Luxembourg Income Study Working Paper Series

Working Paper No. 165

International comparisons of Poverty Intensity: Index Decomposition and Bootstrap Inference

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May 1997

Luxembourg Income Study (LIS), asbl

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June 1997

[1]Paper to be presented at 51st Session of the International Statistical Institute, Istanbul, Turkey, August 22, 1997, Topic 72. We wish to thank our research assistants Lynn Lethbridge and Janice Yates for their excellent research assistance and the Social Sciences and Humanities Research Council of Canada for its financial support under grant #410-93-0557 to Osberg. E-mail: Lars.Osberg@Dal.Ca and Kuan.Xu@Dal.Ca.

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Abstract

This paper proposes an alternative formulation for the Sen-Shorrocks index of poverty intensity for survey data with sampling weights, and decomposes the Sen-Shorrocks index into the poverty rate, the average poverty gap ratio among the poor, and the overall Gini index of poverty gap ratios. This decomposition allows the percentage change in poverty intensity to be approximated as the sum of the percentage changes in the poverty rate and average poverty gap ratio. To account for sampling variation in estimates of poverty intensity, this paper also uses the bootstrap method to compute confidence intervals in international comparisons using Luxemburg Income Study data. Cross-sectional and longitudinal analyses indicate that in the early 1970's poverty intensity in Canada and the U.S. was almost indistinguishable, but in the 1970's Canadian poverty intensity decreased. Large increases in poverty intensity occurred in the 1980's in the United States, the United Kingdom, and Sweden.

1. Introduction

What can we say *for sure* about poverty intensity rankings? How should we interpret these rankings and their changes? International comparisons of poverty have become common in recent years, partly because the availability of data sets such as the Luxemburg Income Study has made the entire social policy debate more international. The outcomes observed in different countries are often viewed as "natural experiments" in the implications of social policy decisions, and international comparisons of social outcomes have expanded the idea of the possible - no longer are social policy debates within countries confined to the examination of purely national experiences. Furthermore, international comparisons have the potential to influence social policy, if national political processes are affected by a sense of shame at bad relative performance in poverty and inequality or if complacency accompanies a "good" ranking.

However, international comparisons of poverty intensity are also often contentious. Because poverty calculations are inevitably based on data which is a *sample* from the population (usually a stratified random sample), international comparisons are more useful if it can be shown that rankings are robust to any sampling variation in the data. Comparisons are also more meaningful if trends in poverty intensity can be decomposed into their underlying contributing factors. The purpose of this study is, therefore: (I) to propose a modified index of poverty intensity which is suitable for survey data with sampling weights; (ii) to introduce a bootstrap-based statistical inference of this index and the Gini index of inequality; (iii) to decompose the index of poverty intensity into three meaningful and familiar poverty measures; and (iv) to apply the above measures to actual data to provide an international comparison of poverty intensity and contributing factors across major industrialized countries and over time.

In the popular debate on poverty, the most commonly used statistic is the poverty rate (the percentage of the population whose incomes lie below the poverty line), but such a measure does not reflect the amount by which the incomes of the poor fall below the poverty line. The average poverty gap ratio (the average percentage difference between poor individuals' incomes and the poverty line) is also a common, simple statistic, but it ignores the number of poor people and the degree of inequality among the poor. Poverty researchers have therefore increasingly turned to measures of poverty intensity which do account jointly for the number of poor, depth of poverty and inequality among the poor.

The measurement of poverty intensity we use was initially advocated by Sen (1976), modified by Shorrocks (1995), and, thus, is called the Sen-Shorrocks index of poverty intensity (the S-S index hereafter). The S-S index is symmetric, replication invariant, monotonic, homogeneous of degree zero in incomes and the poverty line, and normalized to take values in the range [0,1] - all of which are important characteristics. However, the S-S index is a theoretical measure, premised on the assumption that all the individual incomes are known.

In reality, researchers do not have the incomes of the population. Instead, survey data with sampling weights are available. We therefore modify the S-S index to accommodate sampling weights and develop a method of statistical inference for the modified S-S index.

Although the S-S index of poverty intensity is a comprehensive and desirable measure, it currently lacks a straightforward decomposition. We therefore decompose the S-S index into three familiar poverty measures - the poverty rate (sometimes called the headcount ratio), the average poverty gap ratio among the poor, and the overall Gini index of poverty gap ratios. Using this decomposition, we trace changes in poverty intensity to their contributing factors, and rank countries in terms of poverty intensity using the multi-country database of the Luxemburg Income Study.

The remainder of the paper is organized as follows. Section 2.1 suggests an alternative formulation of the S-S index and a related bootstrap method for survey data with sampling weights, and proposes the decomposition method of the S-S index into three meaningful and familiar poverty measures. Section 2.2 briefly discusses data sources, the calculation of equivalent incomes, and the poverty line. Section 3.1 compares the poverty intensity of major developed countries using the modified S-S index and the bootstrap estimates of the sampling variability. Section 3.2 considers the relationship between poverty intensity and inequality. Section 3.3 decomposes the changes in poverty intensity in developed countries into their contributing factors - changes in the poverty rate and average poverty gap ratio. Concluding remarks are given in Section 4.

2. Research Methodology

2.1. The S-S Index: Decomposition and Inference

Since Sen (1976) proposed a poverty index and a set of desirable criteria for evaluating a poverty index in his seminal paper, research on poverty indices has received considerable attention.[2] As the Sen index is not replication invariant, not continuous in individual incomes, and fails to satisfy the transfer axiom, Shorrocks (1995) has recently proposed a modified Sen index (the Sen-Shorrocks index or the S-S index hereafter) for measuring the intensity of poverty, which satisfies the above criteria and represents an advance in the research on poverty intensity measures along the path pioneered by Sen (1976).

Statistical inferences of different poverty measures have been provided by Bishop, Chow and Zheng (1995), Rongve (1995), Preston (1995), and Zheng, Cushing and Chow (1995). Xu (1995) proposed a method of statistical inference for the S-S index. However, Xu (1995) did not consider the sampling weights which are commonly used in large-scale social statistical surveys.

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[2]See, among others, Atkinson (1987), Besley (1990), Blackorby and Donaldson (1980), Donaldson and Weymark (1986), Foster, Greer and Thorbecke (1984), Foster and Shorrocks (1988, and 1991), and Takayama (1979). In addition, Kakwani (1980), Foster (1984), Hagenaars (1986), and Seidl (1988) have provided useful surveys of this literature.

The S-S index is proposed assuming that all the income data of a population are known and nonstochastic. Let *i*th-person's income of the population size N be Y_i such that $Y_1 < Y_2 < ... < Y_N$. and the poverty line be $z > 0$. Let Q ($< N$) be the number of individuals whose income is less than z. For the *i*th poor person, the poverty gap is $z\text{-}Y_i$, and the poverty gap ratio is $(z - Y_i) / z$. The S-S index is defined as [see Shorrocks (1995)]:

(1) $$P(Y;z) \frac{1}{N^2} \sum_{i=1}^{Q} (2N - 2i + 1)\frac{z - Y_i}{z}.$$

It can be regarded as a weighted "average" of individual poverty gap ratios of the poor. The S-S index is desirable because (I) it is symmetric, replication invariant, monotonic, homogeneous of degree zero in individual incomes and the corresponding poverty line, and normalized to take values in the range [0,1]; (ii) it is continuous in individual incomes and consistent with the transfer axiom; and (iii) it admits a geometric interpretation.[3] $P(Y;z)$ can be computed based on Equation (2) if the individual incomes of all members of the population are available.

2.1.1. Decomposition

The decomposition of the Sen-Shorrocks index of poverty intensity [as shown in Shorrocks (1995, p. 1228)] is given by:

(2) $$P(Y;z) = \mu(X)[1 - G(X)],$$

where $\mu(X)$ and $G(X)$ are the average poverty gap ratio and Gini coefficient of poverty gap ratios,

(3) $$X_i = \frac{z - Y_i}{z}, \qquad i = 1,2, ...,N,$$

with the non-poor population's X_i being set to zero.

[3]See Shorrocks (1995) for details regarding the properties of the index.

