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# Interference Exploitation in D2D-enabled Cellular Networks: A Secrecy Perspective

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Abstract—Device-to-device (D2D) communication underlaying cellular networks is a promising technology to improve network resource utilization. In D2D-enabled cellular networks, interference generated by D2D communications is usually viewed as an obstacle to cellular communications. However, in this paper, we present a new perspective on the role of D2D interference by taking security issues into consideration. We consider a large-scale D2D-enabled cellular network with eavesdroppers overhearing cellular communications. Using stochastic geometry, we model such a network and analyze the signal-to-interferenceplus-noise ratio (SINR) distributions, connection probabilities and secrecy probabilities of both the cellular and D2D links. We propose two criteria for guaranteeing performances of secure cellular communications, namely the strong and weak performance guarantee criteria. Based on the obtained analytical results of link characteristics, we design optimal D2D link scheduling schemes under these two criteria respectively. Both analytical and numerical results show that the interference from D2D communications can enhance physical layer security of cellular communications and at the same time create extra transmission opportunities for D2D users.

*Index Terms*—D2D communication, cellular network, physical layer security, link scheduling, stochastic geometry.

## I. INTRODUCTION

Recently, there has been a rapid increase in the demand of local area services and proximity services (ProSe) among the highly-capable user equipments (UEs) in cellular networks. Consequently, device-to-device (D2D) communication, which enables direct communication between UEs that are in proximity, has been proposed as a competitive technology component for next generation cellular networks. The integration of D2D communication to cellular networks holds the promise of many types of advantages [1]: allowing for high-rate low-delay low-power transmission for proximity services, increasing frequency reuse factor and network capacity, facilitating new types of peer-to-peer services, etc. For these reasons, D2D communication has strongly appealed to both academia [2], [3] and industry [4]–[6].

However, the introduction of D2D communication also brings a number of technical challenges, such as device discovery, mode selection and intra-cell interference management. Intra-cell interference, referring to the interference between D2D and cellular links that share the same timefrequency resources within a cell, becomes a major issue in D2D-enabled cellular networks. Especially the interference generated by D2D links, if not properly managed, would severely hamper the performance of cellular links in the network. To guarantee reliable cellular communications in D2D-enabled cellular networks, extensive research has been undertaken on the management of interference generated by D2D communications. To date, most proposed schemes can be categorized into the following three types.

- *Interference avoidance*: Orthogonal time-frequency resource allocation schemes are adopted to avoid interference from D2D links to cellular links [7].
- *Interference coordination*: Intelligent power control and link scheduling schemes are employed to mitigate interference from D2D links to cellular links [8]–[10].
- *Interference cancellation*: Advanced coding and decoding methods are used at cellular and/or D2D links to cancel interfering signals from desired signals [11], [12].

In the above work, the interference generated by D2D communications is purely viewed as an obstacle to cellular communications. However, when privacy and security issues are taken into consideration, such interference may play a completely different role. We consider a D2D-enabled cellular network with eavesdroppers overhearing cellular communications. According to literature on physical layer security [13], [14], perfect secrecy of cellular communications can be achieved at the physical layer by adopting secrecy coding schemes. References [15] and [16] further introduced cooperative jammers which transmit jamming signals to eavesdroppers to improve secrecy capacity. In D2D-enabled cellular networks with eavesdroppers, D2D transmitters can play the role of cooperative jammers for cellular communications if the interference from D2D links to eavesdropping links is more severe than that to cellular links. In this situation, interference generated by D2D links is not harmful to but helpful for secure cellular communications. Based on the above observation, we propose the notion of D2D interference exploitation in D2D-enabled cellular networks, which means that the interference generated by D2D communications can be exploited to enhance secure cellular communications and at the same time create extra transmission opportunities for D2D users. Our prior work [17] put forward this idea and studied the secrecy performance for point-to-point models. In this paper, we extend our analysis to a large-scale D2D-enabled cellular network and investigate the effect of D2D communications on secrecy performance of the cellular network.

The main contributions of this paper are summarized as follows.

(1) We model a large-scale D2D-enabled cellular network in the presence of eavesdroppers via stochastic geometry, and

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derive the general expressions for signal-to-interference-plusnoise ratio (SINR) distributions of typical cellular links, eavesdropping links and D2D links, and the connection probability and secrecy probability of typical cellular links. To illustrate the use of the general expressions, we further derive accurate closed-form bounds for interference-limited case.

(2) Based on the obtained analytical results, we first introduce the *strong* performance guarantee criterion for cellular communications. Under this criterion, the average number of reliable and secure cellular links should not be reduced by introducing D2D links. Then, we analyze the feasible region of D2D scheduling parameters for satisfying the strong guarantee criterion, and design optimal D2D link scheduling schemes within the feasible region to maximize the numbers of cellular links and D2D links respectively.

(3) We also introduce the *weak* performance guarantee criterion for cellular communications. This criterion corresponds to a certain level of performance degradation of cellular links. We analyze the feasible region of D2D scheduling parameters as well as reasonable values of the required minimum connection and secrecy probability of cellular links, and design optimal D2D link scheduling schemes within the feasible region to maximize the numbers of cellular links and D2D links respectively.

The remaining part of this paper is organized as follows. Section II presents the related work. Section III describes the system model. Section IV analyzes SINR distributions, connection probabilities and secrecy probabilities. Sections V and VI investigate optimal D2D link scheduling problems with respect to two different criteria. Section VII presents numerical results and section VIII concludes the paper. A summary of the notations used in this paper is given in Table I.

## II. RELATED WORK

Interference management for D2D communication. Xu et al. [7] proposed a combinatorial auction approach to allocate orthogonal resources between cellular and D2D users. Kaufman et al. [8] presented an opportunistic communication scheme in which the D2D network can communicate as a fully loaded cellular network. Pei and Liang [9] designed a spectrum sharing protocol that enables D2D users to communicate bidirectionally. Fodor et al. [10] introduced a network coding scheme to integrated D2D-cellular networks such that a higher spectral and energy efficiency can be achieved. Min et al. [11] designed an interference cancellation scheme that exploits a retransmission of the interference from the base station. Ma et al. [12] proposed two superposition coding based cooperative relaying schemes to exploit the transmission opportunities for D2D users. All the above work viewed the D2D interference as an obstacle to cellular communications, and focused on the avoidance, mitigation and cancellation of D2D interference. However, in this paper, we present a new perspective on the role of D2D interference and investigate how to exploit such interference.

*Physical layer security for wireless networks.* Security is an important issue in wireless networks due to the open wireless medium [18], [19]. Physical layer security using an information-theoretic point of view has attracted considerable recent attention. Wyner proposed the wire-tap channel model and the concept of perfect secrecy for point-to-point communication in his pioneering work [13]. Csiszár and Körner [14] extended Wyner's results to broadcast channels. Goel et al. [15] and Tang et al. [16] showed that cooperative jamming can improve secrecy capacity of point-to-point systems. The research on physical layer security for large-scale wireless networks focused mainly on connectivity [20], [21], secrecy capacity [22], [23] and capacity scaling laws [24]. In [20], [21], secrecy communication graph and percolation theory were employed to analyze secure connectivity in large-scale wireless networks. In [22] and [23], the throughput cost of achieving a certain level of security in interference-limited networks was analyzed. In [24], the asymptotic behavior of secrecy capacity of ad hoc network was investigated.

Stochastic geometry for wireless networks. As a mathematical tool to study random spatial patterns, stochastic geometry can be used to model and analyze interference, connectivity and coverage in large-scale wireless networks [25]. Recent years, many tractable models have been proposed for analyzing ad hoc [26], [27], cellular [28], [29] and D2D [30]-[32] networks via stochastic geometry. Specially, in [30]-[32], a D2Denabled cellular network was modeled by two independent Poisson point processes, and the SINR distributions of both cellular and D2D links were derived. References [33], [34] studied secrecy performance of ad hoc networks via stochastic geometry. Different from the above work, we consider a more complex scenario that the D2D-enabled cellular network is overheard by eavesdroppers, and study the effect of D2D communications on secure cellular communications. The analysis of such complex scenario is more challenging than that of D2D networks without eavesdroppers [30]-[32] and that of ad hoc networks with eavesdroppers [33], [34].

### **III. SYSTEM MODEL**

In this section, we elaborate on the network model and describe the secrecy coding scheme.

## A. Network Model

We consider a hybrid network consisting of cellular links, D2D links and a set of eavesdroppers that overhear the transmission of cellular links<sup>1</sup> over a large two-dimensional space, as shown in Fig.1. The base stations (BSs) are assumed to be spatially distributed as a homogeneous Poisson point process (PPP)  $\Phi_b$  of intensity  $\lambda_b$ , and an independent collection of cellular users is assumed to be located according to some independent stationary point process  $\Phi_c$ . The downlink scenario is considered for cellular communications, and each cellular user is assumed to connect to its strongest BS instantaneously, i.e. the BS that offers the highest received SINR. We assume there is no intra-cell interference between cellular links due to orthogonal multiple access within a cell.

<sup>&</sup>lt;sup>1</sup>We do not consider the security requirement for D2D links in the model. Therefore, a transceiver pair is not allowed to use D2D mode if it has some security requirement.

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Description					
Poisson point process of base stations					
Poisson point process of D2D links					
Poisson point process of eavesdroppers					
Transmission power of base stations					
Transmission power of D2D users					
Path loss exponent $\left(\delta = \frac{2}{\alpha}\right)$					
SINR threshold for connection of cellular links					
SINR threshold for secrecy of cellular links					
SINR threshold for connection of D2D links					
Connection probability of cellular links					
Secrecy probability of cellular links					
Connection probability of D2D links					
Connection probability requirement for cellular links					
Secrecy probability requirement for cellular links					
Average number of perfect cellular links per unit area					
Average number of perfect D2D links per unit area					

Table I: Notations used in the paper

The spatial locations of eavesdroppers in the network are modeled as a homogeneous PPP  $\Phi_e$  of intensity  $\lambda_e$ . We assume that each cellular link is exposed to all the eavesdroppers and its secrecy data rate is determined by the most detrimental eavesdropper, i.e. the eavesdropper with the highest received SINR of cellular signal. The locations of D2D transmitters in the network are arranged according to a homogeneous PPP  $\Phi_d$ of intensity  $\lambda_d$ , and for a given D2D transmitter, its associated receiver is assumed to be located at a fixed distance l away with isotropic direction. It is noted that other distributions of l can be easily incorporated into the framework, and some discussions are provided in the following section.

