CHAPTER 107

INTERACTION OF PLANE WAVES WITH VERTICAL CYLINDERS

by

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ABSTRACT

This study deals with the interaction of linear, plane water waves with stationary groups of rigid, vertical, circular cylinders under conditions in which the inertial forces on the cylinders dominate over the drag forces. A direct matrix solution as well as multiple scattering as suggested by Twersky (1952) are used to obtain the velocity potential in the vicinity of the cylinders. The groups may consist of a number of cylinders having any geometric arrangement, may have Dirichlet, Neumann, or mixed boundary conditions, and need not have identical diameters. The study represents an extension of the single-cylinder case presented by MacCamy and Fuchs in 1954.

Basic scattering coefficients for 192 different arrangements of two cylinders are obtained with the aid of a Bessel coordinate transformation and a matrix inversion procedure. The resulting potential function is then applied to calculate force components in the direction of wave advance and orthogonal to it. For the cases considered the former departs as much as 65% from the force on a single cylinder and the mass coefficient is found to range from 1.19 to 3.38 - a not insignificant departure from the often used value of 2.0. Furthermore the orthogonal force may be as large as 67% of the single-cylinder force.

INTRODUCTION

As off-shore construction continues to expand around the world, the need for an improved understanding of the effects of water waves on various structures in the sea becomes increasingly evident. Basic to many such problems is the fundamental one involving plane, periodic waves and vertical, circular cylinders, since many such structures include one or more cylindrical legs.

In the present study a general approach to the problem of describing the interaction of linear, plane waves with stationary groups of rigid, vertical circular cylinders is examined. In particular cylinders located in intermediate or deep water and having relatively large diameters (when compared with wave height) are considered in as much as they are representative of a type of off-shore construction that has received much attention recently.

Dean and Harleman (see Ippen (1966)) demonstrate that as the ratio of wave height to cylinder diameter, H/D, diminishes and the ratio of water depth to wave length, h/L, grows, the ratio of inertial force to drag force as described by the Morison Equation (Morison, et al (1950)) increases. For example, if H/D = 1.00 and h/L = 0.40, the inertial force will be ten times as large as the drag force. Many off-shore structures, and in particular those considered in the present study, are therefore subject primarily to inertial forces with drag effect considered negligible.

Under these circumstances classical diffraction theory, which presupposes a frictionless fluid and therefore neglects drag seems to be ideally suited to the solution of problems involving the interaction of plane waves with large cylinders in deep or intermediate-depth water. MacCamy and Fuchs (1954) were the first investigators to apply diffraction theory to this interaction problem. Their study of the diffraction of periodic plane waves about a single circular cylinder led to a new approach to the problem of predicting wave forces on structures.

The present study is an attempt to extend the work of MacCamy and Fuchs to a consideration of wave interaction with more than one vertical cylinder. One approach to the solution of such problems is multiple scattering as suggested by Twersky (1952). A direct matrix method appears to offer more rapid and reliable solutions and is therefore emphasized herein.

THEORETICAL ANALYSIS

Problem Statement. The problem under consideration is the interaction between incoming plane water waves and an arbitrary collection of vertical circular cylinders located in the path of the waves. The following conditions are assumed to prevail:

1. The waves are linear (small amplitude theory), and are not breaking.
2. The bottom is horizontal and impermeable with a depth sufficient for deep water or intermediate depth wave conditions.

3. The cylinders are circular, rigid, vertical, stationary, impermeable and have a relatively large diameter with respect to the wave height.
4. Drag effects are negligible (i.e. the water behaves as an ideal fluid).

A general procedure for determining the velocity potential for any number of cylinders is outlined first and then the specific case of two cylinders, Fig. 1, is analyzed more completely. Details of the analytical procedure are omitted herein, but are described by Spring (1973).

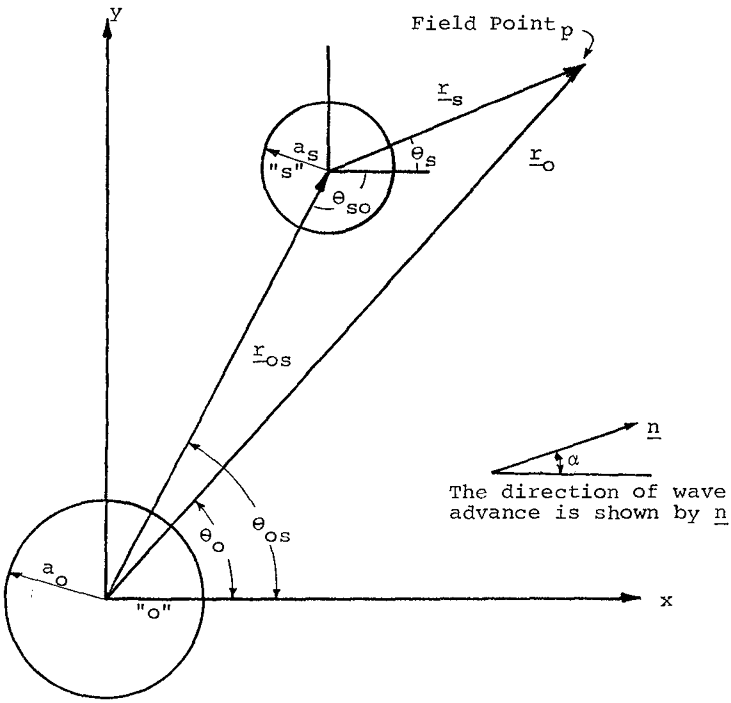

Figure 1. Definition Sketch for Cylinder "o" and Cylinder "s"

