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Produced in the Research Department. Edited by Preston J. Miller, Arthur J. Rolnick, Kathleen S. Rolfe, and Inga Velde. Graphic design by Phil Swenson and typesetting by Terri Desormey, Graphic Services Department.

Address questions to the Research Department, Federal Reserve Bank, Minneapolis, Minnesota 55480 (telephone 612-340-2341).

International Coordination of Macroeconomic Policies: A Welfare Analysis

Preston J. Miller
Vice President and Deputy Director of Research
Research Department
Federal Reserve Bank of Minneapolis

Neil Wallace
Adviser
Research Department
Federal Reserve Bank of Minneapolis
and Professor of Economics
University of Minnesota

Coordination among countries of their monetary and budget policies has been proposed recently as a way to improve the current system of floating exchange rates. This proposal has been prompted by the apparent failure of the floating rate system to eliminate *policy interdependence,* the effects of one country's policies on other countries' economies. Although the floating rate system has seemed to allow countries greater freedom in their choice of monetary and budget policies, in the last few years many countries have complained about being hurt by other countries' policies—especially those of the United States. Our study suggests that, because of policy interdependence, some form of international coordination of macroeconomic policies would, indeed, improve the floating exchange rate system.

This conclusion—that countries would in some sense be better off if they choose macro policies jointly than if they choose those policies independently—is implied by a simple model of a world economy. The ingredients of the model that are crucial for the conclusion are its assumptions about currency and debt markets and about a country's well-being:

- Currencies of different countries are not direct substitutes; each country's currency ends up being held only by its own residents.
- The interest-bearing debt of any one government trades in an integrated world credit market where it competes with debt issued by other governments and by private residents of all countries.
- Government borrowing affects the world interest rate. (Residents of a country don't match changes in their government's borrowing with offsetting changes in their own borrowing, partly because they don't expect the government's outstanding debt to be retired in their lifetime.)
- A country's well-being depends on both the world real interest rate and its own price level because some of its residents borrow at the world rate and some hold wealth in the form of the country's currency.

We think these assumptions are good approximations of conditions in the actual world economy. Moreover, they are present, at least implicitly, in many models. This study is the first, however, to systematically analyze the implications of these assumptions for the issue of international coordination of macroeconomic policies.[1]

A Preview of the Study

Before presenting the formal description and analysis of our model, we describe briefly and informally the model, its defense, and its implications for policy coordination.

The Model

The model is designed to make qualitative predictions about the economic outcomes across countries of alterna-

[1] Coordination of macroeconomic policies has been studied before, but using models with very different basic ingredients. See Cooper 1985 for a survey of previous research.

tive national monetary and budget policies. The outcomes are equilibrium sequences of price levels, interest rates, and lifetime consumption patterns for the residents of different countries. The demands and supplies that appear explicitly in the model are those for bonds, or securities—all of which are traded in one world market—and for base monies, or currencies, one for each country. The *equilibrium* sequences are those which equate demands and supplies of bonds and base monies at each date.

The model has only one good per date and, aside from consumption of that good, only one economic activity: borrowing and lending. We assume that each of several countries is populated by an infinite sequence of identical overlapping generations whose members live two periods. Each generation in each country consists of two groups on opposite sides of the credit market: borrowers and savers. The private supplies and demands for bonds and monies are derived from the behavior of these groups. The borrowers supply private bonds. The savers, besides demanding bonds, demand the base money of their country because they have to; a reserve requirement forces them to hold some fraction of their savings as domestic currency. This restriction ensures that each country's base money is held even when other currencies and securities bear higher returns; it also produces the model's separate currency markets. At the first date of the model, the people in each country who are in the second period of their lives own the initial outstanding stock of their country's monetary base.

The private demands and supplies depend on agents' price expectations, which are assumed to be *rational.* The model has no uncertainty, so this means the expected prices coincide with those that actually prevail.

Each government determines additions to the supply of its bonds and money by its choice of budget and monetary policies. Budget policy is made by choosing the path over time of the real *deficit net-of-interest:* the real value of the government's budget deficit, excluding from government expenditures any interest payments or receipts. (Throughout, we hold real taxes constant, so that a change in budget policy corresponds to a change in real government consumption.) Monetary policy is made by choosing the division of government debt over time between bonds and money. Governments do not buy and sell each other's currencies; in this sense, exchange rates float.

Primarily in order to keep the analysis simple, we consider only budget policies for which the deficit net-of-interest is constant over time and only monetary policies for which the ratio of bonds to base money is constant over time. For these policies, there is an equilibrium which takes a simple form. It has a constant world real interest rate and constant country-specific inflation rates. These imply that the situation of different generations in each country is constant over time.

A Defense of the Model

We defend our model in two ways: by showing that it is internally consistent and by showing that its implications are broadly consistent with recent events.

We show that the model is internally consistent by building it from a theory of individual behavior and proving that under reasonable conditions an equilibrium exists for given policies. The equilibrium assures us that the actions of individual agents are mutually consistent and lead to the aggregate outcomes our model implies.

We show that the model's implications are broadly consistent with recent events by comparing actual economic experience to the model's predictions for the qualitative effects of a change in monetary and budget policies like that implemented recently by the United States and other countries.

For this purpose, we describe the model's implications for the adoption in one country of a permanently easier budget policy together with a nonaccommodating monetary policy, with other countries remaining passive. Specifically, we assume that one country permanently increases its government's real budget deficit net-of-interest and accompanies that with an increase in the ratio of government bonds to base money which keeps unaffected the quantity of base money at the date of the policy change. Passiveness by other countries means that they do not respond with changes in their budget policies (their real government purchases and taxes) or their monetary policies (their ratios of government bonds to base money).

When we compare the model's predictions for this set of policies with actual events, we identify the active country with the United States, the period of the model with four years, and the first period with the four years beginning in 1981. Our representation of policy, at least for the active country, seems to be a reasonable description of what has actually occurred in the United States beginning in 1981. In the four years since then, the U.S. net-of-interest budget deficit rose from its previous average of nearly zero to an average of 2 percent of gross national product, and the ratio of U.S. government bonds to base money rose, roughly, from 4 to 6.

