Internal Dynamics Stabilization of Single-Phase Power Converters with Lyapunov-Based Automatic-Power-Decoupling Control

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Abstract—Single-phase power converters with the active pulsating-power-buffering (PPB) function are essentially highly coupled and nonlinear systems. Advanced control techniques are needed for this emerging class of converters to achieve fast transient response and large-signal stability. Existing control solutions are based on either i) linear control techniques that are operating-point specific or ii) nonlinear control techniques that are generally topology-dependent. The proposed work is an evolved generalized feedback-linearization (FBL) control approach that incorporates the direct Lyapunov control method. The proposed control provides good stabilization of the internal dynamics of the system (which is unavailable with FBL control) while still retaining all the best features of FBL control. A kind of single-phase power conversion system with active PPB is described. It is shown that FBL control naturally destabilizes the system and that the proposed control can globally stabilize the system under various operating conditions whilst yielding fast dynamics.

I. INTRODUCTION

SINGLE-PHASE power converters with the active pulsating-power-buffering (PPB) function are promising candidates for achieving high power density, high energy efficiency, and high reliability (H³) [1]−[4]. The H³ features are particularly attractive for a wide range of applications, such as consumer electronics (laptop adapters and LED drivers), telecom (the power supply unit for data centers and servers) and renewable energy, where power density, efficiency, and reliability are of the utmost figure-of-merits.

The basic operating principles of single-phase power conversion with active PPB can be explained using Fig. 1. As introduced in [5], this new derivative of single-phase converters incorporates a third ripple port (see \( \Delta v_b \) in Fig. 1). By allowing a large voltage ripple \( \Delta v_b \) across \( C_b \) through active PPB control, the required \( C_b \) can be significantly reduced for buffering the double-line-frequency imbalanced power, which is inherent in any single-phase converters [6]. The reduction of the required energy storage enables the elimination of conventional bulk dc-link capacitors (typically electrolytic capacitors, or E-caps) and allows more compact and reliable non-E-caps (e.g. film or ceramic capacitors) to be used in the system, thereby achieving high compactness and long service lifetime.

There is a myriad of circuit configurations reported for single-phase power conversion with active PPB, showcasing superior power density (up to 200 W/in³) and efficiency (up to 98%) [7]−[11]. Subsequently, advancement in the controller design is indispensable for attaining improved system-level performance. As reported in [5], H³ converters are essentially highly nonlinear and coupled, and inherently involve a large-signal operation. Most of the existing control solutions, however, are linear control techniques that are valid only around specific operating points and do not take the nonlinearity and coupling effect into consideration. They are primarily targeted at narrow-load-range steady-state operations and are inapplicable to operations when fast response and large-signal stability are mandated [12]−[14]. In [4], a patent-pending nonlinear feedforward controller that provides excellent large-signal dynamic performance is proposed. However, the controller is topology-specific and the system’s dynamic performance is not theoretically verified. In [5], a nonlinear control approach based on input-output feedback linearization and an automatic-power-decoupling control strategy, namely FBL-APD control, was developed. The effectiveness of the controller was demonstrated with the system’s bandwidth and large-signal stability systematically derived and validated. The FBL-APD control is also applied to other recently proposed circuit configurations [15]−[19].

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demonstrating its versatility. Despite its effectiveness, FBL-APD control, as with other FBL controllers, possess a major limitation of being incapable of ensuring the stability of the system’s internal dynamics, which are unobservable system states [20]. Therefore, the stability of a system with FBL-APD control is essentially determined by the stability of the internal dynamics and is system-dependent.

This work aims to complement the previously proposed FBL-APD controller by tackling the internal dynamics instability challenges. In particular, a Lyapunov-based APD FBL-APD controller by tackling the internal dynamics and is system-dependent.

II. MATHEMATICAL MODELING AND STEADY-STATE ANALYSIS OF AN H^3 SINGLE-PHASE CONVERTER

The H^3 single-phase power converter topology investigated in this work is shown in Fig. 2. It comprises i) a full-bridge active front end converter for ac/dc conversion and ii) a buck-type PPB bridge leg for removing the double-line-frequency voltage ripple from the dc-link [4], [5], [21]. Here, the PPB bridge leg operates in the continuous conduction mode (CCM) of operation and functions as a bidirectional buck/boost converter. Compared to the discontinuous conduction mode (DCM) of operation, a CCM PPB structure can achieve zero voltage switching by operating in the transition current mode (TCM) [22], thereby enabling a higher-efficiency and higher-power-density design.

