International Asset Allocation under Regime Switching, Skew and Kurtosis Preferences*

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JEL code: G12, F30, C32.

Abstract

This paper proposes a new tractable approach to solving asset allocation problems in situations with a large number of risky assets which pose problems for standard numerical approaches. Investor preferences are assumed to be defined over moments of the wealth distribution such as its skewness and kurtosis. Time-variations in investment opportunities are represented by a flexible regime switching process. We develop analytical methods that only require solving a small set of difference equations and can be applied even in the presence of large numbers of risky assets. We find evidence of two distinct bull and bear states in the joint distribution of equity returns in five major regions with correlations that are much higher in the bear state. Ignoring regimes, an unhedged US investor's optimal portfolio is strongly diversified internationally. The presence of regimes in the return distribution leads to a large increase in the investor's optimal holdings of US stocks as does the introduction of predictability in returns from a short US interest rate. Our paper therefore offers a rational explanation of the strong home bias observed in US investors' asset allocation, based on regime switching, skew and kurtosis preferences and predictability from the short US interest rate.

Key words: International Asset Allocation, Regime Switching, Return Predictability, Skew and Kurtosis Preferences, Home Bias.

*We thank seminar participants at HEC Paris, Krannert School of Management at Purdue, University of Washington and Catholic University, Milan for comments. All errors remain our own.

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Abstract

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1. **Introduction**

Despite the increased integration of international capital markets investors continue to hold equity portfolios largely dominated by domestic assets. According to Thomas, Warnock and Wongswan (2004), by the end of 2003 US investors held only 14% of their equity portfolios in foreign stocks at a time where such stocks accounted for 54% of the world market capitalization. Furthermore, there is little evidence that US investors' holdings of foreign stocks has been increasing over the last decade where this share has fluctuated around 10-15%, c.f. Figure 1 in Thomas, Warnock and Wongswan (2004).[1]

This evidence is poorly understood. Calculations reported by Lewis (1999) suggest that a US investor with mean-variance preferences should hold upwards of 40% in foreign stocks or, equivalently 60% in US stocks. Obstfeld and Rogoff (2000) categorize the home bias as one of the six major puzzles in international macroeconomics.

Explanations for the pervasive and persistent home bias include information asymmetries and higher estimation uncertainty for foreign than domestic stocks (Gehrig (1993), Brennan and Cao (1997)), barriers to international investment and transaction costs (Black (1974), Stulz (1981)), hedging demand for stocks with stronger negative or smaller positive correlation with domestic state variables such as inflation risk or non-traded assets (human capital), c.f. Adler and Dumas (1983) and Serrat (2001), and political/country risk (Erb et al. (1996)).[2]

As pointed out by Lewis (1999) and Karolyi and Stulz (2002), the first of these explanations is weakened by the fact that barriers to international investment have come down significantly over the last thirty years and by the large size of gross investment flows. The second explanation is weakened by the magnitude by which foreign stocks should be correlated more strongly with domestic risk factors as compared with domestic stocks. In fact, correlations with deviations from purchasing power parity can exacerbate the home bias puzzle (Cooper and Kaplanis (1994)) as can the strong positive correlation between domestic stock returns and returns on human capital (Baxter and Jermann (1997)). Ahearne, Grivier, and Warnock (2004), confirm that measurable transaction costs fail to explain the observed home bias. It is also not clear that estimation uncertainty provides a good explanation, c.f. Pastor (2000). Finally, political risk seems to apply more to emerging and developing financial markets and is a less obvious explanation for the limited diversification of US investors among the stable Western democracies. Observations such as these lead Lewis (1999, p. 589) to write that "Two decades of research on equity home bias have yet to provide a definitive answer as to why domestic investors do not invest more heavily in foreign assets."

This paper proposes a new explanation for the home bias observed in US investors' equity portfolios. We modify the standard benchmark model that assumes mean-variance preferences over a time-invariant distribution of international stock returns in three ways. First, we allow investor preferences to depend not only on the first two moments of returns but also on third and fourth moments such as skew and kurtosis.

[1] Similar home biases are present in other countries, c.f. French and Poterba (1991) and Tesar and Werner (1994). Cai and Warnock (2004) estimate US investors' foreign equity holdings at a maximum of 24% when the foreign exposure of US firms is taken into account.

[2] More recently, behavioral explanations (e.g. 'patriotism' or a generic preference for 'familiarity') have been proposed, c.f. Coval and Moskowitz (1999) and Morse and Shive (2003) among others. Uppal and Wang (2003) provide theoretical foundations based on heterogeneous ambiguity across domestic and foreign securities. Other papers have explored the effects of heterogeneity in the quality of corporate governance (e.g., investor protection) on international portfolio diversification, e.g. Dahlquist et al. (2003).

This turns out to be important because the co-skew properties of the US stock market portfolio with foreign stocks are quite different from that of most other markets. Our approach follows recent papers such as Harvey and Siddique (2000), Dittmar (2002), Harvey et al. (2004), and Jondeau and Rockinger (2004) that emphasize the need to consider moments beyond the mean and variance in both portfolio choice and asset pricing applications.

Second, we adopt a flexible model that allows the joint distribution of returns on international stock market indexes to vary across regimes that are driven by a Markov switching process and hence captures dynamics in the full return distribution. This follows evidence that stock market volatility tends to cluster through time and correlations vary asymmetrically, strengthening in down markets. There is now a large body of empirical evidence suggesting that returns on stocks and other financial assets can be captured by this class of models.[3] While a single Gaussian distribution generally does not provide an accurate description of stock returns, the regime switching models that we consider have far better ability to approximate the return distribution and can capture outliers, fat tails and skeweness. We find evidence of two regimes in the joint distribution of international stock returns, namely a bear state with high volatility and low mean returns and a bull state with high mean returns and low volatility. Both states are persistent and their presence generates predictability in the distribution of international equity returns. Consistent with evidence reported by Longin and Solnik (1995), Ramchand and Susmel (1998), Ang and Bekaert (2002a), and Butler and Joaquin (2002), return correlations across equity markets strongly depend on the underlying regime and are far higher during bear markets.

Third, we introduce predictability of returns through the short US interest rate process. This is consistent with empirical evidence suggesting that returns in equity and bond markets are predictable by means of state variables such as short interest rates, default- and term premia and the dividend yield.[4]

All three modifications play a role in explaining the home country bias. Regimes in the joint distribution of international equity returns can capture skew and kurtosis and therefore affect the optimal asset allocation of a mean-variance investor differently from that of an investor whose objectives depend on higher order moments. Furthermore, modeling the process governing the short US interest rate jointly with the distribution of stock returns changes the nature of the underlying regimes which in turn affects the asset allocation. In fact, only little is known about the allocation to international markets in the presence of time-variations in investment opportunities and whether predictability makes it more or less difficult to explain the apparent home bias in US investors' portfolios. Ang and Bekaert (2002a) consider bivariate and trivariate regime switching models that capture asymmetric correlations in volatile and stable markets (c.f. Longin and Solnik (2001)), and characterize a US investor's optimal asset allocation under power utility. Our analysis extends Ang and Bekaert's to include a wider set of stock markets as our portfolio selection problem involves five major stock markets, namely the US, Japan, Pacific ex Japan, Continental Europe and the UK, in addition to a risky short-term US bond for a total of six risky assets. Dealing with this number of risky assets creates problems for standard numerical techniques. We therefore propose a new tractable approach to optimal asset allocation that is both convenient to use and offers new insights into asset allocation problems in the presence of regime switching.

[3] See, e.g., Ang and Bekaert (2002a), Ang and Chen (2002), Engel and Hamilton (1990), Guidolin and Timmermann (2005), Gray (1996), Perez-Quiros and Timmermann (2000), Turner, Startz and Nelson (1989), Whitelaw (2001).

[4] See, e.g., Ang and Bekaert (2002a, 2002b), Campbell (1987), Keim and Stambaugh (1986), Fama and French (1988) and Pesaran and Timmermann (1995).

Using this setup, we develop analytical methods for deriving the moments of the wealth distribution. When coupled with a utility specification that incorporates skew and kurtosis preferences, the otherwise complicated numerical problem of optimal asset allocation is reduced to that of solving for the roots of a low-order polynomial. Our solution is closed-form in the sense that it reduces to solving a small number of difference equations.

We apply this method to study an unhedged US investor's allocation to international stocks. Our data on returns in the US, UK, continental European, Japanese and Pacific stock markets confirm the presence of a home-bias puzzle in the standard setting with mean-variance preferences, a single state model and no predictability of returns. In this setup, our sample estimates suggest that an investor should hold only 20% in US stocks, 50% in European stocks and 30% in US T-bills. The presence of bull and bear states raises the portfolio weight on US stocks to 47%, largely at the expense of reducing the weight on European stocks to 25%. The effect of defining preferences over higher order moments such as skew and kurtosis depends very much on the existing asset menu. In the absence of a risk-free asset, the unattractive co-skew properties of US stocks lower the weight assigned to this asset class. This changes in the presence of a risk-free asset and four-moment preferences. For example, assuming predictability from the US short rate, the allocation to US stocks rises to about 74% under four-moment preferences.

A different but related approach is followed by Das and Uppal (2003), who use a multivariate jump-diffusion model in which jumps arrive simultaneously across assets. This captures the stylized fact that large declines are observed simultaneously across international stock markets, leading to systemic risk. Correlated jumps provide an alternative to capturing the existence of (unconditional) skew and excess kurtosis in the empirical distribution of asset returns. In fact, Das and Uppal find that under levels of (relative) risk aversion similar to the ones employed in our paper, it can be optimal to limit the extent of international portfolio diversification. Harvey et al. (2004), propose a Bayesian framework for portfolio choice based on (second and third-order) Taylor expansions of an underlying expected utility functional. They assume that the unconditional distribution of asset returns is a multivariate skewed normal. In their application to an international diversification problem, they find that under third-moment preferences, roughly 50 percent of the equity portfolio should be invested in US stocks.

The plan of the paper is as follows. Section 2 sets up the optimal asset allocation problem for an investor with utility defined through a polynomial function over terminal wealth when asset returns follow a regime switching process. Section 3 provides evidence of regimes in the joint return distribution. Section 4 reports empirical estimates of optimal portfolio weights in the presence of regime dynamics but ignores predictability from state variables such as the US interest rate. Such predictability is introduced in Section 5 while Section 6 discusses the results and reports some robustness checks. Section 7 concludes. An Appendix provides details on the technical results in the paper.

2. The Asset Allocation Problem

This section describes the investor's objectives and the return generating process and goes on to characterize the method used to solve for the optimal asset allocation. We are interested in studying the asset allocation problem at time t for an investor with a T-period investment horizon. Suppose that the investor's utility function $U(W_{t+T};\boldsymbol{\theta})$ only depends on wealth at time $t+T$, W_{t+T}, and its shape is captured through the parameters in $\boldsymbol{\theta}$. The investor maximizes expected utility by choosing among h risky assets whose continuously compounded returns are given by the vector $\mathbf{r}_t^s \equiv (r_{1t}\ r_{2t}\ \dots\ r_{ht})'$. Portfolio weights are collected in the vector $\boldsymbol{\omega}_t \equiv (\omega_{1t}\ \omega_{2t}\ \dots\ \omega_{ht})'$ while $(1-\boldsymbol{\omega}_t'\boldsymbol{\iota}_h)$ is invested in a short-term interest-bearing bond which in period $t+1$ pays a continuously compounded return of r_{ft+1}. The portfolio selection problem solved by a buy-and-hold investor with unit initial wealth then becomes[5]

$$\max_{\boldsymbol{\omega}_t} \quad E_t\left[U(W_{t+T}(\boldsymbol{\omega}_t);\boldsymbol{\theta})\right]$$
$$s.t.\ W_{t+T}(\boldsymbol{\omega}_t) = \left\{(1-\boldsymbol{\omega}_t'\boldsymbol{\iota}_h)\exp\left(R_{t+T}^b\right) + \boldsymbol{\omega}_t'\exp\left(\mathbf{R}_{t+T}^s\right)\right\}, \tag{1}$$

where $\mathbf{R}_{t+T}^s \equiv \mathbf{r}_{t+1}^s + \mathbf{r}_{t+2}^s + \dots + \mathbf{r}_{t+T}^s$ is the vector of continuously compounded equity returns over the $T-$period investment horizon while $R_{t+T}^b \equiv r_{bt+1}+r_{bt+2}+\dots+r_{bt+T}$ is the continuously compounded return on the bond investment. Accordingly, $\exp(\mathbf{R}_{t+T}^s)$ is a vector of cumulated returns. Short-selling can be imposed through the constraint $\omega_{it} \in [0,1]$ for $i = 1,2,\dots,h$.

Classic results on optimal asset allocation were derived for special cases such as power utility with constant investment opportunities or under logarithmic utility, c.f. Merton (1969) and Samuelson (1969). For general preferences there is no closed-form solution to (1). Given the economic importance of problems such as (1), it is not surprising that a variety of approaches have been suggested for their solution. Recent papers that solve (1) under predictability of returns include Ang and Bekaert (2002a), Barberis (2000), Brandt (1999), Brennan, Schwarz and Lagnado (1997), Campbell and Viceira (1999, 2001), Campbell, Chan and Viceira (2003) and Kandel and Stambaugh (1996). These papers generally use approximate solutions or numerical techniques such as quadrature (Ang and Bekaert (2002a)) or Monte Carlo simulations (Barberis (2000), Detemple, Garcia and Rindisbacher (2003)) to characterize optimal portfolio weights. Quadrature methods may not be very precise when the underlying asset return distributions are not Gaussian, as is strongly suggested by empirical research, c.f. Bollerslev et al. (1992), Gallant and Tauchen (1989), and Longin (1996). They also have the problem that the number of quadrature points increases exponentially with the number of assets. Monte Carlo methods can also be computationally expensive to use as they rely on discretization of the state space and use grid methods.[6] Although existing methods have clearly yielded important insights into the solution to (1), they are therefore not particularly well-suited to our analysis of international asset allocation which involves up to six portfolios.

2.1. *Preferences over Moments of the Wealth Distribution*

Building on the work of Scott and Horvath (1980), Harvey and Siddique (2000) and Dittmar (2002) we follow a different approach and study preference functionals that extend mean-variance preferences by taking into account m higher order moments of the wealth distribution. An advantage of this approach is that it allows us to understand how specific moments such as skew or kurtosis affect the investor's asset allocation - a task that is difficult under more conventional preferences such as power utility.

To this end we consider an m-th order Taylor series expansion of a generic utility function $U(W_{t+T};\boldsymbol{\theta})$

[5]Following most papers on portfolio choice (e.g., Ang and Bekart (2001), Barberis (2000), Campbell et al. (2003), Das and Uppal (2003), and Kandel and Stambaugh (1996)), we assume a partial equilibrium framework that treats returns as exogeneous.

[6]In continuous time, closed-form solutions can be obtained under less severe restrictions. For instance Kim and Omberg (1996) work with preferences in the HARA class defined over final wealth and assume that the single risky asset return is mean-reverting.

around some wealth level v_T:

$$U(W_{t+T};\boldsymbol{\theta}) = \sum_{n=0}^{m} \frac{1}{n!} U^{(n)}(v_T;\boldsymbol{\theta})\,(W_{t+T} - v_T)^n + R_m, \tag{2}$$

where the remainder R_m is of order $o((W_{t+T} - v_T)^m)$ and $U^{(0)}(v_T;\boldsymbol{\theta}) = U(v_T;\boldsymbol{\theta})$. $U^{(n)}(.)$ denotes the $n-$th derivative of the utility function with respect to terminal wealth. Suppose the utility function $U(W_{t+T};\boldsymbol{\theta})$ is continuously differentiable with $U'(W_{t+T};\boldsymbol{\theta}) > 0$ (positive marginal utility), $U''(W_{t+T};\boldsymbol{\theta}) < 0$ (strict risk aversion), for all W_{t+T}, and that for all $n \geq 3$ the following conditions hold:

$$\begin{aligned} U^{(n)}(W_{t+T};\boldsymbol{\theta}) &> 0, \\ U^{(n)}(W_{t+T};\boldsymbol{\theta}) &= 0, \text{ or} \\ U^{(n)}(W_{t+T};\boldsymbol{\theta}) &< 0, \end{aligned} \tag{3}$$

Assumption (3) is what Scott and Horvath (1980) call strict consistency for moment preference. It states that the n-th order derivative is either always negative, always positive, or everywhere zero for all possible wealth levels. Under these assumptions, Scott and Horvath show that the following restrictions follow:

$$\begin{aligned} U^{(3)}(W_{t+T};\boldsymbol{\theta}) &> 0 \qquad U^{(4)}(W_{t+T};\boldsymbol{\theta}) < 0 \\ U^{(n\ odd)}(W_{t+T};\boldsymbol{\theta}) &> 0 \qquad U^{(n\ even)}(W_{t+T};\boldsymbol{\theta}) < 0 \end{aligned} \tag{4}$$

In particular, $U^{(3)}(W_{t+T};\boldsymbol{\theta}) < 0$ can be proven to violate the assumption of positive marginal utility, so we must have $U^{(3)}(W_{t+T};\boldsymbol{\theta}) > 0$. Likewise, $U^{(4)}(W_{t+T};\boldsymbol{\theta}) > 0$ would violate the assumption of strict risk aversion. More generally, the strict consistency requirements in (3) imply that all odd derivatives of $U(W_{t+T};\boldsymbol{\theta})$ are positive while all even derivatives are negative.

Provided that (i) the Taylor series in (2) converges; (ii) the distribution of wealth is uniquely determined by its moments; and (iii) the order of sums and integrals can be exchanged, the expansion in (2) extends to the expected utility functional:

$$E_t[U(W_{t+T};\boldsymbol{\theta})] = \sum_{n=0}^{m} \frac{1}{n!} U^{(n)}(v_T;\boldsymbol{\theta}) E_t[(W_{t+T} - v_T)^n] + \hat{R}_m,$$

where $E_t = E[.|\mathcal{F}_t]$ is the conditional expectation given current information, $\mathcal{F}_t$, and $\hat{R}_m$ is a remainder term. We thus have

$$E_t[U(W_{t+T};\boldsymbol{\theta})] \approx \hat{E}_t[U^m(W_{t+T};\boldsymbol{\theta})] = \sum_{n=0}^{m} \frac{1}{n!} U^{(n)}(v_T;\boldsymbol{\theta}) E_t[(W_{t+T} - v_T)^n]. \tag{5}$$

The approximation improves as m gets larger. Many classes of Von-Neumann Morgenstern expected utility functions can be well approximated by a function of the form:

$$\hat{E}_t[U^m(W_{t+T};\boldsymbol{\theta})] = \sum_{n=0}^{m} \kappa_n E_t[(W_{t+T} - v_T)^n], \tag{6}$$

with $\kappa_0 > 0$, and κ_n positive (negative) if n is odd (even). We call (6) an $m-$moment preference functionals.

In practice it is often sufficient to only consider the first four moments of the wealth distribution since the associated derivatives of the utility function and moments of the wealth distribution are more intuitive to interpret. Under non-satiation and risk aversion, $U' > 0$ and $U'' < 0$. Assuming decreasing absolute risk aversion, we further have $U''' > 0$ (investors prefer positive skew) while, as shown by Kimball (1993), decreasing absolute prudence implies that $U'''' < 0$. Preferences under these constraints are referred to as belonging to the standard risk aversion class.

2.2. *The Return Process*

A large empirical literature has documented the presence of persistent 'regimes' in a variety of financial time series. Ang and Bekaert (2002b), Driffill and Sola (1994), Gray (1996) and Hamilton (1988) find evidence of multiple states in the dynamics of interest rates, while Ang and Bekaert (2002a), Guidolin and Timmermann (2005), Longin and Solnik (1995), Perez-Quiros and Timmermann (2000), Turner, Starz and Nelson (1989) and Whitelaw (2001) provide evidence for stock market returns.