The approaching plane waves are conveniently expressed in terms of cylindrical coordinates since circular cylinders are under study. Thus in terms of the coordinate center "s" the incoming wave may be written (see Twersky (1952) and MacCamy and Fuchs (1954)

$$\emptyset_{in}(r_s,\theta_s) = -\mathrm{Re}\Big[f(z,t)\ \exp(ikr_{os}\ \cos(\theta_{os}-\alpha))$$

$$\cdot \sum_{n=-\infty}^{\infty} J_n(kr_s)\ \exp(in(\theta_s-\alpha+\pi/2))\Big] \qquad (1)$$

where the symbols are as defined in Fig. 1, $J_n(kr_s)$ is the Bessel function of the first kind, of order "n" and

$$f(z,t) = \frac{gH}{2\sigma}\ \frac{\cosh[k(h+z)]}{\cosh kh}\ e^{-i\sigma t} \qquad (2)$$

where g = the acceleration due to gravity

H = the wave height

h = the depth of water from the still water level to the bottom

k = the wave number, $2\pi/L$

L = the wave length

T = the period

z = the vertical coordinate, measured positive upward from the still water level

σ = the frequency, $2\pi/T$

The waves scattered by the cylinders have as yet undetermined amplitude but must vanish at large distances from the cylinders due to circular dispersion. Also the waves must be outgoing rather than incoming. Thirdly, the scattered wave expression must be rather general to provide enough flexibility to account for the non-symmetrical scattering of the waves from the cylinder or cylinders. With negative sign on the time exponential, the Hankel function of the first kind (see MacCamy and Fuchs (1954),

$$H_p^{\,1}(kr) = J_p(kr) + iY_p(kr) \qquad (3)$$

will adequately express the radial dependence of the scattered waves. Since there will be no need for any Hankel function of the second kind (which describes incoming circular waves), the superscript on $H_p^{\,1}(kr)$ will be dropped and it will be understood that $H_p(kr)$ = Hankel function of the first kind with argument kr.

The scattered wave from the "sth" cylinder will then be

$$\phi_{sc}(r_s,\theta_s) = \mathrm{Re}\left[f(z,t) \sum_{n=-\infty}^{\infty} A_n^s \, H_n(kr_s) \, e^{in\theta_s}\right] \tag{4}$$

and for the cylinder located at "o"

$$\phi_{sc}(r_o,\theta_o) = \mathrm{Re}\left[f(z,t) \sum_{n=-\infty}^{\infty} A_n^o \, H_n(kr_o) \, e^{in\theta_o}\right] \tag{5}$$

where the A_n's are complex constants (as yet unknown) of the form $a_n + ib_n$, with appropriate superscript.

Since linear waves (small amplitude wave theory) are being considered, the velocity potential at any field point "p" may now be represented by superposing the various wave components to give

$$\phi_p = \mathrm{Re}\Bigg[f(z,t)\Big[\exp(ikr_{os}\cos(\theta_{os}-\alpha)) \cdot \sum_{n=-\infty}^{\infty} -J_n(kr_s)\exp(in(\theta_s-\alpha+\pi/2)) + \sum_{j=1}^{Q}\sum_{n=-\infty}^{\infty} A_n^j \, H_n(kr_j) \, e^{in\theta_j}\Big]\Bigg] \tag{6}$$

where the summation term on j accounts for the potential of the scattered waves from all cylinders present (Q being the number of such cylinders).

The potential function described by Eq. 6 is difficult to handle since each scattered wave is expressed in terms of a different coordinate center. There is however, a Bessel "addition theorem" or coordinate transformation available to express all wave components in terms of one selected coordinate center (see Watson (1966) pp. 359-61, or Abramowitz and Stegun (1965) p. 363).

Referring to Fig. 2, the Bessel coordinate transformation of $C_\nu(\tilde{w})$ (which may be any one of the cylinder functions - J, Y, H^1, H^2 or a derivative or linear combination thereof) from coordinate center 1 to a new coordinate center 2 is given by

$$C_\nu(\tilde{w})\ e^{\pm i\nu\psi} = \sum_{m=-\infty}^{\infty} C_{\nu+m}(Z)\ J_m\ (z)\ e^{\pm im\phi} \tag{7}$$

with the restriction that $|z\ e^{\pm i\phi}| < |Z|$

It should be noted that the restriction is lifted if ν is an integer or zero and the only functions involved are of the first kind.

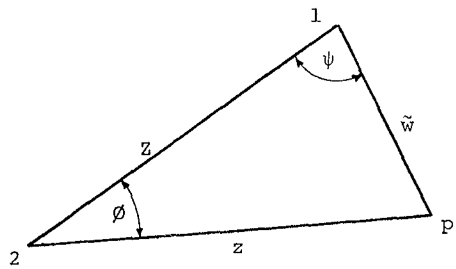

Figure 2. Definition Sketch for Bessel Coordinate Transformation

Direct Solution Versus Multiple Scattering. The final step in the derivation of the velocity potential is the application of the reflection or Neumann boundary conditions at the surface of the rigid, impermeable circular cylinders. These conditions are given by

$$\frac{\partial \phi_p}{\partial r_s} = 0 \ @\ r_s = a \tag{8}$$

for each cylinder and must be applied to evaluate the coefficients A_n^j in Eq. (6). In the instance of water waves scattered by impermeable vertical cylinders, the Neumann condition is applied on each cylinder. It should be noted however, that the method does not require all boundary conditions to be of the same type.

The fundamental difference between the direct approach used herein and the procedure suggested by Twersky (1952) is in the method of application of the cylinder boundary conditions. In the "direct" approach the boundary conditions on all cylinders are applied simultaneously and all unknowns are obtained by means of a single matrix inversion of a set of simultaneous equations. The number of equations increases with the number of cylinders involved in the problem (except for such special cases as an infinite row of cylinders of the same size and spacing); and with the number of terms taken in the summations. The practical limit on the method is therefore the limit of the matrix size which can reasonably be handled by the digital computer available.

In contrast, the multiple scattering approach takes one cylinder at a time and sequentially solves for the scattering coefficients. The sum of the multiple scattering coefficients for a particular cylinder approaches the direct solution if enough orders of scattering are considered.

The Two-Cylinder Problem. The general method described above will be applied in this section to the case of two cylinders as shown on Fig. 1. The two-cylinder case is used as an illustration of the techniques involved. Three, four, six, or more cylinders could likewise be considered. On the other hand it should be noted that the size, orientation and spacing of the cylinders and the direction of wave approach are all arbitrary and can be selected later to make computations for a particular situation.

