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Interfacial Stability with Mass and Heat Transfer

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Abstract

A simplified formulation is presented to deal with interfacial stability problems with mass and heat transfer. For Rayleigh-Taylor stability problems of a liquid-vapor system, it is found that the effect of mass and heat transfer tends to enhance the stability of the system when the vapor is hotter than the liquid, although the classical stability criterion is still valid. For Kelvin-Helmholtz stability problems, however, the classical stability criterion is found to be modified substantially due to the effect of mass and heat transfer.

Interfacial Stability with Mass and Heat Transfer

I. Introduction

In dealing with flow of two fluids divided by an interface, the problem of the interfacial stability is usually studied on the assumption that the fluids are immiscible. Thus there is no mass transfer across the interface. Thermal effects often play only a secondary role. Therefore the effect of heat transfer is also usually neglected. The classical Rayleigh-Taylor stability [1,2] and the Kelvin-Helmholtz stability [3,4,5] as well as other variation of the problem such as the stability of bubble motion in a liquid [6], all belong to this category. However there are situations when the effect of mass and heat transfer across the interface plays an essential role to determine the flow field. For instance, when the fluid is boiling, whether film boiling or pool boiling, the motion of the film and the bubbles depends principally on the effect of mass and heat transfer.

In a previous paper [7], we have formulated the general problem of interfacial fluid flow with mass and heat transfer and applied to the specific problem of Rayleigh-Taylor instability in connection with the problem of film boiling heat transfer. Although explicit dispersion relation was found for the linear problem, the expression is very complicated and is difficult to grasp its essential feature. Moreover, for the specific problem of boiling heat transfer, it is evident from the linear analysis that the study of the nonlinear problem is required in order to really understand its physical mechanism. Therefore it is desirable that a simplified version of the problem, which incorporates the essential effects of mass and heat transfer, can be established and explored first. A more comprehensive study can follow after we have learned enough from the simplified problem.

In the following, we present first a simplified formulation of the problem of the interfacial flow with mass and heat transfer based on a careful investigation of the results of the previous analysis of the more comprehensive formulation. Then the problems of Rayleigh-Taylor stability and Kelvin-Helmholtz stability are studied. The linear dispersion relations are obtained and discussed. The study of the nonlinear stability problem will be presented in a subsequent paper.

II. Formulation of the Problem.

We are essentially concerned with the motion of a fluid with two coexisting phases. Let

$$S(\underset{\sim}{x},t) = 0, \tag{1}$$

represent the interface between these two phases. The interface divides the region into two parts. Each part is occupied by a homogeneous fluid. Within each region, the flow fields are governed by the usual continuity, momentum and energy equation with their respective material parameters. Since the mass is allowed to be transferred across the interface, the usual immiscibility conditions no longer hold. The interfacial conditions, as derived from conservation of mass and momenta[7], become, on $S(\underset{\sim}{x},t) = 0$:

$$\rho^{(1)}\left(\frac{\partial S}{\partial t} + v_i^{(1)}\frac{\partial S}{\partial x_i}\right) = \rho^{(2)}\left(\frac{\partial S}{\partial t} + v_i^{(2)}\frac{\partial S}{\partial x_i}\right), \tag{2}$$

and

$$\rho^{(1)}v_j^{(1)}\left(\frac{\partial S}{\partial t} + v_i^{(1)}\frac{\partial S}{\partial x_i}\right) - \tau_{ij}^{(1)}\frac{\partial S}{\partial x_i}$$

$$= \rho^{(2)}v_j^{(2)}\left(\frac{\partial S}{\partial t} + v_i^{(2)}\frac{\partial S}{\partial x_i}\right) - \tau_{ij}^{(2)}\frac{\partial S}{\partial x_i} - \sigma\left(\frac{1}{R_1} + \frac{1}{R_2}\right)\frac{\partial S}{\partial x_j},$$

$$j = 1,2,3, \tag{3}$$

where ρ is the density of the fluid; v_i, the i^{th} component of the fluid velocity; τ_{ij}, the stress component; σ, the surface tension coefficient; and R_1 and R_2, the two principal radii of the curvature at the point of interest on $S = 0$. The radius of curvature is taken to be positive if the center of curvature lies on the side of fluid (2), and negative if otherwise. The superscripts (1) and (2) designate the fluids in region (1) and (2).

