Intermediate mass black holes in AGN discs – I. Production and growth

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ABSTRACT

Here we propose a mechanism for efficiently growing intermediate mass black holes (IMBH) in discs around supermassive black holes. Stellar mass objects can efficiently agglomerate when facilitated by the gas disc. Stars, compact objects and binaries can migrate, accrete and merge within discs around supermassive black holes. While dynamical heating by cusp stars excites the velocity dispersion of nuclear cluster objects (NCOs) in the disc, gas in the disc damps NCO orbits. If gas damping dominates, NCOs remain in the disc with circularized orbits and large collision cross-sections. IMBH seeds can grow extremely rapidly by collisions with disc NCOs at low relative velocities, allowing for super-Eddington growth rates. Once an IMBH seed has cleared out its feeding zone of disc NCOs, growth of IMBH seeds can become dominated by gas accretion from the active galactic nucleus (AGN) disc. However, the IMBH can migrate in the disc and expand its feeding zone, permitting a super-Eddington accretion rate to continue. Growth of IMBH seeds via NCO collisions is enhanced by a pile-up of migrators.

We highlight the remarkable parallel between the growth of IMBH in AGN discs with models of giant planet growth in protoplanetary discs. If an IMBH becomes massive enough it can open a gap in the AGN disc. IMBH migration in AGN discs may stall, allowing them to survive the end of the AGN phase and remain in galactic nuclei. Our proposed mechanisms should be more efficient at growing IMBH in AGN discs than the standard model of IMBH growth in stellar clusters. Dynamical heating of disc NCOs by cusp stars is transferred to the gas in an AGN disc helping to maintain the outer disc against gravitational instability. Model predictions, observational constraints and implications are discussed in a companion paper (Paper II).

Key words: accretion-discs – planets–disc interactions – protoplanetary discs – binaries:close – galaxies: active – galaxies:nuclei.

1 INTRODUCTION

Extensive evidence exists that supermassive black holes ($>10^6$ M$_\odot$) are found in the centres of most galaxies (e.g. Kormendy & Richstone 1995). Extensive evidence also exists for stellar mass black holes in our own Galaxy (Remillard & McClintock 2006). Stellar mass black holes are expected to form as the end product of high-mass stars. Supermassive black holes, by contrast, have grown to their current size over cosmic time, from much smaller seeds (e.g. Begelman & Rees 1978; Islam, Taylor & Silk 2004;

Portegies-Zwart et al. 2004; Micic, Holley-Bockelmann & Sigurdsson 2011, and references therein). Intermediate mass black holes (IMBH; $\sim 10^2$–10^4 M$_\odot$) may have been the original seeds for supermassive black holes or they may have contributed to fast early growth of such seeds via mergers (e.g. Madau & Rees 2001; Miller & Colbert 2004). Though we expect IMBH should exist, at least as an intermediate stage on the way to a supermassive black hole, observationally the evidence for their existence is scant and ambiguous, especially compared with evidence for supermassive and stellar mass black holes. The low mass end of the supermassive black hole distribution in galactic nuclei may extend down to $\sim 10^5$ M$_\odot$ (Jiang et al. 2011), but below this mass the evidence becomes ambiguous. The ultra-luminous X-ray sources (ULXs) observed outside galactic nuclei (e.g. Winter et al. 2009) may be powered by accretion

on to IMBH (Miller & Colbert 2004). However, ULXs could also be explained by beamed radiation from accreting stellar-mass black holes (King 2009) and power-law-dominated ULXs might be due to background active galactic nuclei (AGN). IMBH have so far been hard to find and constrain in the local Universe, either in our own Galaxy or at low *z*.

AGN are believed to be powered by accretion on to a super-massive black hole. The accretion disc should contain a population of stars and compact objects (collectively nuclear cluster objects, NCOs) that can migrate within and accrete from the disc (e.g. Ostriker 1983; Syer, Clarke & Rees 1991; Artymowicz, Lin & Wampler 1993; Goodman & Tan 2004; Levin 2007; Nayakshin & Sunyaev 2007; McKernan et al. 2011a,b). In McKernan et al. (2011a), we speculated that IMBH seeds may form efficiently in AGN discs due to NCO collisions and mergers, which is quite different from the standard model of stellar mass black holes merging in stellar clusters (e.g. Miller & Hamilton 2002; Miller & Colbert 2004). Here we argue that IMBH production is in fact far more likely and more efficient in AGN discs, with implications for AGN observations, duty cycle and supermassive black hole accretion rates.

In this paper (and its companion Paper II: McKernan et al., in preparation), we discuss semi-analytically the production of intermediate mass black holes in the environment of AGN discs. Discussion of observational predictions of this model of IMBH growth as well as consequences for AGN discs, duty cycles and the demographics of activity in galactic nuclei at low and high redshift will be left to Paper II. In Section 2, we discuss why we think IMBH can be built in AGN discs. In Section 3, we explore mechanisms that will be important in actually growing IMBH in AGN discs, including the competing forces of eccentricity damping and excitation in the disc. The importance of IMBH migration is outlined in Section 4. Section 5 outlines a simple model of IMBH growth in AGN discs and we highlight the remarkable parallel between the growth of IMBH in AGN discs and the growth of giant planets in protoplanetary discs. Finally in Section 6, we outline our conclusions and future work.

2 WHY IMBH CAN BE BUILT IN AGN DISCS

The largest, supermassive, black holes in the Universe ($M_{BH} \sim 10^6$–10^9 M$_\odot$) live in galactic centres (e.g. Kormendy & Richstone 1995). We expect a dense nuclear cluster of objects to surround the supermassive black hole as a result of stellar evolution, dynamical friction, secular evolution and minor mergers (e.g. Morris 1993; Miralda-Escudé & Gould 2000; Merritt 2010). In our own Galaxy, the distributed mass within ~ 1 pc of Sgr A* is ~ 10–30 per cent of the mass of the supermassive black hole (Schödel, Merritt & Eckart 2009). If a large quantity of gas somehow arrives in the innermost pc of a galactic nucleus (e.g. Krolik 1999; Kawawatu, Umemura & Mori 2003; Vittorini, Shankar & Cavaliere 2005; Hopkins & Hernquist 2006; McKernan et al. 2010a; McKernan, Maller & Ford 2010b), it will likely lose angular momentum and accrete on to the central supermassive black hole. But in doing so, gas must also interact with the NCO population. Depending on the aspect ratio of the disc that forms, a few per cent of NCO orbits are likely to coincide with the accretion flow. A small percentage of NCO orbits coincident with the geometric cross-section of a thin disc would lead to an initial population of $\sim 10^3$ M$_\odot$ of NCOs in a pc-scale accretion flow around a SgrA* sized black hole.

NCOs can exchange angular momentum with gas in the disc, and each other, so they can scatter each other and migrate within the disc (see McKernan et al. 2011a). The processes involved are

analogous to protoplanetary disc theory (e.g. Pollack et al. 1996; Armitage 2010). Indeed, physical conditions in the *outskirts* of AGN discs are relatively close to those in protoplanetary discs (McKernan et al. 2011a). The migration of NCOs in the disc will enhance the probability of collisions, mergers and ejections. Under these conditions, IMBH seeds can grow. IMBH seeds will be objects ≥ 10 M$_\odot$ that will not lose very much mass (e.g. stellar mass black holes, hard massive binaries). IMBH 'seedlings' were defined as objects ≥ 10 M$_\odot$ that have grown via mergers (e.g. the merged end-product of a hard binary). IMBH seeds and seedlings located at the semi-major axis a in an AGN disc will maintain a 'feeding zone' within which they may collide with nearly co-orbital disc NCOs. By analogy with proto-planet growth, we define the feeding zone to be $a \pm 4R_H$ where a is the IMBH semi-major axis and $R_H = a(q/3)^{1/3}$ is the IMBH Hill radius, with q the mass ratio of IMBH:supermassive black hole. Once the object gets to ≥ 100 M$_\odot$ we will call the result an IMBH.

The disc NCO population is subjected to dynamical heating from cusp stars and dynamical cooling from gas damping. If gas damping dominates, IMBH seeds and disc NCOs will have their orbits rapidly damped. As a result their collision cross-sections will rapidly increase (since the relative velocity of encounters will be small, particularly in the outer disc). IMBH seedlings will initially accrete disc NCOs within their feeding zone in a 'core accretion' mode of growth. Once nearby NCOs have been scattered or accreted, gas accretion dominates IMBH growth. However, the migration of IMBH seedlings within the disc allows growth to continue via collisions as well as via gas accretion. Thus, we expect IMBH to grow within AGN discs, analogous to the growth of giant planets within protoplanetary discs and we expect that the IMBH growth rate will be much larger than in stellar clusters.