The velocity potential at any field point "p" in terms of cylinder "s" may be formed by superposing the velocity potentials for the incoming plane wave, the scattered wave from cylinder "s" and the scattered wave from cylinder "o" as suggested by Eq. (6). The Bessel coordinate transformation from "o" to "s" and the cylinder boundary on "s" are then applied and the resulting complex expressions are separated into real and imaginary parts, while noting that $A = a + ib$, $H = J + iY$ and $H' = J' + iY'$. This results in two sets of equations in the four sets of unknowns a_p^s, b_p^s, a_p^o, b_p^o, as follows:

$$-J'_{-m}(ka_s)\cos\left[(kr_{os}\cos(\theta_{os}-\alpha)) - m(-\alpha+\pi/2)\right]$$
$$+ (a^s_{-m}J'_{-m}(ka_s) - b^s_{-m}Y'_{-m}(ka_s))$$
$$+ \sum_{n=-\infty}^{\infty} J'_m(ka_s)\left[(a_n^o J_{n+m}(kl_{os}) - b_n^o Y_{n+m}(kl_{os}))\cos(n\theta_{os} + m\theta_{so})\right.$$
$$\left. -(b_n^o J_{n+m}(kl_{os}) + a_n^o Y_{n+m}(kl_{os}))\sin(n\theta_{os} + m\theta_{so})\right] = 0 . \qquad (9a)$$

and

$$-J'_{-m}(ka_s)\ \sin\left[(kr_{os}\cos(\theta_{os}-\alpha)) - m(-\alpha+\pi/2)\right]$$

$$+ (b^s_{-m}\ J'_{-m}(ka_s) + a^s_{-m}\ Y'_{-m}(ka_s))$$

$$+ \sum_{n=-\infty}^{\infty} J'_m(ka_s)\left[(b^o_n\ J_{n+m}(kl_{os}) + a^o_n\ Y_{n+m}(kl_{os}))\cos(n\theta_{os} + m\theta_{so})\right.$$

$$\left. + (a^o_n\ J_{n+m}(kl_{os}) - b^o_n\ Y_{n+m}(kl_{os}))\sin(n\theta_{os} + m\theta_{so})\right] = 0 \qquad (9b)$$

where $m = 0, \pm 1, \pm 2, \ldots\ldots$

In similar fashion the Bessel coordinate transformation from "s" to "o" and boundary condition on cylinder "o" are applied, producing two more equation sets in the same four sets of unknowns a^s_p, b^s_p, a^o_p, b^o_p, as follows

$$-J'_{-m}(ka_o)\ \cos\left[-m(-\alpha+\pi/2)\right] + (a^o_{-m}\ J'_{-m}(ka_o) - b^o_{-m}\ Y'_{-m}(ka_o))$$

$$+ \sum_{n=-\infty}^{\infty} J'_m(ka_o)\left[(a^s_n\ J_{n+m}(kl_{os}) - b^s_n\ Y_{n+m}(kl_{os}))\cos(n\theta_{so} + m\theta_{os})\right.$$

$$\left. -(b^s_n\ J_{n+m}(kl_{os}) + a^s_n Y_{n+m}(kl_{os}))\sin(n\theta_{so} + m\theta_{os})\right] = 0 \qquad (10a)$$

and

$$-J'_{-m}(ka_o)\ \sin\left[-m(-\alpha+\pi/2)\right] + (b^o_{-m}\ J'_{-m}(ka_o) + a^o_{-m}\ Y'_{-m}(ka_o))$$

$$+ \sum_{n=-\infty}^{\infty} J'_m(ka_o)\left[(b^s_n\ J_{n+m}(kl_{os}) + a^s_n\ Y_{n+m}(kl_{os}))\cos(n\theta_{so} + m\theta_{os})\right.$$

$$\left. + a^s_n\ J_{n+m}(kl_{os}) - b^s_n\ Y_{n+m}(kl_{os}))\sin(n\theta_{so} + m\theta_{os})\right] = 0 \qquad (10b)$$

In order to reduce the coefficients and the equations to a finite number, m and n are given a range of -M to +M where the value of M required to maintain a specified precision increases with increasing ka_s and ka_o and decreasing kl_{os}. For the two-cylinder problem this will therefore result in 8M+4 equations and unknowns.

Once the coefficients have been computed the final velocity potential is obtained. In functional form, therefore,

$$\phi = f(z) \sum_m (f_m \cos(m\theta+\sigma t) + g_m \sin(m\theta+\sigma t)) \tag{11}$$

where f_m and g_m are functions of r, and

$$f(z) = \frac{gH}{2} \frac{\cosh[k(h+z)]}{\cosh kh}$$

Pressure on the Cylinder Face. The pressure field may be determined with the aid of the generalized Bernoulli Equation

$$\frac{\partial\phi}{\partial t} + gz + \frac{P}{\rho} + \frac{1}{2} (\nabla\phi)^2 = 0 \tag{12}$$

Neglecting the kinetic energy and considering only the dynamic pressure, P_d, one obtains

$$P_d = -\frac{\partial\phi}{\partial t} = \rho\sigma\, f(z) \sum_m (f_m \sin(m\theta+\sigma t) - g_m \cos(m\theta+\sigma t)) \tag{13}$$

For the pressure on a cylinder face f_m and g_m must be evaluated at the appropriate radial distance.

Horizontal Force Components on the Cylinder Face. By integrating the dynamic pressure over the entire cylinder face the horizontal force is obtained. In general functional form the force components may be expressed as follows:

$$F_x = \frac{\pi H\rho\sigma}{2} f(z)\, R_x \cos(\sigma t+\delta_x) \tag{14a}$$

and

$$F_y = \frac{\pi H\rho\sigma}{2} f(z)\, R_y \cos(\sigma t+\delta_y) \tag{14b}$$

where R_x, R_y, δ_x and δ_y are amplitudes and phase angles which depend on ka_s, ka_o, kl_{os}, and θ_{os}.

ANALYSIS OF THE DATA

Computer Procedure. It is readily apparent that the solution of Eqs. (9) and (10) by matrix inversion requires the services of high speed computers. In this case the UNIVAC 1108 of the Madison Academic Computing Center and the Datacraft 6024 of the Engineering Computing Laboratory, both at the University of Wisconsin-Madison, were employed. The flow diagram describing the program for the solution of the two-cylinder problem is presented in Fig. 3.