Realistically, since the transfer of mass across the interface represents a transformation of the fluid from one phase to another, there is invariably a latent heat associated with the phase change. It is essentially through this interfacial coupling between the mass transfer and the release of latent heat that the motion of fluids is influenced by the thermal effects. Therefore when there is significant mass transfer across the interface, the transfer of heat in the fluid has to be taken into consideration.

Based on a careful investigation of the results from the previous more comprehensive analysis [7], it is reasonable to expect that the amount of the released latent heat depends mainly on the instantaneous position of the interface. More specifically, let us express theinterface by

$$S(\underset{\sim}{x},t) = y - \zeta(x,z,t), \tag{4}$$

where $y = 0$ represents the equilibrium interface. We propose that the interfacial condition for energy transfer can be expressed as

$$L\rho^{(1)}\left(\frac{\partial S}{\partial t} + \underset{\sim}{v}^{(1)}\cdot\nabla S\right) = F(\zeta), \tag{5}$$

where L is the latent heat released when the fluid is transformed from phase (1) to phase (2). The expression $F(\zeta)$ represents essentially the net heat flux from the interface when such phase transformation is taking place. .In general [7], the heat fluxes have to be determined from equations governing the heat transfer in the fluids, thus coupling completely the dynamics and the thermal exchanges in the entire flow region. In this simplified version, the assumption is that F is simply a function of ζ, and moreover F is to be determined from the heat exchange relations in the equilibrium state.

Let us consider a specific equilibrium state. Take two fluids confined between two paralle planes $y = -h_1$ and $y = h_2$. Let the equilibrium interface be located at $y = 0$. (See Figure 1). Fluid (1) occupies the region $-h_1 < y < 0$, while the fluid (2) occupies the region $0 < y < h_2$. Let the temperatures at $y = -h_1$ $y = h_2$ and $y = 0$ be T_1, T_2 and T_0 respectively. The heat flux in the +y direction in regions (1) and (2) are $\frac{K^{(1)}(T_1-T_0)}{h_1}$ and $\frac{K^{(2)}(T_1+T_0)}{h_2}$ respectively. Let us denote

$$F(y) = \frac{K^{(2)}(T_0-T_2)}{h_2 - y} - \frac{K^{(1)}(T_1-T_0)}{h_1 + y} \quad . \tag{6}$$

It is clear that F(0) represents the net heat flux from the inerface into the fluid regions. Since it is an equilibrium state, we have

$$F(0) = 0. \tag{7}$$

We now propose that when the interface is perturbed to become $y = \zeta$, the function F in the equa'ion (5) is given by (6). When there is intense heat exchange and substantial mass transfer, this quasi-equilibrium assumption should be a good approximation. With this simplification, then the dynamical equations are decoupled from the heat equations.

III. The Rayleigh-Taylor Instability.

Consider two incompressible, inviscid fluids confined between two parallel planes $y = -h_1$ and $y = h_2$. Let the interface be given by

$$S = y - \zeta(x,z,t) = 0. \tag{8}$$

Fluid (1) occupies the region $-h_1 < y < \zeta$, while the fluid (2) occupies the region $\zeta < y < h_2$. Assume the flows of the fluids are irrotational, and let the velocity potentials be $\phi^{(1)}$ and $\phi^{(2)}$ respectively. Thus, we have in each fluid region:

$$\nabla^2\phi^{(\alpha)} = 0, \quad \alpha = 1,2, \tag{9}$$

and

$$\frac{p^{(\alpha)}}{\rho^{(\alpha)}} + \frac{1}{2}(\nabla\phi^{(\alpha)})^2 + \Omega + \frac{\partial\phi^{(\alpha)}}{\partial t} = f^{(\alpha)}, \quad \alpha = 1,2. \tag{10}$$

where $\Omega = gy$ is the external gravitational potential, and $f^{(\alpha)}$ are constants.