A. Mathematical Model of the H^3 Single-Phase Converter

The state-space-averaged model of the H^3 single-phase converter can be expressed as

$$\dot{x} = f(x) + g(x) \cdot u,$$

(1)

where

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} i_{ac} \\ v_{dc} \\ i_b \\ v_{ppb} \end{bmatrix}, \quad u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} m \\ d_c \end{bmatrix},$$

$$f(x) = \begin{bmatrix} v_{ac} / L_{ac} \\ -i_{load} / L_{dc} \\ -v_{ppb} / L_b \\ i_b / C_b \end{bmatrix}, \quad g(x) = \begin{bmatrix} -v_{dc} / L_{ac} \\ i_{ac} / C_{dc} \\ -i_b / C_d \\ 0 \end{bmatrix},$$

(2)

Most of the variables in (2) are marked in Fig. 2. $u_1 = m$ is the modulation index of the full-bridge converter and $u_2 = d_c$ is the duty cycle of the PPB bridge leg. Both $u_1$ and $u_2$ are subject to the constraints

$$-1 < u_1 < 1, \quad 0 < u_2 < 1.$$

Clearly, the mathematical model of the system described by (1) indicates that the system is highly nonlinear (due to the multiplication of $u$ and $x$) and highly coupled (between $u$ and $x$).

B. Steady-State Operation of the H^3 Single-Phase Converter

At steady state, the state-space model in (1) can be expressed as

$$\dot{x}^S = f(x^S) + g(x^S) \cdot u^S,$$

(4)

where the variables with a superscript $S$ represent their steady-state values.

Given that $v_{ac}^S = V_{AC} \sin(\omega t)$, $i_{ac}^S = I_{AC} \sin(\omega t)$, $v_{dc}^S = V_{dc}$, and that $i_{load}^S$ is a constant, the input power injected into the full-bridge converter is thus

$$P_{in}^S = (v_{ac}^S - I_{ac}^S i_{ac}^S)^2.$$

The corresponding output power is

$$P_{out}^S = V_{load}^S i_{load}^S.$$

Provided that inductance $L_b$ is sufficiently small such that the energy stored in $L_b$ is negligible, then the energy stored in the PPB circuit is

$$E_b^S = \frac{1}{2} C_b \left(v_{ppb}^S \right)^2.$$

(7)

Applying the principle of energy conservation to the converter, one has

$$\dot{E}_b^S = P_{in}^S - P_{out}^S = -\frac{1}{2} V_{AC} I_{AC} \cos(2\omega t) - \frac{1}{2} \omega L_{ac} I_{ac}^2 \sin(2\omega t).$$

(8)

$v_{ppb}^S$ can now be solved by combining (7) and (8), as

$$v_{ppb}^S = \sqrt{\frac{2E_{i0}}{C_b} - \frac{V_{AC} I_{AC}}{2C_b} \sin(2\omega t) + \frac{L_{ac}^2 I_{ac}^2}{2C_b} \cos(2\omega t)}.$$

(9)

where $E_{i0}$ is the initial energy of the PPB.

Substitution of (9) into (4) yields

$$i_b^S = C_b v_{ppb}^S = \frac{V_{AC} I_{AC} \cos(2\omega t) - \omega L_{ac} I_{ac}^2 \sin(2\omega t)}{2\sqrt{\frac{2E_{i0}}{C_b} - \frac{V_{AC} I_{AC}}{2C_b} \sin(2\omega t) + \frac{L_{ac}^2 I_{ac}^2}{2C_b} \cos(2\omega t)}}.$$

(10)
The expressions of \( i_{ac}^S \), \( i_d^S \), and \( v_p^S \) indicate a large-signal operation with a small \( C_b \) even at the steady state. 

\[
\begin{align*}
\mathbf{u} &= \frac{\mathbf{f}_m - L_u w^S_s}{\sqrt{2}} ,
\mathbf{u}^s &= \mathbf{u}^S i_{ac}^s - i_{load}^s = \frac{V_p^S + L_d b}{v_{dc}^s} .
\end{align*}
\]

The complete time-domain expressions of \( u_1^s \) and \( u_2^s \) can finally be obtained by substituting \( \mathbf{x}^S \) into (11).

### III. Internal Dynamics Instability with Feedback-Linearization-Based APD Control

In [5], a nonlinear controller based on feedback-linearization and an automatic-power-decoupling control strategy (FBL-APD) is proposed for the same \( H^3 \) converter in Fig. 2, except that the PPB operates in DCM. The controller successfully tackles the coupling and the nonlinearity issues of the system and achieves satisfactory steady-state and dynamic performances. However, as will be explained in the following, the same FBL-APD control is inapplicable when the PPB operates in CCM.