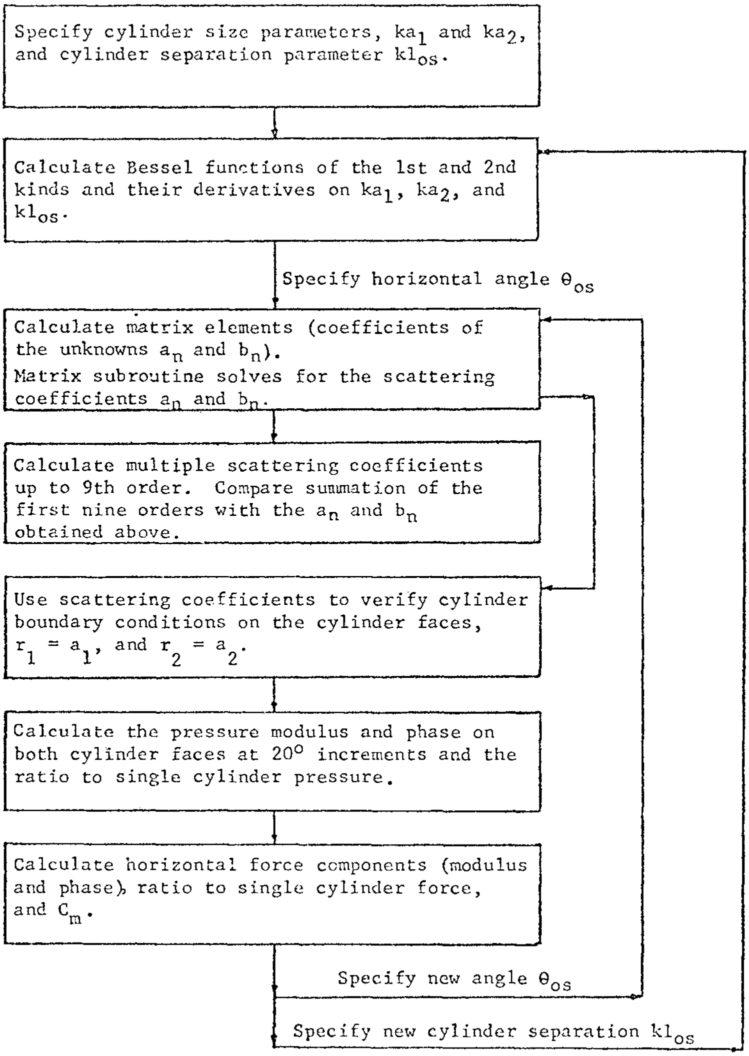

Figure 3. Computer Flow Diagram: Two Cylinders

Calculations were aimed at the determination of the pressure distribution on the faces of the cylinders and the horizontal force components per unit depth. Pressures and forces were also calculated for the single isolated cylinder case of MacCamy and Fuchs (1954). Thus the pressures and forces which are presented herein are expressed as ratios to the single cylinder pressures and forces. These ratios clearly reveal the effect that the presence of the auxiliary cylinder has on the cylinder of interest.

All calculations were accomplished in non-dimensional form in terms of the parameters describing wave length and cylinder size, spacing, and orientation: ka_1, ka_2, kl_{os}, and θ_{os} (where a_1 and a_2 are the radii of cylinders 1 and 2 respectively).

The Two-Cylinder Solution. The primary objectives in studying the two-cylinder case are to illustrate the technique involved, to compare the direct method with the multiple scattering approach, and to lay the groundwork for future extensions. Thus the full range of cylinder sizes and spacings is not considered here.

Two series of calculations were made, both with the incident waves moving in the positive "x" direction ($\alpha = 0°$). In the first series the two cylinders were of equal size, $ka_1 = ka_2 = 0.40$. The horizontal spacing between cylinders, kl_{os}, was varied from 0.80 (cylinders touching) to 9.5 (approximately 1.5 wavelengths apart) in twenty steps, and the angular position of the second cylinder, θ_{os}, was varied from 0° to 90° in steps of 30°. Thus eighty combinations of angle and distance were considered.

In the second series cylinders of unequal size ($ka_1 = 0.40$, $ka_2 = 0.60$) were considered. The parameter kl_{os} was varied between 1.00 and 8.0 while the angular position of the second cylinder was varied between 0° (cylinder one leading) and 180° (cylinder two leading) in steps of 30°, providing 112 different locations of cylinder two in relation to cylinder one. For each of these situations the following items were calculated:

1. Basic scattering coefficients, by both the direct method and the multiple scattering approach.
2. Pressure amplitude and phase angles at 20° increments around the cylinder.
3. Ratio of pressure amplitude to the pressure amplitude on an isolated cylinder at 20° increments around the cylinder.
4. Force components in the two horizontal directions.
5. Ratio of force to that of a single cylinder (x-component only).
6. Equivalent mass coefficient (x-component) as defined by Morison, et al (1950).

Horizontal Force Components. Amplitudes and phase angles of the horizontal force components (R_x, R_y, δ_x, δ_y as given in Eqs. (14)) were calculated along with the ratio to the force on a single cylinder (x-component only). Typical results for two equal diameter cylinders and two unequal diameter cylinders are in Tables 1 and 2, respectively. Furthermore force ratios for the same two cases are shown graphically as follows:

Figure 4: $\theta_{os} = 0°$ and $180°$ Figure 6: $\theta_{os} = 60°$ and $120°$

Figure 5: $\theta_{os} = 30°$ and $150°$ Figure 7: $\theta_{os} = 90°$

The force ratios for both equal- and unequal-size cylinders appear in each figure for easy comparison. Moreover in Fig. 7 force ratios for an infinite row of cylinders ($ka = 0.400$, $\theta_{os} = \pm 90°$) are included.

The orthogonal or y-component of force was also calculated (see Tables 1 and 2), and in one case rose to 67 percent of the x-component force ($ka_1 = 0.400$, $ka_2 = 0.600$, $kl_{os} = 1.00$, $\theta_{os} = 30°$). Although the effect on the maximum resultant was generally less than 10 percent, partly due to the phase differences between the x and y components, in one case the maximum resultant was increased by 50 percent. From a design point of view, either static or dynamic, both force components may be significant.

The Mass Coefficient. The force ratio mentioned above and in Tables 1 and 2 can also be interpreted as the ratio of the mass coefficient for one of two cylinders to the mass coefficient of a single isolated cylinder. Therefore independent calculation of the latter using the equations derived by MacCamy and Fuchs (1954), permits determination of the former. Such values of C_m are also included in Tables 1 and 2.

