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Zhang, Z.Y., Liu, W., Leng, W. et al. (2 more authors) (2016) Interference-plus-Noise Covariance Matrix Reconstruction via Spatial Power Spectrum Sampling for Robust Adaptive Beamforming. *IEEE Signal Processing Letters*, 23. pp. 121-125. ISSN 1558-2361

<https://doi.org/10.1109/LSP.2015.2504954>

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Interference-plus-Noise Covariance Matrix Reconstruction via Spatial Power Spectrum Sampling for Robust Adaptive Beamforming

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Abstract—Recently, a robust adaptive beamforming (RAB) technique based on interference-plus-noise covariance (INC) matrix reconstruction has been proposed, which utilizes the Capon spectrum estimator integrated over a region separated from the direction of the desired signal. Inspired by the sampling and reconstruction idea, in this paper, a novel method named spatial power spectrum sampling (SPSS) is proposed to reconstruct the INC matrix more efficiently, with the corresponding beamforming algorithm developed, where the covariance matrix taper (CMT) technique is employed to further improve its performance. Simulation results are provided to demonstrate the effectiveness of the proposed method.

Index Terms—Covariance matrix reconstruction, matrix taper, robust beamforming, spatial power spectrum sampling.

I. INTRODUCTION

ADAPTIVE beamforming has found many applications ranging from wireless communications, radar, sonar, and speech processing, to medical imaging, radio astronomy, etc [1], [2]. It is well-known that the performance of a standard adaptive beamformer is sensitive to various array manifold errors such as calibration error and direction of arrival (DOA) estimation error for the signal of interest (SOI) [3], [4], [5], [6]. As a solution, various robust adaptive beamforming (RAB) techniques have been proposed in the past decades [1], [7]. The design principles of RAB based on the minimum variance distortionless response (MVDR) criterion were illustrated in [8] and the diagonal loading technique was studied in [5], while the one based on the worst-case optimization was proposed in [9], and steering vector estimation with presumed prior knowledge for RAB was investigated in [10], [11].

In a recent RAB design [12], a method for estimating the interference-plus-noise covariance (INC) matrix to eliminate the power of SOI was proposed, where it first uniformly samples the spatial power spectrum over the full angular range from $-\pi/2$ to $\pi/2$, and then reconstructs the INC matrix by summing up the values over a region separated from the direction of the desired signal. A drawback of this effective RAB method is its high computational complexity due to the large number of samples involved in both spectrum estimation and matrix multiplication/summation. According to [12], it

has a complexity of $\mathcal{O}(M^2S)$ with $S \gg M$, where M is the number of sensors and S the number of samples taken in the summation. Based on this idea, a sparse method was proposed to estimate the INC matrix to reduce the complexity in [13]. On the other hand, to deal with unknown arbitrary-type mismatches, an uncertainty set was employed for INC matrix reconstruction in [10].

To further reduce the computational complexity of the RAB method in [12], in this letter, a low-complexity INC matrix reconstruction method is proposed based on spatial power spectrum sampling (SPSS), and a corresponding beamforming algorithm is developed. The spatial power spectrum sample operation is realized by a proposed sample equation which is derived from the selecting property of the steering vector. The covariance matrix taper (CMT) technique studied in [14] is employed to improve the robustness as well as reinforce the sample equation due to a relatively small size of the array system in practice. With the proposed method, the spatial power spectrum estimation process in [12] can be avoided, making the SPSS based algorithm computationally much more efficient. Simulation results will be provided to demonstrate the effectiveness and robustness of the proposed RAB method.

II. THE SIGNAL MODEL

Consider a uniform linear array (ULA) of M (usually from tens to hundreds [15]) omni-directional sensors, with a half wavelength spacing. One desired signal arrives from the direction θ_s with a power of σ_s^2 , while Q interfering signals impinge upon the array from directions θ_i , $i = 1, 2, \dots, Q$, with their corresponding powers given by σ_i^2 . The $M \times 1$ complex array observation vector at time k can be modeled as

$$\mathbf{x}(k) = \mathbf{s}(k) + \mathbf{i}(k) + \mathbf{n}(k), \quad (1)$$

where $\mathbf{s}(k) = s(k) \mathbf{d}(\theta_s)$, $\mathbf{i}(k)$ and $\mathbf{n}(k)$ are the statistically independent components of the desired signal, interference and noise, respectively, $s(k)$ is the desired signal waveform, and $\mathbf{d}(\theta_s)$ is its steering vector. The steering vector of the ULA has the following general form

$$\mathbf{d}(\theta) = [1 \quad e^{j\pi \sin \theta} \quad \dots \quad e^{j\pi(M-1) \sin \theta}]^T. \quad (2)$$

