# Interference-Limited Mixed MUD-RF/FSO Two-Way Cooperative Networks over Double Generalized Gamma Turbulence Channels

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# Interference-Limited Mixed MUD-RF/FSO Two-Way Cooperative Networks over Double Generalized Gamma Turbulence Channels

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***Abstract*—In this letter, the performance of multiuser-radio frequency/free space optics (RF/FSO) two-way relay network in the presence of interference is investigated. The FSO link accounts for pointing errors and both types of detection techniques, i.e. intensity modulation/direct detection as well as coherent demodulation, which is modeled as double generalized gamma (D-GG) turbulence channel. On the other hand, the multiple users on the RF link are assumed to undergo Nakagami-$m$ fading. Multiple co-channel interferers (CCIs) which corrupt the signal at relay node are modeled using Nakagami-$m$ distribution. Specifically, the exact closed-form expressions for the outage probability (OP) of the overall system is derived. Moreover, the closed form expression for the achievable sum-rate (ASR) of the considered system is presented. In order to simplify the results, the asymptotic approximations of the OP and ASR are derived in terms of elementary functions. The results presented in the paper are validated by Monte-Carlo simulations.**

***Index Terms*—Two-way relaying, multiuser diversity, co-channel interference, double generalized Gamma fading.**

## I. Introduction

THE offerings of free-space optics (FSO) systems such as rapid deployment time, high security, flexibility are limited by the atmospheric turbulence [1]. Besides, utilization of FSO system as an alternative and/or as complement to radio frequency (RF) counterparts has given rise to mixed RF/FSO relay systems [2]. The benefits of robustness to fading of RF signal and high bandwidth available with FSO systems is combined in mixed RF/FSO relaying systems. Additionally, RF systems are low cost and non line of sight communication whereas FSO systems have the advantage of low latency coupled with high data rate. Notably, FSO links offer typically higher speeds than their RF counterparts and as such FSO link can be used as a back-haul for multiplexing multiple RF connections. Therefore, advantages of both RF and FSO systems result in cost effective last-mile connection which increases reliability and connectivity along with increased data rates to the end-user with rapid deployment at low cost. Moreover, conventional one-way-relaying (OWR) is spectrally inefficient since the exchange of information between source to destination requires two complete time-slots. Spectral efficiency can be improved by utilizing two-way-relaying (TWR) scheme [3], where the transmission of information from two nodes takes place simultaneously, through a relay node. In the first phase, the two nodes transmit information simultaneously to the relay node, which is broadcasted by the relay to the designated destinations in the second phase of communication. This brings about an improvement by factor of 2 when compared to OWR strategy.

While most of the known results for the FSO relay-assisted communications have relied on the absence of interference, it is important to note that asymmetric mixed RF/FSO relay systems are inherently vulnerable to the effect of co-channel interference (CCI) due to the involvement of RF links. The recognition of the interference-limited behavior of mixed RF/FSO systems has motivated the authors of [4] to derive the expression of outage probability (OP) and bit error rate (BER) for an interference limited mixed RF/FSO amplify-and-forward (AF) OWR relaying system over double-generalized Gamma (D-GG) turbulence channels. Moreover, the effect of multiple CCIs on the mixed RF/FSO OWR systems have been analyzed in [5] where the authors have considered the presence of channel state information (CSI). On the other hand, impact of multiuser diversity (MUD) on interference limited mixed RF/FSO OWR systems has been indicated by the authors of [6]. Furthermore, research work [7] demonstrates the improvement imparted by MUD scheme on mixed RF/FSO TWR system for interference free transmission. Notably, no work has been reported on the effect of interference over MUD assisted mixed RF/FSO TWR system. This motivates to explore the performance for a bidirectional MUD-RF/FSO system, where the relay node operates in the presence of multiple CCIs. The main contributions of this work are:

1) In this work, performance analysis of interference limited RF/FSO cooperative TWR networks is performed, where user diversity scheme is implemented, such that the relay node selects the user with best channel condition.
2) The closed form expressions for performance metrics such as OP and ergodic sum-rate is derived.
3) The OP and ergodic sum-rate expressions account for both intensity modulation/direct detection (IM/DD) and coherent demodulation schemes on the optical link, in the presence of pointing error.
4) Finally, the derived closed form expression for the OP is expressed asymptotically to validate the proposed work.