Following this literature, suppose that the vector of continuously compounded returns, $\mathbf{r}_t = (r_{1t}, r_{2t}, ..., r_{ht})'$, is generated by a Markov switching vector autoregressive process driven by a common state variable, S_t, that takes integer values between 1 and k:

$$\mathbf{r}_t = \boldsymbol{\mu}_{s_t} + \sum_{j=1}^{p} \mathbf{A}_{j,s_t}\mathbf{r}_{t-j} + \boldsymbol{\varepsilon}_t. \tag{7}$$

Here $\boldsymbol{\mu}_{s_t} = (\mu_{1st}, ..., \mu_{hs_t})'$ is a vector of intercepts in state s_t, $\mathbf{A}_{j,s_t}$ is an $h \times h$ matrix of autoregressive coefficients associated with the jth lag in state s_t, and $\boldsymbol{\varepsilon}_t = (\varepsilon_{1t}, ..., \varepsilon_{ht})' \sim N(\mathbf{0}, \boldsymbol{\Omega}_{s_t})$ is a vector of Gaussian return innovations with zero mean vector and state-dependent covariance matrix $\boldsymbol{\Omega}_{s_t}$:

$$\boldsymbol{\Omega}_{s_t} = E\left[\left(\mathbf{r}_t - \boldsymbol{\mu}_{s_t} - \sum_{j=1}^{p}\mathbf{A}_{j,s_t}\mathbf{r}_{t-j}\right)\left(\mathbf{r}_t - \boldsymbol{\mu}_{s_t} - \sum_{j=1}^{p}\mathbf{A}_{j,s_t}\mathbf{r}_{t-j}\right)'\middle| s_t\right].$$

The state-dependence of the covariance matrix captures the possibility of heteroskedastic shocks to asset returns, which is supported by strong empirical evidence, c.f. Bollerslev et al. (1992). Each state is assumed to be the realization of a first order, homogeneous Markov chain so the transition probability matrix, $\mathbf{P}$, governing the evolution in the common state variable, S_t, has elements

$$\mathbf{P}[i, j] = \Pr(s_t = j | s_{t-1} = i) = p_{ij}, \quad i, j = 1, .., k. \tag{8}$$

Conditional on knowing the state next period, the return distribution is Gaussian. However, since future states are never known in advance, the return distribution is a mixture of normals with the mixture weights reflecting the current state probabilities and the transition probabilities.

There are several advantages to modelling returns as mixtures of Gaussian distributions. As pointed out by Marron and Wand (1992), mixtures of normal distributions provide a flexible family that can be used to approximate many distributions.[7] They can capture skew and kurtosis in a way that is easily characterized as a function of the mean, variance and persistence parameters of the underlying states. They can also accommodate predictability and serial correlation in returns and volatility clustering since they allow the first and second moments to vary as a function of the underlying state probabilities, c.f. Timmermann

[7] Mixtures of normals can also be viewed as a nonparametric approach to modeling the return distribution if the number of states, k, is allowed to grow with the sample size.

(2000). Finally, multivariate regime switching models make it natural for cross-market correlations to vary with the underlying regime, thus confirming Longin and Solnik's (2001) and Ang and Chen's (2002) intuition that asymmetric correlations are key properties of any well-specified multivariate equity return model and that regime switching can generate most of these patterns.

Even in the absence of autoregressive terms, (7)-(8) imply time-varying investment opportunities. For example, the conditional mean of asset returns is an average of the vector of mean returns, $\boldsymbol{\mu}_{s_t}$, weighted by the filtered state probabilities $(\Pr(s_t = 1|\mathcal{F}_t), .., \Pr(s_t = k|\mathcal{F}_t))'$, conditional on information available at time t, $\mathcal{F}_t$. Since these state probabilities vary over time, the expected return will also change. In addition, this setup can readily be extended to incorporate a range of predictor variables such as short term interest rates. This is done simply by expanding the vector $\mathbf{r}_t$ with additional predictor variables, $\mathbf{z}_t$, and modeling the joint process $\mathbf{y}_t = (\mathbf{r}_t' \ \mathbf{z}_t')'$.

2.3. *The Portfolio Allocation Problem*

We next characterize the solution to the investor's optimal asset allocation problem when preferences are defined over moments of terminal wealth (6) while returns follow the regime switching process (7)-(8). We first study the problem under the simplifying assumption of a single risky asset ($h = 1$), a risk-free asset, a regime switching process with two states ($k = 2$) and no autoregressive terms ($p = 0$). For this case, the return process is

$$\begin{aligned} r_t &= \mu_{s_t} + \sigma_{s_t}\varepsilon_t, \quad \varepsilon_t \sim N(0,1), \\ \Pr(s_t &= i|s_{t-1} = i) = p_{ii}, \ i = 1,2 \end{aligned} \tag{9}$$

Concentrating on this case allows us to convey intuition for the more general results. With a single risky asset (stocks) and initial wealth set at unity, the wealth process is

$$W_{t+T} = \{(1-\omega_t)\exp(Tr_f) + \omega_t \exp(R_{t+T})\} \tag{10}$$

where $R_{t+T} \equiv r_{t+1} + r_{t+2} + ... + r_{t+T}$ is the continuously compounded stock return over the T periods and ω_t is the stock holding. For a given value of ω_t, the only unknown component in (10) is the cumulated return, $\exp(R_{t+T}) = \exp(r_{t+1} + ... + r_{t+T})$. Under the assumption of two states, $k = 2$, the nth non-central moment of the cumulated returns is given by[8]

$$\begin{aligned} M_{t+T}^{(n)} &= E\left[(\exp(r_{t+1} + ... + r_{t+T}))^n |\mathcal{F}_t\right] \\ &= \sum_{s_{t+T}=1}^{2} E\left[(\exp(r_{t+1} + ... + r_{t+T}))^n |s_{t+T}, \mathcal{F}_t\right] \Pr(s_{t+T}|\mathcal{F}_t) \\ &\equiv M_{1t+T}^{(n)} + M_{2t+T}^{(n)}, \end{aligned} \tag{11}$$

Using properties of the moment generating function of a log-normal random variable, each of these conditional moments $M_{it+1}^{(n)}$ $(i = 1, 2)$ satisfies the recursions

$$\begin{aligned} M_{it+T}^{(n)} &= E\left[\exp(n(r_{t+1} + ... + r_{t+T-1}))|s_{t+T}\right] E\left[\exp(nr_{t+T})|s_{t+T}, \mathcal{F}_t\right] \Pr(s_{t+T}|\mathcal{F}_t) \\ &= \left(M_{it+T-1}^{(n)} p_{ii} + M_{-i,t+T-1}^{(n)}(1 - p_{-i-i})\right) \exp\left(n\mu_i + \frac{n^2}{2}\sigma_i^2\right), \quad (i = 1,2) \end{aligned}$$

[8]The central moments, $\tilde{M}_{t+T}^{(n)} = E\left[(\exp(r_{t+1} + ... + r_{t+T}) - E[\exp(r_{t+1} + ... + r_{t+T})])^n\right]$., can be derived from the first n non-central moments by expanding $E\left[(\exp(r_{t+1} + ... + r_{t+T}) - E[\exp(r_{t+1} + ... + r_{t+T})|\mathcal{F}_t])^n |\mathcal{F}_t\right]$.

where we used the notation $-i$ for the converse of state i, i.e. $-i = 2$ when $i = 1$ and vice versa. In more compact notation we have

$$\begin{aligned} M_{1t+1}^{(n)} &= \alpha_1^{(n)} M_{1t}^{(n)} + \beta_1^{(n)} M_{2t}^{(n)} \\ M_{2t+1}^{(n)} &= \alpha_2^{(n)} M_{1t}^{(n)} + \beta_2^{(n)} M_{2t}^{(n)}, \end{aligned} \tag{12}$$

where

$$\begin{aligned} \alpha_1^{(n)} &= p_{11} \exp\left(n\mu_1 + \tfrac{n^2}{2}\sigma_1^2\right), \qquad & \beta_1^{(n)} &= (1-p_{22}) \exp\left(n\mu_1 + \tfrac{n^2}{2}\sigma_1^2\right), \\ \alpha_2^{(n)} &= (1-p_{11}) \exp\left(n\mu_2 + \tfrac{n^2}{2}\sigma_2^2\right), \qquad & \beta_2^{(n)} &= p_{22} \exp\left(n\mu_2 + \tfrac{n^2}{2}\sigma_2^2\right). \end{aligned}$$

Equation (12) can be reduced to a set of second order difference equations:

$$M_{it+2}^{(n)} = (\alpha_1^{(n)} + \beta_2^{(n)}) M_{it+1}^{(n)} + (\alpha_2^{(n)}\beta_1^{(n)} - \beta_2^{(n)}\alpha_1^{(n)}) M_{it}^{(n)}, \quad (i = 1, 2). \tag{13}$$

Collecting the two regime-dependent moments into a 2×1 vector $\boldsymbol{\vartheta}_{it+T}^{(n)} \equiv (M_{it+T}^{(n)} \; M_{it+T-1}^{(n)})'$, equation (13) can be written in companion form:

$$\boldsymbol{\vartheta}_{it+T}^{(n)} = \begin{bmatrix} \alpha_1^{(n)} + \beta_2^{(n)} & \alpha_2^{(n)}\beta_1^{(n)} - \beta_2^{(n)}\alpha_1^{(n)} \\ 1 & 0 \end{bmatrix} \boldsymbol{\vartheta}_{it+T-1}^{(n)} \equiv \mathbf{A}^{(n)} \boldsymbol{\vartheta}_{it+T-1}^{(n)}.$$

The elements of $\mathbf{A}^{(n)}$ only depend on the mean and variance parameters of the two states $(\mu_1, \sigma_1^2, \mu_2, \sigma_2^2)$ and the state transition parameters, (p_{11}, p_{22}). Substituting backwards we get the following equation for the ith conditional moment:

$$\boldsymbol{\vartheta}_{it+T}^{(n)} = \left(\mathbf{A}^{(n)}\right)^T \boldsymbol{\vartheta}_{it}^{(n)}.$$

Applying similar principles at $T = 1, 2$ and letting $\pi_{1t} = \Pr(s_t = 1|\mathcal{F}_t)$, the initial conditions used in determining the nth moment of cumulated returns are as follows:

$$\begin{aligned} M_{1t+1}^{(n)} &= (\pi_{1t}p_{11} + (1-\pi_{1t})(1-p_{22})) \exp\left(n\mu_1 + \frac{n^2}{2}\sigma_1^2\right), \\ M_{1t+2}^{(n)} &= p_{11}\left(\pi_{1t}p_{11} + (1-\pi_{1t})(1-p_{22})\right) \exp\left(2n\mu_1 + n^2\sigma_1^2\right) + \\ &\quad + (1-p_{22})\left(\pi_{1t}(1-p_{11}) + (1-\pi_1)p_{22}\right) \exp\left(n(\mu_1+\mu_2) + \frac{n^2}{2}(\sigma_1^2 + \sigma_2^2)\right), \\ M_{2t+1}^{(n)} &= (\pi_{1t}(1-p_{11}) + (1-\pi_1)p_{22}) \exp\left(n\mu_2 + \frac{n^2}{2}\sigma_2^2\right), \\ M_{2t+2}^{(n)} &= p_{22}\left(\pi_{1t}(1-p_{11}) + (1-\pi_1)p_{22}\right) \exp\left(2n\mu_2 + n^2\sigma_2^2\right) + \\ &\quad + (1-p_{11})\left(\pi_{1t}p_{11} + (1-\pi_{1t})(1-p_{22})\right) \exp\left(n(\mu_1+\mu_2) + \frac{n^2}{2}(\sigma_1^2 + \sigma_2^2)\right). \end{aligned} \tag{14}$$

Finally, using (11) we get an equation for the nth moment of the cumulated return:

$$M_{t+T}^{(n)} = M_{1t+T}^{(n)} + M_{2t+T}^{(n)} = \mathbf{e}_1'\boldsymbol{\vartheta}_{1t+T}^{(n)} + \mathbf{e}_2'\boldsymbol{\vartheta}_{2t+T}^{(n)} = \mathbf{e}_1'\left(\mathbf{A}^{(n)}\right)^T \boldsymbol{\vartheta}_{1t}^{(n)} + \mathbf{e}_2'\left(\mathbf{A}^{(n)}\right)^T \boldsymbol{\vartheta}_{2t}^{(n)}, \tag{15}$$

where $\mathbf{e}_i$ is a 2×1 vector of zeros except for unity in the ith place.

Having obtained the moments of the cumulated return process, it is simple to compute the expected utility for any mth order polynomial representation by using (6) and (10):

$$\begin{aligned} \hat{E}_t[U^m(W_{t+T}; \boldsymbol{\theta})] &= \sum_{n=0}^{m} \kappa_n \sum_{j=0}^{n} (-1)^{n-j} v_T^{n-j} \binom{n}{j} E_t[W_{t+T}^j] \\ &= \sum_{n=0}^{m} \kappa_n \sum_{j=0}^{n} (-1)^{n-j} v_T^{n-j} \binom{n}{j} \sum_{i=0}^{j} \binom{j}{i} \omega_t^i M_{t+T}^i \left((1-\omega_t)\exp(Tr_f)\right)^{j-i}. \end{aligned} \tag{16}$$

The first order condition is obtained by differentiating with respect to ω_t :

$$\frac{\partial \hat{E}_t[U^m(W_{t+T};\boldsymbol{\theta})]}{\partial \omega_t} = \sum_{n=0}^{m} \kappa_n \sum_{j=0}^{n} (-1)^{n-j} v_T^{n-j} \binom{n}{j} \sum_{i=1}^{j} i \binom{j}{i} \omega_t^{i-1} M_{t+T}^i \left((1-\omega_t)\exp\left(Tr_f\right)\right)^{j-i} = 0.$$

This takes the form of the roots of an $m-1$ order polynomial in ω_t, which are easily obtained. The optimal solution for ω_t corresponds to the root for which (16) has the highest value.

From this analysis it is clear that the optimal asset allocation depends on the following factors:

1. The current state probabilities $(\pi_t, 1-\pi_t)$ which determine moments of returns at all future points provided that either the mean or variance parameters differ across states ($\mu_1 \neq \mu_2$ or $\sigma_1 \neq \sigma_2$).

2. State transition probabilities (p_{11}, p_{22}) which also affect the speed of mean reversion in the investment opportunity set towards its steady state.

3. Differences between mean parameters (μ_1, μ_2) and variance parameters (σ_1, σ_2) (and more generally covariance parameters) across states. For example, skew in the return distribution can only be induced provided that $\mu_1 \neq \mu_2$, cf. Timmermann (2000).

4. The number of moments of the wealth distribution that matters for preferences, m, in addition to the weights on the various moments.

5. The investment horizon, T.

2.4. *General Results*

In many applications $\mathbf{r}_t$ is a vector of returns on a multi-asset portfolio. The number of states, k, may also exceed two. For the general case with h risky assets, a risk-free asset with constant return r^f and k states, the wealth process is

$$W_{t+T} = \boldsymbol{\omega}_t' \exp\left(\sum_{i=1}^{T} \mathbf{r}_{t+i}\right) + (1 - \boldsymbol{\omega}_t' \boldsymbol{\iota}_h) \exp(r_f T).$$

The moments of the wealth process are complicated to derive and involve many cross-product terms. For example, in the case with only two risky assets, the third moment is

$$E_t[W_{t+T}^3] = E_t \left[\omega_{1t}^3 \exp\left(3\sum_{i=1}^{T} r_{1,t+i}\right) + 3\omega_{1t}^2 \omega_{2t} \exp\left(2\sum_{i=1}^{T} r_{1,t+i} + \sum_{i=1}^{T} r_{2,t+i}\right) + 3\omega_{1t}\omega_{2t}^2 \times \right.$$

$$\left. \times \exp\left(\sum_{i=1}^{T} r_{1,t+i} + 2\sum_{i=1}^{T} r_{2,t+i}\right) + \omega_{2t}^3 \exp\left(3\sum_{i=1}^{T} r_{2,t+i}\right)\right] + 3E_t\left[\omega_{1t}^2 \exp\left(2\sum_{i=1}^{T} r_{1,t+i}\right) + \right.$$

$$\left. + 2\omega_{1t}\omega_{2t} \exp\left(\sum_{i=1}^{T} r_{1,t+i} + \sum_{i=1}^{T} r_{2,t+i}\right) + \omega_{2t}^2 \exp\left(2\sum_{i=1}^{T} r_{2,t+i}\right)\right] (1\text{-}\omega_{1t}\text{-}\omega_{2t}) \exp(r_f T) +$$

$$+ 3E_t \left[\omega_{1t} \exp\left(\sum_{i=1}^{T} r_{1,t+i}\right) + \omega_{2t} \exp\left(\sum_{i=1}^{T} r_{2,t+i}\right)\right] (1\text{-}\omega_{1t}\text{-}\omega_{2t})^2 \exp(2r_f T) + (1\text{-}\omega_{1t}\text{-}\omega_{2t})^3 \exp(3r_f T).$$

In the following we provide a simple, recursive procedure for evaluating the moments of cumulated returns:

Proposition 1. *Under the regime-switching process (7)-(8) and m−moment preferences (6), for a given set of portfolio weights,* $\boldsymbol{\omega}_t$, *the expected utility is given by*

$$\hat{E}_t[U^m(W_{t+T})] = \sum_{n=0}^{m} \kappa_n \sum_{j=0}^{n} (\text{-}1)^{n-j} v_T^{n-j} {}_nC_j E_t[W_{t+T}^j]$$
$$= \sum_{n=0}^{m} \kappa_n \sum_{j=0}^{n} (\text{-}1)^{n-j} v_T^{n-j} \binom{n}{j} \sum_{i=0}^{j} \binom{j}{i} E_t\left[\left(\boldsymbol{\omega}_t' \exp\left(\mathbf{R}_{t+T}^s\right)\right)^i\right]((1\text{-}\boldsymbol{\omega}_t'\boldsymbol{\iota}_h)\exp(Tr^f))^{j-i}.$$

The nth moment of the cumulated return on the risky asset portfolio is

$$E_t\left[\left(\boldsymbol{\omega}_t' \exp\left(\mathbf{R}_{t+T}^s\right)\right)^n\right] = \sum_{n_1=0}^{n} \cdots \sum_{n_h=0}^{n} \lambda(n_1, n_2, ..., n_h) \left(\prod_{i=1}^{h} \omega_i^{n_i}\right) M_{t+T}^{(n)}(n_1, ..., n_h),$$

where $\sum_{i=1}^{h} n_i = n$, $0 \leq n_i \leq n$ $(i = 1, ..., h)$,

$$\lambda(n_1, n_2, ..., n_h) \equiv \frac{n!}{n_1!n_2! \ ... \ n_h!}.$$

and $M_{t+T}^{(n)}(n_1, ..., n_h)$ *can be evaluated recursively, using (A4) in the Appendix.*

The appendix proves this result. Without a risk-free asset, a simpler expression applies:

$$\hat{E}_t[U^m(W_{t+T};\boldsymbol{\theta})] = \sum_{n=0}^{m} \kappa_n \sum_{j=0}^{n} (\text{-}1)^{n-j} v_T^{n-j} \binom{n}{j} E_t\left[\left(\boldsymbol{\omega}_t' \exp\left(\mathbf{R}_{t+T}^s\right)\right)^j\right].$$

Proposition 1 is very convenient to use to derive the expected utility for a vector of portfolio weights, $\boldsymbol{\omega}_t$, of relatively high dimension. The solution is in closed-form in the sense that it reduces the expected utility calculation to a finite number of steps each of which can be solved by elementary operations.

3. Regimes in International Equity Returns

We next turn to the empirical analysis. We first document the presence of regimes in the joint distribution of international stock market returns by considering a selection of the largest international markets, namely the United States, Japan, the United Kingdom, the Pacific region (ex-Japan), and continental Europe, notably Germany and France. More markets could be included but parameter estimation errors are likely to become increasingly important when more markets are included so we do not go beyond five equity portfolios.[9] Following other papers (e.g., De Santis and Gerard (1997), Ang and Bekaert (2002a)), we consider the asset allocation from the perspective of an unhedged US investor. and examine returns in US dollars on Morgan Stanley Capital International (MSCI) indices.[10] When a risk-free asset is included in the analysis it is measured by the 30-day US T-bill rate provided by the Center for Research in Security Prices (CRSP). Our data are monthly and cover the sample period 1975:01 - 2003:12, for a total of 348 observations. Returns are continuously compounded and adjusted for dividends and other non-cash payments to shareholders.

[9] At the end of 2003 these markets roughly represented 98% of the world equity market capitalization.

[10] This is consistent with other authors' finding that US investors predominantly hold large and liquid foreign stocks such as those that dominate the MSCI index, c.f. Thomas, Warnock and Wongswan (2004). Das and Uppal (2003) examine a similar international portfolio problem, assuming a constant US riskless rate.

Table 1 reports summary statistics for the international stock returns and the US T-bill rate. Mean returns are positive and lie in the range 0.008 to 0.013 per month, although they fail to be statistically significant at conventional levels. Return volatilities vary from four to seven percent per month. Comparing the performance across stock markets, US stock returns were characterized by a moderate mean and low volatility, while European markets earned higher mean returns (0.013) but at a higher level of risk. Returns across all stock markets are positively and significantly correlated, with (unreported) correlation estimates varying between 0.31 and 0.63.