Let \mathbf{R} denote the theoretical covariance matrix of the array output vector. Then \mathbf{R} can be expressed as follows:

$$\mathbf{R} = \sigma_s^2 \mathbf{d}(\theta_s) \mathbf{d}^H(\theta_s) + \sum_{i=1}^Q \sigma_i^2 \mathbf{d}(\theta_i) \mathbf{d}^H(\theta_i) + \sigma_n^2 \mathbf{I}, \quad (3)$$

This work was supported in part by the project of State Key Laboratory of Millimeter Waves (K201314).

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where $(\cdot)^H$ denotes Hermitian transpose, and $\sigma_n^2 \mathbf{I}$ is the noise covariance matrix with \mathbf{I} representing the identity matrix and σ_n^2 the noise power. Alternatively, \mathbf{R} can also be formed through the spatial spectrum $\sigma^2(\theta)$ of the array by

$$\mathbf{R} = \int_{\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]} \sigma^2(\theta) \mathbf{d}(\theta) \mathbf{d}^H(\theta) d\theta, \quad (4)$$

In practice, theoretical covariance matrix \mathbf{R} is usually unavailable and the sample covariance matrix (5) is used as an approximation:

$$\mathbf{R}_x = \frac{1}{K} \sum_{k=1}^K \mathbf{x}(k) \mathbf{x}^H(k), \quad (5)$$

where K is number of data snapshots.

Applying the complex weight vector $\mathbf{w} = [w_1, \dots, w_M]^T \in \mathbb{C}^M$ to the received signal vector $\mathbf{x}(k)$, we obtain the beamformer output $y(k)$, given by $y(k) = \mathbf{w}^H \mathbf{x}(k)$. The beamformer output signal-to-interference-plus-noise ratio (SINR) is defined as

$$\text{SINR} = \frac{\sigma_s^2 |\mathbf{w}^H \mathbf{d}(\theta_s)|^2}{\mathbf{w}^H \mathbf{R}_{i+n} \mathbf{w}}, \quad (6)$$

where \mathbf{R}_{i+n} is the INC matrix.

Maximizing (6) subject to a unity constraint to the SOI direction leads to the following optimization problem

$$\underset{\mathbf{w}}{\text{minimize}} \quad \mathbf{w}^H \mathbf{R}_{i+n} \mathbf{w} \quad \text{subject to} \quad \mathbf{w}^H \mathbf{d}(\theta_s) = 1, \quad (7)$$

and the solution is commonly known as the MVDR beamformer or Capon beamformer [11]

$$\mathbf{w}_{opt} = \frac{\mathbf{R}_{i+n}^{-1} \mathbf{d}(\theta_s)}{\mathbf{d}^H(\theta_s) \mathbf{R}_{i+n}^{-1} \mathbf{d}(\theta_s)}. \quad (8)$$

III. THE PROPOSED METHOD

Recall the INC matrix reconstruction method given by[12]:

$$\bar{\mathbf{R}}_{i+n} = \int_{\bar{\Theta}} \hat{P}(\theta) \mathbf{d}(\theta) \mathbf{d}^H(\theta) d\theta \quad (9)$$

where $\hat{P}(\theta) = 1/\mathbf{d}^H(\theta) \mathbf{R}_x^{-1} \mathbf{d}(\theta)$ is the Capon power spectrum estimator and $\bar{\Theta}$ is the angular region excluding the assumed SOI region Θ . The main computational cost is the integration approximation by summation, where S (number of sampled values) times spectrum estimation and vector multiplication operations have to be performed. In the following, by analyzing the selecting property of the steering vector, we give an efficient method to calculate this approximation without incurring the spectrum estimation process.