Here we concentrate on growing IMBH within the AGN disc itself. Of course, it is possible that IMBH already exist in the galactic nucleus when the AGN disc first forms. A top-heavy initial mass function of cusp stars can lead to IMBH seedling formation before low angular momentum gas arrives in the nucleus. IMBH can also arrive from outside the galactic nucleus to interact with the AGN disc since mass segregation and dynamical friction can deliver IMBH to the central parsec of galactic nuclei in a few Gyr from nearby clusters (e.g. McKernan et al. 2011b, see also Paper II).

The physics involved in IMBH formation in AGN discs spans multiple regimes and physical processes and would usefully benefit from detailed numerical simulations. Such simulations require realistic treatments of (amongst others) *N*-body collisions, mergers, accretion, tidal forces, gravitational radiation, special and general relativity, radiative transfer, the magneto-rotational instability and the gravitational instability. However, at present there are no simulations that can adequately address the relevant physics in a self-consistent manner. We take a semi-analytic approach following (e.g. Miller & Hamilton 2002) on the build-up of IMBH in star clusters (e.g. Alexander, Begelman & Armitage 2007) on stellar dynamical heating and cooling, as well as formalism on planet growth from protoplanetary theory (e.g. Pollack et al. 1996; Armitage 2010, and references therein).

3 HOW TO BUILD IMBH IN AGN DISCS

In this section, we will outline the key phenomena involved in growing IMBH seeds in AGN discs. In order to grow into IMBH, seeds must collide with and accrete mass, either NCOs or gas. In Section 3.1, we discuss NCO collision cross-sections in AGN

discs and the importance of eccentricity damping and excitation. In Section 3.2, we outline a model of dynamical heating and cooling of NCO orbits in AGN discs and we discuss the implications for IMBH seed growth. Section 3.3 briefly outlines issues involved in merging binaries, a potentially important channel for producing IMBH seedlings.

3.1 Collision cross-sections in the disc

In the absence of disc gas, NCOs change their orbits only due to weak gravitational interactions, occurring on the (long) relaxation time-scale (see below). The interaction with a gaseous disc gives rise to new effects, namely torques. NCOs embedded in the disc, or crossing it, will have their eccentricities and inclinations damped relative to the disc. Such processes should enhance the stellar density in the disc region and lower the velocity dispersion of NCOs embedded in the disc, in the absence of other important effects. This, in turn, gives rise to a higher rate of encounter and collisions between NCOs in the disc. The collisional cross-section ($\sigma_{\rm coll}$) of compact NCOs of mass M depends on the relative velocity at infinity (v_∞) as

$$\sigma_{\rm coll} \approx \pi r_{\rm p}(2GM/v_\infty^2) \tag{1}$$

in the gravitational focusing regime, where $r_{\rm p}$ is the separation at periastron. In AGN discs, the *relative* velocities involved in close encounters can be very small compared to the velocity dispersion in star clusters (typically $\sim 50\,{\rm km\,s^{-1}}$). For example, NCOs on circularized orbits separated in the disc by $\Delta R \sim 0.01R$ at $R = 10^5 r_{\rm g}$ have relative velocities due to Keplerian shear at periastron of only $\sim 5\,{\rm km\,s^{-1}}$, where $r_{\rm g} = GM_{\rm SMBH}/c^2$ is the gravitational radius of the supermassive black hole of mass $M_{\rm SMBH}$. The disc NCO velocity dispersion varies with radius as $\sigma \approx \sqrt{\bar{e}^2 + \bar{i}^2}\,v_{\rm k}$, where $\bar{e}$, $\bar{i}$, $v_{\rm k}$ are the mean NCO orbital eccentricity, mean NCO orbital inclination and Keplerian velocity, respectively. Fig. 1 shows the NCO velocity dispersion (σ) as a function of radius in a Keplerian AGN disc for a range of mean eccentricities and inclinations. Also shown in Fig. 1 is the typical velocity dispersion in star clusters ($\sim 50\,{\rm km\,s^{-1}}$, red horizontal dashed line). So v_∞ for a typical interaction in a stellar cluster is $\sim 50\,{\rm km\,s^{-1}}$. Fig. 1 shows that the velocity dispersion of NCOs at large disc radii is less than in star clusters for small to moderate NCO orbital eccentricities and inclinations ($e, i \sim 0.01$–0.05). Since the numbers of NCOs should increase with radius, most NCOs should live in the outer disc, where the NCO velocity dispersion should be smallest.

The collisional cross-section of a seed IMBH (mass M) with compact objects (mass m) such as neutron stars, white dwarfs and stellar mass black holes depends on relative velocity as (Quinlan & Shapiro 1989)

$$\sigma_{\rm coll} = 2\pi \left(\frac{85\pi}{6\sqrt{2}}\right)^{2/7} \frac{G^2 m^{2/7} M^{12/7}}{c^{10/7} v_\infty^{18/7}}. \tag{2}$$

Numerically, this can be written as $\sigma_{\rm coll} \approx 2 \times 10^{26} m_{10}^{2/7}$ $M_{50}^{12/7} v_6^{-18/7}\,{\rm cm^2}$ where $v_\infty = 10^6 v_6\,{\rm cm\,s^{-1}}$ and M_{50}, m_{10} are in units of $50\,{\rm M_\odot}$, $10\,{\rm M_\odot}$, respectively (Miller & Hamilton 2002). For M_{50}, $m_{10} = 1$, located $\sim 10^5 r_{\rm g}$ from a supermassive black hole, small eccentricities $e \sim 0.01$ in Fig. 1 lead to collision cross-sections up to an order of magnitude larger than in clusters (for M_{50}, $m_{10} = 1$; $v_6 = 5$ above). However, for large NCO orbital eccentricities ($e \geq 0.1$), IMBH collisions in AGN discs will have smaller cross-sections than in star clusters, over most of the disc. Therefore, if the initial mean eccentricity ($\bar{e}$) of the disc NCO distribution is large,
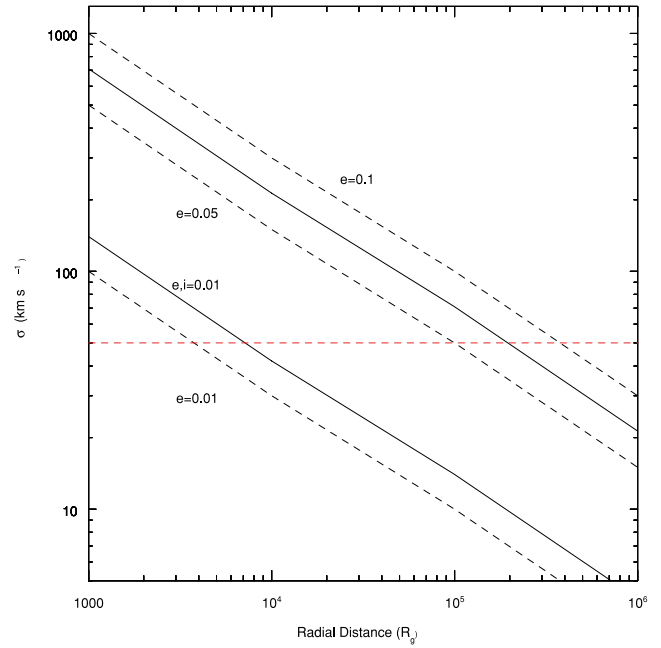

Figure 1. The velocity dispersion ($\sigma \approx \sqrt{e^2 + i^2}\,V_{\rm kep}$) of NCOs in a Keplerian AGN disc. Shown are σ as a function of disc radius, for eccentricity and inclination values of $(e,i) = 0.01, 0.05$ (solid lines) and $e = 0.01, 0.05, 0.1$, with $i = 0$(dashed lines). Also shown (red dashed horizontal line) is the typical velocity dispersion in star clusters ($\sim 50\,{\rm km\,s^{-1}}$). Note that most disc NCOs should live in the outer disc ($> 10^4 r_{\rm g}$) for an NCO population that grows as r^2.

mechanisms for damping orbital eccentricities will be very important in determining whether IMBH growth via collisions is efficient in AGN discs.

