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International Comparisons of Income Inequality: Tests For Lorenz Dominance Across Nine Countries

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I. Introduction

Simon Kuznet's pioneering work on economic development and the distribution of income stimulated great interest in international comparisons of income inequality. While there have been numerous studies, the lack of comparable data and the availability of appropriate statistical tests limits the conclusions that can be drawn concerning international differences in income inequality. Recently, progress has been made on two fronts. The Luxembourg Income Study (LIS) provides researchers with access to large microdata samples with an unparalleled degree of comparability across countries. Important statistical advances have also been made and it is now possible to systematically test for significant differences in comparable income distribution data. We use the LIS data and the new statistical tests to compare the degree of relative inequality in nine countries: Australia, Canada, The Netherlands, Norway, Sweden, Switzerland, West Germany, United Kingdom, and the United States. To illustrate the importance and power of the statistical tests we contrast simple numerical comparisons of Lorenz curves which characterize the established literature on international differences in income inequality to the more complete ordering obtained by appying inference tests for Lorenz dominance.

While the Lorenz curve is well known to be the most general of the inequality measures available, there is considerable pessimism concerning its ability to rank income distributions in terms of Lorenz dominance [see, for example Atkinson (1970, p.258) and Shorrocks (1983, p.3)]. This pessimism is based on numerical comparisons of Lorenz curves which are rarely appropriate. The difficulty with the widely used numerical comparison technique is that sample data are almost always used and numerical comparisons fail to account for sampling variabilty. The recently developed statistical methodology allows the researcher to account for sampling variability and test for differences in Lorenz curves constructed from microdata sample.

The new advances in statistical theory suggest a number of interesting questions to be addressed. For example, can the statistical tests of Lorenz dominance provide a reasonably complete partial order of the degree of relative inequality among countries? Does the statistical ranking vary greatly from the ordering provided by a numerical analysis? If so, is the ordering sensitive to the definition of the recipient unit? Is the degree of inequality less severe on a per capita basis than on a family basis? Finally, which of the LIS countries have the most inequality and the least inequality?

II. Statistical Procedures For Ordering Lorenz Curves

The Lorenz curve is widely acknowledged to be the ethically minimal procedure for ranking the degree of inequality. A potential drawback of this approach is that Lorenz curves provide only a partial order of relative inequality. When Lorenz curves cross, specific judgments about the relative weight given to each positon in the income distribution must be made. To determine the extent to which the Lorenz curve can order income distributions requires a methodology for comparing income distributions.

The standard procedure for drawing conclusions concerning Lorenz dominance involves numerically comparing Lorenz ordinates. Numerical comparisons ignore the fact that Lorenz curves are constructed from sample data which are necessarily subject to sampling variability and error. The simple numerical comparisons of the sample Lorenz ordinates result in a statistical "test" with a probability of a Type I error approaching one. This implies that two Lorenz curves cannot be ranked as equivalent using the numerical method of comparison.[1] The inability to rank two distributions as equal with the numerical method suggests that statistical analysis can provide a more complete and accurate ordering of Lorenz curves.

To order two Lorenz distributions using a statistical test requires the capability of distinguishing between three possible outcomes: Lorenz dominance, Lorenz equivalence, and a Lorenz crossing (no ranking). The problem at hand is different from the standard two sample problem (Ho: F=G vs. H_A: F≠G) because the ranking method must differentitate between Lorenz dominance and noncomparability when the null hypothesis is rejected. The possibility of several departures from the null hypothesis requires a finer partitioning of the rejection region. To partition the rejection region we employ a pairwise multiple comparison of the sample Lorenz ordinates.[2]

Definitions and Sampling Distribution

Let F denote the population distribution function for income, Y. Let $Y(p) := \inf(y : F(y) \geq p)$ be the quantile (inverse distribution) function, where p [0,1] and μ is the mean. Following Gastwirth (1971) the Lorenz curve is then:

$$L(p) = (1/\mu)\int_0^p Y(u)du,$$

where $L(0) = 0$ and $L(1) = 1$. We approximate the Lorenz curve at a fixed set of abscissae, the k fractions, $0 < p_1 < p_2 <, \ldots, p_K = 1$. Let ξ_i be the p_i^{th} population quantile, $F(\xi_i) = p_i$. The conditional (income class) mean and variance for the i^{th} income class are defined as $\mu_i = E(y|\xi_{i-1} \leq y \leq \xi_i)$ and $\sigma_i^2 = E[(y - \mu_i)^2|\xi_{i-1} \leq y \leq \xi_i]$. The cumulative mean and variance of incomes less than ξ_i are $\gamma_i = E(y|y \leq \xi_i)$ and $\lambda_i^2 = E[(y - \gamma_i)^2|y \leq \xi_i]$. Multiplying the cumulative means by the p_k and dividing by the mean μ, defines the vector of Lorenz ordinates,

$$L = (p_1\gamma_1/\mu, \ldots, p_{K-1}\gamma_{K-1}/\mu, p_K\gamma_K/\mu).$$

To construct tests for partial orders of Lorenz curves, we use an important result from Beach and Davidson (1983) on the asymptotic distribution of the Lorenz ordinates.[3] Under the assumption of a strictly monotonic, twice differentiable distribution function with a finite mean and variance,

$N^{1/2}(\hat{L} - L)$ has a limiting k-variate normal distribution with mean zero and variance, V_{ii}, where

$$V_{ii} = (p_i/\mu^2)[\lambda_i^2+(1-p_i)(\xi_i-\gamma_i)^2] + (p_i\gamma_i/\mu^2)^2 - 2(p_i\gamma_i/\mu^3)[\lambda_i^2+(1-p_i)(\xi_i-\gamma_i)].$$

The V_{ii} can be consistently estimated by estimating the ξ_i, γ_i, and λ_i by their sample analogs. It is important to note that these variance estimates do not require the assumption of an arbitrary parametric form for the income distribution.