Returns in all but one market (Japan) are strongly non-normal, as also shown by Das and Uppal (2003) for a shorter sample period. Indeed, most of the stock return series are strongly skewed with fat tails. The strong rejection of normality suggests that a flexible model is required for the joint return distribution in order to accommodate skews and fat tails. Regime switching models are known to provide a flexible representation of many families of distributions so we consider a variety of such models. The first question that arises is of course whether multiple regimes are present in the joint distribution of returns across international stock markets. To answer this we compute the single-state specification test suggested by Davies (1977).[11] Irrespective of the number of states, this test very strongly rejects the linear specification.[12]

To determine the number of regimes, we considered the Schwarz information criterion which is a consistent model selection criterion. This criterion selected a relatively parsimonious two-state specification with 42 parameters and no autoregressive lags.[13] To derive optimal asset allocations, we need to take expectations over the joint distribution of stock returns across regimes, so it is important that the selected model is not misspecified. We therefore conducted specification tests based on the so-called probability integral, c.f. Diebold, Gunther and Tay (1998) which consider the entire conditional probability distribution of returns. We follow Berkowitz (2001) in considering four separate tests for misspecification related to the first four moments of stock returns in addition to any evidence of serial correlation in the normalized residuals.[14] Table 2 shows that a single-state, IID model (with no predictability in asset returns) is strongly rejected for most series, with poor fit particularly for the Pacific and European indices. In contrast, the two-state model produces density forecasts that are particularly good for the US, Japan, and the UK. For the two remaining markets (Pacific and Europe), we see a clear improvement over the single-state model, although at least one test still formally rejects the null that the predictive density is correctly specified. In the case of Pacific returns, it appears that the regime switching model captures most of the evidence of skew and kurtosis; for European stock returns, the tests show moderate evidence of omitted serial correlation and ARCH effects (i.e., serial correlation in squares).

Increasing the number of states, k, to three we obtained evidence (in unreported results) that all return series have properties consistent with a correct return specification. However, this extension gave rise to 22 additional parameters and a much lower value of the Schwarz information criterion,which suggests a worse out-of-sample forecasting accuracy, although a better in-sample fit. Furthermore, Ang and Bekaert's (2002b) Regime Classification Measure (RCM) was significantly lower for $k = 2$ (21.12) than for $k = 3$

[11] Regime switching models have parameters that are unidentified under the null hypothesis of a single state. Standard critical values are therefore invalid in the hypothesis test.

[12] For instance, a likelihood ratio (upper bound) test of the null of $k = 1$ vs. the alternative of $k = 2$ for a model with state-dependent means and variances yields a test statistic of 138.2 which carries a p-value of 0.000.

[13] See Bossaerts and Hillion (1999) for a discussion and application of information criteria to model financial returns.

[14] Further details on the implementation of tests appear in the caption to Table 2.

(25.34), indicating that the simpler two-state model also provides a more precise regime classification.[15] All in all, we opt to use the more parsimonious model characterized by two regimes.

To interpret the two states from an economic perspective, we present parameter estimates in Table 3 and plot the smoothed state probabilities in Figure 1. Figure 1 shows that the probability of state 1 is high around most global recessions (the early 1980s, the 1991-1992 and 2002 recessions), but also rises on many occasions characterized by high volatility in returns and financial downturns (e.g. October 1987, the Asian flu of 1998 and September 2001). Most of the time it is clear which state the markets are in and the state probabilities are far away from 0.5.

Turning to the parameter estimates in Table 3, in the first state mean returns are uniformly negative across all markets and, for Japan, also statistically significant. Regime 1 therefore identifies a global bear market with volatilities well above their sample averages. Returns in the Pacific market appear to be particularly risky with an annualized volatility estimate of 36.9%. Highly volatile returns along with zero or slightly negative mean returns imply that large losses are more likely to occur when equity markets are in this state. Consistent with previous studies (Ang and Bekaert (2002a), Longin and Solnik (1995, 2001) and Karolyi and Stulz (1999)), return correlations are higher in the bear state than in the full sample. Returns in Europe, the US, and the UK seem to be particularly strongly correlated during global bear markets.[16]

Mean returns are positive and highly significant in the second state where they lie in the range 0.015 - 0.019 per month. Volatilities are also lower in this state and, consistent with previous findings, correlations are below their unconditional counterparts, although they remain positive.[17] The second state can thus be characterized as a low-volatility bull state.[18]

The persistence of the bear state (0.83) is considerably lower than that of the bull state (0.95). As a consequence, the average duration of a bear state is six months, while it is 19 months for the bull state. The implied steady state (ergodic) probabilities are 0.24 and 0.76, respectively. These findings are consistent with results in Ramchand and Susmel (1998) and the common perception that bear markets are less frequent and protracted than bull markets. Our finding of large differences in mean returns across the high- and low correlation states echoes Longin and Solnik's (2001) conclusion that correlations vary mostly in response to market trends, and not to volatility per se.

To complete our model specification, we follow Ang and Bekaert (2002a) and test a set of restrictions on the parameters of the two-state model. First, we reject the hypothesis that the means are regime-independent: the likelihood ratio statistic comparing the general model in Table 3 to a restricted two-state model with only a state-dependent covariance matrix is 146.0 with a p-value below 0.001. Second, we reject the null hypothesis that the covariance matrix is identical across regimes: the associated likelihood ratio statistic equals 101.6 with a p-value below 0.001.[19]

[15] This measure is computed as

$$RCM \equiv k^k \times 100 \times T^{-1} \sum_{t=1}^{T} \left[\prod_{i=1}^{k} \Pr(S_t = i|\mathcal{F}_T) \right].$$

$RCM = 0$ when the regime is identified with certainty at each point in the sample, while $RCM > 0$ if there is uncertainty about the state at least one time during the sample. The upper bound for RCM is 100.

[16] Ramchand and Susmel (1998) also find that correlations between US and other world equity markets are significantly higher when the US market is in the highly volatile state.

[17] The exception is the UK for which volatility is higher in regime 2 than in regime 1.

[18] The presence of two very different regimes results in distributions that are likely to exhibit excess kurtosis as a result of mixture effects. Negative skew arises because volatilities tend to be higher when returns are negative.

[19] We also test and fail to reject the hypothesis of no additional heteroskedastic (ARCH) effects in addition to those captured

4. **International Portfolio Holdings**

We next consider empirically the optimal international asset allocation under regime switching. We follow Dittmar (2002) and use $m = 4$ moments in the preference specification. This choice can also be justified on the basis that non-satiation, decreasing absolute risk aversion, and decreasing absolute prudence determine the signs of the first four derivatives of $U(W_{t+T};\boldsymbol{\theta})$, c.f. Kimball (1993).

The weights on the first four moments of the wealth distribution are determined to ensure that our results can be compared to those in the existing literature. Most studies on optimal asset allocation use power utility so we calibrate our coefficients to the benchmark

$$U(W_{t+T};\theta) = \frac{W_{t+T}^{1-\theta}}{1-\theta},\ \theta > 0. \tag{17}$$

For a given coefficient of relative risk aversion, θ, the functional form (17) serves as a guide in setting values of $\{\kappa_n\}_{n=0}^{m}$ in (6) but should otherwise not be viewed as an attempt to approximate results under power utility.[20] Expanding the powers of $(W_{t+T} - v_T)$ and taking expectations, we obtain the following expression for the four-moment preference function:

$$\hat{E}_t[U^4(W_{t+T};\theta)] = \kappa_{0,T}(\theta)+\kappa_{1,T}(\theta)E_t[W_{t+T}]+\kappa_{2,T}(\theta)E_t[W_{t+T}^2]+\kappa_{3,T}(\theta)E_t[W_{t+T}^3]+\kappa_{4,T}(\theta)E_t[W_{t+T}^4], \tag{18}$$

where[21]

$$\begin{aligned}
\kappa_{0,T}(\theta) &\equiv v_T^{1-\theta}\left[(1-\theta)^{-1} - 1 - \frac{1}{2}\theta - \frac{1}{6}\theta(\theta+1) - \frac{1}{24}\theta(\theta+1)(\theta+2)\right] \\
\kappa_{1,T}(\theta) &\equiv \frac{1}{6}v_T^{-\theta}\left[6+6\theta+3\theta(\theta+1)+\theta(\theta+1)(\theta+2)\right] > 0 \\
\kappa_{2,T}(\theta) &\equiv -\frac{1}{4}\theta v_T^{-(1+\theta)}\left[2+2(\theta+1)+(\theta+1)(\theta+2)\right] < 0
\end{aligned}$$

$$\begin{aligned}
\kappa_{3,T}(\theta) &\equiv \frac{1}{6}\theta(\theta+1)(\theta+3)v_T^{-(2+\theta)} > 0 \\
\kappa_{4,T}(\theta) &\equiv -\frac{1}{24}\theta(\theta+1)(\theta+2)v_T^{-(3+\theta)} < 0.
\end{aligned}$$

Notice that the expected utility from final wealth increases in $E_t[W_{t+T}]$ and $E_t[W_{t+T}^3]$, so that higher expected returns and more right-skewed distributions lead to higher expected utility. Conversely, expected utility is a decreasing function of the second and fourth moments of the terminal wealth distribution. Our benchmark results assume that $\theta = 2$, a coefficient of relative risk aversion compatible with the bulk of empirical evidence. Later we present robustness results that allow this coefficient to assume larger values.[22]

by regime-switching. The finding that regime-switching adequately captures heteroskedasticity in monthly returns is consistent with conclusions in Diebold (1986) and Lamoreux and Lastrapes (1990).

[20] As shown by Loistl (1976), the series expansion of power utility converges only for $W_{t+T} \in [0,\, 2E_t(W_{t+T})]$. Such a range is plausible when short-sale restrictions are imposed and T is kept within reasonable bounds.

[21] The notation $\kappa_{n,T}$ makes it explicit that the coefficients of the fourth order Taylor expansion depend on the investment horizon through the coefficient v_T, the point around which the approximation is calculated. We follow standard practice and set the point around which the Taylor series expansion is computed to $v_T = E_t[W_{t+T-1}]$, the expected value of the investor's wealth for a $T-1$ period investment horizon.

[22] Based on the evidence in Ang and Bekaert (2002a) $-$ who show that the optimal home bias is an increasing function of the coefficient of relative risk aversion $-$ this is also a rather conservative choice that allows us to examine the effects on the optimal portfolio choices produced by preferences that account for higher order moments.

A solution to the optimal asset allocation problem can now easily be found from (18) by solving a system of cubic equations in $\hat{\boldsymbol{\omega}}_t$ derived from the first and second order conditions

$$\nabla_{\boldsymbol{\omega}_t} \hat{E}_t[U^4(W_{t+T};\theta)]\Big|_{\hat{\boldsymbol{\omega}}_t} = \mathbf{0}', \quad H_{\boldsymbol{\omega}_t} \hat{E}_t[U^4(W_{t+T};\theta)]\Big|_{\hat{\boldsymbol{\omega}}_t} \text{ is negative definite.}$$

Thus $\hat{\boldsymbol{\omega}}_t$ sets the gradient, $\nabla_{\boldsymbol{\omega}_t} \hat{E}_t[U^4(W_{t+T};\theta)]$, to a vector of zeros and produces a negative definite Hessian matrix, $H_{\boldsymbol{\omega}_t} \hat{E}_t[U^4(W_{t+T};\theta)]$.

4.1. *All-Equity Allocations*

Following common practice (e.g. Ang and Bekaert (2002a)) we first study all-equity allocations. Table 3 showed that the joint distribution of stock returns across international markets is very different in the bull and bear state. It is therefore not surprising that investors' perception of the state probability is a key determinant of their asset holdings. Similarly, the investment horizon, T, is important since the two regimes capture a mean reverting component in stock returns. Investors can be fairly certain that the current state will apply in the short run, particularly the more persistent bull state. Regime switching is, however, likely to occur at longer investment horizons.

These observations are key to understanding Figure 2 which plots the optimal allocation to the various equity markets as a function of the investment horizon using a range of values of the initial probability of being in the bull state. Only the initial state probability is fixed and subsequent state probabilities evolve through the state transitions given by (8). In our baseline scenario we assume $\theta = 2$ but we report results in Section 6 for higher values of this parameter (roughly interpretable as the coefficient of relative risk aversion). We always impose the short-sales constraint $\omega_t^i \in [0, 1]$.

The figure reveals a very interesting interaction between the underlying state probabilities and the investment horizon. Suppose that the initial bull state probability is zero so we are certain to start from the bear state. In this state investors with a short investment horizon hold a fairly balanced portfolio with 35% in US stocks, 26% in European stocks, 17% in Pacific stocks, 15% in UK stocks, and 7% in Japanese stocks. Table 3 helps to explain these choices since US and European equities have the highest mean returns and the lowest volatilities in the bear state. However, there is a 17% chance of switching to the bull state after one period even when starting from the bear state, so it is worthwhile investing in the Pacific region whose stocks are only weakly correlated with stocks in other markets. It is also attractive to invest in the UK market which experiences low volatility in this state. As the investment horizon grows, the more persistent bull state starts to dominate since it has a steady-state probability exceeding 75%. This means that more gets allocated towards markets such as Europe and the UK with attractive returns in the bull state. In contrast, the allocation to US stocks falls below 5% at long horizons. The allocation to Japanese and Pacific stocks is also very low at horizons of six months or longer.

Starting from the bull state, the optimal portfolio choice is very similar to the steady-state values that apply at the longer investment horizons when starting from the bear state. For instance, the long-run portfolio weights are 63%, 33% and 4% for the European, UK, and US markets respectively. When the investor is certain of starting in the bull state and has a one-month horizon, the optimal allocation to these markets is 61%, 39%, and 0%. Demand for US stocks is virtually absent at all horizons in the bull state.

4.2. *The Effect of Higher Moments*

These results confirm the presence of a strong home bias in observed US equity portfolios (vs. our normative results) and suggest a long-run or steady-state equity portfolio dominated by continental European and UK stocks (representing 60% and 30% of the equity portfolio, respectively) with less than 10% allocated to US stocks. Moreover, the demand for US, Japanese, and Pacific stocks is only sizeable for short-term investors in the bear state, which on average occurs one-quarter of the time and has an average duration of only six months.

Higher order moments turn out to be important in explaining these findings. Recall that US stock returns had the second highest unconditional mean and the lowest standard deviation, suggesting that this market is very attractive to a mean-variance investor who does not care about skew or kurtosis.[23] To address the effect of higher order moments, we computed the optimal portfolio weights as a function of T and π (the state probability) under mean-variance ($m = 2$) preferences:

$$\hat{E}_t[U^2(W_{t+T};\theta)] = \kappa_{0,T}(\theta) + \kappa_{1,T}(\theta)E_t[W_{t+T}] + \kappa_{2,T}(\theta)E_t[W_{t+T}^2] \tag{19}$$

where $\kappa_{0,T}(\theta) \equiv v_T^{1-\theta}\left[(1-\theta)^{-1} - 1 - \frac{1}{2}\theta\right]$, $\kappa_{1,T}(\theta) \equiv v_T^{-\theta}(1+\theta) > 0$, and $\kappa_{2,T}(\theta) \equiv -\frac{1}{2}\theta v_T^{-(1+\theta)} < 0$. We also consider optimal allocations under three-moment preferences

$$\hat{E}_t[U^3(W_{t+T};\theta)] = \kappa_{0,T}(\theta) + \kappa_{1,T}(\theta)E_t[W_{t+T}] + \kappa_{2,T}(\theta)E_t[W_{t+T}^2] + \kappa_{3,T}(\theta)E_t[W_{t+T}^3] \tag{20}$$

where now $\kappa_{0,T}(\theta) \equiv v_T^{1-\theta}\left[(1-\theta)^{-1} - 1 - \frac{1}{2}\theta - \frac{1}{6}\theta(\theta+1)\right]$, $\kappa_{1,T}(\theta) \equiv v_T^{-\theta}\left[1+\theta+\frac{1}{2}\theta(\theta+1)\right] > 0$, $\kappa_{2,T}(\theta) \equiv -\frac{1}{2}\theta v_T^{-(1+\theta)}(2+\theta) < 0$, and $\kappa_{3,T}(\theta) \equiv \frac{1}{6}\theta(\theta+1)v_T^{-(2+\theta)} > 0$.

Table 4 compares allocations for $m = 2, 3$ and 4. Stock holdings under mean-variance preferences are very different from those obtained under skew- and kurtosis preferences. Under mean-variance preferences, the bulk of available wealth is shifted away from UK and European stocks towards the US, Japanese and Pacific markets. Starting from steady-state probabilities and going from $m = 2$ to $m = 3$ leads to a reduction in the allocation to US stocks of between 21% and 24%. Under the steady state distribution at the two-year horizon, weights of 46%, 26%, 14%, and 14% are allocated to European, US, Pacific, and Japanese stocks, respectively, while nothing gets invested in the UK. These weights compare with 64%, 3%, 0% and 0% under four-moment preferences. The mean-variance allocations are largely explained by the Sharpe ratios of these markets which are 0.12, 0.12, 0.05, and 0.05, respectively. The UK Sharpe ratio is 0.07 but UK stock returns are unattractive in that they are strongly correlated with returns on European and US stocks.

Interestingly, the optimal weights under a three-moment objective are very similar to those obtained for $m = 4$ suggesting that kurtosis effects are not of first order magnitude. Comparing optimal weights under $m = 3$ and $m = 2$, we see that skew eliminates the demand for Japanese stocks in all states, and reduces the weight on the US and Pacific markets outside the bear state. In contrast, the UK weight increases significantly in all regimes, while the effect on the demand for continental European equities is mixed.

[23] To investigate the effect of skew and kurtosis on the optimal asset allocation, we also inspected the coefficients $\{\kappa_{n,T}\}_{n=1}^4$ tracking the weight on the return moments in the preference specification (18). We found that $\kappa_{1,T}$ and $\kappa_{2,T}$ are of similar magnitude while $\kappa_{3,T}$ has a value roughly half the size of $\kappa_{1,T}$ and $\kappa_{2,T}$. Finally, $\kappa_{4,T}$ took on small negative values, between one fifth and one tenth the size of the coefficients on the first two moments. This suggests that the effect of the third moment is quantitatively similar to that of the first two moments while the fourth moment matters less.

4.3. *Co-Skew and Co-Kurtosis Properties*

Table 4 suggests that the skew and kurtosis properties of US and Japanese stock returns must be responsible for their limited weights under four-moment preferences. Conversely, the positive third and fourth moment properties of UK returns must explain their large weight.

To understand the effect of skew and kurtosis on the optimal asset allocation requires studying the co-skew and co-kurtosis properties at the portfolio level. To this end, define the co-skew of a triplet of stock returns $i, j, l = 1, ..., h$ as:

$$S_{i,j,l} \equiv \frac{E[(r_{it} - E[r_{it}])(r_{jt} - E[r_{jt}])(r_{lt} - E[r_{lt}])]}{\{E[(r_{it} - E[r_{it}])^2]E[(r_{jt} - E[r_{jt}])^2]E[(r_{lt} - E[r_{lt}])^2]\}^{1/2}}. \tag{21}$$

When $i = j = l$, $S_{i,j,l}$ reduces to the third central moment of returns on asset i, which captures the traditional measure of skew, $Skew_i = S_{i,i,i}/\sigma_i^3$. When $i \neq j \neq l$, $S_{i,j,l}$ gives a signed measure of the strength of the linear association among deviations of returns from their means across triplets of asset returns. When only the returns on two assets are involved, $S_{i,j,j}$ will reflect the strength of the linear association between squared deviations from the mean (which is a measure of scale) and signed deviations from the mean for a pair of assets. A security i with negative (positive) $S_{-i,i,i}$ coefficients for the majority of all possible pairs of returns on other securities (denoted as $-i$, i.e. 'not i') is a security that becomes highly volatile when other securities give low (high) returns, and vice-versa. To a risk averse investor this is an unattractive (attractive) feature since risk rises in periods with low returns. A security i with predominantly negative $S_{i,-i,-i}$ coefficients pays low returns when other securities become highly volatile; again this feature is harmful since the security performs poorly when other assets are highly risky.

These effects allow us to explain the finding that aversion to skew in the distribution of final wealth reduces the weights on Japanese, Pacific, and − primarily − US stocks. Table 5 shows that the US portfolio has large negative values of both own-market skew ($S_{US,US,US}$), and co-skews $S_{US,US,j}$, $S_{US,j,j}$, producing either the largest negative or second largest negative sample estimates of these moments across all regions. Hence US stock returns tend to be negative when volatility is high in other markets and they are more volatile when other markets experience negative returns. US stocks therefore provide little or no hedge against adverse return or volatility shocks in other markets. Pacific stocks also generate negative values of these moments which explains their reduced weight in the optimal portfolio when m is raised from two to three. Finally, continental Europe has relatively desirable third moment properties with a positive own-market skew, explaining why this market gets a much larger weight under skew preferences than under mean-variance preferences.

