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Intermediate polars as low-field magnetic cataclysmic variables

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SUMMARY

We present the first detailed calculations of the polarization properties of extended accretion shocks on the surface of a magnetic white dwarf where allowance is made both for field spread and for the change in shock height as a function of specific accretion rate. These results are used to show conclusively that the null detection of circular polarization in most IPs imply fields of less than 5 MG. We suggest that the X-ray properties of MCVs depends critically on the fractional area of the white-dwarf surface over which accretion occurs, and on the type of accretion (smooth or clumpy). We argue that in the known IPs, accretion occurs via a disc. The accretion flow is smooth and a strong shock forms making them a powerful source of hard X-rays. We propose that there is a new class of MCV distinct from the IPs, where the white dwarf is asynchronous and accretes without a disc in which the accretion is clumpy and the radiation is mainly in the EUV region. We argue that a significant fraction of the IPs above the period gap evolve into such systems below the gap and not into AM Hers, as some workers have previously proposed. Any IP–AM Her transitions occur mainly above the period gap at $P_0 > 4$ hr.

1 INTRODUCTION

The magnetic cataclysmic variables (MCVs) have been classified into two groups, the AM Herculis variables (AM Hers) and the intermediate polars (IPs) (see Wickramasinghe 1988). In the AM Hers, the magnetic white dwarf is usually phase-locked into synchronous rotation with the binary period P_0, the only exceptions being V1500Cyg which was discovered as a slightly asynchronous AM Her after a nova event [Stockman, Schmidt & Lamb (1988) and possibly H0538+608, Mason, Liebert & Schmidt (1989)]. Strongly polarized cyclotron emission from accretion shocks near the magnetic poles dominate the radiation in the optical and near-IR bands. Emission in all bands (including X-rays) is modulated at the binary period. Surface field strengths of ~10–16 MG have been measured directly from Zeeman and cyclotron spectroscopy in 11 of the 19 known AM Hers.

In the IPs, the spin period P_s of the magnetic white dwarf is lower than P_0 so that synchronism has not been achieved. The optical and X-ray modulations are now seen at P_s although in many systems variation can also be detected at P_0 and the beat period $(P_s^{-1}-P_0^{-1})^{-1}$. With the exception of BG CMi, measurements of circular polarization in the optical band have led to null (<0.1 per cent) results (see Berriman 1988 for a recent review). Taken at face value the lack of synchronism and the absence of detectable circular polarization would suggest lower fields for most IPs in comparison to the AM Hers.

The above conclusion has been resisted by various groups (Chanmugam & Ray 1984; King, Frank & Ritter 1985) for the following reasons. It has been noted that the AM Hers and IPs have an apparently bimodal distribution with orbital period. While only 2 of the 12 known IPs lie below the period gap, 14 of the 19 AM Hers lie in this same period range. Impressed by these observations the above authors have suggested that most (or all) IPs evolve into AM Hers as they cross the period gap. For this hypothesis to be viable, most (or all) IPs must have magnetic moments that are comparable to those of the AM Hers.

In this paper we argue that the above hypothesis is inconsistent with observations. We investigate further the alternative possibility that the IPs have magnetic moments that are in general lower than those of the AM Hers with minimal overlap as previously concluded by Lamb & Melia (1988) and Warner & Wickramasinghe (1990). We show that a natural consequence of this hypothesis is that most of the IPs above the period gap will evolve into asynchronous discless systems below the period gap. We predict that such systems will emit mainly in the hitherto unobservable EUV region.

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2 IPs AS MCVs WITH LOWER FIELDS THAN AM HERS

2.1 The null detection of circular polarization

The nearly sinusoidal variation of the hard X-ray light curves of some IPs led King & Shaviv (1984) to suggest that in IPs accretion occurs on to a sizeable fraction ($f \sim 0.1$) of the white-dwarf surface in contrast to the AM Hers where $f \sim 10^{-4}$–10^{-5}. They concluded that the lack of circular polarization in IPs was primarily a consequence of field spread in an extended polar cap and dilution due to unpolarized radiation from other components and, was thereby not in conflict with the hypothesis that IPs and AM Hers had similar fields. Chanmugam & Frank (1987) presented calculations of cyclotron emission from an extended polar cap in an underlying dipolar field distribution which appeared to support this point of view. However, Wickramasinghe (1988) noted that the low values of circular polarization calculated by Chanmugam & Frank (1987) were due to a combination of the high fields $B(\sim 75$ MG) and the large values of the size parameter $\Lambda(\sim 10^7)$ (see below) assumed by these authors and not mainly due to the effects of field spread in an extended polar cap. As the field is increased, the optically thicker lower harmonics move into the optical band, resulting in a reduction in the polarization in this band. Likewise, if the optical depth parameter Λ is increased to a point where the relevant harmonics become optically thick ($\Lambda \sim 10^7$ at 20 keV in the optical), the polarization will be reduced. If the parameters were similar to the AM Hers ($B \sim 20$ MG, $\Lambda \sim 10^5$–10^6), field spread has little effect on the net value of circular polarization near the flux peak which then occurs in the optical–IR band. Indeed, the calculations of Wickramasinghe & Ferrario (1988) have shown that field spread could have the opposite effect of increasing the circular polarization at the optically thicker frequencies which would otherwise have zero or low polarization.

The size parameter Λ that enters the polarization calculations as a multiplicative factor in the optical depths is defined by

$$\Lambda = 2.01 \times 10^6 \left(\frac{h}{10^6\ \mathrm{cm}}\right)\left(\frac{N_\mathrm{e}}{10^{16}\ \mathrm{cm}^{-3}}\right)\left(\frac{3\times 10^7\ \mathrm{G}}{B}\right), \tag{1}$$

where h is the thickness of the shock perpendicular to the surface, N_e is the electron number density and B is the magnetic field. The cooling rate is determined by the shock density N_e and the field B so that the shock height, h, is a strong function of these quantities. This dependence is shown in Fig. 1 where we present results of calculations which allow for cyclotron and bremsstrahlung cooling following Wu & Chanmugam (1990) and Wu & Wickramasinghe (1990). For low B and high N_e the shock is bremsstrahlung cooling dominated. $h \propto N_\mathrm{e}^{-1}$ and $\Lambda \sim 2 \times 10^8$ almost independently of electron number density (Wickramasinghe 1988). As N_e decreases or B increases, the shock becomes cyclotron cooling dominated. The shock height decreases and low values of $\Lambda(\sim 10^5)$ are realized. At very lower densities where the ion–ion energy exchange time-scale is longer than the cooling time-scale (beyond the regime of our calculations) the shock is non-hydrodynamic (see Lamb & Masters 1979). The properties of the shock in this regime are not well understood. Clearly in a shock where the specific accretion rate (and hence N_e) varies across its extent, Λ may show large variations. Since the polarization is a strong function of Λ, these variations need to be properly taken into account in calculating the polarized spectrum. For IPs, to our knowledge no previous calculations have included these effects.