Hypotheses and Tests

To partially order two sample Lorenz curves, we propose a test of the differences between two vectors of sample Lorenz ordinates. Given two empirical income distributions with sample sizes N^a and N^b, we let L_i^a and L_i^b denote the i^{th} sample Lorenz ordinates. To locate at which of the k ordinates the Lorenz curves differ, we test the null hypothesis:

$$H_o: L_i^a = L_i^b, \text{ for all } i = 1,2,\ldots,k.$$

An appropriate statistic to test H_o is:

$$Z_i = (\hat{L}_i^a - \hat{L}_i^b)/[\hat{V}_{ii}^a/N^a + \hat{V}_{ii}^b/N^b]^{1/2},$$

for $i = 1,2,\ldots,k$. In large samples, the Z_i are asymptotically normal, i.e., N(0,1).

To make inferences about the population Lorenz curves from the vectors of sample Lorenz ordinates we use the information derived from the k individual Lorenz tests. Since this requires drawing inferences from the union of k disjoint subhypotheses, simultaneous inference procedures are appropriate. We test the null hypothesis of the equality of two Lorenz curves by testing each of the statistics Z_i as a Studentized Maximum Modulus (SMM) variate.[4] That is, an approximately α level test of H_o is to accept the null hypothesis if $|Z_i| < m_\alpha(k,\infty)$ for all i, where $m_\alpha(k,\infty)$ is the upper α critical value of the SMM distribution with ∞ degrees of freedom (in the case of large sample sizes).[5]

If we accept the null hypothesis of equality of the two vectors of Lorenz ordinates, then we rank the two Lorenz curves as equal. The failure to accept H_o [i.e, some $|Z_i| > m_\alpha(k,\infty)$] requires us to differentiate between Lorenz dominance and crossing Lorenz curves. If $\hat{L}_i^a \leq \hat{L}_i^b$ for each pairwise comparison (with a strict inequality at some ordinate), then Lorenz dominance results. If $\hat{L}_i^a > \hat{L}_i^b$ for some i, and $\hat{L}_i^a < \hat{L}_i^b$ for some other i, then a crossing occurs and the two Lorenz curves cannot be ordered.

Finally, Beach and Davison (1983) provide a procedure for tests for significant differences in (lower bound) Gini coefficients. Using the trapezoidal integration formula the Gini coefficient may be estimated as

$$G = (1/k)\sum_{i=1}^{k}(p_i - \hat{L}_i + p_{i-1} - \hat{L}_{i-1}),$$

with the variance, V_G,

$$V_G = [4/(k)^2](\sum_{i=1}^{k} \sum_{i=1}^{k} V_{ij}.$$ [6]

III. An Analysis of Inequality In The LIS Countries

The LIS data set contains national survey data for ten countries collected between 1979 and 1983. The survey data are adjusted for definitional differences in income and the income recipient units. With the exception of the United Kingdom, the sample data are weighted to more precisely represent the underlying populations. Table 1 shows the countries included, the survey coverage, the LIS sample sizes, and the raw data source. We exclude Israel from our study because of non-comparability. The Israeli data excludes both the rural and nonvoting populations. Detailed descriptions of the LIS data sets are provided by O'Higgins et al. (1985) and Buhmann et al. (1988).

An important advantage of the LIS data is that it allows the choice of alternative income concepts and recipient units. We use net cash income, the most comprehensive income concept available, which is defined as market income plus public and private transfers minus direct (income and payroll) taxes. Following the recommendations of Cowell (1984), the family and per capita family income recipient units are used. For example, a family of four with family net cash income of $10,000 is reported as four per capita incomes of $2,500 each.[7]

Tables 2 and 3 provide the necessary information to statistically test for family and per capita family Lorenz dominance. These tables report the decile Lorenz ordinates, Gini coefficients and their standard errors for the nine LIS countries.[8] Since the family is the primary unit of observation in the samples, we calculate the standard errors using the family size as the number of observations in all cases.[9]

Table 4 illustrates the application of the statistical procedures to test for Lorenz dominance. The three possible outcomes dominance, noncomparability (crossing Lorenz curves), and equivalent Lorenz curves are shown. As discussed above, the test of the overall hypothesis of equality of two Lorenz curves is based on the inferences drawn from the individual Lorenz ordinates. To maintain the size of the overall test at no more than five percent, we reject the hypothesis of equality of a pair of individual Lorenz ordinates using critical values drawn from the SMM distribution. In the case of deciles, the critical SMM value for a five percent test of the equality of two Lorenz curves is 2.80.[10]

Table 4a compares the U.S. and The Netherlands family Lorenz curves and shows Lorenz dominance. The test statistics are reported as the Dutch Lorenz ordinate minus the U.S. Lorenz ordinate. The first through eighth deciles all show test statistic values that are greater than 2.80 indicating larger Dutch cumulative income shares. The ninth decile Lorenz ordinates are not significantly different as evidenced by the test statistic value of 2.34. We conclude that The Netherlands Lorenz dominates the U.S. because some Dutch Lorenz ordinates are significantly greater and none is significantly smaller than the corresponding U.S. ordinates. As expected, the Gini coefficient ranking is consistent with the Lorenz analysis. Figure 1 depicts the Lorenz dominance relation showing the Netherlands Lorenz curve lying above (dominating) the U.S. Lorenz curve.