## II. System and Channel Models

Consider a mixed RF/FSO two-way cooperative AF relaying system, where $K$ mobile users on the RF link communicate with the FSO destination node through the bidirectional relay node $R$. In particular, the relay node exchanges information between two source nodes $S_{1,j}$ and $S_2$, where $S_{1,j}$ represents the $j^{th}$-user selected by the relay node based on best channel conditions. The user selection at relay node is performed using the opportunistic scheduling based on the quality of links between $S_{1,j}$ and $R$ nodes. It is assumed that $N$ number of interferers following Nakagami-$m$ distribution, corrupt the signal at relay node. The complete communication takes place in two phases. In the first

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phase $T_1$ of communication, the nodes $S_{1,j}$ and $S_2$, transmit RF and optical signals, to the relay node $R$, respectively. In the second phase $T_2$ of communication, relay node $R$ converts the RF signal from $S_{1,j}$ into optical domain and amplifies the same to transmit the processed signal to node $S_2$. Simultaneously, the relay node converts optical signal from $S_2$ into RF signal and after amplification, forwards the same to $S_{1,j}$. Moreover, it is considered that the channels are reciprocal i.e., the fading coefficients on $S_i \rightarrow R$ and $R \rightarrow S_i$ are identical. The received signal $y_R$ at node $R$ is given by:

$$y_R = \sqrt{P_{1,j}} h_{1,j} x_1 + \sqrt{P_2^{Opt}} g_2 x_2 + \sum_{i=1}^{N} \sqrt{P_{I_{R,i}}} h_{I,r} x_{r,i} + N_{T_1} \quad (1)$$

where all the parameters listed in Table I.

TABLE I: List of Paramters

| Parameter | Definition |
|---|---|
| $P_{1,j}$ | Transmit power on RF link for $j^{th}$-user |
| $h_{1,j}$ | Fading amplitude on RF link for $j^{th}$-user |
| $x_1$ | RF information symbol |
| $P_2^{Opt}$ | Optical power |
| $g_2$ | Irradiance fluctuation on the FSO link |
| $x_2$ | FSO information symbol |
| $P_{I_{R,i}}$ | Interference power |
| $h_{I,r}$ | Interferer Fading amplitude |
| $x_{r,i}$ | Symbol emitted by interferer |
| $N_{T_1}$ | Additive white Gaussian noise (AWGN) |
| $P_R$ | Power pumped by relay node |
| $\eta_1$ | Optical-to-electrical conversion coefficient |
| $\eta_2$ | Electrical-to-optical conversion coefficient |
| $r$ | Type of optical demodulation |
| $G$ | AF gain |
| $N_{01}$, $N_{02}$ | AWGN with zero-mean and variances ${\sigma_n}^2$ |

In the second phase of communication, the relay node amplifies the received signal by $G = \frac{1}{|h_{1,j}|^2 + (\eta_i g_2)^r}$, where $i = 1, 2$ and forwards it to $S_2$ and $S_{1,j}$. Given that the two nodes can perform self-interference cancellation with the knowledge of $h_{1,j}$ and $g_2$ [8, Eq. (18-19)], the received signal at RF node can be estimated as:

$$\tilde{y}_{T_2,RF} = \sqrt{P_R}(\eta_1 g_2)^{\frac{r}{2}} G h_{1,j} x_2 + \sqrt{P_R} h_{1,j} G \sum_{i=1}^{N} \sqrt{P_{I_{R,i}}} h_{I_r} x_{r,i} + N_{01} \quad (2)$$

where the variables are defined in Table I. The constant $r$ in (2) denotes the type of optical demodulation employed where $r = 1$ represents coherent demodulation and $r = 2$ corresponds to IM/DD demodulation. Similarly, the estimated signal at the FSO node can be given as:

$$\tilde{y}_{T_2,FSO} = \sqrt{P_R}(\eta_2 g_2)^{\frac{r}{2}} G[h_{1,j} x_1 + \sum_{i=1}^{N} \sqrt{P_{I_{R,i}}} h_{I_r} x_{r,i}] + N_{02} \quad (3)$$

From (2) and (2), the end-to-end signal-to-interference-plus-noise ratio (SINR) for $S_{1,j} \rightarrow R \rightarrow S_2$ can be given as:

$$\gamma_{T_2,RF} = \frac{P_R G^2 |h_{1,j}|^2 (\eta_1 g_2)^r}{G^2 \sum_{i=0}^{N} P_{I_{R,i}} |h_{I_r}|^2 |h_{1,j}|^2 + {\sigma_n}^2} \quad (4)$$

Similarly, the end-to-end SINR for $S_2 \rightarrow R \rightarrow S_{1,j}$ can be formulated as:

$$\gamma_{T_2,FSO} = \frac{P_R G^2 |h_{1,j}|^2 (\eta_2 g_2)^r}{G^2 \sum_{i=0}^{N} P_{I_{R,i}} |h_{I_r}|^2 (\eta_2 g_2)^r + {\sigma_n}^2} \quad (5)$$

