CHAPTER 162

INTERCOMPARISON OF COASTAL PROFILE MODELS

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Abstract

The present paper briefly presents 6 different models for short term coastal profile modelling for direct incoming waves. The models have been tested against measured profile evolutions from a large wave flume. Features such as wave height distribution, cross shore current profiles and sediment transport are compared and discussed.

Introduction

The described models are all established with the same structure of modules for hydrodynamics, sediment transport and bed level evolution but with different degrees of determinism/empiricism and refinement. The basic structure of the modules and a definition sketch for the coastal profile models are presented in Fig. 1.

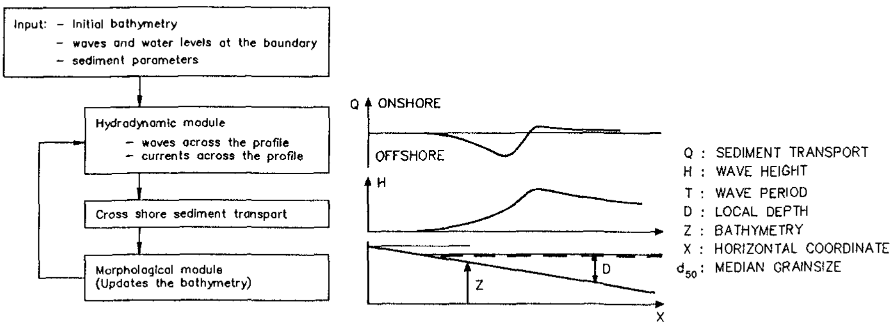

Fig. 1. Basic Structure of the Morphological Models and Definition Sketch.

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Description of the Models

LITCROSS, Danish Hydraulic Institute.

The variation of the wave heights across the profile is determined from the criteria that the wave has either constant energy flux or wave heights decreases in accordance with an empirical relation first suggested by Andersen and Fredsøe (1983):

$$\frac{H}{D} = 0.5 + 0.3 \exp\left(-0.11\frac{\Delta x}{D_B}\right)$$

where Δx is the distance from breaking point and D_B is the depth at the breaking point. The areas of the surface roller of the breaking waves are assumed to correspond to hydraulic jumps apart from a zone just inside the breaker point where the area is assumed to vary according to measurements by Buhr Hansen (1991). Random waves are treated as individual waves with no interaction.

Hydrodynamic modelling. The vertical distributions of currents and turbulence are assumed to be determined by the local depth, wave conditions and sediment properties. The basis for the model is the combined wave current boundary layer model of Fredsøe (1984). In breaking waves a major contribution to the turbulence comes from the loss of energy in waves and surface rollers. This contribution is calculated by the vertical transport equation for turbulent energy, Deigaard et al. (1986). The vertical distribution of the wave period averaged velocities is derived from the distributions of shear stresses and wave period averaged eddy viscosity. The shear stresses include the contributions from breaking waves, Deigaard and Fredsøe (1989), from streaming, determined as outlined by Longuet Higgins (1953), from the increased density due to suspended sediment in case of sloping bed and from a setup of the water surface which is determined such that the total flux including the wave drift and the water carried in the surface rollers equals zero, Svendsen (1984).

The net sediment transport is calculated as bed and suspended load. The instantaneous bed load and nearbed boundary condition for the vertical distribution of suspended sediment are determined as functions of the instantaneous shear stress, Engelund and Fredsøe (1976). The time varying vertical distribution of suspended concentrations is calculated by the vertical diffusion equation, Deigaard et al. (1986). The total wave period averaged net transport is found from

$$q_s = \frac{1}{T}\int_{\text{period}}\int_{\text{depth}} c \cdot u \, dz \, dt + \int_{\text{depth}} u_l \cdot \bar{c} \, dz$$

where u_l is Lagrangian drift. The last term is included to compensate the omission of convective terms in the solution of the concentration field.

The bed level evolutions are determined by the continuity equation for the sediment.
The numerical solution is explicit. A modified Lax-Wendroff scheme has been applied to reduce the numerical diffusion and to obtain a stable solution. LITCROSS is described in more detail in Brøker Hedegaard, Deigaard and Fredsøe (1991).

UNIBEST, Delft Hydraulics.

UNIBEST-TC stands for UNIform BEach Sediment Transport, Time-dependent Cross-shore. It is a direct descendant of the models OSTRAN (Stive and Battjes, 1984) and COSTRAN (Stive, 1986). In Roelvink and Stive (1989), the model has been tested against wave flume measurements and improved on some points. The model aims at predicting long-term development of the profile of beaches that are approximately uniform in alongshore direction, and which are subjected to obliquely incident wave fields, varying water levels and tidal currents. The morphodynamic behaviour due to cross-shore transport only is considered, however, effects of longshore currents on cross-shore transport are accounted for.