The transmission powers are assumed to be  $P_b$  at BSs and  $P_d$  at D2D transmitters. We adopt a unified channel model that comprises standard path loss and Rayleigh fading for both cellular and D2D links: given transmission power P of transmitter  $x_i$ , the received power at receiver  $x_j$  can be expressed as  $Ph ||x_i - x_j||^{-\alpha}$ , where h is the fading factor following an exponential distribution with unit mean, i.e.  $h \sim \exp(1)$ , and  $\alpha > 2$  is the path loss exponent. In later parts of this paper we use parameter  $\delta$  to denote  $\frac{2}{\alpha}$  for brevity of expressions. In addition, the noise power at the receiver is assumed to be additive and constant with value  $\sigma^2$ .

## B. Secrecy Coding

The cellular links are assumed to be eavesdropped in the model. To fight against eavesdropping, each cellular transmitter (say BS for downlink scenario) adopts secrecy coding scheme, such as Wyner code [13], to encode the data before transmission. We assume Wyner code is employed in this paper and thus two kinds of rates, namely the rate of the transmitted codewords  $R_c$  and the rate of the transmitted message  $R_m$  ( $R_c > R_m$ ), need to be determined at the cellular transmitter. The design of  $R_c$  and  $R_m$  should consider both connection and secrecy of the cellular link:

• Connection. If the rate of the transmitted codewords  $R_c$  is above the capacity of the cellular link, the received signal at the cellular receiver can be decoded with an arbitrarily small error, and thus perfect connection of



Figure 1: Network model.

the cellular link can be achieved. Otherwise, connection outage occurs in the cellular link.

• Secrecy. The rate redundancy  $R_c - R_m$  is to provide secrecy. If the rate redundancy is above the capacity of the most detrimental eavesdropping link, the received signal at any eavesdropper provides no information about the transmitted message, and thus perfect secrecy of the cellular link can be achieved. Otherwise, secrecy outage occurs in the cellular link.

Perfect cellular transmission implies that both perfect connection and perfect secrecy of the cellular link are achieved. Since decoding capability is generally determined by the received SINR, the perfect transmission of a cellular link can be defined in terms of received SINRs of both the cellular link and the eavesdropping link:

**Definition 1.** [Perfect transmission] A cellular transmission is said to be *perfect* if SINR<sub>c</sub> >  $T_{\phi}$  and SINR<sub>e</sub> <  $T_{\epsilon}$ , where SINR<sub>c</sub>, SINR<sub>e</sub> denote the received SINRs at the cellular receiver and the most detrimental eavesdropper respectively, and  $T_{\phi}$ ,  $T_{\epsilon}$  represent the corresponding threshold SINR values.

Perfect transmission cannot be always achieved due to timevarying wireless environment. Therefore, in practical networks constraints on connection probability and secrecy probability are usually pre-set to control network performance. Accordingly, we have the following definition:

**Definition 2.**  $[(\phi, \epsilon)$ -Perfect transmission] A cellular transmission is said to be  $(\phi, \epsilon)$ -perfect if  $\mathbb{P}(\mathsf{SINR}_c > T_{\phi}) \ge \phi$  and  $\mathbb{P}(\mathsf{SINR}_e < T_{\epsilon}) \ge \epsilon$ , where  $0 \le \phi, \epsilon \le 1$  denote the required minimum connection probability and minimum secrecy probability respectively.

By the above definitions, perfect transmission is equivalent to (1,1)-perfect transmission. In addition, for  $(\phi, \epsilon)$ -perfect transmission,  $1 - \phi$  and  $1 - \epsilon$  represent the maximum connection outage probability and maximum secrecy outage probability respectively.

## IV. ANALYSIS OF SECRECY TRANSMISSION IN D2D-ENABLED CELLULAR NETWORKS

In this section, we analyze the SINR distributions of typical links in large-scale D2D-enabled cellular networks, and derive the connection probability and secrecy probability of cellular links as well as the connection probability of D2D links. The derived results are the baseline for designing link scheduling schemes in later sections.

#### A. Connection of Cellular Links

Without loss of generality, we conduct analysis on a typical cellular user located at the origin. The distance between the BS located at point x and the typical cellular user is denoted by  $r_x$ , and the fading factor between BS x and the typical cellular user is denoted by  $g_x$  which is i.i.d exponential, i.e.  $g_x \sim \exp(1)$ . Then, the received SINR at the typical cellular user from BS x can be expressed as

$$\operatorname{SINR}_{c}\left(x\right) = \frac{P_{b}g_{x}r_{x}^{-\alpha}}{\sigma^{2} + I_{c}\left(x\right)},$$
(1)

where

$$I_{c}(x) = \sum_{x_{i} \in \Phi_{b} \setminus \{x\}} P_{b}g_{i} \|x_{i}\|^{-\alpha} + \sum_{y_{i} \in \Phi_{d}} P_{d}h_{i} \|y_{i}\|^{-\alpha}$$
(2)

is the cumulative interference from all other BSs that are located at  $x_i$  with fading factor  $g_i$  and D2D transmitters that are located at  $y_i$  with fading factor  $h_i^2$ .

We assume that each cellular user associates with its strongest BS. Thus a cellular user is connected to the network when its SINR from the strongest BS is above the threshold  $T_{\phi}$ , while it is dropped from the network when SINR is below  $T_{\phi}$ . The link that connects the typical cellular user and its strongest BS is referred to as typical cellular link, and the connection probability of the typical cellular link can be defined as

$$p_{con}^{(c)}\left(T_{\phi}\right) \stackrel{\Delta}{=} \mathbb{P}\left[\max_{x \in \Phi_{b}} \mathsf{SINR}_{c}\left(x\right) > T_{\phi}\right].$$
(3)

In the following theorem and proposition, we provide an upper bound on the connection probability and show a sufficient condition under which the bound can be achieved.

**Theorem 1.** *The connection probability of the typical cellular link in D2D-enabled cellular networks is bounded from above by* 

$$p_{con}^{(c)}(T_{\phi}) \leq 2\pi\lambda_{b} \int_{0}^{\infty} \exp\left(-P_{b}^{-1}T_{\phi}r_{x}^{\alpha}\sigma^{2} - \pi r_{x}^{2}T_{\phi}^{\delta}\mu\right)r_{x}\mathrm{d}r_{x},$$

$$(4)$$
where  $\mu = \lambda_{b}\left[1 + \frac{\lambda_{d}}{\lambda_{b}}\left(\frac{P_{d}}{P_{b}}\right)^{\delta}\right]\mathrm{sinc}^{-1}\delta.$ 

*Proof:* The probability that the strongest SINR is above  $T_{\phi}$  equals the probability that at least one SINR is above  $T_{\phi}$ . Therefore, the connection probability of the typical cellular

link under the strongest-BS association model can be derived as follows:

 $p_{co}^{(c)}$ 

$$\begin{split} P_{n}^{()}(T_{\phi}) &\stackrel{\triangle}{=} \mathbb{P}\left[\max_{x \in \Phi_{b}} \mathsf{SINR}_{c}(x) > T_{\phi}\right] \\ &= \mathbb{P}\left[\bigcup_{x \in \Phi_{b}} \mathsf{SINR}_{c}(x) > T_{\phi}\right] \\ &= \mathbb{E}\left[\mathbf{1}\left(\bigcup_{x \in \Phi_{b}} \mathsf{SINR}_{c}(x) > T_{\phi}\right)\right] \\ \stackrel{(a)}{\leq} \mathbb{E}\left[\sum_{x \in \Phi_{b}} \mathbf{1}\left(\mathsf{SINR}_{c}(x) > T_{\phi}\right)\right] \\ &= \mathbb{E}\left[\sum_{x \in \Phi_{b}} \mathbf{1}\left(\frac{P_{b}g_{x}r_{x}^{-\alpha}}{\sigma^{2} + I_{c}(x)} > T_{\phi}\right)\right] \\ \stackrel{(b)}{=} \lambda_{b} \int_{\mathbb{R}^{2}} \mathbb{E}\left[\mathbf{1}\left(\frac{P_{b}g_{x}r_{x}^{-\alpha}}{\sigma^{2} + I_{c}'} > T_{\phi}\right)\right] dx \\ &= \lambda_{b} \int_{\mathbb{R}^{2}} \mathbb{P}\left[\frac{P_{b}g_{x}r_{x}^{-\alpha}}{\sigma^{2} + I_{c}'} > T_{\phi}\right] dx \\ \stackrel{(c)}{=} \lambda_{b} \int_{\mathbb{R}^{2}} e^{-P_{b}^{-1}T_{\phi}r_{x}^{\alpha}\sigma^{2}} \mathbb{E}_{I_{c}'}\left[e^{-P_{b}^{-1}T_{\phi}r_{x}^{\alpha}I_{c}'}\right] dx \\ \stackrel{(d)}{=} \lambda_{b} \int_{\mathbb{R}^{2}} e^{-P_{b}^{-1}T_{\phi}r_{x}^{\alpha}\sigma^{2}} \mathcal{L}_{I_{c}'}\left(P_{b}^{-1}T_{\phi}r_{x}^{\alpha}\right) r_{x} dr_{x}. \end{split}$$

$$(5)$$

(a) follows from the property of union, and the equality holds if at most one BS in the network can provide a SINR above the threshold. In (b),  $I'_c = \sum_{x_i \in \Phi_b} P_b g_i ||x_i||^{-\alpha} + \sum_{y_i \in \Phi_d} P_d h_i ||y_i||^{-\alpha}$ , which includes the term related to the tagged BS x and thereby is quite different from  $I_c(x)$ . The derivation of (b) follows from the Campbell-Mecke Theorem [35]:  $\mathbb{E}\left[\sum_{x \in \Phi} f(x, \Phi \setminus \{x\})\right] = \lambda \int_{\mathbb{R}^2} \mathbb{E}\left[f(x, \Phi)\right] dx$ . (c) follows from the Rayleigh distribution assumption of channel fading and the independence of noise and interference. In (d),  $\mathcal{L}_{I'_c}(\cdot)$  denotes the Laplace transform of  $I'_c$ . To complete the proof, we next derive the expression of  $\mathcal{L}_{I'_c}(s)$ .