#### A. Review of FBL-APD Control

According to the APD control strategy, \( i_{ac} \) and \( v_{dc} \) are selected to form the control output vector \( \mathbf{y} \), i.e.,

\[
\mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} L_u i_{ac} \\ C_d v_{dc} \end{bmatrix} .
\]

(12)

By following the FBL-APD control design procedure in [5], the decoupling control law can be derived as

\[
\begin{align*}
u_1 &= \frac{v_{ac} - y_1}{v_{dc}} ,
u_2 &= \frac{u_1 i_{ac} - v_2 - i_{load}}{i_b} .
\end{align*}
\]

(13)

such that

\[
\begin{align*}
y_1 &= \begin{bmatrix} v_1 \\ y_2 \end{bmatrix} ,
y_2 &= \begin{bmatrix} v_2 \\ y_2 \end{bmatrix} = \mathbf{v} .
\end{align*}
\]

(14)

\( \mathbf{v} \) is a new set of control inputs, with which the original system model in (1) is decoupled and linearized.

If the feedback control law is designed as

\[
\begin{align*}
v_1 &= L_u i_{ac}^p + \alpha_1 \left( L_u i_{ac}^p - y_1 \right) ,
v_2 &= \alpha_2 \left( C_d v_p^p - y_2 \right) .
\end{align*}
\]

(15)

with \( i_{ac}^p (= i_{ac}^s) \) and \( v_p^p (= v_p^s) \) being the references of \( i_{ac} \) and \( v_{dc} \), respectively, the error dynamics of the closed-loop system will be obtained from (14) and (15) as

\[
\begin{align*}
\dot{e}_1 + \alpha_1 e_1 &= 0 ,
\dot{e}_2 + \alpha_2 e_2 &= 0 ,
\end{align*}
\]

(16)

(17)

where \( e_1 = i_{ac}^p - i_{ac} \), \( e_2 = v_{dc}^p - v_{dc} \), and \( \alpha_1 \) are design choices.

Equations (16) and (17) suggest that i) \( i_{ac} \) and \( v_{dc} \) have the simple first-order error dynamics in reference tracking, with bandwidths of \( f_{w1} = \alpha_1 / 2\pi \) and \( f_{w2} = \alpha_2 / 2\pi \), respectively, and ii) \( i_{ac} \) and \( v_{dc} \) are globally and exponentially stable, provided that \( \alpha_1 > 0 \) and \( \alpha_2 > 0 \).

#### B. Stability Analysis of the Internal Dynamics

With FBL-APD control, the dynamics of \( i_b \) and \( v_b \) are not directly controlled. The dynamics of the uncontrolled system states, also known as the internal dynamics, thus can easily affect the stability of the overall system. In the following analysis, it will be shown that the internal dynamics are actually unstable with the control law (13) and (15) despite that \( i_{ac} \) and \( v_{dc} \) have been stabilized.

The internal dynamics of the system are rewritten from (1) as

\[
\begin{align*}
i_b &= -\frac{v_b}{L_b} + \frac{1}{L_b} v_{ac} u_2 ,
v_b &= \frac{1}{C_b} i_b .
\end{align*}
\]

(18)

(19)

By substituting (13) into (18), the dynamics of \( i_b \) can be derived as

\[
\begin{align*}
i_b &= -\frac{v_b}{L_b} + \frac{P_b}{L_b i_b} , \text{ or equivalently } L_b i_b = -v_b + \frac{P_b}{i_b} .
\end{align*}
\]

(20)

where \( p_b = (v_{ac} - v_1) i_{ac} - v_{dc} i_{load} \). According to (14), \( v_1 \) is the voltage drop across \( L_{ac} \) and \( v_2 \) is the current through \( C_d \). Therefore, the physical meaning of \( p_b \) is precisely the instantaneous power absorbed by the PPB circuit. Equation (20) is the expected result as it is simply a Kirchhoff’s Voltage Law (KVL) equation obtained with the PPB circuit, i.e., \( p_d/i_b \) is the average voltage at the node C (see Fig. 2), and \( L_{ac} i_{ac} \) and \( v_p \) are the voltages across \( L_b \) and \( C_b \), respectively. Comparison of (18) and (20) shows that \( u_2 \) can also be expressed in terms of \( p_b \) as

\[
u_2 = \frac{p_b}{v_{ac} i_b} .
\]

(21)

To simplify the analysis, zero dynamics are considered (i.e., when \( e_1 = e_2 = 0 \)). Equation (20) is rewritten as

\[
\begin{align*}
i_b^z &= -\frac{v_b^z}{L_b} + \frac{P_b^z}{L_b i_b^z} ,
\end{align*}
\]

(22)

with

\[
\begin{align*}
p_b^z &= (v_{ac} - L_{ac} \dot{i}_{ac}) i_{ac} - i_{load} v_{dc}^r ,
\end{align*}
\]

(23)

where the variables with a superscript \( Z \) represent their zero dynamics.