For the present study, cross-shore effects only are considered, and the formulations as given in Roelvink and Stive (1989) are used. Mechanisms included here are:

- Wave shoaling and breaking and associated set-up according to Battjes and Janssen (1978);
- Cross-shore current description according to de Vriend and Stive (1987);
- Transition zone effects on the return flow according to Roelvink and Stive (1989); Short wave velocity moments based on Rienecker and Fenton's (1981) Fourier approximation of the stream function method;
- Long wave effects according to the same paper;
- Sediment transport according to Bailard (1981);
- A Fully implicit scheme for the bed evolution.

For the case of regular waves, the wave decay model is adapted simply by setting the fraction of breaking waves to 1 after the wave height exceeds a given fraction of the water depth.

NPM, Hydraulic Research.

NPM (Nearshore Profile Model) is a model for waves, longshore and cross-shore current and sediment transport on uniform beaches for obliquely or direct incoming waves. A brief summary of the physical processes represented in the NPM is given below.

- Wave transformation by refraction (by depth variations and currents), shoaling, Doppler shifting, bottom friction and wave breaking. For random waves, a Battjes and Janssen (1978) framework is used for determining the distribution of wave height and the fraction of time that waves are breaking at any point.
- Wave setup determined from the gradient of wave radiation stress.
- Driving forces for longshore wave-induced currents, determined directly from the spatial rate of wave energy dissipation.
- Longshore currents from pressure-driven tidal forces and wave-induced forces, and the interaction between the two types of current.
- Cross-shore undertow velocities using a three-layer model of the vertical distribution of cross-shore currents (de Vriend and Stive, 1987).
- Transition zone effects (the transition zone is the distance between where a wave starts to break and where turbulence becomes fully developed).

- Cross-shore and longshore sediment transport rates using an 'energetics' approach (Bailard, 1981; Stive, 1986).
- Seabed level changes due to cross-shore sediment transport using a Lax-Wendroff scheme.

Tests (Southgate, 1991) have shown that the model results are particularly sensitive to the transition zone length and to the height above the seabed at which the undertow velocity is taken for input to the sediment transport calculations. Recent improvemnets to the NPM therefore include:

- A reanalysis of transition zone data (O'Shea et. al., 1991) to give a more accurate formula for the transition zone length.
- The use of a concentration-weighted average undertow velocity in the sediment transport calculations.

NPM is described in detail in Southgate and Nairn (1993) and Nairn and Southgate (1993).

WATAN 3, University of Liverpool.

The model WATAN3 consists of a wave sub-model and a sediment sub-model. The wave sub-model (Watanabe and Dibajnia, 1988) comprises a set of two equations which are equivalent to a time-dependent version of the mild-slope equation and contain an additional term to allow for energy dissipation in the surf zone. The latter term represents the rate at which energy is dissipated by breaking and is set to zero wherever the broken waves have reformed inside the surf zone, as well as outside the surf zone. An empirical criterion (Watanabe et al, 1984) is used to determine the point of breaking, and setup and setdown are computed by solving the momentum balance equation.

The sediment sub-model (Ohnaka and Watanabe, 1990) is based on the sediment transport rate due to wave action:

$$Q = (A_W\ (\tau_B - \tau_C) + A_{WB}\ \tau_T)\ F_D\ \hat{u}_B\ /\ (\rho g)$$

where Q is the sediment transport rate, A_W and A_{WB} are coefficients, τ_B is the maximum nearbed shear stress due to wave action, τ_C is the threshold of movement shear stress. τ_T is the shear stress generated by breaker turbulence, F_D is a dimensionless directional function and $\hat{u}_B$ is the maximum nearbed orbital velocity. Having derived the transport rates throughout the computational domain, the former are then modified to allow for bed slope effects, so that:

$$Q_M = Q - \epsilon\ |Q|\ \tan\beta$$

where Q_M is the modified transport rate, ϵ is a coefficient and tan β is the local bed slope. Finally, the bed level changes are computed using the sediment mass conservation equation.

An additional feature of the present version of the sediment sub-model is the inclusion of a breaker transition length, within which turbulence generated by the post-breaking surface roller is distributed throughout the water column. A re-analysis of work carried out by Nairn et al (1990) yielded the following expression for the transition length (O'Shea et al, 1991):

$$L_T = L_B \ (0.56 \ \xi^{-1.47}) \ \tan \beta$$

where L_T is the transition length, L_B is the wave length at breaking and ξ is the Iribarren No. The effects of the transition length are incorporated into the sub-model by switching off the breaker turbulence contribution to the transport rate within the transition length.

SEDITEL, Laboratoire National d'Hydraulique.

SEDITEL computes the wave refraction and shoaling, wave height decay in the surf zone, time-averaged three-dimensional currents (2DV currents here) induced by breaking waves, sediment transport rates, bed evolution.

The wave refraction is derived from the classical Snell's law. The wave height is deduced from the equation of the flux of energy where the dissipation is supposed to be similar to the one of a hydraulic jump in the surf zone. In order to have a good estimation of the wave characteristics in shallow water, non-linear effects are considered in the calculation of the flux of energy, see Péchon (1987).