The observational evidence currently at hand strongly supports the point of view that the majority of IPs have discs. It is accordingly more appropriate to consider cyclotron emission from a ribbon-shaped accretion region rather than from an entire polar cap. We note in this context that if accretion were to occur on to an entire polar cap, the implied low *specific* accretion rate would result in a shock that extends several white dwarf radii above the surface (Fig. 1). Such a geometry would be inconsistent with the hard X-ray light curves which prompted the large polar cap idea in the first place (see Lamb 1988). Furthermore, accretion on to an entire polar cap would require quasi-radial inflow and it is difficult to imagine any mechanism that would do this except for special orientations of the dipole.

For an IP which accretes from a disc with an inner radius r_in, we estimate

$$f = \frac{1}{4}\left(\frac{r_\mathrm{wd}}{r_\mathrm{in}}\right)\left(\frac{\delta}{r_\mathrm{in}}\right), \tag{2}$$

where we have assumed that the field distribution is dipolar and matter couples on to field lines in the region of the disc defined by $r_\mathrm{in} \le r \le r_\mathrm{in} + \delta$. The mean density in the post-shock region is then given by

$$\langle N_\mathrm{e}\rangle = 10^{15}\left(\frac{\dot M}{10^{17}\ \mathrm{g\ s}^{-1}}\right)\left(\frac{P_\mathrm{s}}{10^3\ \mathrm{s}}\right)^{2/3}\left(\frac{r_\mathrm{in}}{\delta}\right)\left(\frac{\omega_\mathrm{s}}{0.35}\right)^{2/3} \times \left(\frac{r_\mathrm{wd}}{10^9\ \mathrm{cm}}\right)^{1/2} \mathrm{cm}^{-3}. \tag{3}$$

Here ω_s is the fastness parameter as defined in Ghosh & Lamb (1979) (see also Section 2.2). The value of δ is quite uncertain depending on the details of the interaction of the field with the disc. An upper limit can be obtained by setting $r_\mathrm{in} + \delta \sim r_\mathrm{c}$, where r_c is the corotation radius. This yields $\delta / r_\mathrm{in} \sim (1 - \omega_\mathrm{s}^\mathrm{c})$. For accretion near the equilibrium disc period ($\omega_\mathrm{s} = \omega_\mathrm{s}^\mathrm{c} \sim 0.35$–$0.8$), δ / r_in could be as high as ~ 0.7 although the coupling is expected to occur mainly near the inner edge of the disc. We estimate $\langle N_\mathrm{e}\rangle \sim 10^{15}$–$10^{16}\ \mathrm{cm}^{-3}$ for an IP with $P_0 \sim 5$ hr and $\dot M = 10^{17}\ \mathrm{g\ s}^{-1}$. That is, the higher accretion rate in comparison to AM Hers is balanced by the larger value of f for disc accretion yielding mean shock densities that are comparable to or lower than the mean shock densities in AM Hers.

We expect the density to be highest in regions of the ribbon that lie directly below the dipole axis in the plane containing the dipole axis and its projection on to the orbital plane. We measure the magnetic longitude ψ from this plane and parameterize the density variations by

$$N_\mathrm{e}(\theta, \psi) = N_\mathrm{e}^0 e^{-(\psi/\Delta\psi)^2} e^{-[(\theta_2 - \theta)/\Delta\theta]^2} \quad \text{for } [\theta_1(\psi) \le \theta \le \theta_2(\psi)] \quad \text{and} \quad (-\psi_0 \le \psi \le \psi_0) \tag{4}$$