3.2 NCO orbital damping and excitation

We begin with a fully analytic approach, demonstrating the relative importance of competing terms and effects. The velocity dispersion (σ) of NCOs in a disc is excited by dynamical heating and is damped by dynamical cooling. Thus,

$$\frac{{\rm d}\sigma}{{\rm d}t} = \Delta Q_+ - \Delta Q_-, \tag{3}$$

where ΔQ_+ is the dynamical heating term and ΔQ_- is the dynamical cooling term. Dynamical heating comes from two sources: the relaxation of disc NCOs through mutual interactions and the dynamical excitation of disc NCOs by cusp NCOs, so

$$\Delta Q_+ = \delta Q_{\rm relax} + \delta Q_{\rm excite}. \tag{4}$$

Considering first the relaxation term, we assume that there are N_1 stars of mass M_1 and velocity dispersion σ_1 in an annulus of width ΔR centred on R. The relaxation time-scale is given by (Alexander et al. 2007)

$$t_{\rm relax} = \frac{2\pi C_1 R \Delta R \sigma^4}{G^2 N m^2 \ln \Lambda \Omega} \tag{5}$$

so

$$\delta Q_{\rm relax} = \frac{\sigma}{t_{\rm relax}} = \frac{D_1}{C_1} \frac{1}{\sigma_1^3}, \tag{6}$$

where

$$D_1 = \frac{G^2 N_1 M_1^2 \ln \Lambda_1}{R \Delta R t_{\rm orb}} \tag{7}$$
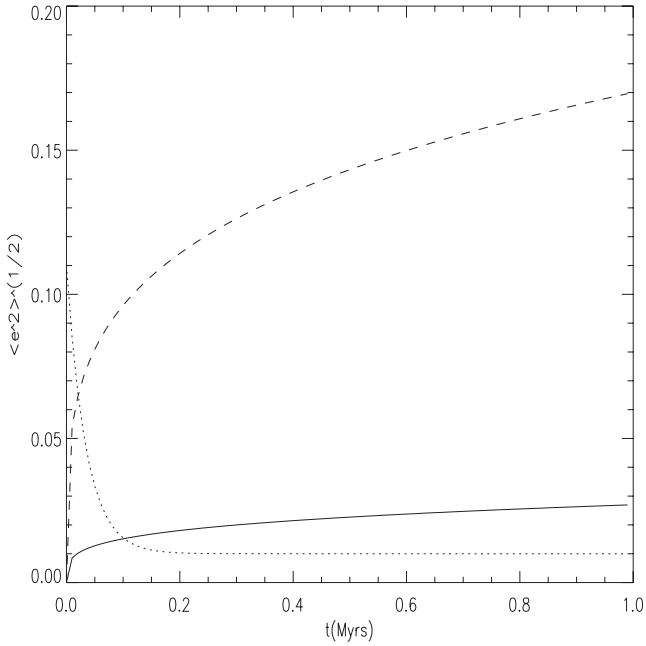
Figure 2. The rms eccentricity as a function of time for an annulus of 10^3 stars of identical mass $0.6\,M_\odot$ located in an AGN disc between 0.05 and 0.15 pc (or $1-3\times10^4 r_g$) around a $10^8\,M_\odot$ supermassive black hole. $\langle e^2\rangle^{1/2}$ scales as $N^{1/4}$, $M^{1/2}$. The solid curve shows the relaxation of the isolated distribution of stars assuming initial circularized orbits. The dashed curve shows relaxation *plus* dynamical heating from a population of $N_i = 100N_1$ cusp stars with $M_i = M_1$ and $\overline\sigma_i \sim 0.5v_k$. The dotted line shows the net domination of exponential damping by disc gas over orbital excitation by cusp stars.

where Ω is the Keplerian frequency, $\ln\Lambda_1$ ($\sim$9) is the Coulomb logarithm and $C_1 \sim 2.2$. The solid curve in Fig. 2 shows the evolution of $\langle e^2\rangle^{1/2}$ due to relaxation alone for a population of $N_1 = 10^3$ stars of mass $M_1 = 0.6\,M_\odot$ in an annulus of width $\Delta R = 0.1$ pc centred on $R = 0.1$ pc (equivalently $1-3\times10^4 r_g$ of an AGN disc around a $10^8\,M_\odot$ supermassive black hole). For these stars, $v_k = 2100\,\mathrm{km\,s^{-1}}$ and so $t_{\mathrm{orb}} = 9\times10^9$ s. Since, for moderate eccentricities,

$$\sigma = \frac{\langle e^2\rangle^{1/2}v_k}{\sqrt2} \tag{8}$$

the solid curve in Fig. 2 follows a $t^{1/4}$ form and rises (limited by the increase in σ to approximately the Keplerian velocity). This solid curve applies to an isolated annulus of stars in the absence of competing effects.

Additional heating is supplied by the cusp population. The cusp stars will transfer kinetic energy to the 'colder' disc population and excite the σ_1 distribution of the disc NCOs (see Perets et al. 2008 and in preparation). Following Alexander et al. (2007), $\delta Q_{\mathrm{excite}}$ has the form

$$\delta Q_{\mathrm{excite}} = \frac{\sigma}{t_{\mathrm{excite}}} = \frac{D_2}{C_2}\frac{\sigma_1}{\overline\sigma_{1i}^4}\left(1 - \frac{E_1}{E_i}\right), \tag{9}$$

where the cusp population $N_i \gg N_1$ has a similar mass function $M_i = M_1$ to the NCOs in the disc and

$$D_2 = \frac{G^2 N_i M_i M_1 \ln\Lambda_{1i}}{R_i \Delta R t_{\mathrm{orb}}} \tag{10}$$

with $\overline\sigma_{1i} = (\sigma_1 + \sigma_i)/2$ and $E_{1,i} = 3M_{1,i}\sigma_{1,i}^2$ is the kinetic energy. Note that t_{excite} is analogous to the t_{relax} term in equation (5) except N is now the cusp population (N_i). So $t_{\mathrm{excite}} \sim (N_1/N_i)t_{\mathrm{relax}} \ll t_{\mathrm{relax}}$.

We assume that on average $\overline\sigma_i \sim \sqrt{\overline{e^2}+\overline{i}^2}v_k \sim 0.5v_k$ so $E_i > E_1$ (i.e. the cusp stars have greater kinetic energy than the disc stars). The dashed curve in Fig. 2 shows the addition of this excitation term to the evolution of $\langle e^2\rangle^{1/2}$, assuming $N_i = 10^2 N_1$ and $C_1/C_2 = 3.5$ (Alexander et al. 2007), with $\ln\Lambda_{1i} \sim \ln\Lambda_1$. Clearly, the curve retains a $t^{1/4}$ dependence, but at larger values of $\langle e^2\rangle^{1/2}$. Thus, dynamical heating by stars in the cusp dominates relaxation by the stars in the disc (see also Perets et al. 2008).

The competing dynamical cooling term ΔQ_- is dominated by gas damping of the disc NCO orbits. Gas drag in AGN discs will tend to reduce small NCO orbital eccentricities and inclinations to much smaller values. Gas at co-orbital Lindblad resonances will damp (e,i) for NCOs with $q \lesssim 10^{-3}$ (e.g. Artymowicz 1993; Ward & Hahn 1994; Cresswell et al. 2007; Bitsch & Kley 2010). Since this mechanism depends on the corotating gas mass, both stellar and compact NCOs and IMBH seeds will have their orbits damped, particularly in the outer disc where most of the disc mass is located. For small eccentricities ($e < 2(H/r)$), orbital eccentricity decays exponentially over time $\tau_e \approx (H/r)^2\tau_{\mathrm{mig}}$, where $h = H/r$ is the disc aspect ratio and τ_{mig} is the migration time-scale (Ward & Hahn 1994; Bitsch & Kley 2010). Thus,

$$\frac{de}{dt} = -\kappa e, \tag{11}$$

which will give us a term linear in σ as the damping term in ΔQ_-. By analogy with the relaxation term ($\sigma/t_{\mathrm{relax}}$ above, we choose $\kappa = 1/t_{\mathrm{damp}}$. The damping time-scale is given by (e.g. Horn et al. 2012)

$$t_{\mathrm{damp}} = \frac{M_{\mathrm{BH}}^2 h_{\mathrm{gas}}^4}{m\Sigma a^2\Omega} \tag{12}$$

where $h_{\mathrm{gas}} = H/R = c_s/v_k$ is the *gas* disc aspect ratio (the disc of stellar NCOs has an aspect ratio $h_{\mathrm{stars}} = \sigma/v_k$ but the disc NCOs are not the source of damping). For larger eccentricities ($e > 2(H/r)$), the eccentricity damping goes as (Bitsch & Kley 2010)

$$\frac{de}{dt} = -\frac{\kappa}{e^2}, \tag{13}$$

where we choose the same normalization $\kappa = 1/t_{\mathrm{damp}}$ as above. So,

$$\Delta Q_- = -\kappa\left[\beta'\sigma + \frac{\beta''}{\sigma^2}\right], \tag{14}$$

where $\beta' = 1$ if $e < 0.1$, zero otherwise and $\beta'' = 1$ if $e > 0.1$, zero otherwise. Thus, our expression for the combined relaxation, excitation and gas damping of the velocity dispersion for an annulus of NCOs is given by

$$\frac{d\sigma}{dt} = \frac{D_1}{C_1}\frac{1}{\sigma_1^3} + \frac{D_2}{C_2}\frac{\sigma_1}{\overline\sigma_{1i}^4}\left(1 - \frac{E_1}{E_i}\right) - \kappa\left[\beta'\sigma + \frac{\beta''}{\sigma^2}\right]. \tag{15}$$