Let  $I'_c = I'_{c-c} + I'_{c-d}$ , where  $I'_{c-c} = \sum_{x_i \in \Phi_b} P_b g_i ||x_i||^{-\alpha}$ and  $I'_{c-d} = \sum_{y_i \in \Phi_d} P_d h_i ||y_i||^{-\alpha}$  denote the interference from cellular links and D2D links respectively. Then it is straightforward to get

$$\mathcal{L}_{I_{c}^{\prime}}\left(s\right) = \mathcal{L}_{I_{c-c}^{\prime}}\left(s\right) \cdot \mathcal{L}_{I_{c-d}^{\prime}}\left(s\right),\tag{6}$$

since  $\mathbb{E}_{I'_c}\left[e^{-sI'_c}\right] = \mathbb{E}\left[e^{-sI'_{c-c}}\right] \cdot \mathbb{E}\left[e^{-sI'_{c-d}}\right]$ . The Laplace transform of  $I'_{c-c}$  is given by

$$\mathcal{L}_{I_{c-c}'}(s) = \mathbb{E}\left[\exp\left(-s\sum_{x_i\in\Phi_b} P_b g_i \|x_i\|^{-\alpha}\right)\right]$$
$$= \mathbb{E}_{\Phi_b,g}\left[\prod_{x_i\in\Phi_b} \exp\left(-sP_b g_i \|x_i\|^{-\alpha}\right)\right]$$
$$= \mathbb{E}_{\Phi_b}\left[\prod_{x_i\in\Phi_b} \mathbb{E}_g\left[\exp\left(-sP_b g_i \|x_i\|^{-\alpha}\right)\right]\right]$$

<sup>&</sup>lt;sup>2</sup>To distinguish different links, in this paper we use  $g \sim \exp(1)$ ,  $h \sim \exp(1)$  to represent the fading factor for links related to cellular transmitter (say BS) and D2D transmitter respectively. It is noted that there is no essential distinction between these two symbols.

$$\stackrel{(e)}{=} \exp\left(-\lambda_b \int_{\mathbb{R}^2} \left(1 - \mathbb{E}_g \left[e^{-sP_b g_i \|x_i\|^{-\alpha}}\right]\right) \, \mathrm{d}x_i\right)$$

$$\stackrel{(f)}{=} \exp\left(-\lambda_b 2\pi \int_{v=0}^{\infty} \frac{v}{1 + s^{-1} P_b^{-1} v^{\alpha}} \, \mathrm{d}v\right)$$

$$\stackrel{(g)}{=} \exp\left(-\lambda_b \pi \left(sP_b\right)^{\delta} \Gamma \left(1 - \delta\right) \Gamma \left(1 + \delta\right)\right)$$

$$\stackrel{(h)}{=} \exp\left(-\frac{\pi \lambda_b P_b^{\delta} s^{\delta}}{\mathrm{sinc} \, \delta}\right).$$

$$(7)$$

(e) follows from the probability generating functional (PGFL) of PPP [35]:  $\mathbb{E}\left[\prod_{x \in \Phi} f(x)\right] = \exp\left(-\lambda \int_{\mathbb{R}^2} (1 - f(x)) \, \mathrm{d}x\right)$ . (f) follows from the double integral in polar coordinates. In (g),  $\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} \, \mathrm{d}t$  is the gamma function and  $\delta = \frac{2}{\alpha}$ . (h) is obtained by using the property of gamma function:  $\Gamma(1+x)\Gamma(1-x) = \frac{\pi x}{\sin \pi x} = \frac{1}{\sin cx}$ , for 0 < x < 1. Similarly, we have

$$\mathcal{L}_{I'_{c-d}}(s) = \exp\left(-\frac{\pi\lambda_d P_d^{\delta} s^{\delta}}{\operatorname{sinc} \delta}\right).$$
(8)

By plugging (7)(8) into (6), we get

$$\mathcal{L}_{I_c'}(s) = \exp\left(-\frac{\pi\lambda_b P_b^{\delta} s^{\delta}}{\operatorname{sinc} \delta} - \frac{\pi\lambda_d P_d^{\delta} s^{\delta}}{\operatorname{sinc} \delta}\right). \tag{9}$$

Therefore,

$$\mathcal{L}_{I_c'}\left(P_b^{-1}T_{\phi}r_x^{\alpha}\right) = \exp\left(-\pi r_x^2 T_{\phi}^{\delta}\mu\right),\tag{10}$$

where  $\mu = \lambda_b \left[ 1 + \frac{\lambda_d}{\lambda_b} \left( \frac{P_d}{P_b} \right)^{\delta} \right] \operatorname{sinc}^{-1} \delta$ . Then by plugging (10) into (5), we complete the proof.

The following proposition provides a sufficient condition under which the equality in (4) holds.

**Proposition 1.** The upper bound on  $p_{con}^{(c)}(T_{\phi})$  is achieved when  $T_{\phi} > 1 (0 dB)$ .

*Proof:* The proof follows from [29, Lemma 1]. According to the lemma, at most one BS can provide a SINR greater than 1 if  $T_{\phi} > 1$ . Therefore, the equality in step (a) of (5) holds and the upper bound on  $p_{con}^{(c)}(T_{\phi})$  is achieved.

By Proposition 1, when  $T_{\phi} > 1$ , the upper bound on  $p_{con}^{(c)}(T_{\phi})$  is the exact value of  $p_{con}^{(c)}(T_{\phi})$ . However, when  $T_{\phi} \leq 1$ , there exists a gap between  $p_{con}^{(c)}(T_{\phi})$  and the upper bound. In the simulation, we will evaluate the gap by numerical results and show the tightness of the upper bound when  $T_{\phi} \leq 1$ . For tractability, in the following part of this paper, we represent the exact value of  $p_{con}^{(c)}(T_{\phi})$  by its upper bound, or equivalently assume  $T_{\phi} > 1$ .

Considering a special case that the network is interferencelimited, we get the following corollary.

**Corollary 1.** In interference-limited D2D-enabled cellular networks, the connection probability of the typical cellular link is bounded from above by

$$p_{con}^{(c)}\left(T_{\phi}\right) \leq \frac{\operatorname{sinc}\delta}{\left[1 + \frac{\lambda_d}{\lambda_b} \left(\frac{P_d}{P_b}\right)^{\delta}\right]T_{\phi}^{\delta}},\tag{11}$$

and the equality holds when  $T_{\phi} > 1$ .

*Proof:* Following from Theorem 1 with  $\sigma^2 \rightarrow 0$ .

*Remark* 1. Corollary 1 shows that  $p_{con}^{(c)}(T_{\phi})$  is inversely correlated with the density ratio  $\lambda_d/\lambda_b$  and transmission power ratio  $P_d/P_b$ . The result is intuitive since given  $\lambda_b$  and  $P_b$ , a larger density and transmission power of D2D communications introduce more severe interference to cellular communications.

## B. Secrecy of Cellular Links

The analysis is conducted based on the typical cellular link that comprises a typical cellular user located at the origin and a typical BS located at  $x_0$ . For a eavesdropper located at z, its distance to the typical BS is denoted by  $r_z$ , and the fading factor for this eavesdropping link is denoted by  $g_z$ ,  $g_z \sim \exp(1)$ . Then, the received SINR at eavesdropper z from the typical BS can be expressed as

$$SINR_e(z) = \frac{P_b g_z r_z^{-\alpha}}{\sigma^2 + I_e(z)},$$
(12)

where

$$I_{e}(z) = \sum_{x_{i} \in \Phi_{b} \setminus \{x_{0}\}} P_{b}g_{i} \|x_{i} - z\|^{-\alpha} + \sum_{y_{i} \in \Phi_{d}} P_{d}h_{i} \|y_{i} - z\|^{-\alpha}$$
(13)

is the cumulative interference from all other BSs (except the typical BS located at  $x_0$ ) that are located at  $x_i$  with fading factor  $g_i$  and D2D transmitters that are located at  $y_i$  with fading factor  $h_i$ .

We assume that each cellular link is exposed to all the eavesdroppers. According to the secrecy requirement, if there exists some eavesdropper z such that  $SINR_e(z)$  is above the threshold  $T_e$ , the cellular link is not secure. Therefore, the secure transmission of a cellular link is determined by its most detrimental eavesdropper, and the secrecy probability of the typical cellular link can be defined as

$$p_{sec}^{(c)}\left(T_{\epsilon}\right) \stackrel{\Delta}{=} \mathbb{P}\left[\max_{z \in \Phi_{e}} \mathsf{SINR}_{e}\left(z\right) < T_{\epsilon}\right].$$
 (14)

The expression of  $p_{sec}^{(c)}(T_{\epsilon})$  is given by the following theorem.