Turning to fat tails in the return distribution, we define the co-kurtosis of a set of four stock returns $i, j, l, q = 1, ..., h$ as:

$$K_{i,j,l,q} \equiv \frac{E[(r_{it} - E[r_{it}])(r_{jt} - E[r_{jt}])(r_{lt} - E[r_{lt}])(r_{qt} - E[r_{qt}])]}{\{E[(r_{it} - E[r_{it}])^2]E[(r_{jt} - E[r_{jt}])^2]E[(r_{lt} - E[r_{lt}])^2]E[(r_{qt} - E[r_{qt}])^2]\}^{1/2}}. \tag{22}$$

When $i = j = l = q$, $K_{i,j,l,q}$ is the coefficient of kurtosis, $Kurt_i = K_{i,i,i,i}/\sigma_i^4$. When $i \neq j \neq l \neq q$, $K_{i,j,l,q}$ gives a signed measure of the strength of the linear association among deviations of returns from their means across four-tuples of asset returns. Three cases are particularly easy to interpret, namely own-market kurtosis, $K_{i,i,i,i}$, when all indices coincide and $K_{i,i,j,j}$ which sheds light on the correlation between volatility shocks across markets. In both cases, large positive values are undesirable: a large value of $K_{i,i,i,i}$ suggests fat tails in the return distribution, while a large value of $K_{i,i,-i,-i}$ shows that volatility tends to be large at the same time in market i as in other markets, thus increasing the overall portfolio risk. $K_{i,i,i,-i}$ measures the signed linear association between cubic and simple deviations from means for a pair of assets. A security i with positive values of $K_{i,i,i,-i}$ becomes skewed to the left when other securities pay below-normal returns and is hence undesirable to risk-averse investors.

Table 5 shows that the sample estimates of the co-volatility are highest for the US and Pacific regions. These regions also produce high estimates of own-market kurtosis as does Europe. Furthermore, the table shows that the steady-state higher order moments implied by our regime switching model closely match their sample counterparts, offering further evidence that the model is not misspecified.[24]

4.4. *Investments with a Risk-Free Asset*

US investors have the option of investing in domestic T-bills so we next consider how the introduction of a risk-free asset affects the optimal allocation to equities. The presence of a risk-free asset will typically lead to significant changes in investors' allocation to international equities since a risk-free asset can be used to separate the problem of controlling the overall level of risk exposure versus the choice of the trade-off in risk across assets, c.f. Ang and Bekaert (2002a).

Figure 3 plots the optimal allocation to stocks and US T-bills as a function of the investment horizon and the initial probability of being in the bull state. At short investment horizons the most highly diversified portfolio emerges when the probability of starting from the bull state is very high. For this case only 30% of the overall portfolio is held in US stocks, 15-20% in Japan and Europe and 3% in the UK market. The remaining 30% is held in US T-bills. When the initial bull state probability is lower, the US market gets a larger share close to 40%, with 35% invested in US T-bills, 20-25% in European stocks and very little held in other markets. The finding that the weight on US stocks is larger in bear than in bull states is related to the lower diversification opportunities implied by the increased correlation coefficients in the bear state.

Interestingly, as shown in Table 4, although the short-term allocation to US stocks varies considerably across states (from 30% in the bull state to 48% in the bear state), in the presence of a risk-free asset this allocation is far more robust across preference specifications. Going from mean-variance preferences to skew or skew and kurtosis preferences only changes the allocation to US equities by one percent when a risk-free asset is available, but changes this allocation by more than 20% in the absence of a risk-free asset.

At longer horizons, the value of the initial state probability continues to have a weak effect on the optimal asset allocation. This is a consequence of the convergence of state probabilities to their steady-state values. Independently of the initial state probability, a US investor commits 40%, 20%, and 1-2% of wealth to US, European, and UK stocks, respectively. The remaining 38% is invested in the domestic riskless asset. Independently of the initial state probability, the long-term investor chooses not to hold Japanese and Pacific stocks so these markets only play a role at the shortest investment horizons.

Finally, Figure 4 plots the first through the fourth moment of cumulated returns on the optimal portfolio as a function of the probability of being in state 2, the bull state. For comparison we also show the moments when portfolio weights are set at their international CAPM (ICAPM), i.e. when weights match the structure of the global market portfolio. When the bull state probability rises, expected returns go up and for a bull state probability exceeding 0.7 is higher under the ICAPM weights than under the optimal portfolio weights.

[24] Das and Uppal (2003) apply similar tests, although limited to own ($i = j = l = q$) skewness and kurtosis coefficients.

As the bull state probability goes from zero to one, the standard deviation of the optimal portfolio declines from 13% to 11%. These values are always below the standard deviation associated with the ICAPM weights. The skew has a U-shaped pattern, reaching its minimum at a bull state probability of 0.7. The skew is always larger−and at worst mildly negative−under the optimal portfolio weights than under the ICAPM weights, where it can take relative large negative values. Kurtosis increases slowly as the bull state probability grows from zero to 0.8 and declines thereafter. It hovers around 3.5 under the optimal portfolio weights and around 5 under the ICAPM weights. Along almost all dimensions the optimal portfolio improves on the ICAPM weights.

5. **Predictability from the Short US Interest Rate**

Many papers have studied stock holdings under return predictability from variables such as dividend yields or short-term interest rates. In particular, a number of studies have considered the leading role of US monetary policy in determining international interest rates as a motivation for models where the short-term US interest rate is a predictor of stock returns across international equity markets (see Obstfeld and Rogoff (1995) for the micro-foundations of such models, Canova and De Nicolo' (2000) and Kim (2001) for corroborating empirical evidence).[25] Following this literature, we extend our model to include the short-term US interest rate (r_{bt}^{US}) as an additional state variable:

$$\mathbf{y}_t = \boldsymbol{\mu}_{s_t} + \sum_{j=1}^{p} \mathbf{A}_{j,s_t}\mathbf{y}_{t-j} + \boldsymbol{\varepsilon}_t, \tag{23}$$

where $\mathbf{y}_t = (\mathbf{r}_t \; r_{bt}^{US})'$ and $\boldsymbol{\varepsilon}_t \sim N(0, \boldsymbol{\Omega}_{s_t})$. The short US interest rate serves not just as the risk-free rate for US investors but also as a state variable that predicts stock returns across international markets − a specification also adopted by Ang and Bekaert (2002a). Panel A of Table 6 presents results from estimating a single-state ($k = 1$) VAR(1) model. Interestingly, in a single-state framework the effect from US short term rates on international equity returns is weak as evidenced by the insignificant regression coefficients of the lagged T-bill rate.

Panel B of Table 6 reports estimates of (23) for a two-state specification.[26] Many of the estimates on the lagged US short rate now become statistically significant, especially in the second state[27] where increases in the short-term US rate (signalling a more restrictive monetary policy) are associated with negative and economically large effects on the conditional mean in four out of five stock markets. Conversely, in state 1 the coefficients of the lagged T-bill rate in the return equations are generally insignificant.

[25] Through a variety of vector-autoregressive identification schemes, Kim (2001) documents that US monetary expansion has a positive international spillover effect that occurs through a reduction of world interest rates, thus stimulating aggregate demand abroad. Additionally, Grilli and Roubini (1996) find evidence that the monetary policy of non-US G-7 countries follows US policy shifts. Conover et al. (1999) and Canova and De Nicolo' (2000) show that innovations in US interest rates have real effects on rates in other countries, while conversely US rates are largely unaffected by shocks to foreign interest rates.

[26] Ang and Bekaert (2002a) estimate a model with time-varying transition probabilities where the short rate follows a discretized square root process. Our model implies that the short rate follows a regime switching process with heteroskedastic shocks.

[27] In the extended model the states have to accommodate dynamics both in equity returns and in the short T-bill rate so there might be benefits from expanding the number of states from two to three. To explore this issue, we estimated a three-state model. However, we found that the Schwartz information criterion strongly penalized the resulting model due to the larger number of parameters so a two-state model continued to be preferred.

Extending our model to include the short US T-bill rate changes the underlying states since the model now has to capture dynamics both in equity returns and in short US interest rates. With probabilities of remaining in states one and two equal to 0.95 and 0.86 the new states remain persistent with implied durations of 18 and 7 months, respectively. This point comes out clearly in Figure 5 which plots the smoothed probabilities. This state is clearly correlated with fluctuations in US interest rates and captures many periods in the 1970s and early 1980s, the bear market of the early 1990s and the recent downturn commencing in 2000. It also captures periods with higher stock returns such as the long interval 1993-2000 marked by low and stable interest rates. In fact, the US interest rate is far more volatile in state 2 with an annualized volatility of 0.48% compared with only 0.17% in state 1.[28]

Table 6 also reveals that pair-wise correlations of stock returns are generally higher in state 1, where they lie in the range 0.3-0.7, than in state 2, where they lie between 0.2 and 0.5. Interestingly, shocks to the short rate and stock returns are essentially uncorrelated in regime 1, and negatively correlated in state 2. These findings for the correlation estimates are similar to those we found for the equity return model reported in Table 3. However, the extended model no longer generates systematic differences across states in the volatility of stock returns.

To calculate the mean return for each state, we used the smoothed state probabilities $\{\Pr(S_t = i|\mathcal{F}_T)\}_{i=1}^{2}$ in Figure 5 as follows:

$$E[\mathbf{y}_t|S_t = i, \mathcal{F}_{t-1}] = \frac{1}{\sum_{t=2}^{T} \Pr(S_t = i|\mathcal{F}_T)} \cdot \sum_{t=2}^{T} \left(\hat{\boldsymbol{\mu}}_i + \hat{\mathbf{A}}_i \mathbf{y}_{t-1}\right) \Pr(S_t = i|\mathcal{F}_T).$$

The estimated mean returns in the two states were

$$\begin{array}{ll} \text{state 1} & E[\mathbf{y}_t|S_t = 1, \mathcal{F}_{t-1}] = (0.008\ 0.004\ 0.007\ 0.010\ 0.008\ 0.004)' \\ \text{state 2} & E[\mathbf{y}_t|S_t = 2, \mathcal{F}_{t-1}] = (0.016\ 0.018\ 0.010\ 0.006\ 0.020\ 0.007)' \end{array}.$$

With an exception for the UK stock market, mean returns in state 1 are between one- and two-thirds of their values in state 2. This ranking applies also to the US T-bill rate, the annualized yield of which is 5.2% in state 1 and 8.3% in state 2. State 1 therefore continues to be characterized by low mean returns, while state 2 sees higher stock returns and higher interest rates.

Interpretation of the states identified by the model fitted to the extended asset menu is complicated by the fact that it balances a variety of forces that played a role during our sample. Reduced and stable global inflation rates drove down nominal interest rates and asset returns during the second half of the sample. At the same time bear markets emerged in some regions (Japan in particular). Although the US stock market experienced high mean returns throughout the 1990s (0.012 per month), these were only marginally above the full-sample estimate (0.011 per month). Mean returns in the other regions were in fact lower during the 1990s (negative in the case of Japan) than during the earlier part of our sample, 1975-1989. Returns were most strongly correlated across markets in the recent period, 2001-2003, which saw negative mean returns in all the markets considered here.

5.1. *Asset Allocation Results*

Table 7 reports the optimal asset allocation for the two-state model (23) that incorporates predictability from the US short rate. Since this model includes the US interest rate as an additional state variable, we

[28] Our finding of a state with volatile but not very persistent interest rates and a state with greater persistence but low volatility matches earlier findings by Gray (1996) and Ang and Bekaert (2002a).

show the optimal portfolio allocation as a function both of the probability of the bull state (state 2) and as a function of the initial value of the US interest rate which we vary from its mean plus or minus two standard deviations. We consider both a short and a long investment horizon, $T = 1, 24$.

At low levels of the US interest rate, nothing gets invested in US T-bills, but the allocation to this asset rises steadily as US interest rates increase. When the US T-bill rate is very low, a rise in this rate leads to a much higher investment in US equities that peaks for values of the interest rate close to its mean. The relationship is not monotonic, however, as further rises in the interest rate lead to a modest decline in the allocation to US stocks as a result of the increased allocation to US T-bills.

Looking across states, for a low initial interest rate the short-term allocation to US equities is highest in the bull state and generally much lower in the bear state. When the T-bill rate is high the allocation to US stocks is very similar across states. In contrast, at the long investment horizon ($T = 24$) allocations tend to be quite similar across initial regime probabilities unless the initial interest rate is very low.

Setting the initial interest rate at its sample average and starting either from the bear state or from steady state probabilities, the optimal short-term portfolio is dominated by US assets. Roughly 70% is held in US stocks and the remaining 30% is held in US T-bills. Starting from the bull state, as much as 80% of the portfolio gets invested in US stocks while 20% is held in European stocks. More diversified portfolios are observed at longer investment horizons where 74% gets invested in US stocks, with 10% going into UK as well as European stocks and slightly less to Pacific stocks.

5.2. *Regimes, Higher Order Moments and Return Predictability*

The effect of regimes on the optimal asset allocation continues to be very large under predictability from the short US interest rate. In the absence of regimes−i.e., under the VAR(1) model−Table 8 shows that the allocation to US stocks is only 37% at the one-month horizon and 43% at the 24 month horizon when the US interest rate is set at its mean. This compares to 73% and 74% under regime switching when the initial state probability is set at its steady state value. When predictability is ignored and a simple IID model is fitted to the data and employed for asset allocation purposes, these (myopic) weights strongly depart from the optimal choices under both regime switching and/or a VAR(1) model: Only 27% of the portfolio is invested in US stocks, 36% in European stocks, and 37% in US T-bills. This result differs from Ang and Bekaert's (2002a) who do not report large differences between myopic and (long run, $T = 24$ months) regime-switching weights using a much smaller asset menu than ours. Myopic weights that reveal large gains from international portfolio diversification are in line with the standard results in papers like Grauer and Hakansson (1987) and Lewis (1999).

The analysis in Section 4 found that, in the absence of a risk-free asset, the introduction of skew and kurtosis effects led to a reduction in the allocation to US stocks. Once a risk-free asset is present, skew and kurtosis have very little effect on allocations. In contrast, the results in the extended model with predictability from a stochastic short US interest rate show that the allocation to US stocks becomes higher under skew and kurtosis effects.

To see why this happens, notice that the optimal portfolio in the extended model is dominated by US stocks and US T-bills. This means that the (co-) skew and kurtosis properties of US stocks that becomes most relevant is based on their joint moments with the US T-bill rate. Stock market portfolios are attractive to US investors if they yield a high Sharpe ratio when combined with the short US T-bill rate and if they have desirable co-skew and co-kurtosis properties with the short rate. In fact, US stocks hedge volatility shocks to the domestic interest rate: $S_{US,TBill,TBill} = -0.05$. Furthermore, the T-bill rate provides a hedge against US equity skew as the correlation between cubed US stock returns and the US T-bill rate is negative, $S_{US,US,US,TBill} = -0.16$. This is important since the distribution of US stock returns is negatively skewed. Similarly, US stock returns provide a hedge against skews in the US T-bill rate, $S_{US,TBill,TBill,TBill} = -0.35$.

6. **Interpretation and Robustness of Results**

To summarize our results so far, the simplest scenario we considered assumed no predictability of returns and mean-variance preferences. We extended this model in three directions. First, by defining preferences over higher order moments such as skew and kurtosis. Second, by allowing for the presence of bull and bear regimes tracking periods with very different mean, variances and correlations between returns and, third, by introducing predictability of equity returns from the short US interest rate. In this section we decompose the results to interpret which effects matter most and investigate interactions between the three model extensions. We also consider the robustness of our results to alternative specifications of investor preferences.

6.1. *Decomposition of Results*

The very different asset allocations that emerge under the various asset menus and model specifications considered thus far suggest a complicated relationship between the effects of regimes in the joint distribution of asset returns, skew and kurtosis preferences and predictability from the short US interest rate. Of course, such effects may be correlated and difficult to differentiate. For example, extending the model to allow for predictability from the US T-bill rate changes the properties of the underlying states (compare Tables 3 and 6) and hence the effect of regime dynamics on the optimal asset allocation.

To review how these extensions affect a US investor's optimal asset allocation, Table 9 presents values of a home-country bias index (HCBI) (see e.g. Thomas et al. (2004)):

$$HCBI(\pi, T) = \frac{\omega^{US}(\pi, T) - \omega^{US}_{WORLD}}{1 - \omega^{US}_{WORLD}},$$

where $\omega^{US}(\pi, T)$ is the weight of US stocks as a proportion of the equity-only portfolio (i.e., excluding US T-bills) assuming an initial state probability of π and an investment horizon of T months, while ω^{US}_{WORLD} is the weight of the US stock market as a portion of the global market value, i.e. the ICAPM weight in a fully integrated global capital market. This index takes a positive value when the weight on the US stock market exceeds its market capitalization and is unity when only US stocks are held in the equity portfolio. Negative values reflect optimal portfolio holdings in US stocks below their weight in the global equity portfolio.

Consistent with the existence of a home country bias relative to the standard model, US investors with mean variance preferences who ignore regimes in returns hold less than the global equity weight in US shares ($HCBI$ equals -0.22). This continues to hold when investors care about skew and kurtosis although the index rises slightly to -0.06.

The introduction of bull and bear states overturns this result. The HCBI index still takes a small negative value in the bull state but is relatively large and positive (0.40) in the bear state under mean-variance preferences. Averaging across the two states by using the steady-state probabilities leads to a positive value of the HCBI index of 0.27 under mean-variance preferences and 0.32 under four-moment preferences. The presence of bull and bear regimes is thus able to generate home bias relative to the simple ICAPM benchmark.

Allowing for predictability from the US interest rate (Panel B) significantly increases the allocation to US stocks and thus raises the HCBI. Interestingly, however, the effect is quite small in the absence of regime switching dynamics in which case the index remains negative at -0.09 and -0.17 under mean-variance and four-moment preferences, respectively. The latter value is actually smaller (more negative) than in the absence of return predictability. Under mean-variance preferences, predictability from the short US rate and regime switching the HCBI becomes 0.31 when starting from steady state probabilities. This is only marginally higher than the value observed in Panel A in the absence of predictability from the US T-bill rate (0.27). Setting interest rates at their historical means, optimal short-term allocations to US stocks under mean-variance preferences, regime switching in returns and predictability from the short interest rate, were 0.66 (0.68) in the bear state, 0.51 (0.80) in the bull state and 0.68 (0.73) in the steady state. Numbers in parentheses show the corresponding values under four-moment preferences. Four-moment preferences generally lead to higher allocations to US stocks in this model, particularly in the bull state.

Once the joint effect of regimes, predictability of returns and four-moment preferences is considered, however, the HCBI increases significantly from its previously negative value in the bull state to a positive value of 0.63 and takes a value of one both in the bear state and under the ergodic state probabilities. This suggests that regime switching dynamics, four-moment preferences and predictability of returns from the short US interest rate all play a role in explaining the home country bias observed in US investors' asset allocation.

6.2. *The importance of State Classifications*

Comparing the results in sections 4 (under a pure-equity model) and 5 (under a mixed equity-T-bill model and predictability from the short rate) highlights some important differences in the optimal asset allocations. Under the simple model in Section 4, in steady state a US investor with skew-kurtosis preferences and a 1-month horizon should hold 41% of her wealth in the US stock market, 21% in European stocks, and 36% in US T-bills (the remaining 2% goes to UK equities). Using the more complicated model of Section 5, the same investor should hold 73% of her portfolio in US stocks and the remaining 27% in T-bills. The magnitude of this difference may appear surprising since, at the 1-month horizon the short US rate is known with certainty. Any differences between these results must therefore be due to differences in state classifications when the short US rate is excluded from the model (Table 4) compared to when it is included (Table 7).

Indeed, the pure-equity model estimated in Section 4 is a tightly parameterized two-state model with no autoregressive terms. In contrast, the mixed equity-short rate model in Section 5 is more complex with a VAR structure. This gives rise to two important differences between the models. First, the presence of autoregressive terms changes the predictive density of stock returns. Second, the underlying regimes implied by the two models are in fact very different. This is clear from the estimates of the transition probability parameters in Tables 3 and 6. For example, although the "bear" states share some similarities across the two models, the mean return estimates in Table 3 are more extreme than those in Table 6.