with

$$\Delta\theta = \xi[\theta_2(\psi) - \theta_1(\psi)]$$

$$\Delta\psi = 2\eta\psi_0.$$

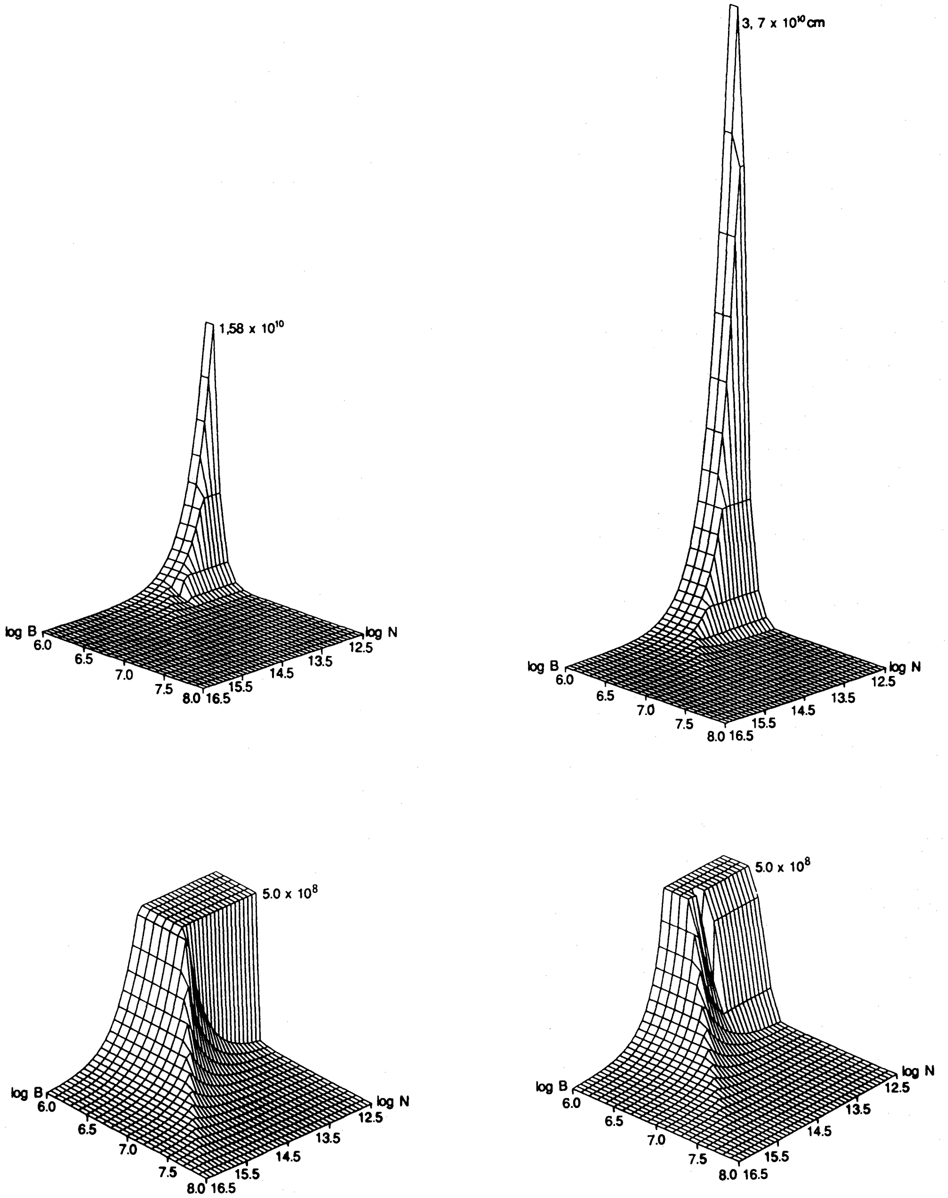

Figure 1. The shock height as a function of magnetic field B and electron number density N_e on two different linear scales. The calculations are for masses of 0.5 $M_\odot$ (right) and 0.8 $M_\odot$ (left).

Here N_e^0, ξ and η are parameters and θ is the magnetic co-latitude. $\theta_1(\psi)$ and $\theta_2(\psi)$ are the extremities of the ribbon at a given ψ calculated from

$$\sin\theta_i = \left(\frac{r_{wd}}{r_i}\right)^{1/2} \frac{\cos\delta_d}{\sqrt{1-\sin^2\delta_d\sin^2\psi}} \quad (i=1,2), \tag{5}$$

where r_1 and r_2 are the inner and the outer radius of the coupling region and δ_d is the dipole inclination. The above parameterization allows for the fact that the highest accretion rate is expected for material that is coupled on to field lines at the inner edge of the disc.

The properties of cyclotron emission from ribbon-shaped extended shocks are calculated using the locally plane approximation of Wickramasinghe (1988) and Ferrario & Wickramasinghe (1990). The calculations also allow for the variations in the shock height in response to changes in the accretion rate in a self-consistent manner as described previously and are accordingly superior to previous models (e.g. Chanmugam & Frank 1987).

We have assumed a white-dwarf mass $M_{wd}=0.6\ M_\odot$ and have carried out calculations for $B_p=5$, 10 and 20 MG for a range of values of $\dot{M}$. The calculations show that the results on flux and polarization at frequencies where cyclotron emission dominates (low harmonic numbers) are not strongly dependent on $\dot{M}$ for $\dot{M}\sim 10^{17}$–10^{18} g s^{-1} for $B\leq 20$ MG. This is a consequence of the fact that for the above range of $\dot{M}$ and value B the shock is mainly in the bremsstrahlung cooling dominated regime with $\Lambda \sim 2\times 10^8$ almost independently of N_e. In the high harmonic optically thin region where free–free emission dominates, the absolute value of the flux is proportional to N_e^2 and does depend on $\dot{M}$. Since we are presently interested in the cyclotron emission properties near the flux peak, we only show results for $\dot{M}=10^{18}$ g s^{-1}.

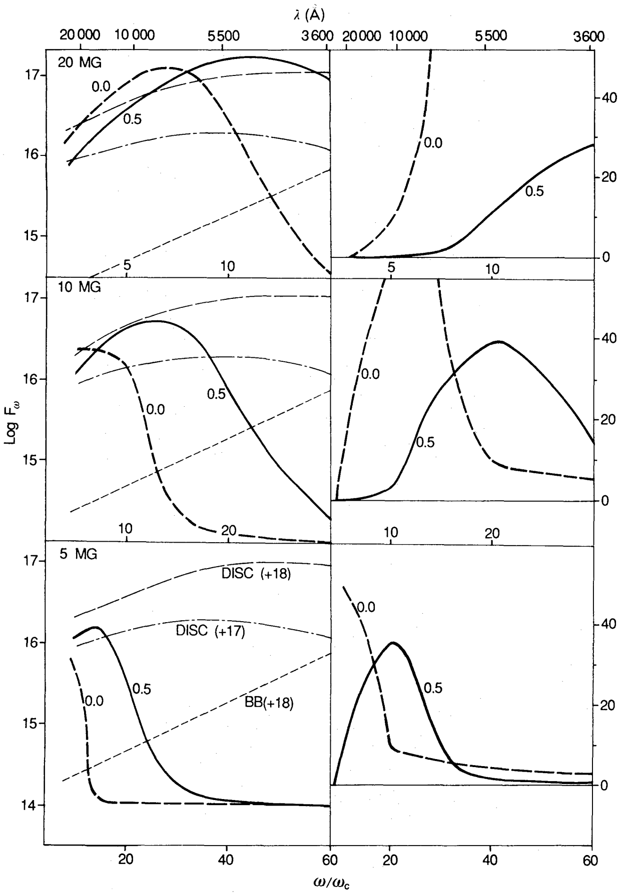

Figure 2. Calculations of the degree of circular polarization and the surface integrated flux ($\mathrm{erg^{-1}\ Hz^{-1}\ sr^{-1}}$) from a structured ribbon shaped shock corresponding to an accretion rate of $\dot{M}=10^{18}\ \mathrm{g\ s^{-1}}$. The dipole axis points towards the observer at phase $\phi=0.0$ (heavy broken line). The circular polarization approaches 95 per cent at phases where it is off scale. We also show the maximum contributions from a disc ($\dot{M}=10^{17}$, $10^{18}\ \mathrm{g\ s^{-1}}$) and from an equivalent blackbody (BB, $10^{18}\ \mathrm{g\ s^{-1}}$) if it is assumed that all the accretion energy is thermalized at the surface of the white dwarf.