The time-averaged currents induced by breaking waves are computed with the three-dimensional model TELEMAC-3D outlined in Lepeintre et al. (1991). In order to establish the equations the instantaneous velocity is separated into three contributions: an unknown mean current, a purely periodic current corresponding to the wave motion, and turbulent fluctuations. In the time-averaged equations some closures are required to express the velocity correlations (see details in Péchon, 1992). They are given by previous works of Svendsen (1984), De Vriend and Stive (1987), Deigaard and Fredsøe (1989).

The sand transport is computed using Bailard's formula, Bailard (1981), but the suspended load efficiency factor is increased in the surf zone to take the breaking effect into account. The proposed expression is:

$$\varepsilon_s' = \left(1 + a \frac{|u_b|}{\sqrt{gD}}\right) \varepsilon_s \quad ;$$

ϵ_s : efficiency factor, non breaking
ϵ'_s: efficiency factor, breaking
a : constant
u_b : mean velocity at the bottom

The bed evolution is computed by solving the continuity equation for the sediment. To reproduce the process of avalanching of dune in the application presented here, the measured amount of sediment is distributed between the crest of the bar and the shoreline before each hydrodynamic computation. Moreover a maximum stability slope of 15/100 is specified out of the breaking zone.

The wave and current patterns are updated when the bottom evolution becomes significant. However, in order to reduce the number of iterations, an additional treatment is performed during the computation of the bottom evolution: considering the bed evolution at the breaking point, its location is moved along the coastal profile and the wave and current characteristics are moved the same way.

REPLA, SOGREAH.

REPLA is a wave-averaged, finite amplitude, current-depth shoaling and refraction model to simulate regular or random wave propagation from deepwater to the shoreline.

The formulation is detailed in Fornerino et al (1992). A set of four equations is to be solved with an iterative procedure because of the influence of the wave height on the wave celerity.

Stokes third order theory is used for small Ursell numbers. For large Ursell numbers a cnoidal second order theory is used. The model can, however, also be run with Stokes first order waves.

The wave breaking criteria derived by Weggel (1972) is used. In presence of an adverse current, this expression is modified following Sakai et al.(1988). The bore model is used to express the energy dissipation. In order to simulate wave reformation after breaking on a bar, the dissipation rate is put to zero when the wave height is less than half the local maximum wave height.

For random waves, two methods are implemented. The parametric approach of Battjes and Janssen (1978) and the individual wave method Mase and Iwagaki (1982); Mizuguchi (1982) in which each class is propagated independently with the regular wave model.

Results from the various Models

The models have been tested against experimental results obtained from the large wave flume in Hannover in 1986 and 1987, Dette and Uliczka (1986) and Dette and Oelerich (1991). The experiment from 1986 was carried out with regular waves. The 1987 experiment was run with irregular waves and included a comprehensive measuring program focusing on waves.

Regular Wave Case

The experiment with regular waves constitutes a severe test of the models due to the fact that all waves are breaking nearly at the same position. This first test gives the opportunity to tune the possible model parameters in the various models. Results in the form of calculated and measured profiles from this first test are presented in Fig. 2.

The calculated profile evolution is the integrated result of the modelling of several physical mechanisms. The interpretation of the differences between the modelled evolutions can therefore only be pointed out after comparison of each individual element in the various models. These comparisons are carried out for a similar case,but now the initial profile is a plane beach with an initial slope of 1:20. Below, the initial wave heights, current fields and sediment transport along this plane beach are considered.

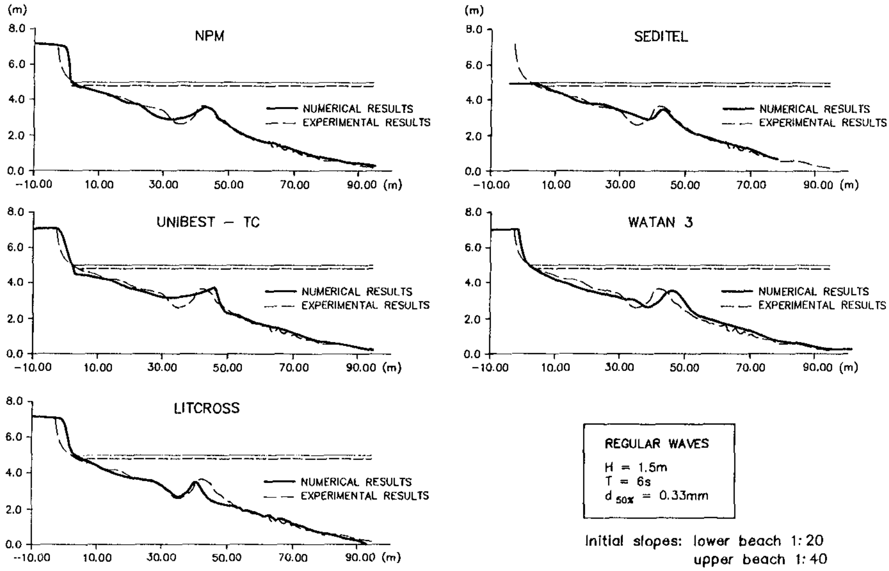

Fig. 2. Comparison of Measured and Calculated Coastal Profiles after 4.3 hours of Exposure.