**Theorem 2.** The secrecy probability of the typical cellular link in D2D-enabled cellular networks is

$$p_{sec}^{(c)}\left(T_{\epsilon}\right) = \exp\left(-2\pi\lambda_{e}\int_{0}^{\infty}e^{-P_{b}^{-1}T_{\epsilon}r_{z}^{\alpha}\sigma^{2}-\pi r_{z}^{2}T_{\epsilon}^{\delta}\mu}r_{z}\,\mathrm{d}r_{z}\right),\tag{15}$$
where  $\mu = \lambda_{b}\left[1+\frac{\lambda_{d}}{\lambda_{b}}\left(\frac{P_{d}}{P_{b}}\right)^{\delta}\right]\mathrm{sinc}^{-1}\delta.$ 

*Proof:* The probability that the most detrimental SINR is below  $T_{\epsilon}$  equals the probability that all the SINRs are below  $T_{\epsilon}$ . Therefore, the secrecy probability of the typical cellular link can be derived as follows:

$$\begin{split} p_{sec}^{(c)}\left(T_{\epsilon}\right) & \stackrel{\triangle}{=} \mathbb{P}\left[\max_{z \in \Phi_{e}} \mathsf{SINR}_{e}\left(z\right) < T_{\epsilon}\right] \\ & = \mathbb{P}\left[\bigcap_{z \in \Phi_{e}} \mathsf{SINR}_{e}\left(z\right) < T_{\epsilon}\right] \\ & = \mathbb{E}_{\Phi_{e},\Phi_{d}}\left[\mathbf{1}\left(\bigcap_{z \in \Phi_{e}} \mathsf{SINR}_{e}\left(z\right) < T_{\epsilon}\right)\right] \end{split}$$

$$\begin{split} \stackrel{(a)}{=} & \mathbb{E}_{\Phi_{e},\Phi_{d}} \left[ \prod_{z \in \Phi_{e}} \mathbf{1} \left( \mathsf{SINR}_{e} \left( z \right) < T_{\epsilon} \right) \right] \\ &= & \mathbb{E}_{\Phi_{e}} \left[ \prod_{z \in \Phi_{e}} \mathbb{E}_{\Phi_{d}} \left[ \mathbf{1} \left( \mathsf{SINR}_{e} \left( z \right) < T_{\epsilon} \mid z \right) \right] \right] \\ &= & \mathbb{E}_{\Phi_{e}} \left[ \prod_{z \in \Phi_{e}} \mathbb{P} \left( \mathsf{SINR}_{e} \left( z \right) < T_{\epsilon} \mid z \right) \right] \\ \stackrel{(b)}{=} & \mathbb{E}_{\Phi_{e}} \left[ \prod_{z \in \Phi_{e}} \left( 1 - e^{-P_{b}^{-1}T_{\epsilon}r_{z}^{\alpha}\sigma^{2}} \mathcal{L}_{I_{e}(z)} \left( P_{b}^{-1}T_{\epsilon}r_{z}^{\alpha} \right) \right) \right] \\ &= & \mathbb{E}_{\Phi_{e}} \left[ \prod_{z \in \Phi_{e}} \left( 1 - e^{-P_{b}^{-1}T_{\epsilon}r_{z}^{\alpha}\sigma^{2}} \mathcal{L}_{I_{e}(z)} \left( P_{b}^{-1}T_{\epsilon}r_{z}^{\alpha} \right) r_{z} \, \mathrm{d}r_{z} \right) \right] \\ \stackrel{(c)}{=} & \exp \left( -2\pi\lambda_{e} \int_{0}^{\infty} e^{-P_{b}^{-1}T_{\epsilon}r_{z}^{\alpha}\sigma^{2}} \mathcal{L}_{I_{e}(z)} \left( P_{b}^{-1}T_{\epsilon}r_{z}^{\alpha} \right) r_{z} \, \mathrm{d}r_{z} \right) \end{aligned}$$
(16)

(a) follows from the property of intersection. (b) follows from the Rayleigh distribution assumption of channel fading and the independence of noise and interference. (c) follows from the probability generating functional (PGFL) of PPP. To complete the proof, we next derive the expression of  $\mathcal{L}_{I_e(z)}(s)$ .

First, shift the coordinates so that eavesdropper z is located at the origin. It is noted that translations do not change the distribution of PPP [22]. Thus  $I_e(z)$  can be replaced by  $I_e$  which equals  $I_e(z=0)$ . Then let  $I_e = I_{e-c} + I_{e-d}$ , where  $I_{e-c} = \sum_{x_i \in \Phi_b \setminus \{x_0\}} P_b g_i ||x_i||^{-\alpha}$  and  $I_{e-d} = \sum_{y_i \in \Phi_d} P_d h_i ||y_i||^{-\alpha}$  denote the interference from cellular links and D2D links respectively. It is straightforward to get

$$\mathcal{L}_{I_{e}}\left(s\right) = \mathcal{L}_{I_{e-c}}\left(s\right) \cdot \mathcal{L}_{I_{e-d}}\left(s\right). \tag{17}$$

Following the steps similar to (7) - (10) and using Slivnyak's Theorem of PPP [35], we have

$$\mathcal{L}_{I_e(z)}\left(P_b^{-1}T_\epsilon r_z^\alpha\right) = \exp\left(-\pi r_z^2 T_\epsilon^\delta \mu\right),\tag{18}$$

where  $\mu = \lambda_b \left[ 1 + \frac{\lambda_d}{\lambda_b} \left( \frac{P_d}{P_b} \right)^{\delta} \right] \operatorname{sinc}^{-1} \delta$ . Plug (18) into (16), then the proof is completed.

By letting  $\sigma^2 \rightarrow 0$  in (15), we get the following result for interference-limited cases.

**Corollary 2.** In interference-limited D2D-enabled cellular networks, the secrecy probability of the typical cellular link is

$$p_{sec}^{(c)}(T_{\epsilon}) = \exp\left(-\frac{\lambda_{e} \operatorname{sinc} \delta}{\lambda_{b} \left[1 + \frac{\lambda_{d}}{\lambda_{b}} \left(\frac{P_{d}}{P_{b}}\right)^{\delta}\right] T_{\epsilon}^{\delta}}\right).$$
(19)

*Remark* 2. By Corollary 2,  $p_{sec}^{(c)}(T_{\epsilon})$  is negatively correlated with the eavesdropper intensity and is positively correlated with the D2D intensity and power. This is because, a larger population of eavesdroppers can reduce the average distance of eavesdropping links, while a larger population of D2D users can generate more interference to eavesdropping links.

## C. Connection of D2D Links

We conduct analysis on a typical D2D link that comprises a typical D2D transmitter and a typical D2D receiver located at distance l away with isotropic direction. Assume that the typical D2D receiver is located at the origin and denote the fading factor for the typical D2D link by  $h_0$ ,  $h_0 \sim \exp(1)$ . Then, the received SINR at the typical D2D receiver can be expressed as

$$\mathsf{SINR}_d = \frac{P_d h_0 l^{-\alpha}}{\sigma^2 + I_d},\tag{20}$$

where

$$I_{d} = \sum_{x_{i} \in \Phi_{b}} P_{b}g_{i} \|x_{i}\|^{-\alpha} + \sum_{y_{i} \in \Phi_{d} \setminus \{y_{0}\}} P_{d}h_{i} \|y_{i}\|^{-\alpha}$$
(21)

is the cumulative interference from all the base stations that are located at  $x_i$  with fading factor  $g_i$  and other D2D transmitters (except the typical D2D transmitter located at  $y_0$ ) that are located at  $y_i$  with fading factor  $h_i$ .

The connection probability of the typical D2D link can be defined as

$$p_{con}^{(d)}\left(T_{\sigma}\right) \stackrel{\simeq}{=} \mathbb{P}\left[\mathsf{SINR}_{d} > T_{\sigma}\right],\tag{22}$$

where  $T_{\sigma}$  is the SINR threshold.

**Theorem 3.** The connection probability of the typical D2D link in D2D-enabled cellular networks is

$$p_{con}^{(d)}\left(T_{\sigma}\right) = \exp\left(-P_{d}^{-1}T_{\sigma}l^{\alpha}\sigma^{2} - \pi l^{2}T_{\sigma}^{\delta}\nu\right), \quad (23)$$

where  $\nu = \lambda_d \left[ 1 + \frac{\lambda_b}{\lambda_d} \left( \frac{P_b}{P_d} \right)^{\delta} \right] \operatorname{sinc}^{-1} \delta.$ 

*Proof:* Based on a fixed distance l, the connection probability can be derived as follows:

$$p_{con}^{(d)}(T_{\sigma}) \stackrel{\Delta}{=} \mathbb{P}\left[\mathsf{SINR}_{d} > T_{\sigma}\right] \\ = \mathbb{P}\left[h_{0} > P_{d}^{-1}T_{\sigma}l^{\alpha}\left(\sigma^{2} + I_{d}\right)\right] \\ = e^{-P_{d}^{-1}T_{\sigma}l^{\alpha}\sigma^{2}}\mathcal{L}_{I_{d}}\left(P_{d}^{-1}T_{\sigma}l^{\alpha}\right).$$
(24)

Following approaches similar to those in previous proofs, we have

$$\mathcal{L}_{I_d}\left(P_d^{-1}T_\sigma l^\alpha\right) = \exp\left(-\pi l^2 T_\sigma^\delta \nu\right),\tag{25}$$

where  $\nu = \lambda_d \left[ 1 + \frac{\lambda_b}{\lambda_d} \left( \frac{P_b}{P_d} \right)^{\delta} \right] \operatorname{sinc}^{-1} \delta$ . Combining (24) and (25), we obtain the result.

**Corollary 3.** In interference-limited D2D-enabled cellular networks, the connection probability of the typical D2D link is

$$p_{con}^{(d)}\left(T_{\sigma}\right) = \exp\left(-\pi l^{2} T_{\sigma}^{\delta} \lambda_{d} \left[1 + \frac{\lambda_{b}}{\lambda_{d}} \left(\frac{P_{b}}{P_{d}}\right)^{\delta}\right] \operatorname{sinc}^{-1} \delta\right).$$
(26)

*Remark* 3. The results of Theorem 3 and Corollary 3 can be extended to cases where *l* is variable. Denote the pdf of *l* as  $f_l(l)$ , then  $p_{con}^{(d)}(T_{\sigma})$  can be computed as  $\int_0^{\infty} \mathbb{P}[\text{SINR}_d > T_{\sigma} \mid l] \cdot f_l(l) \, dl$ . For example, assume *l* is Rayleigh distributed,  $f_l(l) = 2\pi\lambda_d l e^{-\pi\lambda_d l^2}$ , then  $p_{con}^{(d)}(T_{\sigma}) = \lambda_d / (T_{\sigma}^{\delta} \nu + \lambda_d)$  for interference-limited cases.