Note that $\mu(X)$ and $G(X)$ must be differentiated from the more commonly used Gini index of income inequality among the poor or the average income gap ratio among the poor - both of which are components of Sen's original index. The population data set X for $\mu(X)$ and $G(X)$ refer to the poverty gap ratios calculated *for all members of the population*. The poverty gap ratios (X) are truncated at zero, for the non-poor subpopulation. To compute $\mu(X)$ and $G(X)$, the poverty gap ratios must be sorted in ascending order [which implies descending order of income in Equation (3)].[4]

A further decomposition can be based on the fact that $\mu(X)$ is simply the weighted average of the average poverty gap ratio among the poor and the poverty gap ratio among the non-poor (i.e., zero), where the weights are the population proportions (i.e., the poverty rate and one minus the poverty rate, respectively). Let *RATE* be the poverty rate,

(4) $$RATE \quad \frac{Q}{N},$$

and *GAP* the familiar average poverty gap ratio among the poor,

(5) $$GAP \ \frac{1}{Q} \ {}^{N}_{i\,N\,Q\,1} X_i.$$

It is easy to see that

(6) $$\mu(X) = (RATE)\,(GAP) + (1\text{-}RATE)\,(0),$$

$$= (RATE)\,(GAP).$$

Hence, the Sen-Shorrocks index can be further decomposed into:

(7) $$P(Y; z) = (RATE)\,(GAP)\,(1+G(X)).$$

It is useful to transform Equation (7) into the following form:

(8) $$\ln(P(Y; z)) = \ln(RATE) + \ln(GAP) + \ln(1+G(X)).$$

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[4]Note that Equation (11) of Shorrocks (1995), which can be read as implying a descending order of the poverty gap ratio, is a bit misleading.

The overall percentage rate of change in poverty intensity can then be expressed as the sum of the percentage changes in the poverty rate, average poverty gap ratio (among the poor), and Gini index of inequality in the poverty gap ratios (among all people).[5]

$$\ln P(Y; z) = \ln(RATE) + \ln(GAP) + \ln(1+G(X)). \quad (9)$$

One of the problems of the poverty literature is that it can be difficult to interpret measures of poverty intensity, like the Sen-Shorrocks index, which have desirable theoretical properties. Important theoretical advances in poverty measurement have been made in the academic community, but these have had relatively little impact on public debate - perhaps partly because of the limited mathematics background of much of the policy and advocacy community. Equations (7) and (9) provide a straightforward decomposition of the S-S index of poverty intensity which can be readily interpreted by policy makers, social science researchers and the general public. Indeed, as Section 3.3 will demonstrate, since changes in $[1+G(X)]$ are in practice very small, for practical purposes the percentage change in poverty intensity can be approximated as the sum of the percentage changes of the poverty rate and the average poverty gap ratio.

2.1.2. Inference

The data that economists normally use contain the sample incomes of households with sampling weights. Let m households in the sample be ordered by their equivalent incomes in an ascending order and be indexed by I. Let the total number of households whose equivalent income is below the poverty line z be q ($< m$). Let the sample household *equivalent income* of household I, that is shared by all the members of that family, be y_i. Let the number of family members of the ith household be n_i, and the sampling weight[6] of the ith household w_i. Thus the total number of individuals is $\sum_{i=1}^{m} n_i w_i$. To accommodate complex survey data, the following formulation for the S-S index for survey data with sampling weights is proposed below:

$$P(y;z) \quad \frac{1}{[\sum_{l=1}^{m} n_l w_l]^2} \sum_{i=1}^{q} \sum_{j=1}^{n_i w_i} [2(\sum_{l=1}^{m} n_l w_l) - 2(\sum_{k=1}^{i} n_{k-1} w_{k-1}) - 1] \frac{z - y_i}{z}, \quad (10)$$

where $n_0 = 0$, and $w_0 = 0$.

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[5]The term $\ln(1+G(X))$ is an approximate of $G(X)$ based on the first-order Taylor series expansion.

[6] According to the Luxembourg Income Study, the variable of the household weight ("HWEIGHT") contains the sample weight for each sample case in the data set for a particular country. This weight essentially means that this sample case represents that many units within the total population of units.

This modified S-S index maintains all the properties of the original S-S index. In particular, it is symmetric and duplication invariant. These two properties justify our treatment of the tied equivalent incomes of the individuals who come from the same household. According to Sen (1976) and Shorrocks (1995), the incomes used to compute either the Sen poverty index or the S-S index are assumed to have no tied observations. In reality, observed incomes of equal values are not rare. Theoretically, this problem can be circumvented by the property of replication invariance of the S-S index, i.e., if two identical income data sets are merged, the value of the S-S index for the merged new data set does not differ from that for each individual data set. Hence the property of replication invariance allow the S-S index to accommodate the tied observations. Furthermore, the property of symmetry of the S-S index suggests that identical equivalent incomes should be treated symmetrically.

The sample counterpart of Equation (3) is:

(11) $$x_i \quad \frac{z \; y_i}{z}, \qquad i \;\; 1,2, \; , \; \sum_{j \;\; 1}^{m} n_j w_j,$$

with the non-poor individual's x_i being set to zero. To compute sample counterparts of $\mu(X)$, $G(X)$, *RATE*, and *GAP*, sample poverty gap ratios must be sorted in ascending order. The decomposition shown in Equations (7), (8) and (9) can be applied to the survey data with sampling weights when X is replaced by x, and *RATE* and *GAP* are replaced, respectively, by their sample estimates $\hat{RATE}$ and $\hat{GAP}$ where " " denotes an estimate computed from a sample of the population. For the survey data with sampling weights, $\mu(x)$ is given by

(12) $$\mu(x) \quad \frac{\sum_{i \;\; 1}^{m} n_i w_i x_i}{\sum_{i \;\; 1}^{m} n_i w_i},$$

an be decomposed further into $\hat{RATE}$ and $\hat{GAP}$ as

(13) $$\mu(x) \left(\frac{\sum_{i \; n \; q \;\; 1}^{m} n_i w_i}{\sum_{i \;\; 1}^{m} n_i w_i} \right) \left(\frac{\sum_{i \;\; 1}^{m} n_i w_i x_i}{\sum_{i \; n \; q \;\; 1}^{m} n_i w_i} \right) \quad \hat{RATE} \quad \hat{GAP} \; .$$

$G(x)$ in this case should be defined as

$$G(x) \;1\; \sum_{i\;1}^{m}\left(\frac{w_i n_i}{\sum_{j\;1}^{m} w_j n_j}\right)\left(\frac{\sum_{k\;1}^{i} w_k n_k x_k \;\; \sum_{k\;1}^{i\;1} w_k n_k x_k}{\sum_{j\;1}^{m} w_j n_j x_j}\right). \tag{14}$$

In this paper, we propose a bootstrap procedure to compute the standard deviation of the modified S-S index estimator. According to Xu (1995), the linear combination of the poverty gap ratios can be viewed as a linear combination of order statistics following Stigler (1969, 1974) and Ghosh (1971). But the analytical variance (or standard deviation) of $P(y;z)$ is complex. One may want to rely on the computing intensive bootstrap method to compute the bootstrap variance (or standard deviation) as proposed by Efron (1979, 1982) and Efron and Tibshirani (1986). Essentially, each individual household in the survey data has equal probability to be drawn in the resampling.

To compute the bootstrap standard deviation of the modified S-S index estimator, we resample both equivalent incomes and the sampling weights associated with them. We generate a random integer t, from a uniform distribution defined over the support from zero to the total number of the households m . Then we use this random integer to draw the tth household equivalent income, the number of members of the tth household, and the sampling weight. The new sample of size m is denoted by $\{y_i^*, w_i^*, n_i^*\}_{i=1}^{m}$. The new sample can then be used to compute a new S-S index denoted as $P(y^{*t},z^{*t})$. Repeating this process T times (e.g. T=200) gives $P(y^{*1},z^{*1})$, $P(y^{*2},z^{*2})$,..., $P(y^{*T},z^{*T})$. The bootstrap variance is computed as the sample variance from the large number of the standard S-S index estimates from the resampling. We denote the sampling variance of $P(y;z)$ as $^2(P(y;z))$, see Efron (1982, Chapter 8) for details. We can approximate a 95% confidence interval by adding two bootstrap standard deviations on each side of the S-S index estimate when ranking the examined countries.[7]

2.2. Data, Equivalent Income, and Poverty Line

In this paper, we analyze changes of poverty intensity over time for the following countries: Australia (1981, 1985, and 1989), Austria (1987), Belgium (1985, 1988, and 1992), Canada (1971, 1975, 1981, 1987, 1991 and 1994), Denmark (1987 and 1992), Finland (1987 and 1991), France (1979, 1981, and 1984), Germany (1981, 1983, and 1984), Hungary (1991), Ireland (1987), Israel (1979,

[7]It should be noted that the S-S index estimator is "non-pivotal" in the sense of Hall (1992, p. 14) in that its sampling distribution depends upon some unknown parameters(s). The bootstrap sampling distribution may differ from the true sampling distribution by an error. The error is in the order of 1/[sample size] if the statistic is pivotal; it is in the order of $1/\sqrt{[sample\ size]}$ if the statistic is non-pivotal [see Hall (1992, pp. 83-85)]. Given the sample sizes used here are very large, the bootstrap error becomes negligible.

1986, and 1992), Italy (1986 and 1991), Luxembourg (1985), the Netherlands (1983, 1987, and 1991), Norway (1979, 1986, and 1991), Poland (1986, 1992), Russia (1992, 1995), Spain (1980, 1990), Sweden (1975, 1981, 1987, and 1992), Switzerland (1982), Taiwan (1986, 1991), United Kingdom (1979, 1986 and 1991), and United States (1974, 1979, 1986, 1991, and 1994).[8]

This paper assumes that income is shared within families. However, the focus of welfare comparisons is the distribution of income among persons. We therefore calculate the "equivalent income" of all individuals, and measure poverty intensity in terms of equivalent income. In the literature, a number of equivalence scales have been used to account for the economies of scale of household consumption [Burkhauser et al. (1996), and Phipps and Garner (1994)].