The figures 3, 4, and 5 show wave heights across the plane profile, vertical distribution of horizontal wave averaged velocities, for Seditel also the vertical velocity component just shoreward of the breaker point and the vertical flow pattern, and the initial cross shore sediment transport. Already the comparison of wave heights show large spreading across the surf zone. This spreading is reflected in deviations in the assumed cross section area of the surface rollers. The differences in the vertical distribution of horizontal velocities exist due to differences in the formulation of the vertical distribution of shear stresses, eddy viscosities and the area of surface rollers.

The above comparisons illustrates differences in the models, but suffer unfortunately from lack of measured data. The calculated initial transport rates highlight the fact that the bar forms at a 'critical' position, seen from a model view point where the onshore transport under the non breaking waves turn into offshore transport inside the surf zone.

Further, although discrepancies exist between the distribution of horizontal velocities calculated by the one DV models and the 2 DV model, Seditel, the 2 DV results indicate that the order of magnitude of vertical velocities shoreward of the breaker point are comparable with the settling velocity of sand. Therefore, in a narrow zone inside the breaker point, the sediment transport models which take into account only velocities parallel to the bottom might underestimate the amount of suspended sediment.

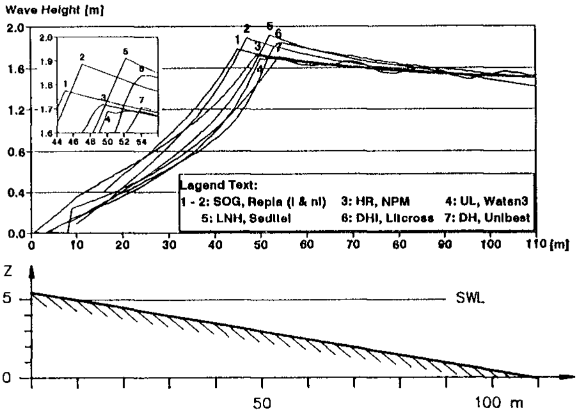

Fig. 3. Wave heights as calculated by 6 different models.

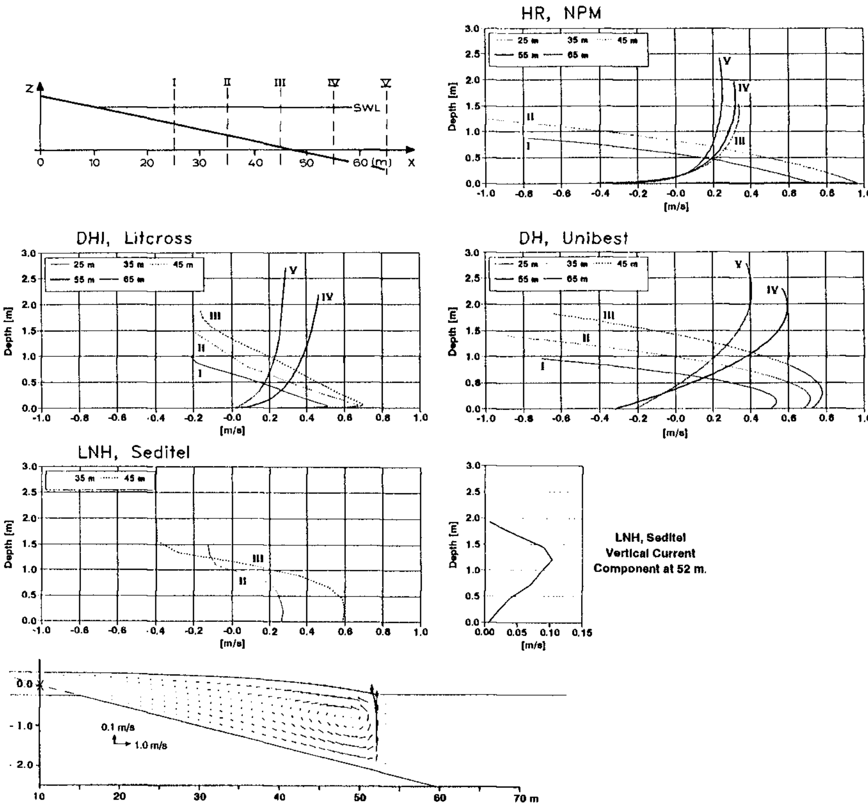

Fig. 4. Velocity components and the 2 DV current field as calculated by Seditel.

