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Interactive approach in multicriteria analysis based on stochastic dominance

by

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Abstract: The paper considers a discrete stochastic multicriteria problem. This problem can be defined by a finite set of actions A, a set of attributes X and a set of evaluations E. It is assumed that the performance probability distributions for each action on each attribute are known. A new procedure for such a problem is proposed. It is based on two concepts: stochastic dominance and interactive approach. Stochastic dominance is employed for comparing evaluations of actions with respect to attributes. The STEM methodology is employed in the dialogue procedure between decision maker and decision model. In each step a candidate action a_i is generated. The decision maker examines evaluations of a_i with respect to attributes and selects the one that satisfies him/her. Then the decision maker defines the limit of concessions, which can be made on average evaluations with respect to this attribute. The procedure continues until a satisfactory action is found.

Keywords: multiple criteria analysis, interactive approach, uncertainty modeling, stochastic dominance.

1. Introduction

Rational decision making is possible if we are able to evaluate possible actions. In many real-world decision problems decision maker (DM) can evaluate the performance of actions only in a probabilistic way. In such situations the comparison of two actions leads to the comparison of two probability distributions.

Mean-risk analysis and stochastic dominance (SD) are the main concepts used for comparing uncertain prospects. The former is based on two criteria: one measuring expected outcome (usually mean) and the second representing variability of outcomes. Various risk measures are proposed: variance

(Markowitz, 1952a), semivariance (Markowitz, 1959), probability of loss, risk measures based on below-target returns (Fishburn, 1977) and others. Mean-risk analysis is usually used to model preferences of a risk-averse decision-maker and as is noticed by Ogryczak and Ruszczyski (1999) this model is not capable of modeling even the entire gamut of risk-averse preferences. Although the model of risk-averse preferences is widely exploited in decision theory, it is not suitable for all situations. The paradox of risk-seeking in choices between negative prospects was noticed by Markowitz (1952b) and justified by experiments conducted by Kahneman and Tversky (1979).

The concept of SD was proposed in 1930s. The fundamental papers on SD theory were published in 1969-70 (Hadar and Russell, 1969; Hanoch and Levy, 1969; Rothschild and Stiglitz, 1970; Whitmore, 1970). Initially, SD was used in models of risk-averse preferences only. Goovaerts (1984) and Zaras (1989) proposed rules for risk seeking decision-makers. Thus, stochastic dominance rules can be employed in modeling preferences of various groups of decision-makers: risk-averse and risk-seeking.

Real-world decision problems usually involve multiple and conflicting objectives. Numerous approaches have been proposed in multicriteria decision making for the last 40 years. Prominent among these developments has been the methodology known as interactive approach. Various reasons have been mentioned for implementing this technique. First of all, it is pointed out that a limited amount of preference information is required from DM as compared to other methodologies. It is only assumed that DM is able to define attributes that influence his/her preferences and to provide local preference information to given solution or small subset of solutions. Thus, it is not necessary to ask DM to make a lot of hypothetical choices between alternatives like in approaches based upon multiattribute value (utility) theory. As such choices are often of no practical applicability, it is not easy to motivate DM to consider and evaluate them. On the other hand it is also indicated that as in interactive procedures DM actively participates in all phases of the problem solving process, he/she puts much reliance in the generated solution and in consequence, the final solution has a better chance of being implemented.

First interactive techniques for deterministic continuous multiple criteria problems were proposed in early 1970s. One of the first was the Step Method (STEM) proposed by Benayoun et al. (1971). Other approaches were proposed by Geoffrion et al. (1972), Zionts and Wallenius (1976), Nijkamp and Spronk (1980), Wierzbicki (1980, 1982), Steuer (1986), Korhonen and Lakso (1986) and many others. Procedures for discrete problems have also been presented: Roy (1976), Spronk (1981), Zionts (1981), Korhonen et al. (1984), Vanderpooten (1989), Lotfi et al. (1992), Habenicht (1992), Sun and Steuer (1996). All these techniques and a lot of others are applicable under the circumstances of certainty. Up to now only few methods have been devised for the case of risk.

