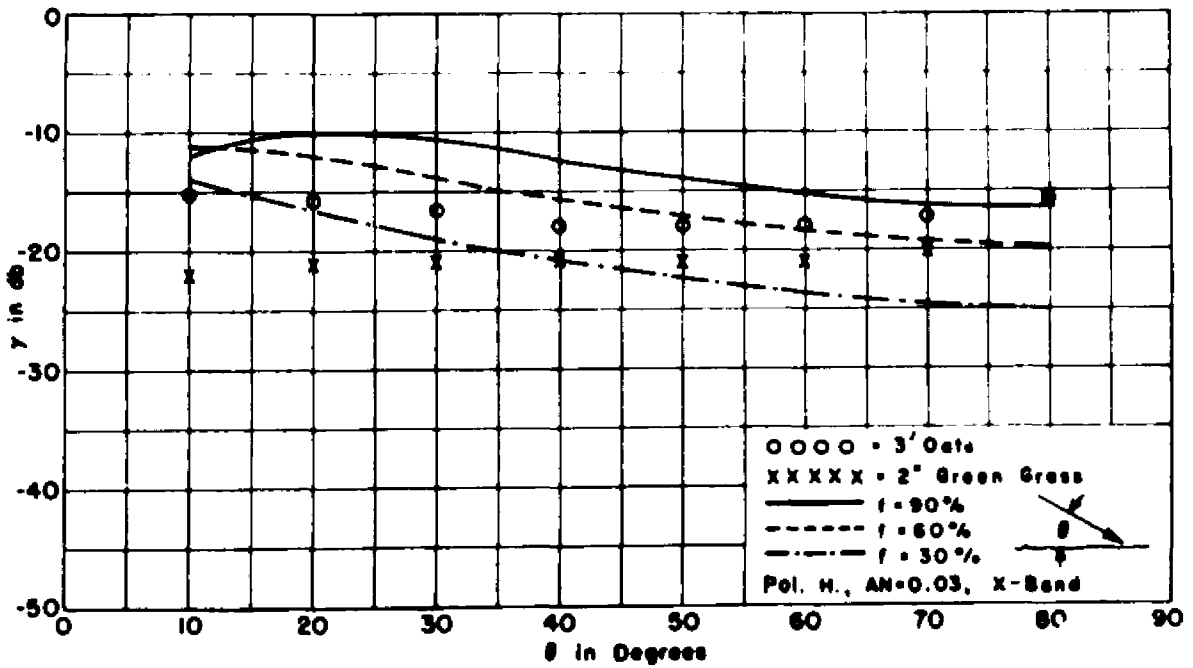
Fig. 23. Calculated and measured return from vegetation at
3 cm wavelength. Dependence on water content.
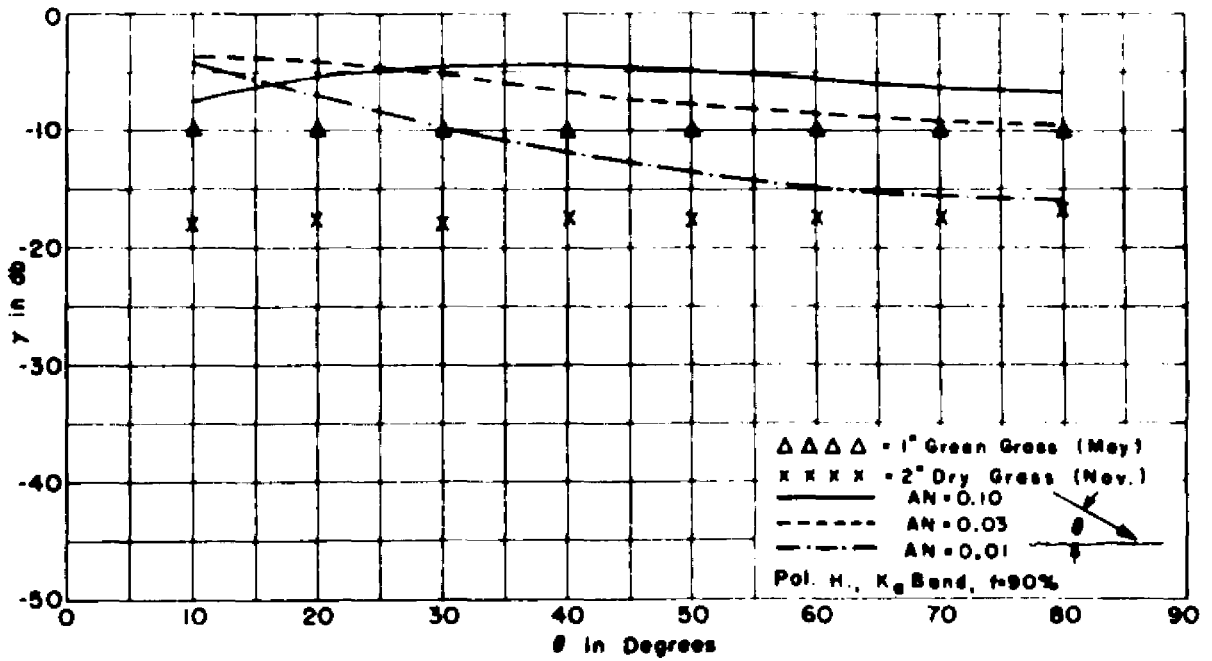
Fig. 24. Calculated and measured return from vegetation at
8.6 mm wavelength. Dependence on AN.
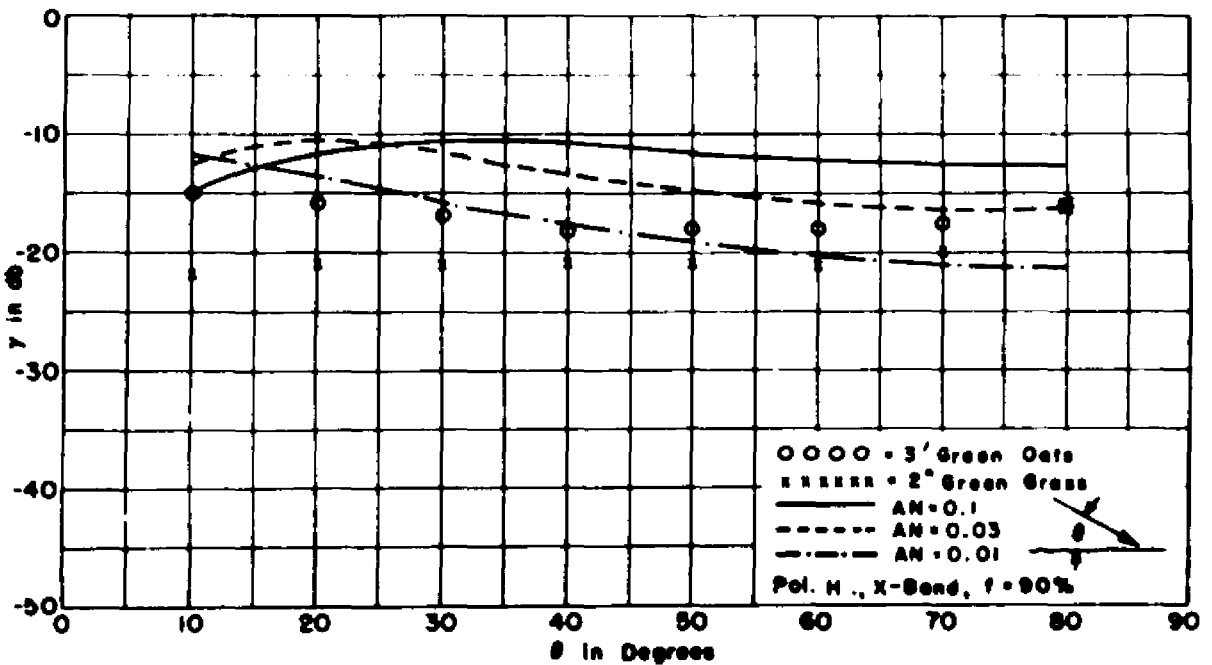
Fig. 25. Calculated and measured return from vegetation at
3 cm wavelength. Dependence on AN.
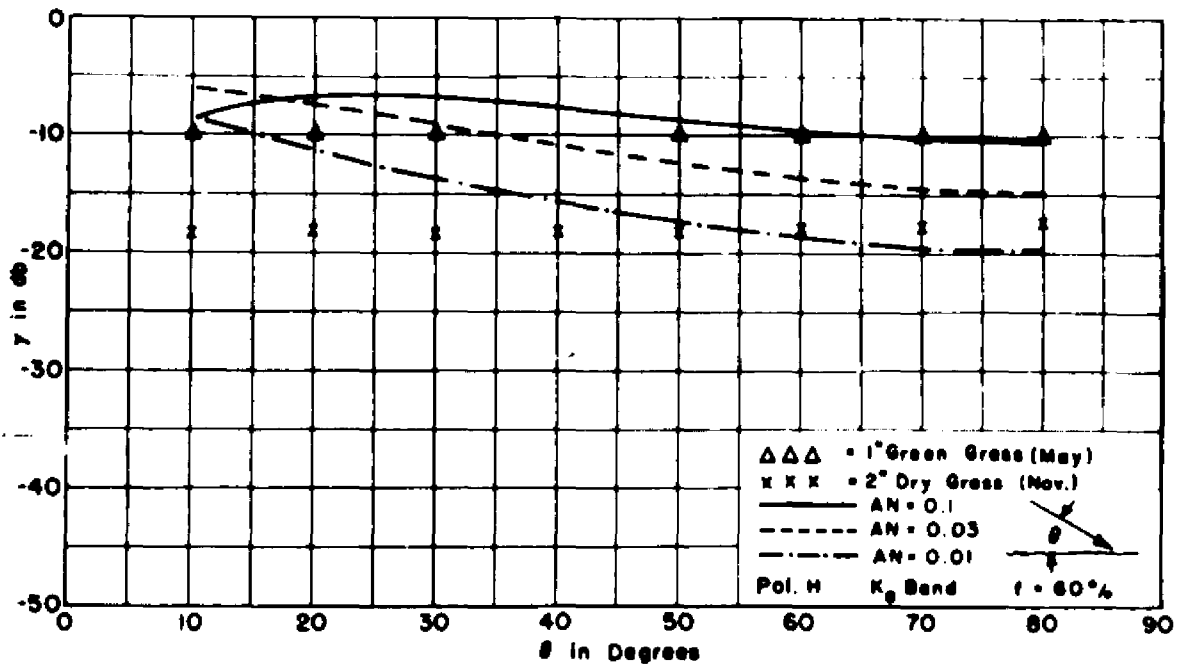
Fig. 26. Calculated and measured return from vegetation at
8.6 mm wavelength. Dependence on AN.
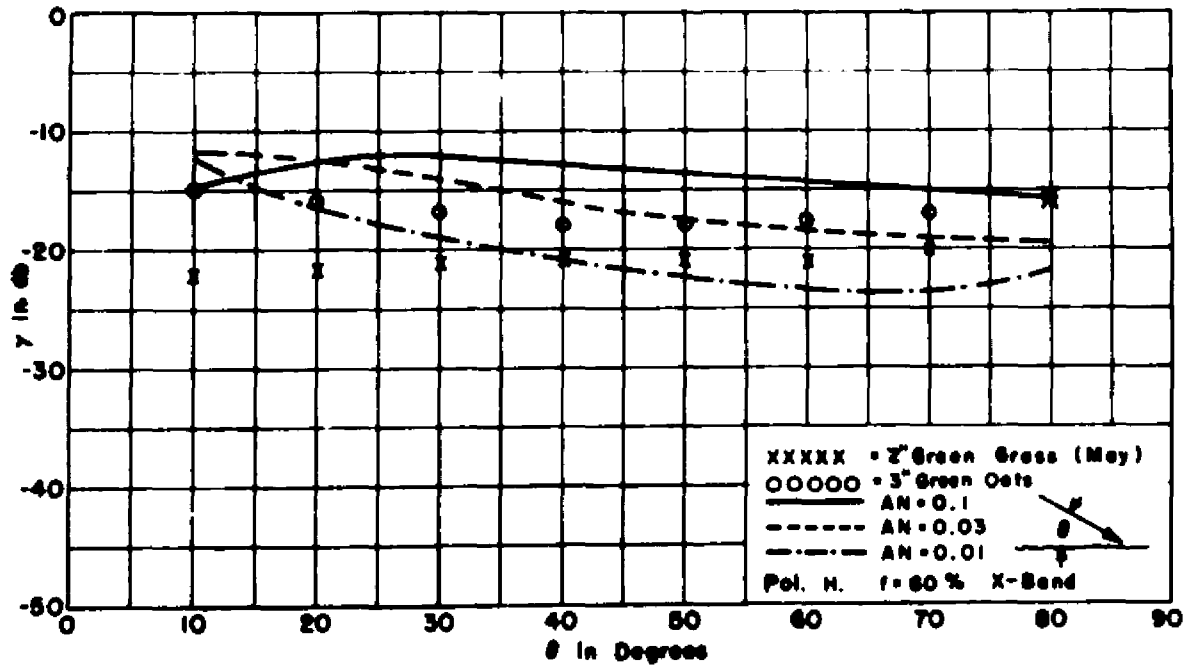
Fig. 27. Calculated and measured return from vegetation at