A. The selecting property of the steering vector, sample matrix and sample equation

The inner product of two steering vectors is given by

$$f(\alpha; \alpha_0) = \frac{1}{M} \mathbf{d}^H(\alpha_0) \mathbf{d}(\alpha) = \frac{1}{M} \sum_{k=0}^{M-1} e^{jk\pi[\sin(\alpha) - \sin(\alpha_0)]} \quad (10)$$

where $\alpha, \alpha_0 \in [-\pi/2, \pi/2]$. Let $x = M/2[\sin(\alpha) - \sin(\alpha_0)] \in [(-1 - \sin(\alpha_0))M/2, (1 - \sin(\alpha_0))M/2]$, then (10) can be

rewritten as $f(x) = 1/M \sum_{k=0}^{M-1} e^{j(2\pi/M)kx}$, and $f(x)$ can be seen as a time-domain signal corresponding to an M -point discrete rectangular function in the frequency domain. So we can obtain that

$$f(x) = \frac{1}{M} \cdot \frac{\sin(\pi x)}{\sin(\frac{\pi}{M}x)} e^{j\frac{M-1}{M}\pi x}. \quad (11)$$

When M is large enough, $f(x)$ will approximate a sinc function, i.e. $f(x) = \text{sinc}(\pi x) = \sin(\pi x)/\pi x$. As $x = M/2[\sin(\alpha) - \sin(\alpha_0)]$, unless α is very close to α_0 , x will be very large and $f(x)$ will have a very small value. Then we can conclude that when M is big enough, $f(\alpha; \alpha_0)$ will approximate a Kronecker delta function, i.e.

$$f(\alpha; \alpha_0) \approx \delta_{\alpha, \alpha_0} = \begin{cases} 1, & \alpha = \alpha_0 \\ 0, & \alpha \neq \alpha_0 \end{cases}. \quad (12)$$

This is called *the selecting property of the steering vector* in this letter. Fig. 1 shows the relationship between M and the selecting property of the steering vector.

Moreover, for equation (11), when $x = 0$, we have $f(x) = 1$; when $x \in Z = \{z | z \in [(-1 - \sin(\alpha_0))M/2, (1 - \sin(\alpha_0))M/2], z \in \mathbb{Z}, z \neq 0\}$, we have $f(x) = 0$. There are $M - 1$ such values in the set Z , i.e. $f(x)$ has $M - 1$ zeros, and we denote them as $x_k, k = 1, 2, \dots, M - 1$. As $x = M/2[\sin(\alpha) - \sin(\alpha_0)]$, the zeros of $f(\alpha; \alpha_0)$, denoted as $\alpha_k, k = 1, 2, \dots, M - 1$, can be easily obtained by x_k as $\alpha_k = \arcsin(2x_k/M + \sin(\alpha_0))$.

Additionally, from (10), we can show that any two of the steering vectors of $(\alpha_0, \{\alpha_k\}_{k=1}^{M-1})$ are orthogonal to each other. Therefore, the steering vectors of $(\alpha_0, \{\alpha_k\}_{k=1}^{M-1})$ span the M -dimensional complex space.

Then, we define a matrix using $(\alpha_0, \{\alpha_k\}_{k=1}^{M-1})$:

$$\mathbf{D} = \frac{1}{M} \sum_{\alpha_k \in \Omega} \mathbf{d}(\alpha_k) \mathbf{d}^H(\alpha_k) \quad (13)$$

where Ω is a specified angular sector. When M is large enough, we have the result for the Hermitian matrix $\mathbf{D} \cdot \mathbf{R} \cdot \mathbf{D}$ given in (14). From (14), we can see that when Ω covers the whole region, $\mathbf{D} \cdot \mathbf{R} \cdot \mathbf{D}$ can be considered as an M -point approximation to the covariance matrix \mathbf{R} . We refer to $(\alpha_0, \{\alpha_k\}_{k=1}^{M-1})$, \mathbf{D} , and equation (14) as *watch points*, *sample matrix*, and *sample equation*, respectively.

To estimate the INC matrix, we can remove the assumed angle sector for SOI, i.e. let $\Omega = \bar{\Theta}$. Then an approximation of the INC matrix can be obtained by $\hat{\mathbf{D}} \cdot \mathbf{R} \cdot \hat{\mathbf{D}}$, where $\hat{\mathbf{D}} = 1/M \sum_{\alpha_k \in \bar{\Theta}} \mathbf{d}(\alpha_k) \mathbf{d}^H(\alpha_k)$. In practice, since \mathbf{R} is not available, we can replace \mathbf{R} by \mathbf{R}_x , i.e. $\hat{\mathbf{D}} \cdot \mathbf{R}_x \cdot \hat{\mathbf{D}}$.

In this way, we have avoided the estimation $\hat{P}(\theta)$ of the spatial power spectrum in (9). However, when $M \ll \infty$, there will be a large error in the estimation by $\hat{\mathbf{D}} \cdot \mathbf{R}_x \cdot \hat{\mathbf{D}}$, because the selecting property of the steering vector is not ideal and the watch points spacing is not dense enough. To improve it, a taper operation is needed, as detailed in the next subsection.