The issues raised by different equivalence scales are important, but to keep this paper focussed, we simply use the "OECD" equivalence scale, which calculates the equivalent income of each household member as:

(15) $$EI \quad Y/(1 \quad .7(N_a \quad 1) \quad .5N_c).$$

Here Y is total household income (Disposable Personal Income), N_a is the number of the adults in the household and N_c is the number of the children under age 18.

As Hagenaars (1991) and many others have noted, there has long been a debate on how best to conceptualize poverty. In very poor countries, where many people may be continually hungry, poverty can best be seen in absolute terms, but in developed countries we take the view that social norms within each country as to a minimally adequate standard of living differ across countries and change over time and are in fact heavily influenced by the prevailing average standard of living [see Osberg (1984), pp. 61-73]. We therefore adopt the commonly accepted standard of half the median equivalent income as the poverty line in each country, at each point in time.[9]

We therefore assume that within all the sampled countries, at all dates: (i) household income (after tax) is equally shared among all household members , (ii) the OECD equivalence scale adequately accounts for economies of scale in family consumption; and (iii) the poverty line is represented by half the median equivalent income. Clearly, these are strong assumptions. Sharif and Phipps (1994) have, for example, demonstrated the sensitivity of child poverty in Canada to alternative assumptions about the intra-household distribution of resources, and one could plausibly argue that sharing norms within families vary over time and across countries. There is a considerable literature on intra household

[8]Rose and McAllister (1996) argue convincingly that money income has been a poor measure of command over goods and services in Eastern Europe, and that its relative importance is changing rapidly in the economic transition process. For this reason, our money income poverty intensity calculations for the transition economies are presented separately, in Appendix 2.

[9]We note that this does *not* imply either that poverty cannot be eliminated or that poverty and inequality are identical issues, since the fraction of a population below half the median is a characteristic of only the lower tail of the distribution of income - see Section 3.2.

allocation, equivalence scales and poverty lines, but we make these assumptions in order to focus attention on issues which have, thus far, been neglected in the literature.[10]

3. International Comparisons

As column 1 of Table 1 indicates, international comparisons of poverty and inequality are based on *samples*, of widely varying sizes, drawn from national populations. Within countries, individual observations are often sampled with widely varying probabilities.[11] Since the inclusion, or exclusion, of a particular observation can enter with widely varying weights, estimates of population characteristics are potentially subject to "the luck of the draw" in sampling variability. In the literature, it has been common for point estimates of poverty and inequality to be reported without comment.[12]

As column 2 of Table 1 indicates, the actual raw number of observations below the poverty line can be, in some countries, rather small [e.g., 67 in Luxemburg (1985), 121 in the Netherlands (1987)] but in other instances the number of poor observations is large [e.g., 10,762 in the U.S. (1994); this number outnumbers the entire sample size in most other countries]. To account for the impact of

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[10]Parenthetically, we would note that measures of poverty intensity can be somewhat sensitive to data handling conventions. Although Rawls (1971) has argued that, for ethical reasons, public policy should emphasize most the well-being of the least well off, in practice poverty researchers face a quandary in deciding how to treat very low incomes. In some data sets, individuals with negative incomes appear. For example, the 1992 Swedish micro-data contains an income of - 3, 533, 937 Kroner [= -$641,000 (Cdn)] and the 1987 Danish date contains an income of - 656,959 Kroner [= -$143,000 (Cdn.)]. Researchers commonly delete such observations, because they would otherwise totally dominate measures of inequality or poverty which emphasize the well-being of the very poor (e.g., the Atkinson family of inequality measures) and because one can suspect that they are really stockbrokers (or other business people) with large capital losses in a particular year, but who really have substantial wealth. It is less clear how to deal with individuals who record zero money income, which may be a valid measure of money income (e.g., for members of monastic religious orders) but which may also be a measurement error. As Table A.1 in Appendix A indicates, in the LIS data base, countries differ quite a lot in the percentage of the sample with zero recorded money income. Although this makes little difference to a measure of aggregate inequality such as the Gini Index, it can make a difference to the S-S index of poverty intensity. Table A.1 also reveals that some countries, in some years, sanitize their data by removing zero income observations. To keep our data comparable, and because real income must be positive (to ensure the consumption required for survival), we therefore delete all observations with zero recorded income.

[11]Although LIS documentation indicates that the Belgian data is a straight random sample, other countries use a stratified random sample design, with sampling weights varying by as much as a factor of 391 in Norway (1991), 184 in Canada (1981), etc.

[12]Wolfson (1986) is an exception, which demonstrated the sensitivity of measures of income inequality to sampling variability by comparing the impact of dropping the top 5 income observations in different years of the Canadian Survey of Consumer Finance.

Table 1
Poverty Intensity and Inequality in LIS Data

Country	No. of Observations	No. of Observations Below poverty line	Sen-Shorrocks Index	Standard Deviation of 200 Bootstraps	Gini Index	Standard Deviation of 200 Bootstraps
Austria (87)	11147	718	0.0148	0.00111	0.210	0.00256
Luxembourg (85)	2044	67	0.0158	0.00311	0.238	0.00448
Germany (83)	42746	1222	0.0170	0.00081	0.260	0.00168
Finland (91)	11740	594	0.0190	0.00118	0.224	0.00183
Belgium (92)	3779	148	0.0195	0.00220	0.221	0.00338
Finland (87)	11856	498	0.0195	0.00126	0.220	0.00177
Belgium (85)	6446	260	0.0206	0.00177	0.230	0.00268
Norway (86)	4967	167	0.0207	0.00214	0.222	0.00291
Belgium (88)	3751	148	0.0208	0.00237	0.235	0.00395
Norway (91)	8059	249	0.0211	0.00231	0.222	0.00433
The Netherlands	4670	143	0.0235	0.00274	0.266	0.00687
Germany (84)	5159	323	0.0240	0.00254	0.253	0.00475
Germany (81)	2852	150	0.0247	0.00340	0.251	0.00466
Norway (79)	10307	432	0.0249	0.00175	0.232	0.00236
Sweden (75)	10268	672	0.0257	0.00180	0.214	0.00178
The Netherlands	4097	121	0.0259	0.00335	0.262	0.00419
Sweden (81)	9564	1196	0.0286	0.00216	0.194	0.00200
Taiwan (86)	16434	1267	0.0312	0.00116	0.318	0.00312
United Kingdom(79)	6758	387	0.0324	0.00265	0.264	0.00298
Switzerland (82)	6877	506	0.0349	0.00227	0.326	0.00727
The Netherlands	4326	202	0.0353	0.00390	0.272	0.00560
Denmark (92)	12798	843	0.0355	0.00174	0.224	0.00253
Sweden (92)	12435	782	0.0372	0.00198	0.222	0.00211
Taiwan (91)	16434	1598	0.0377	0.00126	0.309	0.00255
Denmark (87)	12340	931	0.0382	0.00186	0.243	0.00248

Table 1 (Continued) Poverty Intensity and Inequality in LIS Data						
Sweden (87)	9476	944	0.0390	0.00165	0.211	0.00290
France (84)	12579	834	0.0431	0.00221	0.298	0.00296
Ireland (87)	3259	241	0.0465	0.00482	0.325	0.00595
United Kingdom	7091	408	0.0479	0.00348	0.293	0.00320
Israel (92)	5212	474	0.0480	0.00369	0.325	0.00387
France (79)	10989	638	0.0482	0.00266	0.304	0.00355
Spain (90)	21100	1796	0.0526	0.00195	0.307	0.00335
Canada (94)	38938	4176	0.0538	0.00145	0.287	0.00166
Israel (79)	2271	191	0.0543	0.00553	0.332	0.00649
Canada (91)	21566	2478	0.0561	0.00248	0.285	0.00296
United Kingdom	7030	798	0.0562	0.00308	0.338	0.00538
Italy (86)	8020	715	0.0562	0.00396	0.310	0.00465
Australia (85)	8369	630	0.0586	0.00338	0.296	0.00293
Israel (86)	4997	417	0.0593	0.00401	0.333	0.00405
Canada (87)	11924	1609	0.0595	0.00300	0.290	0.00351
France (81)	3639	235	0.0597	0.00488	0.305	0.00507
Australia (81)	16938	1543	0.0620	0.00249	0.286	0.00191
Canada (81)	15045	1880	0.0634	0.00246	0.290	0.00212
Australia (89)	16244	1519	0.0648	0.00259	0.306	0.00223
Spain (80)	23917	2437	0.0690	0.00218	0.327	0.00419
Canada (75)	26247	3820	0.0757	0.00179	0.296	0.00153
United States (79)	15773	2421	0.0972	0.00291	0.311	0.00224
United States (74)	12189	1759	0.0990	0.00330	0.325	0.00278
Canada (71)	25318	4295	0.1020	0.00246	0.330	0.00189
United States (91)	15912	2644	0.1162	0.00288	0.348	0.00226
United States (86)	12477	2051	0.1185	0.00344	0.343	0.00266
United States (94)	65282	10762	0.1246	0.00154	0.371	0.00137

sampling variability on poverty comparisons (which is likely to differ widely in potential importance in LIS data) we report in column 3 of Table 1 our calculation of the S-S index for each data set as it appears in LIS and in column 4 the standard deviation of 200 bootstraps.