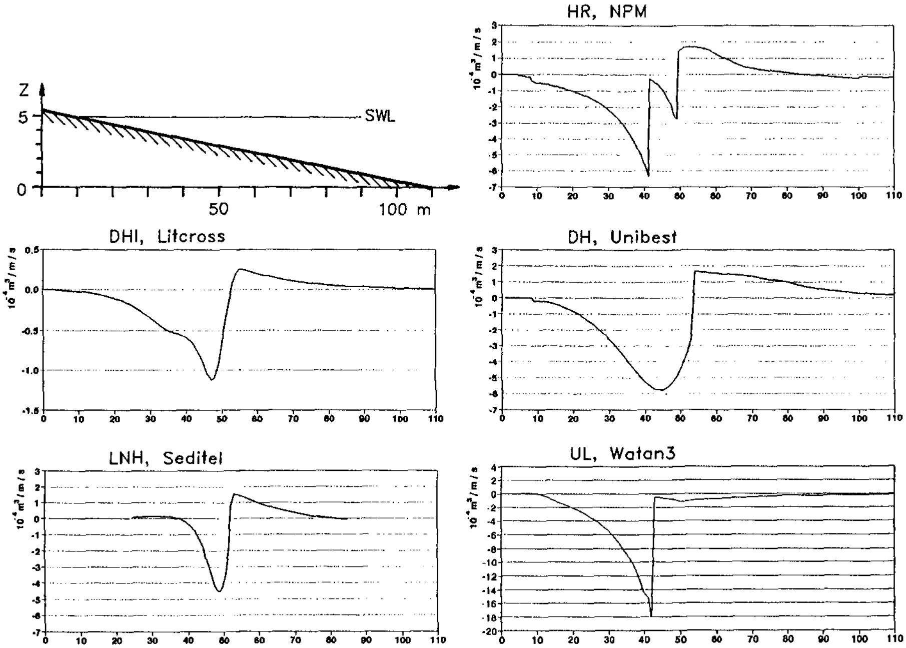

Fig. 5. Cross shore sediment transport calculated by NPM, Unibest, LITCROSS, Seditel and Watan 3.

It appears that the cross shore transport models when applied in a morphological calculation sequence in combination with more or less smoothing, either on the calculated transport or on the bed levels between updates, are able to reproduce to a certain extent the evolution of a breaker bar. In the case of regular waves it seems clear that the mechanisms just shoreward of the breaker point are essential for the bar evolution.

For irregular waves these complex mechanisms are expected to be less important for the profile evolution. In the following the models are compared with measurements in the case of irregular waves.

Irregular Waves Case

This test was carried out with irregular waves, Jonswap spectrum, with H_s = 1.5m and T_p = 6 s at 5 m depth in front of the wavemaker. The profile was built out of natural well sorted sand with a mean diameter of 0.22 mm. The test was subdivided into 45 runs of 780 s each. The wavemaker was stopped after each run. The wave generation was first order with no long waves reflection compensation. The observed bathymetry, the wave heights (H_{RMS}), the sediment transport derived from successive observations of the bed evolution and the wave energy spectrum as measured at three positions are presented in Fig. 6.

In this case a pronounced bar is not formed but the relatively steep profile is flattened out. The wave heights in the surfzone decrease concurrently with the flattening of the profile, i.e. the energy is dissipated further and further offshore. These tendencies are reflected in the cross-shore transport rates which are largest at the beginning of the test. From the spectral analysis of wave energy it appears that the spectra become double-peaked while the waves approach the beach, i.e. in the nearshore area low frequency waves become more important, see figure 6. From analysis of correlations between bound long waves and the observed low frequency waves it seems clear that the major part of the low frequency energy come from reflections in the flume.

In figures 7 and 8 measured and simulated variations of the wave heights across the profile for the observed bathymetries in runs 2, 14 and 32 are compared. These results are produced by REPLA applying both the parametric approach and the individual wave method with both linear and non linear wave theory. With the first approach it appears that when γ is adjusted corresponding to the highest wave heights, the heights inside the surf zone are underestimated for runs 2 and 14. The individual wave method is seen also to underestimate the near shore wave heights. It might be necessary to include mechanisms as the extra variations of the water level and flow due to the long waves and the opposing undertow in the wave modules.

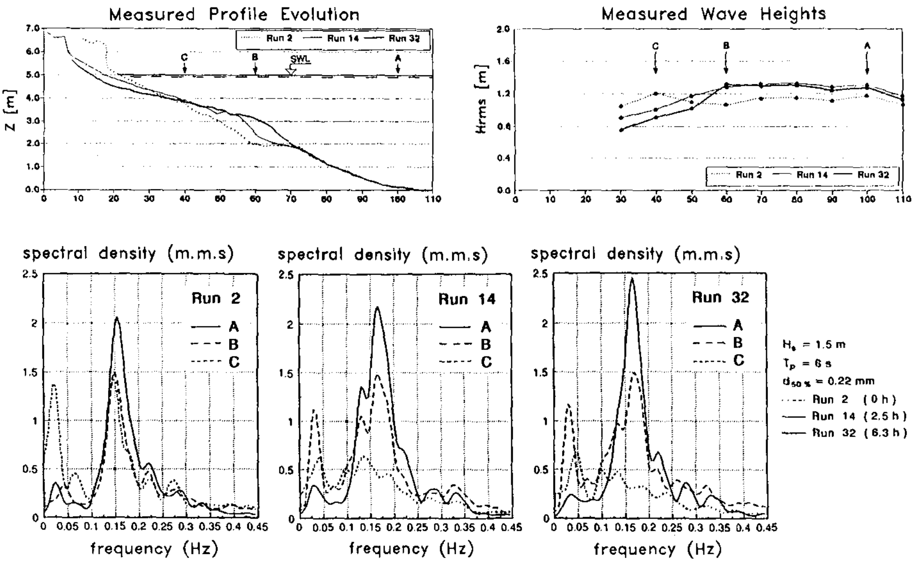

Fig. 6. Observed bathymetries, measured H_{RMS}, and wave energy spectrum at 3 positions.