B. SPSS INC matrix reconstruction

As just mentioned, for the estimation given in (14), when M is not large enough, the spacing between two adjacent watch

$$\begin{aligned}
\mathbf{D} \cdot \mathbf{R} \cdot \mathbf{D} &= \frac{1}{M} \sum_{\alpha_i \in \Omega} \mathbf{d}(\alpha_i) \mathbf{d}^H(\alpha_i) \cdot \left(\int_{\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]} \sigma^2(\theta) \mathbf{d}(\theta) \mathbf{d}^H(\theta) d\theta \right) \cdot \frac{1}{M} \sum_{\alpha_k \in \Omega} \mathbf{d}(\alpha_k) \mathbf{d}^H(\alpha_k) \\
&= \sum_{\alpha_i \in \Omega} \sum_{\alpha_k \in \Omega} \mathbf{d}(\alpha_i) \left\{ \int_{\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]} \sigma^2(\theta) \left[\frac{1}{M} \mathbf{d}^H(\alpha_i) \mathbf{d}(\theta) \right] \left[\frac{1}{M} \mathbf{d}^H(\theta) \mathbf{d}(\alpha_k) \right] d\theta \right\} \mathbf{d}^H(\alpha_k) \\
&\approx \sum_{\alpha_i \in \Omega} \sum_{\alpha_k \in \Omega} \mathbf{d}(\alpha_i) \left[\int_{\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]} \sigma^2(\theta) \delta_{\theta, \alpha_i} \delta_{\theta, \alpha_k} d\theta \right] \mathbf{d}^H(\alpha_k) \\
&= \sum_{\alpha_k \in \Omega} \sigma^2(\alpha_k) \mathbf{d}(\alpha_k) \mathbf{d}^H(\alpha_k).
\end{aligned} \tag{14}$$

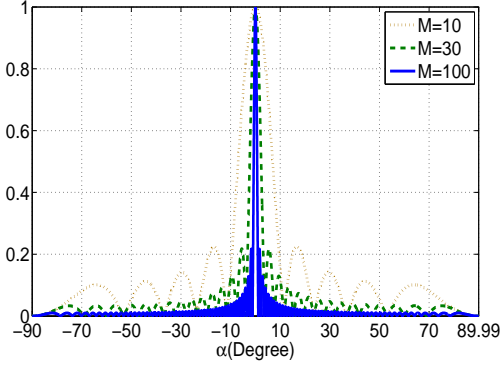


Fig. 1. $|f(\alpha, 0)|$ versus α .

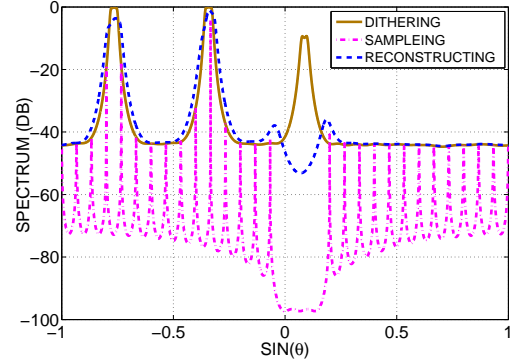


Fig. 2. Effect of the first three steps in the SPSS algorithm.

points may be too large to sample the power information of interfering signals accurately. So we need to dither the power of interferences to their neighborhood for robustness as well as for the sample equation to catch more critical power information of the interfering signals to some degree. To achieve this, the covariance matrix tapering technique introduced in [14] is employed to modify the estimated INC matrix. In particular, we here use the Malloux-Zatman (MZ) taper defined as follows [14],

$$\mathbf{T}_{MZ} = [a_{mn}]_{M \times M} = [\text{sinc}((m-n)\Delta/\pi)], \quad \Delta > 0 \tag{15}$$

where Δ corresponds to the width of the dithering area. For the matrix \mathbf{R} , the ‘tapered matrix’ is given by $\mathbf{R} \circ \mathbf{T}_{MZ}$, where ‘ \circ ’ denotes the Hadamard product. As pointed out in [14], ‘the MZ taper is equivalent to the introduction of a uniformly distributed coherent phase dither.’

As for the choice of Δ , it should satisfy a requirement that, for a signal whose arriving angle is located between two adjacent watch points, the power of the arriving signal should be dithered to one of the adjacent watch points by the taper operation. In this paper, $\Delta = \sin^{-1}(2/M)$ is chosen.