The interfacial condition (2), (3) and (5) now becomes

$$\rho^{(1)}[\frac{\partial S}{\partial t} + (\nabla\phi^{(1)})\cdot(\nabla S)] = \rho^{(2)}[\frac{\partial S}{\partial t} + (\nabla\phi^{(2)})\cdot(\nabla S)], \tag{11}$$

$$\begin{aligned}\rho^{(1)}[(\nabla\phi^{(1)})\cdot(\nabla S)][\frac{\partial S}{\partial t} + (\nabla\phi^{(1)})\cdot(\nabla S)] &= \\ = \rho^{(2)}[(\nabla\phi^{(2)})\cdot(\nabla S)][\frac{\partial S}{\partial t} &+ (\nabla\phi^{(2)})\cdot(\nabla S)] \\ + [p^{(2)} - p^{(1)} - \sigma(\frac{1}{R_1} &+ \frac{1}{R_2})]|\nabla S|^2,\end{aligned} \tag{12}$$

and

$$L\rho^{(1)}[\frac{\partial S}{\partial t} + (\nabla\phi^{(1)})(\nabla S)] = F(\zeta). \tag{13}$$

In the equilibrium state we can set $\phi^{(1)} = \phi^{(2)} = 0$, $f^{(1)} = f^{(2)} = 0$, $p^{(1)} = -\rho^{(1)}gy$, $p^{(2)} = -\rho^{(2)}gy$, and the interface is specified to be $y = 0$. The equilibrium temperature distribution is the same as that discussed in the last section.

Now let us perturb the interface from $y = 0$ to $y = \zeta e^{i(kx-\omega t)}$ For small ζ, we see that in order to satisfy the boundary conditions that the normal velocities vanish at $y = -h_1$ and $y = h_2$, the perturbed velocity potentials are given by

$$\phi^{(1)} = A_1 \cosh k(y+h_1), \tag{14}$$

and

$$\phi^{(2)} = A_2 \cosh k(y-h_2), \tag{15}$$

where the factor $e^{i(kx-\omega t)}$ is suppressed in writing for simplicity as with the subsequent expressions.

Neglecting the nonlinear terms, we obtain from (10) the expression of the pressures on $S = 0$:

$$p^{(1)} = -\rho^{(1)} \frac{\partial\phi^{(1)}}{\partial t} - \rho^{(1)}g\zeta = i\omega\rho^{(1)} A_1 \cosh kh_1 - \rho^{(1)}g\zeta, \tag{16}$$

and

$$p^{(2)} = i\omega\rho^{(2)} A_2 \cosh kh_2 - \rho^{(2)}g\zeta. \tag{17}$$

The linearized interfacial conditions (11) and (12) then lead to

$$\rho^{(1)}(A_1 k \sinh kh_1 + i\omega\zeta) = \rho^{(2)}(i\omega\zeta - A_2 k \sinh kh_2), \tag{18}$$

and

$$\rho^{(1)}[i\omega A_1 \cosh kh_1 - g\zeta] = \rho^{(2)}[i\omega A_2 \cosh kh_2 - g\zeta] + \sigma k^2\zeta, \tag{19}$$

since

$$\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \left(\frac{\partial^2\eta}{\partial x^2} + \frac{\partial^2\eta}{\partial z^2}\right) \quad \text{when } \eta \text{ is small.}$$