The results of our calculations are shown in Fig. 2. The models have $\delta_\mathrm{d}=30°$, $\psi_0=180°$, $r_1=10\ r_\mathrm{wd}$, $r_2=20\ r_\mathrm{wd}$, $\xi=0.25$, $\eta=0.5$. $N_\mathrm{e}^0=3\times10^{17}\ \mathrm{cm^{-3}}$ which yields $\dot{M}=10^{18}\ \mathrm{g\ s^{-1}}$. The orbital inclination $i=30°$ and the results are for phases $\phi=0.0$ and 0.5, where $\phi=0.0$ corresponds to the phase that the dipole axis points mostly directly towards the observer.

The $B=20$ MG model shows that large values of circular polarization are predicted near the cyclotron flux peak at all viewing angles. The peak shifts from the blue to the near-IR as ϕ changes from 0.5 to 0.0 so that the polarized peak will be clearly apparent in optical–IR observations. The cyclotron peak shifts towards redder wavelengths and occurs at 2 μm near 5 MG. If the peak in the *polarized flux* can be established, the field can be determined quite accurately almost independently of $\dot{M}$. At low fields the radiation in the optical band is free–free emission from the shock which has a flat spectrum and a degree of circular polarization of $\mathrm{O}(\omega_\mathrm{c}/\omega)$. The possible importance of free–free emission in IPs was first noted by Wickramasinghe (1988).

Sources of radiation which may dilute the cyclotron flux are the heated photosphere, the accretion disc and the companion star. An upper limit to the former can be estimated from

$$F_\omega^{bb}=2\times10^7\left(\frac{f}{10^{-2}}\right)\left(\frac{r_\mathrm{wd}}{10^9\ \mathrm{cm}}\right)^2\left(\frac{\omega}{\omega_\mathrm{c}}\right)^2 \times\left(\frac{B}{2\times10^7\ \mathrm{G}}\right)\mathrm{erg\ s^{-1}\ Hz^{-1}\ sr^{-1}} \tag{6}$$

with

$$T=1.2\times10^5\left(\frac{\dot{M}}{10^{16}\ \mathrm{g\ s^{-1}}}\right)^{1/4}\left(\frac{10^{-2}}{f}\right)^{1/4}\left(\frac{10^9\ \mathrm{cm}}{r_\mathrm{wd}}\right)^{3/4} \times\left(\frac{M_\mathrm{wd}}{M_\odot}\right)^{1/4}\mathrm{K}, \tag{7}$$

which is obtained by assuming that the accretion energy is totally thermalized over a fractional area, f, of the white-dwarf surface. If the heating is due to the X-ray and cyclotron luminosity, the background flux may be only a fraction (~0.5) of what is given by (6). The curves that are obtained by setting $f\sim10^{-2}$, the total fractional area of accretion in the present model, are shown as the dotted lines in Fig. 2. The cyclotron peak stands far above the blackbody flux for $B=20$ MG. We should point out that even if this component were to dominate (as it does at lower fields) it is not clear that it will significantly decrease the polarization. The radiation from the heated photosphere will also have a degree of circular polarization of $\mathrm{O}(\omega_\mathrm{c}/\omega)$ whether scattering is important or not (Wickramasinghe 1988).

It is more difficult to estimate the contribution from a disc. An *upper limit* can be obtained by assuming that the disc is viewed face on and emits as a blackbody at the local effective temperature of the disc. The results of such a calculation for a disc with inner and outer radii $r_\mathrm{in}=10\ r_\mathrm{wd}$ and $r_\mathrm{out}=30\ r_\mathrm{wd}$, respectively, are shown in Fig. 2 for $\dot{M}=10^{18}$ and $10^7\ \mathrm{g\ s^{-1}}$.

Clearly the disc can effectively dilute the polarized emission over most of the optical–IR region but only at low fields. In particular the high harmonic free–free emission from the shock and heated photosphere which has a circular polarization of $\mathrm{O}(\omega_\mathrm{c}/\omega)$ can be effectively masked. However, it is more difficult to mask the strongly circularly polarized IR cyclotron peak. The absence of evidence for circular polarization in these bands must indicate fields of less than 5 MG. A stronger limit can be placed only if a more realistic estimate can be made of the disc luminosity.

Our calculations show that for fields $B\lesssim5$ MG the pulsed fraction in the optical cannot come from the accretion shock. The vast majority of IPs have a degree of circular polarization of less than 0.1 per cent in the optical. The pulsed fraction ranges from 0.2–30 per cent (Berriman 1988). If we assume that the pulsed component originates from the heated photosphere with a degree of circular polarization of $\mathrm{O}(\omega_\mathrm{c}/\omega)$ expected for scattering, the vast majority of IPs should have fields of less than 2 MG. For instance, a 50 per cent pulsed fraction with a degree of circular polarization that is less than 2 per cent would imply a field of 2 MG. However, this argument is not particularly strong since it is possible that the accretion curtain makes some contribution to the pulsed optical pulsed flux (Hellier *et al.* 1987).

We conclude that the lack of polarization over an extended wavelength region covering the optical–IR bands must imply either significantly higher fields than the AM Hers with the cyclotron peak in the far UV or significantly lower fields with the peak at $\lambda > 2\ \mu$m. The high field solution does not solve the IP–AM Her evolution problem as posed by previous investigators and is anyway inconsistent with the lack of synchronism in IPs. Furthermore, such IPs cannot evolve into AM Hers below the gap since the known AM Hers do not have high fields.

The case of BG CMi, the only IP with measured circular polarization, needs some comment. This system shows circular polarizations of -0.239 ± 0.3 per cent (I), $-1.74 \pm 0.26(J)$ and $-4.24 \pm 1.78(H)$ (Penning, Schmidt & Liebert 1986; West, Berriman & Schmidt 1987). The low values of polarization in the optical must indicate that the radiation in this region is free–free emission (see Wickramasinghe 1988) implying fields of <10 MG. A field lower than 10 MG is also indicated by the requirement that the peak of the polarized flux occurs longward of the H-band. Since the peak in the polarized flux is undetermined and the phase dependence is unknown more detailed modelling is unwarranted at the present time. It is significant that BG CMi, which is likely to have the strongest field of the known IPs, has a field that is lower than the lowest field detected in AM Hers, namely in EF Eri. Wickramasinghe, Achilleos, Wu & Boyle (1990) estimate $B_d = 9$ MG for EF Eri based on a single measurement of a Zeeman shifted π component of H_α.