To separate the asset allocation effect of differences in state classifications from the effect of differences emerging from the autoregressive component, we performed the following exercise: we use the state probabilities from the simple model and the estimated transition probability matrix obtained in the last step of the EM algorithm employed in Section 3. These probabilities are then used to perform the last of the (feasible, two-step) GLS estimation leading to maximum likelihood estimates of intercepts, VAR, and covariance matrix coefficients in the model underlying Table 6 (c.f. Hamilton (1990)):

$$\begin{aligned} \widehat{\boldsymbol{\beta}} &= (\mathbf{Z}'\hat{\mathbf{W}}^{-1}\mathbf{Z})^{-1}\mathbf{Z}'\hat{\mathbf{W}}^{-1}(\mathbf{1}_k \otimes \mathbf{y}) \\ \hat{\boldsymbol{\Omega}}_s &= (\hat{\boldsymbol{\varepsilon}}_s'\tilde{\Xi}_s\hat{\boldsymbol{\varepsilon}}_s)/\sum_{t=2}^{T} p(s). \end{aligned} \tag{24}$$

Here $\widehat{\boldsymbol{\beta}}$ collects the unknown parameters, $\mathbf{y} \equiv [\mathbf{y}_2' \ \mathbf{y}_3' \ \ldots \ \mathbf{y}_T']'$, $\hat{\boldsymbol{\varepsilon}}_s \equiv [(\mathbf{y}_2 - \mathbf{Z}_{s_2=i}\widehat{\boldsymbol{\beta}})' \ (\mathbf{y}_3 - \mathbf{Z}_{s_3=i}\widehat{\boldsymbol{\beta}})' \ \ldots \ (\mathbf{y}_T - \mathbf{Z}_{s_T=i}\widehat{\boldsymbol{\beta}})']'$,

$$\underset{N(T-1)k \times kN(N+1)}{\mathbf{Z}} \equiv \begin{bmatrix} \mathbf{Z}_1 \\ \mathbf{Z}_2 \\ \vdots \\ \mathbf{Z}_k \end{bmatrix}, \quad \underset{N(T-1)\times kN(N+1)}{\mathbf{Z}_i} \equiv \begin{bmatrix} [\mathbf{e}_i' \ \mathbf{e}_i' \otimes \mathbf{y}_1'] \otimes \mathbf{I}_N \\ [\mathbf{e}_i' \ \mathbf{e}_i' \otimes \mathbf{y}_2'] \otimes \mathbf{I}_N \\ \vdots \\ [\mathbf{e}_i' \ \mathbf{e}_i' \otimes \mathbf{y}_{T-1}'] \otimes \mathbf{I}_N \end{bmatrix},$$

$$\underset{N(T-1)k \times N(T-1)k}{\hat{\mathbf{W}}^{-1}} \equiv \begin{bmatrix} \Xi_1 \otimes \hat{\boldsymbol{\Omega}}_1^{-1} & \cdots & \mathbf{O} \\ \vdots & \ddots & \vdots \\ \mathbf{O} & \cdots & \Xi_k \otimes \hat{\boldsymbol{\Omega}}_k^{-1} \end{bmatrix},$$

and $\boldsymbol{\Xi}_i \equiv diag\{p(s_2 = i), p(s_3 = i), ..., p(s_T = i)\}$ is a $(T-1) \times (T-1)$ matrix that collects on its main diagonal the smoothed state probabilities obtained from the estimation of the two-state model in Table 3. The result is a model based on the same state definition as the regime switching model in Section 3, which accommodates the VAR(1) structure and includes the US T-bill rate as an additional variable.

Table 10 shows optimal asset allocation results conditional on these state probabilities and parameter estimates. Unsurprisingly, the weights in this Table lie between their respective values in Table 4 (pure equity model) and those in Table 7 (mixed equity-short rate model). For instance, if we focus on the steady-state probabilities, set $T = 1$ month and fix the interest rate at its sample mean, 50% gets invested in US stocks (vs. 41% in Table 4 and 73% in Table 7), 11% goes to Continental Europe (vs. 21% in Table 4 and 0% in Table 7), and 39% is held in T-bills (vs. 36% in Table 4 and 27% in Table 7). The state classification underlying Table 4 is thus rather more favorable to short-term investments in European equities and less favorable to US stocks. The remaining differences between the two sets of allocations reflect differences in the predictive densities due to the conditioning on autoregressive terms in the mixed model from Section 5.

6.3. *Robustness Analysis*

We next investigate the robustness of our results to a range of assumptions made in the baseline scenario. As shown by Adler and Dumas (1983), the optimal weight on foreign stocks can be decomposed into two terms. The first term reflects the inverse of the investor's coefficient of risk aversion times the expected excess return on foreign over domestic stocks, weighted by the variance of this excess return. The second term is the global minimum variance portfolio. As the investor's risk aversion changes, so does the relative weights on these two portfolios and consequently the degree of international diversification.

We first consider the effect of changing the coefficient of risk aversion from $\theta = 2$ in the baseline scenario to values of $\theta = 5$ (high) and $\theta = 10$ (very high). Ang and Bekaert (2002a) and Das and Uppal (2003) have found that changes in risk aversion have first-order effects on their conclusions on the importance of either regime shifts or systemic (jump) risks. Table 11 shows the effect of such changes. As θ increases the allocation to both European and US equities declines and the allocation to the risk-free asset increases across all investment horizons. In the absence of predictability of returns the largest effect of θ on the asset allocation is seen when θ is changed from five to ten. In this case allocations to US stocks decline by 3-4%. When the short US rate is introduced as a predictor variable, the largest effect emerges when θ changes from two to five. This results in a decline of 10% in the allocation to US stocks and a commensurate increase in the allocation to T-bills.

To make our results comparable to those reported in the literature which assume power utility, we also compare results under four-moment preferences to those under constant relative risk aversion. Table 12 shows the results. In the bear state the allocation to US stocks is 2-4% higher under power utility while conversely the allocation to UK stocks tends to be lower. Hence differences between results computed under power utility and under four-moment preferences appear to be relatively minor. This makes our findings more comparable to existing results.

6.4. *Rebalancing*

To keep the analysis simple, so far we have ignored the possibility of portfolio rebalancing. In this section we relax this assumption and allow the investor to rebalance every $\varphi = \frac{T}{B}$ months at B equally spaced points t, $t+\frac{T}{B}$, $t+2\frac{T}{B}$, ..., $t+(B-1)\frac{T}{B}$. This requires determining the portfolio weights at the rebalancing times ω_b $(b = 0, 1, ..., B-1)$. Cumulated wealth can be factored out as a product of interim wealth at the rebalancing points:

$$W_{t+T} = \prod_{b=1}^{B} \frac{W_{t+\varphi b}(\omega_{b-1})}{W_{t+\varphi(b-1)}(\omega_{b-2})}, \tag{25}$$

where

$$\frac{W_{t+\varphi b}(\omega_{b-1})}{W_{t+\varphi(b-1)}(\omega_{b-2})} = \left\{(1-\boldsymbol{\omega}'_{b-1}\boldsymbol{\iota}_h)\exp\left(R^b_{\varphi(b-1)+1\to\varphi b}\right) + \boldsymbol{\omega}'_{b-1}\exp\left(\mathbf{R}^s_{\varphi(b-1)+1\to\varphi b}\right)\right\},$$

and $\mathbf{R}^s_{\varphi(b-1)+1\to\varphi b} \equiv \mathbf{r}^s_{t+\varphi(b-1)+1} + \mathbf{r}^s_{t+\varphi(b-1)+2} + ... + \mathbf{r}^s_{t+\varphi b}$, $R^b_{\varphi(b-1)+1\to\varphi b} \equiv r_{bt+\varphi(b-1)+1} + r_{bt+\varphi(b-1)+2} + ... + r_{bt+\varphi b}$ are the cumulated risky and riskless returns between periods$\varphi(b-1)+1$ and φb. By the law of iterated expectations, the following decomposition holds:

$$\begin{aligned}
M^{(n)}_{t+T} &= E_t[W^n_{t+T}] = E_t\left[\prod_{b=1}^{B}\left(\frac{W_{t+\varphi b}(\boldsymbol{\omega}_{b-1})}{W_{t+\varphi(b-1)}(\boldsymbol{\omega}_{b-2})}\right)^n\right] \\
&= E_t\left\{(W_{t+\varphi}(\boldsymbol{\omega}_t))^n E_{t+\varphi}\left[\left(\frac{W_{t+2\varphi}(\boldsymbol{\omega}_{t+\varphi})}{W_{t+\varphi}(\boldsymbol{\omega}_t)}\right)^n E_{t+2\varphi}\left(\left(\frac{W_{t+3\varphi}(\boldsymbol{\omega}_{t+2\varphi})}{W_{t+2\varphi}(\boldsymbol{\omega}_{t+\varphi})}\right)^n ...\right)\right]\right\} \\
&= M^{(n)}_{0\to\varphi}(\boldsymbol{\omega}_0)E_t\left\{M^{(n)}_{\varphi\to2\varphi}(\boldsymbol{\omega}_1)E_{t+\varphi}\left[M^{(n)}_{2\varphi\to3\varphi}(\boldsymbol{\omega}_2)E_{t+2\varphi}\left(M^{(n)}_{3\varphi\to4\varphi}(\boldsymbol{\omega}_3)...\right)\right]\right\}
\end{aligned} \tag{26}$$

where $E_{t+\varphi(b-1)}[\cdot]$ is shorthand notation for $E\left[\cdot|\mathcal{F}_{t+\varphi(b-1)}\right]$ and $M^{(n)}_{\varphi(b-1)\to\varphi b}(\boldsymbol{\omega}_{b-1})$ is the n-th (noncentral) moment of the cumulated portfolio returns between $t+\varphi(b-1)+1$ and $t+\varphi b$, calculated on the basis of time $t+\varphi(b-1)$ information:

$$\begin{aligned}
M^{(n)}_{\varphi(b-1)\to\varphi b}(\boldsymbol{\omega}_{b-1}) &\equiv E_{t+\varphi(b-1)}\left[\left(\frac{W_{t+\varphi b}(\boldsymbol{\omega}_{b-1})}{W_{t+\varphi(b-1)}(\boldsymbol{\omega}_{b-2})}\right)^n\right] \\
&= E_{t+\varphi(b-1)}\left[\left((1-\boldsymbol{\omega}'_{b-1}\boldsymbol{\iota}_h)\exp\left(R^b_{\varphi(b-1)+1\to\varphi b}\right) + \boldsymbol{\omega}'_{b-1}\exp\left(\mathbf{R}^s_{\varphi(b-1)+1\to\varphi b}\right)\right)^n\right].
\end{aligned}$$

The decomposition in (26) shows that future moments of wealth depend on future portfolio choices, $\boldsymbol{\omega}_b$.

We use the following recursive strategy to solve the asset allocation problem under m-moment preference functionals and rebalancing:

1. Solve the time $T-\varphi$ problem

$$\hat{\boldsymbol{\omega}}_{B-1} \equiv \arg\max_{\boldsymbol{\omega}_{B-1}} \sum_{n=0}^{m} \kappa_n(\boldsymbol{\theta}) \hat{E}_{T-\varphi}\left[M^{(n)}_{T-\varphi\rightarrow T}(\boldsymbol{\omega}_{B-1})\right].$$

   Here $\hat{E}_{T-\varphi}[\cdot]$ is shorthand notation for $E[\cdot|\mathcal{F}_{T-\varphi}]$ calculated on the basis of the filtered state probabilities for time $T-\varphi$.

2. Solve the time $T-2\varphi$ problem

$$\hat{\boldsymbol{\omega}}_{B-2} \equiv \arg\max_{\boldsymbol{\omega}_{B-2}} \sum_{n=0}^{m} \hat{E}_{T-2\varphi}\left[\lambda_n^{B-1}(\boldsymbol{\theta}) M^{(n)}_{T-2\varphi\rightarrow T-\varphi}(\boldsymbol{\omega}_{B-2})\right],$$

   where $\lambda_n^{B-1}(\boldsymbol{\theta}) \equiv \kappa_n(\boldsymbol{\theta})\hat{E}_{T-\varphi}[M^{(n)}_{T-\varphi\rightarrow T}(\hat{\boldsymbol{\omega}}_{B-1})]$ and $\hat{E}_{T-\varphi}[M^{(n)}_{T-\varphi\rightarrow T}(\hat{\boldsymbol{\omega}}_{B-1})]$ is the n-th noncentral moment of the optimal wealth process calculated under the solution found in 1.[29]

3. Solve the problem backward by iterating on steps 1 and 2 up to time $t+\varphi$, to generate a sequence of optimal portfolio choices $\{\hat{\boldsymbol{\omega}}_i\}_{i=1}^{B-1}$. The optimal time t asset allocation, $\boldsymbol{\omega}_0 \equiv \omega_t$, is then found by solving

$$\boldsymbol{\omega}_0 \equiv \arg\max_{\boldsymbol{\omega}_0} \sum_{n=0}^{m} \hat{E}_t\left[\lambda_n^1(\boldsymbol{\theta}) M^{(n)}_{t\rightarrow t+\varphi}(\boldsymbol{\omega}_0)\right]$$

   where

$$\lambda_n^1(\boldsymbol{\theta}) \equiv \kappa_n(\boldsymbol{\theta})\hat{E}_{t+\varphi}[M^{(n)}_{t+\varphi\rightarrow t+2\varphi}(\boldsymbol{\omega}_b)]. \tag{27}$$

In practice, this algorithm replaces a complex multiperiod program with a sequence of simpler, buy-and-hold portfolio choice problems (each with horizon φ) in which the original moment coefficients $\{\kappa_n(\boldsymbol{\theta})\}_{n=0}^m$ are recursively replaced with products of (random) variables representing the conditional moments of future wealth weighted by the corresponding coefficients in the preference functional (18).

Table 13 reports optimal portfolio weights in the presence of predictability from the US short rate. To simplify the analysis we report results only for two investment horizons ($T = 6$ and 24 months) and a few rebalancing frequencies ($\varphi = 1$, 3, 6, 12 months and $\varphi = T$, the buy-and-hold benchmark of Table 7). In these simulations, the US interest rate is set at its (regime-specific) mean. As already noted in the literature (c.f., Brandt (1999) and Guidolin and Timmermann (2004)), rebalancing opportunities give investors incentives to exploit current information more aggressively. This effect is stronger when rebalancing occurs more frequently, i.e. when φ is small. Stock allocations under rebalancing are large and always exceed two-thirds of current wealth. Starting from state 1, very frequent rebalancing ($\varphi = 1$ and 3 months) reduces the allocation to US stocks, while Pacific stocks emerge as an attractive investment. However, when the economy starts from the bull state or when the initial state is unknown and steady-state probabilities are used, a large bias towards US stocks emerges for all possible values of φ. In fact, under frequent rebalancing a

[29]Maximizing $\hat{E}_{T-2\varphi}[\lambda_n^{B-1}(\boldsymbol{\theta})\ M^{(n)}_{T-2\varphi\rightarrow T-\varphi}(\boldsymbol{\omega}_{B-2})]$ implies that the conditional correlation between optimal wealth at time $T-2\varphi$ and portfolio returns between $T-\varphi$ and T affects portfolio weights.

US investor with four-moment preferences should hold even more in US securities than under no rebalancing. For example, for $T = 24$, 100% of wealth goes into domestic securities, comprising between 70% and 80% in stocks. Furthermore, under the steady-state probabilities the optimal portfolio essentially consists of US stocks for $\varphi \leq 3$ and all values of T. Starting from state 2 and assuming a moderately long rebalancing frequency (6 months and longer), a considerable bias towards US stocks emerges. All told, regime switching combined with preferences that reflect aversion against fat tails and negative skew seem to explain the home bias under a range of assumptions about the rebalancing frequency.

7. Conclusion

This paper proposed a new method for deriving optimal portfolio weights when investors' preferences can be represented through a finite order polynomial and the return distribution follows a flexible Markov switching process that can be extended to allow for predictability from state variables such as the short interest rate. We showed how to characterize the mean, variance, skew and kurtosis (as well as other moments of arbitrarily high order) of the wealth distribution in the form of solutions to simple difference equations. When coupled with a utility specification that incorporates skew and kurtosis preferences, our method greatly reduces the otherwise numerically challenging problem of solving for the optimal asset allocation in the presence of a large number of risky assets.

We found several interesting results. If time-variations in the joint distribution of international stock market returns are ignored, the (co-) skew and (co-) kurtosis properties of US stock returns do not help to explain US investors' home bias in the absence of a risk-free rate. In fact they exacerbate this bias as US stocks have undesirable (co-)skew properties. Allowing for predictability of returns through the short US interest rate and introducing a risk-free rate changes the optimal asset allocation significantly and brings it closer in line with actual portfolio holdings. Skew and kurtosis effects now lead to an increase in the allocation to US stocks due to their desirable co-skew properties with the US interest rate.

De Santis and Gerard (1997), Lewis (1999) and others have reported high estimates for the utility costs incurred by US investors due to their apparent failure to hold internationally diversified equity portfolios. Our paper shows that under plausible assumptions on investor preferences and accounting for the distinctly non-normal properties of equity return distributions, these costs may not be as high as previously thought.

Several extensions would be of interest in future work. Most obviously, the asset menu could be extended to include other asset classes such as domestic and foreign long-term bonds. This is consistent with the fact that most of the finance literature on the home bias has mainly focussed on equity holdings. However, some papers (e.g. Campbell, Chan, and Viceira (2003) and Guidolin and Timmermann (2004)) have shown that predictability and regime shifts have important implications for the allocation of wealth across bonds and stocks. Furthermore, recent studies have considered the effects of systemic jumps and event risks on optimal portfolio choices, c.f. Das and Uppal (2003) and Liu, Longstaff, and Pan (2003). Our framework with regime shifts have implications that appear to be consistent with their results on gains from international portfolio diversification and this represents an interesting extension for future work.

Appendix

This appendix derives Proposition 1 and shows how to extend the results to include autoregressive terms in the return process. To derive the n-th moment of the cumulated return on the risky asset holdings in the general case with multiple assets (h) and states (k), notice that

$$E_t\left[\left(\boldsymbol{\omega}_t'\exp\left(\mathbf{R}_{t+T}^s\right)\right)^n\right] = E_t\left[\sum_{n_1=1}^{n}...\sum_{n_h=1}^{n}\lambda(n_1,n_2,...,n_h)(\omega_1^{n_1}\times..\times\omega_h^{n_h})\times \exp\left(\sum_{i=1}^{T}r_{1t+i}\right)^{n_1}....\times\exp\left(\sum_{i=1}^{T}r_{ht+i}\right)^{n_h}\right]. \quad \text{(A1)}$$

where the powers $0\leq n_i\leq n$ $(i=1,...,h)$ satisfy the summing-up constraint $\sum_{i=1}^{h}n_i=n$ and the coefficients λ are given by

$$\lambda(n_1,n_2,...,n_h)\equiv\frac{n!}{n_1!n_2!\ ...\ n_h!}.$$

The sum in (A1) involves $\binom{h+n-1}{n}$ terms and requires solving for moments of the form

$$\begin{aligned}M_{t+T}^{(n)}(n_1,n_2,...,n_h) &= E_t\left[\exp\left(\sum_{i=1}^{T}r_{1t+i}\right)^{n_1}\times....\times\exp\left(\sum_{i=1}^{T}r_{ht+i}\right)^{n_h}\right]\\ &= E_t\left[\exp\left(\sum_{l=1}^{h}n_l\sum_{i=1}^{T}r_{l,t+i}\right)\right]. \quad \text{(A2)}\end{aligned}$$

(A2) can be decomposed as follows

$$M_{t+T}^{(n)}(n_1,n_2,...,n_h)=\sum_{i=1}^{k}M_{i,t+T}^{(n)}(n_1,n_2,...,n_h), \quad \text{(A3)}$$

where for $i=1,...,k$,

$$M_{i,t+T}^{(n)}(n_1,n_2,...,n_h)=E_t\left[\exp\left(\sum_{l=1}^{h}n_l\sum_{i=1}^{T}r_{lt+i}\right)|s_{t+T}=i\right]\Pr(s_{t+T}=i).$$

Each of these terms satisfies the recursions

$$\begin{aligned}M_{i,t+T}^{(n)}(n_1,n_2,...,n_h) &= \sum_{g=1}^{k}M_{g,t+T-1}^{(n)}(n_1,n_2,...,n_h)E_t\left[\exp\left(\sum_{l=1}^{h}n_l r_{lt+T}\right)|s_{t+T}=i,\mathcal{F}_t\right]p_{gi}\\ &= \sum_{g=1}^{k}p_{gi}M_{g,t+T-1}^{(n)}(n_1,n_2,...,n_h)\exp\left(\sum_{l=1}^{h}n_l\mu_{il}+\sum_{l=1}^{h}\sum_{u=1}^{h}n_l n_u\frac{\sigma_{i,lu}}{2}\right). \quad \text{(A4)}\end{aligned}$$

where μ_{il} is the mean return of asset l in state i and $\sigma_{i,lu}=e_l'\Omega_i e_u$ is the covariance between r_{lt+T} and r_{ut+T} in state $i=1,\ 2,...,\ k$. This is a generalization of the result in (12).

Finally, using (A1) and (A2), we get an expression for the $n-$th moment of the cumulated return:

$$E_t\left[\left(\boldsymbol{\omega}_t'\exp\left(\mathbf{R}_{t+T}^s\right)\right)^n\right]=\sum_{n_1=0}^{n}\cdots\sum_{n_h=0}^{n}\lambda(n_1,n_2,...,n_h)(\omega_1^{n_1}\times...\times\omega_n^{n_h})M_{t+T}^{(n)}(n_1,...,n_h). \quad \text{(A5)}$$

Expected utility can now be evaluated in a straightforward generalization of (16):

$$\begin{aligned}\hat{E}_t[U^m(W_{t+T};\boldsymbol{\theta})] &= \sum_{n=0}^{m}\kappa_n\sum_{j=0}^{n}(-1)^{n-j}v_T^{n-j}{}_nC_jE_t[W_{t+T}^j]\\ &= \sum_{n=0}^{m}\kappa_n\sum_{j=0}^{n}(-1)^{n-j}v_T^{n-j}\binom{n}{j}\sum_{i=0}^{j}\binom{j}{i}E_t[(\boldsymbol{\omega}_t'\exp\left(\mathbf{R}_{t+T}^s\right))^i]\left((1-\boldsymbol{\omega}_t'\boldsymbol{\iota}_h)\exp\left(Tr^f\right)\right)^{j-i}.\end{aligned}$$

Inserting (A5) into this expression gives a first order condition that takes the form of an $m-1$th order polynomial in the portfolio weights.