Under the assumption of Nakagami-$m$ distribution, the fading coefficients on the RF link, for the $k^{th}$ user, follow the probability density function (PDF) given by [9] $f_{\gamma_k}(\gamma) = \frac{\beta^{m_{RF}}}{\Gamma(m_{RF})} \gamma^{m_{RF}-1} \exp(-\beta\gamma)$, where $m_{RF}$ is the shape parameter, $\beta = \frac{m_{RF}}{\bar{\gamma}_k}$ and $\bar{\gamma}_k$ is the average SNR of the $k^{th}$–user. Furthermore, the PDF of instantaneous SNR on the RF link with $K$ users can be obtained using the definition of ordered statistics as $f_{\gamma_{SR},j}(\gamma) = K[F_{\gamma_k(\gamma)}]^{K-1} f_{\gamma_k}(\gamma)$. The closed form expression for the cumulative distribution function (CDF) can be obtained using $F_{\gamma_{RF}}(\gamma) = \int_0^{\gamma} f_{\gamma_{SR},j}(y) dy$. For integer values of $m_{RF}$, utilizing [6, Eq. (2)] along with [10, Eq. (3.381.1) and (8.352.1)], with some mathematical manipulations, the closed-form expression for the CDF can be expressed as:

$$F_{\gamma_{RF}}(\gamma) = \sum_{n_1=0}^{K-1} \sum_{n_2=0}^{n_1(m_{RF}-1)} A_1 \left( 1 - e^{(-\mathcal{B}_0 \gamma)} \sum_{l=0}^{m_{RF}+n_2-1} \frac{(\mathcal{B}_0 \gamma)^l}{l!} \right) \quad (6)$$

where $\mathcal{B}_0 = \beta(n_1 + 1)$ and $A_1$ is given by:

$$A_1 = \frac{K(m_{RF} + n_2 - 1)!}{\Gamma m_{RF}} (-1)^{n_1} \binom{K-1}{n_1} \zeta_{n_1 n_2}(m_{RF})(n_1+1)^{-n_2-m_{RF}} \quad (7)$$

where $\zeta_{n_1 n_2}(m_{RF})$ is the coefficient of multinomial expansion which can be recursively obtained using the relation $\zeta_{n_1 n_2} = \sum_{b=n_1-x+1}^{n_1} \frac{\zeta_{b n_2 - 1}}{n_1 - b} I_{[0,(n_2-1)(x-1)]}$ where $I_{[a,b]}$ is the indicator function as defined in [9, Eq. (9.120)].

On the other hand, it is assumed that $R \rightarrow S_{FSO}$ link encompasses the turbulence-induced fading $I_a$ with pointing errors $I_p$ such that $I = I_a I_p$. The atmospheric turbulence fading $I_a$ on the FSO link is modeled as double generalized Gamma (D-GG) fading model [2]. The irradiance $I_a = I_x I_y$, such that $I_x \sim GG(\alpha_1, \beta_1, \Omega_1)$ and $I_y \sim GG(\alpha_2, \beta_2, \Omega_2)$. $\beta_1$ and $\beta_2$ are shaping parameters, whereas $\alpha_1$ , $\Omega_1$, $\alpha_2$ and $\Omega_2$ are calculated based on the variances of the small and large scale fluctuations. The PDF of SNR on the FSO link can be expressed as [2]:

$$f_{\gamma_{FSO}}(\gamma) = \frac{\mathcal{D}_1}{r\gamma} G_{1,\lambda+\sigma+1}^{\lambda+\sigma+1,0} \left[ \mathcal{D}_2 z^y \left( \frac{\gamma}{\mu_r} \right)^{\frac{y}{r}} \middle| \begin{matrix} \tau_2 \\ \tau_1 \end{matrix} \right] \quad (8)$$