2.2 The P_0–P_s relationship

Additional support for our point of view that IPs have low fields and accrete via discs comes from considering the orbital period–spin period relationship which has recently been discussed by Warner & Wickramasinghe (1990). We first consider some standard ideas on accretion flows in binary systems.

Material which flows from the inner Lagrangian point first forms a free stream which falls towards the white dwarf in a well-defined trajectory as shown by Lubow & Shu (1975). The stream follows a ballistic trajectory approaching a minimum distance $\varpi_{\min}$ from the white dwarf, orbits around the white dwarf, and then intersects with itself forming a ring at the circularization radius $\varpi_{\max}$. The ring then spreads through viscosity to form a disc. For the orbital periods ($P_0 \sim 2$–8 hr) presently under consideration $\varpi_{\max}/\varpi_{\min} \sim 1.71$–$1.75$ and $\varpi_{\min} \sim 0.13$–$0.22\ r_{\rm L}$, where $r_{\rm L}$ is the equivalent spherical Roche lobe radius given by

$$\frac{r_{\rm L}}{r_{\rm wd}} = 34\left(\frac{P_0}{4\ \rm hr}\right)^{2/3}\left(\frac{M_{\rm wd}}{0.6\ M_\odot}\right)^{1/3}\left(\frac{10^9\ \rm cm}{r_{\rm wd}}\right). \tag{8}$$

Setting $\omega_{\max} = \alpha r_{\rm L} (0.22 < \alpha < 0.38)$ we can write

$$\frac{\varpi_{\max}}{r_{\rm wd}} = 10\left(\frac{\alpha}{0.3}\right)\left(\frac{P_0}{4\ \rm hr}\right)^{2/3}\left(\frac{M_{\rm wd}}{0.6\ M_\odot}\right)^{1/3}\left(\frac{10^9\ \rm cm}{r_{\rm wd}}\right). \tag{9}$$

If the magnetic field of the white dwarf is strong enough to affect the flow of the free stream before $\varpi_{\min}$ is reached a disc may not form. For disc accretion, Lamb and Melia (1988, hereafter LM) give

$$r_\mu^{(d)} = 0.52\ r_\mu^{(0)}. \tag{10}$$

$r_\mu^{(0)}$ is the so-called 'spherical magnetic radius' given by

$$\frac{r_\mu^{(0)}}{r_{\rm wd}} = 110\left(\frac{\mu}{10^{34}\ \rm G\ cm^3}\right)^{4/7}\left(\frac{\dot M}{10^{16}\ \rm g\ s^{-1}}\right)^{-2/7}\left(\frac{M_{\rm wd}}{0.6\ M_\odot}\right)^{-1/7} \times\left(\frac{10^9\ \rm cm}{r_{\rm wd}}\right). \tag{11}$$

The problem of calculating a magnetic radius for stream accretion has been discussed by Hameury, King & Lasota (1986) who suggest a value

$$r_\mu^{(s)} = 0.37\ r_\mu^{(0)}. \tag{12}$$

For stream accretion, the stream may follow a ballistic trajectory up to much smaller radii than given by $r_\mu^{(s)}$ due to the presence of screening currents, before being totally captured by the field lines (LM).

If $r_\mu^s < \varpi_{\min}$ and $r_\mu^d < \varpi_{\max}$, a ring will form at a radius $\varpi_{\max}$ and spread inwards to form a disc with inner radius equal to $r_\mu^{(d)}$. On the other hand, if $r_\mu^{(d)} > \varpi_{\max}$ a disc can never form since the stream cannot circularize at the radius appropriate to its angular momentum. For the same reason a disc once formed will disrupt if $r_\mu^{(d)} > \varpi_{\max}$ during subsequent evolution – for instance due to a change in $\dot M$ (LM).

Consider a white dwarf which spins with a period $P_{\rm s}$ (angular frequency $\Omega_{\rm s}$) and accretes from a disc. For such a star the inner radius of the disc $r_{\rm in} \sim r_\mu^{(d)}$. Ghosh & Lamb (1979) have shown that the net torque exerted on the white dwarf can be written as a function of the fastness parameter (see also Elsner & Lamb 1977)

$$\omega_{\rm s} = \frac{\Omega_{\rm s}}{\Omega_{\rm K}(r_\mu^{(d)})} = \frac{P_{\rm K}(r_\mu^{(d)})}{P_{\rm s}}, \tag{13}$$

where $P_{\rm K}$ and $\Omega_{\rm K}$ are, respectively, the Keplerian period and angular velocity at the inner edge of the disc. For a critical value $\omega_{\rm s} = \omega_{\rm s}^c$ accretion will occur at zero torque on to the white dwarf. The corresponding value $\Omega_{\rm s} = \Omega_{\rm s}^c = \omega_{\rm s}^c \Omega_{\rm K}(r_\mu^d)$ is referred to as the equilibrium spin frequency. If $0 \le \omega_{\rm s} \le \omega_{\rm s}^c$, the white dwarf will spin up while if $\omega_{\rm s}^c < \omega_{\rm s} \le 1$ it will spin down. If $\omega_{\rm s} > 1$ accretion will cease and the material will be centrifugally driven away from the white dwarf. The value of $\omega_{\rm s}^c$ has been difficult to estimate theoretically but is expected to lie between 0.35 and 0.85 (Ghosh & Lamb 1979).

The corotation radius $r_{\rm c}$ defined as the point where gravitational and centrifugal forces balance is given by $r_{\rm c} = (GM/\Omega_{\rm s}^2)^{1/3}$. It follows from (13) that $r_\mu^{(d)} = \omega_{\rm s}^{2/3} r_{\rm c}$. The spin period of a white dwarf which accretes from a disc with inner radius $r_{\rm in} = \omega_{\max} = r_\mu^{(d)}$ is then given by

$$P_{\rm s}^{(d)} = 2095\left(\frac{\alpha}{0.3}\right)^{3/2}\left(\frac{0.35}{\omega_{\rm s}}\right)\left(\frac{P_0}{4\ \rm hr}\right)\ \rm s. \tag{14}$$

From our previous discussion we expect systems which accrete from a disc with zero torque to lie below the curves given by (14) with $\omega_{\rm s} = \omega_{\rm s}^c$. We refer to this curve as the equilibrium disc–discless boundary. Systems above the curve will be discless or have discs and be accreting with a non-zero torque ($\omega_s < \omega_s^c$) spinning up towards its equilibrium line.