The model's predictions for the direction that various economic variables change in response to such policy changes are shown in Table 1. These predictions roughly

Table 1

The International Effects of Easing a Country's Budget Policy Without Monetary Accommodation

Variable	Active Country [United States]	Passive Countries: Debtor Government	Passive Countries: Creditor Government
Real Interest Rate	+	+	+
Inflation Rate	+	+	−
Nominal Interest Rate	+	+	+
Initial Price Level	0	+	−
Initial Value of Currency Relative to Active Country's	n.a.	−	+
Initial Current Account Deficit	+	−	+ or −

+ = increases   0 = no change
− = decreases   n.a. = not applicable

match recent experience. Since 1981, we have seen higher real interest rates, an enhanced value of the U.S. dollar relative to the currencies of debtor governments, and a higher U.S. current account deficit. (The higher inflation rate prediction for the United States is a prediction of the future course of the price level which assumes no subsequent change in the real deficit net-of-interest or in the ratio of bonds to money.)

Implications for Policy Coordination

For some purposes, policymakers might want more than qualitative predictions or descriptions of the effects of given policies. They might want to know what policies other countries would adopt in various circumstances. A country contemplating a change in policy surely would want to know if and how other countries would respond. It could also want to know whether it could end up in a better position if it joined with other countries to choose monetary and budget policies than if they all proceeded, in some sense, independently. A standard mathematical theory, called *game theory,* deals with such questions. We apply that theory to the choice of monetary policy in our model.

To apply it, we need to add to the model assumptions about the objectives or goals pursued by countries. We assume that countries try to do the best they can for their residents. In our model, this is complicated by the fact that each country is populated by groups of people whose interests do not coincide.

In an equilibrium with a constant real interest rate and constant inflation rates, each country is populated by three well-defined groups. There is a group of people who, at the date of the policy choice, own assets denominated in domestic currency and are thus better off the lower the starting price level. There is another group, the current and future savers, whose savings earn a return which is a weighted average (determined by the reserve requirement) of the real return on the country's monetary base and that on securities. These people are better off the higher is this average real return. Finally, there is the group of current and future borrowers who borrow at the real return on securities and who, therefore, are better off the lower is that real return. This correspondence between predicted paths for price levels and the interest rate and the well-being of these different groups implies conflicts of interest—for example, between a country's borrowers, who prefer an easier monetary policy, and the initial owners of its monetary base, who prefer a tighter monetary policy.

An objective for a country must weight these competing interests. The objective we adopt, maximization of a social welfare function, assumes this is done. The social welfare function of a country describes how the government trades off the well-being of its three different groups as it makes policy decisions.

Using a social welfare function for each country, we compare what happens if each country chooses a monetary policy that maximizes that function taking as given the choices of the monetary policies of other countries—our way of describing noncooperation—with what happens if countries choose their monetary policies jointly, or cooperatively. We find, for the special case of identical countries, that every country can do better if all countries cooperate than if they do not. This doing better, of course, is in terms of the country's social welfare function. Every person in each country is not better off under cooperation. Rather, given the way social welfare functions (the governments) weight the different groups in each country, cooperation results in a higher value of each social welfare function, but at the expense of at least one group in each country.

Although we demonstrate a gain from cooperation only in a very special model, the result is likely to hold in any model which shares these crucial features of ours: separate country-by-country markets in currencies and an integrated world credit market in which government borrowing affects the world interest rate. These features produce policy interdependence: one country's policy affects others through the effects of its borrowing on the world real interest rate. The features also produce a kind of asymmetry: a country's choice of monetary policy affects its borrowers and borrowers in other countries in the same way, but affects the initial owners of currency and savers in its country differently than those in other countries. This asymmetry is what produces the gain from cooperation.

The Model

Our model is a multi-country version of the single-country model used in Wallace 1984. Here we describe the model and study the equilibrium for a special case in which the saving behavior of individuals in each country takes a particularly simple form.[2]

A Typical Country

Each country in our model has a private sector and a public sector.

□ *Private Demands and Supplies*

Each country k (for $k = 1, 2, \ldots, K$) is populated by overlapping generations, the members of which live two periods. At each date t (where t is an integer) a new generation, generation t, appears. Its members are present in the economy at t (when they are young) and at $t+1$ (when they are old). We assume that people do not move between countries, a standard assumption in models of international trade.

The model contains one good at each date, the time t good, that is common to all countries.[3] This good can be costlessly and instantly transported from one country to another. There is, however, no production; the time t good cannot be produced or used to produce any other good (that is, any good available at any other time than t).

Each member of generation t has preferences over private consumption of the time t and time $t + 1$ goods, preferences that are representable by a utility function or an indifference curve map of the usual sort. These preferences are unaffected by government consumption. Each such member also has an income stream or endowment consisting of some amount of the time t good and some amount of the time $t + 1$ good.

We assume that different generations are identical and that within each country each generation has a special kind of diversity. Each generation consists of two groups of people. Members of one group, called *lenders* (or savers), are identical and have preferences and endowments that lead them to want to lend (or save) at most rates of return. Members of the other group, called *borrowers* (or dissavers), are also identical and have preferences and endowments that lead them to want to borrow (or dissave) at most rates of return.

These assumptions imply that the competitive desired trades by the members of each group in country k can be described as functions of the terms of trade between the time t good and the time $t + 1$ good faced by the members of each group. We let $S^k(\cdot)$ denote the aggregate supply function or curve of the time t good (or desired lending or saving) of the lender group of generation t in country k and let $D^k(\cdot)$ denote the aggregate demand curve for the time t good (or desired borrowing or dissaving) of the borrower group for generation t in country k. In each case, the argument of the function is the intertemporal terms of trade which we express by the price of the time t good in units of the time $t + 1$ good (the gross real rate of return) faced by the members of the respective group. In general, as we will see, lenders, who are subject to a reserve requirement, face a different and lower rate of return than borrowers. We assume that $D^k(\cdot)$ is decreasing where it is positive.[4]

Since we will be describing how this economy evolves over time from the initial (or current) date, which we label $t = 1$, we need to add to the above description of the competitive behavior of the young of each generation in country k a description of the behavior of the country's people who are in the second period of their lives at $t = 1$, the initial (or current) old. We assume that their preferences are such that they try to consume as much of the time 1 good as they can and that they are endowed, or start, with some of the time 1 good and some nominally denominated assets (assets valued in terms of the current price level). Their implied competitive behavior is very simple: they supply all their assets at any positive price in terms of the time 1 good.