with $\mathcal{D}_1 = \frac{\xi^2 \sigma^{\beta_1 - \frac{1}{2}} \sigma^{\beta_2 - \frac{1}{2}} (2\pi)^{1 - \frac{\sigma+\lambda}{2}}}{\Gamma(\beta_1)\Gamma(\beta_2)}$, $\mathcal{D}_2 = \frac{\beta_1^{\sigma} \beta_2^{\lambda}}{\lambda^{\lambda} \sigma^{\sigma} \Omega_1^{\sigma} \Omega_2^{\lambda}}$ where $\tau_1 = \left[ \frac{\xi^2}{y}, \Delta(\sigma : \beta_1), \Delta(\lambda : \beta_2) \right]$, $\tau_2 = \left[ 1 + \frac{\xi^2}{y} \right]$ and $\mu_r = \frac{(\eta \mathbb{E}(I))^r}{N_0}$ [2]. Considering $\mathcal{D}_3 = \prod_{g=1}^{\sigma+\lambda} \Gamma\left( \frac{1}{y} + \tau_{0,g} \right)$, ($\tau_{x,g}$ denoting $g^{th}$-term in $\tau_x$) $z$ can be given as $z = \frac{\mathcal{D}_1 \mathcal{D}_3}{(\mathcal{D}_2)^{1/y}(1+\xi^2)}$, where $\tau_0 = [\Delta(\sigma : \beta_1), \Delta(\lambda : \beta_2)]$ with $\Delta(z : x)$ defined as $[\frac{x}{z}, \frac{x+1}{z}, \ldots, \frac{x+z-1}{z}]$. Moreover, $G_{p,q}^{m,n}\left[ . \middle| \right]$ is the Meijer-G function defined in [10, Eq. (9.301)] and $y = \alpha_2 \lambda$. The pointing error parameter $\xi$ is defined as the ratio between the equivalent beam width $\omega_{eq}$ and pointing error jitter standard deviation $\sigma_s$, given by relation $\xi = \frac{\omega_{eq}^2}{\sigma_s}$ [2]. Further, the CDF of FSO channel over D-GG atmospheric turbulence with pointing errors can be formulated as [2]

$$F_{\gamma_{FSO}}(\gamma) = \mathcal{D}_4 G_{r+1,n+1}^{n,1} \left[ \mathcal{D}_5 \left( \frac{\gamma}{\mu_r} \right)^y \middle| \begin{matrix} 1, \tau_3 \\ \tau_4, 0 \end{matrix} \right] \quad (9)$$

where $n = r(\lambda + \sigma + 1)$, $\mathcal{D}_4 = \frac{\xi^2 \sigma^{\beta_1 - \frac{1}{2}} \lambda^{\beta_2 - \frac{1}{2}} (2\pi)^{1 - \frac{r(\sigma+\lambda)}{2}} r^{(\beta_1+\beta_2-2)}}{y\Gamma(\beta_1)\Gamma(\beta_2)}$, $\mathcal{D}_5 = \left( \frac{\mathcal{D}_2 z^y}{r^{\sigma+\lambda}} \right)^r$, $\tau_3 = [\{\Delta(r : \tau_2)\}]$ and $\tau_4 = [\{\Delta(r : \tau_1)\}]$ comprising of $n$ terms, with $\{\Delta(z : \tau_i)\} = [\Delta(z : \tau_1), \ldots, \Delta(z : \tau_i)]$.

## III. PERFORMANCE ANALYSIS

In this section, the investigation of outage performance and sum-rate of the bidirectional relay system based on the aforementioned channel models is presented.

### A. Exact Outage Probability

The outage probability (OP) is an important performance indicator of wireless communication system. OP is defined

$$P_{out}=\sum_{n_1=0}^{K-1}\sum_{n_2=0}^{n_1(m_{RF}-1)}A_2\left[\frac{y^{Nm_1}}{(2\pi)^{\frac{y-1}{2}}}G_{y+1+r,n+1}^{n,y+1}\left[\mathcal{D}_6\gamma_{th}^y\middle|\begin{smallmatrix}1,\tau_3,\tau_5\\ \tau_4,0\end{smallmatrix}\right]-\sum_{m=0}^{n_2+m_{RF}-1}\mathcal{D}_7\gamma_{th}^m H_{\mathbf{x}_1}^{\mathbf{x}_2}\left[\begin{matrix}\tau_6\\ —\end{matrix}\middle|\begin{matrix}—\\ (0,1)\end{matrix}\middle|\begin{matrix}\tau_7\\ \tau_8\end{matrix}\middle|\mathcal{B}_1,\mathcal{B}_2\right]\right] \quad (13)$$

as the probability that the instantaneous SNR falls below a pre-defined threshold $\gamma_{th}$. For the considered TWR system, the OP can be defined as:

$$\begin{aligned}P_{out}=&F_{e_2e}(\gamma)|_{\gamma_{th}}=\Pr\left(\min[\gamma_{T_2,RF},\gamma_{T_2,FSO}]<\gamma_{th}\right)\\ &=1-\Pr\left(\frac{P_R\min\left[|h_{1,j}|^2,(\eta g_2)^r\right]}{\sum_{i=0}^{N}P_{I_{R,i}}|h_{I_r}|^2}>\gamma_{th}\right)\end{aligned} \quad (10)$$