3 cm wavelength. Dependence on AN.
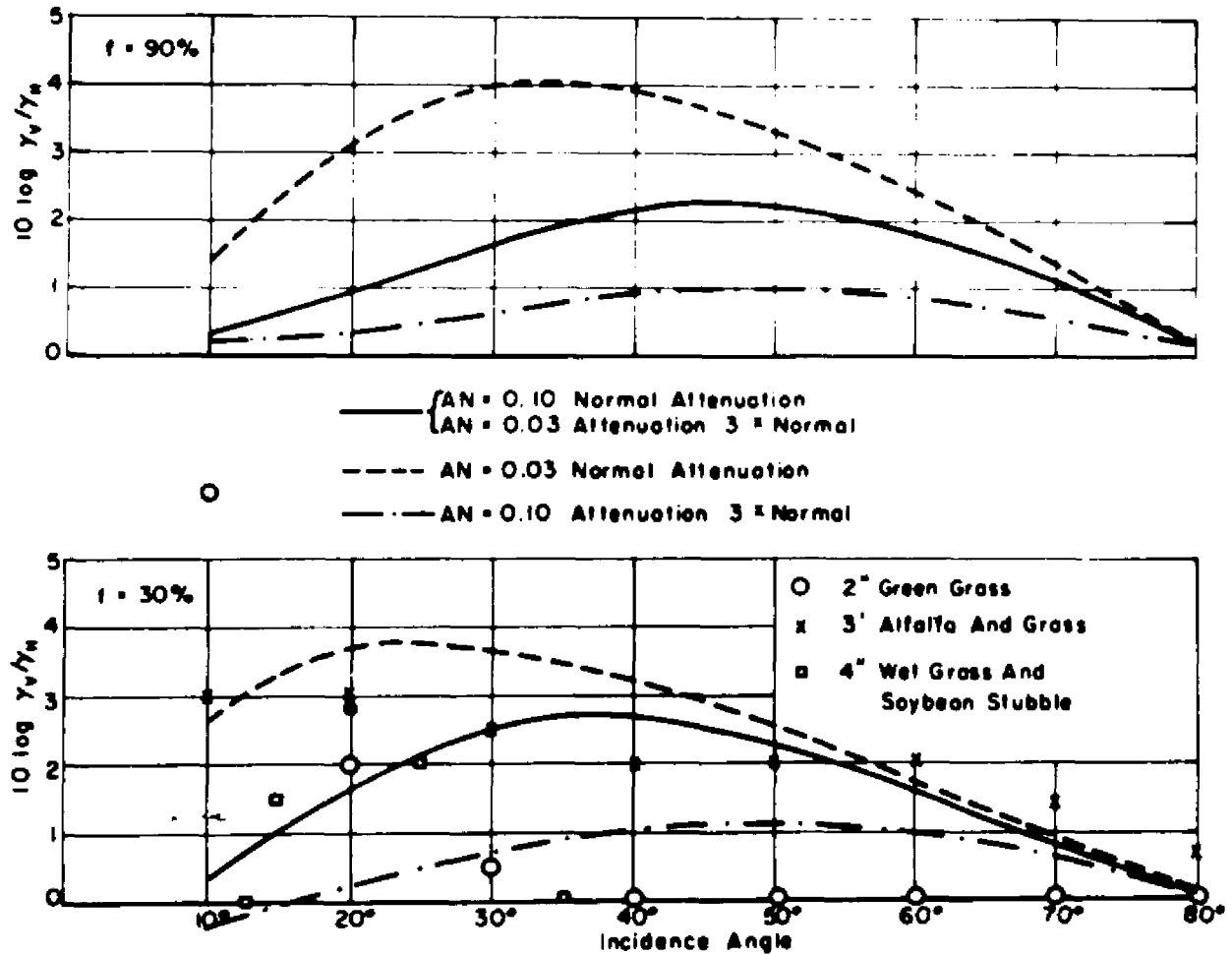
Fig. 28. Calculated return from vegetation at K_a-band. Ratio of return for vertical polarization to return for horizontal polarization for two values of water content and various densities of vegetation.
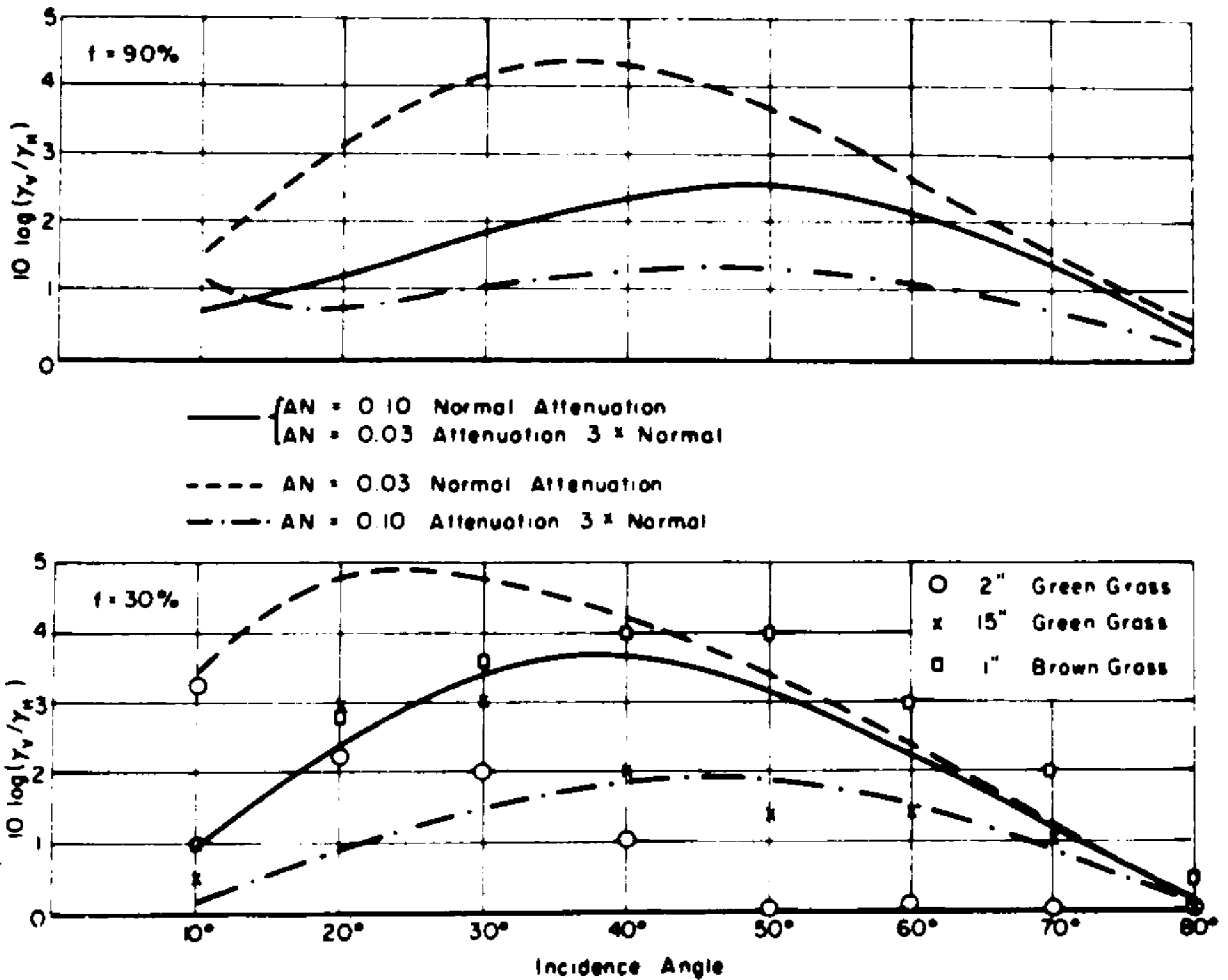
Fig. 29. Calculated return from vegetation at X-band. Ratio
of return for vertical polarization to return for
horizontal polarization for two values of water
content and various densities of vegetation.
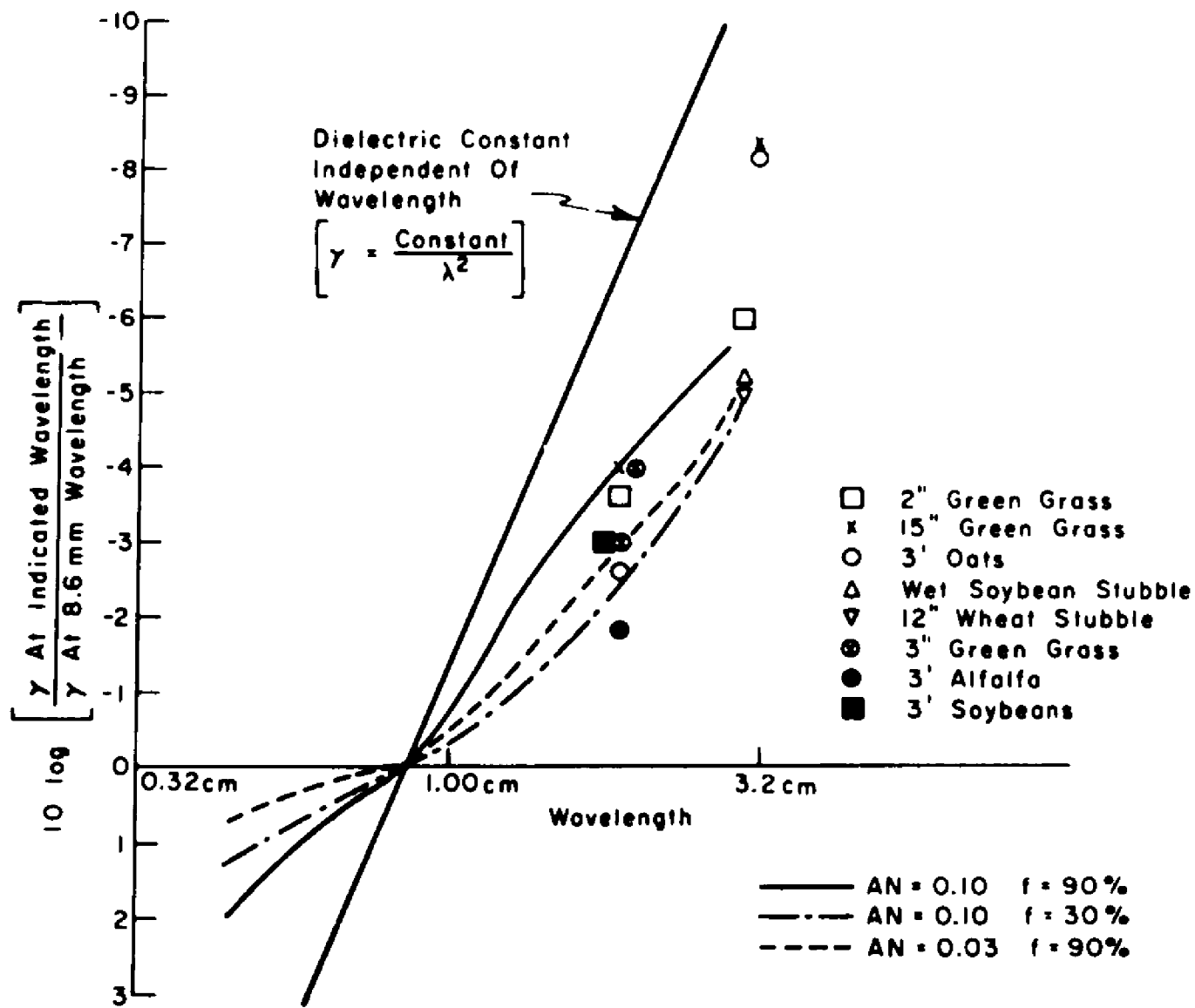
Fig. 30. Calculated and measured wavelength dependence of the return from vegetation, at 45° incidence angle.