The aim of this paper is to present an interactive procedure for discrete decision making problems under risk. On the one hand we base on stochastic dominance rules, on the other on the STEM method. In each step rankings of actions with respect to attributes are constructed. The SD rules are employed in this phase. Next, a candidate action is chosen and the DM is asked to analyze its evaluations with respect to attributes and to decide which of them is satisfying. The DM is also expected to indicate minimal acceptable level of the average evaluation with respect to the considered attribute. The procedure continues until the satisfying action is generated.

The paper is structured as follows. A discrete multiattribute decision making problem is formulated in Section 2. Next section deals with the SD approach and the relationships between SD rules and the decision-maker's utility function. In Section 4 the interactive procedure is presented. Section 5 gives an example. The last section contains conclusions.

2. Discrete multiattribute decision making problem under risk

A discrete multiattribute decision making problem may be conceived as a model (A, X, E), where A is a finite set of actions a_i, $i = 1, 2, ..., n$, X is a finite set of attributes X_k, $k = 1, 2, ..., m$ and E is the set of evaluations of actions with respect to attributes e_{ik}, $i = 1, 2, ..., n$, $k = 1, 2, ..., m$. In the stochastic case considered in this work, the evaluations of actions with respect to attributes are probability distributions and as a result the comparison of two actions is equivalent to the comparison of two vectors of probability distributions. It is assumed that attributes are defined in such a way that a larger value is preferred to a smaller one ("more is better") and that the distribution functions $f_i(x_k)$ are known.

Various concepts for solving such a problem have been proposed. The classical approach is based on the multiattribute utility theory. In order to employ this technique one has to estimate DM's multiattribute utility function. Unfortunately, the assessment of such a function is not easy. Keeney and Raiffa (1976) show that if additive independence condition is verified, then multiattribute comparison of two actions can be decomposed to n one-attribute comparisons. Thus, estimating one-attribute utility functions and assessing the form of the aggregate function can solve the problem. In practice, however, both estimating one-attribute utility functions and assessing the aggregate function is difficult.

A lot of other techniques for a multiattribute decision making problem under risk have also been proposed. Saaty and Vargas (1987) present the analytical hierarchly process (AHP) procedure that includes uncertainty. Various methods based on the outranking approach have also been presented. Jacquet-Lagreze (1977) constructs a probabilistic preference relation and probabilistic indifference relation for each pair of actions and for each attribute. These relations are then integrated in a degree of credibility and processed as in ELECTRE III. The procedure proposed by Dendrou et al. (1980) is based on probabilities that one action will dominate another. Such probabilities are obtained by the Bayesian approach and used for generation of joint probabilities, which are ranked in the decreasing order. Martel et al. (1986) use fuzzy outranking relation on the set of actions. This relation is obtained from an overall index of confidence and overall index of doubt regarding the supposition that an action is at least as good as another. D'Avignon and Vincke (1988) construct strength and weakness distributions for all actions, two partial rankings and a final ranking based on a measure of the proposition "action a_i outranks a_j" and a measure of the risk level for the same proposition.

3. Stochastic dominance in multiattribute analysis

Stochastic dominance approach is based on the multiattribute utility theory's axioms. Thus, it is assumed that the preferences of the DM can be explained by a multiattribute utility function. Instead of estimating this function, however, we base on SD rules. If certain assumptions on the type of the decision-maker's utility functions are fulfilled, then stochastic dominance rules are equivalent to the expected utility rules.

Two groups of SD rules for two classes of DM's utility function are usually considered. The first one includes first degree stochastic dominance - FSD (see Appendix A for definitions of SD relations and Appendix B for decision rules), second degree stochastic dominance - SSD, and third degree stochastic dominance - TSD. The FSD rule is equivalent to the expected utility maximisation rule for increasing utility functions ($u' > 0$). Two other rules are more restrictive: the SSD rule may be employed in the case of increasing concave utility functions ($u' > 0$ and $u'' \geq 0$), while TSD rule is for the decreasing absolute risk aversion (DARA) utility function. Thus FSD/SSD/TSD rules are equivalent to expected utility maximisation rule for risk-averse preferences.