## D. Performance Guarantee for Cellular Transmissions

By Corollary 2 and 3, the secrecy probability of cellular links  $p_{sec}^{(c)}(T_{\epsilon})$  and the connection probability of D2D links  $p_{con}^{(d)}(T_{\sigma})$  can be enhanced by increasing  $\lambda_d$  and/or  $P_d$ . However, by Corollary 1, increasing  $\lambda_d$  and/or  $P_d$  would reduce the connection probability of cellular links  $p_{con}^{(c)}(T_{\phi})$ . Therefore, the scheduling parameters of D2D links  $(\lambda_d, P_d)$ should be carefully designed to improve the performance of D2D communications and meanwhile guarantee a certain performance level of cellular communications.

We propose the following two performance guarantee criteria for cellular communications:

• *Strong guarantee criterion*: the probability of perfect cellular transmissions<sup>3</sup> should not to be reduced by introducing D2D communications, i.e.,

$$p_{con}^{(c)}(T_{\phi}) p_{sec}^{(c)}(T_{\epsilon}) \ge p_{con}^{(c)(0)}(T_{\phi}) p_{sec}^{(c)(0)}(T_{\epsilon}), \quad (27)$$

where  $p_{con}^{(c)(0)}(T_{\phi})$  and  $p_{sec}^{(c)(0)}(T_{\epsilon})$  denote the connection probability and secrecy probability of cellular links in the absence of D2D links respectively.

• Weak guarantee criterion:  $(\phi, \epsilon)$ -perfect cellular transmission should be guaranteed after introducing D2D communications, i.e.,

$$p_{con}^{(c)}(T_{\phi}) \ge \phi \text{ and } p_{sec}^{(c)}(T_{\epsilon}) \ge \epsilon,$$
 (28)

where  $\phi \epsilon \leq p_{con}^{(c)(0)}(T_{\phi}) p_{sec}^{(c)(0)}(T_{\epsilon})$ .

The first criterion requires that introducing D2D links to cellular networks should not degrade the performance of cellular links, while the second criterion can tolerate a certain level of performance degradation of cellular links. In the following sections, we explore D2D link scheduling issues under both the strong and weak guarantee criteria.

## V. OPTIMAL D2D LINK SCHEDULING UNDER STRONG GUARANTEE CRITERION FOR CELLULAR LINKS

Based on the derived analytical results in previous section, we design D2D link scheduling schemes, which determine the intensity and power of D2D links, under the strong performance guarantee criterion for cellular links. For purposes of mathematical tractability, we consider the interference-limited scenario where  $\sigma^2 \rightarrow 0$ .

## A. Feasible Region of D2D Scheduling Parameters

Let  $\mathcal{A}$  be a Borel set of  $\mathbb{R}^2$  with unit measure, i.e.  $|\mathcal{A}| = 1$ . Hence, the intensity measure of the set of perfect cellular transmissions in  $\mathcal{A}$  can be defined as

$$N_{c}\left(\mathcal{A}\right) \stackrel{\triangle}{=} \mathbb{E}\left[\sum_{i \in \Phi_{b}} \mathbf{1}_{\mathcal{A}}^{per}\left(i\right)\right],\tag{29}$$

where

$$\mathbf{1}_{\mathcal{A}}^{per}\left(i\right) = \begin{cases} 1, & if \ i \in \mathcal{A} \ is \ perfect \ transmission, \\ 0, & otherwise. \end{cases}$$
(30)

<sup>3</sup>Here we approximate the probability of perfect cellular transmissions by  $p_{con}^{(c)}(T_{\phi}) \cdot p_{sec}^{(c)}(T_{\epsilon})$  using the assumption that the interference at each point is independent, and its validity is evaluated in numerical results. It is noted that such assumption is also used in some stochastic geometry literature [33].

The indicator function  $\mathbf{1}_{\mathcal{A}}^{per}(i)$  allows to count the number of perfect cellular links in  $\mathcal{A}$ . Therefore,  $N_c(\mathcal{A})$  is a measure of the average number of perfect cellular links in  $\mathcal{A}$ . Due to the stationary of PPPs,  $N_c(\mathcal{A})$  is independent of  $\mathcal{A}$ . Hence, we use  $N_c$  to denote the average number of perfect cellular links per unit area, and

$$N_c = \lambda_b p_{con}^{(c)} \left( T_\phi \right) p_{sec}^{(c)} \left( T_\epsilon \right).$$
(31)

By Corollary 1 and 2, we have

$$N_c = \lambda_b a x \exp\left(-bx\right),\tag{32}$$

where  $a = \frac{\sin c \delta}{T_{\phi}^{\delta}} > 0$ ,  $b = \frac{\lambda_e \sin c \delta}{\lambda_b T_e^{\delta}} > 0$ ,  $x = \frac{1}{1 + \frac{\lambda_d}{\lambda_b} \left(\frac{P_d}{P_b}\right)^{\delta}} \in (0, 1]$ . From (32) we can see that D2D communications introduce a factor of  $\frac{\lambda_d}{\lambda_b} \left(\frac{P_d}{P_b}\right)^{\delta}$  into the performance metrics of cellular communications. By letting  $\frac{\lambda_d}{\lambda_b} \left(\frac{P_d}{P_b}\right)^{\delta} = 0$ , i.e. x = 1, in (32), we get

$$N_c^{(0)} = \lambda_b a \exp\left(-b\right),\tag{33}$$

where the superscript (0) indicates the case in the absence of D2D links in the network.

By comparing (27) and (31), we can see that the strong guarantee criterion (27) is equivalent to

$$N_c \ge N_c^{(0)}.\tag{34}$$

The following lemma shows the feasible region of D2D scheduling parameters under criterion (34).

**Lemma 1.** The feasible region constrained by the strong guarantee criterion is

$$\mathcal{F}_{str} = \left\{ (\lambda_d, P_d) : \\ \begin{cases} \lambda_d P_d^{\delta} \le \left( -\frac{b}{W_p(-be^{-b})} - 1 \right) \lambda_b P_b^{\delta}, & if \ b > 1 \\ \lambda_d = P_d = 0, & if \ b \le 1 \end{cases} \right\},$$
(35)

where  $W_{p}(\cdot)$  is the real-valued principal branch of Lambert W-function.

*Proof:* The proof comprises two steps. In the first step, we derive the solution to  $N_c = N_c^{(0)}$ , and in the second step, we further derive the solution to  $N_c \ge N_c^{(0)}$  based on the results obtained in the first step.

(1) Evaluate the equation  $N_c = N_c^{(0)}$ . By (32) and (33),  $N_c = N_c^{(0)}$  is equivalent to

$$e^{b(1-x)} = \frac{1}{x}.$$
 (36)

The derivation of the solution to (36) is as follows:

$$e^{b(1-x)} = \frac{1}{x}$$

$$\Rightarrow -bxe^{-bx} = -be^{-b}$$

$$\stackrel{(a)}{\Rightarrow} ye^{y} = -be^{-b}$$

$$\stackrel{(b)}{\Rightarrow} y = W_{p} (-be^{-b}) \bigcup W_{m} (-be^{-b})$$

$$\stackrel{(c)}{\Rightarrow} x = -\frac{1}{b} W_{p} (-be^{-b}) \bigcup -\frac{1}{b} W_{m} (-be^{-b}) . (37)$$

(a) employs a change of variable y = -bx. In (b), the solution of y has two branches due to  $-be^{-b} \in \left(-\frac{1}{e}, 0\right)$ . W<sub>p</sub>(·) and W<sub>m</sub>(·) are the real-valued principal branch and the other branch of Lambert W-function respectively [36]. It is noted that W<sub>p</sub>(·) and W<sub>m</sub>(·) are also denoted as W<sub>0</sub>(·) and W<sub>-1</sub>(·) in some literature. (c) makes an inverse variable change that  $x = -\frac{1}{b}y$ .

By examining the two branches of the solution in (37), we find that  $-\frac{1}{b} W_p(-be^{-b}) \in (0,1]$  and  $-\frac{1}{b} W_m(-be^{-b}) \in [1,\infty)$ . Considering  $0 < x \le 1$ , we reject the second branch and obtain the final solution to  $N_c = N_c^{(0)}$ :

$$x_0 = -\frac{1}{b} \operatorname{W}_{\mathbf{p}} \left( -be^{-b} \right). \tag{38}$$

(2) Evaluate  $N_c \ge N_c^{(0)}$ , which is equivalent to

$$xe^{-bx} \ge e^{-b}.$$
 (39)

Let  $f(x) = xe^{-bx}$  ( $0 < x \le 1$ ). Then, (39) is equivalent to

$$f(x) \ge f(1). \tag{40}$$

Next, we derive the solution to (40). Taking the derivative of f(x) with respect to x, we get

$$f'(x) = (1 - bx) e^{-bx}.$$
 (41)

Three cases are considered according to the value of b.

Case 1: b < 1. In this case, f'(x) > 0 and thereby f(x) monotonically increases in  $0 < x \le 1$ . Therefore, the solution to (40) is x = 1, i.e.,

$$\mathcal{F}_{str} = \left\{ (\lambda_d, P_d) : \lambda_d = P_d = 0 \right\}.$$
(42)

Case 2: b = 1. It is easy to verify that the solution is also (42).