Table 4b compares the Dutch and Norwegian family Lorenz curves and shows a statistically significant crossing. The Dutch Lorenz ordinates are significantly larger than the corresponding Norwegian Lorenz ordinates at the third through fifth deciles, while Norway dominates The Netherlands at the seventh through ninth deciles. When some Lorenz ordinates are greater and some are smaller the two Lorenz curves cross and the degree of relative inequality is noncomparable by the Lorenz criteria. In contrast, the more ethically restrictive Gini coefficients suggest that there is no significant difference in relative family inequality.[11] Figure 2 shows that the two Lorenz curves cross between the fifth and sixth deciles.

The usefulness of the statistical procedures is most clearly illustrated in Table 4c which shows the equivalence of two Lorenz curves. Application of the numerical comparison method precludes the possibility of equal Lorenz curves and in the case the Dutch and Swiss per capita Lorenz curves would result in a conclusion that a crossing exists. However, application of the inference tests fails to reject the null hypothesis of equality and we conclude that there is no significant difference between any two pairs of ordinates when the Dutch and Swiss per capita family Lorenz curves are compared. The Ginis are consistent with the Lorenz curves in that they too are not significantly different. Again, it should be emphasized that the failure to account for sampling variability will result in overestimating the number of Lorenz curves that cross.

Tables 5a and 5b present the results of pairwise numerical and statistical comparisons of the family and per capita family Lorenz curves. For each country, the first position in each element shows the results of a numerical comparison while the second element shows the results of the statistical tests. A "+" means that a country in a row Lorenz dominates a country in a column. A "-" denotes a country in a row is dominated by a country in a column. A "0" means that the two Lorenz distributions are equal, while a "X" denotes crossing Lorenz curves.

Focusing first on the family definition of the recipient unit, the most striking result in Table 5a is that of the total of 17 numerical crossings only The Netherlands and Norway is significant. While the numerical analysis can rank slightly more than half of the family comparisons, more than 97 percent of the pairwise comparisons can be ranked when we account for sampling variation. Of the 16 remaining crossings, six are ranked as no different while ten provide dominance results.

For the per capita definition of the recipient unit Table 5b shows ten numerical crossings whereas the statistical analysis provides a complete ordering. Additionally, in three cases (Switzerland vs. Canada, the U.K. vs. Germany, and Sweden vs. Germany) the numerical approach suggests a dominance relationship while the statistical procedure fails to reject the null hypothesis of equality. In 13 out of the 36 pairwise comparisons the statistical procedures rank the Lorenz curves differently than the numerical comparisons.

The information in Tables 5a and 5b is summarized in Table 6. This table highlights the differences in the rankings obtained with the statistical and numerical methods. Using numerical comparisons, almost one-half of the pairwise family Lorenz comparisons and slightly more than one quarter of the per capita Lorenz comparisons resulted in crossings. In contrast, the statistical analysis results in a complete per capita Lorenz ordering and an almost complete family Lorenz ordering (one crossing). An important conclusion is that when sampling variabilty is taken into account the Lorenz curve is a powerful tool for evaluating relative inequality.

The information in Tables 5a and 5b can be used to construct an inequality ordering. Figures 3 and 4 present the LIS countries ranked from more equal at the top to less equal at the bottom on a family and per capita basis, respectively. Figure 3 shows that family incomes are most equally distributed in Sweden while the U.S. has the greatest degree of family inequality. The statistical ordering reveals several interesting dominance relations. Germany dominates the U.S. and is dominated by Canada, but is not statistically different from any other country. Similarly, Norway and The Netherlands are Lorenz dominated by Sweden, and dominate all other countries, but together they are noncomparable due to statistically significant intersection of Lorenz curves.

Figure 4 presents the percapita Lorenz orderings. Again, Sweden has the greatest equality of incomes and the U.S. the greatest inequality. Germany dominates the U.S. and The Netherlands and is not statistically different from any other country. The Norwegian and Dutch Lorenz curves do not cross on a per capita basis. Interestingly, Norway has the second most equal income on a per capita basis while having the second most unequal distribution of family incomes.

The differences in Figures 3 and 4 demonstrate that the definition of the income receiving unit has an important impact on the international Lorenz orderings. In eleven comparisons, the two measures of the recipient unit provide inconsistent orderings. For example, Canada dominates Australia when the family definition of the recipient unit is used while Australia dominates Canada in terms of per capita Lorenz curves. A ranking reversal (from dominated to dominating) occurs as the recipient unit changes in the following six pairwise comparisons: The Netherlands against Australia, Canada, and the U.K.; Australia against Canada and Switzerland; and the U.K. against Switzerland. In four pairwise comparisons, The Netherlands against both Switzerland and Germany, and Canada also against Switzerland and Germany the ranking changes from equality to dominance as the recipient unit changes. Finally, as noted above, unlike the family Lorenz curves the per capita Dutch and Norwegian Lorenz curves do not cross.