For $\langle e^2\rangle^{1/2} < 2h$ (Bitsch & Kley 2010), equation (15) has the general form

$$\frac{d\sigma}{dt} = \left[\frac{\sigma}{t_{\mathrm{relax}}} + \frac{\sigma}{t_{\mathrm{excite}}}\right] - \frac{\sigma}{t_{\mathrm{damp}}} = \frac{A}{\sigma^3} - \kappa\sigma, \tag{16}$$

where we have combined the dynamical heating terms into a single general A/σ^3 term. Since $t_{\mathrm{excite}} \ll t_{\mathrm{relax}}$, at steady state, $d\sigma/dt = 0$, and so $t_{\mathrm{damp}} \sim t_{\mathrm{excite}}$ and e takes the general form

$$e^4 \sim \frac{4G^2 N_c m M_{\mathrm{BH}}^2 h_{\mathrm{gas}}^4 \ln\Lambda}{2\pi C_2 \Sigma a^2 R \Delta R v_k^4}, \tag{17}$$

where N_c is the number of stars in the cusp. So, for a population of $10^3 \times 0.6\,M_\odot$ stars located at $1-3\times10^4 r_g$ in an AGN disc, with $h_{\mathrm{gas}} \sim 10^{-2}$ and a cusp population of $N_c = 10^5$ stars, equilibrium eccentricity is $e \sim 0.01$. In Fig. 2, we plot (dotted line) the

evolution of e over time assuming $i \sim 0$. From this we see that disc NCOs should rapidly settle down to near circular orbits ($\langle e^2 \rangle^{1/2} \sim 0.01$) within ~ 0.1 Myr. Therefore, collision cross-sections (σ_{coll}) of IMBH seeds in AGN discs should rapidly become much larger than typical collision cross-sections in star clusters and it is gas damping that makes the difference.

We expect gas damping to become even more dominant if the NCO disc population declines ($\dot{N}_-$) due to mergers, accretion and scatterings. From equipartition of energy, we expect that $\dot{N}_-$ will be dominated by low mass stars. At moderate inclinations, these NCOs can be captured fairly quickly by the disc again (such that $\dot{N}_+$ increases), if the gas damps the orbital inclination efficiently (Artymowicz et al. 1993). As $\dot{N}_+$ increases, the system is driven towards a dynamical equilibrium when $\dot{N}_- \approx \dot{N}_+$.

One important point to note from the above discussion is that the dynamical heating of the NCOs by cusp stars (ΔQ_+) gets transferred to the AGN disc *gas*. The stability of the outskirts of the AGN disc is a well-known and unsolved problem (e.g. Sirko & Goodman 2003); dynamical heating of disc NCOs by cusp stars is a new, additional source of disc heating which will contribute to maintaining the outer disc against gravitational instability. A self-consistent calculation of the disc heating requires a disc model (e.g. Sirko & Goodman 2003) and is beyond the scope of this paper, but see Paper II. Nevertheless, we can see that a large density of NCOs in a galactic nucleus will strongly excite the orbits of disc NCOs (δQ_{excite} is large). Gas damping (ΔQ_-) will naturally transfer much of this dynamical energy to the disc gas. Thus, disc luminosity must increase and the disc itself will puff up. The scale height increase will be a function of the density of NCOs in the nucleus. Therefore, one prediction of our model is that among nuclei with similar supermassive black hole masses, those with denser stellar cusps should generate more luminous AGN discs (see Paper II).

So far we have discussed low mass stars. However, we are interested in higher mass IMBH seeds. For simplicity let us assume a steep NCO mass function ($\mathrm{d}N/\mathrm{d}M \propto M^{-3}$) with two mass bins. The low mass population NCOs are $0.6\,\mathrm{M}_\odot$ stars (N_l in number); thus, the high mass NCO population is $10^{-3}N_l \times 10\,\mathrm{M}_\odot$ stellar mass black holes. For a total initial disc NCO mass of $10^3(10^4)\,\mathrm{M}_\odot$, the distribution is $1.65 \times 10^3(10^4)$ low mass stars and $1(10)$ stellar mass black holes. Alexander et al. (2007) show that a low mass population of stars will diffuse out of the disc more than the high mass population of stars and in fact damp the orbits of the high mass stars, as expected from equipartition. Thus, for small initial values of $\langle e^2 \rangle^{1/2}$ among disc NCOs, we expect potential IMBH seeds in AGN discs to evolve to even smaller eccentricities than the equilibrium value of $e \sim 0.01$ calculated above. Recall that small eccentricities imply large σ_{coll}, allowing IMBH seedlings to grow rapidly via collisions.

3.3 Binary mergers in the disc

Depending on the recent star formation history of a given galactic nucleus, massive binaries are likely to be rare in AGN discs. However, if there is even *one* in the initial AGN disc, it will have the largest collisional cross-section of any disc NCO and should undergo the largest number of interactions (e.g. Portegies-Zwart et al. 1999; Fregeau et al. 2004). A massive binary, if present, is therefore the most likely IMBH seed and should arise frequently enough to be of astrophysical interest (for similar ideas concerning planetesimal growth through binary-single interactions in a protoplanetary disc, see e.g. Perets (2011)). In this section, we briefly consider some of the issues involved in binary mergers in an AGN disc and we

contrast the merger efficiency with that found in star clusters. For ease of comparison, we consider the $50\,\mathrm{M}_\odot + 10\,\mathrm{M}_\odot$ binary from (Miller & Hamilton 2002).

An unequal mass binary ($M > m$, separation a_{bin}) in the disc is considered hard if its binding energy (GMm/a_{bin}) is greater than the kinetic energy ($m\sigma^2 = m(\bar{e}^2 + \bar{i}^2)v_k^2$) of a typical interacting NCO. The collisional cross-section of such a binary is given by (Perets 2011)

$$\sigma_{coll} \approx \pi a_{bin}^2 \left(\frac{v_c}{v_\infty} \right)^2 \left(\frac{r_N/10^6\,\mathrm{cm}}{a_{bin}/2.14 \times 10^8\,\mathrm{cm}} \right), \quad (18)$$

where $v_c = (G/\mu(Mm/a_{bin}))^{1/2}$ is the critical velocity separating hard and soft binaries, with $\mu = (M_{bin} \times M_N)/(M_{bin} + M_N)$ the reduced total mass of the binary (M_{bin}) and interacting NCO (M_N). The time to merge for a binary of reduced mass $\mu = mM/(M + m)$ with $M > m$, semi-major axis a_{bin} and eccentricity e_{bin} is

$$\tau_{merge} \approx 3 \times 10^8\,\mathrm{M}_\odot^3(\mu M^2)^{-1}(a_{bin}/R_\odot)^4 \left(1 - e_{bin}^2 \right)^{7/2}\,\mathrm{yr} \quad (19)$$

and the typical semi-major axis separation for a merger time of τ_6 Myr (assuming $e_{bin} \sim 0$) is

$$a_{bin} \approx 3 \times 10^{11} \tau_6^{1/4} M_{50}^{1/2} m_{10}^{1/4}\,\mathrm{cm}, \quad (20)$$

where M_{50}, m_{10} are the masses in units of $50\,\mathrm{M}_\odot$ and $10\,\mathrm{M}_\odot$, respectively (Miller & Hamilton 2002). However, the above discussion neglects the gas disc.

Baruteau, Cuadra & Lin (2011) carried out hydrodynamic simulations of a binary in an AGN disc and found that binaries harden rapidly due to interaction with their own migratory spiral wakes. Baruteau et al. (2011) also found that e_{bin} is damped rapidly with inward migration. The rate of binary hardening ($\dot{a}_{bin}$) scales with the disc surface density such that $a_{bin}/\dot{a}_{bin} \ll \tau_{bin}$, the binary migration time-scale. Massive binaries with initial separation $a_{bin} \sim 0.3R_H$ end up at half this separation within 10 orbits of the supermassive black hole, where R_H is the Hill radius $= (q/3)^{1/3}a$, with a the semi-major axis of the binary centre of mass and q the mass ratio of the reduced mass binary to the supermassive black hole. For a constant rate of hardening ($\dot{a}_{bin}$), a M_{50}, m_{10} migrating binary separated by $\sim 3 \times 10^{11}(10^{13})$ cm (or $3\,R_\odot(2\,\mathrm{au})$) at $10^5 r_g$ will merge in $<0.1(2)$ Myr, which is a very small fraction of the AGN disc lifetime. So, because of the presence of a gas disc, it is easier to harden binaries in AGN discs than in star clusters.