On the other hand, by solving (3) and (18), the viable range of \( i_b^z \) can be determined as

\[
-\frac{v_b^z}{L_b} < i_b^z < \frac{v_b^z - v_{dc}^r}{L_b} .
\]

(24)

With (22) and (24), the phase plane of \( i_b^z \) can be drawn. To simplify the analysis, it is assumed that \( i_{ac}^r, v_p^r, v_{ac} \), and \( i_{load} \) are constant. This is justified by the fact that the dynamics of \( i_b \) is significantly faster than the dynamics of \( i_{ac}^r, v_p^r, v_{ac} \), and \( i_{load} \) due to a small \( L_b \). Furthermore, (23) indicates that \( p_b^z \) can also be regarded as a constant. The phase plane of \( i_b \) is now drawn in Fig. 3 by considering the following two scenarios:

- **Scenario 1:** When \( p_b^z > 0 \), the PPB circuit is absorbing energy from the dc bus. By equating \( i_b^z \) to zero in (22), the equilibrium point of \( i_b^z \) is calculated as \( i_b^z = p_b^z/v_{dc}^z \), which is positive. Fig. 3(a) shows that \( i_b^z \) will converge to \( i_b^r(0) > 0 \) (\( i_b^r(0) \) is the initial value of \( i_b^z \)), but will decrease unboundedly if \( i_b^z(0) < 0 \), i.e., \( i_b^z \) is merely locally stable.
Scenario 2: When \( p_z^E < 0 \), the PPB circuit is injecting energy to the dc bus and \( i_b^E \) is negative. Fig. 3(b) indicates that \( i_b^Z \) will converge to zero if \( i_b^Z(0) > i_b^E \), but will decrease unboundedly if \( i_b^Z(0) < i_b^E \), i.e. \( i_b^Z \) is globally unstable.

The time-domain responses of \( i_b^Z \) given different \( i_b^Z(0) \) are further simulated and the results are shown in Fig. 5. The simulation is conducted on the PPB circuit solely where \( C_b \) and \( C_{ac} \) are replaced by two voltage sources \( V_b \) and \( V_{ac} \), respectively. The schematic diagram of the simulated PPB circuit (controlled according to (21)) is shown in Fig. 4, where \( V_{ac} = 400 \text{ V} \), \( V_b = 250 \text{ V} \), and \( p_z^E \) is set as 1 kW and −1 kW for Scenario 1 and 2, respectively. From Fig. 5, the following observations can be made:

- In Scenario 1, \( i_b^E = 4 \text{ A} \). Fig. 5(a) depicts that all the five curves with \( i_b^Z(0) > 0 \) converge to 4 A while the other two with \( i_b^Z(0) < 0 \) decrease unboundedly.

- In Scenario 2, \( i_b^E = -4 \text{ A} \). Fig. 5(b) depicts that all the five curves with \( i_b^Z(0) > -4 \text{ A} \) converge to zero while the other three with \( i_b^Z(0) < -4 \text{ A} \) decrease unboundedly.

The simulation results are a good match with the above discussion.

The simulation waveforms of the overall system with FBL-APD control (according to (13) and (15)) are shown in Fig. 6. It is clearly noted that:

- the system’s direct control outputs, i.e., \( i_{ac} \) and \( v_{dc} \), are also highly unstable as the instability of the internal dynamics severely distorts the reference for \( i_{ac} \) and turns the system into abnormal operation.

These simulated waveforms clearly demonstrate the incapability of the conventional FBL-APD control techniques when applied to control the target power converter.
IV. PROPOSED NONLINEAR APD CONTROL WITH LYAPUNOV DIRECT METHOD

Equation (18) shows that the internal dynamics of \( i_b \) is determined by \( u_2 \) only. To stabilize \( i_b \), a different \( u_2 \) from that in (21) is needed. Modification of \( u_2 \) will not affect the dynamics of \( v_{dc} \), but will alter the dynamics of \( v_{dc} \) according to (1), meaning that the stability of \( v_{dc} \) is no longer guaranteed. It is therefore desirable to develop a new control law of \( u_2 \) that can ensure the stability of \( v_{dc} \) and \( i_b \) simultaneously while still retaining a simple first-order and decoupled dynamics of \( v_{dc} \) as that described in (17). This problem is to be addressed by the proposed Lyapunov-based APD (LP-APD) control described as follows.

A. Stabilization of \( v_{dc} \) and \( i_b \)

The LP-APD control uses a two-step approach to stabilize \( v_{dc} \) and \( i_b \). Firstly, the Lyapunov direct method is used to ensure that \( v_{dc} \) converges to \( v_{dc}^R \) and \( i_b \) to \( i_b^R \) (signified by \( v_{dc} \rightarrow v_{dc}^R \) and \( i_b \rightarrow i_b^R \)). \( i_b^R \) is a virtual signal in a reference system, which we shall define shortly. Secondly, as \( v_{dc}^R \rightarrow v_{dc}^S \), \( v_{dc} \rightarrow v_{dc}^S \) is obtained, verifying the stability of \( v_{dc} \). We then merely need to verify that \( i_b^R \rightarrow i_b^S \) such that \( i_b \rightarrow i_b^S \) and the stability of \( i_b \) is ensured.