CHAPTER IV

APPLICATIONS TO THE RADIOMETRIC
PROPERTIES OF TERRAIN

A. Introduction - Kirchhoff's Law

Another interesting application of the surface models developed in Chapter II is to the radiometric properties (or apparent temperatures) of natural surfaces at microwave frequencies. These have recently become of interest in connection with observations from artificial satellites, and may have some importance for an accurate determination of the heat balance of the earth. It was the intention, in formulating the complete models in Chapter II, to be able to determine these radiometric properties, in particular the emissivity, directly from the surface properties. There do exist certain special surfaces for which the emissivity may be calculated directly (for example, a perfectly flat surface of arbitrary dielectric constant). However, for non-uniform surfaces in general, in appears necessary to invoke Kirchhoff's radiation law in order to calculate the emissivity from the scattering coefficients Y. An understanding of this relation between the scattering coefficients and the apparent temperature of natural surfaces is particularly important at microwave frequencies. Here the scattering coefficients can be measured rather easily, whereas it may be difficult, if not impossible, to separate

the several contributions to the apparent temperature by measurement of thermal radiation alone.

Unfortunately, the usual statement of Kirchhoff's law is not adequate for the purpose. To remedy this situation, von Fragstein[27] has recently given a critical discussion of the form the law must take for both smooth and diffuse surfaces, and the relation between it and the reciprocity properties of the scattering coefficients. However, his discussion was for the optical case, and ignored the polarization properties of the surface. It will thus be necessary to extend his results to take account of these polarization effects, in order to provide a statement of Kirchhoff's law in its most general form.

Before carrying out the derivation it will be convenient to define the parameters that will be used to characterize the ability of a surface to emit thermal radiation.

These are, in the terminology of the review article by Rutgers,[28] the emission coefficients $e_h(\theta_o\phi_o)$ and $e_v(\theta_o\phi_o)$. where

(59) $$e_h(\theta_o\phi_o) = \frac{\begin{bmatrix} \text{Power emitted with horizontal polarization by} \\ \text{unit area of surface into element of solid angle} \\ d\Omega_o \text{ in the direction } \theta_o\phi_o. \end{bmatrix}}{\begin{bmatrix} \text{Power emitted with horizontal polarization by} \\ \text{unit area of black body at same temperature} \\ \text{into same element of solid angle in same} \\ \text{direction.} \end{bmatrix}}$$

An analogous definition holds for $e_v(\theta_o\phi_o)$, for vertically polarized radiation.

Throughout this chapter, it is to be understood that the thermal radiation referred to occupies a narrow band of frequencies Δf.

To obtain the desired relation between the emission coefficient and the absorption coefficient, defined earlier in Eq. (8), consider a surface (see Fig. 31) in temperature equilibrium with black body
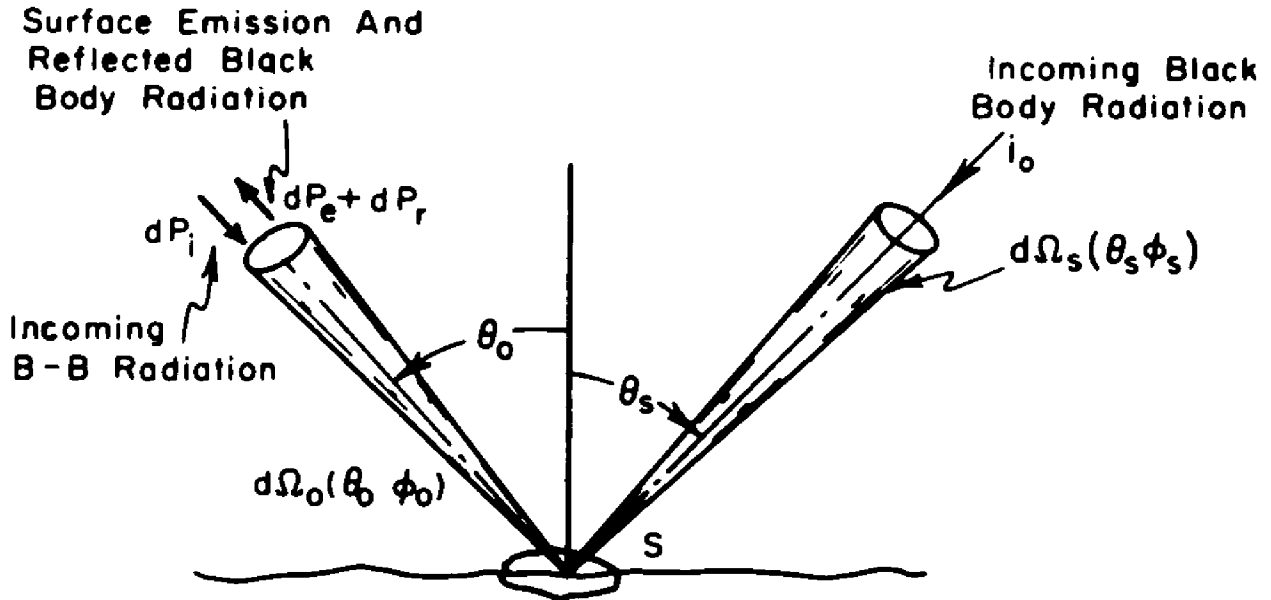
Fig. 31. Geometry for derivation of Kirchhoff's law.

radiation in the half space above it. Under this equilibrium condition, it will be assumed that just as much energy of a given polarization leaves the surface in a given direction as falls upon it from the same direction with the same polarization. To state this assumption quantitatively, let i_0 be the power density per unit solid angle (watts-meter^{-2}-steradian^{-1}) of the black body radiation in a specific polarization state. (Thus the total power density per unit

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solid angle of the black body radiation is $2i_o$). Then i_o is a function of frequency and the equilibrium temperature but not of the direction or polarization of the radiation. It refers to the energy in the frequency interval Δf. The power dP_i incident with horizontal polarization on an element of surface of area S from a range of solid angle $d\Omega_o$ in direction $\theta_o \phi_o$ is

(60) $\qquad dP_i = i_o \, d\Omega_o \, S \cos \theta_o$.

The power leaving the surface is the sum of two parts, the thermal emission from the surface, and the external black body radiation reflected by the surface. The power dP_e emitted by the same element of surface into the same range of solid angle with horizontal polarization is, from the definition of emission coefficient,

(61) $\qquad dP_e = e_h(\theta_o \phi_o) \, i_o \, d\Omega_o \, S \cos \theta_o$.

The power reflected by the surface into $d\Omega_o$ in the direction $\theta_o \phi_o$ with horizontal polarization, due to black body radiation coming from the range of solid angle $d\Omega_s$ in the direction $\theta_s \phi_s$ is, from Eq. (2),

(62) $\qquad d^2 P_r = \underbrace{i_o \, d\Omega_s \, S \cos \theta_s}_{\substack{\text{incident power} \\ \text{from } \theta_s \phi_s}} \underbrace{\left[\gamma_{hh}(s, o) + \gamma_{vh}(s, o) \right]}_{\substack{\text{fraction reflected} \\ \text{into } d\Omega_o}} \frac{d\Omega_o}{4\pi}$.

Thus the power dP_r reflected into $d\Omega_o$ with horizontal polarization is

(63) $\qquad dP_r = \dfrac{i_o d\Omega_o S}{4\pi} \displaystyle\int \left[\gamma_{hh}(s,o) + \gamma_{vh}(s,o) \right] \cos\theta_s \, d\Omega_s.$

Unless otherwise specified, integration is over the hemisphere.

Assuming the incident power of given polarization is balanced by the sum of the emitted and reflected power of the same polarization,

(64) $\qquad dP_i = dP_e + dP_r.$

Consequently,

(65) $\qquad 1 = e_h(\theta_o \phi_o) + \dfrac{1}{4\pi} \displaystyle\int \dfrac{\cos\theta_s}{\cos\theta_o} \left[\gamma_{hh}(s,o) + \gamma_{vh}(s,o) \right] d\Omega_s.$

By the reciprocity relations of Eq. (5) this becomes

(66) $\qquad 1 = e_h(\theta_o \phi_o) + \dfrac{1}{4\pi} \displaystyle\int \left[\gamma_{hh}(o,s) + \gamma_{hv}(o,s) \right] d\Omega_s.$

The third term in Eq. (66) is just the albedo defined by Eq. (7), which is related to the absorption coefficient by Eq. (8). Thus on substituting Eqs. (7) and (8) into (66), one finds

(67a) $\qquad e_h(\theta_o \phi_o) = a_h(\theta_o \phi_o)$.

A similar relation

(67b) $\qquad e_v(\theta_o \phi_o) = a_v(\theta_o \phi_o)$

is easily derived for vertical polarization.

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Equations (67a) and (67b) constitute the desired generalization of Kirchhoff's Law, taking account of both the polarization properties of the surface and the angular dependence of the coefficients. Furthermore, through Eqs. (7) and (8), the emission coefficients can be found from the scattering coefficients Y of the surface.

By comparing the derivation leading to Eq. (67) with that of von Fragstein.[27] the different assumptions in the two derivations become clear. The latter postulated only that there was a balance between the total power incident on, emitted and reflected by the surface, whereas Eq. (64) required that each polarization state separately satisfied a power balance. On the other hand, in deriving Eq. (67), no assumptions were made about the polarization properties of the surface, whereas von Fragstein assumed, without stating the fact, that

$$(68) \qquad Y_{hh}(o,s) + Y_{hv}(o,s) = Y_{vh}(o,s) + Y_{vv}(o,s) \; .$$

There is one other point in which the above derivation differs from the case where polarization is not taken into account. Von Fragstein pointed out that his derivation, which made use of what he referred to as the Sommerfeld-Pfrang reciprocity relation, could be reversed. That is, by assuming the validity of Kirchhoff's law, he could prove the Sommerfeld-Pfrang reciprocity relation. However the argument leading to Eq. (67) cannot be reversed to obtain

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the reciprocity relations of Eq. (5). At most, it can be shown that the pairs of coefficients entering Eq. (68) satisfy a relation like Eq. (5). This is because Eq. (67) provides only two relations between the four coefficients Y.

B. Apparent Surface Temperatures

In practice, real terrain surfaces are not in equilibrium with black-body radiation in the half-space above them. Thus the total radiation emanating from the surface will be different from that emitted by a black body at the same temperature. That is, the surface will have an "apparent temperature" different from its actual temperature. The concept of the apparent temperature of a surface may be defined in the following manner.