Given the changes in the inequality ordering across countries as the recipient unit is changed, it is of interest to make comparisons of Lorenz curves across income recipient units within countries. Using the information in Tables 2 and 3, we can test for Lorenz dominance across recipient units in each of the nine LIS countries. The test results reveal that incomes are more equally distributed on a per capita basis in Australia, Norway, Sweden, Switzerland, and the U.K. In contrast, incomes are more equally distributed on a family basis in The Netherlands and not significantly different across recipient units in West Germany. Interestingly, the U.S. and Canadian family and per capita Lorenz curves have a significant crossing with per capita cumulative income shares larger at the bottom of the distribution and smaller at the top. Thus, many of the cross country differences in the Lorenz ordering as the recipient unit is changed are traceable to the fact that there is no consistent pattern within countries between the relative distributions of family and per capita family incomes. Thus, international differences in variations in the size of families as the level of income changes has important implications for cross country comparisons of income inequality.[12]

IV. Conclusions

A priori there is reason to believe that the ability of to order income distributions with Lorenz curves has been seriously understated. This necessarily follows from the fact that simple numerical comparisons of the sample Lorenz ordinates preclude the finding of the equivalence of any two Lorenz curves. Ignoring the possibility of Lorenz equality increases the number of Lorenz crossings observed.

The empirical results of this study confirms this prediction of a more complete ranking using a statistical ranking procedure. With numerical comparisons, we find that almost one-half of the pairwise family Lorenz comparison and more than a quarter of the per capita Lorenz comparisons result in crossings. In contrast, the statistical analysis provides a complete percapita Lorenz ordering and an almost complete family Lorenz ordering (one crossing). An important conclusion is that when sampling variabilty is taken into account the Lorenz curve is a powerful tool for evaluating relative inequality.

On both a family and a per capita family basis, Sweden Lorenz dominates all of the other LIS countries. On the other hand, the United States is Lorenz dominated by all other countries using either definition of the recipient unit. Australia and Canada are generally Lorenz dominated by the European countries, while West Germany is generally found to be not significantly different from the other LIS countries. We find that the Lorenz ordering of LIS countries is sensitive to the definition of the income recipient unit. Comparisons within individual LIS countries reveals that there is no consistent pattern or relationship across countries between the Lorenz curves for the family definition of the recipient unit and the per capita definitions. Of all the LIS countries, The Netherlands ranking is found to be most sensitive to the income receiving unit choosen.

FOOTNOTES

1. A simple example can illustrate this point. Suppose two vectors of sample Lorenz ordinates, L^a = (.1, .25, .75, 1.0) and L^b = (.09, .25, .751, 1.0). A numerical comparison will result in a Lorenz crossing whereas it is quite likely that there are no differences in the underlying populations.

2. These procedures are developed in Bishop, Chakraborti, and Thistle (1988) and Bishop, Formby, and Thistle (1988).

3. For similar asymptotic results, see Goldie (1977) and Gail and Gastwirth (1978).

4. See Miller (1981) for a discussion of the SMM distribution. Beach and Richmond (1985) use the SMM distribution to test for the equality of two vectors of income shares.

5. To implement the test, tables for the percentiles of the SMM distribution by Stoline and Ury (1979) can be used.

6. See Beach and Davidson (1983) for an expression for the covariance terms.

7. While the family unit essentially assumes the marginal impact of an additional member is 0, the per capita family measure assumes that the marginal impact is 1. Thus, any family based equivalence scale will lie between these

two measures. For a discussion of the definition of the family in the various countries, Buhmann et al.

8. The choice of deciles is, of course, arbitrary. However, given a fixed sample size increasing the number of quantiles does not necessarily improve the quality of the overall test for Lorenz dominance.

9. Table 2a is comparable to Table 2(b) of O'Higgins et al. The quintile shares are equivalent while the Ginis differ slightly due to differences in the algorthims used.

10. The five percent critical value for testing for differences between two Ginis is 1.96.

11. As is well known, ranking crossing Lorenz curves requires specifying an explicit tradeoff between the upper and lower income classes.

12. In fact, six out of the eleven differences across recipient units involved comparisons with The Netherlands which is the only country to show unambiguously less equal incomes on a per capita basis.

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TABLE 2

LORENZ ORDINATES AND STANDARD ERRORS FOR FAMILY INCOMES

DECILE	AUSTRALIA	CANADA	NETHERLANDS	NORWAY	SWEDEN	SWITZERLAND	W.G.	U.K.	U.S.
1	0.0192 (0.0003)	0.0179 (0.0004)	0.0237 (0.0011)	0.0220 (0.0024)	0.0287 (0.0035)	0.0153 (0.0037)	0.0245 (0.0022)	0.0230 (0.0003)	0.0126 (0.0008)
2	0.0544 (0.0005)	0.0539 (0.0006)	0.0724 (0.0012)	0.0625 (0.0010)	0.0800 (0.0012)	0.0577 (0.0016)	0.0654 (0.0058)	0.0578 (0.0005)	0.0453 (0.0006)
3	0.1047 (0.0007)	0.1059 (0.0008)	0.1328 (0.0013)	0.1193 (0.0010)	0.1412 (0.0008)	0.1154 (0.0017)	0.1194 (0.0105)	0.1062 (0.0009)	0.0941 (0.0008)
4	0.1708 (0.0008)	0.1736 (0.0009)	0.2032 (0.0014)	0.1908 (0.0013)	0.2124 (0.0007)	0.1868 (0.0021)	0.1844 (0.0162)	0.1726 (0.0013)	0.1582 (0.0009)
5	0.2526 (0.0008)	0.2567 (0.0009)	0.2843 (0.0014)	0.2776 (0.0016)	0.2942 (0.0004)	0.2703 (0.0027)	0.2593 (0.0227)	0.2560 (0.0014)	0.2384 (0.0009)
6	0.3503 (0.0006)	0.3558 (0.0007)	0.3775 (0.0015)	0.3801 (0.0018)	0.3866 (0.0006)	0.3660 (0.0034)	0.3456 (0.0302)	0.3548 (0.0011)	0.3360 (0.0007)
7	0.4644 (0.0003)	0.4713 (0.0002)	0.4860 (0.0015)	0.4984 (0.0021)	0.4964 (0.0011)	0.4746 (0.0043)	0.4464 (0.0390)	0.4699 (0.0005)	0.4525 (0.0005)
8	0.5998 (0.0008)	0.6058 (0.0008)	0.6128 (0.0014)	0.6331 (0.0026)	0.6311 (0.0015)	0.6006 (0.0053)	0.5639 (0.0492)	0.6045 (0.0009)	0.5914 (0.0010)
9	0.7642 (0.0010)	0.7675 (0.0008)	0.7661 (0.0015)	0.7879 (0.0034)	0.7921 (0.0016)	0.7515 (0.0066)	0.7022 (0.0612)	0.7667 (0.0009)	0.7595 (0.0012)
10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
GINI	0.3439 (0.0018)	0.3383 (0.0020)	0.3082 (0.0036)	0.3057 (0.0045)	0.2875 (0.0046)	0.3323 (0.0074)	0.3578 (0.0470)	0.3377 (0.0028)	0.3624 (0.0022)