1) Step 1

According to the Lyapunov stability theory [23]–[25], to ensure \( v_{dc} \rightarrow v_{dc}^R \) and \( i_b \rightarrow i_b^R \) (or \( e_2 \rightarrow 0 \) and \( e_3 \rightarrow 0 \)), a Lyapunov function candidate \( V(e_2, e_3) \) should be found.

Analogous to (18), we define a reference system as

\[
i_b^R = -\frac{v_{dc}}{L_b} + \frac{1}{L_b} v_{dc} u_2^R,
\]

where \( u_2^R \) is the duty ratio of the PPB bridge leg in the reference system.

Assuming that \( L_b u_2^R \) is sufficiently small, \( u_2^R \) is solved from (25) as

\[
u_2^R = \frac{L_b u_2^R + v_{dc}}{v_{dc}} \approx v_{dc}.
\]

The error dynamics of \( i_b \) is obtained by subtracting (18) from (25) as

\[
\dot{e}_3 = \frac{1}{L_b} v_{dc} e_{a2} \quad \text{with} \quad e_{a2} = u_2^R - u_2.
\]

The functions of \( e_2 \) and \( e_3 \) are designed as \( V(e_2, e_3) = V_1(e_2) + V_2(e_3) \) with \( V_1(e_2) = 0.5 C_d e_2^2 \), \( V_2(e_3) = 0.5 L_d e_3^2 \).

According to the Lyapunov’s direct method, \( (e_2, e_3) = (0, 0) \) is a globally stable operating point if \( V(e_2, e_3) \) is a Lyapunov function candidate, i.e.,

\[
\dot{V}(e_2, e_3) = \dot{V}_1(e_2) + \dot{V}_2(e_3) \leq 0,
\]

where

\[
\dot{V}_1(e_2) = -C_d e_2 \dot{v}_dc = e_2 \left[ i_{load} - \frac{i_{ac}}{v_{dc}} (v_{ac} - v_1) + i_2 u_2 \right].
\]

A sufficient condition for (29) is to achieve \( \dot{V}_1(e_2) \leq 0 \) and \( \dot{V}_2(e_3) \leq 0 \) simultaneously. It can be seen that if

\[
e_{a2} = -\frac{\beta e_3}{v_{dc}}
\]

with \( \beta_1 > 0 \), then \( \dot{V}_2(e_3) \) will become

\[
\dot{V}_2(e_3) \geq -\beta_1 e_3^2 \leq 0.
\]

Combination of (28) and (32) leads to

\[
u_2 = u_2^R - e_{a2} = v_b + \beta_1 \left( i_b^R - i_b \right).
\]

By substituting \( u_2 \) into (30), \( \dot{V}_1(e_2) \) is further obtained as

\[
\dot{V}_1(e_2) = e_2 \left[ \frac{i_{load} - (v_{ac} - v_1) i_{ac}}{v_{dc}} + \left( \frac{v_b + \beta e_3}{v_{dc}} \right) i_b \right].
\]

Provided that the dynamics of \( i_b \) is significantly faster than that of \( v_{dc} \) (referred to as Condition A hereafter), \( i_b = i_b^R \) (or \( e_3 = 0 \) can be assumed in the calculation of \( \dot{V}_1(e_2) \)). Equation (35) then becomes

\[
\dot{V}_1(e_2) = e_2 \left[ \frac{i_{load} - (v_{ac} - v_1) i_{ac}}{v_{dc}} + v_b^R \right].
\]

If

\[
i_b^R = \frac{(v_{ac} - v_1) i_{ac} - i_{load} v_{dc} - \beta_2 i_v v_{dc}}{v_b}
\]

with \( \beta_2 > 0 \), \( \dot{V}_1(e_2) \) will become

\[
\dot{V}_1(e_2) = -\beta_2 e_2^2 \leq 0.
\]

Therefore, (29) is fulfilled, ensuring that \( v_{dc} \rightarrow v_{dc}^R \) and \( i_b \rightarrow i_b^R \).

2) Step 2

As we have proved above that \( i_{ac} \rightarrow i_{ac}^S \), \( v_{dc} \rightarrow v_{dc}^S \) and \( i_b \rightarrow i_b^R \), (37) becomes

\[
i_b^R = \frac{\left( v_{ac} - L_{ac} i_{ac}^S \right) i_{ac} - v_{dc}^S i_{load}}{v_b}
\]

at steady state, \( i_b^R \) can then be solved from (19) and (39) as

\[
n_{b}^R = \frac{-V_{ac} I_{ac} \cos(2\omega t) - \alpha L_{ac} I_{ac}^2 \sin(2\omega t)}{2E_{ac} - \frac{V_{ac} I_{ac} \sin(2\omega t) + L_{ac} I_{ac}^2}{2C_{b}} \cos(2\omega t)}
\]

which is bounded. Therefore, together with the results in Step 1, we have proved that \( v_{dc} \) and \( i_b \) are stable and converge to their steady-state values, respectively, with the control laws of (34) and (37).