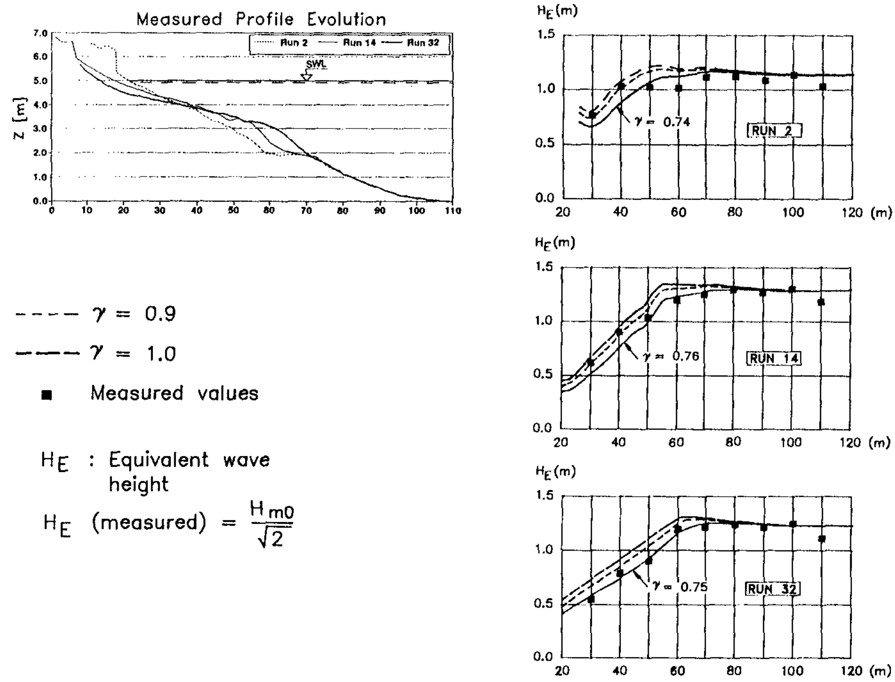

Fig. 7. Measured and calculated equivalent wave heights. Calculations by REPLA applying Battjes and Janssen's parametric approach.

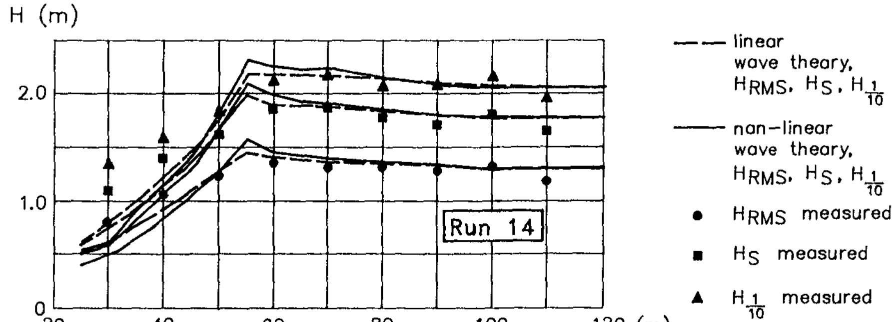

Fig. 8. Measured and calculated H_{RMS}, H_S and $H_{1/10}$. Calculations by REPLA applying the individual wave approach with linear and non linear wave theories.

Figure 9 shows the measured and calculated cross-shore transport. It is seen that the models are very sensitive to even small humps in the bed and that the observed relatively large initial offshore transport is not very well reproduced. The Watan 3 model has been run with 5 wave components only which obviously gives a very scattered transport pattern.

Figure 10 shows results of morphological modelling of the coastal profile. The modelled evolutions obviously suffer from the underestimation of the initial offshore transport capacity and the lack of description of the erosion in the steep dune front.