TABLE 3

LORENZ CURVES AND STANDARD ERRORS FOR PERCAPITA INCOME

DECILE	AUSTRA LIA	CANADA	NETHER LANDS	NORWAY	SWEDEN	SWITZER LAND	W.G.	U.K.	U.S.
1	0.0259 (0.0004)	0.0252 (0.0005)	0.0228 (0.0010)	0.0344 (0.0018)	0.0365 (0.0028)	0.0232 (0.0033)	0.0349 (0.0034)	0.0371 (0.0005)	0.0175 (0.0010)
2	0.0736 (0.0006)	0.0714 (0.0006)	0.0696 (0.0011)	0.0910 (0.0009)	0.0971 (0.0011)	0.0700 (0.0015)	0.0837 (0.0081)	0.0895 (0.0006)	0.0575 (0.0007)
3	0.1328 (0.0006)	0.1301 (0.0007)	0.1264 (0.0012)	0.1569 (0.0010)	0.1682 (0.0007)	0.1261 (0.0015)	0.1409 (0.0136)	0.1510 (0.0008)	0.1118 (0.0008)
4	0.2014 (0.0007)	0.1983 (0.0008)	0.1928 (0.0014)	0.2315 (0.0013)	0.2481 (0.0006)	0.1918 (0.0020)	0.2059 (0.0199)	0.2220 (0.0010)	0.1788 (0.0009)
5	0.2791 (0.0007)	0.2773 (0.0009)	0.2697 (0.0015)	0.3139 (0.0016)	0.3365 (0.0003)	0.2679 (0.0026)	0.2791 (0.0270)	0.3030 (0.0011)	0.2579 (0.0009)
6	0.3690 (0.0006)	0.3682 (0.0009)	0.3583 (0.0015)	0.4060 (0.0019)	0.4354 (0.0005)	0.3567 (0.0033)	0.3614 (0.0349)	0.3944 (0.0011)	0.3503 (0.0009)
7	0.4751 (0.0005)	0.4738 (0.0009)	0.4639 (0.0015)	0.5101 (0.0023)	0.5461 (0.0009)	0.4610 (0.0042)	0.4568 (0.0441)	0.4992 (0.0011)	0.4589 (0.0008)
8	0.6026 (0.0004)	0.5999 (0.0008)	0.5907 (0.0013)	0.6314 (0.0028)	0.6712 (0.0011)	0.5869 (0.0052)	0.5674 (0.0547)	0.6230 (0.0009)	0.5879 (0.0005)
9	0.7626 (0.0007)	0.7566 (0.0008)	0.7501 (0.0009)	0.7802 (0.0033)	0.8164 (0.0012)	0.7431 (0.0064)	0.7022 (0.0676)	0.7739 (0.0007)	0.7489 (0.0003)
10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
GINI	0.3156 (0.0037)	0.3980 (0.0039)	0.3311 (0.0068)	0.2689 (0.0141)	0.2289 (0.0036)	0.3346 (0.0070)	0.3335 (0.0547)	0.2814 (0.0061)	0.3461 (0.0037)

Table 4

Examples of Inference Based Lorenz Dominance, Crossings, and Equivalence

4a Lorenz Dominance -- Family Incomes

Decile	Netherlands	U.S.	Test Statistics
1	0.0237	0.0126	8.1701
2	0.0724	0.0453	17.3324
3	0.1328	0.0941	20.3873
4	0.2032	0.1582	20.1120
5	0.2843	0.2384	18.2395
6	0.3775	0.3360	15.0331
7	0.4860	0.4525	11.5708
8	0.6128	0.5914	7.3239
9	0.7661	0.7590	2.3791
10	1.0000	1.0000	-
Gini	0.3082	0.3624	-12.7435

4b Lorenz Crossing -- Family Incomes

Decile	Netherlands	Norway	Test Statistics
1	0.0237	0.0220	0.6452
2	0.0724	0.0625	3.7651
3	0.1328	0.1193	4.9664
4	0.2032	0.1908	4.3403
5	0.2843	0.2776	2.2086
6	0.3775	0.3801	-0.8159
7	0.4860	0.4984	-3.5909
8	0.6128	0.6331	-5.4233
9	0.7661	0.7879	-5.2383
10	1.0000	1.0000	-
Gini	0.3082	0.3057	-0.4497