Warner & Wickramasinghe (1990) first drew attention to the fact that the IPs appeared to lie close to the line corresponding to the equilibrium spin period for a disc with $r_{in} = \omega_{min}$ in the log P_s–log P_0 diagram and referred to this line as the critical disc line. The disc–discless boundary (14) with $\omega_s = \omega_s^c$ is displaced parallel to this line and has a more immediate physical interpretation. The observations and the various boundaries are shown in Fig. 3. All except two IPs lie close to, or are below, the equilibrium disc–discless boundary. One of the two objects which lies above this boundary, namely Ex Hya, is eclipsing and is observed to have a disc. It can be interpreted as a low μ system that is in the process of spinning up towards equilibrium. We view the location of the majority of IPs below the disc–discless boundary as additional evidence in support of our point of view that IPs have discs.

The apparent concentration of IPs near this boundary cannot be explained by the theory of spin equilibrium of a rotating compact magnetic star in the presence of a disc as first discussed by Ghosh & Lamb (1979) without some modification. On this theory $\Omega_s = \omega_s^c \Omega_K(r_\mu^{(d)})$ at equilibrium so that we expect a spread in Ω_s corresponding to the expected spread in μ below the values appropriate for disc formation. The value of ω_s^c is determined by balancing the spin up torque exerted by the forward swept field lines in the inner regions of the disc ($r_{in} \le r \le r_c$) with the spin down torque exerted by the backward swept field lines in the outer regions of the disc ($r_c \le r \le r_s$), where r_s is the screening radius outside which the field of the compact star is screened by currents flowing in the disc. An assumption that is implicit in these calculations is that $r_s \le r_{out}$, the outer radius of the disc which is related to the size of the Roche lobe (typically $r_{out} \sim 0.7\ r_c$). In our case this condition is violated at the upper end of the range of μ^s consistent with disc accretion.

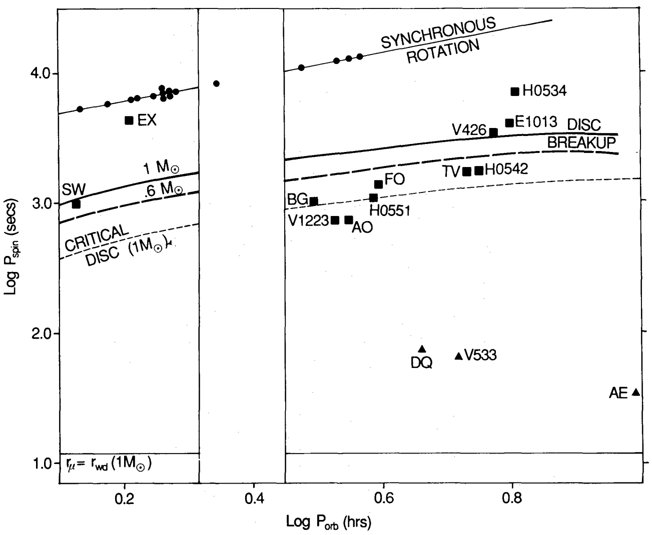

Figure 3. The log (spin period)–log (orbital period) diagram for known IPs (filled squares), DQ Hers (filled triangles) and AM Hers (filled circles) following Warner & Wickramasinghe (1990). The various lines are discussed in the text.

The spin down torque will then need to be calculated as the torque exerted by the backward swept field lines in the outer region of the disc ($r_c \le r \le r_{out}$), and a torque perhaps of the MHD type due to the interaction of the unscreened part of the stellar magnetic field with the secondary. Although the results of calculations of the spin equilibrium under such conditions are presently unavailable, it seems likely that such objects will have a spin that is above the value given by the Ghosh & Lamb (1979) theory and below an equilibrium disc–discless boundary of the type discussed previously. The precise location of this boundary will of course also depend on the results of the full calculations.

Other interpretations of the P_0–P_s observations may also be possible. Thus Warner & Wickramasinghe (1990) attempted to interpret the narrowness of the sequence in terms of the inferred variations in mass transfer on time-scales of $\sim 10^3$ yr in CVs claiming that the variations drove an MCV between disced and discless states with the systems with a sufficiently low μ (IPs) lying mainly in a disced state.

3 IP AND AM HER EVOLUTION

Our conclusion that IPs have, in general, a lower magnetic moment than the AM Hers indicates that the IPs above the period gap cannot evolve into AM Hers below the period gap as previously proposed by King *et al.* (1985) and Chanmugam & Ray (1984). In this section we briefly reconsider the question of the evolution of MCVs in the light of the results of Section 2.

A necessary condition for synchronization is $R = G_{mag}/G_{acc} > 1$, where G_{acc} and G_{mag} are, respectively, the accretion (spin-up) and magnetic torques exerted on the white dwarf. An additional requirement is that the synchronization time-scale must be shorter than the evolutionary time-scale (LM). The accretion torque is calculated from

$$G_{acc} = \dot{M} r_L^2 \Omega_0. \tag{15}$$

Estimates of G_{mag} available in the literature vary by several orders of magnitude as discussed in the recent review by LM. Following Hameury *et al.* (1986) we write

$$G_{mag} = \beta \left(\frac{\mu \mu'}{a^3} \right) \tag{16}$$

which leads to

$$R = 20\,\beta \left(\frac{M_1}{M_\odot} \right)^{-5/3} (1+q)^{-1} \left(\frac{4\ \mathrm{hr}}{P_0} \right)^{7/3}, \tag{17}$$

where μ' (a magnetic moment) and β are parameters whose values depend on the specific model adopted, and $q = M_2/M_1$. In the version of the vacuum dipole–dipole torque model discussed by Campbell (1983), $\beta \sim 1$ and $\mu' = \mu$. LM have argued that the torque calculated according to this theory is likely to be too large since Campbell overestimates the energy that is dissipated by turbulent diffusion in the companion star. The MHD torque model of Lamb *et al.* (1983), on the other hand, predicts a torque that is closer to what is required to explain the AM Hers (LM) though, in general, somewhat lower. In what follows, we adopt the version of this model when the intrinsic magnetic moment μ_2

of the companion is assumed to be negligible. For this model $\mu' = \mu$ and $\beta \sim \varepsilon\gamma(R_2/a)^2$ which can be written as

$$\beta = 0.21\,\varepsilon\gamma q^{2/3}(1+q)^{-2/3}, \tag{18}$$

where γ is the pitch angle and ε is the fractional area of the secondary (of radius R_2) threaded by the field lines of the white dwarf. Adopting the usual values $\gamma \sim 1$ and $\varepsilon \sim 1$ (for a low mass secondary) $\beta \sim 1/10$ for $M_1 = 1\ M_\odot$ at $P_0 = 4$ hr. The synchronous–asynchronous boundary which results from the criterion $R = 1$ for the above parameters is shown in Fig. 4.