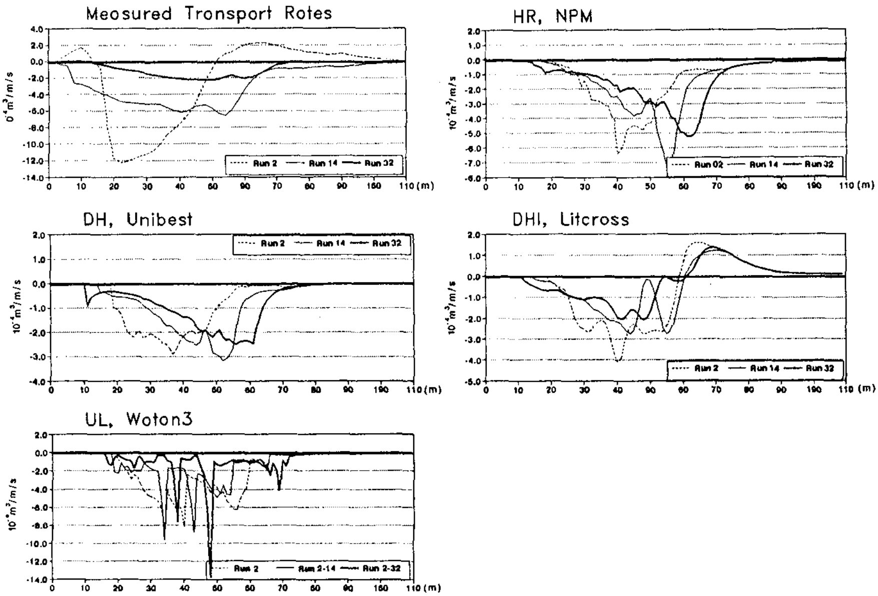

Fig. 9. Measured and calculated cross-shore transport rates. These calculations are carried out with the observed bathymetries.

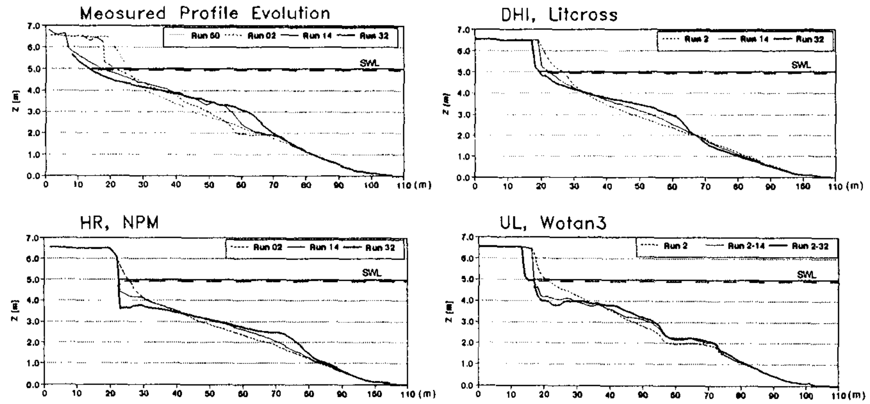

Fig. 10. Measured profile evolution. Simulated evolution. HR, NPM and DHI, LITCROSS from 'initial' to Run 32. UL, WATAN 3 from Run 2 to Run 32.

Concluding remarks

Large differences exist between the coastal profile models presented above. However, based on the experience gained through the comparisons with experimental data the following general comments are given:

- The models generally underestimate the offshore transport on relatively steep profiles. One reason seems to be related to poor wave description in cases where reflections and long waves, free and bound, exist.
- The swash zone processes and dune erosion are not described in the models. In combination with the above mentioned underestimation of transport on steep slopes the exchange of material from the dune to the bar is too slow in the case of a steep, initial profile.
- The vertical velocities close to the breaker point seem to reach a significant order of magnitude and might influence the bar formation, at least in the case of a closed flume. Generally, the velocity field in the area just before and after the (average) break point is still understood rather poorly.

It is noted that with the outlined concept of a coastal profile model the pronounced breaker bar in regular waves and the flattening of a steep profile in irregular waves can be modelled. In nature the coastal profiles are formed by 3D phenomena. It seems that the understanding of cross-shore processes has now reached a stage where it is relevant to extend the models into 3D to be able to judge where the 'weakest point' appears and where most effort in the future should be spent.

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References

Andersen, O.H. and Fredsøe, J. (1983): Transport of sediment along the coast. Prog. Rep. 59, ISVA, Techn. Univ. of Denmark, pp. 33-46.
Bailard, J.A. (1981). An Energetics Total Load Sediment Transport Model for Plane Sloping Beach, J. Geophys. Res., Vol. 86, No. CII.
Battjes, J.A. and Janssen J.P.F.M. (1978) Energy Loss and Set-up due to Breaking of Random Waves, Proc. 16th Int. Conf. on Coastal Eng, ASCE, Hamburg.
Brøker Hedegaard, I., Deigaard, R., Fredsøe, J. (1991). Onshore/Offshore Sediment Transport and Morphological Modelling of Coastal Profiles. Proc. Coastal Sediments '91, Seattle.
Buhr Hansen, J. (1991). Air entrainment in surf zone waves, 3rd Int. conf. on Coastal and Port Eng. in Dev. Countries, Mombasa, 1991.