Case 3: b > 1. In this case, f'(x) > 0 when  $x \in (0, \frac{1}{b})$ and f'(x) < 0 when  $x \in (\frac{1}{b}, 1]$ . Thus f(x) monotonically increases in  $0 < x < \frac{1}{b}$ , and monotonically decreases in  $\frac{1}{b} < x \le 1$ . In addition, by the results obtained in step (1), f(x) = f(1) holds at point  $x_0 = -\frac{1}{b} W_p(-be^{-b})$ and  $x_1 = -\frac{1}{b} W_m(-bx_0e^{-bx_0}) = 1$ , where  $x_0 < x_1 = 1$ . Therefore, the solution to (40) is  $x \in [x_0, 1]$ , i.e.,

$$\mathcal{F}_{str} = \left\{ (\lambda_d, P_d) : \lambda_d P_d^{\delta} \le \left( -\frac{b}{W_p \left( -be^{-b} \right)} - 1 \right) \lambda_b P_b^{\delta} \right\}.$$
(43)

Combining the results of case 1-3, we complete the proof. *Remark* 4. Lemma 1 shows that, under the strong guarantee criterion, the performance of cellular links is hampered by D2D links if  $\lambda_e \leq \frac{T_e^{\delta}}{\operatorname{sinc} \delta} \lambda_b$ , and is enhanced otherwise. This is because, for D2D links, their interfering effect on cellular links is critical when the eavesdropper intensity is small, while their jamming effect on eavesdropping links becomes dominant when the intensity is large.

### B. D2D Link Scheduling Schemes

Based on the derived feasible region of D2D scheduling parameters, we study the D2D link scheduling problems for optimizing network performance. Note that we focus only on the case of b > 1, since when  $b \le 1$  D2D communications should be blocked by the network.

The first problem is how to obtain the maximum average number of perfect cellular transmissions per unit area, i.e.,

$$\max_{(\lambda_d, P_d)} N_c, \quad \text{s.t.} \ (\lambda_d, P_d) \in \mathcal{F}_{str}.$$
(44)

Lemma 2. The optimal solution of problem (44) is

$$\mathcal{F}_{str}' = \left\{ \left(\lambda_d, P_d\right) : \lambda_d P_d^{\delta} = \left(b - 1\right) \lambda_b P_b^{\delta} \right\}.$$
(45)

*Proof:* According to the proof of Lemma 1,  $f'(x) = 0 \Rightarrow x = \frac{1}{b}$ , and  $f''(\frac{1}{b}) < 0$ . Therefore, the optimal solution is  $x = \frac{1}{b}$ .

Lemma 2 shows that there exist a series of  $(\lambda_d, P_d)$  pairs that can achieve maximum  $N_c$ . Next, we further study which pair(s) among them can achieve the optimal D2D performance.

We employ the average number of perfect D2D links per unit area as the metric for D2D communications, which is defined as

$$N_d = \lambda_d p_{con}^{(d)} \left( T_\sigma \right). \tag{46}$$

By Corollary 3, we have

$$N_d = \lambda_d \exp\left(-\frac{c}{1-x}\lambda_d\right),\tag{47}$$

where  $c = \frac{\pi l^2 T_{\sigma}^{\delta}}{\operatorname{sinc} \delta} > 0$ . Then, the optimization problem is **P1**:

$$\max_{(\lambda_d, P_d)} N_d, \quad \text{s.t.} \ (\lambda_d, P_d) \in \mathcal{F}'_{str}.$$
(48)

Theorem 4. The optimal solution of Problem 1 is

$$S_{1} = \left\{ (\lambda_{d}^{*}, P_{d}^{*}) : \lambda_{d}^{*} = \frac{b-1}{bc}, P_{d}^{*} = (bc\lambda_{b})^{\frac{1}{\delta}} P_{b} \right\}.$$
 (49)

*Proof:* The constraint of P1 is equivalent to  $x = \frac{1}{b}$ , thereby  $N_d = \lambda_d \exp\left(-\frac{bc}{b-1}\lambda_d\right)$ , which depends only on  $\lambda_d$ . Similar to the proof of Lemma 1,  $N_d$  monotonically increases in  $\lambda_d \in \left(0, \frac{b-1}{bc}\right)$ , and monotonically decreases in  $\lambda_d \in \left(\frac{b-1}{bc}, \infty\right)$ . Therefore, the maximum value of  $N_d$  is obtained at point  $\lambda_d^* = \frac{b-1}{bc}$ . By (45), we have  $P_d^* = (bc\lambda_b)^{\frac{1}{\delta}} P_b$ .

Relaxing the constraint of Problem 1 to the strong guarantee criterion (27), we have

$$\max_{(\lambda_d, P_d)} N_d, \quad \text{s.t.} \ (\lambda_d, P_d) \in \mathcal{F}_{str}.$$
(50)

**Theorem 5.** The optimal solution of Problem 2 is

$$S_{2} = \left\{ (\lambda_{d}^{*}, P_{d}^{*}) : \lambda_{d}^{*} = \frac{b + W_{p} \left( -be^{-b} \right)}{bc}, \\ P_{d}^{*} = \left( -\frac{bc\lambda_{b}}{W_{p} \left( -be^{-b} \right)} \right)^{\frac{1}{\delta}} P_{b} \right\}.$$
(51)

*Proof:* The constraint of P2 is equivalent to  $x \in [x_0, 1]$ . Then,  $N_d = \lambda_d \exp\left(-\frac{c}{1-x}\lambda_d\right) \leq \lambda_d \exp\left(-\frac{c}{1-x_0}\lambda_d\right)$ , and the equality holds iff  $x = x_0$ . Similar to the proof of Lemma 1, the maximum value of the upper bound of  $N_d$  is obtained at point  $\lambda_d^* = \frac{1-x_0}{c}$ . By  $x = x_0$ ,  $P_d^* = \left(\frac{c\lambda_b}{x_0}\right)^{\frac{1}{\delta}} P_b$ . Theorem 4 and 5 give two D2D link scheduling schemes,  $S_1$  and  $S_2$ , under the strong performance guarantee criterion for cellular links. For schemes  $S_1$ ,

$$N_c^{(1)} = \frac{\lambda_b a}{b} e^{-1},\tag{52}$$

$$N_d^{(1)} = \frac{b-1}{bc} e^{-1},$$
(53)

and for schemes  $\mathcal{S}_2$ ,

$$N_c^{(2)} = \lambda_b a e^{-b},\tag{54}$$

$$N_d^{(2)} = \frac{b + W_p(-be^{-b})}{bc}e^{-1}.$$
 (55)

By observing (52) - (55), we can get that

$$N_c^{(1)} > N_c^{(2)} = N_c^{(0)},\tag{56}$$

$$N_d^{(1)} < N_d^{(2)}.$$
(57)

The results show that schemes  $S_1$  can achieve the optimal performance for cellular communications, while  $S_2$  can provide a higher performance level for D2D communications.

## VI. OPTIMAL D2D LINK SCHEDULING UNDER WEAK GUARANTEE CRITERION FOR CELLULAR LINKS

In this section, we study the D2D link scheduling problems under the weak performance guarantee criterion for cellular links. Two D2D link scheduling schemes are designed for interference-limited networks.

#### A. Feasible Region of D2D Scheduling Parameters

The weak guarantee criterion (28) requires the minimum connection probability  $\phi$  and secrecy probability  $\epsilon$  for cellular links. The following lemma gives the feasible region of D2D scheduling parameters under this criterion.

**Lemma 3.** The feasible region constrained by the weak guarantee criterion is

$$\mathcal{F}_{weak} = \left\{ \left(\lambda_d, P_d\right) : \left(\frac{b}{\ln\frac{1}{\epsilon}} - 1\right)\lambda_b P_b^{\delta} \le \lambda_d P_d^{\delta} \\ \le \left(\frac{a}{\phi} - 1\right)\lambda_b P_b^{\delta} \right\}.$$
 (58)

Proof: By  $p_{con}^{(c)}(T_{\phi}) \geq \phi$  and (11), we have  $x \geq \frac{1}{a}\phi$ , where  $x = \frac{1}{1 + \frac{\lambda_d}{\lambda_b} \left(\frac{P_d}{P_b}\right)^{\delta}}$  is defined in (32). By  $p_{sec}^{(c)}(T_{\epsilon}) \geq \epsilon$ and (19), we have  $x \leq \frac{1}{b} \ln \frac{1}{\epsilon}$ . Therefore,  $\frac{1}{a}\phi \leq x \leq \frac{1}{b} \ln \frac{1}{\epsilon}$ .  $\blacksquare$ *Remark* 5. By Lemma 1 and 3,  $\mathcal{F}_{str}$  exists only for b > 1, while  $\mathcal{F}_{weak}$  exists for any b. This is due to the fact that, comparing to the strong guarantee criterion (27), the weak guarantee criterion (28) relaxes the secrecy constraint and hence provide extra transmission opportunities for D2D users.

Before investigating the D2D link scheduling problems, we should determine the feasible values of  $\phi$  and  $\epsilon$ . A reasonable range of  $(\phi, \epsilon)$  is given by

$$\mathcal{R} = \left\{ (\phi, \epsilon) : 0 \le \phi \le a, \ e^{-b} \le \epsilon \le 1, \right.$$

$$\phi \epsilon \le a e^{-b}, \ \frac{1}{\phi} \ln \frac{1}{\epsilon} \ge \frac{b}{a} \bigg\},$$
 (59)

where the first two conditions correspond to the fact that  $0 \leq p_{con}^{(c)}(T_{\phi}) \leq a$  and  $e^{-b} \leq p_{sec}^{(c)}(T_{\epsilon}) \leq 1$ , the third condition corresponds to the definition of weak guarantee criterion that  $\phi \epsilon \leq p_{con}^{(c)(0)}(T_{\phi}) p_{sec}^{(c)(0)}(T_{\epsilon})$ , and the last one corresponds to the implied condition in Lemma 3 that  $\frac{1}{a}\phi \leq \frac{1}{b}\ln\frac{1}{\epsilon}$ . In the following analysis, we assume  $(\phi, \epsilon) \in \mathcal{R}$ .