Moreover, the PDF of the total interference-to-noise ratio (INR) $\sum_{i=1}^{L}\gamma_{I_r,i}$ can be expressed as [9]:

$$f_{I_r}(\gamma)=\left[\frac{m_1}{\Omega_{I1}}\right]^{m_1L}\frac{\gamma^{m_1L-1}}{\Gamma(m_1L)}\exp\left(-\frac{m_1L}{\Omega_{I1}}\gamma\right) \quad (11)$$

where $m_1$ is Nakagami-$m$ fading parameter and $\Omega_{I1}$ is the average interference to noise ratio (INR) at the relay node. Defining $Y\triangleq P_R\min\left[|h_{1,j}|^2,(\eta g_2)^r\right]$, and considering statistical independence between $\gamma_{T_2,RF}$ and $\gamma_{T_2,FSO}$, the OP can be further calculated as:

$$\begin{aligned}P_{out}&=1-\int_0^\infty\Pr\left(Y>z\gamma_{th}\right)f_{I_r}(z)dz\\ &=\int_0^\infty F_{\gamma_{RF}}(z\gamma_{th})F_{\gamma_{FSO}}(z\gamma_{th})f_{I_r}(z)dz\end{aligned} \quad (12)$$

Appropriately substituting (6), (9) and (11) into (12), while invoking [14, Eq. (07.34.21.0013.01)] and [11, Eq. (2.3)], with some mathematical manipulations, closed-form expression for the OP is derived as given in (13), where $H_{\mathbf{h}_1}^{\mathbf{h}_2}[\mathcal{A}_1,\mathcal{A}_2]$ is the bivariate Fox's H-function as defined in [11, Eq. (1.1)]. The order of H-function can be formulated as $\{\mathbf{x}_1,\mathbf{x}_2\}=\{(0,1:1,0:n,1),(1,0:0,1:r+1,n+1)\}$. Moreover, in (13), the argument of Meijer-G is given by $\mathcal{D}_6=\mathcal{D}_5\left(\frac{y\Omega_{I1}}{m_1\mu_r}\right)^y$, whereas the arguments of Fox's H-function are derived to be $\mathcal{B}_1=\frac{\Omega_{I1}}{m_1}\mathcal{B}_0\gamma_{th}$ and $\mathcal{B}_2=\mathcal{D}_5\left(\frac{\Omega_{I1}\gamma_{th}}{\mu_r m_1}\right)^y$. The constant $A_2=A_1\mathcal{D}_4\frac{(m_{RF}+n_2-1)!}{\Gamma(m_1N)}$, $\mathcal{D}_7=\frac{(\mathcal{B}_0)^m}{m!}\left(\frac{\Omega_{I1}}{m_1}\right)^{-m}$, while various parameters involved in (13) can be defined as $\tau_5=[\Delta(y:1-m_1N)]$, $\tau_6=(1-m-m_1N,1:y)$, $\tau_7=\left[(1,1),(\tau_3,[1]_{\text{length}(\tau_3)})\right]$ and $\tau_8=\left[(\tau_4,[1]_{\text{length}(\tau_4)}),(0,1)\right]$, where $[1]_{\text{length}(x)}$ denotes array of all $1^s$ of length $x$. The bivariate Fox's-H function can be evaluated numerically using the efficient MATLAB implementation as provided in [12].

### B. Asymptotic Outage Probability

The exact expression derived in (13) fails to offer quick insights into the performance of overall system. A simpler asymptotic (high SNR) expression for OP can be developed to get more insights about system's performance. According to [13, Theorem (1.7) and Theorem (1.11)], the asymptotic expansion of H-function can be obtained as the residue of complex integration at the poles nearest to contour of integration. Assuming that $\mathbf{p_n}=\min(\tau_4)$ is the dominant pole closest to contour, the Meijer-G function can be formulated as:

$$G_{y+1+r,n+1}^{n,y+1}\left[\mathcal{D}_6\gamma_{th}^y\middle|\begin{smallmatrix}1,\tau_3,\tau_5\\ \tau_4,0\end{smallmatrix}\right]\underset{\mu_r\to\infty}{\simeq}\Lambda_1\gamma_{th}^{-y\mathbf{p_n}} \quad (14)$$