Binaries will also encounter field NCOs in the disc as they migrate. This can either result in hardening to merger or disruption. The probability that a binary is disrupted per unit time is $1/t_{dis}$ where

$$t_{dis} = \frac{9|E|^2}{16\sqrt{\pi}\nu G^2 m^4 \sigma} \left(1 + \frac{4m\sigma^2}{15|E|} \right) \left[1 + \exp\left(\frac{3|E|}{4m\sigma^2} \right) \right], \quad (21)$$

where the field NCO has mean mass m and velocity dispersion σ, ν is the number density of field NCOs, $E = -GMm/a_{bin}$ is the binding energy of the binary and the average energy change per interaction is $\sim -0.2m\sigma^2$ (Binney & Tremaine 1987). In stellar clusters $\sim 10^2$ field interactions are required to harden massive binaries to merger (Miller & Hamilton 2002), where the relative velocities at close encounters are approximately the velocity dispersion ($\sim 50\,\mathrm{km\,s^{-1}}$) in star clusters. In AGN discs, the number of interactions required to harden a binary to merger depends on the mean eccentricity ($\bar{e}$) of the field NCO orbits. The probability of binary disruption ($1/t_{dis}$) increases as $\bar{e}$ increases. However, for already hard binaries (large $|E|$), fewer interactions (of energy $-0.2m\sigma^2$) are required for merger in gas discs. Thus, hard binaries will continue to harden due to inward Type I migration (Baruteau et al. 2011) *and* due to interactions with field NCOs in the disc.

For a $60\,M_{\odot}$ unequal mass binary (M_{50}, m_{10}) at $10^5 r_g$ interacting with a population of NCOs having $\bar{e} = 0.01$, the energy per binary interaction $(-0.2m\sigma^2)$ is $\sim$4 per cent of the typical interaction energy in stellar clusters (where $\sigma \sim 50\,\mathrm{km\,s^{-1}}$). So for highly damped NCO orbits, binary interactions are likely to be 'soft'. The number of interactions depends on the disc NCO surface density. If this binary is already hard, it could merge within a few orbits, i.e. orders of magnitude faster than binary merger time-scales in stellar clusters.

4 NCO MIGRATION AND COLLISIONS

The gas in the AGN disc exerts a net torque on NCOs. This means that individual NCOs (and IMBH) will migrate within the gas disc, enhancing the probability of collision and merger. Migration is mostly inwards in discs, although sometimes outwards. This means that IMBH can migrate into new regions of the disc, in search of new NCO 'victims'. The IMBH feeding zone (approximately $a \pm 4R_H$) moves with the migrating IMBH. This is analogous to a giant planet core continuing to collide with planetesimals as it migrates through a protoplanetary disc (Alibert et al. 2004). Migration can stall in discs, leading to a pile-up (overdensity of disc NCOs). In this case, rapid merger leading to rapid IMBH growth may occur, by analogy with pile-up in protoplanetary discs (Horn et al. 2012). A detailed calculation of this scenario will be carried out in future work. NCOs and IMBH may also migrate on to the central supermassive black hole, just as protoplanets may migrate on to a central star. So the issue of IMBH survival in AGN discs parallels the survival of giant planets in protoplanetary discs.

4.1 Type I IMBH migration

NCOs with mass ratios $q \leq 10^{-4}$ of the mass of the central supermassive black hole will undergo migration in the disc analogous to Type I protoplanetary migration, on a time-scale of (Paardekooper et al. 2010)

$$\tau_I = \frac{1}{N} \frac{M}{q \Sigma r^2} \left(\frac{H}{r}\right)^2 \frac{1}{\omega}, \tag{22}$$

where M is the central mass, q is the ratio of the satellite (NCO) mass to the central (supermassive black hole) mass, Σ is the disc surface density, H/r is the disc aspect ratio and ω is the satellite angular frequency. The numerical factor N depends on the ratio of radiative to dynamical time-scales and is a function of the power-law indices of Σ, T and entropy (Lyra, Paardekooper & Mac Low 2010; Paardekooper et al. 2010). Note that the Type I migration time-scale decreases with increasing migrator mass at a given radius so more massive NCOs will migrate more quickly at a given disc radius. Binary NCOs face exactly the same torques and will migrate on the same time-scale, but q and r in equation (22) are replaced with the ratio of the reduced mass of the binary to the supermassive black hole and a, the location of the binary centre of mass in the AGN disc, respectively.

Sirko & Goodman (2003) model an AGN disc including all the parameters that we require for the calculation of migrator time-scales as a function of radius around a $10^8\,M_{\odot}$ supermassive black hole. Although in principle, Sirko & Goodman (2003) model a disc out to $10^7 r_g$, they regard their disc as effectively truncating at $\sim 10^5 r_g$. This disc also *requires* a constant mass accretion rate $(\dot{M})$ and a constant disc viscosity (α) at all radii over the disc lifetime, which are obvious simplifications. Nevertheless using the simple AGN disc of Sirko & Goodman (2003) as our disc model, we can
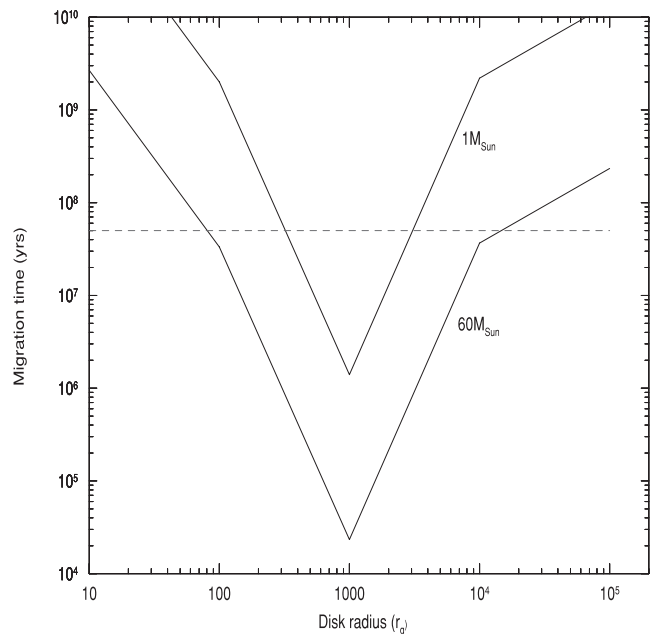

Figure 3. Estimates of Type I migration time-scales for $1\,M_{\odot}$ NCOs (upper curve) and $60\,M_{\odot}$ IMBH seedlings (lower curve) as a function of radius in the AGN disc modelled by Sirko & Goodman (2003). Also shown (red dashed horizontal line) is a fiducial AGN lifetime of 50 Myr. Most disc NCOs should live in the outer disc ($>10^4 r_g$) for an NCO population that grows as r^2. Note that migration is fastest around $10^3 r_g$ in the disc, where Σ is largest and the disc is thinnest (H/r is smallest). There is a possibility of NCO pile-up due to migration at smaller disc radii, as the corotating mass of gas drops dramatically in the disc interior.

estimate migration time-scales semi-analytically. Fig. 3 shows the Type I migration time-scales of a fiducial $1\,M_{\odot}$ NCO (upper curve) and a $60\,M_{\odot}$ IMBH seed (lower curve) as a function of disc location. The curves in Fig. 3 are generated by choosing $N \sim 3$ in equation (22) and assuming that Σ and H/r have the form of the curves in fig. 2 of Sirko & Goodman (2003). Also marked in Fig. 3 is an approximate AGN lifetime of 50 Myr (red dashed line). Evidently, substantial changes of NCO orbital radius can occur even for low mass NCOs in the inner disc ($<10^4 r_g$) over the AGN lifetime (few $\times$ 10 Myr). Larger mass migrators (stellar mass black holes, binaries, large mass stars or seed IMBH) are likely to have migration time-scales roughly comparable with the AGN disc lifetime. Therefore, as IMBH seedlings grow in mass, they should migrate in the disc and encounter low mass NCOs at low relative velocities. If migration stalls for a given NCO, either inwards at small disc radii or outwards at large disc radii, interactions are possible at incredibly low relative velocities (v_∞). Note that spiral density waves from migrating NCOs should not strongly perturb the NCO migrations or their orbits (Horn et al. 2012). Here we assume that seed IMBH migrate independently. Of course, resonant capture can occur, both between IMBHs and NCOs and between multiple IMBHs, analogous to resonances between Jupiter and its moons, or between Neptune and Pluto. Given the low mass ratios and high migration speeds, an assumption of independent migration seems reasonable, but future simulations involving multiple migrators are required to test this assumption.