Now we can expand $F(\zeta)$ about $\zeta = 0$ by

$$F(\zeta) = F'(0)\zeta + \frac{F''(0)}{2}\zeta^2 + \dots ,$$

since $F(0) = 0$. From (6) we obtain

$$F'(0) = G\left(\frac{1}{h_2} + \frac{1}{h_1}\right), \tag{20}$$

where $G = \dfrac{K^{(2)}(T_0-T_2)}{h_2} = \dfrac{K^{(1)}(T_1-T_0)}{h_1}$ is the equilibrium heat flux from the plane $y = -h_1$ to $y = h_2$. Thus the linearized interfacial condition (13) becomes

$$\rho^{(1)}(A_1 k \sinh kh_1 + i\omega\zeta) = \alpha\zeta, \tag{21}$$

where

$$\alpha = \frac{G}{L}\left(\frac{1}{h_2} + \frac{1}{h_1}\right). \tag{22}$$

Although the above formulation can be applied quite generally, the physical system we keep in mind in the background is the liquid-vapor system.

Now the vapor phase is usually hotter then the liquid phase, therefore α is always positive. Because if the fluid (2) is liquid and the fluid (1) is vapor, then L is positive and G is positive since $T_1 > T_0 > T_2$. If the fluid (1) is liquid and the fluid (2) is vapor then L and G both are negative.

Eliminating A_1, A_2 and ζ from (18), (19) and (21), we obtain the dispersion relation:

$$\omega^2[\rho^{(1)} \cosh kh_1 \sinh kh_2 + \rho^{(2)} \cosh kh_2 \sinh kh_1]$$
$$+ i\alpha\omega \sinh k(h_1+h_2) + [gk(\rho^{(2)}-\rho^{(1)}) - \sigma k^3]\sinh kh_1 \sinh kh_2 = 0. \tag{23}$$

When $\alpha = 0$, we recover the classical Rayleigh-Taylor dispersion relation. Let us denote

$$a = \alpha \sinh k(h_1+h_2) \Big/ 2(\rho^{(1)} \cosh kh_1 \sinh kh_2 + \rho^{(2)} \cosh kh_2 \sinh kh_1) ,$$

$$b = \frac{[gk(\rho^{(2)}-\rho^{(1)}) - \sigma k^3] \sinh kh_1 \sinh kh_2}{(\rho^{(1)} \cosh kh_1 \sinh kh_2 + \rho^{(2)} \cosh kh_2 \sinh kh_1)} .$$

Thus (23) can be rewritten as

$$\omega = -ia \pm [-a^2-b]^{1/2}$$

Recalling that a factor of $e^{i(kx-\omega t)}$ is attached to each perturbed quantity, thus when $b > 0$, the system is unstable, since one root of ω is positive imaginary. However, since $a > 0$, the growth rate of the instability is reduced from that of the classical case when $a = 0$. When $b < 0$, the system is stable. But in contrast to the classical case, there is no permanent periodi-wave state, and the system will settle down to an asymptotic equilibrium because of the evaporation effect.

In most physical situations, kh_2 is very large, then the expressions of a and b are simplified to

$$a = \frac{\alpha}{2}(\sinh kh_1 + \cosh kh_1) \Big/ (\rho^{(1)}\cosh kh_1 + \rho^{(2)}\sinh kh_1).$$

$$b = [gk(\rho^{(2)}-\rho^{(1)}) - \sigma k^3]\sinh kh_1 \Big/ (\rho^{(1)}\cosh kh_1 + \rho^{(2)}\sinh kh_1).$$

In many cases, the ratio $\rho^{(2)}/\rho^{(1)}$ is also very large, then the expressions of a and b can be further simplified to

$$a = \alpha(1 + \coth kh_1)/2\rho^{(2)},$$

$$b = gk[1 - \frac{\sigma k^2}{g\rho^{(2)}}].$$

Although a direct comparison between the present result and the results from the previous more general formulation is not easy to make. It may be seen that the present result agrees with some of the limiting cases from the general result as discussed in the previous works [7,8]. It is noteworthy that the effects of mass and transfer are revealed through a single parameter α in this simplified version. It would be interesting to see whether experimental data can indeed be correlated by this parameter.

