

A. Nepomnyashchy I. Simanovskii  
J.C. Legros

# Interfacial Convection in Multilayer Systems

With 187 Illustrations

 Springer

# Contents

<b>Preface</b> .....	v
<b>1 Introduction</b> .....	1
1.1 Motivation of the Problem .....	1
1.2 Mathematical Models .....	2
1.2.1 Sharp-Interface Approach .....	2
1.2.2 One-Layer Model .....	4
1.2.3 Two-Layer Model .....	10
1.2.4 Three-layer Model .....	14
<b>2 Types of Convective Instability in Systems with an Interface</b> .....	16
2.1 The Problem of Stability .....	16
2.2 Rayleigh-Bénard Convection .....	18
2.2.1 Linear Stability .....	18
2.2.2 Nonlinear Flow Regimes .....	23
2.3 Anticonvection .....	33
2.4 Stationary Marangoni Patterns .....	36
2.4.1 Exact Formulas .....	36
2.4.2 Short-Wave Marangoni Patterns .....	37
2.4.3 Long-Wave Marangoni Patterns; The Case of Poorly Conducting Boundaries .....	41
2.4.4 Long-Wave Deformational Instability .....	42
2.5 Marangoni Waves in Systems with a Nondeformable Interface .....	45
2.5.1 Oscillatory Marangoni Instability .....	45
2.5.2 Competition between Marangoni and Rayleigh Instability Mechanisms .....	52
2.5.3 Mode Mixing of Interfacial and Internal Waves .....	71
2.5.4 Oscillatory Instability in the Presence of a Thermal Gradient and a Surfactant .....	74
2.6 Marangoni Waves in Systems with a Deformable Interface .....	77
2.6.1 The Transverse Marangoni Instability in One-Layer Systems ..	78

2.6.2	The Limit of Large $Ga$ and $M$ .....	80
2.6.3	Linear Theory of Transverse Instability: Numerical Results .....	81
2.6.4	Nonlinear Theory of Transverse Instability .....	84
2.6.5	Oscillations Generated by a Surfactant .....	90
2.6.6	Transverse and Longitudinal Marangoni Instabilities in the Case of the Mass Transfer .....	96
<b>3</b>	<b>Bénard Problem in Multilayer Systems with Undeformable Interfaces</b> .....	<b>100</b>
3.1	General Equations and Boundary Conditions .....	100
3.2	Linear Stability Theory .....	102
3.2.1	Marangoni Convection. The Case of a Symmetric System and Equal Layer Thicknesses .....	103
3.2.2	Onset of Marangoni Convection in Nonsymmetric Three-Layer Systems .....	108
3.2.3	Combined Action of Marangoni and Rayleigh Instability Mechanisms .....	111
3.3	Nonlinear Simulations .....	117
3.3.1	Marangoni Convection. The Case of a Symmetric System .....	119
3.3.2	Marangoni Convection. The Case of a Nonsymmetric System ..	129
3.3.3	Rayleigh Convection .....	141
3.3.4	Mixed Rayleigh-Marangoni Convection .....	148
3.3.5	Anticonvection .....	150
3.4	Space Experiments .....	152
3.4.1	Experiment Description .....	152
3.4.2	Experiment I .....	155
3.4.3	Experiment II .....	160
<b>4</b>	<b>Bénard Problem in Multilayer Systems with Deformable Interfaces</b> .....	<b>164</b>
4.1	Formulation of the Problem .....	164
4.2	Linear Stability Analysis .....	165
4.2.1	Long-Wave Asymptotics .....	167
4.2.2	Neutral Stability Curves .....	171
4.3	Nonlinear Theory .....	175
4.3.1	Derivation of the Amplitude Equations .....	175
4.3.2	Traveling Wave Solutions .....	183
4.3.3	Results of Numerical Simulations .....	189
<b>5</b>	<b>Stability of Flows</b> .....	<b>194</b>
5.1	Mechanisms of Instabilities .....	194
5.1.1	Purely Thermocapillary Flows .....	194
5.1.2	Flows Under Combined Action of Thermocapillarity and Buoyancy .....	196
5.2	Thermocapillary Flows in Two-Layer Systems .....	197

5.2.1 Basic Equations and Boundary Conditions . . . . .	197
5.2.2 Stationary Flow Profiles . . . . .	199
5.2.3 Linear Stability Theory . . . . .	201
5.2.4 Nonlinear Patterns . . . . .	208
5.3 Buoyancy-thermocapillary Convection in Two-layer Systems . . . . .	214
5.4 Buoyancy-thermocapillary Convection in Three-layer Systems . . . . .	222
5.4.1 Formulation of the Problem . . . . .	222
5.4.2 Results of Numerical Simulations . . . . .	225
5.5 Deformational Instabilities of Thermocapillary Flows in Three-Layer Systems . . . . .	229
5.5.1 Formulation of the Model . . . . .	232
5.5.2 Derivation of the Interface Evolution Equations . . . . .	235
5.5.3 Linear Stability Analysis . . . . .	236
5.5.4 Weakly Nonlinear Model . . . . .	240
<b>6 Flows in Ultra-Thin Films . . . . .</b>	<b>242</b>
6.1 Lubrication Approximation . . . . .	242
6.2 Intermolecular Forces . . . . .	243
6.3 Generalized Cahn-Hilliard Equation . . . . .	246
6.3.1 Model Formulation . . . . .	246
6.3.2 Film Rupture . . . . .	246
6.3.3 Film on an Inclined Plane . . . . .	248
6.3.4 Two-Layer Films . . . . .	248
6.3.5 Evaporating Films . . . . .	251
6.4 Diffuse-Interface Models . . . . .	252
<b>7 Outlook . . . . .</b>	<b>255</b>
7.1 Extension of the Linear Stability Theory. Influence of Lateral Boundaries . . . . .	255
7.2 Three-Dimensional Convective Flows . . . . .	257
7.3 Deformation of the Interface . . . . .	258
7.4 Transition to Chaos and Interfacial Turbulence . . . . .	259
7.5 Multicomponent Fluids . . . . .	260
7.6 Chemical Reactions . . . . .	262
7.7 Porous Layers . . . . .	263
7.8 Contact Line Dynamics . . . . .	265
7.9 Feedback Control of Interfacial Instabilities . . . . .	266
7.10 Biological Surface-Tension-Driven Flows . . . . .	270
<b>References . . . . .</b>	<b>273</b>
<b>Index . . . . .</b>	<b>303</b>