Within an enclosure whose walls are maintained at a fixed temperature T, the black-body radiation is characterized by the quantity $i_o(T,\lambda)$, the power density per unit solid angle of radiation of a specific polarization, with frequencies in the interval Δf. For sufficiently high temperatures and long wavelengths, $i_o(T\lambda)$ is given by the Rayleigh-Jeans approximation[30]

(69) $i_o(T,\lambda) = \dfrac{kT\Delta f}{\lambda^2}$ watts-meter^{-2}-steradian^{-1}

where k is Boltzmann's constant. For arbitrary temperature and wavelength, the Planck[31] distribution law must be used.

Because of the unique relation between i_o and the temperature T of the enclosure, the radiation itself is also said to have temperature T. Consider now any arbitrary thermal radiation travelling in a specified direction with specified polarization with frequencies in the small interval Δf. Then the apparent temperature of this arbitrary thermal radiation is defined as the temperature of the blackbody radiation which has the same power density per unit solid angle. To extend the concept to a surface, it is only necessary to postulate that the apparent temperature of a surface when viewed from some specified direction (with a detector of specified polarization) is equal to the apparent temperature of the radiation emanating from the surface in the prescribed direction with the prescribed polarization.

There are three principal contributions to the apparent temperature of natural terrain surfaces (see Fig. 32).

The first is the thermal radiation emitted by the surface. It is customary to assume that as long as the surface has a well-defined temperature (T_g), its emission coefficient is independent of conditions external to the surface.

The second contribution to the apparent temperature is due to the reflection of the diffuse thermal radiation (T_h or T_v) impinging on the surface from the upper hemisphere. This diffuse radiation is,
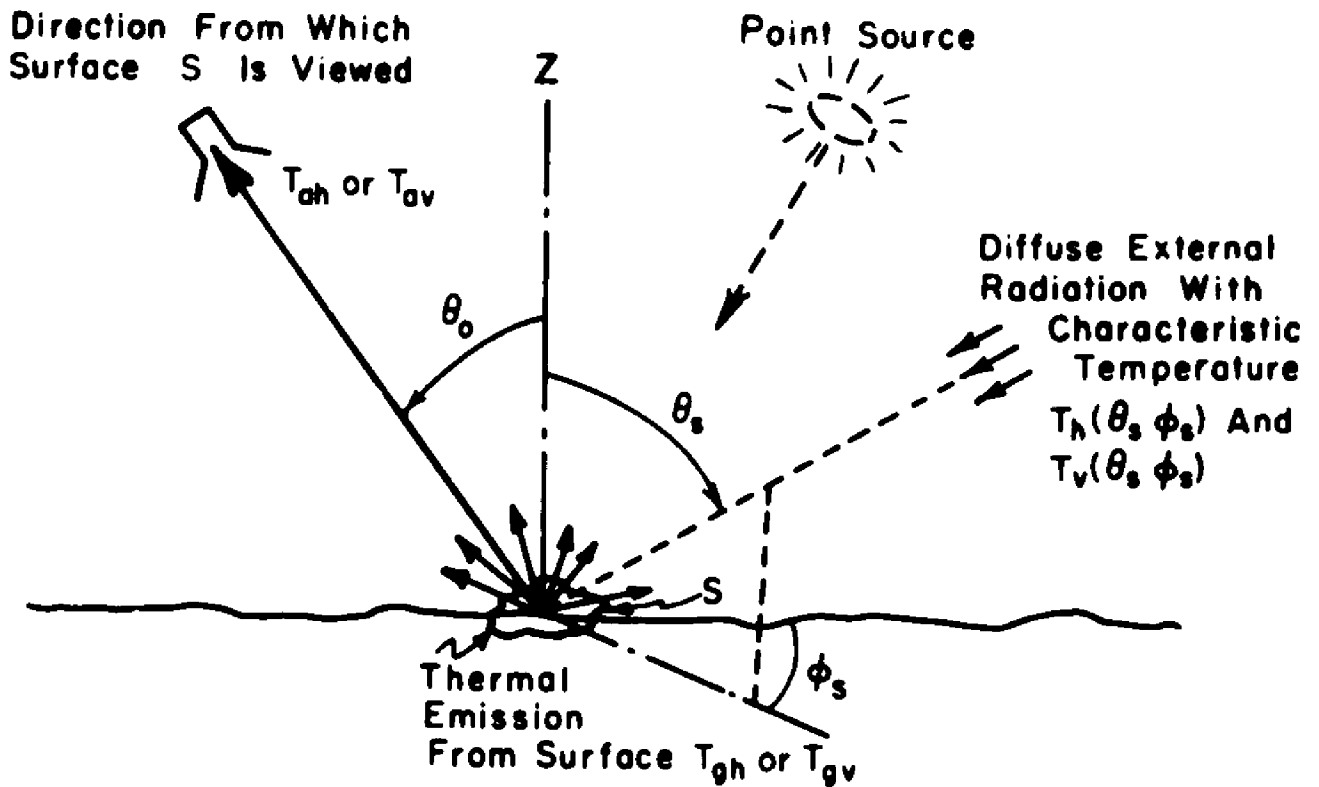
Fig. 32. Contributions to the apparent temperature of a surface. in practice, due primarily to radiation from the earth's atmosphere at microwave wavelengths shorter than 10 cm, and to extraterrestrial (or cosmic) radio noise at wavelengths longer than 100 cm.[30]

The third contribution to the apparent surface temperature is due to radiation from point, or quasi-point sources scattered by the surface. The most prominent of these is the sun.

The power due to these three principal contributions adds directly to give the total power emanating from the surface. Thus, for conditions under which the Rayleigh-Jeans approximation (Eq. (69)) may be used, the apparent temperature of the surface may also be expressed as a sum of contributing temperatures, viz:

(70) $T_{ah} = T_{gh} + T_{sh} + T_{ph}$

$T_{av} = T_{gv} + T_{sv} + T_{pv}$

where

$T_{ah}(\theta_o \phi_o)$, $T_{av}(\theta_o \phi_o)$ are the _total_ apparent temperatures of a surface for radiation of horizontal or vertical polarization respectively, when the surface is viewed from the direction θ_o, ϕ_o (see Fig. 32).

T_{gh}, T_{gv} are the contributions to the apparent temperature due solely to the thermal radiation _emitted_ by the surface.

T_{sh}, T_{sv} are the contributions to the apparent temperature due to the scattering of the diffuse external radiation.

T_{ph}, T_{pv} are the contributions to the apparent temperature due to the bistatic scattering of radiation from point or quasi-point sources.

The first contribution, that due to thermal emission from the ground, is easily calculated from the definition, Eq. (59), of the emission coefficient. Clearly,

$T_{gh}(\theta_o \phi_o) = e_h(\theta_o \phi_o) T_g$

(71)

$T_{gv}(\theta_o \phi_o) = e_v(\theta_o \phi_o) T_g$

where T_g is the actual temperature of the ground.

The second contribution, due to the external diffuse radiation may be calculated as follows. Let the apparent temperature of this radiation arriving from the direction $\theta_s \phi_s$ be $T_h(\theta_s \phi_s)$, $T_v(\theta_s \phi_s)$ for the horizontally and vertically polarized components, respectively. Then from the definition of the scattering coefficients, Eq. (2), and the fact that the apparent temperature of radiation is assumed to be proportional to its power density/steradian, it is found that,

$$(72) \qquad T_{sh}(\theta_0 \phi_0) = \frac{1}{4\pi} \int \left[T_h(\theta_s \phi_s) \gamma_{hh}(s,o) + T_v(\theta_s \phi_s) \gamma_{vh}(s,o) \right] \cdot \frac{\cos \theta_s}{\cos \theta_0} \, d\Omega_s \, .$$

From the reciprocity relations, Eq. (5), this becomes

$$(73a) \qquad T_{sh}(\theta_0 \phi_0) = \frac{1}{4\pi} \int \left[T_h(s) \gamma_{hh}(o,s) + T_v(s) \gamma_{hv}(o,s) \right] d\Omega_s \, .$$

Similarly

$$(73b) \qquad T_{sv}(\theta_0 \phi_0) = \frac{1}{4\pi} \int \left[T_v(s) \gamma_{vv}(o,s) + T_h(s) \gamma_{vh}(o,s) \right] d\Omega_s \, .$$

The third contribution, that due to point sources, will be discussed in the next section.

Before considering the detailed application of the foregoing results to actual terrain surfaces, it may be of interest to give formulas for the apparent temperatures of some highly idealized surface models much used in the past.[20]

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(i) **Black Body**

$$e_h = e_v = 1$$

(74)
$$\gamma_{hh} = \gamma_{hv} = \gamma_{vh} = \gamma_{vv} = 0$$

$$T_{ah} = T_{av} = T_g \ .$$

The apparent temperature of a black body is equal to its actual temperature. It absorbs all radiation falling on it, and reflects none.

(ii) **Gray Body**

The emission coefficients for a gray body are, like those of a black body, independent of polarization and direction:

(75)
$$e_h(\theta_o \phi_o) = e_v(\theta_o \phi_o) = e \ .$$

Here e is a constant, $e < 1$. The scattering coefficients for a gray body have not been previously defined. However, to maintain the similarity between gray and black body radiation, it seems plausible to require the γ's to be independent of polarization, and to follow Lambert's law. Thus

(76)
$$\gamma_{hh}(o, s) = \gamma_{hv}(o, s) = \gamma_{vh}(o, s) = \gamma_{vv}(o, s) = \gamma_g \cos \theta_s$$

where γ_g is a constant. Then the apparent temperature of a gray body is

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(77) $$T_{ah} = T_{av} = e\, T_g + \frac{Y_g}{4\pi} \int \left[T_h(s) + T_v(s)\right] \cos\theta_s\, d\Omega_s \; .$$

As may be seen by letting $T_h = T_v = T_g$, the relation between e and Y_g is

(78) $$e = 1 - (Y_g/2) \; .$$

(iii) Flat Surface

The emission coefficients for a flat surface have long been known.[31] Because the reflection is specular, they may be calculated from the standard form of Kirchhoff's law, and the Fresnel coefficients. For a surface of arbitrary complex dielectric constant $\epsilon' = \epsilon_1 - i\epsilon_2$ the apparent temperatures are

(79)
$$T_{ah}(\theta_0\phi_0) = \left[1 - |R_h(\theta_0)|^2\right] T_g + |R_h(\theta_0)|^2 T_h(\theta_0, \phi_0 + \pi)$$

$$T_{av}(\theta_0\phi_0) = \left[1 - |R_v(\theta_0)|^2\right] T_g + |R_v(\theta_0)|^2 T_v(\theta_0, \phi_0 + \pi) \; ,$$

where R_h, R_v are the Fresnel coefficients

(80)
$$R_h = \frac{\cos\theta_0 - (\epsilon' - \sin^2\theta_0)^{\frac{1}{2}}}{\cos\theta_0 + (\epsilon' - \sin^2\theta_0)^{\frac{1}{2}}}$$

$$R_v = \frac{\epsilon'\cos\theta_0 - (\epsilon' - \sin^2\theta_0)^{\frac{1}{2}}}{\epsilon'\cos\theta_0 + (\epsilon' - \sin^2\theta_0)^{\frac{1}{2}}} \qquad .$$

There is an important qualification to the foregoing discussion. The effects of atmospheric attenuation have been ignored. Consequently the apparent temperatures as calculated from the above formulas apply only near enough to the surface so that there is no appreciable attenuation between the surface and the observer at the frequency of interest. If this condition is not fulfilled, the thermal radiation due to the intervening medium must also be taken into account.