The generalization of the results to include autoregressive terms is straightforward. To keep the notation simple, suppose $k=2$. Using (7) the n-th noncentral moment satisfies the recursions

$$\begin{aligned} M_{i,t+T}^{(n)} &= M_{i,t+T-1}^{(n)}(n) p_{ii} \exp\left(n\mu_i + n\sum_{j=1}^{p} a_{j,i} E_t[r_{t+T-j}] + \frac{n^2}{2}\sigma_i^2\right) + \\ & + M_{-i,t+T-1}^{(n)}(n)(1-p_{-i-i}) \exp\left(n\mu_i + n\sum_{j=1}^{p} a_{j,i} E_t[r_{t+T-j}] + \frac{n^2}{2}\sigma_i^2\right) \end{aligned}$$

or

$$\begin{aligned} M_{1,t+1}^{(n)} &= \tilde{\alpha}_1^{(n)} M_{1,t}^{(n)} + \tilde{\beta}_1^{(n)} M_{2,t}^{(n)} \\ M_{2,t+1}^{(n)} &= \tilde{\alpha}_2^{(n)} M_{1,t} + \tilde{\beta}_2^{(n)} M_{2,t}^{(n)}, \end{aligned}$$

where now

$$\begin{aligned} \tilde{\alpha}_1^{(n)} &= p_{11} \exp\left(n\mu_1 + n\sum_{j=1}^{p} a_{j,1} E_t[r_{t+T-j}] + \frac{n^2}{2}\sigma_1^2\right) \\ \tilde{\beta}_1^{(n)} &= (1-p_{22}) \exp\left(n\mu_1 + n\sum_{j=1}^{p} a_{j,1} E_t[r_{t+T-j}] + \frac{n^2}{2}\sigma_1^2\right) \end{aligned}$$

$$\begin{aligned} \tilde{\alpha}_2^{(n)} &= (1-p_{11}) \exp\left(n\mu_2 + n\sum_{j=1}^{p} a_{j,2} E_t[r_{t+T-j}] + \frac{n^2}{2}\sigma_2^2\right) \\ \tilde{\beta}_2^{(n)} &= p_{22} \exp\left(n\mu_2 + n\sum_{j=1}^{p} a_{j,2} E_t[r_{t+T-j}] + \frac{n^2}{2}\sigma_2^2\right). \end{aligned}$$

Subject to these changes, the earlier methods can be used with the only difference that terms such as $\exp\left(n\mu_i + \frac{n^2}{2}\sigma_i^2\right)$ have to be replaced by

$$\exp\left(n\mu_1 + n\sum_{j=1}^{p} a_{j,i} E_t[r_{t+T-j}] + \frac{n^2}{2}\sigma_1^2\right).$$

The term $\sum_{j=1}^{p} a_{j,i} E_t[r_{t+T-j}]$ may be decomposed in the following way:

$$\sum_{j=1}^{p} a_{j,i} E_t[r_{t+T-j}] = I_{\{j>T\}} \sum_{j=1}^{p} \left(I_{\{j\geq T\}} a_{j,i} r_{t+T-j} + I_{\{j<T\}} a_{j,i} E_t[r_{t+T-j}]\right),$$

where $E_t[r_{t+1}], ..., E_t[r_{t+T-1}]$ can be evaluated recursively, c.f. Doan et al. (1984):

$$\begin{aligned}
E_t[r_{t+1}] &= \pi_{1t}\left(\mu_1 + \sum_{j=1}^{p} a_{j,1} r_{t-j}\right) + (1-\pi_{1t})\left(\mu_2 + \sum_{j=1}^{p} a_{j,2} r_{t-j}\right) \\
E_t[r_{t+2}] &= \boldsymbol{\pi}_t' \mathbf{P} \mathbf{e}_1 \left(\mu_1 + \sum_{j=1}^{p} a_{j,1} E_t[r_{t+1}]\right) + (1-\boldsymbol{\pi}_t' \mathbf{P} \mathbf{e}_1)\left(\mu_2 + \sum_{j=1}^{p} a_{j,2} E_t[r_{t+1}]\right) \\
& \ldots \\
E_t[r_{t+T-1}] &= \boldsymbol{\pi}_t' \mathbf{P}^{T-1} \mathbf{e}_1 \left(\mu_1 + \sum_{j=1}^{p} a_{j,1} E_t[r_{t+T-2}]\right) + (1-\boldsymbol{\pi}_t' \mathbf{P}^{T-1} \mathbf{e}_1)\left(\mu_2 + \sum_{j=1}^{p} a_{j,2} E_t[r_{t+T-2}]\right).
\end{aligned}$$

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Table 1

Summary Statistics for International Stock Returns and 1-month US T-bill Yields

The table reports basic moments for monthly MSCI total stock return series (including dividends, adjusted for stock splits, etc.) for a few international aggregate portfolios. The last row concerns the yield of 1-month US Treasury Bill (source: CRSP). The sample period is 1975:01 – 2003:12. All returns are expressed in US dollars currencies. ρ_1 denotes the first-order autocorrelation coefficient.

Portfolio	Mean	St. Dev.	Skewness	Kurtosis	Jarque-Bera	ρ_1
MSCI United States	0.0106	0.0446	-0.7023	5.7260	136.36**	0.006
MSCI Japan	0.0082	0.0649	0.0736	3.4636	3.430	0.073
MSCI Pacific ex-Japan	0.0083	0.0693	-2.2124	20.9027	4,931.3**	0.042
MSCI United Kingdom	0.0088	0.0514	-0.5406	4.4772	48.593**	0.061
MSCI Europe ex-UK	0.0125	0.0636	0.7781	9.8693	719.34**	0.048
US 1-month T-bills	0.0051	0.0025	0.9874	4.4959	89.005**	0.932

* denotes 5% significance, ** significance at 1%.

Table 2

Density Specification Tests for Regime Switching Models

This table reports tests for the transformed z-scores generated by multivariate regime-switching models

$$\mathbf{r}_t = \boldsymbol{\mu}_{s_t} + \sum_{j=1}^{p} A_{js_t}\mathbf{r}_{t-j} + \Sigma_{s_t}\,\boldsymbol{\varepsilon}_t$$

where $\mathbf{r}_t$ are monthly nominal MSCI stock index returns (in US dollars) for the US, Japan, Pacific (ex-Japan), the United Kingdom, and Europe (ex-UK). $\boldsymbol{\mu}_{s_t}$ is the intercept vector in state s_t, A_{js_t} is the matrix of autoregressive coefficients at lag $j \geq 1$ in state s_t and $\boldsymbol{\varepsilon}_t \sim$ I.I.D. $N(0,\mathbf{I}_5)$. s_t is governed by an unobservable, first-order Markov chain that can assume k distinct values (states). The sample period is 1975:01 – 2003:12. The tests are based on the principle that under the null of correct specification of the model, the probability integral transform of the one-step-ahead standardized forecast errors should follow an IID uniform distribution over the interval (0,1). A further Gaussian transform described in Berkowitz (2001) is applied to perform Likelihood ratio tests of the null that (under correct specification) the transformed z-scores, z^*_{t+1}, are IIN(0,1) distributed. In particular, given the transformed z-score model

$$z^*_{t+1} = \mu + \sum_{j=1}^{p}\sum_{i=1}^{l} \rho_{ij}(z^*_{t+1-i})^j + \sigma e_{t+1},$$

LR_2 tests the hypothesis of zero mean and unit variance under the restriction $p = l = 0$; LR_3 tests the joint hypothesis of zero mean, unit variance, and $\rho_{11}= 0$ under $p = l = 1$; LR6 tests the joint null of zero mean, unit variance, and $\rho_{11}= \rho_{12}= \rho_{21}= \rho_{22}= 0$ with $p = l = 2$.

Model	Jarque-Bera test	LR_2	LR_3	LR_6
MSCI US total equity returns				
Linear	136.369 (0.000)	**0.000 (1.000)**	8.210 (0.042)	17.442 (0.008)
Two-state	**1.688 (0.430)**	**0.662 (0.718)**	**5.378 (0.146)**	**10.610 (0.101)**
MSCI Japan total equity returns (in USD)				
Linear	**3.431 (0.000)**	**0.001 (1.000)**	**4.116 (0.249)**	**10.724 (0.097)**
Two-state	**5.928 (0.052)**	**0.386 (0.824)**	**5.940 (0.115)**	**9.690 (0.138)**
MSCI Pacific ex-Japan total equity returns (in USD)				
Linear	4,931.1 (0.000)	**0.002 (0.999)**	11.970 (0.007)	17.048 (0.009)
Two-state	222.88 (0.000)	**1.804 (0.406)**	**5.168 (0.160)**	**9.264 (0.159)**
MSCI United Kingdom total equity returns (in USD)				
Linear	48.589 (0.000)	**0.000 (1.000)**	10.686 (0.014)	17.500 (0.008)
Two-state	**0.189 (0.910)**	**0.788 (0.674)**	**7.016 (0.071)**	**12.134 (0.059)**
MSCI Europe ex-UK total equity returns (in USD)				
Linear	719.27 (0.000)	**0.002 (0.999)**	53.088 (0.000)	75.228 (0.000)
Two-state	47.528 (0.000)	**3.676 (0.159)**	13.956 (0.003)	31.304 (0.000)

Table 3

Estimates of a Two-State Switching Model

This table reports maximum likelihood estimates for a single state model and a two-state regime switching model (MSIH(2,0)) fitted to monthly nominal MSCI stock index returns (in US dollars) for the US, Japan, Pacific (ex-Japan), the United Kingdom, and Europe (ex-UK). The regime switching model takes the form:

$$\mathbf{r}_t = \boldsymbol{\mu}_{s_t} + \Sigma_{s_t}\boldsymbol{\varepsilon}_t,$$

where $\boldsymbol{\mu}_{s_t}$ is the intercept in state s_t and $\boldsymbol{\varepsilon}_t \sim$ I.I.D. $N(0, \mathbf{I}_5)$ is an unpredictable return innovation. The sample period is 1975:01 – 2003:12. For second moments, we report on the main diagonal monthly volatilities, and correlations off the diagonals.

Panel A – Single State Model					
	U.S.	Japan	Pacific ex-Japan	United Kingdom	Europe ex-UK
1. Mean excess return	0.0106	0.0082	0.0083	0.0088	0.0125
2. Correlations/Volatilities					
US	0.0445***				
Japan	0.3139**	0.0648***			
Pacific ex-Japan	0.5471***	0.3673**	0.0692***		
United Kingdom	0.5908***	0.4810***	0.5452***	0.0513***	
Europe ex-UK	0.5419***	0.3876**	0.5656***	0.6331***	0.0635***

Panel B – Two State Model					
	U.S.	Japan	Pacific ex-Japan	United Kingdom	Europe ex-UK
1. Mean excess return					
Bear State	-0.0024	-0.0257**	-0.0117	-0.0112	-0.0040
Bull State	0.0189***	0.0188***	0.0145***	0.0151***	0.0178***
2. Correlations/Volatilities					
Bear state:					
US	0.0591***				
Japan	0.3708**	0.0604***			
Pacific ex-Japan	0.6331***	0.4726***	0.1064***		
United Kingdom	0.7709***	0.5330***	0.6783***	0.0697***	
Europe ex-UK	0.7862***	0.3864**	0.7127***	0.8596***	0.0564***
Bull state:					
US	0.0373***				
Japan	0.2308*	0.0625***			
Pacific ex-Japan	0.4362**	0.2941*	0.0507***		
United Kingdom	0.4059**	0.4255**	0.3748**	0.0421***	
Europe ex-UK	0.4453**	0.3590**	0.5442***	0.5556***	0.0647***

3. Transition probabilities	Bear State	Bull State
Bear State	0.8304***	0.1696
Bull State	0.0529	0.9471***

* denotes 10% significance, ** significance at 5%, *** significance at 1%.

Table 4

Effects of the order *m* on Optimal Portfolio Choices

The table reports the optimal allocation to international stocks as a function of perceived probability of a bull regime (regime 2) for three choices of *m*: *m*=2 (mean-variance preferences), *m*=3 (three-moment preference functional), and *m*=4 (four-moment functional). The coefficients of the objective function are evaluated by interpreting the objective as a Taylor approximation (around v_T) to power utility with constant relative risk aversion equal to θ=2.

		All-Equity Allocation					**Allocation w/Conditionally Riskless Asset**					
	m	U.S.	Japan	Pacific ex-Jpn	UK	EU	U.S.	Japan	Pacific ex-Jpn	UK	EU	US 1m T-bills
Single state model (Gaussian IID)												
	All values	0.27	0.18	0.17	0.19	0.19	0.27	0.00	0.00	0.00	0.36	0.37
Bear state (π = 0)												
T=1	*m* = 2	0.27	0.00	0.18	0.00	0.55	0.48	0.00	0.00	0.02	0.23	0.27
	m = 3	0.23	0.00	0.19	0.13	0.45	0.45	0.00	0.00	0.00	0.22	0.33
	m = 4	0.35	0.07	0.17	0.15	0.26	0.43	0.00	0.04	0.00	0.25	0.28
T=6	*m* = 2	0.29	0.00	0.19	0.00	0.52	0.46	0.00	0.00	0.03	0.25	0.26
	m = 3	0.15	0.00	0.17	0.17	0.51	0.44	0.00	0.00	0.00	0.23	0.33
	m = 4	0.12	0.00	0.02	0.29	0.57	0.41	0.00	0.00	0.00	0.23	0.36
T=24	*m* = 2	0.33	0.00	0.20	0.00	0.47	0.45	0.00	0.00	0.04	0.28	0.23
	m = 3	0.05	0.00	0.00	0.28	0.67	0.44	0.00	0.00	0.00	0.23	0.33
	m = 4	0.04	0.00	0.00	0.32	0.64	0.40	0.00	0.00	0.01	0.20	0.39
Ergodic state probs. (π = 0.76)												
T=1	*m* = 2	0.21	0.17	0.13	0.00	0.49	0.43	0.00	0.00	0.00	0.28	0.29
	m = 3	0.00	0.00	0.00	0.32	0.68	0.42	0.00	0.00	0.00	0.25	0.33
	m = 4	0.00	0.00	0.00	0.33	0.67	0.41	0.00	0.00	0.02	0.21	0.36
T=6	*m* = 2	0.24	0.15	0.13	0.00	0.48	0.44	0.00	0.00	0.00	0.28	0.28
	m = 3	0.00	0.00	0.00	0.32	0.68	0.43	0.00	0.00	0.00	0.24	0.33
	m = 4	0.00	0.00	0.00	0.33	0.67	0.41	0.00	0.00	0.02	0.21	0.36
T=24	*m* = 2	0.26	0.14	0.14	0.00	0.46	0.45	0.00	0.00	0.00	0.29	0.26
	m = 3	0.05	0.00	0.00	0.30	0.65	0.43	0.00	0.00	0.00	0.24	0.33
	m = 4	0.03	0.00	0.00	0.33	0.64	0.40	0.00	0.00	0.01	0.20	0.39
Bull state (π = 1)												
T=1	*m* = 2	0.17	0.28	0.08	0.00	0.47	0.30	0.15	0.00	0.00	0.26	0.29
	m = 3	0.00	0.00	0.00	0.34	0.66	0.30	0.15	0.00	0.01	0.22	0.32
	m = 4	0.00	0.00	0.00	0.39	0.61	0.29	0.18	0.00	0.03	0.17	0.33
T=6	*m* = 2	0.18	0.29	0.09	0.00	0.44	0.43	0.02	0.00	0.00	0.29	0.26
	m =	0.00	0.00	0.00	0.33	0.67	0.42	0.00	0.00	0.01	0.25	0.32
	m = 4	0.00	0.00	0.00	0.37	0.63	0.39	0.04	0.00	0.03	0.20	0.34
T=24	*m* = 2	0.24	0.18	0.13	0.00	0.45	0.45	0.00	0.00	0.00	0.30	0.25
	m = 3	0.04	0.00	0.00	0.30	0.66	0.43	0.00	0.00	0.00	0.24	0.33
	m = 4	0.03	0.00	0.00	0.34	0.63	0.40	0.00	0.00	0.01	0.20	0.39

Table 5

Sample and Implied Co-Skewness Coefficients

The table reports average sample co-skewness coefficients,

$$S_{i,j,l} \equiv \frac{E[(r_i - E[r_i])(r_j - E[r_j])(r_l - E[r_l])]}{\{E[(r_i - E[r_i])^2]E[(r_j - E[r_j])^2]E[(r_l - E[r_l])^2]\}^{1.5}} \quad K_{i,j,l,b} \equiv \frac{E[(r_i - E[r_i])(r_j - E[r_j])(r_l - E[r_l])(r_b - E[r_b])]}{\{E[(r_i - E[r_i])^2]E[(r_j - E[r_j])^2]E[(r_l - E[r_l])^2]E[(r_b - E[r_b])^2\}^2} \quad (i, j, l = \text{US, JP, Pac, UK, EU})$$

across equity markets and compares them with the co-skewness coefficients implied by a two-state regime switching model:

$$\mathbf{r}_t = \boldsymbol{\mu}_{s_t} + \Sigma_{s_t} \boldsymbol{\varepsilon}_t,$$

where $\boldsymbol{\varepsilon}_t \sim$ I.I.D. $N(0, \mathbf{I}_5)$ is an unpredictable return innovation. In this case, the coefficients are calculated under ergodic state probabilities, employing simulations (10,000 trials) and averaging across simulated sample of length equal to the available data (1975:01 – 2003:12, 348 observations). In the table the indices *j* and *l* refer to market different from the one under consideration, indexed by *i*.

		Co-Skewness				Co-Kurtosis						
		$S_{i,j,l}$	$S_{i,i,j}$	$S_{i,j,j}$	$S_{i,i,i}$	$S_{i,j,l,q}$	$S_{i,i,j,l}$	$S_{i,i,j,j}$	$S_{i,i,i,j}$	$S_{i,j,j,j}$	$S_{i,j,j,l}$	$S_{i,i,i,i}$
i = US	Sample	-0.39	-0.56	-0.50	-0.70	1.84	2.69	4.16	3.71	5.31	2.81	5.73
	MS – ergodic	-0.28	-0.34	-0.28	-0.43	1.92	2.68	3.92	4.00	3.90	2.61	7.41
i = Japan	Sample	-0.23	-0.06	-0.25	0.07	1.28	0.90	1.39	1.04	2.34	1.59	3.46
	MS – ergodic	-0.23	-0.12	-0.25	-0.01	1.66	1.88	2.81	2.35	2.90	2.06	5.96
i = Pacific	Sample	-0.43	-0.99	-0.47	-2.21	1.88	4.04	5.33	9.09	3.98	2.57	20.90
	MS – ergodic	-0.28	-0.42	-0.28	-0.55	1.95	2.92	4.08	4.79	3.78	2.59	9.69
i = UK	Sample	-0.37	-0.45	-0.44	-0.54	1.80	2.28	3.44	2.98	4.97	2.75	4.48
	MS – ergodic	-0.29	-0.38	-0.29	-0.49	1.99	2.97	4.27	4.58	4.34	2.76	7.72
i = EU	Sample	-0.28	0.01	-0.39	0.79	1.79	2.46	3.53	4.25	4.47	2.66	9.87
	MS – ergodic	-0.20	-0.05	-0.20	0.06	1.87	2.16	3.30	3.09	3.90	2.59	6.03

Table 6

Estimates of a Two-State Switching Model – Predictability from the US 1-month T-bill

This table reports estimates for a single state and a two-state VAR(1) regime switching model (MSIAH(2,1)) when the 1-month US T-bill rate predicts subsequent stock returns. The regime switching model takes the form:

$$\mathbf{y}_t = \boldsymbol{\mu}^*_{s_t} + \mathbf{A}_{s_t}\mathbf{y}_{t-1} + \boldsymbol{\Sigma}^*_{s_t}\boldsymbol{\varepsilon}_t,$$

where $\boldsymbol{\varepsilon}_t \sim \textbf{\textit{I.I.D.}}\ N(\mathbf{0}, I_6)$ is an unpredictable return innovation. The sample period is 1975:01 – 2003:12.