The second group of SD rules includes FSD, second degree inverse stochastic dominance - SISD, and two kinds of third degree inverse stochastic dominance - TISD1 and TISD2. The SISD rule is equivalent to the expected utility maximisation rule for increasing convex utility function ($u' > 0$ and $u'' \geq 0$), while TISD1 and TISD2 are limited to increasing absolute risk aversion (INARA) utility functions. Thus, FSD/SISD/TISD1/TISD2 rules are equivalent to expected utility maximisation rule for risk-seeking preferences.

In this work we assume that additive independence condition formulated by Keeney and Raiffa (1976) is fulfilled. In such a case additive multiattribute utility function can be used. Thus, the multiattribute comparison of two actions can be decomposed into n one-attribute comparisons. As these comparisons are based on SD rules, they are expressed in terms of "a_i is at least as good as a_j" in relation to each attribute and for all pairs $(a_i, a_j) \in A \times A$. The following question arises: how the SD concept can be implemented in modelling DM's global preferences? Huang et al. (1978) propose the Multiattribute Stochastic Dominance (MSD) rule. According to this rule, action a_i is at least as good as a_j in the sense of MSD, if and only if the evaluations of action a_i

dominate corresponding evaluations of a_j according SD rules with respect to all attributes. In practice this rule is rarely verified. Zaras and Martel (1994) suggest weakening the unanimity condition and accepting a majority attribute condition. They propose MSD$_r$ - multiattribute stochastic dominance rule for a reduced number of attributes. The approach of Zaras and Martel is based on the observation that people tend to simplify the multiattribute problem by taking into account only the most important attributes. The procedure consists of two steps. In the first one SD relations are verified for each pair of actions with respect to all attributes. The second phase is aggregation of the multiple attributes. The ELECTRE I methodology is used to obtain the final ranking of actions. First, actions are classified considering all attributes. If the ranking is not clear enough then new classification is constructed taking all but the least important attribute into account. The procedure proceeds until the DM accepts the ranking and the majority condition is valid. Employment of this technique is possible if the DM is able to express his/her preferences in a way that makes it possible to set values of weighting coefficients expressing importance of attributes. This task is usually inconvenient and time consuming. Moreover, as only the most important attributes are taken into account when final ranking is constructed, it can happen that an action very weakly evaluated with respect to less important attribute is chosen as a final solution.

One can avoid such problems by employing the interactive approach. The DM is expected to analyze evaluations of a proposed solution and specify which of them satisfies him/her. Thus, the DM has to provide only a limited amount of preference information, which is based on a solution or a subset of solutions that are known to be feasible.

4. Interactive procedure based on SD rules

A new interactive technique for a discrete stochastic multicriteria problem is proposed here. It is based on two concepts: SD and interactive approach. The idea of our technique comes from the STEM method (Benayoun et al., 1971). In the STEM method the concept of the ideal solutions is used. The elements of the ideal solution are the maximum values of the attributes, which are individually attainable within the set of actions. In the STEM method a candidate action is generated and presented to the DM in each step. It is the one that is the closest to the ideal solution according to the minimax rule. If the DM accepts the proposal, then the procedure ends, otherwise the DM is asked to define the margins of relaxation for these attributes, whose values are already satisfactory. Then new set of actions is generated taking into account the restrictions defined by the DM. The procedure continues until an action with satisfactory attribute evaluations is found.

In our technique, like in the STEM method, a candidate action a_i is generated. We apply SD rules for doing this. In the dialogue phase of the procedure average evaluations are used. The DM examines average evaluations of a candidate a_i with respect to attributes and selects the attribute with respect to which the candidate action is satisfactory for him/her. Then the DM defines the limit of concessions, which can be made on average evaluations of the attribute X_k.