IV. The Kelvin-Helmholtz Instability.

Let us now consider the case that the primary flow state is given by two uniform streams moving with uniform velocities U_1 and U_2 in the x-direction, confined again in the same region as in Section III. Then in this primary flow state we have for $\alpha = 1,2$:

$$\phi^{(\alpha)} = U_\alpha x,$$

$$p^{(\alpha)} = -\rho^{(\alpha)} gy,$$

and

$$f^{(\alpha)} = \frac{\rho^{(\alpha)}}{2} U^2 .$$

The interface is again $y = 0$, and the temperature distribution the same as that discussed in Section II.

After a small perturbation of the interface $S = 0$ from $y = 0$ to $y = \zeta e^{i(kx-\omega t)}$, then the velocity potentials become

$$\phi^{(1)} = U_1 x + A_1 \cosh k(y+h_1)e^{i(kx-\omega t)}, \tag{24}$$

and

$$\phi^{(2)} = U_2 x + A_2 \cosh k(y-h_2)e^{i(kx-\omega t)}. \tag{25}$$

Neglecting the nonlinear terms in A_1, A_2 and ζ, and suppressing the expression $e^{i(kx-\omega t)}$, we obtain from (10) that the pressures on $S = 0$ are given by

$$p^{(1)} = i(\omega-kU_1)\rho^{(1)} A_1 \cosh kh_1 - \rho^{(1)} g\zeta, \tag{26}$$

and

$$p^{(2)} = i(\omega-kU_2)\rho^{(2)} A_2 \cosh kh_2 - \rho^{(2)} y\zeta . \tag{27}$$

Substituting (24) - (27) into (11) - (13), and neglecting the non-linear terms, we obtain

$$\rho^{(1)}[A_1 k \sinh kh_1 + i(\omega-kU_1)\zeta] = \rho^{(2)}[i(\omega-kU_2)\zeta - A_2 k \sinh kh_2], \tag{28}$$

$$\rho^{(1)}[i(\omega-kU_1)A_1 \cosh kh_1 - g\zeta] = \rho^{(2)}[i(\omega-kU_2)A_2 \cosh kh_2 - g\zeta] + \sigma k^2\zeta, \quad (29)$$

and

$$\rho^{(1)}[A_1 k \sinh kh_1 + i(\omega-kU_1)\zeta] = \alpha\zeta. \quad (30)$$

Eliminating A_1, A_2 and ζ from the equations (28) - (30), we obtain the following dispersion relation:

$$\omega^2(\nu^{(1)}+\nu^{(2)}) - 2\omega k(\nu^{(1)}U_1+\nu^{(2)}U_2) + k^2(\nu^{(1)}U_1^2+\nu^{(2)}U_2^2)$$
$$-[gk(\rho^{(1)}-\rho^{(2)}) + \sigma k^3]\sinh kh_1 \sinh kh_2 + i\alpha[(\frac{\nu^{(1)}}{\rho^{(1)}} + \frac{\nu^{(2)}}{\rho^{(2)}})\omega$$
$$- k(\frac{\nu^{(1)}U_1}{\rho^{(1)}} + \frac{\nu^{(2)}U_2}{\rho^{(2)}})] = 0, \quad (31)$$

where

$$\nu^{(1)} = \rho^{(1)} \cosh kh_1 \sinh kh_2,$$

and

$$\nu^{(2)} = \rho^{(2)} \sinh kh_1 \cosh kh_2.$$

Equation (31) can be solved to give

$$\omega = \frac{1}{(\nu^{(1)}+\nu^{(2)})}[k(\nu^{(1)}U_1 + \nu^{(2)}U_2) - \frac{i\alpha}{2}(\frac{\nu^{(1)}}{\rho^{(1)}} + \frac{\nu^{(2)}}{\rho^{(2)}} \pm \sqrt{W}], \quad (32)$$

where

$$W = -k^2\nu^{(1)}\nu^{(2)}(U_1-U_2)^2 - \frac{\alpha^2}{4}(\frac{\nu^{(1)}}{\rho^{(1)}} + \frac{\nu^{(2)}}{\rho^{(2)}})^2$$
$$+ (\nu^{(1)}+\nu^{(2)})[gk(\rho^{(1)}-\rho^{(2)}) + \sigma k^3]\sinh kh_1 \sinh kh_2$$
$$+ i\alpha k\nu^{(1)}\nu^{(2)}(U_1-U_2)(\frac{1}{\rho^{(1)}} - \frac{1}{\rho^{(2)}}). \quad (33)$$