where $\Lambda_1=\frac{\mathcal{D}_7^{-\mathbf{p_n}}\prod_{j=1}^{n}\Gamma(\tau_{4,j}+\mathbf{p_n})}{\mathbf{p_n}\prod_{j=y+2}^{y+1+r}\Gamma(\tau_{3,j}+\mathbf{p_n})}\prod_{j=1}^{y}\Gamma(1-\tau_{5,j}-\mathbf{p_n})$. Similarly, by expressing the bivariate Fox's H-function in complex integral form using [11, Eq. (1.1)], and making use of identity [10, Eq. (9.113)], the asymptotic representation can be given as:

$$\begin{aligned}&H_{\mathbf{x}_1}^{\mathbf{x}_2}\left[\begin{matrix}\tau_6\\ —\end{matrix}\middle|\begin{matrix}—\\ (0,1)\end{matrix}\middle|\begin{matrix}\tau_7\\ \tau_8\end{matrix}\middle|\mathcal{B}_1,\mathcal{B}_2\right]\underset{\mu_r\to\infty}{\simeq}\Gamma\left(m+m_1N-y\mathbf{p_n}\right)\\ &\times{}_1F_0\left(m+m_1N-y\mathbf{p_n};—;-\frac{\Omega_{I1}\mathcal{B}_0}{m_1}\gamma_{th}\right)\Lambda_1\left\{\frac{\gamma_{th}}{y}\right\}^{-y\mathbf{p_n}}\end{aligned} \quad (15)$$

where ${}_1F_0\left(a;—;b\right)$ is the Gaussian hypergeometric function as defined in [10, Eq. (9.111)]. Plugging (14) and (15) into (13), the asymptotic high SNR approximation of OP can be obtained.

### C. Achievable Sum Rate

In this section, the achievable sum-rate (ASR) offered by the proposed model is derived. The sum-rate of the system can be defined as the throughput over all the channel realizations. The ASR of wireless fading channels can be defined as:

$$\mathcal{R}=\frac{1}{2}\mathbb{E}[log_2(1+\gamma_{T_2,RF})]+\frac{1}{2}\mathbb{E}[log_2(1+\gamma_{T_2,FSO})]=\mathcal{R}_1+\mathcal{R}_2 \quad (16)$$

where $\mathbb{E}(.)$ denotes the expectation operator. The transmission rate for the link $S_{1,j}\to R\to S_2$ can be obtained as:

$$\mathcal{R}_1=\frac{1}{2}\int_0^\infty log_2(1+\gamma)f_{\gamma_{T_2,RF}}(\gamma)d\gamma \quad (17)$$

Firstly, a closed-form expression for $F_{\gamma_{T_2,RF}}(\gamma)$ can be attained using $F_{\gamma_{T_2,RF}}(\gamma)=\int_0^\infty F_{\gamma_{RF}}(\gamma y)f_{I_r}(y)dy$. Further to this, placing (6) and (11) and applying [10, Eq. (3.381.4)], $F_{\gamma_{T_2,RF}}(\gamma)$ can be derived, which can be differentiated to obtain $f_{\gamma_{T_2,RF}}(\gamma)$, which can be represented as:

$$\begin{aligned}f_{\gamma_{T_2,RF}}(\gamma)=&\frac{1}{2}\sum_{n_1=0}^{K-1}\sum_{n_2=0}^{n_1(m_{RF}-1)}\sum_{m=0}^{m_{RF}+n_2-1}A_3\left[\frac{(m+m_1N)\mathcal{B}_0}{m!}\gamma^m\right.\\ &\left.\times\left[\mathcal{B}_0\gamma+\frac{m_1}{\Omega_{I1}}\right]^{-(m+m_1N)}\left\{\left[\mathcal{B}_0\gamma+\frac{m_1}{\Omega_{I1}}\right]^{-1}-m\gamma^m\right\}\right]\end{aligned} \quad (18)$$

where $A_3=A_1\frac{(m_{RF}+n_2-1)!}{\Gamma(m_1N)}\left(\frac{m_1}{\Omega_{I1}}\right)^{m_1N}\frac{\Gamma(m+m_1N)\mathcal{B}_0{}^m}{m!}$. After representing $log_2(1+\gamma)=G_{2,2}^{1,2}\left[\gamma\middle|\begin{smallmatrix}1,1\\ 1,1\end{smallmatrix}\right]$ with the aid of [14, Eq. (07.34.03.0456.01)], substituting $f_{\gamma_{T_2,RF}}(\gamma)$ into (17) and applying [14, Eq. (07.34.03.0271.01) and (07.34.21.0013.01)], with some mathematical manipulations, the closed-form expression of $\mathcal{R}_1$ can be obtained as given in below:

$$\begin{aligned}\mathcal{R}_1=&\frac{1}{2}\sum_{n_1=0}^{K-1}\sum_{n_2=0}^{n_1(m_{RF}-1)}\sum_{m=0}^{m_{RF}+n_2-1}A_3\left[\left(\frac{\mathcal{B}_0\Omega_{I1}}{m_1\delta}\right)G_{3,3}^{3,2}\left[\mathcal{B}_3\middle|\begin{smallmatrix}\tau_9\\ \tau_{10}\end{smallmatrix}\right]\right.\\ &\left.-mG_{3,3}^{3,2}\left[\mathcal{B}_3\middle|\begin{smallmatrix}\tau_{11}\\ \tau_{12}\end{smallmatrix}\right]\right]\end{aligned} \quad (19)$$

where $\mathcal{B}_3=\frac{\Omega_1\mathcal{B}_0}{m_1\delta}$, whereas the parameters are defined as: $\tau_9=[-m-m_1N,-m-1,-m]$, $\tau_{10}=[0,-m-1,-m-1]$, $\tau_{11}=[1-m-1,-m,-m]$ and $\tau_{12}=[0,-m,-m]$. Similarly, the effective CDF on the FSO link can be obtained by substituting (9) and (11) in $F_{\gamma_{T_2,FSO}}(\gamma)=\int_0^\infty F_{\gamma_{FSO}}(\gamma y)f_{I_r}(y)dy$, which can be evaluated using [14, Eq. (07.34.21.0013.01)]. The resulting expression can be further differentiated using [14, Eq. (07.34.20.0017.02)] to express the PDF of $\gamma_{T_2,FSO}$. Plugging the PDF, thus obtained, into the relationship $\mathcal{R}_2=\frac{1}{2}\int_0^\infty log_2(1+\gamma)f_{\gamma_{T_2,FSO}}(\gamma)d\gamma$, the closed-form expression of $\mathcal{R}_2$ can be obtained as given below:

$$\mathcal{R}_2=\frac{log_2(e)\mathcal{D}_4(y)^{Nm_1-\frac{3}{2}}}{2\delta\Gamma(m_1N)(2\pi)^{\frac{3}{2}(y-1)}}G_{4y+r+1,n+3y+1}^{n+2y,3y+1}\left[\frac{\mathcal{D}_7}{\delta^y}\middle|\begin{smallmatrix}\tau_{13}\\ \tau_{14}\end{smallmatrix}\right] \quad (20)$$

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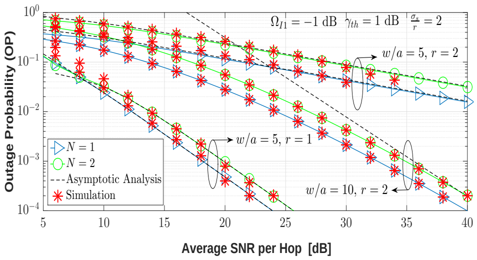


Fig. 1: Effect of type of demodulation, number of interferers, $N$, and pointing error on outage probability performance.

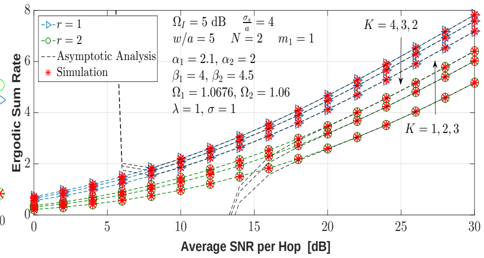


Fig. 2: Effect of type of demodulation and number of users, $K$, on the achievable ergodic sum rate.

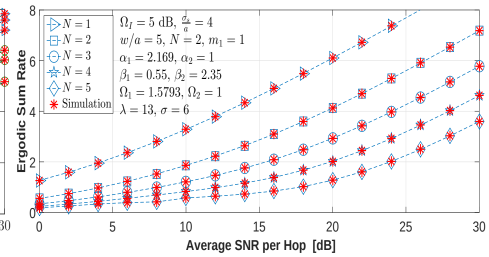


Fig. 3: Effect of varying number of interferers, $N$, on the achievable ergodic sum rate of the two-way relaying networks.