Let us assume following notation:

F_{ik} - cumulative distribution function representing evaluation of action a_i with respect to attribute X_k,

μ_{ik} - average evaluation of action a_i with respect to attribute X_k,

SD - stochastic dominance relation consistent with DM's utility function.

The operation of the proposed procedure is as follows:

1. Identify SD relations between actions with respect to attributes, calculate average evaluations of actions with respect to attributes μ_{ik}, $i = 1, ..., n$, $k = 1, ..., m$;
2. $l := 1$, $A_1 := A$, $K := \{1, ..., m\}$;
3. Identify candidate action a_i:

   $$a_1 := \arg \min_{a_j \in A_l} \max_{k \in K} \{d_{jk}^l\}$$

   where d_{jk}^l is the number of actions $a_i \in A_l$ such that the evaluation of a_i dominate the evaluation of a_j with respect to attribute X_k according to SD relation consistent with DM's utility function:

   $$d_{jk}^l = \text{card } D_{jk}^l$$
   $$D_{jk}^l = \{a_i : a_i \in A_l;\ F_{ik} \text{ SD } F_{jk}\}.$$

   In the case of a tie choose any a_i minimising the value of $\max_{k \in K} \{d_{jk}^l\}$.
4. Present the data to the DM:
   - the average evaluations of the candidate action a_i with respect to all attributes - μ_{ik}, $k = 1, ..., m$,
   - the values of d_{ik}^l for $k = 1, ..., m$,
   - the maximal average evaluations μ_k^* for $k = 1, ..., m$:

     $$\mu_k^* = \max_{i: a_i \in A_l} \{\mu_i\}.$$
5. Ask the DM whether he/she is satisfied with the candidate action. If the answer is YES - the final solution is action a_i - go to 9, else - go to 6.
6. Ask the DM whether the candidate action is satisfactory with respect to at least one attribute. If the answer is YES - go to 7, else - it is impossible to find an action with satisfactory attribute evaluations by the procedure - go to 9.
7. Ask the DM to select the attribute with respect to which the candidate action is satisfactory for him/her, say attribute X_k, and to define δ_k - the limit of concessions, which can be made on average evaluations of the attribute X_k.

8. Generate the set $A_{l+1} := \{a_j : a_j \in A_l,\ \mu_{jk} \geq \delta_k\}$, assume $l := l + 1$, $K := K \setminus \{k\}$, if $K = \emptyset$ then $K = \{1, ..., m\}$, go to 3.
9. The end of the procedure.

Comments:

Step 2: K is the set of attributes that are considered when a candidate solution is generated. Once the DM accepts the evaluation of the candidate solution with respect to X_k, the number of this attribute is removed from K. If K is empty and the satisfactory solution has not been identified, then again all attributes are included in K (step 7).

Step 3: As the evaluations are probability distributions, so we are not able to generate candidate action in the same way as is done in the STEM method. We apply SD rules instead: the distance from the ideal solution is measured by the number of actions with evaluations dominating the evaluation of the considered action according to SD relation.

Step 4: Two types of data are presented to the DM. The number of evaluations dominating the evaluation of candidate action with respect to attribute X_k provide the information on the position of the this action in a "ranking" of actions with respect to X_k, constructed according to SD rules. At the same time the average evaluation μ_{ik} together with μ_k^* inform the DM about the distance between the best action with respect to X_k and the proposed action. Thus, the DM is able to evaluate the candidate action and decide whether he/she accepts the evaluation of the candidate action with respect to X_k.

Step 6: If the DM does not accept any evaluation of the candidate action for any $k = 1, ..., m$ then it is not possible to identify the solution of the problem by the proposed procedure. In such case there is no attribute evaluation to compromise on.

Step 7: In order to define the limit of concessions for the attribute X_k the DM is asked to define the minimal value of average evaluation for that attribute. Obviously, as the DM accepts the evaluation of a_i with respect to X_k, so δ_k should not be greater than μ_{ik}.

5. Illustrative example

To illustrate our procedure let us consider a project selection problem. Twenty proposals are evaluated with respect to three attributes: X_1, X_2, and X_3. The evaluations of actions with respect to attributes are presented in Table 1.