C. Point Source Effects

In this section, the contribution of point sources to the apparent surface temperature will be calculated. This may be done most easily by the artifice of replacing the point source by an equivalent source subtending a small solid angle Ω_p, producing radiation with i_p watts-meter^{-2}-steradian^{-1}. That is, if the actual point source produces radiation of intensity I_p watts-meter^2 at the surface, then $I_p = i_p \Omega_p$. Both I_p and i_p refer to radiation of specific polarization in the frequency interval Δf. Then from Eq. (2) the power density scattered by an element of surface S toward the observing direction $\theta_o \phi_o$ is

$$(81) \qquad I_s = \left[i_p \Omega_p \right] \gamma(s, o) \frac{S}{4\pi R^2} \cos \theta_s .$$

where (s) refers to the direction of the point source. But

$S \cos \theta_0 / 4\pi R^2$ is the solid angle subtended by S at the viewing position, so that the intensity per unit solid angle of the scattered radiation is

$$(82) \qquad i_s = \frac{i_s}{S \cos \theta_0} \; 4\pi R^2 = \frac{i_p \Omega_p}{4\pi} \; \gamma(s,o) \; \frac{\cos \theta_s}{\cos \theta_0} \qquad .$$

Making use of the reciprocity relation, and taking polarization into account,

$$(83) \qquad i_{sh} = \frac{(i_p \Omega_p)}{4\pi} \; \left[\gamma_{hh}(o,s) + \gamma_{hv}(o,s) \right] \qquad .$$

Because of the relation Eq. (69) between i and T, the contribution of the quasipoint source to the apparent temperature becomes

$$(84) \qquad T_{ph} = T_p \left(\frac{\Omega_p}{4\pi} \right) \left[\gamma_{hh}(o,s) + \gamma_{hv}(o,s) \right]$$

where T_p is the apparent temperature of the radiation i_p. For true point sources, only the product $T_p \Omega_p$ is known.

It may be of interest to estimate the contribution of the sun to the apparent surface temperature at microwave frequencies. At K_a-band, the apparent temperature of the sun is about $10^{4\,\circ}K$ and a typical value of γ is 0.1. Thus T_{sh} due to the sun is about $0.01^\circ K$. It seems likely that unless great accuracy is required, the effects of the sun on apparent temperature may be ignored for diffuse surfaces.

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D. Application to the Apparent Temperature of Some Natural Surfaces

In this section, the results of IVA and IVB will be used to estimate the apparent temperatures of two surfaces for which some measurements are available.

The first surface of interest is an asphalt roadway, the apparent temperature of which was measured[36] at wavelength of 4.3 mm. The measurements could be interpreted in terms of the model for a slightly rough surface developed in Chapter II. However, such a model is unnecessarily complicated for interpreting apparent temperature measurements. This is because the scattering pattern of such a surface is predominantly specular, with a reflection coefficient not very different from that for a perfectly flat surface. The diffuse part of the scattering pattern (see, e.g., Eqs. (25), (26)), though of vital importance for predicting the back scattering, contains very little power. Elementary considerations show, in fact, that the ratio of the power in the complete diffuse pattern (integrated over the hemisphere) to the power in the specular beam has an order of magnitude of $\psi(2|R|^2)$ where R is the appropriate Fresnel coefficient. For the roadway surfaces presented in Figs. 10-19, this amounts to a few percent. Thus the diffuse scattering will be ignored, and the apparent temperature will be calculated from the results of IVB(iii).

To do so, it is necessary to estimate the dielectric constant of the surface, and the apparent "sky" temperatures $T_v(\theta_0)$ and $T_h(\theta_0)$. At 4.3 mm wavelength, the latter is due primarily to radiation from the earths atmosphere. A convenient formula, based on a simple model which represents the atmosphere as a uniform horizontal layer, is[37, 38]

$$(85) \qquad T_h(\theta_0) = T_v(\theta_0) = T_a(1 - r^{\sec \theta_0})$$

where r is the fraction of power of the given wavelength transmitted through the atmosphere at normal incidence, and T_a is the temperature of the model atmosphere. Taking $r = 0.63$, a value appropriate to the dry air of Texas, the predictions of Eq. (85) are in fair agreement with the sky temperatures measured by Straiton.[36]

With this assumption about the sky temperature, the apparent temperature of a flat or slightly rough surface becomes

$$T_{ah}(\theta_0) = \left[1 - |R_h|^2\right]T_g + |R_h|^2 \, T_a(1-r^{\sec \theta_0})$$

$$(86)$$

$$T_{vh}(\theta_0) = \left[1 - |R_v|^2\right]T_g + |R_v|^2 \, T_a(1 - r^{\sec \theta_0}) \ .$$

Using $\epsilon' = 4$ and $r = 0.63$, the predictions of Eq. (86) are shown in Fig. 33. The measurements of Straiton, Britt and Tolbert[36] on an asphalt surface are included for comparison. It will be seen
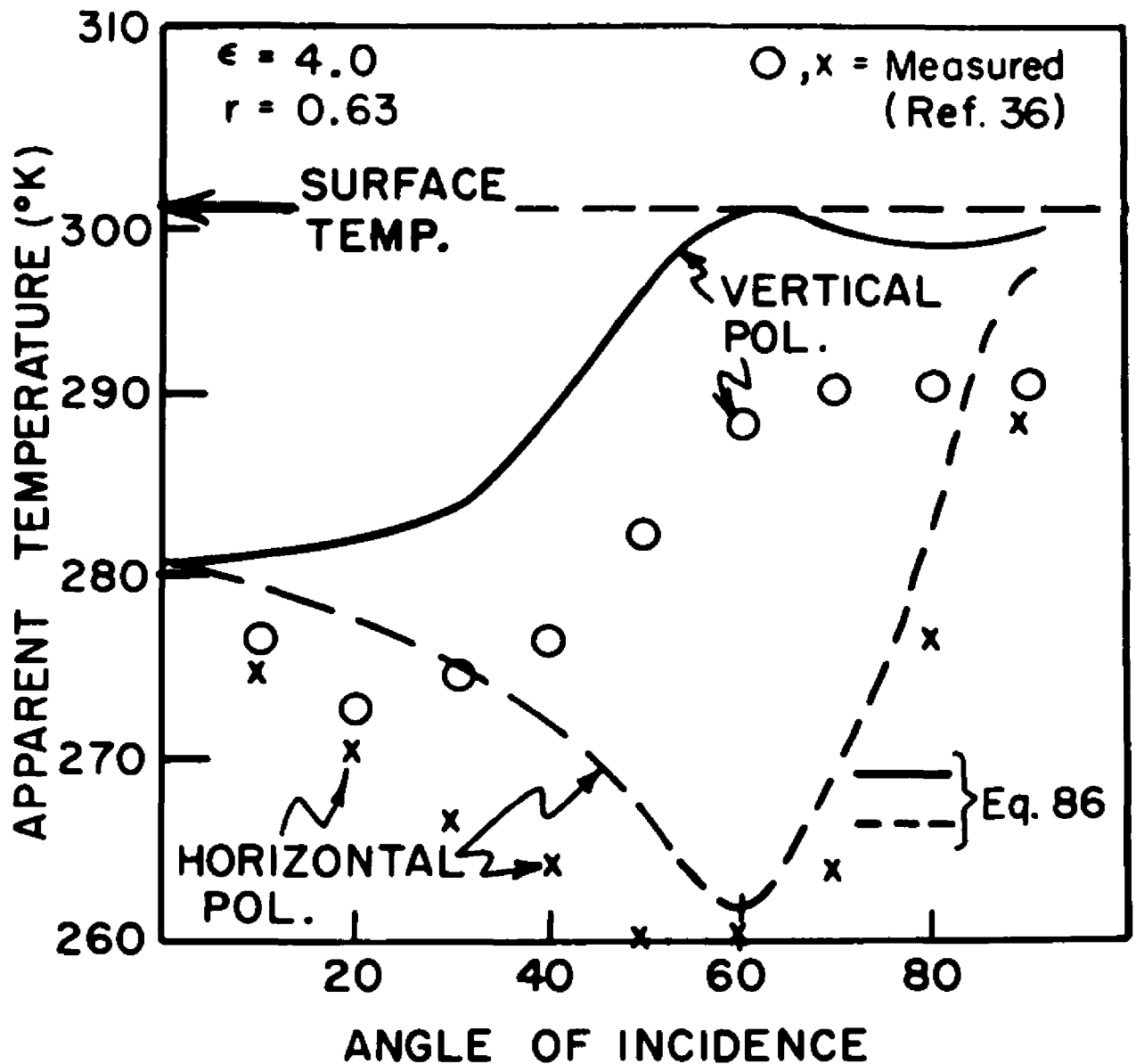
Fig. 33. Measured and calculated apparent temperature
of an asphalt surface.

that apart from a systematic error of about $10^\circ K$, both the polari-

zation dependence and the angular dependence of the measured

temperatures are fairly well represented by the model.

A second surface measured by Straiton was one composed of

grass and weeds, for which the lossy cylinder model of IIC should be

appropriate. Again, the complexity of Eq. (41) makes it desirable

to introduce certain simplifying assumptions for a qualitative calculation. Consider first the general formula for the apparent temperature of a surface with sky temperature given by Eq. (85), viz.,

$$
(87) \qquad T_{ah} = T_g \left[1 - (4\pi)^{-1} \int \left[\gamma_{hh}(o, s) + \gamma_{hv}(o, s) \right] d\Omega_s \right]
$$
$$
+ T_a (4\pi)^{-1} \int \left[\gamma_{hh}(o, s) + \gamma_{hv}(o, s) \right] \left[1 - r^{\sec \theta_s} \right] d\Omega_s \ ,
$$

with a similar formula for vertical polarization. In the first place, measurements reported by Campbell[35] indicate that $\gamma_{hv} \simeq (1/4)\gamma_{hh}$ so that the term γ_{hv} may be dropped or accounted for by a small increase in γ_{hh}. Furthermore, measurements by Taylor[17] show that the back scattering from vegetation surfaces tends to be independent of incidence angle. Thus a plausible choice for the integrands in Eq. (87) might be either

$$
(88a) \qquad \gamma_{hh}^{(1)}(o, s) = \gamma_1 \qquad \left[\text{constant} \right]
$$

or, secondly,

$$
(88b) \qquad \gamma_{hh}^{(2)}(s, o) = \gamma_2 \qquad \left[\text{constant} \right]
$$

where, by Eq. (5),

$$
(88c) \qquad \gamma_{hh}^{(2)}(o, s) = \frac{\cos \theta_s}{\cos \theta_o} \ \gamma_2 \ .
$$

Here the superscripts (1) and (2) identify the special choices of γ.

In order to compare the predictions of Eqs. (87) and (88) with the measurements, it is convenient to calculate ΔT, the difference between the actual temperature and the apparent temperature of the surface. For the two special choices of Y given by Eq. (88), the temperature differences are

$$\Delta T_1 = T_g - T_{ah}^{(1)} = \frac{Y_1}{2}\left[T_g - T_a F_1(r)\right]$$

(89)

$$\Delta T_2 = T_g - T_{ah}^{(2)} = \frac{Y_2}{4\cos\theta_0}\left[T_g - T_a F_2(r)\right]$$

where

$$F_1(r) = \int_0^{\pi/2}\left[1 - r^{\sec\theta_s}\right]\sin\theta_s\,d\theta_s$$

(90)

$$F_2(r) = 2\int_0^{\pi/2}\left[1 - r^{\sec\theta_s}\right]\cos\theta_s\,\sin\theta_s\,d\theta_s\ .$$

These functions may be evaluated in terms of the exponential integral, and are plotted in Fig. 34.