### B. D2D Link Scheduling Schemes

We first investigate how to obtain the maximum value of  $N_c$ , i.e.,

$$\max_{(\lambda_d, P_d)} N_c, \quad \text{s.t.} \ (\lambda_d, P_d) \in \mathcal{F}_{weak}.$$
(60)

Lemma 4. The optimal solution of problem (60) is

$$\begin{aligned} \mathcal{F}'_{weak} &= \{ (\lambda_d, P_d) : \\ \begin{cases} \lambda_d P_d^{\delta} &= \left(\frac{b}{\ln \frac{1}{\epsilon}} - 1\right) \lambda_b P_b^{\delta}, & if \ b \le 1 \ or \ b > 1, \epsilon > e^{-1} \\ \lambda_d P_d^{\delta} &= \left(\frac{a}{\phi} - 1\right) \lambda_b P_b^{\delta}, & if \ b > 1, \phi > \frac{a}{b} \end{cases} \} . \\ \lambda_d P_d^{\delta} &= (b-1) \lambda_b P_b^{\delta}, & if \ b > 1, \epsilon \le e^{-1}, \phi \le \frac{a}{b} \end{aligned}$$

$$\end{aligned}$$

*Proof:* Consider two cases: b < 1 and b > 1.

(1)  $b \leq 1$ . In this case,  $N_c$  monotonically increases in  $\frac{1}{a}\phi \leq x \leq \frac{1}{b} \ln \frac{1}{\epsilon}$ . Therefore, the solution of (60) is  $x = \frac{1}{b} \ln \frac{1}{\epsilon}$ .

(2) b > 1. Three cases are considered according to the values of  $\frac{1}{a}\phi$  and  $\frac{1}{b}\ln\frac{1}{\epsilon}$ . Case 1:  $\frac{1}{b}\ln\frac{1}{\epsilon} < \frac{1}{b}$ , i.e.,  $\epsilon > e^{-1}$ . In this case,  $N_c$  monotonically increases in  $\frac{1}{a}\phi \le x \le \frac{1}{b}\ln\frac{1}{\epsilon}$ . Therefore, the solution is  $x = \frac{1}{b}\ln\frac{1}{\epsilon}$ . Case 2:  $\frac{1}{a}\phi > \frac{1}{b}$ , i.e.,  $\phi > \frac{a}{b}$ . In this case,  $N_c$  monotonically decreases in  $\frac{1}{a}\phi \le x \le \frac{1}{b}\ln\frac{1}{\epsilon}$ . Therefore, the solution is  $x = \frac{1}{b}h$ . Case 3:  $\frac{1}{a}\phi > \frac{1}{b}$ . In this case,  $N_c$  monotonically decreases in  $\frac{1}{a}\phi \le x \le \frac{1}{b}\ln\frac{1}{\epsilon}$ . Therefore, the solution is  $x = \frac{1}{a}\phi$ . Case 3:  $\frac{1}{a}\phi \le \frac{1}{b} \le \frac{1}{b}\ln\frac{1}{\epsilon}$ , i.e.,  $\epsilon \le e^{-1}$ ,  $\phi \le \frac{a}{b}$ . In this case,  $N_c$  monotonically increases in  $\frac{1}{a}\phi < x < \frac{1}{b}$  and monotonically decreases in  $\frac{1}{a}\phi < x < \frac{1}{b}\ln\frac{1}{\epsilon}$ . Therefore, the solution is  $x = \frac{1}{a}b$ . Combining the above results, we complete the proof.

Lemma 4 shows that there exist a series of  $(\lambda_d, P_d)$  pairs that can achieve maximum  $N_c$ . Next, we further study which pair(s) among them can achieve maximum  $N_d$ .

$$\max_{(\lambda_d, P_d)} N_d, \quad \text{s.t.} \ (\lambda_d, P_d) \in \mathcal{F}'_{weak}.$$
(62)

**Theorem 6.** The optimal solution of Problem 3 is

$$S_{3} = \left\{ \left(\lambda_{d}^{*}, P_{d}^{*}\right) : \\ \begin{cases} \lambda_{d}^{*} = \frac{b - \ln \frac{1}{e}}{bc}, P_{d}^{*} = \left(\frac{bc}{\ln \frac{1}{e}}\lambda_{b}\right)^{\frac{1}{\delta}}P_{b}, & \text{if } b \le 1 \text{ or } b > 1, \epsilon > e^{-1} \\ \lambda_{d}^{*} = \frac{a - \phi}{ac}, P_{d}^{*} = \left(\frac{ac}{\phi}\lambda_{b}\right)^{\frac{1}{\delta}}P_{b}, & \text{if } b > 1, \phi > \frac{a}{b} \\ \lambda_{d}^{*} = \frac{b - 1}{bc}, P_{d}^{*} = (bc\lambda_{b})^{\frac{1}{\delta}}P_{b}, & \text{if } b > 1, \epsilon \le e^{-1}, \phi \le \frac{a}{b} \end{cases} \right\}.$$
(63)

*Proof:* Similar to the proof of Theorem 4, the maximum value of  $N_d$  is obtained at point  $\lambda_d^* = \frac{1-x}{c}$ , where x is the solution of problem (60) given in Lemma 5.

Relaxing the constraint of Problem 3 to the weak guarantee criterion (28), we have

$$\max_{(\lambda_d, P_d)} N_d, \quad \text{s.t.} \ (\lambda_d, P_d) \in \mathcal{F}_{weak}.$$
(64)

	$\lambda_d$	$P_d$	$N_c$	$N_d$	
$\mathcal{S}_1$	$\frac{b-1}{bc}$	$(bc\lambda_b)^{rac{1}{\delta}} P_b$	$\frac{\lambda_b a}{b} e^{-1}$	$\frac{b-1}{bc}e^{-1}$	
$\mathcal{S}_2$	$\frac{b + W_{p}\left(-be^{-b}\right)}{bc}$	$\left(-\frac{bc\lambda_b}{\mathrm{W}_{\mathrm{p}}\left(-be^{-b}\right)}\right)^{\frac{1}{\delta}}P_b$	$\lambda_b a e^{-b}$	$\frac{b + W_{\rm p}\left(-be^{-b}\right)}{bc}e^{-1}$	
$\mathcal{S}_3$	$\begin{cases} \frac{b-\ln\frac{1}{c}}{bc}, & if \ case \ 1\\ \frac{a-\phi}{b-1}, & if \ case \ 2\\ \frac{b-1}{bc}, & if \ case \ 3 \end{cases}$	$\begin{cases} \left(\frac{bc}{\ln\frac{1}{\epsilon}}\lambda_b\right)^{\frac{1}{\delta}}P_b, & if \ case \ 1\\ \left(\frac{ac}{\phi}\lambda_b\right)^{\frac{1}{\delta}}P_b, & if \ case \ 2\\ (bc\lambda_b)^{\frac{1}{\delta}}P_b, & if \ case \ 3 \end{cases}$	$\begin{cases} \frac{\lambda_b a \epsilon}{b} \ln \frac{1}{\epsilon}, & if \ case \ l \\ \lambda_b \phi \exp \left(-\frac{b}{a} \phi\right), & if \ case \ 2 \\ \frac{\lambda_b a}{b} e^{-1}, & if \ case \ 3 \end{cases}$	$\begin{cases} \frac{b-\ln\frac{1}{\epsilon}}{bc}e^{-1}, & if \ case \ 1\\ \frac{a-\phi}{bc}e^{-1}, & if \ case \ 2\\ \frac{b-\alpha_1}{bc}e^{-1}, & if \ case \ 3 \end{cases}$	
$\mathcal{S}_4$	$\frac{a-\phi}{ac}$	$\left(rac{ac}{\phi}\lambda_b ight)^{rac{1}{\delta}}P_b$	$\lambda_b \phi \exp\left(-rac{b}{a}\phi ight)$	$\frac{a-\phi}{ac}e^{-1}$	
In the table (1) $a = \frac{\sin \delta}{b} = \frac{\lambda_e \sin \delta}{b} = \frac{\lambda_e \sin \delta}{c} = \frac{\pi l^2 T_{\sigma}^3}{c}$					

Table II: A summary of the proposed D2D link scheduling schemes

In the table, (1)  $a = \frac{\sin c}{T_{\phi}^{\delta}}, b = \frac{\lambda e \sin c}{\lambda_b T_{\epsilon}^{\delta}}, c = \frac{\pi e}{\sin c} \frac{x_{\sigma}}{\delta};$ (2) case 1:  $b \leq 1 \text{ or } b > 1, \epsilon > e^{-1}$ , case 2:  $b > 1, \phi > \frac{a}{b}$ , case 3:  $b > 1, \epsilon \leq e^{-1}, \phi \leq \frac{a}{b}$ .

#### **Theorem 7.** The optimal solution of Problem 4 is

$$\mathcal{S}_4 = \left\{ (\lambda_d^*, P_d^*) : \lambda_d^* = \frac{a - \phi}{ac}, P_d^* = \left(\frac{ac}{\phi}\lambda_b\right)^{\frac{1}{\delta}} P_b \right\}.$$
(65)

Proof: The constraint of P4 is equivalent to  $x \in \left[\frac{1}{a}\phi, \frac{1}{b}\ln\frac{1}{\epsilon}\right]$ . Then,  $N_d = \lambda_d \exp\left(-\frac{c}{1-x}\lambda_d\right)$  $\leq$  $\lambda_d \exp\left(-\frac{c}{1-\frac{1}{a}\phi}\lambda_d\right)$ , and the equality holds iff  $x = \frac{1}{a}\phi$ . Therefore, the maximum value of  $N_d$  is obtained at point  $\lambda_d^* = \frac{1 - \frac{1}{a}\phi}{c}$ . By  $x = \frac{1}{a}\phi$ , we have  $P_d^* = \left(\frac{ac}{\phi}\lambda_b\right)^{\frac{1}{\delta}}P_b$ . Theorem 6 and 7 give two D2D link scheduling schemes,