4c Lorenz Equivalence -- Per Capita Incomes

Decile	Netherlands	Switzerland	Test Statistics
1	0.0228	0.0232	-0.1216
2	0.0696	0.0700	-0.1317
3	0.1264	0.1261	0.0670
4	0.1928	0.1918	0.2530
5	0.2697	0.2679	0.4268
6	0.3583	0.3567	0.3528
7	0.4639	0.4610	0.5623
8	0.5907	0.5869	0.6373
9	0.7501	0.7431	0.9982
10	1.0000	1.0000	-
Gini	0.3311	0.3346	-0.3346

Table 5
Pairwise Numerical and Inference Based Comparisons
of Family and Per Capita Lorenz Distributions of Income

5a Family Income

	Australia	Canada	Netherlands	Norway	Sweden	Switzerland	W.G.	U.K.	U.S.
Australia	**								
Canada	x+	**							
Netherlands	++	x+	**						
Norway	++	++	xx	**					
Sweden	++	++	++	x+	**				
Switzerland	x+	++	--	--	--	**			
W.G.	x0	x+	x0	x0	-0	x0	**		
U.K.	++	x+	x-	x-	--	x-	x0	**	
U.S.	--	--	--	--	--	x-	x-	--	**

5b Per Capita Income

	Australia	Canada	Netherlands	Norway	Sweden	Switzerland	W.G.	U.K.	U.S.
Australia	**								
Canada	--	**							
Netherlands	--	--	**						
Norway	++	++	++	**					
Sweden	++	++	++	++	**				
Switzerland	--	-0	x0	--	--	**			
W.G.	x0	x0	x+	x0	-0	x0	**		
U.K.	++	++	++	x-	x-	++	+0	**	
U.S.	--	--	--	--	--	x-	x-	--	**

Note: A "+" means the country in the row dominates the country in the column.

A "-" means the country in the row is dominated by the country in the column.

An "x" means the distributions cross.

A "0" means the distributions are equivalent.

The first position in each element shows the Lorenz ordering based upon a naive numerical comparison.

The second position in each element shows the Lorenz ordering based upon statistical inference using a five percent test.

Table 6

Summary of Numerical and Statistical Lorenz Comparisons

	Numerical Comparisons		Statistical Comparisons	
	Number	Percent	Number	Percent
Family Income				
Dominance	19	52.8	28	80.5
Crossings	17	47.2	1	2.8
Equivalence	0	0.0	6	16.7
	36		36	
Per Capita Income				
Dominance	26	72.2	28	77.8
Crossings	10	27.8	0	0.0
Equivalence	0	0.0	8	22.2
	36		36	