It is of interest to note that C_m may differ significantly from 2.0. In the limited range of this study C_m varies from 1.192 to 3.380 as shown in Table 2.

General Observations. The following general observations can be made about the cases studied:

1. The force ratios are periodic in the spacing parameter kl_{os}, with attenuated amplitude, not unlike that of Bessel functions.
2. The effect of a neighboring cylinder on the force of the first cylinder is very significant with as much as a 59 percent increase for two cylinders of equal size at $\theta_{os} = 90°$ and in contact; and up to 42 percent decrease in the instance of two unequal cylinders at $\theta_{os} = 0°$ and in contact.
3. Cylinders lined up in the direction of wave advance generally exhibit greater variation in the x-component of force than do cylinders lined up orthogonal to the wave advance vector.

Table 1. THEORETICAL FORCE; TWO EQUAL-DIAMETER CYLINDERS $ka(1) = ka(2) = 0.400$

Cylinder Separation kl_{os}	Horizontal Angle θ_{os}	Cylinder No. 1 Force R_x	Phase δ_x	Coeff C_m	Force R_y	Phase δ_y	Force Ratio	Cylinder No. 2 Force R_x	Phase δ_x	Coeff C_m	Force R_y	Phase δ_y	Force Ratio
0.8000	0°	0.5904	178.9	1.476	0.0000	----	0.7187	0.6829	123.9	1.707	0.0000	----	0.8313
	30	0.7113	164.7	1.778	0.3084	- 51.5	0.8659	0.8396	130.8	2.099	0.3036	- 55.9	1.0220
	60	1.0731	153.6	2.683	0.2937	- 33.1	1.3063	1.1625	143.5	2.906	0.3374	- 56.7	1.4151
	90	1.3045	157.9	3.261	0.1075	78.2	1.5880	1.3045	157.9	3.261	0.1075	-101.8	1.5880
1.0000	0	0.6368	-177.2	1.592	0.0000	----	0.7752	0.7395	111.8	1.849	0.0000	----	0.9002
	30	0.6719	172.9	1.680	0.1702	- 56.2	0.8179	0.8032	121.7	2.008	0.1428	- 56.4	0.9777
	60	0.8493	161.1	2.123	0.1694	- 28.2	1.0339	0.9525	141.9	2.381	0.1704	- 64.2	1.1594
	90	1.0052	163.7	2.513	0.0922	67.3	1.2236	1.0052	163.7	2.513	0.0922	-112.7	1.2236
1.3000	0	0.7212	-174.1	1.803	0.0000	----	0.8780	0.7937	94.5	1.984	0.0000	----	0.9661
	30	0.7001	178.9	1.750	0.1201	- 73.5	0.8523	0.8165	106.8	2.041	0.0767	- 59.8	0.9940
	60	0.7694	166.0	1.924	0.1269	- 36.8	0.9366	0.8916	134.5	2.229	0.0980	- 78.8	1.0854
	90	0.9000	166.1	2.250	0.0811	50.6	1.0955	0.9000	166.1	2.250	0.0811	-129.5	1.0955
1.6000	0	0.8158	-174.1	2.039	0.0000	----	0.9930	0.8249	76.9	2.062	0.0000	----	1.0041
	30	0.7605	-178.2	1.901	0.0987	- 95.7	0.9258	0.8289	91.4	2.072	0.0501	- 58.7	1.0090
	60	0.7460	170.2	1.865	0.1091	- 52.2	0.9081	0.8702	126.1	2.175	0.0621	- 93.6	1.0593
	90	0.8545	167.5	2.136	0.0736	32.6	1.0402	0.8545	167.5	2.136	0.0736	-147.4	1.0402
2.0000	0	0.9217	-178.3	2.304	0.0000	----	1.1219	0.8417	53.2	2.104	0.0000	----	1.0245
	30	0.8505	-178.7	2.126	0.0830	-130.1	1.0352	0.8340	70.8	2.085	0.0411	- 57.1	1.0151
	60	0.7558	174.3	1.890	0.0954	- 78.3	0.9200	0.8553	114.5	2.138	0.0354	-108.1	1.0411
	90	0.8233	168.9	2.058	0.0654	7.6	1.0021	0.8233	168.9	2.058	0.0654	-172.4	1.0021
2.5000	0	0.9675	173.6	2.419	0.0000	----	1.1778	0.8363	24.0	2.091	0.0000	----	1.0180
	30	0.9195	176.4	2.299	0.0703	-177.0	1.1193	0.8247	45.3	2.062	0.0423	- 74.8	1.0039
	60	0.7971	176.6	1.993	0.0833	-115.2	0.9703	0.8407	100.0	2.102	0.0227	-110.6	1.0234
	90	0.8046	170.4	2.011	0.0560	- 23.8	0.9794	0.8046	170.4	2.011	0.0560	156.2	0.9794
3.0000	0	0.9102	166.7	2.276	0.0000	----	1.1080	0.8212	- 4.2	2.053	0.0000	----	0.9997
	30	0.9143	170.7	2.286	0.0612	134.6	1.1130	0.8101	20.7	2.025	0.0410	-106.4	0.9861
	60	0.8377	176.2	2.094	0.0741	-153.6	1.0197	0.8280	85.7	2.070	0.0237	-119.6	1.0079
	90	0.7977	171.7	1.994	0.0477	- 53.8	0.9711	0.7977	171.7	1.994	0.0477	126.2	0.9711