where $\tau_{13} = [0, \Delta(y:1), 1, \tau_5, \tau_3]$ and $\tau_{14} = [\tau_5, \Delta(y:1), 1, 0]$. Finally, on substituting (19) and (20) into (16), the expression of ASR can be established. Furthermore, the asymptotic approximation of Meijer-G function involved in $\mathcal{R}_1$ can be developed considering the fact that as $\bar{\gamma}_k \to \infty$, the argument $\mathcal{B}_3 \to 0$. Invoking [14, 07.34.06.0006.01], the Meijer-G in $\mathcal{R}_1$ can be approximated as:

$$G_{3,3}^{3,2}\left[\mathcal{B}_3\middle| \begin{matrix}\tau_9\\ \tau_{10}\end{matrix}\right] \underset{\bar{\gamma}_k\to\infty}{\simeq} \sum_{i=1}^{3} \frac{\prod_{j=1, j\neq i}^{3}\Gamma(\tau_{10,j}-\tau_{10,i})}{\Gamma(\tau_{9,3}-\tau_{10,i})}\mathcal{B}_3^{\tau_{10,i}} \times \prod_{j=1}^{2}\Gamma(1-\tau_{9,j}+\tau_{10,i}) \quad (21)$$

Similar approach can be opted to represent $G_{3,3}^{3,2}\left[\mathcal{B}_3\middle| \begin{matrix}\tau_{11}\\ \tau_{12}\end{matrix}\right]$ asymptotically. Additionally, in order to simplify the expression of $\mathcal{R}_2$, it can be noted that as $\mu_r \to \infty$, the argument of Meijer-G in (20) diminishes, i.e., $\mathcal{D}_6 \to 0$, which can be approximated using [14, 07.34.06.0006.01] as:

$$G_{4y+r+1,n+3y+1}^{n+2y,3y+1}\left[\frac{\mathcal{D}_7}{\delta^y}\middle| \begin{matrix}\tau_{13}\\ \tau_{14}\end{matrix}\right] \underset{\mu_r\to\infty}{\simeq} \sum_{i=1}^{n+2y} \frac{\prod_{j=1, j\neq i}^{n+2y}\Gamma(\tau_{14,j}-\tau_{14,i})}{\prod_{j=3y+2}^{4y+r+1}\Gamma(\tau_{13,j}-\tau_{14,i})} \times \left(\frac{\mathcal{D}_7}{\delta^y}\right)^{\tau_{14,i}} \frac{\prod_{j=1}^{3y+1}\Gamma(1-\tau_{13,j}+\tau_{14,i})}{\prod_{j=n+2y+1, j\neq i}^{n+3y+1}\Gamma(1-\tau_{14,j}+\tau_{14,i})} \quad (22)$$

Substituting the approximations of Meijer-G function in (19) and (20), the asymptotic representation of ASR can be obtained.

## IV. Numerical Results

In this section, numerical examples are shown to demonstrate the findings of the research work, together with Monte-Carlo simulations. Fig. 1 illustrates the OP of mixed RF/FSO fixed-gain AF TWR systems versus the average SNR per hop in moderate (i.e., $\alpha_1 = 2.1, \alpha_2 = 2, \beta_1 = 4, \beta_2 = 4.5, \Omega_1 = 1.0676, \Omega_2 = 1.06$) turbulence conditions. Fig. 1 also investigates the effect of strong (i.e., $w/a = 5$) and weak (i.e., $w/a = 10$) pointing errors on the system performance. OP deteriorates by increasing the number of interferers, i.e., $N$. At high SNR, the asymptotic expansion matches very well with its exact counterpart, which confirms the validity of our mathematical analysis for different parameter settings. It can also be observed that coherent detection ($r = 1$) outperforms IM/DD ($r = 2$) in turbulent environments. Additionally, the impact of type demodulation schemes, along with the number of RF users, on the ASR has been demonstrated in Fig. 2. It can be noted that increasing $K$ provides a remarkable improvement in the system performance. Finally, the deterioration introduced by interference on ASR of the considered TWR relay network is quantified in Fig. 3. In the plot, the set of parameters assumed to model turbulence on the FSO link is given as: $\alpha_1 = 2.169, \alpha_2 = 1, \beta_1 = 0.55, \beta_2 = 2.35, \Omega_1 = 1.5793, \Omega_2 = 1$. It is evident in the plot that the average rate improves as the number of interferers, $N$, reduces.

## V. Conclusion

In this work, performance analysis for a mixed MUD-RF/FSO TWR network in the presence of interference is presented. Specifically, the exact and asymptotic closed-form expressions for OP and ASR have been derived in order to demonstrate the effect of various model parameters on overall performance of the considered system. The analysis is extended to provide asymptotic OP expression. The derived results account for both IM/DD and coherent demodulation techniques on the optical link. This letter quantifies the effect of interference on mixed RF/FSO TWR systems along with improvement due to user diversity on the RF link, which can be useful during practical implementation of future wireless systems.

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