For comparison purposes, column 5 reports the Gini index of inequality of equivalent incomes, calculated for all persons and column 6 reports the standard deviation of 200 bootstraps of the Gini index. Since the Gini index tends to be mid range sensitive and is calculated for the entire (poor plus non-poor) sample, it is not surprising that the standard deviation of the Gini is noticeably smaller, as a percentage of the Gini index, than is the case with the S-S index.

3.1. Poverty Intensity Rankings

Figures 1 to 4 present the results of Table 1 graphically, in order to highlight the range of statistical uncertainty surrounding poverty intensity comparisons. Figures 1 to 3 split the LIS data by decade of observations, while Figure 4 illustrates the rather different trends in poverty intensity within North America (in Canada and the United States).

It is noteworthy that the 1970's LIS data of Figure 1 show a fairly clear dichotomy between a "European" and a "North American" level of poverty intensity. Norway (1979) and Sweden (1979) have least poverty intensity, while the UK (1979) is very close. In the early 1970's, Canada's point estimate of poverty intensity exceeds that of the U.S., but the statistical uncertainty surrounding the estimates of Canada (1971) and the U.S. (1974) indicates that no clear judgement is possible.

The 1980's data of Figure 2 is, however, clear in indicating that by 1986 the U.S. had moved into a class by itself. Canada, like Australia, had moved into the high end of a continuum of "European-style" poverty intensity. Austria, Germany and the Benelux and Scandinavian countries rank low in poverty intensity, depending somewhat on the year of observation for their precise ranking (Notably, Sweden is not particularly exemplary among this group). France, Italy, Israel, Australia and Canada are at the top end of European style poverty intensity - but there is enough statistical uncertainty to caution against being more exact in country rankings.

Luxemburg Income Study data for the 1990's is not, as yet, as complete as that available for the 1980's, but it does demonstrate a continuation of the trend for the United States to be a clear outlier in poverty intensity. The European countries, and Canada, are clustered in a quite different range of poverty intensity, with Finland, Belgium and Norway clearly doing a better job in reducing poverty intensity than other European countries.

The purpose of including Figure 4 is *not* to encourage the distressingly common Canadian tendency to complacency. Although Canada looks good in comparisons of poverty intensity with the United States, Canada looks relatively bad compared to almost all other countries. Rather, the purpose of including Figure 4 is to make the point that choices are possible in the