The disc–discless boundary in Fig. 4 is calculated for $M = 1\ M_\odot$ and is determined by the criterion $r_\mu^d = \omega_{\max}$, where r_μ^d is the magnetic radius for disc accretion and $\omega_{\max}$ is the circularization radius. Using the standard expressions for r_μ^d and $\omega_{\max}$ (LM and Section 2) this criterion can be written as

$$\frac{\mu_{34}}{\sqrt{M_{17}}} > 0.27\left(\frac{\alpha}{0.3}\right)\left(\frac{P_0}{4\ \mathrm{hr}}\right)^{7/6}\left(\frac{M_{\mathrm{wd}}}{M_\odot}\right)^{5/6}. \tag{19}$$

We have no accurate determinations of the μ^s of known IPs. Estimates based on the observed spin period and the current value of $\dot M$ may not be appropriate for reasons given in Section 2. We use our basic premise, namely that the IPs have discs and lower μ^s than the AM Hers to locate them below the disc–discless boundary. The ranges adopted for $\dot M$ (5×10^{15}–5×10^{17} g s^{-1} above and 10^{15}–10^{16} g s^{-1} below the period gap) are within the limits implied by the observed luminosities. For the AM Hers we have assumed that the magnetic moments are in the range $\mu_{34} = 0.6$–2 as given by recent estimates of field strength and mass. These estimates are generally larger than used by previous investigators. We also assume $\dot M = 10^{16}$ g s^{-1} above the gap and $\dot M \sim 2$ 10^{15}–10^{16} g s^{-1} below the gap as implied by the observed luminosities.

Fig. 4 leads to the conclusion that a significant fraction of the known IPs with periods $P_0 > 3$ hr will evolve into asynchronous discless systems below the gap. Only a very small fraction will evolve into AM Hers. This conclusion is inevitable if the IPs are located in the diagram in a position appropriate to the observed lack of polarization (low μ's) and the presence of discs. On our picture the discless asynchronous systems form a new class of MCV with properties that are distinct from those of the known IPs and whose existence has so far not been recognized. We expect these general conclusions to hold despite the intrinsic uncertainties in locating the boundaries in Fig. 4 although the presence of such uncertainties makes it impossible to estimate the expected relative numbers. For instance, these may be additional synchronization torques (e.g. if the secondary has an intrinsic magnetic moment $\mu_2 > \mu$) that have not been accounted for and r_μ^d could be uncertain by a factor of ~ 2 (LM).

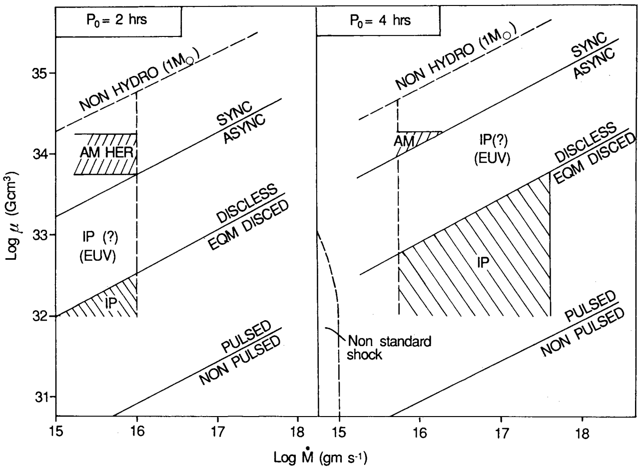

Figure 4. The log (magnetic moment)–log (accretion rate) diagram. The known IPs (shaded area) are drawn in on the assumption that they have discs. A significant fraction of the IPs will evolve into discless systems (IP, EUV) below the period gap and not into AM Hers.

As discussed previously, the discless accretors will have fields that are lower than those of the AM Hers but not low enough to allow the formation of a disc. Recent studies have shown that in many AM Hers accretion occurs over extended bands on the white-dwarf surface. The angular widths in magnetic colatitude and longitude are typically $\Delta\theta \sim 10°$–$30°$ and $\Delta\psi \sim 30°$–$120°$ (Wickramasinghe *et al.* 1990; Wickramasinghe, Ferrario, Cropper & Bailey 1990). If we extrapolate to the asynchronous discless accretors, it is reasonable to expect accretion over a band of similar width in magnetic colatitude extending over $\Delta\psi \sim 360°$ in magnetic longitude. The fractional area over which accretion occurs could thus be as high as $f \sim 0.01$, larger than for an AM Her.

One can only speculate on the nature of the accretion flow in asynchronous discless systems. The more rapid rotation of the white dwarf may result in the stream breaking up into smaller fragments via the Kelvin–Helmholtz instability. As a consequence, the accretion may be clumpier than in AM Hers with most of the accretion energy being thermalized and radiated from the white dwarf surface at the blackbody temperature given by (7). We envisage the emission of asynchronous discless IPs to be similar to the emission due to clumpy accretion from the second pole in AM Hers except that due to the larger area over which accretion occurs, the energy is now radiated in the EUV region. We therefore predict that these systems will be discovered in the forthcoming *ROSAT* survey as a new class of MCV in the hitherto unobserved 0.01–0.1 eV spectral region.

MCVs are detected through their X-ray properties. At the one extreme we have the synchronous and discless AM Hers which are erratic in their X-ray behaviour switching between hard and soft X-ray modes of emission depending on whether one or two poles are active and on the balance between the two poles. We attribute this erratic behaviour to the discless nature of these systems. At the other extreme there are the IPs which are more consistent in their X-ray properties, which we hypothesize is a consequence of the fact that they are disced accretors. The asynchronous discless accretors, which according to our scenario most probably outnumber the known IPs below the gap must, by virtue of their mode of accretion, do not distinguish themselves as X-ray sources. With this hypothesis it may be possible to explain the observed decrease in the relative number of IPs below the period gap without having to postulate high fields for these systems.