B. Dynamics Analysis of the Overall System

According to (1), the error dynamics of \( i_{ac} \) is only related to \( u_1 \). As \( u_1 \) needs no modification, the dynamics of \( i_{ac} \) is the same as (16), i.e.,

\[
\dot{e}_1 + \alpha e_1 = 0.
\]
which is stable with a bandwidth of $f_{BW1} = \alpha_1/2\pi$.

The error dynamics of $v_{dc}$ is obtained by substituting (34) and (37) into (1) as

$$\dot{e}_2 + \frac{\beta_2}{C_{dc}} e_2 + \Delta = 0 \quad (42)$$

with

$$\Delta = \frac{1}{C_{dc}} \left( \frac{\beta_2}{v_b} - 1 \right) \left( \beta_2 e_2 + i_{\text{load}} + \frac{v_b}{v_{dc}} \alpha_1 i_{ac} - u_{ac} \right). \quad (43)$$

The error dynamics in (42) differs from that in (17) with an additional nonlinear term $\Delta$, confirming that the dynamics of $v_{dc}$ is altered with the LP-APD control.

However, notice that $\Delta = 0$ on Condition A. Thus by setting $\beta_2 = C_{dc} \alpha_2$, (42) becomes

$$\dot{e}_2 + \alpha_1 e_2 = 0, \quad (44)$$

which is the same as (17). This result is highly favorable because it indicates that the first-order and decoupled dynamics of $v_{dc}$ can still be approximately retained with the LP-APD control on Condition A.

The error dynamics of $i_b$ is derived by substituting (32) into (27) as

$$\dot{e}_3 + \frac{\beta_3}{L_b} e_3 = 0, \quad (45)$$

which again describes a first-order response with a bandwidth of $f_{BW1} = \beta_3/2\pi L_b$.

Condition A can, therefore, be achieved by selecting appropriate $\alpha_2$ and $\beta_1$ such that

$$\frac{\alpha_2}{2\pi} \ll \frac{\beta_1}{2\pi L_b} \quad \text{or} \quad \beta_1 \gg \alpha_1 L_b. \quad (46)$$

Finally, according to (19), the error dynamics of $v_b$ is

$$\dot{e}_3 = \frac{1}{C_b} e_3. \quad (47)$$

As $i_{ac} \to i_{ac}^s$, $v_{dc} \to v_{dc}^s$, $i_b \to i_b^s$ at steady state, we can conclude that $v_b \to v_b^s$ according to the principle of conservation of energy, and that $v_b$ is stable.

The complete control law of the proposed LP-APD control is summarized as

$$u_1 = \frac{v_{ac} - v_1}{v_{dc}}, \quad (48a)$$

$$u_2 = \frac{v_b + \beta_3 e_3}{v_{dc}} = \frac{1}{C_b} \left( u_{ac} \alpha_1 i_{ac} - v_2 - i_{\text{load}} - \alpha_1 \hat{C}_e e_2 - i_{\text{load}} \right) + \frac{v_b - \beta_1 i_b}{v_{dc}}. \quad (48b)$$

with $v_1$ and $v_2$ given in (15). Fig. 7 depicts the overall control schematic diagram with the unity power factor control included.

C. Design Considerations

To present a more comprehensive evaluation of the proposed controller in a practical setting, the impacts of component tolerances on the controller performance are analyzed. For simplicity, the above representations, i.e. $L_{ac}$, $C_{dc}$, $L_b$, $C_b$, are reused to denote the exact values of these parameters, while their respective measurements are signified as $\hat{L}_{ac}$, $\hat{C}_e$, $\hat{L}_b$, $\hat{C}_b$ with $\hat{L}_{ac} = L_{ac} + \Delta L_{ac}$, $\hat{C}_e = C_{dc} + \Delta C_{dc}$, $\hat{L}_b = L_b + \Delta L_b$, $\hat{C}_b = C_b + \Delta C_b$.

$$\dot{\hat{L}}_b = L_b + \Delta L_b, \quad \dot{\hat{C}}_b = C_b + \Delta C_b. \quad (49)$$

Then the proposed control law alters from (48) to

$$u_1 = \frac{v_{ac} - \hat{L}_{ac}^{\hat{g}} v_1 - \alpha_1 \hat{L}_{ac} \alpha_1 e_1}{v_{dc}}, \quad (48a)$$

$$u_2 = \frac{\beta_1 \left( u_{ac} \alpha_1 \hat{C}_e e_2 - i_{\text{load}} \right)}{v_{dc}} + \frac{v_b - \beta_1 i_b}{v_{dc}}. \quad (50)$$