Because we expect NCOs in the AGN disc to migrate differentially, we expect the migrators to encounter each other. As large mass NCOs migrate inwards across the orbits of less-massive NCOs, if the gas has damped $\bar{e}$ (see Section 3.1 above) the relative

velocities (v_∞) will be low and the collision cross-section (with gravitational focusing) will be large. However, different large mass NCOs will have different outcomes from multiple interactions. Large stars could have a shorter life expectancy if there are many NCO interactions increasing the odds of merger and supernova. Stellar mass black holes and IMBH seedlings will tend to shred low mass main sequence or giant stellar NCOs as they migrate inwards, although they can swallow compact NCOs whole as tidal forces shred compact objects only after the compact object crosses the innermost stable circular orbit (ISCO). Inward migrating binaries will tend to harden and scatter lower-mass NCOs until merger (see below).

Although a majority of Type I migration is directed inwards, Type I migration can also occur outwards in protoplanetary discs. For eccentricities $e < 0.02$, migration may be outwards rather than inwards (Bitsch & Kley 2010). Migration can also stall on both inward and outward migrations, depending on the temperature and density of the adiabatic disc (e.g. Veras & Armitage 2004; Lyra et al. 2010). So, as we consider a population of NCOs migrating in an AGN disc and interacting with each other, we do so with the caveat that a fraction of the NCOs may be migrating outwards or stalled. From equation (22), the migration time-scale gets longer at small disc radii (r), where Σ decreases and (H/r) increases due to disc heating. Migration may even stall or cease at small disc radii, particularly for large migrator masses, when the corotating disc mass becomes less than the migrator mass (e.g. Syer & Clarke 1995; Armitage 2007). However, conditions in the hot inner disc may be dramatically different to the outer disc. If there is an abrupt transition to an optically thin accretion region, or a disc trucation or cavity, migration will stall. In this case, NCO pile-up can occur, potentially leading to mergers and ejections. Recent *N*-body simulations of protoplanetary discs suggest that migrator pile-up in regions of the disc where inward and outward torques balance results in very rapid merging (Horn et al. 2012). If migrator pile-up occurs in AGN discs, it could favour the rapid building of IMBH seedlings. A stalled IMBH seed can merge with and scatter piled-up NCO migrators. Using a simple equipartition of energy, an IMBH seedling of mass M stalled at $10^2 r_g$ in an AGN disc with $\bar{e}, \bar{i} \sim 0.01$ could scatter in-migrating NCOs (m) to $\sigma \sim (M/m)$ 400 km s^{-1}. At such hypervelocities, small mass NCOs can be ejected into a galactic halo (see Paper II).

If disc NCOs migrate inwards, their rate of migration decreases in the inner disc ($<10^3 r_g$) as the disc surface density and corotating disc mass drops (see also Fig. 3 above). Although conditions in the innermost AGN disc are dramatically different from those in the outskirts, it is useful to consider the possibility of NCO pile-ups. Evidently, if the AGN inner disc truncates at some radius and then becomes a geometrically thick, optically thin advection flow, NCO migrators will stall near the disc truncation radius. As NCOs build up over time, the chances of interaction increase and hypervelocity scatterings become likely. The conditions may be similar to the migration trap for *N* protoplanets in protoplanetary discs, where rapid merger is possible (Horn et al. 2012). In this case, IMBH could form in the inner disc with distinctive observational signatures (see Paper II for details). Of course, migration traps can also occur as a result of out-migration.

A majority of NCOs in galactic nuclei will have orbits that do not coincide with the plane of the AGN disc ($i \gtrsim 0.05$) and will instead punch through the disc periodically. These NCOs can interact with each other, the disc and the migrating NCOs in the disc. NCOs on orbits with small radii will eventually decay into the plane of the disc over the AGN disc lifetime (Artymowicz et al. 1993), growing the NCO disc population ($\dot{N}_+$), particularly at small disc

radii. Resonant relaxation and the Kozai mechanism will also allow non-disc NCOs to trade eccentricity with inclination and migrate into the disc over time (e.g. Rauch & Tremaine 1996; Subr & Karas 2005; Chang 2009). On the other hand, NCO interactions within the disc that lead to ejection should keep ejected NCOs at relatively low inclinations, thereby increasing the probability of re-capture by the disc (growing $\dot{N}_+$). A disc capture rate of $\sim 10^{-4}$ of the non-disc NCO population over the lifetime of the AGN disc corresponds to the capture of $\sim 10^3$ M$_\odot$ of non-disc NCOs mostly in the inner AGN disc, over the ~ 50 Myr disc lifetime around a 10^8 M$_\odot$ supermassive black hole.

Unlike protoplanets in discs around stars, a large number of NCOs in AGN discs should have retrograde orbits. The behaviour of retrograde NCOs will depend on the efficiency of angular momentum transfer between the NCO and the disc gas. On one hand, if the coupling between NCO and gas is strong, the retrograde NCO rapidly loses angular momentum and falls into the central supermassive black hole very quickly. On the other hand, if the coupling is very weak, the disc gas can move fast enough past the NCO that the NCO can persist in the disc for a long time without migrating. In this case, prograde NCOs will migrate and encounter retrograde NCOs.

4.2 Type II IMBH migration

An IMBH that grows large enough by accreting gas can open a gap in the AGN disc (Syer & Clarke 1995). This phenomenon is analogous to gap opening by massive planets in protoplanetary discs (Armitage 2010). For typical disc parameters ($H/r \sim 0.05$, $\alpha = 0.01$), the mass ratio required to open a gap is $q \sim 10^{-4}$. However, there is a strong dependence on disc viscosity for both the profile and depth of the gap (Crida, Morbidelli & Masset 2006; Muto, Suzuki & Inutsuka 2010). To open a gap in the disc requires low disc viscosity (Crida et al. 2006)

$$\alpha < 0.09 q^2 \left(\frac{H}{r}\right)^{-5}, \tag{23}$$

where α is the disc viscosity parameter (Shakura & Sunyaev 1973). In the AGN disc modelled by Sirko & Goodman (2003), $H/r \sim 0.05$ on average between 10^2 and $10^5 r_g$ and $\alpha = 0.01$ is fixed. An IMBH with $q > 3 \times 10^{-4}$ ($> 3 \times 10^4$ M$_\odot$) will clear a gap in this disc. The gap will close by pressure if $(H/r) > (q/\alpha)^{1/2}$ and by accretion if $(H/r) > (q^2/\alpha)^{1/5}$ (Syer & Clarke 1995). Thus, a gap opened by a 3×10^4 M$_\odot$ IMBH in the AGN disc modelled by Sirko & Goodman (2003) will be closed by pressure and/or accretion in the outermost and innermost parts of the disc where (H/r) and α are large. In a more viscous type of accretion flow ($\alpha \gtrsim 0.1$, e.g. advection dominated), an IMBH might not open a gap in the disc. Whether an IMBH opens a gap will have major implications for observational signatures in AGN, but we defer that discussion to Paper II.

An IMBH that opens a gap will tend to migrate on the viscous disc time-scale (Type II migration) given by

$$\tau_{\text{II}} = \frac{1}{\alpha} \left(\frac{h}{r}\right)^{-2} \frac{1}{\omega}. \tag{24}$$

From equation (24), for $\alpha \sim 0.01$ and $H/r \sim 0.05$ (approximately the conditions across the AGN disc modelled by Sirko & Goodman 2003), the Type II migration time-scale is $\sim 10^4 \times$ the orbital time-scale. So, at $10^4(10^5) r_g$, the Type II migration time-scale is $\sim 1(30)$ Myr. Evidently a gap-opening IMBH can migrate on time-scales shorter than the AGN lifetime across the disc. We therefore have to ask whether any gap-opening IMBH will survive

the AGN disc. This is analogous to a major problem encountered in proto-planetary disc theory. The migration of some gap-opening migrators must somehow stall before accretion on to the central mass, otherwise no Jupiter-mass planets would be observed. One solution to this problem is that Type II migration can stall once the corotating disc mass is less than the migrator mass. This condition could arise due to disc drainage on to the supermassive black hole, or a change in the surface density profile of the disc. In the Sirko & Goodman (2003) disc, this radius is $\sim 10^4 r_g$ for a $3 \times 10^4 \, M_\odot$ IMBH. Once Type II migration stalls, it can resume but at a much slower rate, once the migrators' angular momentum is exported to the local disc (e.g. Syer & Clarke 1995; Armitage 2010).