Figure 1
1970's Country Rankings by S-S Poverty Index

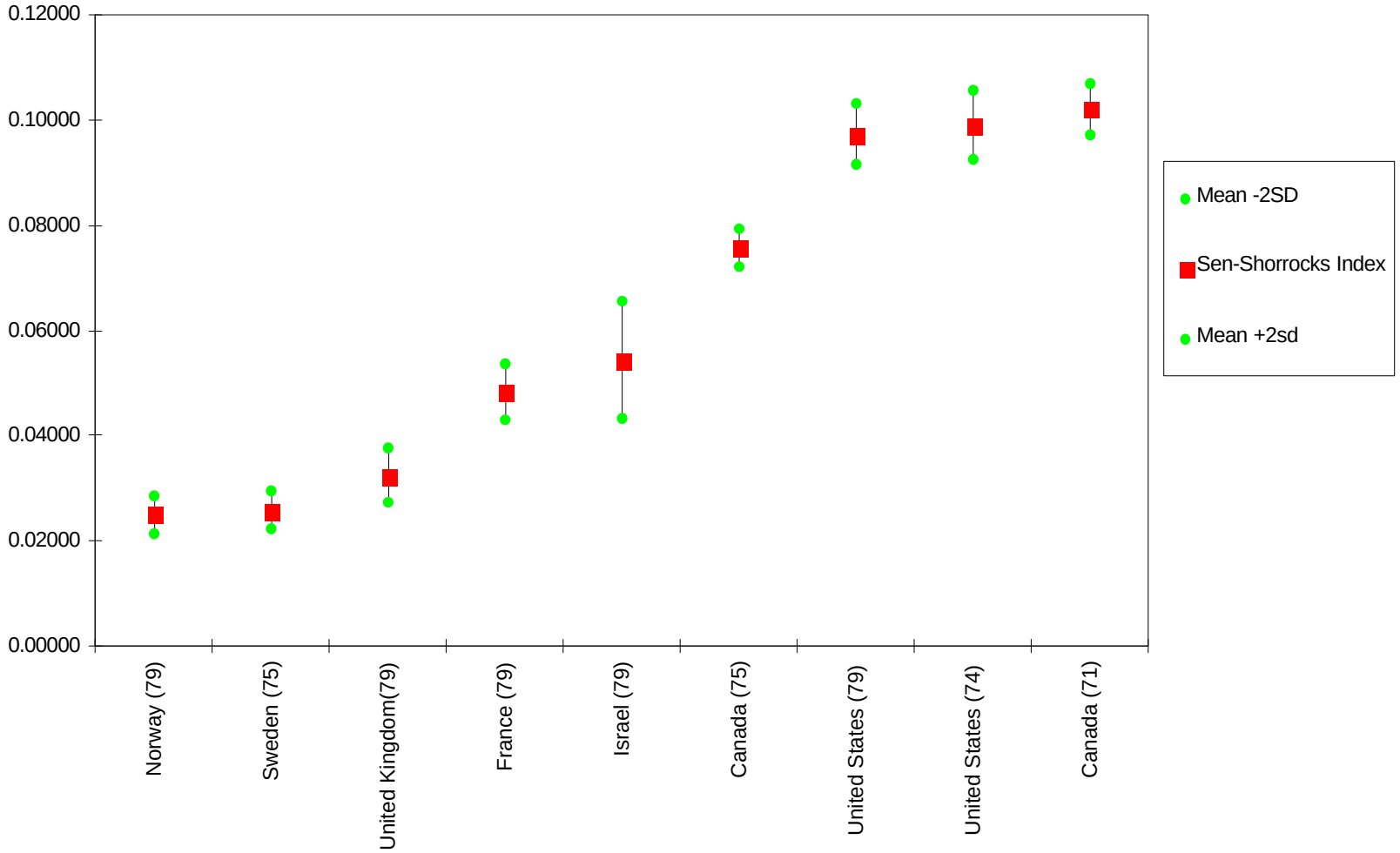


Figure 2
1980s Country Ranking by S-S Poverty Index

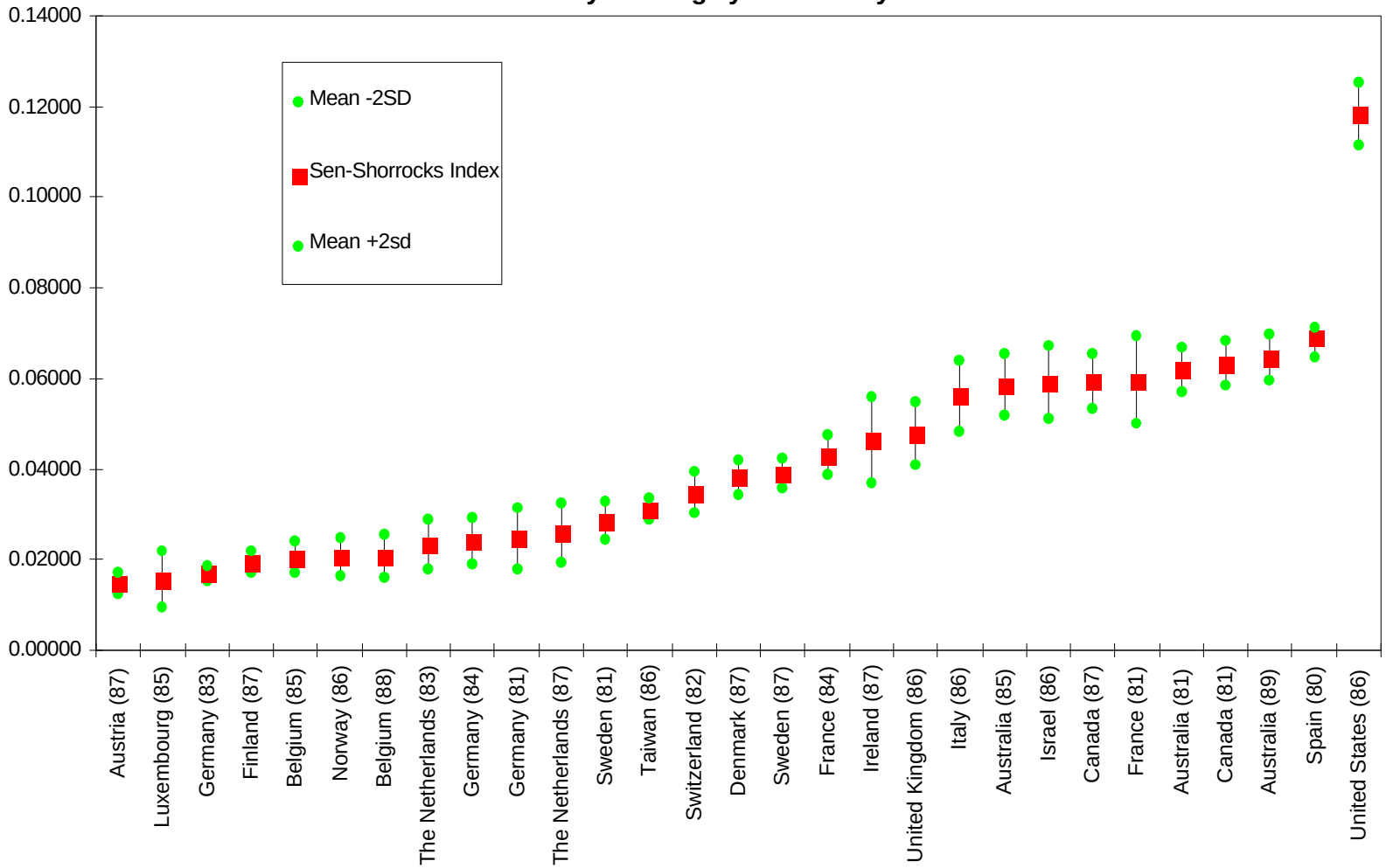


Figure 3
1990s - Country Rankings by S-S Poverty Index

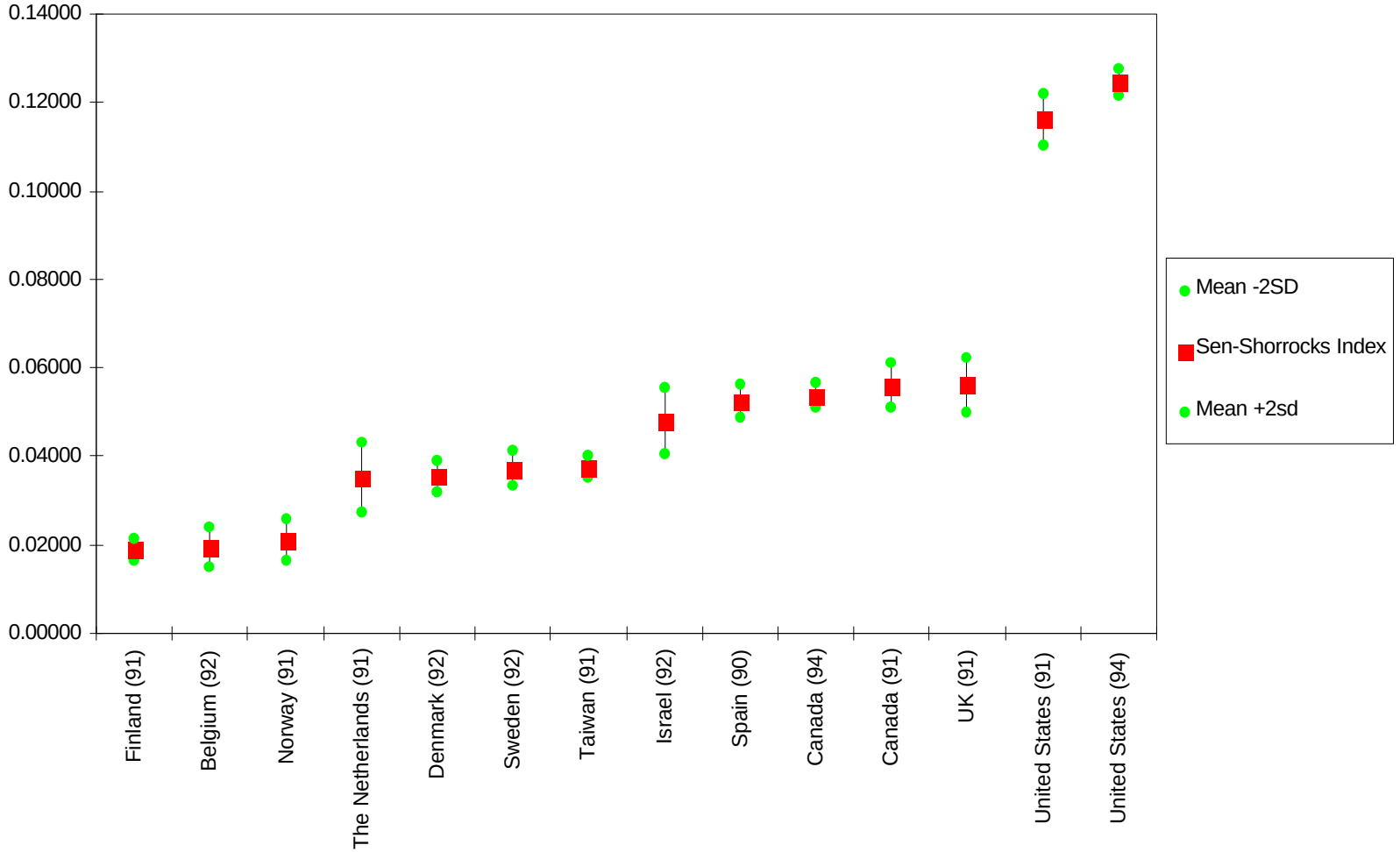
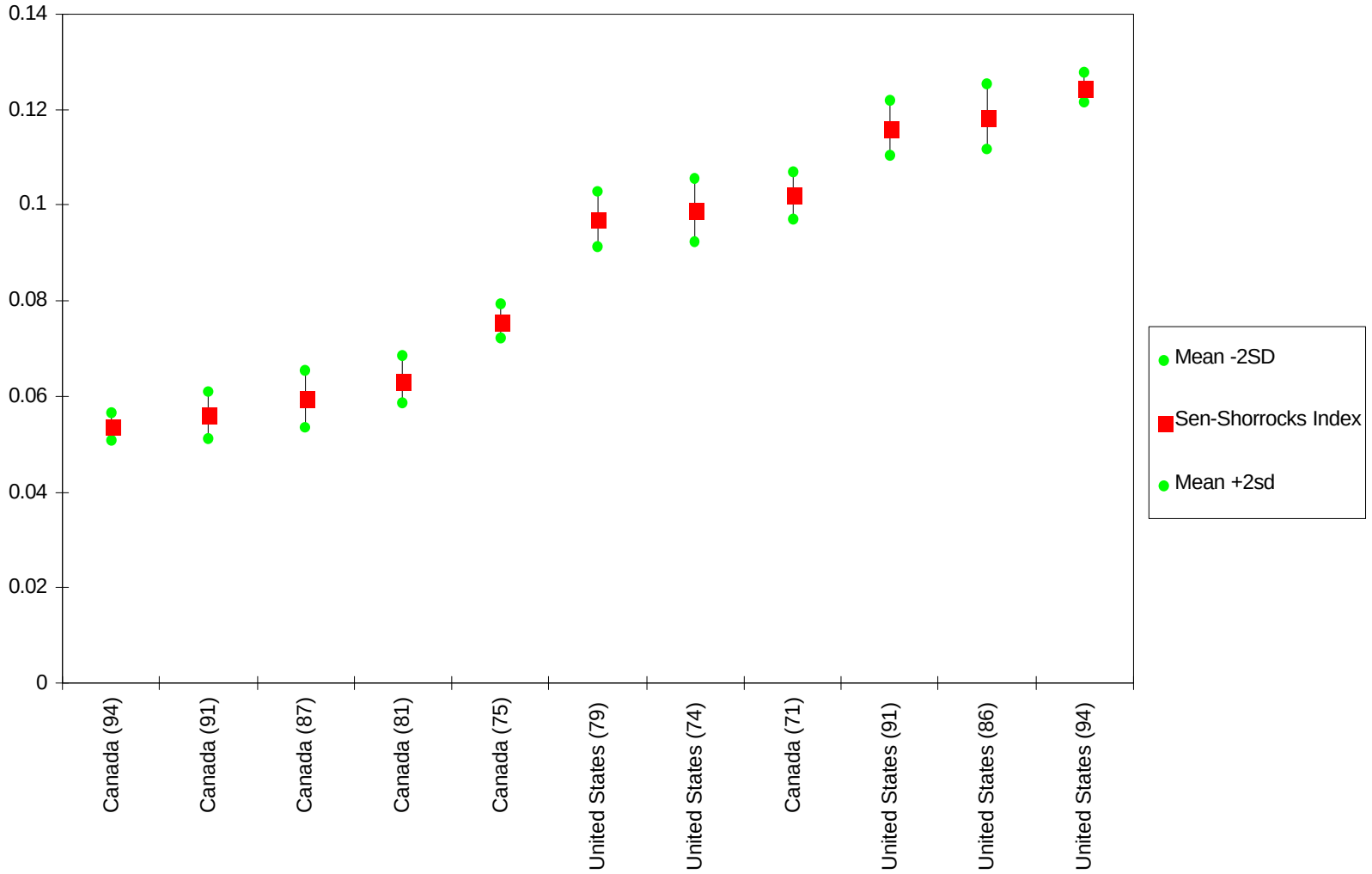


Figure 4
Canada-United States Comparison



social policies which affect the intensity of poverty. Canada and the United States are very similar in economic organization, income levels, statistical data collection methods, etc., but as Card and Freeman (1993) have documented, Canadian and American social policy diverged in the 1970's and 1980's, and these differences matter. Starting from a position of statistical indistinguishability in poverty intensity in the early 1970's, Canadian data up to 1994 show a clear trend to reduced poverty intensity, while U.S. data shows an equally clear trend to increased poverty intensity.