Table 2. THEORETICAL FORCE: TWO CYLINDERS OF UNEQUAL DIAMETER $ka(1) = 0.400$, $ka(2) = 0.600$

Cylinder Separation kl_{os}	Horizontal Angle θ_{os}	Cylinder No. 1 Force R_x	Phase δ_x	Coeff C_m	Force R_y	Phase δ_y	Force Ratio	Cylinder No. 2 Force R_x	Phase δ_x	Coeff C_m	Force R_y	Phase δ_y	Force Ratio
1.0000	0°	0.4769	-166.3	1.192	0.0000	-----	0.5805	1.0569	109.1	1.761	0.0000	-----	0.9184
	30	0.5629	166.6	1.407	0.3784	- 56.5	0.6852	1.1717	116.7	1.953	0.2551	- 79.4	1.0182
	60	1.0151	146.0	2.538	0.3651	- 35.6	1.2356	1.4176	132.2	2.363	0.3274	- 82.2	1.2319
	90	1.3521	150.7	3.380	0.1729	73.7	1.6459	1.5049	150.6	2.508	0.1529	-110.1	1.3077
	120	1.2106	161.2	3.026	0.4658	139.3	1.4736	1.2571	175.8	2.095	0.2516	155.1	1.0923
	150	0.8065	166.2	2.016	0.4175	159.2	0.9818	0.9769	-148.1	1.628	0.2951	149.4	0.8489
	180	0.6035	164.3	1.509	0.0000	-----	0.7346	0.9421	-127.7	1.570	0.0000	-----	0.8187
1.3000	0	0.6560	-159.9	1.640	0.0000	-----	0.7985	1.1445	90.4	1.908	0.0000	-----	0.9946
	30	0.5943	-174.1	1.486	0.2244	- 70.5	0.7235	1.1702	101.8	1.950	0.0927	- 83.7	1.0168
	60	0.7279	158.7	1.820	0.2364	- 35.9	0.8860	1.2561	127.8	2.093	0.1549	-100.8	1.0915
	90	0.9750	158.9	2.437	0.1582	54.9	1.1868	1.2560	158.1	2.093	0.1352	-129.2	1.0914
	120	0.9539	168.4	2.385	0.2307	136.0	1.1612	1.0783	-165.1	1.797	0.1583	170.3	0.9370
	150	0.7833	168.1	1.958	0.1887	174.6	0.9535	0.9983	-124.2	1.664	0.1695	155.7	0.8675
	180	0.7179	163.3	1.795	0.0000	-----	0.8739	1.0417	-106.9	1.736	0.0000	-----	0.9052
1.7000	0	0.9009	-164.3	2.252	0.0000	-----	1.0967	1.2088	65.3	2.015	0.0000	-----	1.0504
	30	0.7717	-168.5	1.929	0.1774	-101.9	0.9394	1.1979	80.3	1.996	0.0403	- 59.3	1.0409
	60	0.6695	170.3	1.674	0.2015	- 56.9	0.8150	1.2216	116.8	2.036	0.0738	-128.8	1.0615
	90	0.8581	162.8	2.145	0.1462	29.6	1.0445	1.1751	160.9	1.958	0.1209	-156.6	1.0211
	120	0.8959	170.2	2.240	0.1347	128.4	1.0905	1.0372	-147.2	1.729	0.1354	164.4	0.9013
	150	0.8109	167.4	2.027	0.1125	-166.3	0.9871	1.1088	-100.7	1.848	0.1314	147.7	0.9635
	180	0.8016	162.3	2.004	0.0000	-----	0.9758	1.2083	- 84.6	2.014	0.0000	-----	1.0499
2.1000	0	1.0662	-175.0	2.666	0.0000	-----	1.2979	1.2184	40.2	2.031	0.0000	-----	1.0587
	30	0.9366	-173.4	2.342	0.1525	-139.0	1.1412	1.1961	58.6	1.994	0.0538	- 46.0	1.0394
	60	0.7121	178.1	1.780	0.1808	- 85.3	0.8669	1.2042	104.9	2.007	0.0270	-147.4	1.0464
	90	0.8071	165.6	2.018	0.1318	3.5	0.9825	1.1416	162.6	1.903	0.1069	174.7	0.9920
	120	0.8708	170.4	2.177	0.0861	125.1	1.0600	1.0626	-131.6	1.771	0.1229	149.2	0.9234
	150	0.8160	166.1	2.040	0.0950	-149.5	0.9934	1.2251	- 82.5	2.042	0.1120	132.4	1.0645
	180	0.8294	160.9	2.073	0.0000	-----	1.0096	1.3313	- 67.1	2.219	0.0000	-----	1.1569