It is clear that when $\alpha = 0$, the dispersion relation is reduced to that of the classical Kelvin-Helmholtz problem. When $U_1 = U_2 = U$, the expression of ω is the same as that of the last section, except for an additive term kU to take care of the streaming of the fluid.

Thus when $|U_1-U_2|$ is small, the behavior of the flow system differs little from the Rayleigh-Taylor case.

The stability criterion may be determined by the condition that ω is real. Thus we obtain from either (32) or (33), the following critical condition:

$$J \equiv (\nu^{(1)}+\nu^{(2)})[gk(\rho^{(1)}-\rho^{(2)}) + \sigma k^3]\sinh kh_1 \sinh kh_2$$
$$- k^2(U_1-U_2)^2\nu^{(1)}\nu^{(2)}[1 + (\frac{\rho^{(2)}-\rho^{(1)}}{\rho^{(2)}\nu^{(1)}+\rho^{(1)}\nu^{(2)}})^2\nu^{(1)}\nu^{(2)}] = 0. \tag{34}$$

The system is stable if $J \geq 0$, and unstable if $J < 0$. The expression of J differs from that of the classical Kelvin-Helmholtz problem by the additional last term. It is somewhat surprising that the parameter α does not appear in the expression. Thus this expression is valid even for infinitesimal α, and yet when $\alpha = 0$, the last term is absent. There is no anomaly, however, if we look at the growth rate of the instability. When α is infinitesimally small, the additional effect on the growth rate of the instability is also infinitesimally small.

The additional last term in the expression J is always negative. Thus it is always a destabilizing term. It is not a small term unless the density difference or the velocity difference is small. When $\rho^{(2)} \gg \rho^{(1)}$ (or $\rho^{(1)} \gg \rho^{(2)}$), the term in the bracket is of the order of $\frac{\rho^{(2)}}{\rho^{(1)}}$ (or $\frac{\rho^{(1)}}{\rho^{(2)}}$). Therefore the modification on Kelvin-Helmholtz stability can be very large for such two fluid systems as water-vapor or water-air.

We can also obtain from (34) the critical value of U_1-U_2 such that the system is stable for all k if $|U_1-U_2|$ does not exceed this value. In order to bring out the salient feature without involving two much computation, let us take both h_1 and h_2 to be infinitely large. Then we have

$$J = (\rho^{(1)}+\rho^{(2)})[gk(\rho^{(1)}-\rho^{(2)}) + \sigma k]$$
$$- k^2(U_1-U_2)^2\rho^{(1)}\rho^{(2)}[1 + \frac{(\rho^{(1)}-\rho^{(2)})^2}{4\rho^{(1)}\rho^{(2)}}].$$

When $\rho^{(1)}>\rho^{(2)}$, the system is thus stable for all k, if

$$(U_1-U_2)^2 < \frac{8}{\rho^{(1)}+\rho^{(2)}}[\sigma g(\rho^{(1)}-\rho^{(2)})]^{1/2}. \qquad (35)$$

In contrast, the classical Kelvin-Helmholtz criterion is [5]

$$(U_1-U_2)^2 < \frac{2(\rho^{(1)}+\rho^{(2)})}{\rho^{(1)}\rho^{(2)}}[\sigma g(\rho^{(1)} - \rho^{(2)})]^{1/2}. \qquad (36)$$

When $\rho^{(1)} >> \rho^{(2)}$, the right hand sides of (35) and (36) differ by a factor of $4\frac{\rho^{(2)}}{\rho^{(1)}}$.