Using Figs. 20 and 30, and other results of Taylor to extrapolate the back-scattering coefficient from vegetation surfaces to 4.3 mm the following values of Y_1 and Y_2 may be estimated:

$$Y_1 = Y_2 = 0.1 \text{ for grass and weeds - vertical polarization}$$

$$Y_1 = Y_2 = 0.126 \text{ for grass and weeds - horizontal polarization}$$

$$F_1(r) = \int_0^1 (1 - r^{1/x})\, dx$$

$$= 1 - r - \ell n\left(\frac{1}{r}\right) Ei(\ell n\, r)$$

$$F_2(r) = 2\int_0^1 (1 - r^{1/x})\, x\, dx$$

$$= 1 - r + r\ell n\left(\frac{1}{r}\right) + \left[\ell n\left(\frac{1}{r}\right)\right]^2 Ei(\ell n\, r)$$

$F_2(r) \rightarrow$ $\leftarrow F_1(r)$

F(r)

r = Fraction Transmitted
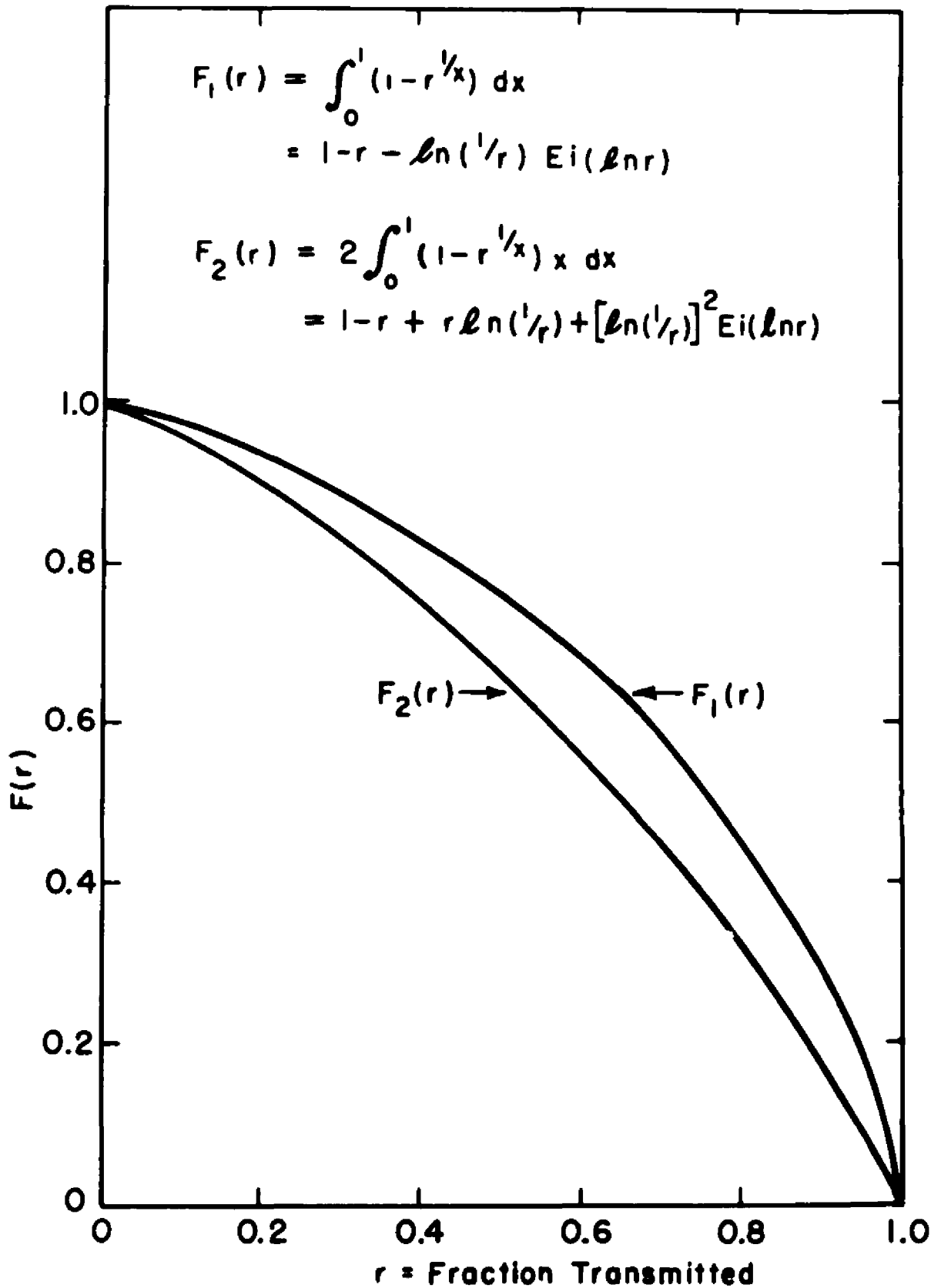
Fig. 34.   The functions $F_1(r)$ and $F_2(r)$ vs r.

$$\gamma_1 = \gamma_2 = 0.2 \text{ for damp grass and weeds.}$$

The second and third values are 1 db and 3 db larger than the first. With these choices of γ, and the value $r = 0.63$ used previously, the predictions of Eq. (89) are shown in Figs. 35-37. These figures also show the measured temperatures.[36] Again there is qualitative agreement between the measurements and the model, which could be brought into surprisingly good quantitative agreement by using the mean of ΔT_1 and ΔT_2, i.e., by choosing

$$(91) \qquad \gamma(0, s) = \frac{1}{2} \gamma_1 \left[1 + \frac{\cos \theta_s}{\cos \theta_0} \right]$$

for vegetation surfaces.

Unfortunately, there is not sufficient experimental data to check this and the previous flat surface model in the same detail as the back-scattering. Nevertheless, the qualitative features of the apparent surface temperatures for these two classes of surface can be explained by using very simple idealization of the surface models developed earlier in this report.

Fig. 35. Measured and calculated apparent temperature
of vegetation covered surface.

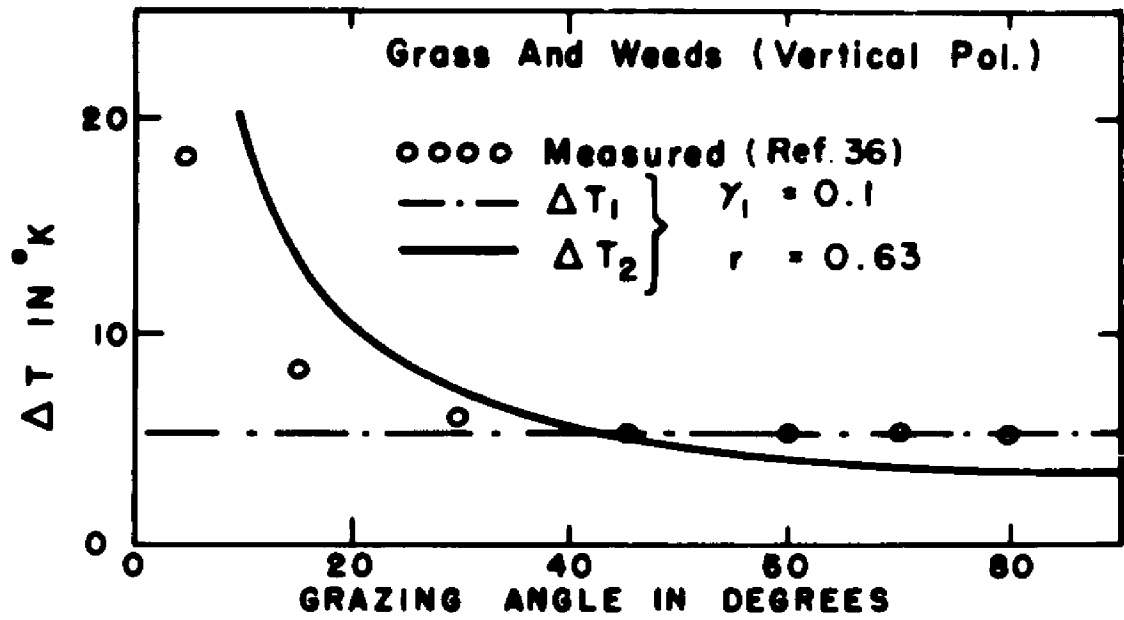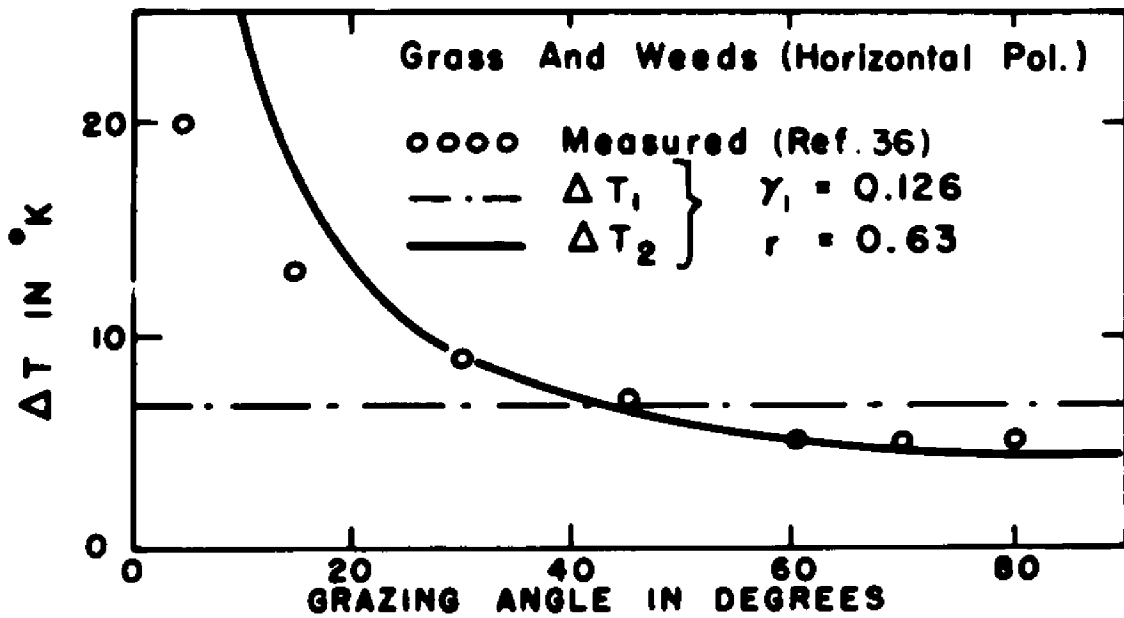Fig. 36. Measured and calculated apparent temperature
of vegetation covered surface.
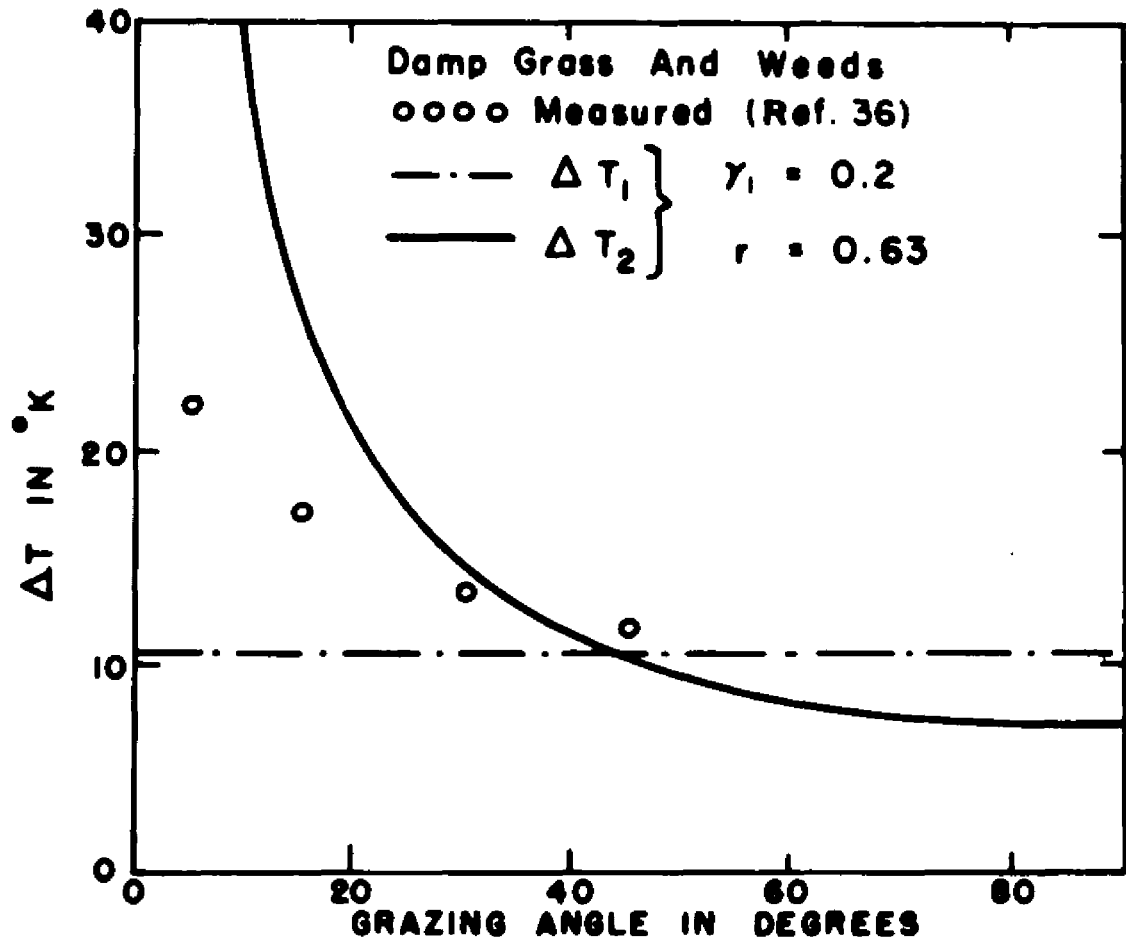
Fig. 37. Measured and calculated apparent temperature
of vegetation covered surface.