These assumptions imply some simple relationships between prices, including rates of return, and the well-being of individuals in country k: the initial old are better off the more valuable are their nominally denominated assets at time 1; lenders, or savers, in any generation t (for $t \geq 1$) are better off the higher the rate of return they earn on savings; borrowers in any generation t (for $t \geq 1$) are

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[2]The special case should make our presentation accessible to undergraduate students of economics—at least those whose background includes intermediate microeconomic theory and some calculus.

[3]Under well-known conditions, the single time t good can be interpreted as a composite good. See, for example, Kareken and Wallace 1981, p. 210.

[4]If the arguments of borrowers' utility functions are normal goods, then $D^k(\cdot)$ is decreasing where it is positive. For a more detailed description of the derivation of the $S^k(\cdot)$ and $D^k(\cdot)$ functions, see Wallace 1984, pp. 16–18, or the section on the derivation of demand in any intermediate price theory text.

better off the lower the rate of return at which they can borrow. It is in terms of these relationships that we will describe how one country's policy affects other countries and how cooperation can or cannot improve welfare.

□ Government Policy

Each country in the model has a budget policy and a monetary policy. Budget policy is a sequence of the real net-of-interest deficit (the difference between real government consumption and real taxes, the latter of which we hold fixed throughout). Monetary policy is sequences of the monetary base and the interest-bearing government debt which finance the deficit. Consistent with this, we write the cash flow constraint of country k's combined budget and monetary authority as

$$(1) \qquad G_t^k = p_t^k(H_{t+1}^k - H_t^k) + p_t^k(P_t^k B_{t+1}^k - B_t^k)$$

which must hold for all dates $t \geq 1$. Here G_t^k, measured in units of the time t good, is government k's real deficit net-of-interest. The first term on the right side of equation (1) is the value in terms of the time t good of government k's addition to its outstanding monetary base, and the second term is the value of its addition to its outstanding debt, which consists of one-period, zero coupon (pure discount) bonds. Specifically, the variables on the right side of equation (1) are defined this way:

H_t^k = The country k monetary base that generation $t - 1$ starts with at time t.

p_t^k = The time t price of a unit of the country k monetary base in units of the time t consumption good ($1/p_t^k$ = the country k price level at time t).

B_t^k = The nominal face value, in terms of the country k monetary base, of the maturing country k government bonds owned by members of generation $t - 1$ at time t.

P_t^k = The price at time t, in terms of the country k monetary base, of a bond that pays one unit of the country k monetary base at time $t + 1$. [The country k nominal interest rate at t is $(1/P_t^k) - 1$.]

To insure that the monetary base of country k has value in equilibrium and that its bonds can bear nominal interest in equilibrium ($P_t^k < 1$), we assume that country k imposes (and is able to costlessly enforce) a reserve requirement on its residents' saving. Any resident of country k that saves a positive amount must hold a fraction λ^k of that amount in the form of country k base money. This requirement implies that the gross real rate of return faced by country k lenders at time t is the following weighted average: $\bar{r}_t^k = \lambda^k R_t^k + (1-\lambda^k)r_t$, where R_t^k is the gross real rate of return on the country k monetary base, namely, p_{t+1}^k/p_t^k, and r_t is the gross real rate of return on loans, the single real return on loans in all countries.

As discussed more fully in Wallace 1984 (p. 19), this reserve requirement is intended to capture in a simple way the role played by legal restrictions on private borrowing and lending in actual economies. Taken literally, it is an accurate description of an economy in which all individual lending, or saving, by residents of a country must take the form of accounts at banks or financial intermediaries, and these institutions must hold some fraction of the amount in those accounts in the form of the country's base money, but can otherwise hold assets in any form they want. If these institutions operate competitively and costlessly, then the rate they pay on their liabilities (their deposits) is a weighted average of the rate they earn on reserves and the rate they earn on loans, the weighted average described above as that facing private lenders.

World Equilibrium

Before formally describing the conditions for equilibrium in our world of K countries, we must describe some conditions on prices and interest rates, *arbitrage* conditions, that are implicit in the above description of individual trading opportunities. The first involves arbitrage between goods and monies, and the second involves arbitrage among securities.

As noted above, we are assuming that the single good in our world economy can be costlessly transported between countries. The first arbitrage condition—commonly known as *purchasing power parity*—is that the prices of the monies of any two countries in terms of the good and the exchange rate between the two monies are such that no gains can be made from the following set of transactions: selling the good in country k, using the resulting country k money to buy country k' money, and using the country k' money to buy the good in country k'. As the reader can verify, the condition that no gain be possible from such transactions is that the exchange rate $e_t^{k,k'}$, the price of country k money in units of country k' money at date t, be equal to the ratio of prices of monies:

$$(2) \qquad e_t^{k,k'} = p_t^k/p_t^{k'}.$$

The second arbitrage condition is that interest rates facing borrowers in the K countries of our world economy are such that they imply the same terms of trade at any date t between the time t good and the time $t + 1$ good. This condition is implied by the assumption that anyone in any country can borrow and lend in any other country subject only to the reserve requirement on positive saving. Under this assumption, we cannot have an equilibrium in which the real return on loans in one country exceeds that in another country because in such a situation no saver would want to hold securities bearing the lower return (a demand consistent with reserve requirements) and every borrower would want to borrow at the lower rate. Together, these imply an excess supply of the securities bearing the lower return and an excess demand for those bearing the higher return, which, of course, cannot be an equilibrium. Thus, the assumption that individuals and governments can borrow and lend anywhere subject only to the reserve requirement implies that our world economy has only one real rate of return on loans. In the notation introduced above, it implies a single real rate of return on loans, r_t.

For prices, including interest rates and exchange rates, that satisfy these two arbitrage conditions, the real trading opportunities facing individuals are those we have described—essentially, trading present consumption for future consumption or, equivalently, trading present consumption for assets which are promises of future consumption.