We conclude this section by noting that the possible importance of discless accretion in MCVs was first discussed by Hameury *et al.* (1986). However, their basic hypothesis, namely that the *known IPs* were such systems, differs from ours. Our conclusion that the known IPs will not evolve into AM Hers below the period gap had been reached previously by LM who estimated the magnetic moments of IPs from their spin behaviour and also argued that the known IPs were likely to have low fields.

4 CONCLUSIONS

We have presented the first detailed calculations of the polarization expected from ribbon shaped accretion shocks on the surface of a magnetic white dwarf where allowance is made for the dependence of shock height on the specific accretion rate. These calculations show clearly that the lack of polarization in IPs over an extended wavelength region must imply fields of less than ~5 MG, significantly lower than in the AM Hers.

We have also argued that the location of the IPs in the log P_s–log P_0 diagram indicates that they are low field objects that accrete via discs.

We have considered the implications of our result to current ideas on the evolution of MCVs. We argue that a consistent picture can be constructed if it is assumed that a significant fraction of IPs above the period gap evolve into asynchronous discless accretors below the gap and not into AM Hers as previously proposed by some investigators. We propose that these form a new class of MCV distinct from the IPs which emit mainly at EUV energies. These systems may be detected as EUV sources in the forthcoming *ROSAT* survey.

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REFERENCES

Berriman, G., 1988. In: *Polarized Radiation of Circumstellar Origin,* p. 281, eds Coyne, G. V., Moffat, A. F. J., Tapia, S., Mogalhaes, A. M., Schults-Landbeck, R. E. & Wickramasinghe, D. T., Vatican Observatory, Vatican City.
Campbell, C. G., 1983. *Mon. Not. R. astr. Soc.,* **205,** 1031.
Chanmugam, G. & Frank, J., 1987. *Astrophys. J.,* **320,** 746.
Chanmugam, G. & Ray, A., 1984. *Astrophys. J.,* **285,** 252.
Elsner, R. F. & Lamb, F. K., 1977. *Astrophys. J.,* **215,** 897.
Elsner, R. F. & Lamb, F. K., 1984. *Astrophys. J.,* **278,** 326.
Farrario, L. & Wickramasinghe, D. T., 1990. *Astrophys. J.,* **357,** 589.
Ghosh, P. & Lamb, F. K., 1979. *Astrophys. J.,* **234,** 296.
Hameury, J. M., King, A. R. & Lasota, J. P., 1986. *Mon. Not. R. astr. Soc.,* **218,** 695.
Hameury, J. M., King, A. R. & Lasota, J. P., 1989. *Mon. Not. R. astr. Soc.,* **237,** 845.
Hellier, C., Mason, R. O., Rosen, S. R. & Cordova, F. A., 1987. *Mon. Not. R. astr. Soc.,* **228,** 463.
King, A., Frank, J. & Ritter, H., 1985. *Mon. Not. R. astr. Soc.,* **213,** 181.
King, A. & Shaviv, G., 1984. *Mon. Not. R. astr. Soc.,* **211,** 883.
Lamb, D. Q., 1988. In: *Polarized Radiation of Circumstellar Origin,* p. 151, eds Coyne, G. V., Moffat, A. F. J., Tapia, S., Mogalhaes, A. M., Schults-Landbeck, R. E. & Wickramasinghe, D. T., Vatican Observatory, Vatican City.
Lamb, D. Q. & Masters, A. R., 1979. *Astrophys. J.,* **234,** L117.
Lamb, D. Q. & Melia, F., 1988. In: *Polarized Radiation of Circumstellar Origin,* p. 45, eds Coyne, G. V., Moffat, A. F. J., Tapia, S., Mogalhaes, A. M., Schults-Landbeck, R. E. & Wickramasinghe, D. T., Vatican Observatory, Vatican City.
Lamb, F. K., Aly, J. J., Cook, M. C. & Lamb, D. Q., 1983. *Astrophys. J.,* **274,** L71.
Lubow, S. H. & Shu, F. H., 1975. *Astrophys. J.,* **198,** 383.
Mason, P. A., Liebert, J. & Schmidt, G. D., 1989. *Astrophys. J.,* **346,** 941.
Penning, W., Schmidt, G. D. & Liebert, J., 1986. *Astrophys. J.,* **301,** 885.
Schmidt, G., Stockman, H. S. & Grandi, S., 1986. *Astrophys. J.,* **300,** 804.
Schmidt, G. & Liebert, J., 1987. *Astrophys. Space Sci.,* **131,** 549.
Stockman, H. S., Schmidt, G. & Lamb, D. Q., 1988. *Astrophys. J.,* **332,** 282.
Warner, B. & Wickramasinghe, D. T., 1991. *Mon. Not. R. astr. Soc.,* **248,** 370.
West, S. C., Berriman, G. & Schmidt, G. D., 1987. *Astrophys. J.,* **322,** L35.
Wickramasinghe, D. T., 1988. In: *Polarized Radiation of Circumstellar Origin,* p. 3, eds Coyne, G. V., Moffat, A. F. J., Tapia, S., Mogalhaes, A. M., Schults-Landbeck, R. E. & Wickramasinghe, D. T., Vatican Observatory, Vatican City.
Wickramasinghe, D. T. & Ferrario, L., 1988. *Astrophys. J.,* **334,** 412.
Wickramasinghe, D. T., Achilleos, N., Wu, K. & Boyle, B. J., 1990. *IAU Circ. No. 4962.*
Wickramasinghe, D. T., Ferrario, L., Cropper, M. & Bailey, J., 1990. *Mon. Not. R. astr. Soc.,* submitted.
Wickramasinghe, D. T., Bailey, J., Meggitt, S. M. A., Ferrario, L., Hough, J. & Tuohy, I. R., 1990. *Mon. Not. R. astr. Soc.,* submitted.
Wu, K. & Chanmugam, G., 1990. *Astrophys. J.,* **354,** 625.
Wu, K. & Wickramasinghe, D. T., 1990. *Mon. Not. R. astr. Soc.,* **246,** 686.