CHAPTER V
CONCLUSIONS

It has been shown that the "bi-static" or "differential" scattering coefficients Y_{hh}, etc., may be used to determine a number of properties of the interaction of electromagnetic radiation with natural surfaces, in particular the back-scattering (radar cross-section) the albedo, and the absorption coefficient. By making use of the reciprocity relations satisfied by the scattering coefficients, it is then possible to derive Kirchhoff's radiation law in a form which accounts for the angular dependence and polarization dependence of the emission and absorption coefficients, and predicts them in terms of the Y's. Thus it has been shown that once the Y's are known, both the scattering and the radiometric properties of the surface may be found in terms of them. In applying the general results to actual terrain surfaces, two models have been suggested for two commonly encountered types of surface, for which the scattering coefficients have been calculated in detail. At the same time, it is shown that the models offer a convenient method for describing and classifying the surfaces in a rational and quantitative manner.

The first model, applicable to slightly rough, continuous surfaces, solves the scattering problem by perturbation methods, and gives results for the angular and polarization dependence of the back-scattering coefficients and (in idealized form) for the apparent

temperatures that are in good quantitative agreement with measure-ments on asphalt and concrete surfaces. One may thus infer that such surfaces are adequately represented by the model at microwave frequencies, and can be characterized by their mean square surface roughness, surface correlation function, and complex dielectric constant. The second model, proposed to represent surfaces covered with vegetation in the form of long thin lossy cylinders, solves the scattering problem in the single-scattering approximation and gives results in qualitative agreement with measurements of both the back-scattering coefficients and the apparent temperatures of actual vegetation covered surfaces. Insofar as this model is successful it implies that the surface may be described in terms of the number, size, distribution and complex dielectric constant of the individual cylinders.

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APPENDIX 1
SCATTERING FROM A SLIGHTLY ROUGH SURFACE
FOR VERTICAL POLARIZATION

In this appendix, the results of S. O. Rice[16] for horizontal polarization will be extended to the case of vertical polarization. The notation, except where otherwise specified, will be that of Rice's article. The equations are given the same number as the corresponding equations in the article.

If a vertically polarized incident wave is incident on a perfectly flat surface with angle of incidence θ_0 (for coordinate system, see Fig. 2), then the electric fields above and below the surface are

$$
\begin{aligned}
E_x^+ &= Y\, e^{-\sigma\alpha x}(e^{i\beta Yz} - R\,e^{-i\beta Yz}) \\
(7\text{-}2) \qquad E_y^+ &= 0 \qquad\qquad\qquad\qquad\qquad\qquad z > 0 \\
E_z^+ &= \alpha\, e^{-\sigma\alpha x}(e^{i\beta Yz} + R\,e^{-i\beta Yz})
\end{aligned}
$$

$$
\begin{aligned}
E_x^- &= T\, Y'\, e^{-\tau\alpha'x + \tau Y'z} \\
E_y^- &= 0 \qquad\qquad\qquad\qquad z < 0 \\
E_z^- &= \alpha'\, T\, e^{-\tau\alpha'x + \tau Y'z}
\end{aligned}
$$

where, in this appendix only, T and R are the Fresnel coefficients

for vertical polarization

$$T = \frac{2\sigma/\tau}{1 + (\gamma'\sigma/\gamma\tau)} \qquad R = \frac{1 - (\gamma'\sigma/\gamma\tau)}{1 + (\gamma'\sigma/\gamma\tau)} \quad ;$$

and where

$$Y = \cos\theta_0 \qquad \alpha = \sin\theta_0$$
$$Y' = (1 - \alpha'^2)^{\frac{1}{2}} \qquad \alpha' = (\sigma/\tau)\alpha \ .$$

The other quantities are defined on p. 18. When the equation of the separating surface is $z = f(x, y)$ the electric fields may be written

$$E_x = \begin{cases} E_x^+ + \sum A_{mn} E(m, n; z) & z > f \\ E_x^- + \sum G_{mn} F(m, n; z) & z < f \end{cases}$$

$$(7\text{-}3) \qquad E_y = \begin{cases} E_y^+ + \sum B_{mn} E(m, n; z) & z > f \\ E_y^- + \sum H_{mn} F(m, n; z) & z < f \end{cases}$$

$$E_z = \begin{cases} E_z^+ + \sum C_{mn} E(m, n; z) & z > f \\ E_z^- + \sum L_{mn} F(m, n; z) & z < f \ . \end{cases}$$

Here $E(m, n; z)$ (see Eq. (15)) represents a plane wave in the region above the surface and $F(m, n; z) = \exp\left[-ia(mx+ny) + icz\right]$ represents a plane wave in the region below the surface. The quantities $a, m, n,$ c, etc. are given in Chapter II.

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At the surface $z = f$ the tangential components of the electric and magnetic fields must be continuous. That is, to $O(f^3)$, the four quantities

(7-6)
$$
\begin{cases}
E_x - N_x^2\, E_x - N_x E_z \\[2em]
E_y - N_x N_y E_x - N_y E_z
\end{cases}
$$

(7-7)
$$
\begin{cases}
H_x - N_x N_y H_y - N_x H_z \\[2em]
H_y - N_y^2 H_y - N_y\, H_z
\end{cases}
$$

must be continuous. After considerable algebraic reduction, the four equations arising from these four conditions become, to $O(f^2)$:

$$
e^{-\sigma\alpha x}\left[1-f_x^2\right]\left[\frac{2fU\alpha^2}{\tau} + f^2\,\gamma' U\right]
$$

$$
+ f_x\, e^{-\sigma\alpha x}\left[\alpha V + f^2\alpha^3\,\frac{\sigma}{\tau}\,U\right]
$$

$$
+(1-f_x^2)\left[A_{mn}\,E(m,n,z) - G_{mn}\,F(m,n,z)\right]
$$

$$
+ f_x\left[C_{mn}\,E(m,n,z) - I_{mn}\,F(m,n,z)\right] = 0
$$

(7-11)
$$
- f_x f_y\, e^{-\sigma\alpha x}\left[\frac{2fU\alpha^2}{\tau} + f^2\gamma' U\right]
$$

$$
+ f_y\, e^{-\sigma\alpha x}\left[\alpha V + f^2\alpha^3\,\frac{\sigma}{\tau}\,U\right]
$$

$$
- f_x f_y\left[A_{mn}\,E(m,n,z) - G_{mn}\,F(m,n,z)\right]
$$

$$
+\left[B_{mn}\,E(m,n,z) - H_{mn}\,F(m,n,z)\right]
$$

$$
+ f_y\left[C_{mn}\,E(m,n,z) - I_{mn}\,F(m,n,z)\right] = 0
$$

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$$-f_x f_y \left[e^{-\sigma \alpha x} \right] \left[2f\gamma' U + f^2 _\tau U \right]$$

$$+ \left[ib\, B_{mn} - ian\, C_{mn} \right] E(m,n,z) + \left[ic\, H_{mn} + ian\, I_{mn} \right] F(m,n,z)$$

$$-f_x f_y \left[iam\, C_{mn} - ib\, A_{mn} \right] E(m,n,z) + f_x f_y \left[ic\, G_{mn} + iam\, I_{mn} \right]$$

$$\cdot\ F(m,n,z)$$

$$+ f_x \left[ian\, A_{mn} - iam\, B_{mn} \right] E(m,n,z) - f_x \left[ian\, G_{mn} - iam\, H_{mn} \right]$$

$$\cdot\ F(m,n,z) = 0$$

$$(1 - f_y^2) e^{-\sigma \alpha x} \left[2f\gamma' U + f^2 _\tau U \right]$$

$$+ (1 - f_y^2) \left[iam\, C_{mn} - ib\, A_{mn} \right] E(m,n,z)$$

$$- (1 - f_y^2) (ic\, G_{mn} + iam\, I_{mn}) F(m,n,z)$$

$$+ f_x \left[ian\, A_{mn} - iam\, B_{mn} \right] E(m,n,z) - f_x \left[ian\, G_{mn} - iam.\, H_{mn} \right]$$

$$\cdot\ F(m,n,z) = 0$$

where $V = T(\tau/\sigma - \sigma/\tau)$ and $U = T(\sigma^2 - \tau^2)/2$. Expanding the coefficients as $A'_{mn} + A^2_{mn} \cdots$, etc., utilizing (7-12) and replacing $f \exp(-ia\nu x)$ by its Fourier series expansion, the first order terms lead to the equations

$$A'_{mn} - G'_{mn} = \left[- \frac{2U\alpha^2}{\tau} + ia\alpha\, V(m-\nu) \right] P(m-\nu, n)$$

$$B'_{mn} - H'_{mn} = ian\alpha\, V\, P(m-\nu, n)$$

(7-13)

$$b B'_{mn} - an\, C'_{mn} + c\, H'_{mn} + an\, I'_{mn} = 0$$

$$am\, C'_{mn} - b\, A'_{mn} - c\, G'_{mn} - am\, I'_{mn} = 2i\, U\gamma' P(m-\nu, n),$$

which, with the divergence relations

$$am\ A_{mn} + an\ B_{mn} + b\ C_{mn} = 0$$

(7-17)

$$am\ G_{mn} + an\ H_{mn} - c\ L_{mn} = 0,$$

provide the six equations for the six "unknowns" $A_{mn} \cdots L_{mn}$. The solution is

(7-21) $$A'_{mn} = \frac{2iUP(m-\nu, n)}{dD}\left[-\gamma'(a^2 n^2 + bc) + ab\ \frac{\tau}{\sigma}\ am\right]$$

$$B'_{mn} = \frac{2iUP(m-\nu, n)}{dD}\left[a^2 nm\ \gamma' - a\ \frac{\tau}{\sigma}\ b\ an\right]$$

$$C'_{mn} = \frac{2iUP(m-\nu, n)}{dD}\left[amc\ \gamma' - a\ \frac{\tau}{\sigma}\ (a^2 m^2 - a^2 n^2)\right]$$

$$G'_{mn} = A'_{mn} + \frac{2U\alpha}{\tau}\ (a + i(am - a\nu)/\sigma)\ P(m-\nu, n)$$

$$H'_{mn} = B'_{mn} + \frac{2ian\alpha\ U}{\sigma\tau}\ P(m-\nu, n)$$

$$L'_{mn} = \frac{am}{c}\ G'_{mn} + \frac{an}{c}\ H'_{mn}\ .$$

APPENDIX II
THE RECIPROCITY RELATIONS SATISFIED
BY THE SCATTERING COEFFICIENTS

Because of the decisive role played by the reciprocity relations,
Eq. (5), in deriving Kirchhoff's law, it may be desirable to provide
a more detailed proof than that given in Chapter II.