Table 1. Evaluations of actions with respect to attributes

X_1	Projects																			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1		0.2		0.4	0.2			0.4		0.2	0.6						0.4		0.4	
2	0.4	0.2		0.6						0.4	0.4		0.4			0.6		0.2	0.4	0.2
3	0.4	0.2	0.2		0.6		0.4	0.4		0.2		0.2	0.2		0.2	0.2	0.6	0.4	0.2	0.6
4	0.2	0.4	0.2		0.2	0.4		0.2	0.6	0.2		0.4	0.2	0.8	0.6	0.2		0.4		0.2
5			0.6			0.6	0.6		0.4			0.4	0.2	0.2	0.2					
X_2	Projects																			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1			0.4	0.2		0.6			0.2	0.4	0.2			0.2	0.4		0.6			
2			0.4						0.2	0.2	0.4	0.2		0.2	0.6		0.4	0.2		
3	0.2	0.4	0.2	0.4		0.2	0.4		0.4	0.4	0.2		0.2			0.2		0.2	0.2	0.2
4	0.6			0.4	0.6	0.2	0.4	0.8	0.2		0.2	0.6	0.4	0.6		0.2		0.6	0.4	0.6
5	0.2	0.6			0.4		0.2	0.2				0.4	0.4			0.6			0.4	0.2
X_3	Projects																			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1		0.2			0.6	0.6		0.2	0.4			0.6	0.4	0.2		0.8				
2		0.2	0.2			0.4	0.2	0.2					0.4	0.2	02	0.2				
3	0.2	0.4	0.6		0.4		0.2	0.4		0.2		0.2	0.2	0.2				0.2		0.4
4	0.6	0.2	0.2	0.6			0.6	0.2	0.6	0.2	0.2	0.2		0.4	0.6		0.8	0.2	0.8	
5	0.2			0.4						0.6	0.8				0.2		0.2	0.6	0.2	0.6

To apply our approach, it is necessary to establish FSD relations for each pair of actions with respect to each attribute. As attributes are defined in the domain of gains, so we assume that DM's utility function is DARA. Thus, FSD/SSD/TSD rules can be used to model DM's preferences.

Step 1: SD relations are identified and average evaluations m_{ik} are calculated (Table 2).

Step 2: $l := 1$, $A_1 := A$, $K := \{1, 2, 3\}$

Step 3: Values of d_{ik}^1 are calculated (Table 2); action a_{20} is selected as the candidate action.

Step 5: Let us assume that the candidate action is not satisfactory for the DM.

Step 6: Let us assume that the candidate action is satisfactory for the DM with respect to attribute X_2.

Step 7: The DM defines $\delta_2 = 3.4$ as the minimum average evaluation with respect to attribute X_2.

Step 8: The set of actions satisfying the condition formulated by the DM is generated: $A2 = \{a_1, a_2, a_5, a_7, a_8, a_{12}, a_{13}, a_{16}, a_{18}, a_{19}, a_{20}\}$; $l = 2$; $K = \{1, 3\}$.

Table 2. μ_{ik} and d^1_{ik}

Project	μ_{ik}			d^1_{ik}			$\max_{k\in\{1,2,3\}}\{d^1_{ik}\}$
	X_1	X_2	X_3	X_1	X_2	X_3	
1	2.8	4.0	4.0	10	5	6	10
2	2.8	4.2	2.6	12	5	12	12
3	4.4	1.8	3.0	2	16	10	16
4	1.6	3.0	4.4	18	11	1	18
5	2.8	4.4	1.8	11	0	17	17
6	4.6	2.0	1.4	0	16	18	18
7	4.2	3.6	3.4	5	9	9	9
8	2.4	4.2	2.6	15	1	12	15
9	4.4	2.6	2.8	1	13	12	13
10	2.4	2.0	4.4	14	15	2	15
11	1.4	2.4	4.8	19	14	0	19
12	4.2	3.8	2.0	4	8	15	15
13	3.2	4.2	1.8	8	3	15	15
14	4.2	3.0	2.8	2	12	11	12
15	4.0	1.6	3.8	5	17	8	17
16	2.6	4.4	1.2	11	1	19	19
17	2.2	1.4	4.2	16	19	2	19
18	3.2	3.4	4.4	7	10	2	10
19	1.8	4.2	4.2	17	3	2	17
20	3.0	4.0	4.2	8	5	6	8
μ^*_k	4.6	4.4	4.8				

Step 3: Values of d^2_{ik} are calculated (Table 3); action a_{18} is selected as candidate action.