Now we can describe what we mean by a competitive, perfect foresight equilibrium for this world economy. *Competitive* means that people treat prices as beyond their control when they choose quantities. *Perfect foresight* here means that anticipated rates of return on assets equal actual or realized rates of return or, more particularly, that at each date t the young correctly anticipate the price of the monies of the different countries in terms of goods at the next date. *Equilibrium* means that all markets clear at each date. From now on, we will refer to a competitive, perfect foresight equilibrium as simply an *equilibrium.*

The formal definition of equilibrium that we give below is valid only for values of r_t for which the borrowers of each country actually borrow or, more precisely, only for values of r_t for which $D^k(r_t) \geq 0$ for every k. The following notation allows us to state this condition concisely. Let $\tilde{r}^k$ be such that $D^k(\tilde{r}^k) = 0$, and let $\tilde{r}$ be the smallest of the $\tilde{r}^k$ for $k = 1, 2, \ldots, K$. Then $D^k(r) \geq 0$ for all k if $r \leq \tilde{r}$. This condition appears as part of the following definition of an equilibrium:

DEFINITION. *Given each country's reserve requirement λ^k, its initial nominal indebtedness including its base money $H_1^k + B_1^k$ (a total which is assumed positive), sequences for its real net-of-interest deficit G_t^k, and a sequence for its base money H_{t+1}^k, an equilibrium consists of a sequence for r_t satisfying $r_t \leq \tilde{r}$ and sequences for each country for p_t^k, P_t^k, R_t^k, $\bar{r}_t^k$ and B_{t+1}^k that for all $t \geq 1$ satisfy equation* (1), *the cash flow constraint for each country, and*

$$(3) \qquad \textstyle\sum_k[S^k(\bar{r}_t^k) - D^k(r_t)] = \sum_k[p_t^k(H_{t+1}^k + P_t^k B_{t+1}^k)]$$

$$(4) \qquad \bar{r}_t^k = \lambda^k R_t^k + (1-\lambda^k) r_t$$

$$(5) \qquad R_t^k = p_{t+1}^k / p_t^k$$

$$(6) \qquad r_t = p_{t+1}^k / p_t^k P_t^k$$

$$(7) \qquad r_t \geq R_t^k$$

$$(8) \qquad p_t^k H_{t+1}^k \geq \lambda^k S^k(\bar{r}_t^k)$$

where for each country k (4)–(8) *must hold and either* (7) *or* (8) *must hold at equality.*

Equation (3) says that world net private saving—the sum over countries of each country's saving supplied at the weighted average of the return on its base money and the return on securities less each country's private borrowing—must equal the total world value of government liabilities. Equations (4), (5), and (6) define the returns facing savers and borrowers in each country and contain our perfect foresight assumption—namely, that the returns that determine choices at t match the actual returns. Note that (6) implies that the ratio of a country's gross inflation rate, p_t^k/p_{t+1}^k, to its gross nominal interest rate, $1/P_t^k$, is the same for all countries. Inequalities (7) and (8) and the accompanying proviso are related to the reserve requirement. Inequality (7) says that the return on loans is at least as great as that on the base money of each country. If it were not, then unlimited gains could be made by borrowing and using the proceeds to acquire base money, activities which would not violate the reserve requirement. That being so, no equilibrium can violate (7). Inequality (8) expresses the reserve requirement: the value of country k base money must be at least as great as the required fraction λ^k times gross saving of the residents of country k. The proviso arises in this way. If $r_t > R_t^k$, then wealth maximization implies that country k residents and everyone else hold no more of country k base money than the minimum required, which is to say that (8) holds at equality. Alternatively, if the value of country k's base money exceeds the minimum required to be held $[p_t^k H_{t+1}^k > \lambda^k S^k(\bar{r}_t^k)]$, then wealth maximization implies that the return on country k base money is as great as the return on securities, which is (7) at equality.[5]

Instead of trying to study all possible equilibria for this economy for arbitrary sequences of government policies, we study a limited class of policies and potential equilibria under those policies. We study only policies for which each country's real net-of-interest deficit is a constant, $G_t^k = G^k$, and each country's ratio of government bonds to base money is a constant, $B_{t+1}^k/H_{t+1}^k = \beta^k$. For such policies, we attempt to describe only those equilibria for which all real variables are constant over time, equilibria we call *stationary* equilibria. For such policies, we formally define a stationary equilibrium as follows:

DEFINITION. *Given* λ^k, $H_1^k + B_1^k > 0$, G^k, *and* β^k *for each k, a stationary equilibrium consists of a scalar* $r \leq \tilde{r}$ *and of scalars* R^k, $\bar{r}^k$, h^k, b^k, *and* p_1^k *for each k, where* h^k *denotes a constant real value of the country k monetary base,* $p_t^k H_{t+1}^k$, *and* b^k *denotes a constant real value of the government bonds of country k,* $p_t^k P_t^k B_{t+1}^k$, *that satisfy*

(9) $\quad G^k = (1-R^k)h^k + (1-r)b^k$

(10) $\quad \sum_k [S^k(\bar{r}^k) - D^k(r)] = \sum_k (h^k+b^k)$

(11) $\quad \bar{r}^k = \lambda^k R^k + (1-\lambda^k)r$

(12) $\quad r \geq R^k$

(13) $\quad h^k \geq \lambda^k S(\bar{r}^k)$

(14) $\quad G^k = h^k + b^k - p_1^k(H_1^k+B_1^k)$

where either (12) *or* (13) *must hold at equality.*

Note that equation (9) is the stationary version of the country k cash flow constraint, equation (1), and that (14) comes from that constraint for the first date, $t = 1$. For constant real sequences, this definition of an equilibrium and the earlier one are equivalent.

Below we make assumptions that imply that stationary equilibria are necessarily *binding,* equilibria for which (13) holds at equality. We focus on binding stationary equilibria because we suspect that they are the relevant ones for the current world economy.[6] Our approach to studying binding stationary equilibria is to solve equations (9)–(11) and equation (13) at equality for the h^k, b^k, and rates of return and then to verify that the implied solution satisfies (12). If it does and if it implies a positive p_1^k using equation (14), then it is a valid solution.