Substituting (50) into (1) gives the system’s actual dynamics

$$\dot{i}_b = \frac{\hat{L}_{ac}^{\hat{g}}}{L_{ac}} \alpha_1 e_1, \quad \dot{v}_{dc} = \frac{\hat{C}_b}{C_{dc}} \alpha_1 e_2, \quad \dot{i}_b = \frac{\beta_1}{L_b} e_1, \quad \dot{v}_b = \frac{1}{C_b} i_b. \quad (51)$$

$e_1$ is solved from (51) as

$$e_1 = e_1(0) + \frac{\gamma_1}{L_{ac}} \int_{t_0}^{t} \hat{L}_{ac}^{\hat{g}} v_{ac} \sin(\omega t + \arctan(\frac{\xi}{\omega L_{ac}})). \quad (52)$$

Equation (52) shows that $e_1$ comprises two parts, one related to the initial error $e_1(0)$ and the other one caused by inaccurate knowledge of $L_{ac}$. The latter part is generally very small in magnitude owing to small $\Delta L_{ac}/L_{ac}$ and large $\alpha_1/\omega$, while the former part will dissipate exponentially with a bandwidth of

$$f_{BW1} = \frac{\hat{L}_{ac}^{\hat{g}} \alpha_1}{2\pi} = \frac{\hat{L}_{ac}^{\hat{g}}}{L_{ac}} \cdot \hat{f}_{BW1}, \quad (53)$$

where $\hat{f}_{BW1}$ represents the designed bandwidth of the $i_{ac}$ control loop.

Similarly, the impacts of the uncertainties of $C_{dc}$, $L_b$, $C_b$ can also be analyzed quantitatively. The results are summarized in Table I.

V. Simulations and Experimental Results
To examine the performance of the proposed LP-APD control, a 2-kW model of the H\(^3\) single-phase converter was first simulated in PSIM and a downsized 300-W prototype was also constructed. The system’s specifications in both the simulation and the experiments are listed in Table II. In particular, the bandwidths of the controller in both cases are designed identical as \(f_{BW1} = 2.5 \text{ kHz}\), \(f_{BW2} = 400 \text{ Hz}\), \(f_{BW3} = 2 \text{ kHz}\).

### Table I

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Impacts of Component Tolerances</th>
</tr>
</thead>
<tbody>
<tr>
<td>(L_{ac})</td>
<td>1. Bandwidth of (x_i) control loop: (f_{BW1} = \frac{L_{ac}}{L_{ac} f_{BW1}}). 2. Steady-state error of (x_i).</td>
</tr>
<tr>
<td>(C_{dc})</td>
<td>Bandwidth of (x_i) control loop: (f_{BW2} = \frac{C_{dc}}{C_{dc} f_{BW2}}).</td>
</tr>
<tr>
<td>(L_b)</td>
<td>Bandwidth of (x_i) control loop: (f_{BW3} = \frac{L_b}{L_b f_{BW3}}).</td>
</tr>
<tr>
<td>(C_b)</td>
<td>No impact.</td>
</tr>
</tbody>
</table>

### Table II

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Simulation</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated power</td>
<td>2 kW</td>
<td>300 W</td>
</tr>
<tr>
<td>Switching frequency</td>
<td>25 kHz</td>
<td>25 kHz</td>
</tr>
<tr>
<td>Ac port (v_{ac})</td>
<td>220 V (RMS) / 50 Hz</td>
<td>220 V (RMS) / 50 Hz</td>
</tr>
<tr>
<td>(L_{ac})</td>
<td>1 mH</td>
<td>7 mH</td>
</tr>
<tr>
<td>Dc port (V_{dc})</td>
<td>400 V</td>
<td>400 V</td>
</tr>
<tr>
<td>(C_{dc})</td>
<td>20 (\mu F)</td>
<td>20 (\mu F)</td>
</tr>
<tr>
<td>Ripple port (C_b)</td>
<td>200 (\mu F)</td>
<td>50 (\mu F)</td>
</tr>
<tr>
<td>(L_b)</td>
<td>0.3 mH</td>
<td>1.87 mH</td>
</tr>
<tr>
<td>Controller (f_{BW1})</td>
<td>2.5 kHz</td>
<td>2.5 kHz</td>
</tr>
<tr>
<td>Coefficients (f_{BW2})</td>
<td>400 Hz</td>
<td>400 Hz</td>
</tr>
<tr>
<td>(f_{BW3})</td>
<td>2 kHz</td>
<td>2 kHz</td>
</tr>
</tbody>
</table>

The steady-state waveforms of the proposed LP-APD controller are illustrated in Fig. 8. At the input port, the waveform of \(i_{ac}\) has a low total harmonic distortion (THD) of 0.6% and almost no phase displacement with respect to \(v_{ac}\), testifying a good regulation of \(i_{ac}\) and a unity power factor. At the output port, \(v_{dc}\) is regulated at 400 V with a peak-to-peak ripple of about 9 V (2.3%). The instantaneous power difference between the input and the output is buffered by the PPB, leading to a large-signal voltage variation of \(v_b\) in Fig. 8. Finally, the waveform of \(i_b\) confirms CCM operation of the PPB.