Table 3. μ_{ik} and d^2_{ik}

Project	μ_{ik}			d^2_{ik}			$\max_{k\in\{1,3\}}\{d^2_{ik}\}$
	X_1	X_2	X_3	X_1	X_2	X_3	
1	2.8	4.0	4.0	5	5	2	5
2	2.8	4.2	2.6	7	5	5	7
5	2.8	4.4	1.8	6	0	9	9
7	4.2	3.6	3.4	1	9	4	4
8	2.4	4.2	2.6	9	1	5	9
12	4.2	3.8	2.0	0	8	7	7
13	3.2	4.2	1.8	3	3	7	7
16	2.6	4.4	1.2	6	1	10	10
18	3.2	3.4	4.4	2	10	0	2
19	1.8	4.2	4.2	10	3	0	10
20	3.0	4.0	4.2	3	5	2	3
μ^*_k	4.2	4.4	4.4				

Step 5: Let us assume that the candidate action is not satisfactory for the DM.
Step 6: Let us assume that the candidate action is satisfactory for the DM with respect to attribute X_3.
Step 7: The DM accepts $\delta_3 = 3.4$ as the minimum average evaluation with respect to attribute X_3.
Step 8: The set of actions satisfying the condition formulated by the DM is generated: $A_3 = \{a_1, a_7, a_{18}, a_{19}, a_{20}\}$; $l = 3$; $K = \{1\}$.
Step 3: Values of d_{ik}^3 are calculated (Table 4); action a_7 is selected as the candidate action.
Step 5: Let us assume that the DM accepts the candidate action as the final solution.

Table 4. μ_{ik} and d_{ik}^3

Project	μ_{ik}			d_{ik}^3			$\max_{k\in\{1\}} \{d_{ik}^3\}$
	X_1	X_2	X_3	X_1	X_2	X_3	
1	2.8	4.0	4.0	3	1	2	3
7	4.2	3.6	3.4	0	3	4	0
18	3.2	3.4	4.4	1	4	0	1
19	1.8	4.2	4.2	4	0	0	4
20	3.0	4.0	4.2	2	1	2	2
μ_k^*	4.2	4.2	4.4				

6. Conclusions

Several motivations have been mentioned for employing interactive methodology in multiple criteria decision making. It is usually pointed out that this approach requires limited amount of preference information from the DM as compared to other approaches. It is also indicated that as the DM is more closely involved in the process of problem solving, the final solution has a better chance of being implemented.

Up to now interactive techniques have rarely been implemented in stochastic environment. The main reason for this is that in stochastic case it is not easy for the DM to analyze action's evaluations and to compare them with evaluations of other actions. Thus, the DM is often incapable of providing local preference information that is required in interactive techniques.

In this paper a new methodology for a discrete stochastic multicriteria decision making problem was proposed. The approach combines two concepts: stochastic dominance and interactive approach. Taking into account the preferences deduced from the SD rules in relation to each attribute, we use the dialogue procedure to identify the final solution of the problem. Two types of data are presented to the DM: the number of evaluations dominating the evaluation of a considered action according to SD rules in relation to each attribute and the average evaluations of actions with respect to each attribute. Thus, the DM is able to provide local preference information either by accepting the considered action, or by constraining the set of admissible actions from which subsequent candidate actions are selected.

Appendix A

Notation:

$F(x), G(x)$ - cumulative distribution functions,
$\bar{F}(x), \bar{G}(x)$ - decumulative distribution functions.