If equilibria are binding, we can reduce equations (9)–(11) and (13) at equality, $3K + 1$ equations, to $K + 1$ equations in $K + 1$ unknowns, r and the R^k. From the definitions of h^k and b^k, we have

(15) $\quad b^k/h^k = \beta^k P_t^k.$

Since, by (5) and (6), $P_t^k = R_t^k/r_t$, a constant in a stationary equilibrium, we can rewrite (15) as

(16) $\quad b^k = h^k \beta^k R^k/r.$

Then, upon substituting the right sides of (16) and (13) at equality into (9) and (10) we have, respectively,

(17) $\quad G^k = \lambda^k S^k(\bar{r}^k)[(1-R^k) + (1-r)\beta^k R^k/r]$

(18) $\quad \sum_k [S^k(\bar{r}^k) - D^k(r)] = \sum_k [\lambda^k S^k(\bar{r}^k)(1+\beta^k R^k/r)].$

If we use (11) to replace $\bar{r}^k$ by the weighted average of R^k and r, then the resulting versions of equations (17) and (18) are the $K + 1$ equations in the $K + 1$ unknowns, r and R^k for each k, that we referred to above. Moreover, as noted above, if the solution for these $K + 1$ equations satisfies (12) and is such that (14) can be solved for a positive p_1^k for each k, then the solution is a valid binding equilibrium.

A Special Case

Since (17) and (18) are complicated equations for general functions $S^k(\cdot)$ and $D^k(\cdot)$, we will study in detail only a special case of the model, one in which each $S^k(\cdot)$ function is a constant, denoted S^k, which does not depend on the return, $\bar{r}^k$.[7] This case is easy to study because for it, as we now show, equations (17) and (18) can be rewritten as a set of completely *recursive* equations, equations which

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[5] Two conditions in this definition depend on the restriction $r_t \leq \tilde{r}$. Without it, the argument of $D^k(\cdot)$ in (3) is not necessarily r_t and the right side of (8) would have to be $\lambda^k S^k(\bar{r}_t^k) + \lambda^k \max[0, -D^k(\bar{r}_t^k)]$.

[6] One significant feature of nonbinding equilibria is perfect substitution among the monetary bases of the different countries. See Kareken and Wallace 1981 for a discussion of the consequences of such substitution.

[7] If lenders have a utility function of the Cobb-Douglas form and if their endowment is entirely in the form of income when they are young, then $S^k(\cdot)$ is a constant fraction of that income (and does not depend on any rate of return).

can be solved one at a time.

We begin by solving equation (17) for R^k/r, obtaining

$$(19) \quad R^k/r = (1-G^k/\lambda^k S^k)/[r(1+\beta^k) - \beta^k].$$

Solving (17) in this way is valid if $r(1+\beta^k) - \beta^k \neq 0$. Below we present conditions that insure that the implied solution for r is such that this holds. Then, if we substitute the right side of (19) into the right side of (18) and at the same time impose the constant saving assumption, we can write the result as

$$(20) \quad E(r) = F(r; \beta, G)$$

where

$$E(r) \equiv \textstyle\sum_k[(1-\lambda^k)S^k - D^k(r)]$$

$$F(r; \beta, G) \equiv \textstyle\sum_k\{(\lambda^k S^k - G^k)\beta^k/[r(1+\beta^k) - \beta^k]\}$$

and where $\beta = (\beta^1, \beta^2, \ldots, \beta^K)$ and $G = (G^1, G^2, \ldots, G^K)$. Note that $E(r)$, an increasing function of r, is the world private excess demand for securities if the reserve requirement is binding in every country. The function $F(r; \beta, G)$ can be interpreted as the supply of securities by all the governments, a supply implied by the stationary versions of their cash flow constraints, bindingness of all the reserve requirements, and the choices of government portfolios, the β^k. If equation (20), which contains only one unknown, r, can be solved, then its solution can be used in equation (19) to find R^k. It can also be used to find p_1^k, the country k initial value of base money, from the following equation:

$$(21) \quad p_1^k = (\lambda^k S^k - G^k)r(1+\beta^k)/[r(1+\beta^k) - \beta^k].$$

Equation (21) is obtained from (14)—with $H_1^k + B_1^k = 1$—by substituting for h^k and b^k from (13) and (16) at equality and for R^k/r from (19).[8]

The propositions we want to establish, mainly about solutions to equations (19)–(21), are implied by the following assumptions:

Assumption 1. *$\lambda^k S^k > G^k \geq 0$ and $\beta^k > -1$ for all k.*

Assumption 2. $\Sigma_k[S^k - D^k(1)] < 0$.

Assumption 3. *$\tilde{r} > 1$ and*
$E(\tilde{r}) > [\Sigma_k(\lambda^k S^k - G^k)]/(\tilde{r}-1)$.

Assumption 4. *$rD'(r)/D(r) < -1$,*
where $D(r) \equiv \Sigma_k D^k(r)$.

The first part of Assumption 1 places bounds on the net-of-interest deficit; the upper bound is such that the deficit can be financed with $\beta^k = 0$; the lower bound says that, net of interest, the budget is not in surplus. The second part of Assumption 1 limits ratios of government debt to base money to those that keep the sum of the monetary base and the face value of government debt positive. Assumption 2 says that net private saving is negative at $r \leq 1$, that is, at negative and zero real interest rates. Together, these two assumptions have the following consequence:

Proposition 1. *Under Assumptions 1 and 2, any stationary equilibrium has $r > 1$ and is a binding equilibrium.*

(Proofs of Propositions 1–3 appear in Appendix A.)

Assumption 3 assures that we get a binding equilibrium with $r < \tilde{r}$. It assures that no matter how large are the β^k—that is, no matter how tight monetary policies are—there is an equilibrium with $r < \tilde{r}$.[9] Note that if countries are identical, so that, among other things, $D^k(\tilde{r}) = 0$ for all k, then Assumption 3 is implied by the simple condition $1 - \lambda > \lambda/(\tilde{r}-1)$.