#### 1) Steady-State Performance

Firstly, step-up/down changes of \(i_{ac}, v_{ac}\) and \(i_b\) (i.e. \(x_1\), \(x_2\) and \(x_3\)) are conducted to verify the control bandwidth design analysis in Section IV-B. The results are displayed in Fig. 8(a)–(f). First-order responses of \(i_{ac}\), \(v_{dc}\) and \(i_b\) can be clearly observed from Fig. 9 for both reference step-up and step-down. The respective settling times of \(i_{ac}\), \(v_{dc}\) and \(i_b\) are measured in Fig. 9 to be around 300 \(\mu s\), 1 ms and 300 \(\mu s\), which are very close to their theoretical values of 318 \(\mu s\), 2.0 ms and 398 \(\mu s\). Note that the slight differences between the estimated and the theoretical settling time are mainly due to i) the large switching ripple of \(i_b\), ii) the double-line-frequency ripple of \(v_{dc}\), iii) the time-varying nature of \(i_{ac}\) and \(i_b\). These disturbances make precise estimation of the settling time difficult.

Secondly, load step change tests are carried out, and the waveforms are displayed in Fig. 10. In Fig. 10(a), a negative spike of 23 V appears in \(v_{dc}\) as the load steps up from 0 to 2 kW. With the proposed LP-APD controller, \(v_{dc}\) quickly...
recovered and settled back to its reference 400 V within 1 ms. In Fig. 10(b), a positive spike of 21 V in $v_{dc}$ is observed due to sudden load removal and the fast response of $v_{dc}$ is also demonstrated.

Fig. 10 Transient waveforms of the system in the processes of (a) load step up and (b) load step down.

B. Experimental Results

1) Steady-State Performance

Fig. 11 shows the steady-state operating waveforms of the power converter at the input port, the output port and the ripple port with the proposed LP-APD control at full load (300 W). All the waveforms in Fig. 11 match well with the simulation results in Fig. 8. The THD of $i_{ac}$ is measured at 2.21% and the peak-to-peak ripple voltage of $v_{dc}$ is measured at 8 V (2% of the average $v_{dc}$), demonstrating good regulations of the line current and the output voltage. The stability of the internal dynamics, i.e., $i_b$, is also confirmed.

2) Transient Performance

Fig. 12 depicts the dynamic responses of the system as the load is step changed between 0 W and 300 W. Fig. 12(a) illustrates that $v_{dc}$ is almost immune to the step-up load change and remains its tight regulation at 400 V. As a sudden increase of the output power leads to a sudden power imbalance between the input and output, the buffer energy in the PPB circuit is released to the dc bus immediately to compensate the power imbalance (see $v_b$ in Fig. 12(a)) in an automatic fashion, similar to the result in [5]. The fast responses and robustness of the proposed control are also validated by Fig. 12(b) as the load is suddenly cut off.

Fig. 13 further illustrates the transient waveforms of the converter as the output voltage reference is changed between 380 V and 420 V. $v_{dc}$ is shown to have a fast response and follow a typical first-order response. The settling time of $v_{dc}$ is around 2 ms, which is a good match to both the theoretical value (2 ms) and the simulation result (1 ms) in Fig. 9. Additionally, the fast response of $i_{ac}$ and the PPB function are also demonstrated in Fig. 13.

VI. CONCLUSIONS

In this study, the control of single-phase power converters that possess the active pulsating-power-buffering (PPB) function is investigated. A prior-art generalized nonlinear controller that combines the feedback linearization (FBL) theory and the automatic-power-decoupling (APD) control strategy, or an FBL-APD controller, is applied to a type of single-phase converters with active PPB. The internal dynamics instability phenomenon is demonstrated both mathematically and using simulations. To solve the instability problem, an evolved FBL-APD controller that incorporates the direct Lyapunov control method, or an LP-APD controller, is proposed, where the system’s internal dynamics are utilized to formulate the Lyapunov energy function $V(x)$. Theoretical analysis is presented to show that the proposed LP-APD control can well stabilize the internal dynamics while still retaining the best features of the FBL-APD control.
Simulation and experimental waveforms successfully confirm
the feasibility of this new control approach. The proposed
control technique, as a complement of the generalized FBL-
APD control, can be very useful for controlling emerging
single-phase converters with active PPB whilst enriching
the existing FBL control theory.

Fig. 12. Transient waveforms of the system with the proposed LP-APD control as the output power changes (a) from 0 W to 300 W, and (b) from 300 W to 0 W.

Fig. 13. Transient waveforms of the system with the proposed LP-APD control as the output voltage reference changes (a) from 380 V to 420 V, and (b) from 420 V to 380 V.

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