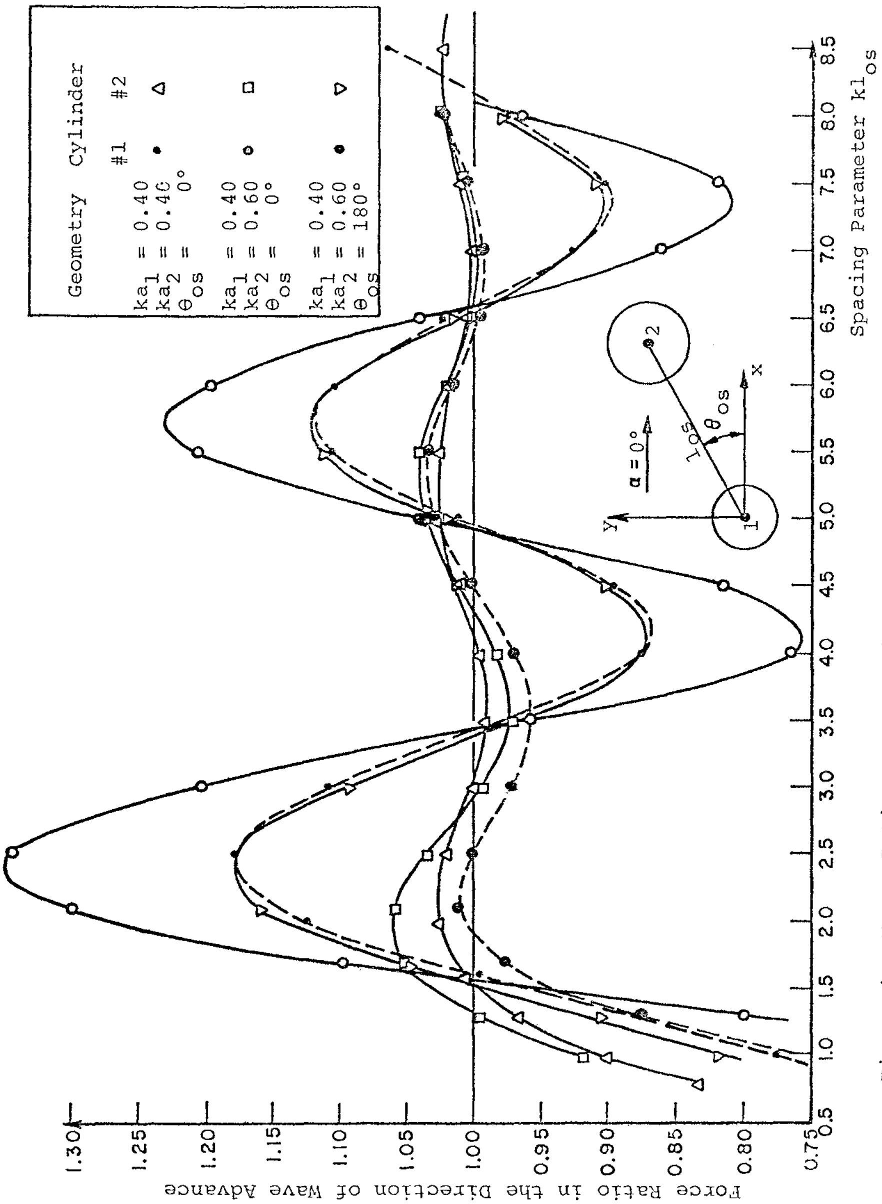

Figure 4. Force Ratio Versus Cylinder Separation, Two Cylinders, $\theta_{os} = 0°$ and $180°$

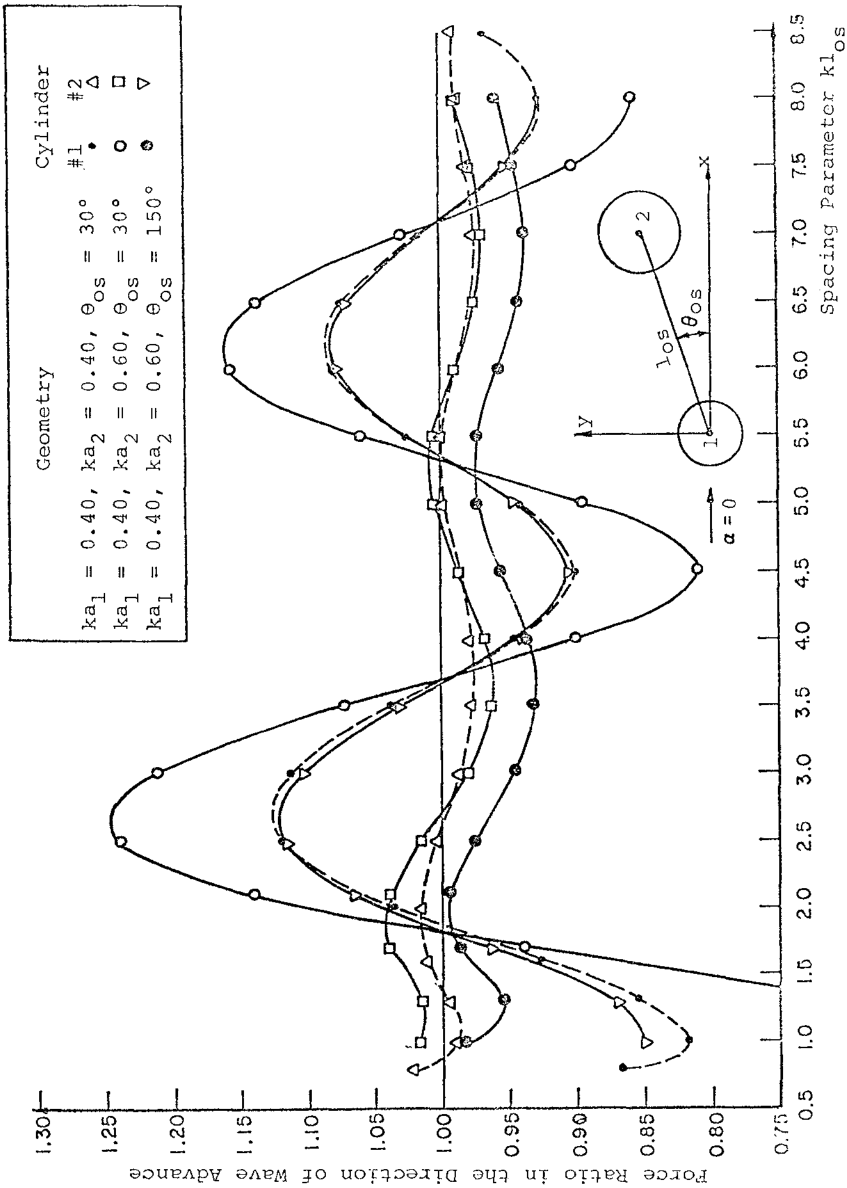

Figure 5. Force Ratio Versus Cylinder Separation, Two Cylinders, θ_{os} = 30° and 150°