Proposition 2. *Under Assumptions 1–3, a binding equilibrium with $r < \tilde{r}$ exists.*

Proposition 2 leaves open the possibility that there are several solutions to equation (20) and, hence, several binding equilibria with $r < \tilde{r}$. The next proposition shows that the elasticity condition, Assumption 4, rules out this possibility.

Proposition 3. *Under Assumptions 1–3 and either Assumption 4 or the existence of an equilibrium with $F(r; \beta, G) \geq 0$, equation (20) has a unique solution with $r < \tilde{r}$.*

The arguments in the proofs of Propositions 2 and 3 (in Appendix A) imply that the functions $E(r)$ and $F(r; \beta, G)$ are essentially as shown in Figures 1 and 2. That is, $F(r; \beta, G)$ crosses $E(r)$ only once and from above on the left. Thus, under the assumptions of Proposition 3, we can define the unique value of $r < \tilde{r}$ that satisfies (20) as a function of β and G, say,

$$(22) \quad r = \phi(\beta, G).$$

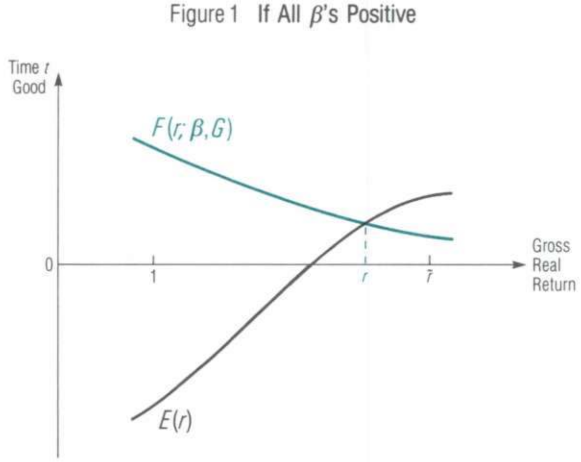
---

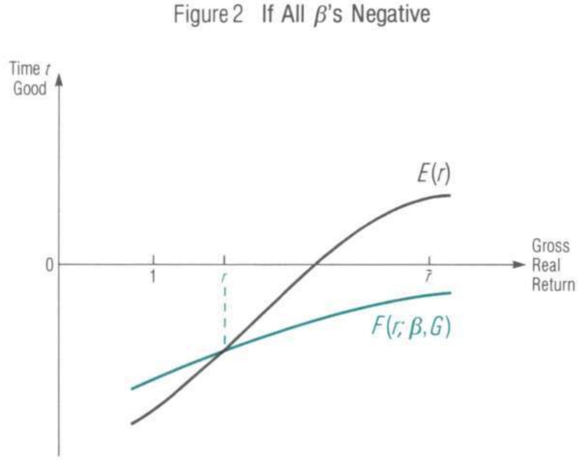
[8] Setting $H_1^k + B_1^k = 1$ for all k saves space and is innocuous. It amounts to no more than choosing monetary units of the different countries in a particular way.

[9] Examples of economies with $\tilde{r}$ as large as we want are easy to produce. For example, if every borrower in every country has a Cobb-Douglas utility function which weights consumption when young and when old equally and has the same lifetime income pattern, say, w_1^b when young and w_2^b when old, then $\tilde{r} = w_2^b/w_1^b$.

Figures 1 and 2

Possible Real Interest Rate Solutions to Equation (20)





By direct substitutions into (19) and (21), we get the corresponding unique solutions for R^k and p_1^k. Then, given the solutions for p_1^k, we use (2) to solve for $e_1^{k,k'}$.

The Effects of a Policy Change: The Model's Predictions vs. Recent Events

Existence of equilibria under given policies in our model indicates a kind of internal consistency. Now we want to describe a kind of external consistency as well—to show, that is, that the model's predictions for policies like those adopted in the world in the last few years roughly agree with what has in fact occurred. We adopt the assumptions of Proposition 3 and generate the model's qualitative predictions by determining how the unique world equilibrium changes when one nation's budget and monetary policies change.

We characterize the policies actually adopted in recent years as the adoption of a more expansionary budget policy in the face of a nonaccommodating monetary policy in the United States coupled with a passive policy response in the rest of the world. Labeling the United States *country 1,* we represent the adoption of a more expansionary budget policy as a permanent increase in the U.S. budget deficit net-of-interest G^1. Given this change in budget policy, we represent a nonaccommodating U.S. monetary policy as a permanent change in β^1 which keeps the monetary base in the period of the policy change, H_2^1, at what it otherwise would have been.[10] We represent the passive policy response in the rest of the world as no change in the rest of the model's G's and β's.

In order to make the qualitative predictions of our combined budget and monetary policy experiment better understood, we first describe the model's predictions for each policy change alone. This exercise also shows that our model cannot explain recent events by a change in just U.S. budget policy, but instead gives monetary policy a prominent role.

The structure of the model and the solutions indicate that all effects on other countries of a change in country 1's budget and monetary policies result from a change in the real interest rate.[11] Because of this, we solve once for the effects of a change in the real interest rate on the inflation rate, the nominal interest rate, the price level, and the current account deficit of a passive country—one whose G and β are fixed. Then, to determine the effects of a change in country 1's policies on other countries, we need only use equation (20) to determine how that change affects the real interest rate.

The analysis continues with an examination of own-

[10]Note that the United States cannot adhere to the original path of H^1 for all time. This is not a stationary policy, because it requires in each period an increase in β^1 and, hence, an increase in r. In a finite number of periods such a policy would cause the interest payments on the debt to exceed potential tax revenue.

[11]In terms of Figures 1 and 2, we determine how changes in β and G shift the curve $F(r;\ \beta,\ G)$ and, hence, move the intersection of $E(r)$ and $F(r;\ \beta,\ G)$.

country effects of policy changes. These effects are generally different from the cross-country effects because of some direct effects on the government's budget in addition to the interest rate change. Then, given the own-country and cross-country effects, we calculate the change in the own-country's exchange rate—the value of its currency relative to other countries'—and the change in its balance on current account.