 $S_3$  and  $S_4$ , under the weak performance guarantee criterion for cellular links. For schemes  $S_3$ ,

$$N_c^{(3)} = \begin{cases} \frac{\lambda_b a\epsilon}{b} \ln \frac{1}{\epsilon}, & \text{if } b \le 1 \text{ or } b > 1, \epsilon > e^{-1} \\ \lambda_b \phi \exp\left(-\frac{b}{a}\phi\right), & \text{if } b > 1, \phi > \frac{a}{b} \\ \frac{\lambda_b a}{b} e^{-1}, & \text{if } b > 1, \epsilon \le e^{-1}, \phi \le \frac{a}{b} \end{cases}$$

$$\tag{66}$$

$$N_{d}^{(3)} = \begin{cases} \frac{b-\ln\frac{1}{\epsilon}}{bc}e^{-1}, & if \ b \le 1 \ or \ b > 1, \epsilon > e^{-1} \\ \frac{a-\phi}{ac}e^{-1}, & if \ b > 1, \phi > \frac{a}{b} \\ \frac{b-1}{bc}e^{-1}, & if \ b > 1, \epsilon \le e^{-1}, \phi \le \frac{a}{b} \end{cases},$$
(67)

and for schemes  $S_4$ ,

$$N_c^{(4)} = \lambda_b \phi \exp\left(-\frac{b}{a}\phi\right),\tag{68}$$

$$N_d^{(4)} = \frac{a - \phi}{ac} e^{-1}.$$
 (69)

By observing (66) - (69), we can get that

$$N_c^{(3)} \ge N_c^{(4)},$$

$$N_c^{(3)} < N_c^{(4)}$$
(70)
(71)

$$N_d^{(6)} \le N_d^{(4)}.$$
 (71)

The results show that schemes  $S_3$  can achieve the optimal performance for cellular communications, while  $S_4$  can provide a higher performance level for D2D communications.

Remark 6.  $N_c$ ,  $N_d$  of schemes  $S_3$ ,  $S_4$  depend on specific values of  $\phi$  and  $\epsilon$ . Thus it is difficult to compare  $N_c$ ,  $N_d$  of  $S_1, S_2$  with those of  $S_3, S_4$  by analytical results. A summary of the proposed D2D link scheduling schemes is shown in Table II. In the simulation, we provide numerical illustration for performance comparison of these schemes.

#### VII. SIMULATION RESULTS

In this section, we first present some simulation results to validate the proposed model and analytical results, and then provide some numerical results on performances of D2D link scheduling schemes.

#### A. Validation of Analytical Results

As all the link scheduling schemes are designed based on the probabilities derived in section IV, in this part we validate the analytical results of these probabilities by simulations. The simulations employ PPP model with main simulation parameters  $\alpha = 3, l = 0.05, T_{\phi} = 0.25, T_{\epsilon} = 0.5, T_{\sigma} =$ 0.5,  $\lambda_b = 2$ ,  $\lambda_e = 6$ , and the results are shown in Fig.2. Fig.2(a) shows  $p_{con}^{(c)}$ ,  $p_{sec}^{(c)}$ ,  $p_{con}^{(d)}$  versus  $\lambda_d$ . As can be observed from the curves, the analytical results of the probabilities that are derived in Corollary 1-3, are in quite good agreement with corresponding simulation results. This fact confirms that our proposed framework closely matches the practical D2Denabled cellular network with eavesdroppers.  $p_{per}^{(c)}$ , which denotes the probability of perfect cellular transmissions (see (27) and footnote 3), is another critical metrics for the network. In the analysis, we approximate  $p_{per}^{(c)}$  by  $p_{con}^{(c)} \cdot p_{sec}^{(c)}$ . However, in fact,  $p_{per}^{(c)} < p_{con}^{(c)} \cdot p_{sec}^{(c)}$ , since the interference at each point is not independent in practical networks. Fig.2(b) plots  $p_{per}^{(c)}$  and  $p_{con}^{(c)} \cdot p_{sec}^{(c)}$  in different scenarios. As expected, when  $P_b$  is small and  $\lambda_d$  is large, the difference between  $p_{per}^{(c)}$  and  $p_{con}^{(c)} \cdot p_{sec}^{(c)}$  is large. It is because small transmission power of cellular links, which can be regarded correspondingly as large transmission power of D2D links, and large intensity of D2D links lead to stronger correlation of interference generated by D2D links. This result implies that the approximation is more precise with larger  $P_b$  and smaller  $\lambda_d$ . Considering that in practical networks, the transmission power of base stations is usually much larger than that of D2D terminals, and the intensity of D2D links can be properly controlled by the network, the approximation error of  $p_{per}^{(c)}$  is not large.

### B. Performance Evaluation of Proposed Schemes

In this part we provide some numerical results to evaluate the performances of the proposed link scheduling schemes. Fig.3 shows  $N_c, N_d$  versus  $\lambda_d$  and  $P_d$  under the strong



Figure 2: Validation of analytical results of  $p_{con}^{(c)}$ ,  $p_{sec}^{(c)}$ ,  $p_{con}^{(d)}$  and  $p_{per}^{(c)}$ .



Figure 3:  $N_c$ ,  $N_d$  under the strong guarantee criterion.

guarantee criterion. In the figure, the feasible region  $\mathcal{F}'_{str}$  and the boundary of the feasible region  $\mathcal{F}_{str}$  are shown by the marked curves, and the optimal  $\lambda_d$  and  $P_d$  derived in scheme  $S_1$  and  $S_2$  are shown by the marked points. Fig.3(a) shows that when  $(\lambda_d, P_d)$  is below  $\mathcal{F}'_{str}$ ,  $N_c$  increases as  $\lambda_d$ ,  $P_d$  increase; however, when  $(\lambda_d, P_d)$  is above  $\mathcal{F}'_{str}$ ,  $N_c$  decreases as  $\lambda_d, P_d$ increase. This result reflects that when  $\lambda_d$ ,  $P_d$  are small, the effect of interference from D2D links to eavesdropping links is dominant, but when  $\lambda_d$ ,  $P_d$  are large, the effect of interference to cellular links becomes dominant. As we can observe from Fig.3,  $N_c^{(1)}$  is the maximum value of  $N_c$  within  $\mathcal{F}'_{str}$  and  $N_d^{(2)}$ is the maximum value of  $N_d$  within  $\mathcal{F}_{str}$ , which matches the analysis in section V. Furthermore, the figure shows that the transmission power and intensity of D2D links of scheme  $S_1$ are smaller than those of scheme  $S_2$  respectively. This suggests that larger transmission power and intensity of D2D links are desirable for improving D2D performances.

Fig.4 shows  $N_c, N_d$  of scheme  $S_1$  and  $S_2$  versus  $\lambda_b$  with different  $\lambda_e$ . We can see that for both the schemes, as  $\lambda_b$ 

increases,  $N_c$  increases while  $N_d$  reduces. In addition, for both the schemes,  $N_c$  is smaller when  $\lambda_e$  is larger, since a larger population of eavesdroppers lowers the probability of secrecy cellular transmissions. However, for both the schemes,  $N_d$  is larger when  $\lambda_e$  is larger. This is because when the intensity of eavesdroppers increases, larger transmission power and intensity of D2D links are required to guarantee the performance of cellular transmissions, which creates more transmission opportunities for D2D links. Furthermore, comparing between the curves in the figure, we can see that when  $\lambda_b$  is larger and  $\lambda_e$  is smaller, the performance of scheme  $S_1$  approaches that of scheme  $S_2$ .

Fig.5 compares  $N_c$ ,  $N_d$  of the proposed schemes  $S_1 - S_4$ , where we consider three cases of  $\phi$ ,  $\epsilon$  for scheme  $S_3$ ,  $S_4$  in the numerical results. By Fig.5, in case 1,  $N_c$  of scheme  $S_3$ ,  $S_4$  are smaller than that of scheme  $S_1$ ,  $S_2$  respectively, while  $N_d$  of scheme  $S_3$ ,  $S_4$  are larger than that of scheme  $S_1$ ,  $S_2$  respectively; in case 2 and 3,  $N_c$ ,  $N_d$  of scheme  $S_3$ equal or approximately equal those of scheme  $S_1$  respectively,



Figure 4:  $N_c$ ,  $N_d$  of schemes  $S_1$ ,  $S_2$ .

while  $N_c$ ,  $N_d$  of scheme  $S_4$  are larger than those of scheme  $S_2$ respectively. This result suggests that by adjusting parameter  $\phi$ and  $\epsilon$ , different performance levels of cellular and D2D links can be achieved. In addition, it is noted that in the simulation *b* is above 1. For the scenario  $b \leq 1$ , the strong guarantee criteria blocks D2D communications, but the weak guarantee criteria admits some D2D links to the network and scheme  $S_3$ ,  $S_4$  are optimal link scheduling schemes. Therefore, scheme  $S_3$ ,  $S_4$  are applicable to a broader range of system parameters, comparing with scheme  $S_1$ ,  $S_2$ .

### VIII. CONCLUSIONS

In this paper, we focus on a large-scale D2D-enabled cellular network in which cellular communications are overheard by eavesdroppers, and propose a framework for modeling such a network via stochastic geometry. We derive the expressions for SINR distributions and connection and secrecy probabilities of cellular and D2D links, based on which we further design optimal D2D link scheduling schemes under both strong and weak performance guarantee criteria for cellular communications. By investigating both analytical and numerical results, we find out that the interference from D2D communications can be exploited to enhance physical layer security of cellular communications and meanwhile create extra transmission opportunities for D2D users. This study provides a new perspective on the role of the interference generated by D2D communications.

The main limitation of current model is that the mode (cellular mode or D2D mode) of each user is preset. A major area of future work is to study the scenario where each user can change its communication mode. Another possible extension of this work is to consider eavesdropper collusion in the network, and design D2D link scheduling schemes that are robust to colluding eavesdropping.

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Figure 5: Performance comparison of schemes  $S_1 - S_4$ . For scheme  $S_3, S_4$ , three cases of  $\phi$ ,  $\epsilon$  are considered. Case 1:  $\phi = e^{-b}$ ,  $\epsilon = 0.5$ , Case 2:  $\phi = \frac{1.5}{b}$ , Case 3:  $\phi = \frac{a}{2b}$ ,  $\epsilon = 0.2$ .

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