3.2. Poverty Intensity and Inequality

Measures of poverty intensity, particularly those based on a poverty line drawn at half the median equivalent income, are sometimes attacked under the misconception that they are "merely" another aspect of income inequality. However, although there is certainly a correlation between income inequality and poverty intensity, the relationship is far from direct. Figure 5 presents a plot of the relationship between the Sen-Shorrocks index of poverty intensity and the Gini index of inequality in individual equivalent income. Clearly, the details of the design of social policy, and of the functioning of low wage labour markets, matter considerably for the well-being of the poor even at comparable levels of aggregate inequality. Some countries [e.g., Switzerland (1982)] do a considerably better job in preventing intense poverty than might be expected from examination of the Gini index of income inequality. Furthermore, as Figure 6 indicates, in comparisons of the inequality of equivalent income (as measured by the Gini index) in the 1980's, there is much more of a continuum in the international data than is observed in the comparable Figure 2, which plots poverty intensity. The United States ranks high in income inequality, but is not nearly as much of an outlier in Figure 6 as in Figure 2.

3.3. Decomposing Trends in Poverty Intensity

Table 2 decomposes the level of the Sen-Shorrocks index as per Equation (8) and the changes observed in poverty intensity as per Equation (10). In Table 2, column 1 (the level of poverty intensity) is reported for both the LIS countries with multiple observations over time and for those with single observations - it is the *product* of column 2 (the poverty rate), column 3 (the average poverty gap ratio among the poor) and column 4 (the inequality in poverty gap ratios among all people). Column 5 reports the proportionate *change* in poverty intensity, in a given year of a country's data compared to the immediately preceding observation of that country - it is the *sum* of column 6 (the proportionate change in the poverty rate), column 7 (the proportionate change in the average poverty gap ratio of the poor) and column 8 (the change in inequality of the poverty gap ratio among all people).

It is noteworthy that, in practice, percentage changes in $\ln(1+G(x))$ are always an order of magnitude smaller than percentage changes in the poverty rate and the average poverty gap

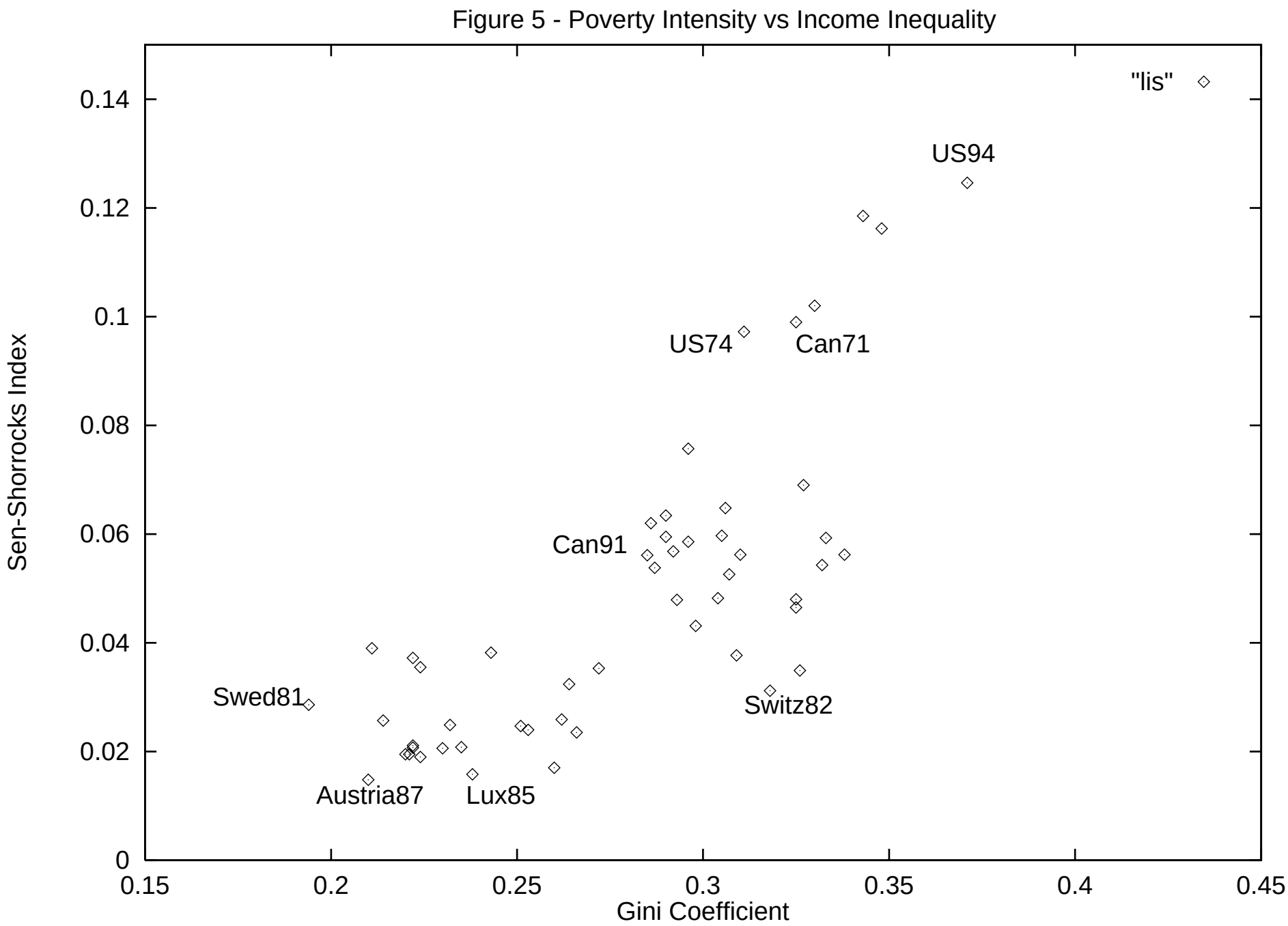


Figure 5 - Poverty Intensity vs Income Inequality
Sen-Shorrocks Index
Gini Coefficient
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.15
0.2
0.25
0.3
0.35
0.4
0.45
"lis"
US94
US74
Can71
Can91
Swed81
Switz82
Austria87
Lux85