In our calculation of a country's current account balance, we assume that all foreign debt contracted at time $t-1$ by individuals of generation $t-1$ is paid off at time t at the gross real interest rate r_{t-1}. Thus, at time t, the real value of total foreign claims on country k, K_t^k, is the excess of total domestic borrowing at time t—by individual residents and the government, $D^k(r_t)$ and $p_t^k(H_{t+1}^k + P_t^k B_{t+1}^k)$, respectively—over total domestic saving, S^k:

$$(23) \quad K_t^k = D^k(r_t) + p_t^k(H_{t+1}^k + P_t^k B_{t+1}^k) - S^k.$$

The current account deficit, C_t^k, measures the increase over time in real foreign claims on country k, which can be written using (1) as

$$(24) \quad C_t^k \equiv K_t^k - K_{t-1}^k = D^k(r_t) + G^k + p_t^k(H_t^k + B_t^k) - S^k - K_{t-1}^k.$$

Two points can be made from (24) about a country's real current account deficit. First, in a stationary equilibrium, K_t^k must be constant for $t \geq 1$, so that the current account deficit must be zero for $t \geq 2$. However, since K_1^k can be different from K_0^k, C_1^k can be different from zero. Second, by substituting for terms on the right side of (24), we see that the current account deficit measures the excess of consumption in country k—by the old, the young borrowers, the young lenders, and the government—over total endowments (the country k trade deficit) plus net interest payments on foreign debt $(r_{t-1}-1)K_{t-1}^k$. We assume in our policy experiments that $(r_0-1)K_0^k$ is unaffected by the choice of policy at time 1.

The effects of a change in the real interest rate on a passive country's inflation rate, nominal interest rate, price level, and current account deficit are calculated by differentiating equations (19), (21), and (24) holding its policies G and β fixed. In these calculations we assume Assumptions 1–4 hold. (The derivatives are displayed in Appendix B, and the effects of an increase in the real interest rate are displayed in Table 2.)

The results in Table 2 are best understood in terms of the impact of a higher real interest rate on a passive government's budget. If the government is a debtor, then a higher real interest rate raises its interest payments and forces it to increase the rate at which it is issuing both money and debt. If it is a creditor, then the reverse occurs.

The model's qualitative predictions for the effects of a policy change in country 1 are determined in two steps. First we obtain general expressions for the changes in the real interest rate and the own-country inflation rate, nominal interest rate, price level, current account deficit, and exchange rates by differentiating equations (20), (19), (21), (24), and (2). (These derivatives are also displayed in Appendix B.) In the second step, we identify country 1 as the United States and, in addition to Assumptions 1–4, we make two assumptions. We assume, quite realistically, that the U.S. government is a debtor, so that $\beta^1 > 0$, and that the governments of all other countries collectively are debtors in the sense that

$$\textstyle\sum_{k=2}^{K} \chi^k \beta^k (1+\beta^k) \geq 0$$

where

$$\chi^k \equiv (\lambda^k S^k - G^k)/[r(1+\beta^k) - \beta^k]^2 > 0$$

by Assumption 1. The discussion that follows invokes these additional assumptions.

Tighter U.S. Monetary Policy

The first policy we evaluate is a tighter monetary policy in country 1, which we have identified as the United States. We increase β^1, the ratio of government bonds to money in the United States, holding all other β's and all G's constant. The predictions from our model are combined

Table 2

The Effects of a Higher Real Interest Rate r on a Passive Country k

Variable	Debtor Government ($\beta^k > 0$)	Creditor Government ($\beta^k < 0$)
Inflation Rate ($1/R^k-1$)	+	−
Nominal Interest Rate (r/R^k-1)	+	+
Price Level ($1/p_1^k$)	+	−
Current Account Deficit (C_1^k)	−	+ or −

+ = increases − = decreases

Table 3

The Effects of a Tighter U.S. Monetary Policy

Variable	United States ($\beta^1>0$)	Other Countries: Debtor Government ($\beta^k>0$)	Other Countries: Creditor Government ($\beta^k<0$)
Real Interest Rate (r)	+	+	+
Inflation Rate ($1/R^k-1$)	+	+	−
Nominal Interest Rate (r/R^k-1)	+	+	+
Price Level ($1/p_1^k$)	−	+	−
Value of Currency Relative to Dollar ($1/e_1^{1,k}$)	n.a.	−	−
Current Account Deficit (C_1^k)	+	−	+ or −

+ = increases − = decreases n.a. = not applicable

with the results from Table 2, and all are displayed in Table 3.

The results in Table 3 have an intuitive explanation. An increase in β^1 decreases the amount of money and increases the amount of government bonds outstanding in the United States. The drop in money produces a lower price level and thus a higher real value of government bonds. The increase in the supply of real debt in the world capital market causes the real interest rate to rise and generates the cross-country effects discussed earlier. Some of the additional real debt is purchased by foreign residents, causing the U.S. current account deficit to rise. With more bonds outstanding at a higher real interest rate, the real interest expense on government debt rises and forces the U.S. government to make greater use of the inflation tax.

Easier U.S. Budget Policy With Monetary Accommodation . . .

Now we evaluate an easier budget policy in the United States. We increase G^1, the U.S. real budget deficit net-of-interest, which corresponds to an increase in real government consumption, holding constant all β's and all other G's. The predictions from our model are combined with the results from Table 2, and all are displayed in Table 4.

The effects in the United States of an easing in its budget policy seem counterintuitive. They are explained by the monetary accommodation implied by a fixed β^1 in the face of a higher G^1. At the initial real interest rate, an increase in G^1 leaves the demand for government bonds, $E(r)$, unaffected. The increase in G^1 affects the supply, however, in a way that depends on β^1. When $\beta^1 > 0$, the higher deficit with a fixed β^1 implies sufficient monetary expansion to reduce the real value of government debt. The excess demand for bonds causes the real interest rate to fall, and that generates the cross-country effects discussed earlier. The monetary accommodation results in a higher U.S. price level, and the decline in U.S. borrowing causes the U.S. current account deficit to shrink. Although the interest expense on U.S. government debt falls due to both a lower real stock of bonds and a lower real interest rate, the fall does not offset the increase in the budget deficit net-of-interest. With a higher deficit inclusive of interest, the U.S. government must make greater use of the inflation tax.