If we simply assume that the IMBH can undergo Type II in-migration without stalling, the IMBH will collide with NCOs at radii interior to its starting position $10^4 (10^5) r_g$ before accreting on to the supermassive black hole in 1(30 Myr). Ignoring gas accretion, the IMBH can swallow up to 5 per cent (50 per cent) of the uniformly distributed disc NCO population if it starts migrating at $10^4 (10^5) r_g$. So, IMBH growth via NCO merger can be as much as $\sim 5000 \, M_\odot / 30$ Myr (starting at $10^5 r_g$ and 10^4 NCOs in the disc). This growth rate is due to NCO mergers only and does not include growth due to gas accretion (see Section 5.2 below). If the IMBH stalls permanently at $\sim 10^4 r_g$, only a small number of remaining migrators (~ 2 per cent of the remaining disc NCO population) migrate inwards to merge with the IMBH within 50 Myr.

To sum up, for a powerlaw stellar mass function ($\sim M^{-3}$), most NCOs should be low mass stars ($< 1 \, M_\odot$) with a small fraction of compact NCOs (mostly white dwarfs, some neutron stars and stellar mass black holes). The largest mass NCOs (and seeds for IMBHs) are likely to be small in number, and consist of stellar mass black holes or massive binaries. IMBH seedlings will undergo Type I migration in the disc. IMBH migration means that they can maintain a feeding zone of disc NCOs as they 'catch up' with the much more slowly migrating low mass disc NCOs. This migration of the feeding zone is precisely analogous to the situation expected for migrating giant planets (e.g. Alibert et al. 2004). Some of the classic problems of protoplanetary migration (e.g. how to stop migrators from accreting on to the central object, or how to get migrators moving once stalled) will also apply to IMBH seeds in AGN discs. Nevertheless, low relative velocity encounters due to migration will result in large NCO collision cross-sections and will help grow IMBH seeds via core accretion.

5 A MODEL OF IMBH GROWTH IN AGN DISCS

In this section, we will draw together much of the above discussion and construct a simple model of IMBH growth in AGN discs. The starting point for our model is a stellar mass black hole. This $10 \, M_\odot$ black hole can accrete gas from the AGN disc, migrate within the disc and collide with disc NCOs. We follow the approach of models of giant planet growth in protoplanetary discs (Pollack et al. 1996; Alibert et al. 2004). In Section 5.1, we discuss the growth of the IMBH seedling as a result of collision with disc NCOs within the IMBH feeding zone. In Section 5.2, we discuss the growth of the IMBH seedling as the result of gas accretion.

5.1 The parallel with 'core accretion'

We considered a simple model of the growth of a $10 \, M_\odot$ IMBH seed embedded in an AGN disc around a $10^8 \, M_\odot$ black hole. The disc NCO initial population is $(10^3) 10^4 \, M_\odot$, with a mass function of $dN/dM \propto M^{-3}$ (as discussed above mostly $0.6 \, M_\odot$ stars). We

assume that the IMBH seed 'feeds' on NCOs within its accretion zone, given by $a \pm \delta a$ where $\delta a \sim 4 R_H$, analogous to the feeding zone of giant planet cores (Pollack et al. 1996). The maximum (isolation) mass that the IMBH seed can attain by feeding on all the NCOs within its accretion zone is

$$M_{\text{ISO}} = M_I + 16\pi a^2 \Sigma_0 \left(\frac{q}{3} \right)^{1/3}, \tag{25}$$

where Σ_0 is the mean initial NCO surface density and q is the mass ratio of the IMBH seed to the supermassive black hole. If there are $10^4 \, M_\odot$ NCOs initially in the disc around a $10^8 \, M_\odot$ black hole, the mean initial NCO surface density is $\Sigma_0 \sim 3.5$ g cm^{-2}. For an IMBH seed of $M_I = 10 \, M_\odot$, equation (25) implies $M_{\text{ISO}} \sim 10 + 9(900) \, M_\odot$ at $10^4 (10^5) r_g$. So, in principle, a stellar mass black hole in the outer disc could grow to many times its original mass just by accreting low mass disc NCOs. This process is analogous to the growth of giant planet cores by planetesimal accretion (Pollack et al. 1996; Armitage 2010).

Assuming small eccentricities ($\langle e^2 \rangle^{1/2} = \Delta a / a$) for disc NCOs, we are in a shear-dominated regime and the rate of mass growth of the IMBH seed may be approximated by the form of giant planet core growth as (Armitage 2010)

$$\frac{dM}{dt} = \frac{9}{32} \frac{(\Delta a)^2}{\langle i^2 \rangle^{1/2} a R_H} \nu \Sigma_0 \Omega \sigma_{\text{coll}}, \tag{26}$$

where $\langle i^2 \rangle^{1/2}$ is the rms inclination for the NCO distribution, ν is the relative local overdensity of disc NCOs and σ_{coll} is the collision cross-section as given by equation (1). Thus, if $\Delta a = \langle e^2 \rangle^{1/2} a$ and if we assume $\langle e^2 \rangle^{1/2} \sim 2 \langle i^2 \rangle^{1/2}$, with $v_\infty \sim \sigma \approx \langle e^2 \rangle^{1/2} v_k$, the rate of IMBH seed growth via core accretion within its feeding zone is

$$\frac{dM}{dt} = \frac{9}{8} \frac{\nu \Sigma_0 \Omega \pi r_p}{e(q/3)^{1/3}} \frac{2GM}{v_k^2}, \tag{27}$$

where r_p is the periastron. The periastron for compact object collisions with an IMBH seed is $r_p \sim R_\odot$ (Miller & Hamilton 2002). Substituting into equation (27), where we assume that $\langle e^2 \rangle^{1/2} \sim 0.01$ is the equilibrium eccentricity, we find that for a $10 \, M_\odot$ IMBH seed at $\sim 2 \times 10^4 r_g$ with $\Sigma_0 = 3.5$ g cm^{-2} and $\nu = 1$, $dM/dt \sim 10^{-7} \, M_\odot$ yr^{-1}, which is approximately half the Eddington rate of growth. This is a very high accretion rate for a black hole, exceeding inferred accretion rates from gas discs in most Seyfert AGN (McKernan, Yaqoob & Reynolds 2007). Of course, Σ_0 is the average surface density assuming a uniform distribution of disc NCOs and that the mass ratio of disc NCOs to gas in the disc is ~ 1 per cent. If instead the mass ratio is a factor of a few larger, or the surface density distribution of NCOs is non-uniform, the accretion rate of disc NCOs by IMBH can be substantially *super-Eddington*. Equally, if gas damping is more efficient than outlined above so that equilibrium is reached at $\bar{e} < 0.01$ (e.g. due to a lower ratio of cusp population to disc NCOs), we could also reach super-Eddington rates of IMBH growth via mergers. From our earlier discussion, it is easy to envisage regions of the disc where NCOs tend to pile-up, leading to non-uniform distributions of disc NCOs. For example, if there is a region of the disc where inward and outward torques balance (as in the scenario outlined by Horn et al. 2012), or migration stalls due to a change in the aspect ratio, or the mass of corotating gas drops. In these cases, we could write $\Sigma_0 \sim \nu 3.5$ g cm^{-2}, where ν is an overdensity factor (which could locally be >100 in a pile-up scenario such as in Horn et al. 2012 and lead to highly super-Eddington growth). Note that a growing IMBH seedling avoids problems in protoplanetary coagulation theory, such as fracturing and sticking efficiency. For IMBH seedlings, nearby objects will either be captured (at high efficiency) or they will escape.
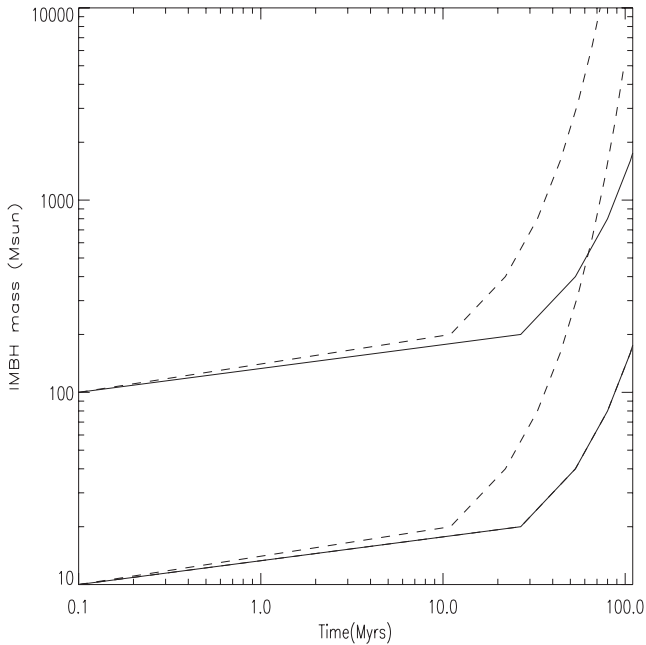
Figure 4. The growth over time of a $10\,M_\odot$ IMBH seed and a $100\,M_\odot$ IMBH located $2 \times 10^4 r_g$ from a $10^8\,M_\odot$ supermassive black hole in an AGN disc. The solid (dashed) curves show IMBH growth as it accretes disc NCOs within $\leq 4R_H$ at a rate $\times$ 0.5(2.5) Eddington, assuming $\bar{e} = 0.01$ and an initial NCO surface density $\Sigma_0 \sim 3.5(18)\,\mathrm{g\,cm^{-2}}$. The dashed curves could equivalently be generated with $\Sigma_0 \sim 7\,\mathrm{g\,cm^{-2}}$ and $\bar{e} = 0.004$. The gas accretion rate is assumed to be at the Eddington rate, for a total accretion rate of $\times$ 1.5(3.5) Eddington and a mass doubling time of $\sim$27(11) Myr, respectively. We assume the IMBH migrates in the disc and moves its feeding zone, so that it may continue to undergo collisions with NCOs at up to super-Eddington rates (analogous to continued collision and migration in protoplanetary discs; Alibert et al. 2004). In order to clear out $>10^3\,M_\odot$ of disc NCOs from this choice of initial disc location, *outward* migration is required. Once the IMBH reaches $10^4\,M_\odot$, we assume the disc NCO population has been cleared out and the IMBH opens a gap in the disc, accreting gas at an Eddington rate thereafter. In this picture, between 1/3 and 2/3 of the IMBH mass is actually due to gas accretion.