Figure 6
Cross Country Gini Comparisons 1980's

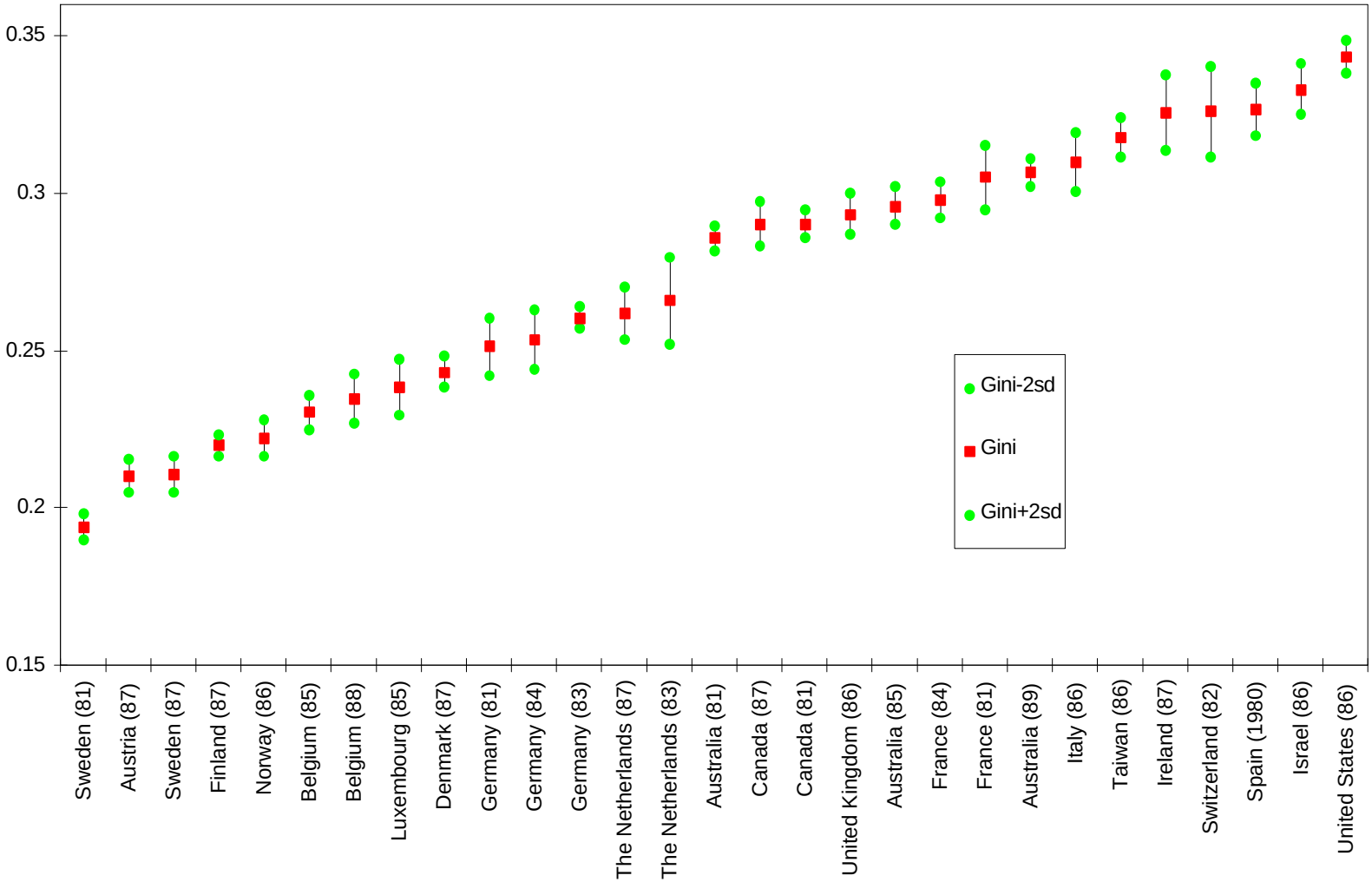


Table 2
Decomposition of the Sen-Shorrocks Index

		S-S Index (P)	Decomposition of Level			ln(P)	Decomposition of Change		
			Rate	Gap	(1+G(x))		ln(Rate)	ln(Gap)	ln(1+G(x))
Finland	**87**	0.020	0.041	0.243	1.978				
	91	0.019	0.041	0.234	1.979	-0.026	0.010	-0.036	0.001
Belgium	**85**	0.021	0.044	0.237	1.979				
	88	0.021	0.047	0.224	1.977	0.008	0.067	-0.058	-0.001
	92	0.019	0.045	0.221	1.978	-0.066	-0.051	-0.015	0.001
Norway	**79**	0.025	0.041	0.307	1.978				
	86	0.021	0.037	0.285	1.980	-0.185*	-0.112	-0.074	0.001
	91	0.021	0.035	0.303	1.982	0.020	-0.043	0.063	0.001
The Netherlands	**83**	0.023	0.040	0.299	1.978				
	87	0.026	0.048	0.271	1.976	0.098	0.200	-0.101	-0.001
	91	0.035	0.047	0.380	1.975	0.309*	-0.030	0.339	0.000
Denmark	**87**	0.038	0.064	0.302	1.969				
	92	0.036	0.053	0.340	1.972	-0.074	-0.194	0.119	0.001
Sweden	**75**	0.026	0.043	0.303	1.977				
	81	0.029	0.047	0.309	1.972	0.107	0.089	0.021	-0.003
	87	0.039	0.058	0.344	1.966	0.309*	0.205	0.107	-0.003
	92	0.037	0.052	0.363	1.969	-0.048	-0.104	0.054	0.002
Taiwan	**86**	0.031	0.086	0.186	1.953				
	91	0.038	0.096	0.201	1.943	0.187*	0.115	0.078	-0.005
France	**79**	0.048	0.081	0.305	1.954				
	81	0.060	0.096	0.318	1.948	0.214*	0.176	0.041	-0.003
	84	0.043	0.080	0.276	1.957	-0.326*	-0.190	-0.140	0.005
Israel	**79**	0.054	0.138	0.205	1.919				
	86	0.060	0.130	0.238	1.921	0.092	-0.060	0.151	0.001
	92	0.048	0.117	0.212	1.931	-0.215*	-0.106	-0.115	0.005
Spain	**80**	0.069	0.119	0.300	0.929				
	90	0.053	0.096	0.281	0.947	-0.272*	-0.214	-0.067	0.009

Table 2 (Continued)
Decomposition of the Sen-Shorrocks Index

	S-S Index (P)	Decomposition of Level			ln(P)	Decomposition of Change		
		Rate	Gap	(1+G(x))		ln(Rate)	ln(Gap)	ln(1+G(x))
Canada 71	0.102	0.149	0.359	1.914				
75	0.076	0.123	0.318	1.929	-0.299*	-0.186	-0.120	0.008
81	0.063	0.113	0.290	1.935	-0.177*	-0.089	-0.091	0.003
87	0.060	0.109	0.281	1.937	-0.063	-0.031	-0.033	0.001
91	0.056	0.107	0.271	1.938	-0.059	-0.022	-0.037	0.000
94	0.054	0.105	0.265	1.940	-0.042	-0.021	-0.022	0.001
United Kingdom 79	0.032	0.067	0.245	1.966				
86	0.048	0.081	0.300	1.960	0.390*	0.190	0.204	-0.003
91	0.056	0.127	0.229	1.934	0.160*	0.442	-0.269	-0.013
Australia 81	0.062	0.100	0.319	1.947				
85	0.059	0.091	0.329	1.951	-0.057	-0.089	0.030	0.002
89	0.065	0.102	0.329	1.943	0.101	0.106	-0.001	-0.004
United States 74	0.099	0.146	0.355	1.913				
79	0.097	0.155	0.328	1.909	-0.018	0.065	-0.081	-0.002
86	0.118	0.180	0.349	1.888	0.198*	0.145	0.063	-0.011
91	0.116	0.177	0.346	1.892	-0.020	-0.013	-0.009	0.002
94	0.125	0.183	0.360	1.889	0.070	0.031	0.040	-0.001
Austria 87	0.015	0.010	0.185	1.979				
Luxembourg 85	0.016	0.043	0.184	1.981				
Switzerland 82	0.035	0.070	0.253	1.965				
Ireland 87	0.046	0.098	0.242	1.955				
Italy 86	0.056	0.109	0.266	1.938				

* Change in the Sen-Shorrocks Index is significant at the 95% confidence level.

ratio.[13] Since inequality in the poverty gap ratios among all people (i.e. $1+G(x)$) does not change much, changes in poverty intensity are dominated by changes in the poverty rate and the average poverty gap ratio. Hence, to a first approximation, the percentage change in poverty intensity is the sum of the percentage change in the poverty rate and the percentage change in the average poverty gap ratio of the poor.

Given our earlier discussion of the importance of sampling variability, we would not want to ascribe unwarranted importance to small changes in poverty intensity. We therefore use our estimates of the bootstrap variance reported in Table 1 to indicate (with an asterisk) the changes in poverty intensity within countries that are statistically significant at a 95% level of confidence (i.e., differ by more than two standard deviations from the prior years's estimate). Only thirteen of 30 observed changes in poverty intensity pass this fairly stringent test.

However, Table 2 does have one clearly negative result - there is no clear trend to greater poverty intensity - no "immutable natural law" of greater polarization - to be observed in the Luxemburg Income Study data. Simply counting the number of times one observes in Table 2 a decrease in poverty intensity (16) compared to the number of times an increase in poverty intensity is observed (14), the result is pretty much a draw. The number of statistically significant declines (6) and increases (7) are nearly matched. Even in an increasingly globalized international economy, different social choices and different social outcomes are to be observed.

A second implication of Table 2 is the potentially precarious economic position of the poor. In a number of countries, there have been quite large increases in poverty intensity, albeit with differing underlying causes. The 31% increase in Dutch poverty intensity between 1987 and 1991 was driven entirely by an increase in the average poverty gap ratio among the poor - but although Sweden experienced a similar increase in poverty intensity between 1981 and 1987, two-thirds of that increase in poverty intensity was due to an increase in the poverty rate. The 39% increase in poverty intensity in the United Kingdom between 1979 and 1986 was almost equally due to an increase in the poverty rate and increase in the average poverty gap ratio among the poverty. The 1986 to 1991 period saw a further 16% increase in poverty intensity, driven entirely by a substantial increase in the poverty rate. Countries experiencing significant declines in poverty intensity include Norway (between 1979 and 1986) France (between 1981 and 1984) , Israel (between 1986 and 1992) and Spain (between 1980 and 1990).