Deigaard, R., Fredsøe, J. (1989). Shear Stress Distribution in Dissipative Water Waves. Coastal Eng. Vol. 13, pp. 357-378.
Deigaard, R., Fredsøe, J., and Hedegaard, I.B. (1986). Suspended Sediment in the Surf Zone. J. Waterway, Port, Coastal, and Ocean Eng., ASCE, Vol. 112, No. 3.
Dette, H., Uliczka, K. (1986). Seegangsserzeugte Wechselwirkung zwischen Vorland und Vorstrand sowie Küstenschutzbauwerk. Technischer Bericht Nr. 3-SBF 205/TP A6. Universität Hannover.
Dette, H. and Oelerich (1991): Internal MAST-G6M report.
De Vriend, H.J. and **Stive M.J.F.** (1987). Quasi-3D Modelling of Nearshore Currents, Coastal Engineering, Vol II, pp. 565-601.
Engelund, F. and Fredsøe, J. (1976). A Sediment Transport Model for Straight Alluvial Channels. Nordic Hydrology, Vol. 7.
Fornerino, M., P. Sauvaget and L. Hamm (1992). Numerical modelling of finite amplitude wave shoaling in presence of currents in Hydraulic Engineering Software IV, Fluid flow modelling, eds. W.R.Blain and E. Cabrera, Computational Mechanics publ., Southhampton, pp. 587-598.
Fredsøe, J. (1984). Turbulent Boundary Layer in Wave and Current Motion, J. Hydraulic Engineering, ASCE, Vol. 110, No. 8.
Lepeintre, F., Gest, B., Hervouet, J-M., Péchon, Ph., (1991), 'MITHRIDATE (former name for TELEMAC-3D): a finite element code to solve 3D free surface flow problems'. 2nd International Conference on Computer Modelling in Ocean engineering, Barcelone.
Longuet-Higgins, M.S. (1953). Mass Transport in Water Waves, Philos. Trans. R. Soc. London, Ser. A, 245.
Mase, H. and **Y. Iwagaki** (1982). Wave height distributions and wave grouping in surf zone 'Proc. Int. Conf. on Coastal eng., ASCE, 58-76.
Mizuguchi, M. (1982). Individual wave analysis of irregular wave deformation in the nearshore zone. Proc. Int. Conf. on Coatal Eng., ASCE, 485-504.
Nairn, R.B. and Southgate, H. N. (1993). "Deterministic profile modelling of nearshore processes. Part 2. Sediment transport and beach profile development", Coastal Engineering (to appear).
Nairn, R.B., Roelvink, J.A. and Southgate, H.N. (1990), Transition Zone Width and Implications for Modelling Surfzone Hydrodynamics, Proc. Twenty-Second Coastal eng. Conf., ASCE, pp 68-81.
O'Shea, K., Nicholson, J. and O'Connor, B.A. (1991). The Transition Zone Length in Cross-Shore Sediment Transport Modelling, Report CE/1/91. Department of Civil Eng., University of Liverpool.
Ohnaka, S. and Watanabe, A. (1990), 'Modeling of Wave-Current Interaction and Beach Change', Proc. Twenty-Second Coastal Engineering Conf., ASCE, pp. 2443-2456.
Péchon, P. (1987), 'Modelling of longshore currents with a non-linear wave theory'. ASCE Speciality Conference on Coastal Hydrodynamics, Delaware, 1987.
Péchon, P., (1992), Numerical modelling of wave-driven currents and sediment transport in the surfzone. Internal report EDF-LNH HF.42/92.15.
Rienecker, M.M. and J.D. Fenton (1981). A Fourier approximation method for steady water waves. J. Fluid Mech., Vol. 104, pp. 119-137.
Roelvink, J.A. and Stive M.J.F. (1989). Bar-generating Cross-shore Flow Mechanisms on a Beach, JGR Vol. 94, No. C4, pp. 4785-4800.
Sakai, S., K. Hirayama and H. Saeki (1988). A new parameter for wave breaking with opposing current on sloping bed. Proc. Int. Conf. on Coastal Eng., ASCE, 1035-1044.
Southgate, H.N. and Nairn, R. B. (1993). "Deterministic profile modelling of nearshore processes. Part 1. Waves and Currents", Coastal Engineering (to appear).
Southgate, H.N. (1991) Beach Profile Modelling: Flume Data Comparisons and Sensitivity Tests, coastal Sediments '91, Seattle, USA.
Stive, M.J.F. and Battjes, J.A. (1984). 'A Model for Offshore Sediment Transport'. Proc. 19th Int. Conf. Coastal Eng., pp. 1420-1436, ASCE, New York.
Stive, M.J.F. (1986). A Model for Cross-shore Sediment Transport, Proc. 20th ICCE, ASCE, Taiwan.
Svendsen, L.A. (1984). Mass Flux and Undertow in a Surf Zone. Coastal Engineering, Vol.8, No.4.
Watanabe, A., Hara, T. And Horikawa, K. (1984). Study on Breaking Condition for Compound Wave Trains, Coastal Entineering in Japan, vol. 27, pp. 71-82.
Watanabe, A. and Dibajnia, M. (1988). A Numerical Model of Wave Deformation in Surf zone, Proc. Twenty-First Coastal Engineering Conf., ASCE, pp. 578-587.
Weggel, J.R. (1972) 'Maximum breaker height for design', Proc. Int. Conf. on Coastal Eng., ASCE, 419-432.