	Panel A – Single State Model					
	U.S.	Japan	Pacific ex-JPN	UK	Europe ex-UK	US T-bill
1. Intercept	0.0084	0.0066	0.0144*	0.0107*	0.0097	0.0003**
2. VAR(1) US T-bill Coeff.	0.3845	-0.1155	-1.6217	-0.5828	0.1570	0.9418***
3. Correlations/Volatilities						
US	0.0439***					
Japan	0.3273**	0.0641***				
Pacific ex-Japan	0.5494***	0.3591*	0.0674***			
United Kingdom	0.5939***	0.4779***	0.5318***	0.0502***		
Europe ex-UK	0.5378***	0.4048***	0.5567***	0.6382***	0.0591***	
US 1-month T-bill rate	-0.1014**	-0.0007	-0.0874*	-0.0515	-0.0883	0.0009
	Panel B – Two State Model					
	U.S.	Japan	Pacific ex-JPN	UK	Europe ex-UK	US T-bill
1. Intercept						
Regime 1	0.0042	0.0041	0.0153*	0.0016	0.0048	9.20e-05
Regime 2	0.0300***	0.0330***	0.0125***	0.0197***	0.0414***	0.0013*
2. VAR(1) US T-bill Coeff.						
Regime 1:	0.9622*	-0.2266	-1.9106	1.7046*	0.6513*	0.9696***
Regime 2:	-1.9897**	-2.7532**	1.4225*	-2.2268**	-3.1629***	0.8394***
3. Correlations/Volatilities						
Regime 1:						
US	0.0438***					
Japan	0.3397**	0.0651***				
Pacific ex-Japan	0.6115***	0.3212*	0.0667***			
United Kingdom	0.6875***	0.4471**	0.5477***	0.0501***		
Europe ex-UK	0.6740***	0.4388***	0.5825***	0.7194***	0.0493***	
US 1-month T-bill rate	-0.0037	-0.0308	-0.0891	-0.0528	-0.0593	0.0005**
Regime 2:						
US	0.0415***					
Japan	0.2188*	0.0547***				
Pacific ex-Japan	0.4135**	0.4640***	0.0663***			
United Kingdom	0.3392**	0.5363***	0.5573***	0.0471***		
Europe ex-UK	0.3242**	0.3418**	0.5719***	0.5493***	0.0755***	
US 1-month T-bill rate	-0.2842**	-0.1058*	-0.0995*	-0.1478**	-0.1854**	0.0014***
4. Transition probabilities	Regime 1			Regime 2		
Regime 1	0.9457***			0.0543		
Regime 2	0.1368			0.8632***		

* denotes 10% significance, ** significance at 5%, *** significance at 1%.

Table 7

International Portfolio Weights under Regime Switching in the US Short Rate – Effects of State Probabilities and the Initial Interest Rate

The table reports optimal portfolio choices calculated under a two-state regime switching VAR(1) model in which the US short-term rate may predict future stock returns:

$$\mathbf{y}_t = \boldsymbol{\mu}^*_{s_t} + \mathbf{A}_{s_t}\mathbf{y}_{t-1} + \boldsymbol{\Sigma}^*_{s_t}\boldsymbol{\varepsilon}_t,$$

where $\boldsymbol{\varepsilon}_t \sim$ ***I.I.D.*** $\boldsymbol{N}(\mathbf{0}, \mathrm{I}_6)$ is an unpredictable return innovation. Optimal asset allocations across international equity markets are calculated for a two investment horizons. In the table header, mean and standard deviations are regime-specific and are calculated by simulation. The five (annualized) levels of the US short-term rate are then [0.3 3.2 6.1 9.0 11.9], [1.4 4.0 6.6 9.2 11.8], and [0.4 3.3 6.1 9.0 11.9] percent per annum, respectively.

		Mean – 2 × SD.	Mean – 1 × SD.	Mean	Mean + 1 × SD.	Mean + 2 × SD.
Panel A - T = 1 month		**Regime 1 (π = 0)**				
	United States	0.00	0.47	0.68	0.66	0.63
	Japan	0.00	0.00	0.00	0.00	0.00
	Pacific (ex-Japan)	1.00	0.17	0.00	0.00	0.00
	United Kingdom	0.00	0.00	0.00	0.00	0.00
	Europe (ex-UK)	0.00	0.06	0.00	0.00	0.00
	1-month US T-bills	0.00	0.30	0.32	0.34	0.37
		Ergodic probs. (π = 0.29)				
	United States	0.61	0.56	0.73	0.68	0.63
	Japan	0.04	0.00	0.00	0.00	0.00
	Pacific (ex-Japan)	0.15	0.15	0.00	0.00	0.00
	United Kingdom	0.03	0.08	0.00	0.00	0.00
	Europe (ex-UK)	0.15	0.18	0.00	0.00	0.00
	1-month US T-bills	0.00	0.00	0.27	0.32	0.37
		Regime 2 (π = 1)				
	United States	0.80	0.74	0.80	0.68	0.62
	Japan	0.06	0.03	0.00	0.00	0.00
	Pacific (ex-Japan)	0.01	0.00	0.00	0.00	0.00
	United Kingdom	0.00	0.00	0.00	0.00	0.00
	Europe (ex-UK)	0.15	0.26	0.20	0.06	0.00
	1-month US T-bills	0.00	0.00	0.00	0.26	0.38
Panel B - T = 24 months		**Regime 1 (π = 0)**				
	United States	0.36	0.67	0.74	0.74	0.70
	Japan	0.08	0.00	0.00	0.00	0.00
	Pacific (ex-Japan)	0.30	0.14	0.09	0.00	0.00
	United Kingdom	0.05	0.08	0.11	0.00	0.00
	Europe (ex-UK)	0.21	0.11	0.06	0.00	0.00
	1-month US T-bills	0.00	0.00	0.00	0.26	0.30
		Ergodic probs. (π = 0.29)				
	United States	0.42	0.67	0.74	0.74	0.70
	Japan	0.07	0.00	0.00	0.00	0.00
	Pacific (ex-Japan)	0.25	0.13	0.08	0.00	0.00
	United Kingdom	0.05	0.08	0.10	0.00	0.00
	Europe (ex-UK)	0.16	0.12	0.10	0.00	0.00
	1-month US T-bills	0.00	0.00	0.00	0.26	0.30
		Regime 2 (π = 1)				
	United States	0.63	0.71	0.75	0.75	0.69
	Japan	0.01	0.00	0.00	0.00	0.00
	Pacific (ex-Japan)	0.14	0.10	0.08	0.00	0.00
	United Kingdom	0.06	0.07	0.08	0.00	0.00
	Europe (ex-UK)	0.21	0.12	0.09	0.00	0.00
	1-month US T-bills	0.00	0.00	0.00	0.25	0.31

Table 8

Optimal Portfolio Choices under a Single-State Gaussian Model

The table reports optimal portfolio choices calculated under a single-state VAR(p) model, p = 0, 1:

$$\mathbf{y}_t = \mu^* + A\ \mathrm{y}_{t-1} + \Sigma^* \boldsymbol{\varepsilon}_t,$$

where $\boldsymbol{\varepsilon}_t \sim \boldsymbol{I.I.D.}\ \boldsymbol{N}(\mathbf{0}, \mathrm{I}_N)$ is an unpredictable return innovation. When the US short-term rate is allowed to predict future stock returns, p = 1. In this case, calculations are performed initializing the interest rate at its regime-specific mean (calculated by simulation). When the US short-term is taken as pre-determined (conditionally riskless), p = 0 and portfolio weights do not depend on the investment horizon (myopic case).

	U.S.	**Japan**	**Pacific ex-Japan**	**UK**	**Europe ex-UK**	**US 1-month T-bills**
	Panel A - Conditionally Riskless Asset - Single state model (Gaussian IID)					
IID - Myopic	0.27	0.00	0.00	0.00	0.36	0.37
	Panel B - Predictability from the US 1-month T-bill					
	US short-rate = Mean - 2× SD (0.5% per annum)					
VAR - T = 1	0.00	0.00	0.61	0.00	0.00	0.39
VAR - T = 24	0.00	0.00	0.62	0.00	0.00	0.38
	US short-rate = Mean - SD (3.3% per annum)					
VAR - T = 1	0.15	0.06	0.49	0.15	0.15	0.00
VAR - T = 24	0.20	0.05	0.37	0.22	0.16	0.00
	US short-rate = Mean (6.1% per annum)					
VAR - T = 1	0.37	0.05	0.25	0.16	0.17	0.00
VAR - T = 24	0.43	0.04	0.22	0.15	0.16	0.00
	US short-rate = Mean + SD (9.0% per annum)					
VAR - T = 1	0.71	0.00	0.00	0.00	0.00	0.29
VAR - T = 24	0.75	0.00	0.00	0.00	0.00	0.25
	US short-rate = Mean + 2× SD (11.8% per annum)					
VAR - T = 1	0.68	0.00	0.00	0.00	0.00	0.32
VAR - T = 24	0.73	0.00	0.00	0.00	0.00	0.27

Table 9

Optimal Home Bias in US Equity Holdings – Alternative Statistical Models and Preferences

For a variety of assumptions on the state beliefs, the investment horizon, preferences, and the underlying model for asset returns, the table calculates and reports the following index of home bias in US equity portfolios:

$$HCBI(\pi,T) \equiv \frac{[(\hat{\omega}^{US}(\pi,T)/(1-\hat{\omega}^{T-bill}(\pi,T))) - \omega^{US}_{world}]}{1-\omega^{US}_{world}}$$

where $\hat{\omega}^{US}(\pi,T)$ and $\hat{\omega}^{T-bill}(\pi,T)$ are the optimal weights of US stocks and 1-month T-bills, when belief are described by π and the investment horizon is T; ω^{US}_{world} is the total weight of US stocks on the aggregate world market capitalization (0.46 at the end of 2003, see Thomas et al. (2004)). The index takes a positive (negative) value when optimal portfolio choices imply a US equity weight below (above) ω^{US}_{world}; when the asset allocation implies that only US equities should be held, HCBI(π,T) = 1. When the US short-term rate predicts future equity returns, calculations are performed initializing the interest rate at its regime-specific mean. Numbers in brackets are percentage weights of US equities on the equity portfolio.

	Investment Horizon			
	T = 1	**T = 6**	**T = 12**	**T = 24**
Panel A - Conditionally Riskless Asset				
	Four-Moment Preferences			
MS - Bear state (π = 0)	0.254 [0.597]	0.327 [0.637]	0.334 [0.641]	0.359 [0.654]
MS - Ergodic probs. (π = 0.76)	0.325 [0.636]	0.334 [0.640]	0.327 [0.639]	0.359 [0.654]
MS - Bull state (π = 1)	-0.049 [0.433]	0.250 [0.595]	0.322 [0.634]	0.359 [0.654]
IID – Myopic	-0.058 [0.429]	-0.058 [0.429]	-0.058 [0.429]	-0.058 [0.429]
	Mean-Variance Preferences			
MS - Bear state (π = 0)	0.400 [0.676]	0.348 [0.648]	0.306 [0.625]	0.290 [0.616]
MS - Ergodic probs. (π = 0.76)	0.270 [0.606]	0.280 [0.611]	0.264 [0.603]	0.274 [0.608]
MS - Bull state (π = 1)	-0.069 [0.423]	0.254 [0.597]	0.264 [0.603]	0.274 [0.608]
IID – Myopic	-0.215 [0.306]	-0.215 [0.306]	-0.215 [0.306]	-0.215 [0.306]
Panel B - Predictability from the US 1-month T-bill				
	Four-Moment Preferences			
MS – Regime 1 (π = 0)	1.000 [1.000]	1.000 [1.000]	1.000 [1.000]	0.519 [0.740]
MS - Ergodic probs. (π = 0.29)	1.000 [1.000]	1.000 [1.000]	1.000 [1.000]	0.519 [0.740]
MS – Regime 2 (π = 1)	0.630 [0.801]	0.667 [0.820]	0.537 [0.752]	0.537 [0.752]
VAR(1)	-0.167 [0.359]	-0.130 [0.384]	-0.111 [0.386]	-0.056 [0.434]
	Mean-Variance Preferences			
MS – Regime 1 (π = 0)	0.333[0.647]	0.333 [0.647]	0.370 [0.670]	0.370 [0.670]
MS - Ergodic probs. (π = 0.29)	0.315 [0.630]	0.333 [0.640]	0.333 [0.640]	0.352 [0.650]
MS – Regime 2 (π = 1)	-0.019 [0.453]	0.130 [0.530]	0.222 [0.582]	0.278 [0.608]
VAR(1)	-0.093 [0.409]	-0.111 [0.400]	-0.111 [0.400]	-0.093 [0.409]

Table 10

International Portfolio Weights under Regime Switching in the US Short Rate – VAR(1) Switching Model Based on the State Definition in Table 3

The table reports optimal portfolio choices calculated under a two-state regime switching VAR(1) model in which the US short-term rate may predict future stock returns:

$$\mathbf{y}_t = \boldsymbol{\mu}^*_{s_t} + \mathbf{A}_{s_t}\mathbf{y}_{t-1} + \boldsymbol{\Sigma}^*_{s_t}\boldsymbol{\varepsilon}_t,$$

where $\boldsymbol{\varepsilon}_t \sim \textbf{\textit{I.I.D. N}}(\mathbf{0}, I_6)$ is an unpredictable return innovation. In the table header, mean and standard deviations are regime-specific and are calculated by simulation. The five (annualized) levels of the US short-term rate are then [0.3 3.2 6.1 9.0 11.9], [1.4 4.0 6.6 9.2 11.8], and [0.4 3.3 6.1 9.0 11.9] percent per annum, respectively. The underlying two-state model is estimated by calculating GLS estimates based on the smoothed state probabilities implied by the simpler two-state VAR(0) model for equity returns in Table 3.

Panel		Mean – 2 × SD.	Mean – 1 × SD.	Mean	Mean + 1 × SD.	Mean + 2 × SD.
Panel A - T = 1 month				**Regime 1 (π = 0)**		
	United States	0.72	0.57	0.47	0.40	0.40
	Japan	0.00	0.00	0.00	0.00	0.00
	Pacific (ex-Japan)	0.00	0.00	0.00	0.00	0.00
	United Kingdom	0.00	0.00	0.00	0.00	0.00
	Europe (ex-UK)	0.01	0.05	0.06	0.07	0.03
	1-month US T-bills	0.27	0.38	0.47	0.53	0.57
				Ergodic probs. (π = 0.76)		
	United States	0.65	0.58	0.50	0.43	0.42
	Japan	0.00	0.00	0.00	0.00	0.00
	Pacific (ex-Japan)	0.09	0.00	0.00	0.00	0.00
	United Kingdom	0.08	0.00	0.00	0.00	0.00
	Europe (ex-UK)	0.18	0.10	0.11	0.12	0.09
	1-month US T-bills	0.00	0.32	0.39	0.45	0.49
				Regime 2 (π = 1)		
	United States	0.46	0.68	0.73	0.71	0.69
	Japan	0.04	0.00	0.00	0.00	0.00
	Pacific (ex-Japan)	0.16	0.00	0.00	0.00	0.00
	United Kingdom	0.07	0.00	0.00	0.00	0.00
	Europe (ex-UK)	0.27	0.11	0.00	0.00	0.00
	1-month US T-bills	0.00	0.21	0.27	0.29	0.31
Panel B - T = 24 months				**Regime 1 (π = 0)**		
	United States	0.52	0.69	0.68	0.64	0.60
	Japan	0.00	0.00	0.00	0.00	0.00
	Pacific (ex-Japan)	0.13	0.00	0.00	0.00	0.00
	United Kingdom	0.08	0.00	0.00	0.00	0.00
	Europe (ex-UK)	0.27	0.06	0.01	0.00	0.00
	1-month US T-bills	0.00	0.25	0.31	0.36	0.40
				Ergodic probs. (π = 0.76)		
	United States	0.51	0.59	0.60	0.67	0.63
	Japan	0.00	0.00	0.00	0.00	0.00
	Pacific (ex-Japan)	0.14	0.00	0.00	0.00	0.00
	United Kingdom	0.08	0.00	0.00	0.00	0.00
	Europe (ex-UK)	0.27	0.18	0.12	0.00	0.00
	1-month US T-bills	0.00	0.23	0.28	0.33	0.37
				Regime 2 (π = 1)		
	United States	0.57	0.71	0.73	0.72	0.70
	Japan	0.01	0.00	0.00	0.00	0.00
	Pacific (ex-Japan)	0.12	0.00	0.00	0.00	0.00
	United Kingdom	0.06	0.00	0.00	0.00	0.00
	Europe (ex-UK)	0.24	0.08	0.02	0.00	0.00
	1-month US T-bills	0.00	0.21	0.25	0.28	0.30

Table 11

Optimal Home Bias in US Equity Holdings – Effects of Risk Aversion

The table reports optimal portfolio weights under regime switching when the coefficients of the objective function are evaluated by interpreting the objective as a Taylor approximation (around v_T) to power utility with constant relative risk aversion equal to θ:

$$E[U^m(W_{t+T};\theta)] = \kappa_0(v_T;\theta) + \sum_{j=1}^{m} \kappa_j(v_T;\theta) M_{t+T}^{(j)} \ .$$

In both panels, the weights are calculated assuming the state probabilities are held at their ergodic levels. In panel B, the US 1-month T-bill is random and its initial value is set to the full-sample mean of 6.1% (in annualized terms).

	Risk aversion	US	Japan	Pacific ex-Japan	UK	Europe ex-UK	US 1-month T-bills
			Panel A – Portfolio weights w/Conditionally Riskless Asset				
T = 1	θ = 2	0.41	0.00	0.00	0.02	0.22	0.35
	θ = 5	0.40	0.00	0.00	0.00	0.20	0.40
	θ = 10	0.38	0.00	0.00	0.00	0.18	0.44
T = 6	θ = 2	0.41	0.00	0.00	0.02	0.21	0.36
	θ = 5	0.40	0.00	0.00	0.00	0.19	0.41
	θ = 10	0.37	0.00	0.00	0.00	0.17	0.46
T = 12	θ = 2	0.40	0.00	0.00	0.02	0.21	0.37
	θ = 5	0.39	0.00	0.00	0.00	0.18	0.43
	θ = 10	0.35	0.00	0.00	0.00	0.16	0.49
T = 24	θ = 2	0.40	0.00	0.00	0.01	0.20	0.39
	θ = 5	0.38	0.00	0.00	0.00	0.17	0.45
	θ = 10	0.34	0.00	0.00	0.00	0.15	0.51
			Panel B - Predictability from the US 1-month T-bill				
T = 1	θ = 2	0.73	0.00	0.00	0.00	0.00	0.27
	θ = 5	0.63	0.00	0.00	0.00	0.00	0.37
	θ = 10	0.60	0.00	0.00	0.00	0.00	0.40
T = 6	θ = 2	0.74	0.00	0.01	0.02	0.00	0.23
	θ = 5	0.65	0.00	0.00	0.00	0.00	0.35
	θ = 10	0.62	0.00	0.00	0.00	0.00	0.38
T = 12	θ = 2	0.75	0.00	0.03	0.03	0.01	0.18
	θ = 5	0.66	0.00	0.00	0.00	0.00	0.34
	θ = 10	0.62	0.00	0.00	0.00	0.00	0.38
T = 24	θ = 2	0.74	0.00	0.08	0.10	0.08	0.00
	θ = 5	0.69	0.00	0.00	0.00	0.00	0.31
	θ = 10	0.64	0.00	0.00	0.00	0.00	0.36

Table 12

Comparing Optimal Portfolio Choices under Four-Moment Preferences vs. Standard Power Utility.

This table compares (all-equity) optimal portfolio weights under a four-moment preference functional,

$$E[U^m(W_{t+T};\theta)] = \kappa_0(v_T;\theta) + \sum_{j=1}^{m} \kappa_j(v_T;\theta) M_{t+T}^{(j)} ,$$

vs. the weights calculated (by simulation, using 60,000 independent draws) under a standard power utility (CRRA) objective,

$$\boldsymbol{E}[\boldsymbol{U}(\boldsymbol{W}_{t+T};\theta)] = \frac{\boldsymbol{W}_{t+T}^{1-\theta}}{1-\theta} .$$

Returns are generated from a two-state regime-switching model with structure:

$$\mathbf{r}_t = \boldsymbol{\mu}_{s_t} + \Sigma_{s_t} \boldsymbol{\varepsilon}_t ,$$

where $\boldsymbol{\varepsilon}_t \sim$ I.I.D. $N(0, \mathbf{I}_5)$ is an unpredictable return innovation. θ is set to 2 throughout.