Table 2 is also interesting for the light it sheds on the sources on the diverging trends in poverty intensity in Canada and the United States. The largest changes in the intensity of poverty in Canada came in the 1970's, particularly between 1971 and 1975, following introduction of the Canada Pension Plan and Guaranteed Income Supplement for senior citizens and Canada Assistance Plan, plus the liberalization of unemployment insurance.

---

[13]Since $(1+G(x))$ measures inequality in the poverty gap ratios among all people, most of whom are non-poor (i.e., their poverty gap ratios are set to zero), this number is always large, but nearly constant within countries [although it does differ somewhat *across* countries].

Reductions in poverty intensity in the 1980's and 1990's were much more modest and not statistically significant at a 95% level of confidence. In both the 1970's and the 1980's, declines in Canadian poverty intensity were split fairly evenly between declines in the poverty rate and decreases in the average poverty gap ratio among the poor. Unfortunately, it is unlikely that trends to decreased poverty intensity in Canada are continuing, since the period since 1994 has seen substantial cuts in the generosity of Social Assistance and Unemployment Insurance.

The U.S. data on poverty intensity indicates that the big change in U.S. poverty - an increase of 20% in poverty intensity - occurred in the early 1980's, between 1979 and 1986. About seven tenths of that increase in poverty intensity is ascribable to an increase in the poverty rate, with the remainder being due to an increase in the average poverty gap ratio among the poor. The recession of the early 1980's, coinciding with cuts in welfare benefits, evidently hit the poor of the U.S. rather hard.

4. Conclusion

There is some degree of uncertainty in international comparisons of poverty intensity, partly due to the potential for sampling variation in estimation, and partly due to the difficulties in interpreting the indicators of poverty intensity. Bootstrap methods can be used to assess the potential importance of the sampling variation. Although some rankings of poverty intensity may be indeterminate, others are unambiguous.

In the international data, the clear divergence between the U.S. and Europe in poverty intensity is especially noteworthy. Decomposition of the Sen-Shorrocks index of poverty intensity indicates that most of this change occurred in the early 1980's, largely due to an increase in the poverty rate. Canada has followed a different path from the U.S., with large declines in poverty intensity in the 1970's, and in the early 1990's sat at the high end of a “European” continuum of poverty intensity.

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Appendix A

Table A.1

Country	Percent of sample with zero incomes	Gini Index			Sen-Shorrocks Index		
		Include zero Incomes	Exclude zero incomes	Percent Difference	Include Zero Incomes	Exclude zero incomes	Percent Difference
Austria (87)	0.000	0.210	0.210	0.000	0.0148	0.0148	0.000
Luxembourg (85)	0.244	0.239	0.238	0.372	0.0180	0.0158	12.542
Germany (83)	0.002	0.260	0.260	0.012	0.0171	0.0170	0.491
Finland (91)	0.009	0.224	0.224	0.013	0.0191	0.0190	0.277
Belgium (92)	1.099	0.227	0.221	2.845	0.0354	0.0195	45.058
Finland (87)	0.000	0.220	0.220	0.000	0.0195	0.0195	0.000
Belgium (85)	0.371	0.232	0.230	0.544	0.0237	0.0206	12.874
Norway (86)	0.121	0.223	0.222	0.314	0.0225	0.0207	7.794
Belgium (88)	0.741	0.238	0.235	1.270	0.0283	0.0208	26.674
Norway (91)	0.173	0.224	0.222	0.683	0.0249	0.0211	15.297
The Netherlands (83)	1.994	0.288	0.266	7.750	0.0810	0.0235	71.040
Germany (84)	0.000	0.253	0.253	0.000	0.0240	0.0240	0.000
Germany (81)	0.210	0.252	0.251	0.219	0.0262	0.0247	5.454
Norway (79)	0.194	0.233	0.232	0.408	0.0273	0.0249	8.809
Sweden (75)	0.088	0.215	0.214	0.687	0.0293	0.0257	12.294
The Netherlands (87)	0.510	0.264	0.262	0.736	0.0308	0.0259	15.984
Sweden (81)	0.344	0.195	0.194	0.758	0.0322	0.0286	10.997
Taiwan (86)	0.000	0.318	0.318	0.000	0.0312	0.0312	0.000
United Kingdom(79)	0.015	0.264	0.264	0.015	0.0325	0.0324	0.326
Switzerland (82)	2.149	0.329	0.326	0.951	0.0436	0.0349	19.963
The Netherlands (91)	0.689	0.275	0.272	1.300	0.0446	0.0353	20.894
Denmark (92)	0.513	0.226	0.224	1.031	0.0412	0.0355	13.775
Sweden (92)	0.008	0.222	0.222	0.050	0.0374	0.0372	0.693
Taiwan (91)	0.000	0.309	0.309	0.000	0.0377	0.0377	0.000
Denmark (87)	0.644	0.246	0.243	1.038	0.0446	0.0382	14.275
Sweden (87)	0.148	0.212	0.211	0.463	0.0414	0.0390	5.687

Table A.1 (Continued)

Country	Percent of sample with zero incomes	Gini Index			Sen-Shorrocks Index		
		Include zero Incomes	Exclude zero incomes	Percent Difference	Include Zero Incomes	Exclude zero incomes	Percent Difference
France (84)	0.293	0.299	0.298	0.465	0.0468	0.0431	7.838
France (79)	0.109	0.305	0.304	0.154	0.0494	0.0482	2.415
Spain (90)	0.251	0.308	0.307	0.364	0.0555	0.0526	5.282
Canada (94)	0.259	0.289	0.287	0.851	0.0607	0.0538	11.289
Israel (79)	0.000	0.332	0.332	0.000	0.0543	0.0543	0.000
Canada (91)	0.231	0.286	0.285	0.385	0.0588	0.0561	4.653
United Kingdom (91)	0.368	0.340	0.338	0.861	0.0644	0.0562	12.718
Italy (86)	0.025	0.310	0.310	0.077	0.0569	0.0562	1.160
Australia (85)	0.452	0.298	0.296	0.604	0.0632	0.0586	7.328
Israel (86)	0.060	0.333	0.333	0.093	0.0604	0.0593	1.922
Canada (87)	0.193	0.291	0.290	0.357	0.0622	0.0595	4.291
France (81)	0.000	0.305	0.305	0.000	0.0597	0.0597	0.000
Australia (81)	0.482	0.287	0.286	0.561	0.0661	0.0620	6.160
Canada (81)	0.430	0.292	0.290	0.633	0.0680	0.0634	6.729
Australia (89)	0.533	0.309	0.306	0.684	0.0702	0.0648	7.675
Spain (80)	0.229	0.327	0.326	0.244	0.0696	0.0690	0.947
Canada (75)	1.041	0.300	0.296	1.277	0.0853	0.0757	11.212
United States (79)	0.755	0.315	0.311	1.171	0.1062	0.0972	8.495
United States (74)	0.684	0.327	0.325	0.719	0.1048	0.0990	5.547
Canada (71)	2.153	0.336	0.330	1.942	0.1183	0.1020	13.728
United States (91)	0.693	0.350	0.348	0.643	0.1215	0.1162	4.360
United States (86)	0.677	0.346	0.343	0.717	0.1246	0.1185	4.941
United States (94)	0.936	0.375	0.371	0.860	0.1328	0.1246	6.169

Appendix 2 - Transition Economies

Table 1

Country	No. of Observations	No. of Observations Below poverty line	S-S Index	Standard Deviation of 200 Bootstraps	Gini Index	Standard Deviation of 200 Bootstraps
Hungary (1991)	1994	109	0.0501	0.00583	0.280	0.00698
Poland (1986)	10645	749	0.0382	0.00197	0.262	0.00238
Poland (1992)	6596	389	0.0550	0.00398	0.302	0.00398

Table 2

	S-S Index (P)	Decomposition of Level			ln(P)	Decomposition of Change		
		Rate	Gap	(1+G(x))		ln(Rate)	ln(Gap)	ln(1+G(x))
Hungary (1991)	0.050	0.076	0.336	1.954				
Poland (1986)	0.038	0.084	0.233	0.953				
Poland (1992)	0.055	0.102	0.277	0.941	0.363*	0.198	0.171	-0.006

Appendix A.1

Country	Percent of sample with zero incomes	Gini Index			Sen-Shorrocks Index		
		Include zero Incomes	Exclude zero incomes	Percent Difference	Include Zero Incomes	Exclude zero incomes	Percent Difference
Hungary (1991)	0.238	0.287	0.280	2.233	0.0665	0.0501	24.564
Poland (1986)	0.009	0.262	0.262	0.023	0.0384	0.0382	0.418
Poland (1992)	0.091	0.303	0.302	0.168	0.0561	0.0550	2.116