Figure 1

Lorenz Dominance

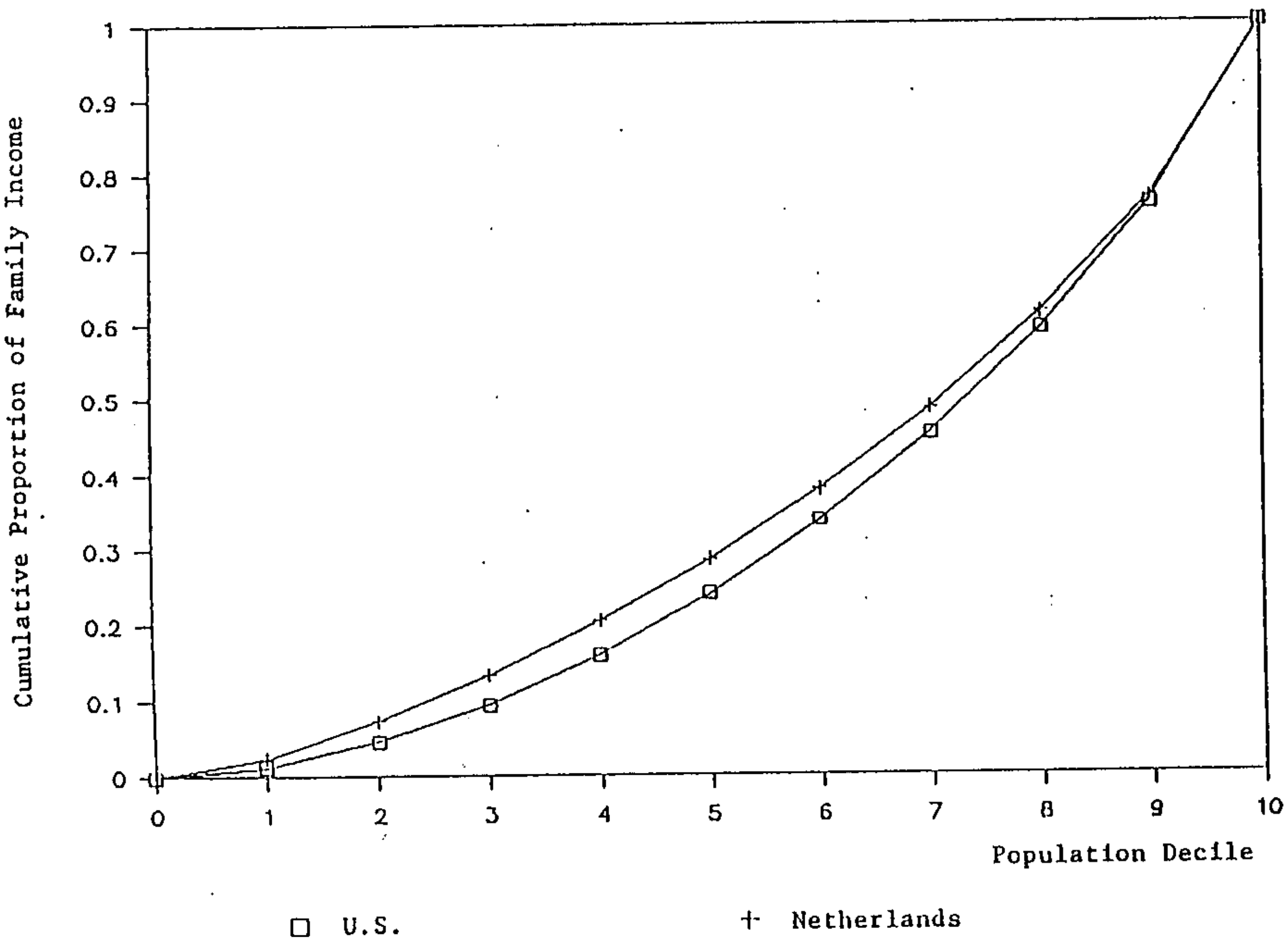


Figure 2

Lorenz Crossing

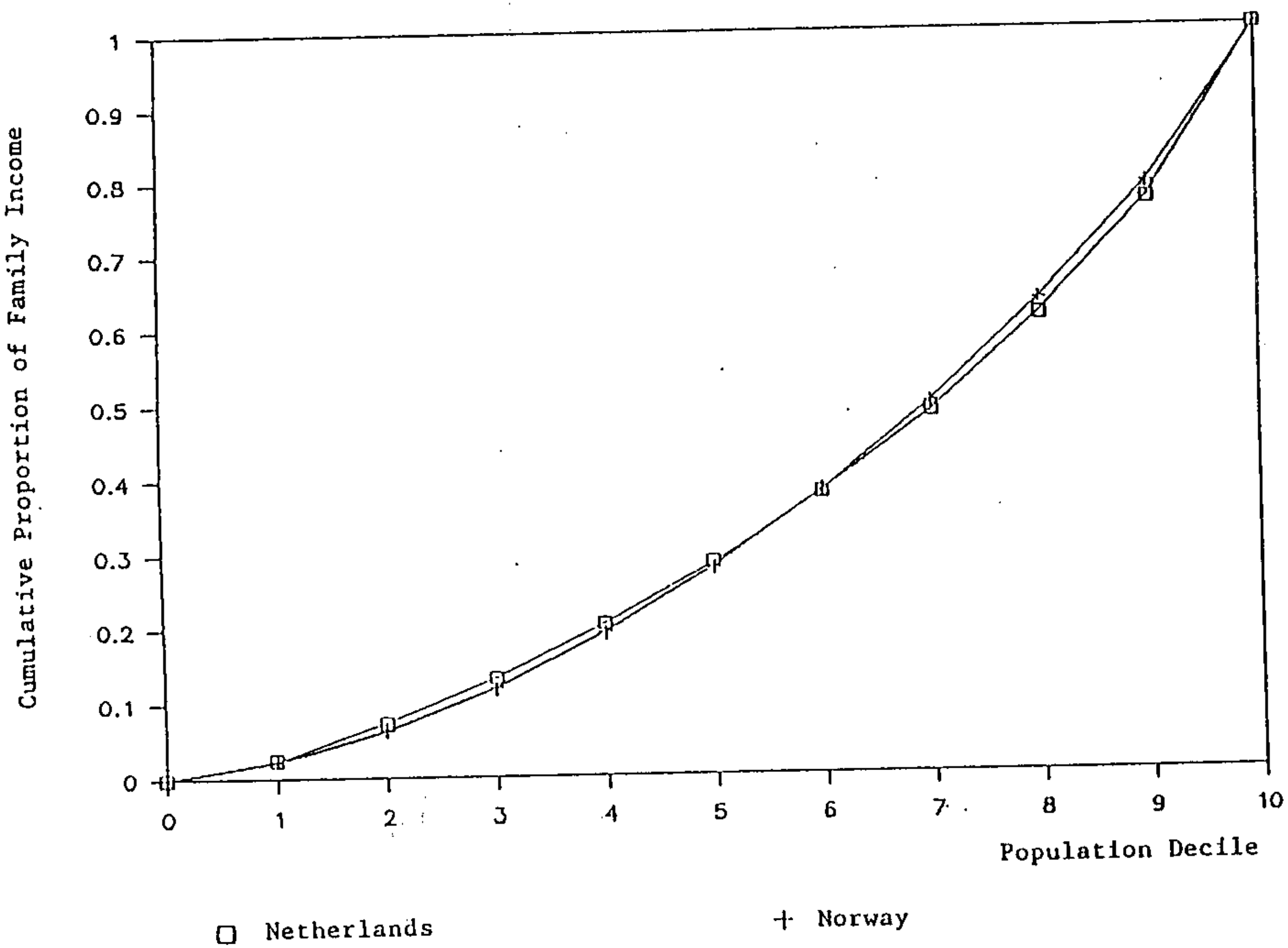
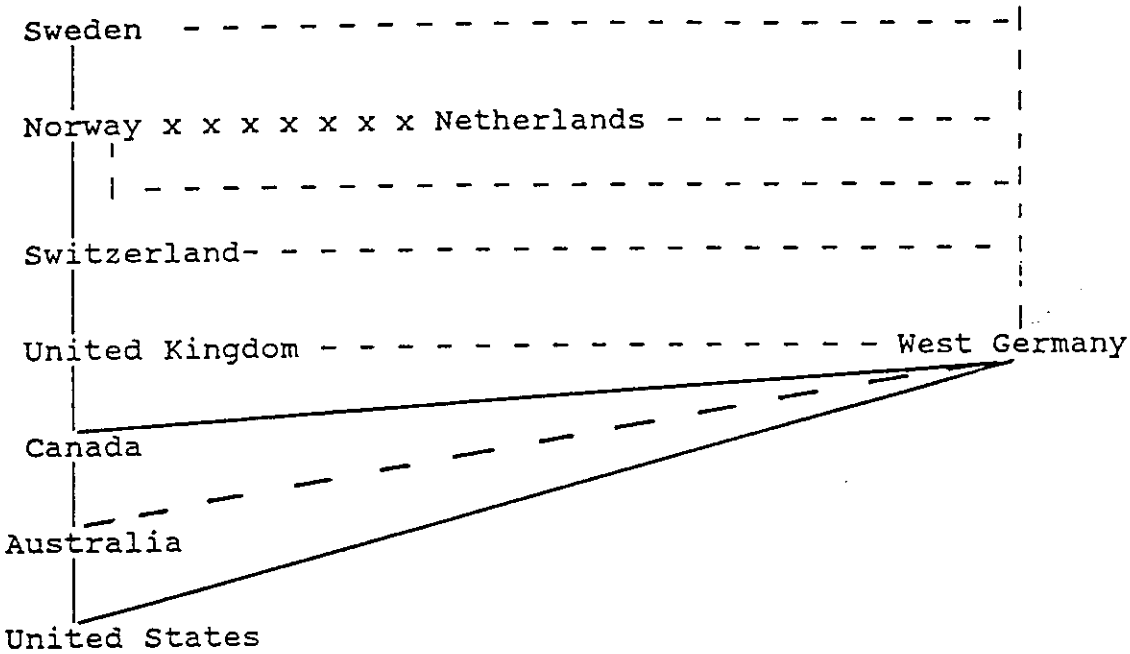