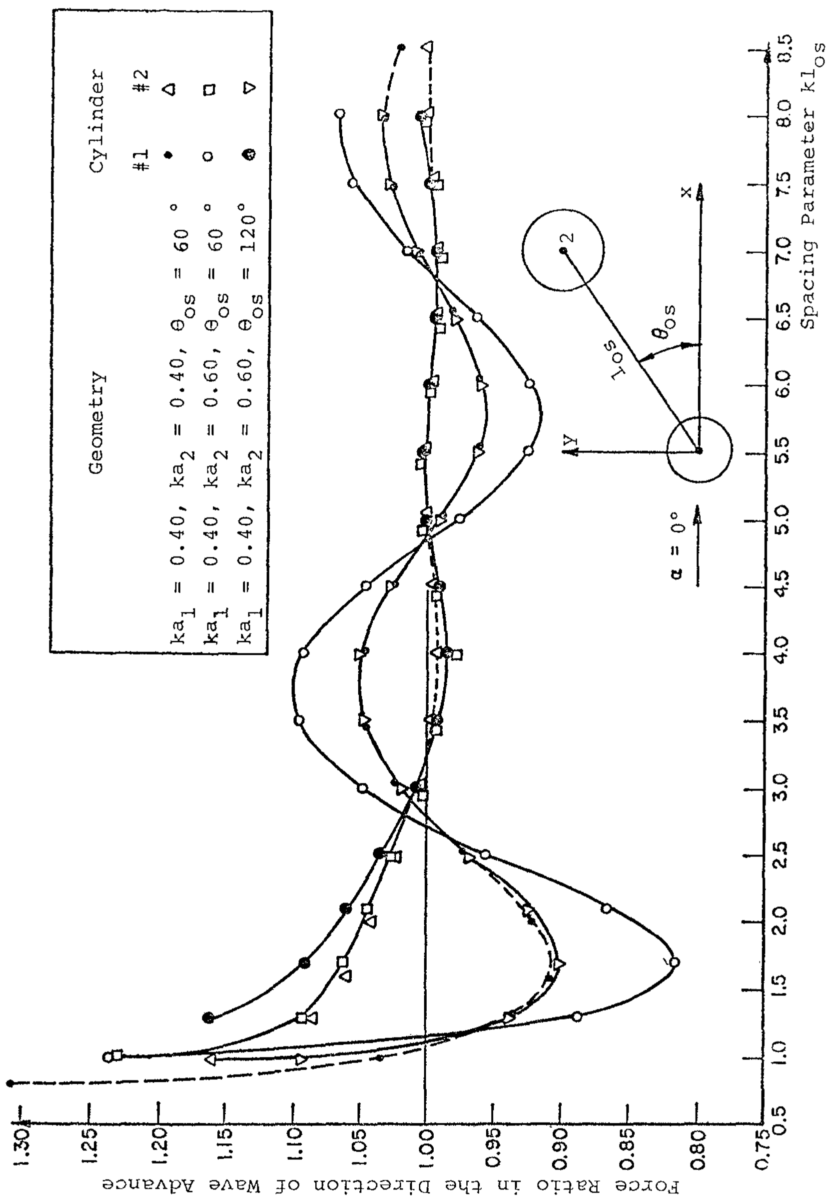

Figure 6. Force Ratio Versus Cylinder Separation, Two Cylinders, $\theta = 60°$ and $120°$

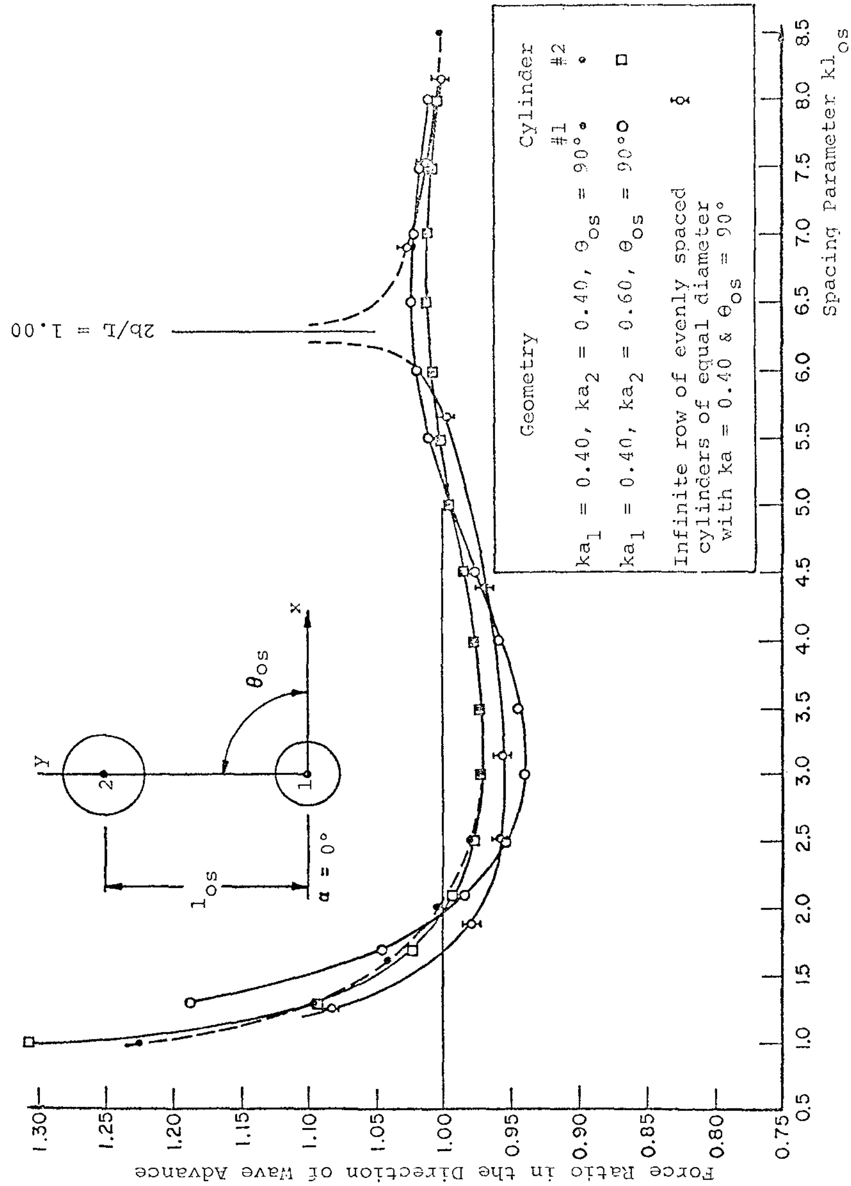

Figure 7. Force Ratio Versus Cylinder Separation, Two Cylinders, $\Theta_{os} = 90°$

Single Isolated Cylinder Pressure and Force. By way of verification, the pressure amplitudes for single, isolated cylinders with ka = .5 and ka = 1.0 were checked against those published by Wiener (1947) for sound diffraction by a single rigid cylinder. Agreement to the 4th decimal place was obtained.

CONCLUSIONS

1. A means of calculating the pressures and forces on a cluster of vertical circular cylinders is developed. The method employs diffraction theory, but avoids multiple scattering techniques, in favor of a direct, matrix solution.
2. Theoretical calculations for the force in the direction of wave advance reveal as much as a 60% departure from the force on a single isolated cylinder in the instance of two equal-diameter cylinders and as much as a 65% departure for two cylinders of unequal diameter. The force on a given cylinder is thus significantly affected by the presence of neighboring cylinders. The mass coefficient, C_M, is found to range from 1.19 to 3.38, significant departures from the often assumed value of 2.0
3. The component of force perpendicular to the direction of wave advance is found to be very significant when the cylinders are close together, rising in one case to 67% of the force component in the direction of wave advance. Although the effect on the maximum resultant is generally less than 10%, in one case a 50% increase is found. Both force components may be significant for design of cylinders used for offshore tower supports.
4. The method is not restricted to water waves, but can also be applied to other cases of scalar scattering in acoustics or electromagnetics.

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