Additionally, considering that the spectrum of the sampled matrix is discrete, \mathbf{T}_{MZ} should be adopted again to dither the power of watch points into their neighborhood to obtain a relatively continuous spatial spectrum. And this finish the reconstruction of INC matrix.

C. The SPSS-based beamforming algorithm

Based on the discussions above, the proposed SPSS-based beamforming algorithm can be described in four steps: dithering, sampling, reconstructing, and weighting. Fig. 2 shows the power spectrums of the output matrices in the first three steps, where $\Theta = [-1^\circ, 11^\circ]$ is used and the Capon power spectrum

estimator is adopted. It can be seen that the reconstructed INC matrix can effectively restrain the power of SOI, as well as maintain the information of interferences and noise.

- 1) Step 1: (Dithering) Specify a certain Δ for \mathbf{T}_{MZ} to taper the sample covariance matrix \mathbf{R}_x , i.e. $\mathbf{R}_T = \mathbf{R}_x \circ \mathbf{T}_{MZ}$.
- 2) Step 2: (Sampling) Develop the required sample matrix $\hat{\mathbf{D}}$; then sample \mathbf{R}_T using the sample equation, i.e. $\hat{\mathbf{R}}_{i+n} = \hat{\mathbf{D}} \cdot \mathbf{R}_T \cdot \hat{\mathbf{D}}$.
- 3) Step 3: (Reconstructing) Use \mathbf{T}_{MZ} in Step 1 again, to dither the power of each watch point to its neighborhood, and obtain a continuous spatial spectrum, i.e. $\tilde{\mathbf{R}}_{i+n} = \hat{\mathbf{R}}_{i+n} \circ \mathbf{T}_{MZ}$;
- 4) Step 4: (Weighting) Substitute the reconstructed INC matrix $\tilde{\mathbf{R}}_{i+n}$ and presumed DOA of SOI, θ_p , into the Capon beamformer (8) to obtain the weight vector, i.e.

$$\mathbf{w} = \frac{\tilde{\mathbf{R}}_{i+n}^{-1} \mathbf{d}(\theta_p)}{\mathbf{d}^H(\theta_p) \tilde{\mathbf{R}}_{i+n}^{-1} \mathbf{d}(\theta_p)}. \tag{16}$$

It can be seen that the main computational cost of the proposed algorithm is the matrix inversion operation in Step 4. Its overall computational complexity is $\mathcal{O}(M^3)$ in contrast to $\mathcal{O}(SM^2)$ with $S \gg M$ for the algorithm in [12]. As an example to show the significantly reduced computational complexity by the proposed method, we run the two algorithms using MATLAB 2009a on a Windows XP SP3 PC with dual core 3.07GHz Intel Core i3 CPU and 3.36GB memory. With $M = 30$, $K = 60$, and $S = 300$, the required CPU time for the beamformer in [12] is around 14.6ms, while it takes the proposed one only about 0.6ms with no code optimization.

IV. SIMULATION RESULTS

In our simulations, we consider a ULA with $M = 30$ omnidirectional sensors, with zero-mean and unity variance

spatially and temporally white Gaussian noise. Two interfering sources with random waveforms arrive from DOA angles of -50° and -20° , respectively. The interference-to-noise ratio (INR) at each sensor is 30dB. The desired signal impinges on the array from the presumed direction $\theta_p = 5^\circ$. For each simulation, 500 Monte-Carlo runs are performed.

Our proposed SPSS-based beamformer is compared with the worst-case-based beamformer [9], the beamformer in [16], the sequential quad-ratic programming (SQP) based beamformer in [17], and the beamformer in [12]. For the SPSS-based beamformer and the beamformer in [12], the general angular location of the desired signal is assumed to be within the interval $\Theta = [\theta_p - 6^\circ, \theta_p + 6^\circ]$; $\alpha_0 = 0^\circ$ is used in (10)¹, and $\Delta = \sin^{-1}(2/M)$ is used in (15). The value $\delta = 0.1$ and 20 dominant eigenvectors of the matrix $\mathbf{C} = \int_{\Theta} \mathbf{d}(\theta)\mathbf{d}^H(\theta)\mathbf{d}(\theta)$ are used in the SQP based beamformer, while $\epsilon = 0.3$ is used for the worst-case-based beamformer.

