Interference Alignment — A New Look at Signal Dimensions in a Communication Network

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Syed A. Jafar

University of California, Irvine, USA syed@uci.edu



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Interference Alignment — A New Look at Signal Dimensions in a Communication Network

Syed A. Jafar

University of California, Irvine, CA 92697, USA, syed@uci.edu

Abstract

This monograph introduces to the reader the idea of interference alignment, traces its origins, reviews a variety of interference alignment schemes, summarizes the diverse settings where the idea of interference alignment is applicable and highlights the common principles that cut across these diverse applications. The focus is on theoretical aspects.

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Interference alignment is a radical idea that has recently emerged out of the capacity analysis of interference networks. In a relatively short time, this concept has challenged much of the conventional wisdom about the throughput limits of both wired and wireless networks. A representative example is the wireless interference channel with Ktransmitter-receiver pairs where, because of interference alignment, each user is simultaneously able to send at a data rate equal to half of his interference-free channel capacity to his desired receiver, even though the number of users K can be arbitrarily large, thus showing that the interference channel is not fundamentally interferencelimited. While the remarkable benefits of interference alignment have so far been shown mostly under idealized assumptions such as global channel knowledge, bandwidth expansion, unlimited resolution, high signal strengths and significant delays, the idea has garnered rapidly increasing interest in the communication, signal processing, networking and information theory communities and has produced an array of surprising and fundamental insights into the number of accessible signaling dimensions in both wired and wireless communication networks. A diversity of tools from linear algebra, algebraic geometry, diophantine approximation theory as well as coding and

2 Introduction

traditional Shannon theory continue to be the basis for an increasing variety of interference alignment schemes that include spatial alignment, lattice alignment, asymptotic alignment, asymmetric complex signal alignment, opportunistic alignment, ergodic alignment, aligned interference neutralization, blind alignment and retrospective alignment schemes. Applications include wireless interference networks, X networks, cellular networks, two-way communication networks, multicast and compound networks, multihop multiflow networks, tactical communication networks, cognitive radio networks, distributed data storage networks, index coding networks, and wired multiple unicast networks.

The goal of this monograph is to provide both a tutorial and a survey of the state-of-the-art on the topic of interference alignment. The majority of the paper is written to be accessible to a graduate student working in communication, signal processing, networking or information theory. The focus is on theoretical aspects. The presentation style of this monograph is informal, favoring broad intuition over mathematical rigor, and the finer details are mostly left to the references.

This monograph is organized into six sections, starting with this section — the introduction. Section 2 introduces the fundamental idea of interference alignment in its simplest form — in the language of elementary linear algebra. The origins of interference alignment are the topic of Section 3 where the earliest applications of interference alignment are reviewed. Section 4 explores the challenges faced by interference alignment schemes and the ingenious solutions that get around those challenges. Section 5 reviews the diverse settings, including both wireless and wired networks, where interference alignment has been applied while highlighting how many of these diverse problems can be reduced to the same essential problem for which a systematic solution already exists. Each subsection of this section ends with pointers to one or more open problems. Section 6 concludes the paper. An Appendix is included at the end which provides a rudimentary introduction to the Degrees-of-Freedom (DoF) and Generalized Degrees of Freedom (GDoF) metrics that are necessary to appreciate the concepts discussed throughout the paper.

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