Stochastic dominance relations are defined as follows:

Definition 1:
$F(x)$ FSD $G(x)$ if and only if $F(x) \neq G(x)$ and $H_1(x) = F(x) - G(x) \leq 0$ for all $x \in [a, b]$.

Definition 2:
$F(x)$ SSD $G(x)$ if and only if $F(x) \neq G(x)$ and $H_2(x) = \int_a^x H_1(y)dy \leq 0$ for all $x \in [a, b]$.

Definition 3:
$F(x)$ TSD $G(x)$ if and only if $F(x) \neq G(x)$ and $H_3(x) = \int_a^x H_2(y)dy \leq 0$ for all $x \in [a, b]$.

Definition 4:
$\bar{F}(x)$ SISD $\bar{G}(x)$ if and only if $\bar{F}(x) \neq \bar{G}(x)$ and $\bar{H}_2(x) = \int_x^b \bar{H}_1(y)dy \geq 0$ for all $x \in [a, b]$, where: $\bar{H}_1 = \bar{F}(x) - \bar{G}(x)$.

Definition 5:
$\bar{F}(x)$ TISD1 $\bar{G}(x)$ if and only if $\bar{F}(x) \neq \bar{G}(x)$ and $\bar{H}_3(x) = \int_x^b \bar{H}_2(y)dy \geq 0$ for all $x \in [a, b]$.

Definition 6:
$\bar{F}(x)$ TISD2 $\bar{G}(x)$ if and only if $\bar{F}(x) \neq \bar{G}(x)$ and $\tilde{H}_3(x) = \int_a^x \bar{H}_2(y)dy \geq 0$ for all $x \in [a, b]$.

Appendix B

Notation: $u(x)$ - utility function.

Rule 1 (Hadar, Russel, 1969):
If $H_1(x) \leq 0$ for all $x \in [a, b]$ then $E_F[u(x)] - E_G[u(x)] \geq 0$ for all $u(x) \in U_1$, where $U_1 = \{u(x) : u'(x) > 0\}$.

Rule 2 (Hadar, Russel, 1969):
If $H_2(x) \leq 0$ for all $x \in [a, b]$ then $E_F[u(x)] - E_G[u(x)] \geq 0$ for all $u(x) \in U_2^1$, where $U_2^1 = \{u(x) : u'(x) > 0,\ u''(x) \leq 0\}$.

Rule 3 (Whitmore, 1970):
If $\mu_F \geq \mu_G$ and $H_3(x) \leq 0$ for all $x \in [a, b]$ then $E_F[u(x)] - E_G[u(x)] \geq 0$ for all $u(x) \in U_3^1$, where $U_3^1 = \{u(x) : u'(x) > 0,\ u''(x) \leq 0,\ u'''(x) \geq 0$ and $u'(x) \cdot u'''(x) \geq [u''(x)]^2\}$.

Rule 4 (Goovaerts, 1984):
If $\bar{H}_2(x) \geq 0$ for all $x \in [a, b]$ then $E_F[u(x)] - E_G[u(x)] \geq 0$ for all

$u(x) \in U_2^2$, where $U_2^2 = \{u(x) : u'(x) > 0,\ u''(x) \geq 0\}$.

Rule 5 (Goovaerts, 1984):

If $\bar{H}_3(x) \geq 0$ for all $x \in [a, b]$ then $E_F[u(x)] - E_G[u(x)] \geq 0$ for all $u(x) \in U_3^2$ where $U_3^2 = \{u(x) : u'(x) > 0,\ u''(x) \geq 0,\ u'''(x) \geq 0$ and $u'(x) \cdot u'''(x) \leq [u''(x)]^2\}$.

Rule 6 (Zaras, 1989):

If $\tilde{H}_3(x) \geq 0$ for all $x \in [a, b]$ then $E_F[u(x)] - E_G[u(x)] \geq 0$ for all $u(x) \in U_3^2$, where $U_3^2 = \{u(x) : u'(x) > 0,\ u''(x) \geq 0,\ u'''(x) \leq 0\}$.

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