Consider two antennas, (a) and (b) (see Fig. 38) at distances
R_a and R_b, respectively, from the surface element S. Suppose, for
the purposes of this proof, that the antennas have the following
characteristics:

(i) The antennas are connected to transmission lines (real
characteristic impedances Z_a and Z_b, respectively)
matched in both directions, and supporting only one
mode, so that a well defined current and voltage exist
at some reference plane in each line.

(ii) The antennas are of high gain, and are directed at S;
thus it is assumed that the only interaction between
them is due to scattering from the surface S.

These two special assumptions are not necessary, but greatly simplify
the following proof. For the general proof, the only requirement
on the antennas is that there exist reference planes in the feed lines
where the voltage and current can be defined.
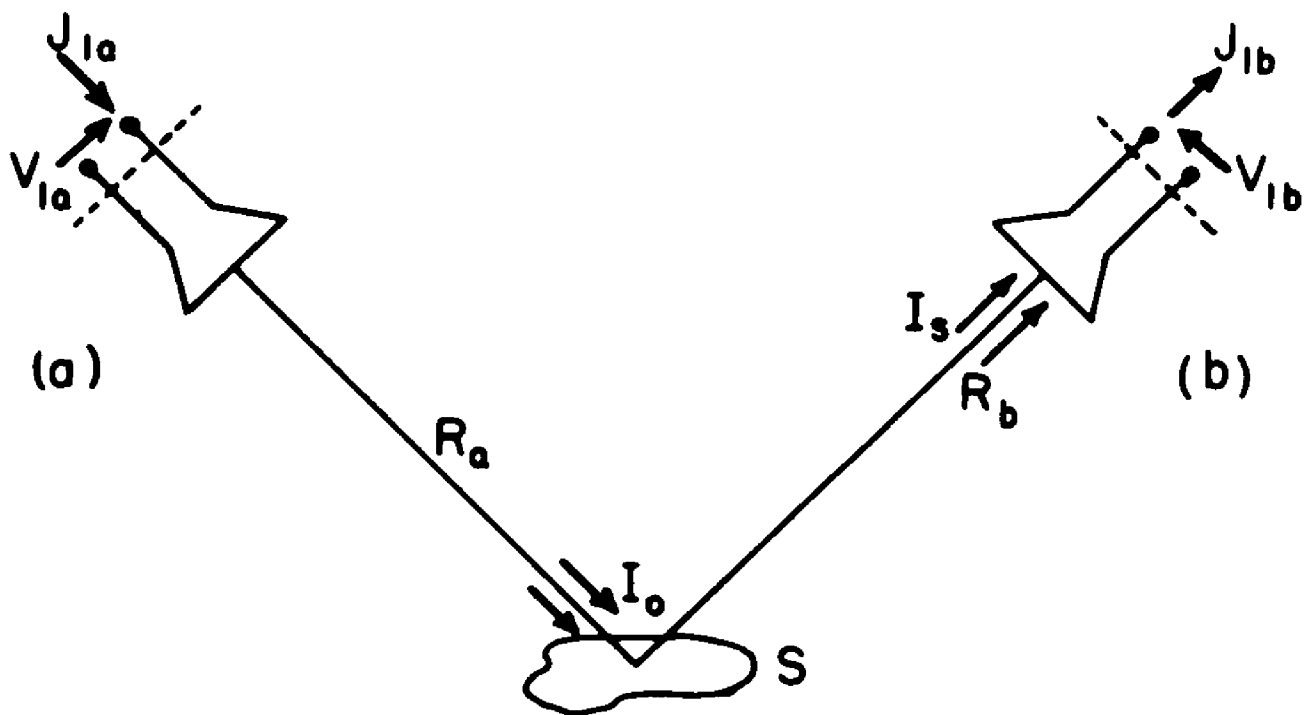
Situation (1);   (a) Is Transmitting
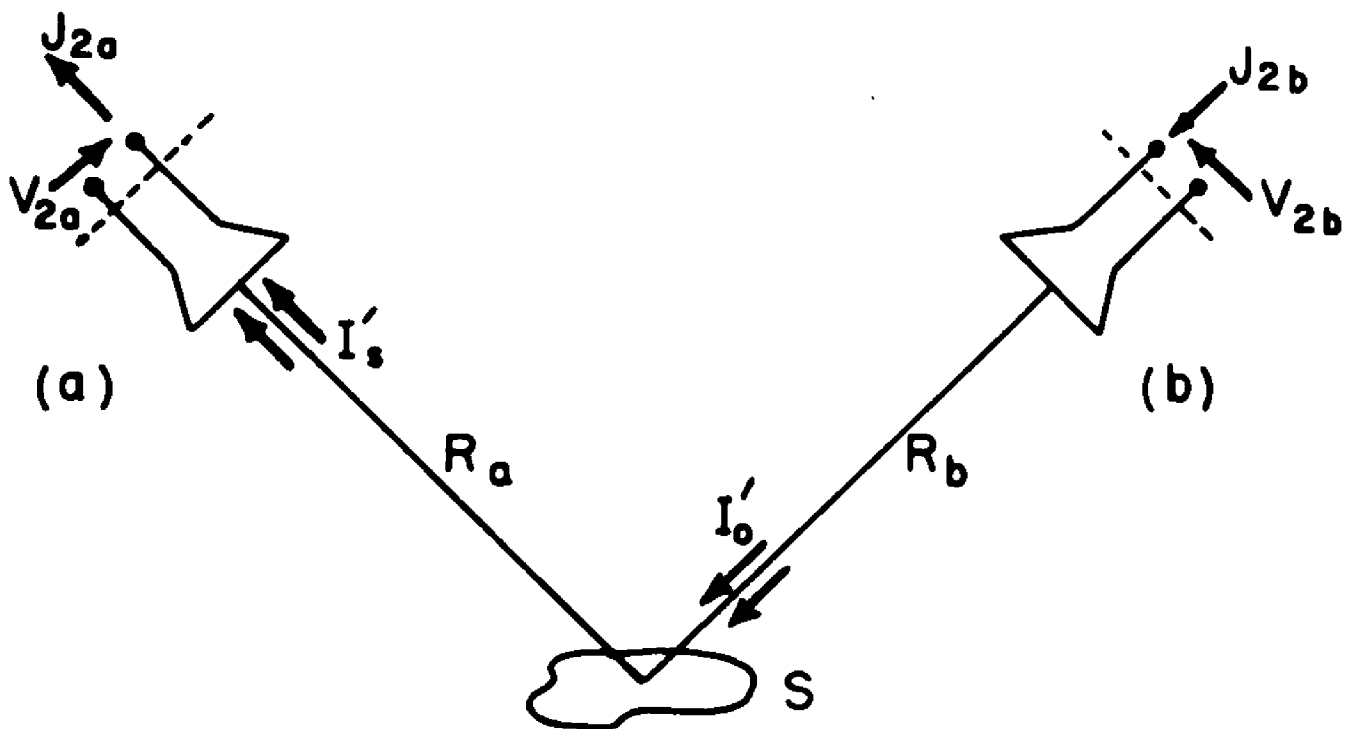
Situation (2);   (b) Is Transmitting

Fig. 38.   Geometry for reciprocity theorem.

Now consider the following two situations, illustrated in Fig. 38, viz.:

(1) A voltage V_{1a} is maintained across the reference terminals of (a) acting as a transmitter. The current at the reference terminals of (b) is measured, and found to be J_{1b}.

(2) A voltage V_{2b} is maintained across the reference terminals of (b) acting as a transmitter. The current at the reference terminals of (a) is measured and found to be J_{2a}.

It is a consequence of the Lorentz reciprocity theorem (Reference 20, p. 59) that the above defined voltages and currents satisfy the equation

$$V_{1a} J_{2a} = V_{2b} J_{1b} \ .$$

This is referred to as the Rayleigh reciprocity theorem (Reference 20, p. 19). An extensive discussion of the relation between the two theorems, the proper definition of voltage and current in antenna feed lines, and the justification for representing two antennas coupled by a scatterer as a four terminal network is given in the second chapter of Reference 20. It is sufficient here to note that the scattering region S may be of any complexity, as long as the constitutive

parameters (dielectric constant, permeability, conductivity) are everywhere at most a real symmetric tensor. Thus gyrotropic media (ferrites, etc.) are not permitted. Also, because the super-position principle is used, the parameters ϵ, μ, g must be independ-end of field strength. Thus ferromagnetics, rectifying oxide coatings, etc. are not permitted.

The voltages and currents just defined must now be related to the power densities depicted in both Fig. 1 and Fig. 38. In situation (1), the total power radiated by (a) is just $(2Z_a)^{-1}|V_{1a}|^2$, so that the power density I_0 at the surface S is found from

$$(2Z_a)^{-1}|V_{1a}|^2 = 4\pi R_a^2 I_0 / g_a(\theta_0\phi_0)$$

where $g_a(\theta_0\phi_0)$ is the gain of (a) in the direction of the surface. (see Reference 20, pp. 2 and 50). It is assumed that (a) is de-signed to produce radiation of the specific polarization of interest at S. On the other hand, the power received by (b) is $(1/2)Z_b|J_{1b}|^2$. Thus if I_s is the power density in the scattered wave at the aperture of (b), then

$$I_s A_b(\theta_s\phi_s) = (1/2)Z_b|J_{1b}|^2$$

where $A_b(\theta_s\phi_s)$ is the receiving aperture of (b) in the scattering direction (see Reference 20, pp. 3 and 50). It is assumed that (b) is designed to receive the polarization of interest.

In situation (2), the power density I_o' at the surface S, and I_s' at the aperture of the receiver (a), may be found similarly,

$$(2Z_b)^{-1} |V_{2b}|^2 = 4\pi R_b^2 I_o'/g_b(\theta_s \phi_s)$$

$$(1/2) Z_a |J_{2a}|^2 = A_a(\theta_o \phi_o) I_s' \; .$$

Now it is also a consequence of the Lorentz reciprocity theorem (applied to isolated antennas, without the scatterer) that the gain of an antenna in a given direction for radiation of a specific polarization is related to its receiving aperture for radiation from the same direction with the same polarization by the following equation,

$$\frac{g_a(\theta_o \phi_o)}{A_a(\theta_o \phi_o)} = \frac{g_b(\theta_s \phi_s)}{A_b(\theta_s \phi_s)} = \frac{4\pi}{\lambda^2} \; .$$

This is known as the antenna pattern reciprocity theorem, and is proved on p. 51 of Reference 20.

When the relations between voltage, current and power density are substituted into the Rayleigh reciprocity theorem, it is found that

$$R_a^2 \frac{I_o}{I_s} \frac{A_a(\theta_o \phi_o)}{g_a(\theta_o \phi_o)} = R_b^2 \frac{I_o'}{I_s'} \frac{A_b(\theta_s \phi_s)}{g_b(\theta_s \phi_s)} \; .$$

Thus using the antenna pattern reciprocity theorem, there results

$$R_a^2 (I_o/I_s) = R_b^2 (I_o'/I_s') \; .$$

If it is assumed that $R_a = R_b$, and that (a) and (b) are identical (except for polarization), so that $I_0 = I_0'$, then $I_s = I_s'$. This is just the relation required on p. 9 to prove the reciprocity relations for the scattering coefficients.

AUTOBIOGRAPHY

I, William Heller Peake, was born on the tenth of June, 1926, in Binghamton, New York. I received the Bachelor of Science degree from the Massachusetts Institute of Technology in 1946, and the Master of Arts degree from Columbia University in 1951. Since 1952 I have been employed at the Antenna Laboratory in the Department of Electrical Engineering of The Ohio State University.