Fig. 4 shows the analytic growth of a $10\,M_\odot$ stellar mass black hole (IMBH seed) and a $100\,M_\odot$ IMBH in an AGN disc around a $10^8\,M_\odot$ supermassive black hole. The lower curve in each case corresponds to $\nu = 1$ (no overdensity, fiducial numbers) and the upper curves correspond to a moderate overdensity $\nu \sim 5$ of disc NCOs (or equivalently, a slight overdensity, $\nu = 2$, and a moderately lower mean eccentricity $\bar{e} = 0.004$). The fiducial mass doubling time for a black hole accreting gas at the Eddington rate (assuming 10 per cent efficiency) is 4×10^7 yr. In the case of IMBH seeds growing via collisions in AGN discs, and assuming a gas accretion rate of $\sim$ Eddington, the total growth rates are $\times$ 1.5(3.5) Eddington for $\nu = 1(5)$. At $3.5\times$ Eddington growth rates, the mass doubling time could be as little as $\sim$11 Myr. Once the IMBH reaches its isolation mass, it will then grow via accretion from the gas disc, at a much slower rate (Eddington or a fraction thereof). However, with an increased mass, the IMBH will have a shorter Type I migration time-scale (see Section 4 above). Thus, in $\sim$11 Myr, the IMBH will have migrated inwards or outwards in the disc and the size of the feeding zone will have grown. So, a super-Eddington mode of accretion via collisions could continue (dashed curve in Fig. 4). This process is analogous to the migration of giant planet cores and their feeding zones in protoplanetary discs (Alibert et al. 2004).

5.2 IMBH and gas accretion: runaway growth

A big difference between IMBH growth in stellar clusters and in AGN discs is that in the latter, gas can damp orbits quite effectively. So mergers tend to be more frequent, and the IMBH seeds can continuously accrete dense gas. Thus, we expect IMBH growth in AGN discs to involve growth by merger (as in stellar clusters) but we also expect growth by gas accretion. Torques from the gas will cause the IMBH to migrate and enter new feeding zones, analogous to the situation in protoplanetary discs (Alibert et al. 2004). If the IMBH grows large enough ($q = 10^{-4}$ or $10^4\,M_\odot$ around a $10^8\,M_\odot$ supermassive black hole), the IMBH can open a gap in the gas disc and the rate of gas accretion will drop. Note that although we concentrate on building a single IMBH, multiple IMBH seedlings (10100$\,M_\odot$) are likely to appear in the disc (assuming $\mathrm{d}M/\mathrm{d}t \propto M^{-3}$, see discussion above).

One problem will be in preventing an IMBH from migrating on to the supermassive black hole. Outward migration and the stalling of migration due to a drop in disc surface density or a change in the disc aspect ratio are possible solutions, but as with protoplanetary disc theory, this theoretical problem is complicated and remains unsolved for now. A sufficiently massive gap-opening IMBH ($\geq 10^4\,M_\odot$ around a $10^8\,M_\odot$ supermassive black hole) will grow if the AGN disc is particularly long lived (>50 Myr), or if there is a large local disc NCO overdensity (ν), or if gas damping is particularly efficient so that equilibrium eccentricity is $\bar{e} < 0.01$. Alternatively, an IMBH which survives a period of AGN activity could grow to gap-opening size via the mechanisms outlined here, during a later, independent AGN phase. Earlier in the history of the Universe (at $z \sim 2$), the time between individual AGN phases should be much smaller (e.g. Vittorini et al. 2005; Doherty et al. 2006; Shankar, Weinberg & Miralda-Escude 2009; McKernan et al. 2010b). So, if they survive, large-mass (gap-opening) IMBH could grow rapidly in galactic nuclei over a few 100 Myr and observational signatures of IMBH in galactic nuclei may be common at higher redshift (see Paper II).

Of course, as the gas disc is consumed or blown away, other mechanisms will come into play. For planetesimals in a late-stage protoplanetary disc, planet–planet interactions, the Kozai mechanism or resonant relaxation can increase $\bar{e}, \bar{i}$ (e.g. Rauch & Tremaine 1996; Subr & Karas 2005; Chang 2009). By analogy with protoplanetary discs, such mechanisms will certainly apply in the late stages of an AGN disc when most gas has been drained. However, we do not consider these mechanisms in more detail here since damping due to dense gas disc should dominate such effects (see Paper III for further discussion).

6 CONCLUSIONS

We show that it is possible to efficiently grow IMBH from stars and compact objects within an AGN disc. NCOs in the AGN disc are subject to two competing effects: orbital excitation due to cusp dynamical heating and orbital damping due to gas in the disc. For a simple, semi-analytic model, we show that gas damping dominates such that equilibrium eccentricities of disc NCOs are $e \sim 0.01$. In this case, IMBH seedling formation via NCO collision is more efficient in the AGN disc than in stellar clusters (the standard model for IMBH formation). If, as we expect, gas damping dominates, then the dynamical heating of disc NCOs by cusp stars is transmitted to the gas disc. This is a new, additional source of heating of the outer disc that can help counter the well-known gravitational instability ($Q \leq 1$) of the outer disc.

Stellar mass black holes and hard massive binaries are likely IMBH seeds. IMBH seedlings grow by collisions with disc NCOs within their feeding zone ($a \pm 4R_{\rm H}$) at near Eddington rates, as well as via gas accretion. IMBH seedlings will migrate within the AGN disc and so continue to feed on disc NCOs as they accrete gas. If there are regions of modest over-density of NCOs in the disc, IMBH seedling growth via collisions can be super-Eddington and a $10\,{\rm M}_\odot$ IMBH seed orbiting a $10^8\,{\rm M}_\odot$ supermassive black hole can grow to $\sim 300\,{\rm M}_\odot$ in less than the fiducial AGN disc lifetime. An over-density of disc NCOs can occur in regions of the disc where e.g. outward torques and inward torques balance, or where the aspect ratio changes, or where IMBH migration stalls.

The largest IMBH will open gaps in AGN discs, analogous to giant planets in protoplanetary discs. Gap-opening IMBH are more likely to arise if gas damping is very efficient (equilibrium disc NCO eccentricity is $\bar{e} < 0.01$), or if the disc is long lived ($>50\,{\rm Myr}$), or disc NCO surface density is moderately high ($>15\,{\rm g\,cm^{-2}}$), or if there is an IMBH seedling which survived a previous AGN phase (analogous to the survival of planets in protoplanetary discs). Our model of IMBH growth in AGN discs strongly parallels the growth of giant planets in protoplanetary discs. We leave a discussion of model predictions, observational constraints and implications of efficient IMBH growth in AGN discs to Paper II.

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