A. Example 1: Random direction mismatch for SOI and interference

In this example, the direction mismatch error is assumed to be randomly and uniformly distributed in $[-4^\circ, 4^\circ]$ for both the SOI and interferences as in [12]. S is kept at 300 to calculate the integration in the beamformer [12]. Here the random DOAs change from run to run but remain fixed from snapshot to snapshot. Fig. 3a depicts the output SINR of the beamformers versus the input SNR. The number of snapshots is fixed to be $K = 2M = 60$. It can be seen that the performance of the SPSS-based beamformer is very close to that of the beamformer in [12] and outperforms the other beamformers when SNR is larger than 0dB. In Fig. 3b, the output SINR is shown with respect to the number of snapshots K , with a fixed SNR for the desired signal at 10dB. Again the proposed beamformer has a similar performance to the beamformer in [12], but much better than the remaining ones.

B. Example 2: Performance versus number of sensors

In the second example, we compare the performance between the SPSS-based beamformer and the beamformer in [12] against the number of sensors. Considering the same mismatched situation in Example 1, we vary M from 20 to 80, while the SNR of SOI and K are kept at 20dB and $2M$ respectively. In all the simulations, $S = 3601$ is chosen to get the best performance for the beamformer in [12]. It can be seen from Fig. 4 that, when using 22 or more sensors, the deviation between the two beamformers is within 0.7dB, which means the approximation by our proposed beamformer has been good enough to reach a similar performance to [12], but with a much lower computational complexity.

V. CONCLUSION

In this letter, an SPSS-based method has been proposed to reconstruct the INC matrix in a computationally efficient

¹Note that for the choice of α_0 , it can take any value as long as at least one watch point α_i (as a result of the choice of α_0) is within the desired angular sector Θ . When this is satisfied, the power of SOI will be excluded in the reconstructed INC matrix.

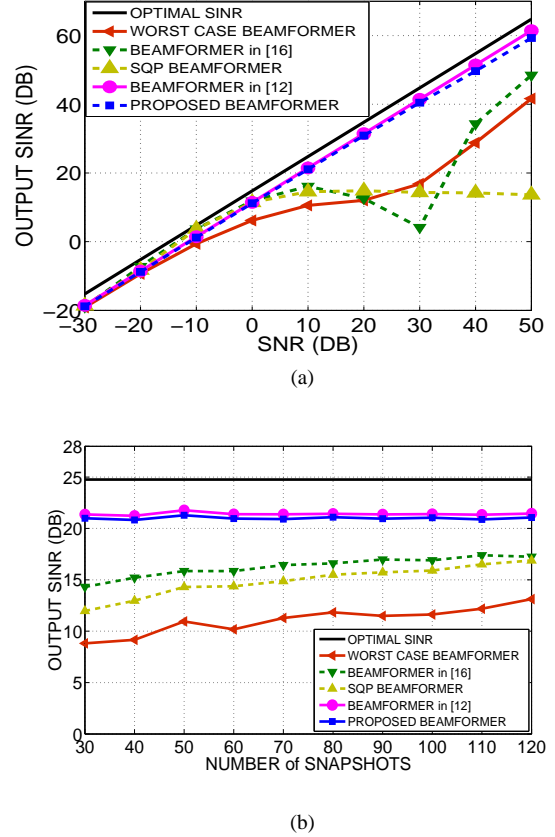


Fig. 3. First example: (a) Output SINR versus input SNR; (b) output SINR versus number of snapshots.

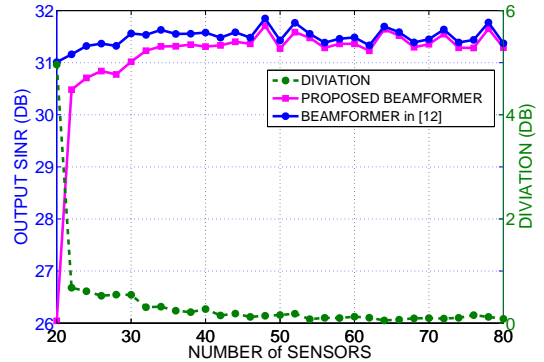


Fig. 4. Second example. Output SINR versus number of sensors.

way, with the corresponding robust beamforming algorithm developed. The computational complexity of the proposed beamformer is $\mathcal{O}(M^3)$, which in general is much smaller than $\mathcal{O}(M^2S)$ ($S \gg M$) of a previously proposed reconstruction method. In particular the spatial spectrum estimation process has been avoided. Simulation results have demonstrated that the proposed beamformer can achieve a very similar performance to its high-complexity version.

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