For the case of the air-water system, the classical result (36) yields

$$|U_1-U_2| < 65^{\circ} \text{ cm/sec}, \quad \text{(or 23 km/hr)}$$

while the relation (35) yields

$$|U_1-U_2| < 47 \text{ cm/sec}. \quad \text{(or 1.7 km/hr)}$$

As we apply the new result to the phenomenon of the surface waves generated by the wind on the ocean, we may make the following interpretation. When the wind speed is below 1.7 km/hr, the sea would be calm. Waves begin to appear when the wind speed exceeds 1.7 km/hr. As α is indeed extremely small for this case, the growth rate for the instability is so small that the amplitude of the wave remains very small. The growth rate becomes significant only when the wind speed exceeds the classical value, i.e. 23 km/hr. Then white caps start to appear on the ocean surface as observed[5].

V. Summary.

To sum up, we have presented a simplified formulation for the interfacial stability problems with mass and heat transfer. For the case of Rayleigh-Taylor stability problem, it is found that the simplified version retains the essential feature that although the effect of mass and heat transfer tends to reduce the growth rate of the instability, the criterion for stability is still the same as the classical result. For this formulation, the effects of mass and heat transfer are revealed through one single parater α. Thus correlation of experimental data would be greatly facilitated by this simplification. For such physical problems as film boiling, nonlinear effects are essential. Then the simplified formulation is even more valuable for the difficult analysis of the nonlinear stability. The study of nonlinear Rayleigh-Taylor stability with mass and heat transfer will be reported tin a subsequent paper.

For the case of Kelvin-Helmholtz stability problem, a remarkable result is that the classical stability criterion is substantially modified when the effect of mass and heat transfer is taken into consideration, and the modification is independent of the parameter α. The result is less surprising from the perspective of the growth rate of the instability. The growth rate of the instability is indeed small if α is small, when the system is classically stable. Experimental verification and a detailed analysis from a more comprehensive formulation are both desirable for fuller understanding of the problem.

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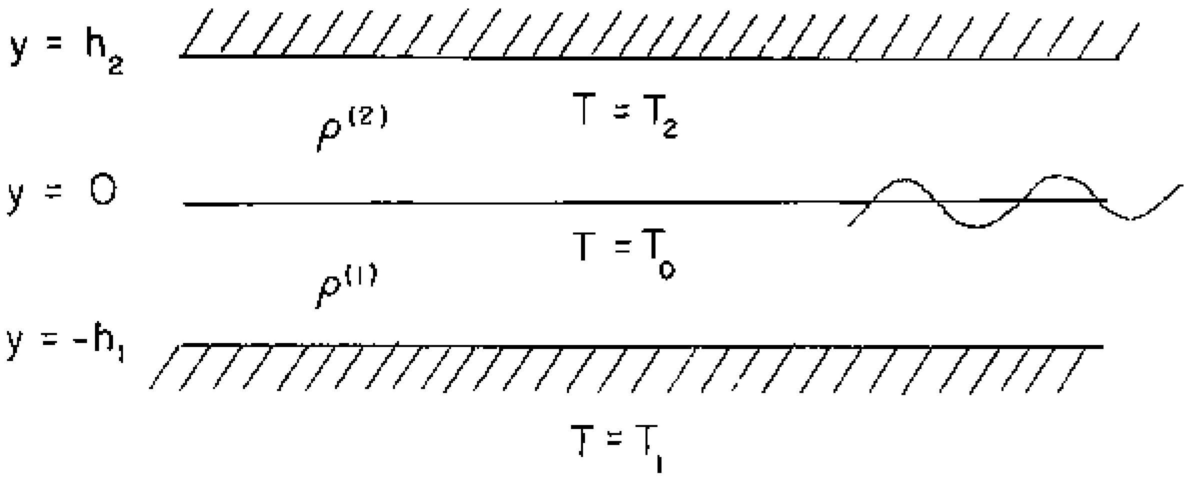

Figure 1

The equilibrium configuration of the fluid system