	US	Japan	Pacific ex-JPN	UK	Europe ex-UK
			Bear state (π = 0)		
T = 1 – Four moment	0.35	0.07	0.17	0.15	0.26
T = 1 – CRRA	0.39	0.09	0.15	0.12	0.25
T = 12 – Four moment	0.06	0.00	0.00	0.30	0.64
T = 12 – CRRA	0.11	0.01	0.00	0.25	0.63
T = 24 – Four moment	0.04	0.00	0.00	0.32	0.64
T = 24 – CRRA	0.06	0.00	0.00	0.27	0.67
			Bull state (π = 1)		
T = 1 – Four moment	0.00	0.00	0.00	0.39	0.61
T = 1 – CRRA	0.00	0.00	0.00	0.42	0.58
T = 12 – Four moment	0.00	0.00	0.00	0.36	0.64
T = 12 – CRRA	0.00	0.00	0.00	0.39	0.61
T = 24 – Four moment	0.03	0.00	0.00	0.34	0.63
T = 24 – CRRA	0.02	0.00	0.00	0.37	0.61

Table 13

International Portfolio Weights under Regime Switching in the US Short Rate – Effects of Rebalancing

The table reports optimal portfolio choices calculated under a two-state regime switching VAR(1) model in which the US short-term rate may predict future stock returns:

$$\mathbf{y}_t = \boldsymbol{\mu}^*_{s_t} + \mathbf{A}_{s_t}\mathbf{y}_{t-1} + \boldsymbol{\Sigma}^*_{s_t}\boldsymbol{\varepsilon}_t,$$

where $\boldsymbol{\varepsilon}_t \sim \boldsymbol{I.I.D.}\ \boldsymbol{N}(\mathbf{0}, \mathrm{I}_6)$ is an unpredictable return innovation. Optimal asset allocations across international equity markets are calculated for two investment horizons. The (annualized) US short-term rate is set at its unconditional mean. In the table header, we report different, alternative rebalancing frequencies (φ, in months). The 'buy-and-hold column' corresponds to a rebalancing frequency identical to the investment horizon, T.

		φ = 1 month	φ = 3 months	φ = 6 months	φ = 12 months	Buy-and-hold
Panel A - T = 6 month				**Regime 1 (π = 0)**		
	United States	0.00	0.21	0.68	0.68	0.68
	Japan	0.00	0.00	0.00	0.00	0.00
	Pacific (ex-Japan)	0.67	0.46	0.00	0.00	0.00
	United Kingdom	0.00	0.00	0.00	0.00	0.00
	Europe (ex-UK)	0.00	0.00	0.00	0.00	0.00
	1-month US T-bills	0.33	0.33	0.32	0.32	0.32
				Ergodic probs. (π = 0.29)		
	United States	0.69	0.72	0.73	0.73	0.73
	Japan	0.00	0.00	0.00	0.00	0.00
	Pacific (ex-Japan)	0.02	0.00	0.00	0.00	0.00
	United Kingdom	0.00	0.00	0.00	0.00	0.00
	Europe (ex-UK)	0.00	0.00	0.00	0.00	0.00
	1-month US T-bills	0.29	0.28	0.27	0.27	0.27
				Regime 2 (π = 1)		
	United States	0.73	0.78	0.80	0.80	0.80
	Japan	0.00	0.00	0.00	0.00	0.00
	Pacific (ex-Japan)	0.00	0.00	0.00	0.00	0.00
	United Kingdom	0.00	0.00	0.00	0.00	0.00
	Europe (ex-UK)	0.05	0.08	0.20	0.20	0.20
	1-month US T-bills	0.22	0.14	0.00	0.00	0.00
Panel B - T = 24 months				**Regime 1 (π = 0)**		
	United States	0.00	0.05	0.21	0.56	0.74
	Japan	0.00	0.00	0.00	0.00	0.00
	Pacific (ex-Japan)	0.71	0.67	0.61	0.27	0.09
	United Kingdom	0.00	0.00	0.00	0.04	0.11
	Europe (ex-UK)	0.00	0.00	0.00	0.02	0.06
	1-month US T-bills	0.29	0.28	0.18	0.11	0.00
				Ergodic probs. (π = 0.29)		
	United States	0.74	0.74	0.76	0.75	0.74
	Japan	0.00	0.00	0.00	0.00	0.00
	Pacific (ex-Japan)	0.00	0.00	0.02	0.02	0.08
	United Kingdom	0.00	0.00	0.03	0.08	0.10
	Europe (ex-UK)	0.00	0.00	0.04	0.05	0.10
	1-month US T-bills	0.26	0.26	0.15	0.10	0.00
				Regime 2 (π = 1)		
	United States	0.78	0.78	0.77	0.76	0.75
	Japan	0.00	0.00	0.00	0.00	0.00
	Pacific (ex-Japan)	0.00	0.00	0.01	0.05	0.08
	United Kingdom	0.00	0.00	0.03	0.07	0.08
	Europe (ex-UK)	0.00	0.00	0.01	0.02	0.09
	1-month US T-bills	0.22	0.22	0.19	0.10	0.00

Figure 1

Smoothed Probabilities of a Bear State in a Two-Regime Model

This figure plots smoothed probabilities for the two-state MSIH (2,0) model

$$\mathbf{r}_t = \boldsymbol{\mu}_{s_t} + \Sigma_{s_t} \boldsymbol{\varepsilon}_t,$$

to monthly nominal MSCI stock index returns (in US dollars) for the US, Japan, Pacific (ex-Japan), the United Kingdom, and Europe (ex-UK).

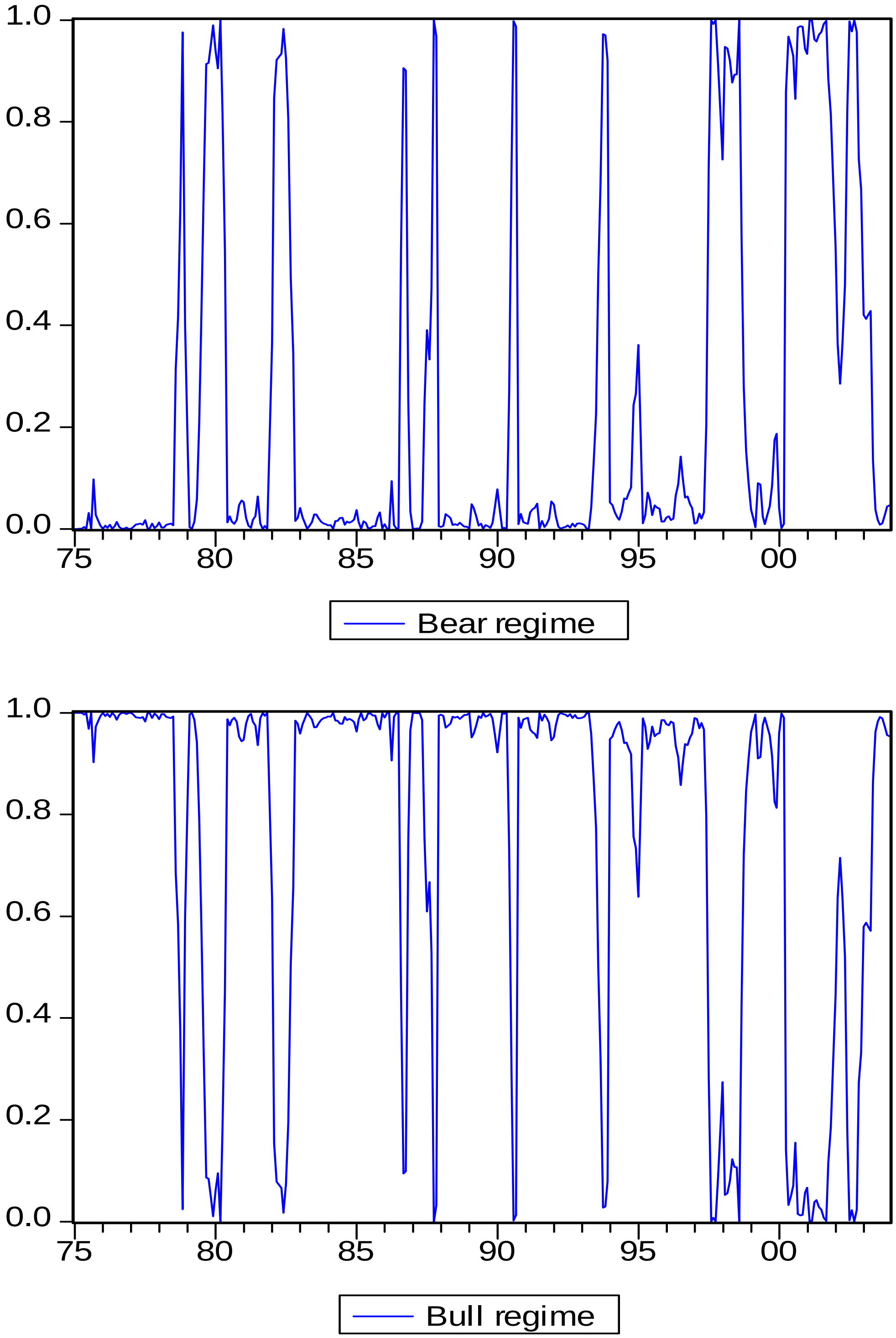

Figure 2

Optimal All-Equity International Portfolio Allocation – Effects of Bull State Probabilities

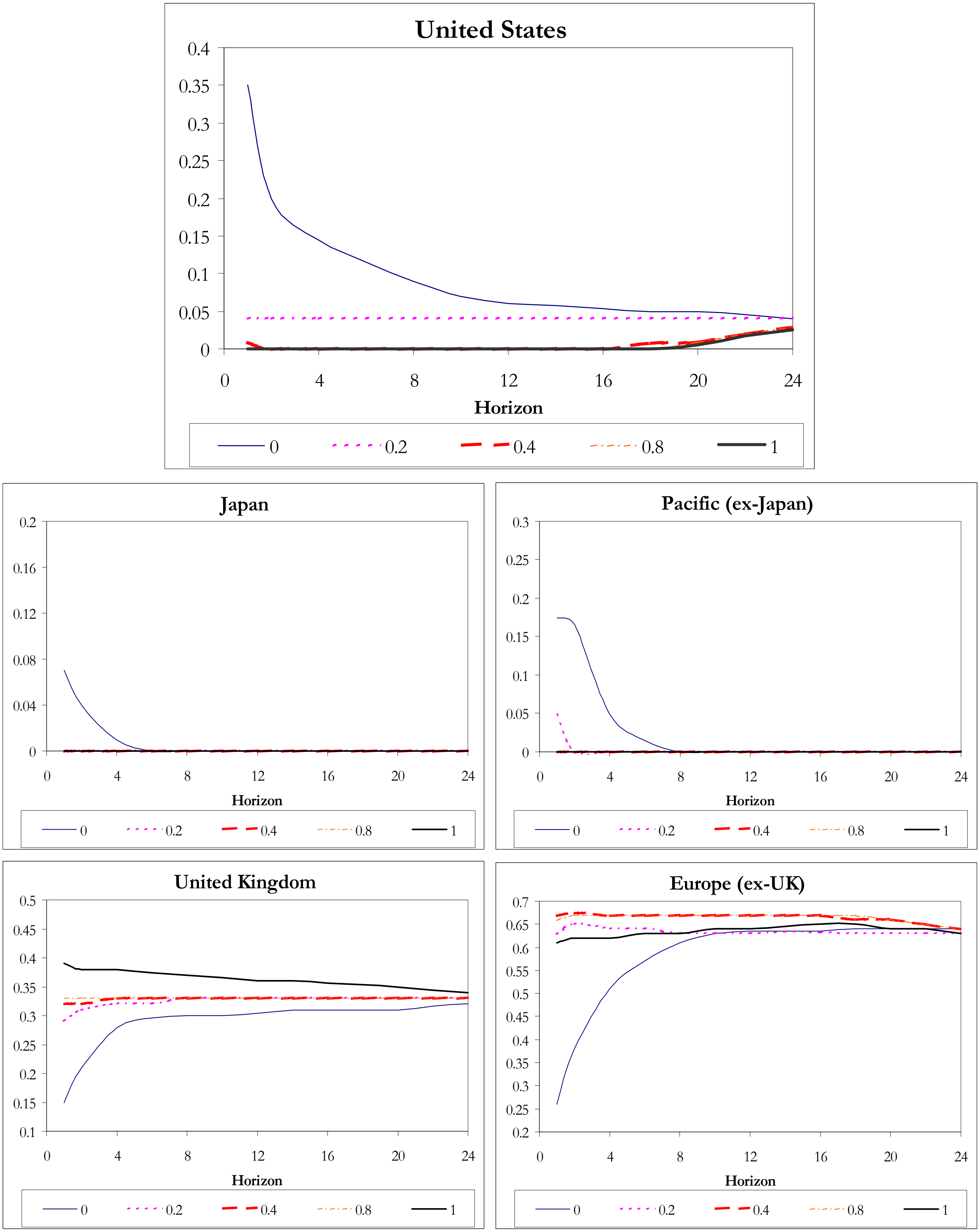

Figure 3

Optimal International Portfolio Allocation with a Riskless Asset – Effects of Bull State Probabilities

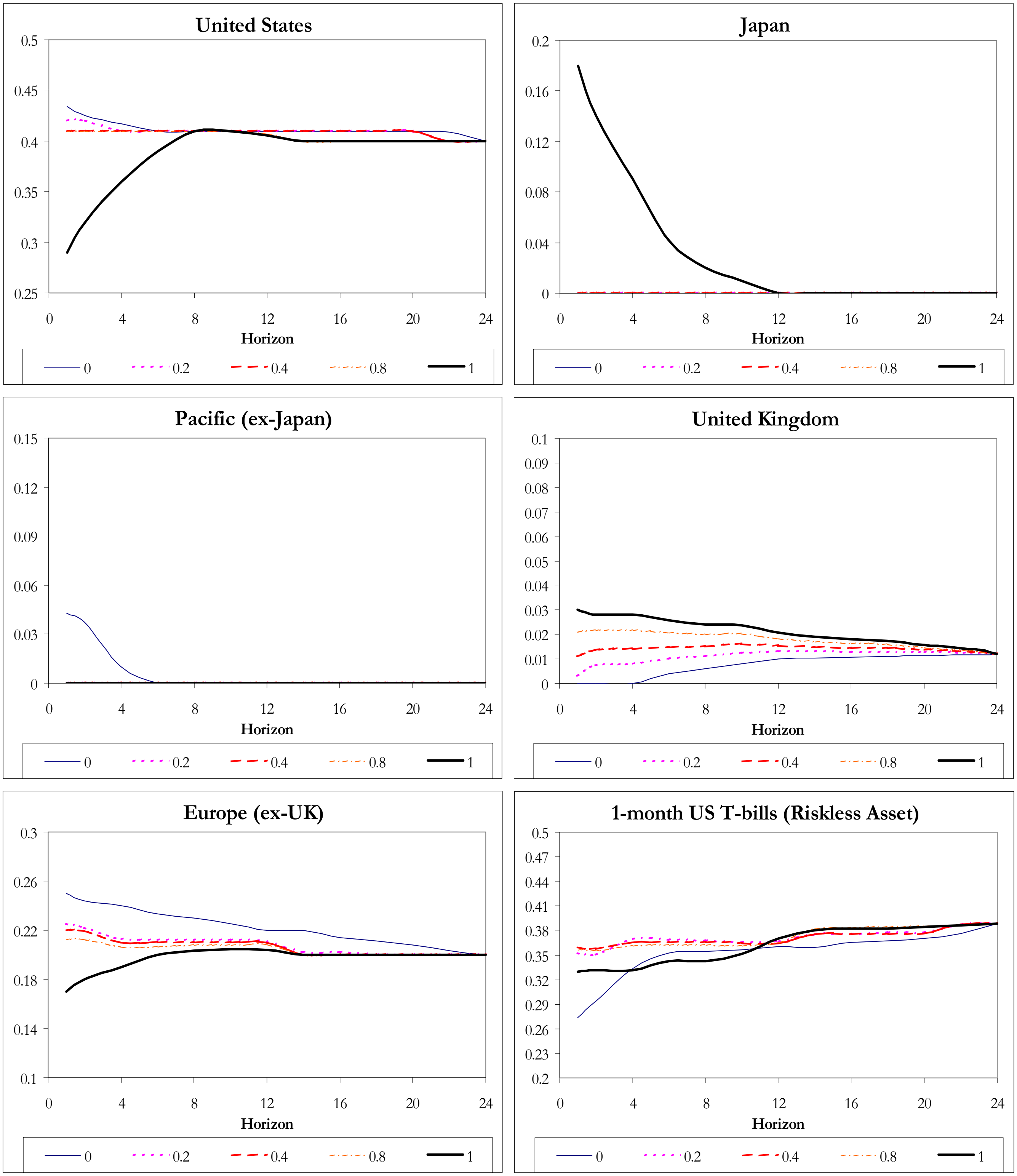

Figure 4

Moments of Wealth/Portfolio Returns under Optimal Weights vs. World Market Shares – Conditionally Riskless Asset

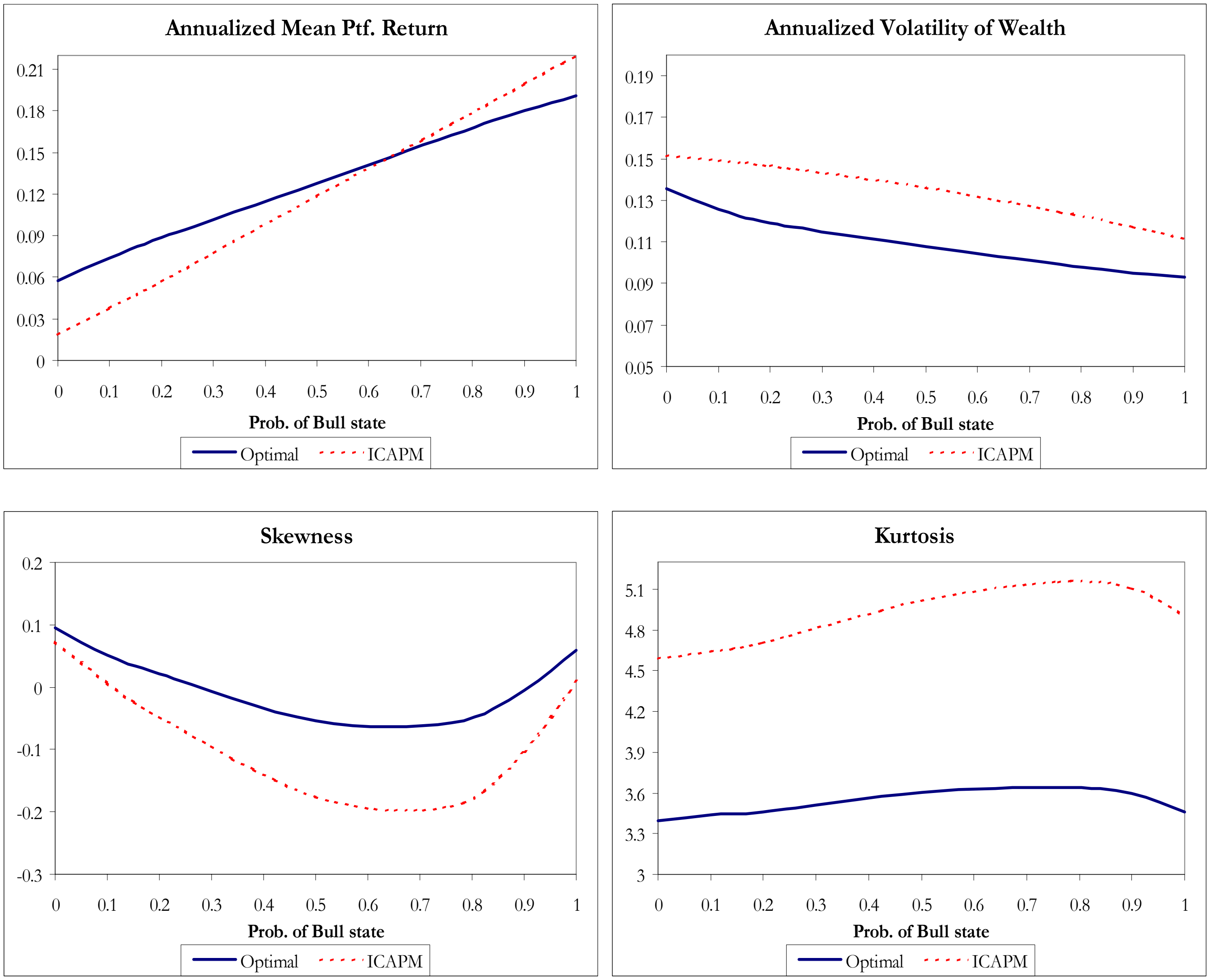

Figure 5

Smoothed Probabilities of a Bear State in a Two-Regime Model – Predictability from the US 1-month T-bill

This figure plots smoothed probabilities for the two-state MSIAH (2,1) model

$$\mathbf{y}_t = \boldsymbol{\mu}^*_{s_t} + \mathbf{A}_{s_t}\mathbf{y}_{t-1} + \boldsymbol{\Sigma}^*_{s_t}\boldsymbol{\varepsilon}_t,$$

to monthly nominal MSCI stock index returns (in US dollars) for the US, Japan, Pacific (ex-Japan), the United Kingdom, Europe (ex-UK), and US 1-month T-bills.