Figure 3
Lorenz Distributions of Family Income--Ordered
by Statistically Significant Differences



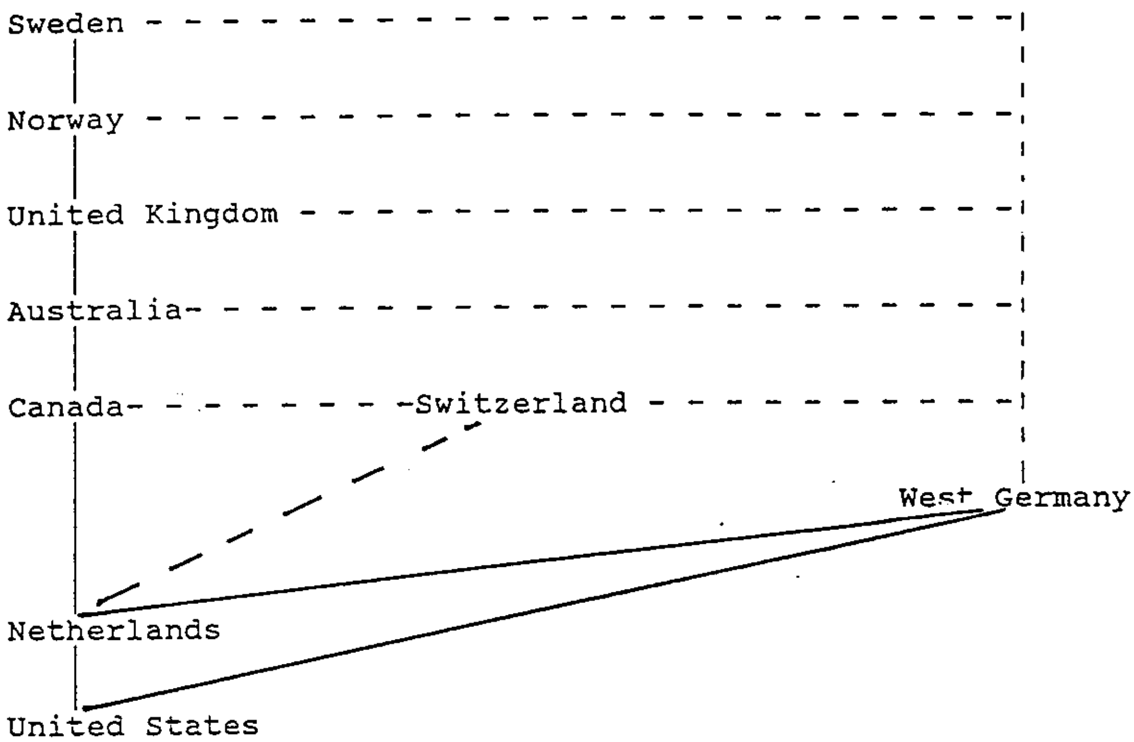
Note: Solid lines connecting two countries indicates statistically significant Lorenz dominance.

Dashed lines indicate statistically equivalent Lorenz curves.

A line of xxxx's indicate a statistically significant crossing.

Figure 4

Lorenz Distributions of Per Capita Income--Ordered by Statistically Significant Differences



Note: Solid lines connecting two countries indicates statistically significant Lorenz dominance.

Dashed lines indicate statistically equivalent Lorenz curves.