. . . And Without Monetary Accommodation

Finally, we evaluate an easier budget policy with a non-accommodating monetary policy in the United States—what seems to represent actual U.S. policies in the past four years. It is a combination of the previous two experiments, involving an increase in β^1 and an increase in G^1. Since the effects of such increases are often of opposite signs, the effects of the combined policy experiment cannot simply be deduced from the previous experiments.

In this experiment we increase G^1 and let the model determine the increase in β^1 required to keep the initial money stock H_2^1 unchanged from what it otherwise would have been. All other β's and G's are held constant. To do this experiment, we use equation (1) for the first date to solve for β^1 as a function of G^1 and H_2^1, namely,

$$\beta^1 = (r/R^1)(G^1/\lambda^1 S^1+\psi)$$

where

$$\psi \equiv (H_1^1+B_1^1-H_2^1)/H_2^1.$$

We next substitute this expression for β^1 into (19), (20), and (21) to get, respectively,

$$(25)\quad R^1/r = (1+\psi)/r - (\lambda^1 S^1\psi+G^1)/\lambda^1 S^1$$

$$(26)\quad E(r) = \textstyle\sum_{k=2}^{K}(\lambda^k S^k-G^k)\{\beta^k/[r(1+\beta^k) - \beta^k]\} + \lambda^1 S^1(1+\psi) + G^1$$

$$(27)\quad p_1^1 = \lambda^1 S^1(1+\psi)/(H_1^1+B_1^1).$$

Our policy experiment then translates into determining

Table 4

The Effects of an Easier U.S. Budget Policy With Monetary Accommodation

Variable	United States ($\beta^1>0$)	Other Countries: Debtor Government ($\beta^k>0$)	Other Countries: Creditor Government ($\beta^k<0$)
Real Interest Rate (r)	−	−	−
Inflation Rate ($1/R^k-1$)	+	−	+
Nominal Interest Rate (r/R^k-1)	+	−	−
Price Level ($1/p_1^k$)	+	−	+
Value of Currency Relative to Dollar ($1/e_1^{1,k}$)	n.a.	+	+
Current Account Deficit (C_1^k)	−	+	+ or −

+ = increases − = decreases n.a. = not applicable

the effects in this new system of an increase in G^1 holding ψ (and hence H_2^1) constant.

By combining these results with the cross-country effects in Table 2, we get the Table 1 results that we described in the preview to the paper. As we discussed there, these predictions seem generally consistent with actual experience.

These results highlight how the effects of a budget policy change depend on the accompanying monetary policy. When we hold constant H_2^1—rather than β^1 as in the previous experiment—the effects of an increase in G^1 seem more in accord with intuition. When H_2^1 is held constant, the initial price level is unaffected by the increase in G^1. That increase must then increase the supply of real debt in the world capital market, and this causes a rise in the real interest rate, which in turn leads to the cross-country effects in Table 1. Some of the real debt is bought by foreign residents, so that the U.S. current account deficit increases. With a higher deficit net-of-interest and higher interest expense, the United States must make greater use of the inflation tax.

Choosing Monetary Policies: Cooperation vs. Noncooperation

Countries obviously interact in our model, in the sense that one country's policy choices affect residents of other countries. Here we take all budget policies as given and consider whether cooperation among countries in choosing monetary policies, the β^k, would be desirable for the world. We address this question by comparing what happens if countries cooperate in choosing monetary policies with what happens if they do not.

We make the comparison using the following definitions of *not cooperating* and *cooperating*. *Not cooperating* will mean that each country k chooses its own monetary policy, β^k, to maximize its social welfare function taking as given the monetary policies of all the other countries, β^j for all $j \neq k$.[12] The outcome of noncooperation will be described by a vector $\hat{\beta} = (\hat{\beta}^1, \hat{\beta}^2, \ldots, \hat{\beta}^K)$ that simultaneously satisfies these conditions for all countries. In the terminology of game theory, such an outcome is called a *Nash equilibrium. Cooperating* will mean that all the β^k are chosen to maximize a weighted average of the social welfare functions of the individual countries. We will show that these definitions and our model imply that cooperation is desirable in the sense that it can produce a higher value of every country's social welfare function than does not cooperating.

Our first task, then, is to describe the social welfare function of a country. Since we are considering only stationary equilibria, the country k social welfare function can be expressed as a function of three arguments: the welfare of a country k initial old person, that of a country k saver in any generation, and that of a country k borrower in any generation. Moreover, since current old persons are better off the higher is p_1^k, since savers are better off the higher is $\bar{r}^k = \lambda^k R^k + (1-\lambda^k)r$, and since borrowers are better off the higher is $1/r$, we can express country k's welfare as a function of those three variables, namely, as a function $u^k(p_1^k, \bar{r}^k, 1/r)$ where u^k is increasing in each of its arguments and is in other respects like an ordinary utility function.[13]

The next step is to express social welfare for country k in terms of the monetary policy parameters, the vector $\beta = (\beta^1, \beta^2, \ldots, \beta^K)$. This is done by substituting the solutions for p_1^k, $\bar{r}^k$, and r that we found earlier into the expression for u^k. To do this, we use (22) with the argument G suppressed—$r = \phi(\beta)$—and express the solutions for R^k and p_1^k, as implied by (19) and (21), as

$$(28) \quad R^k = \phi_2^k(\beta) \equiv \phi(\beta)(1-G^k/\lambda^k S^k) \div [\phi(\beta)(1+\beta^k) - \beta^k]$$

[12]If the world has many similar countries, so that each is a small part of the world economy, then taking other countries' monetary policies as given is approximately the same as taking the world interest rate r as given, as unaffected by the choice of β^k.

[13]In particular, we assume that the upper contour sets of u^k are strictly convex.

$$(29)\quad p_1^k = \phi_3^k(\beta) \equiv \phi(\beta)(\lambda^k S^k - G^k)(1+\beta^k) \div [\phi(\beta)(1+\beta^k) - \beta^k].$$

Then, recalling that $\bar{r}^k = \lambda^k R^K + (1-\lambda^k)r$, we let

$$(30)\quad V^k(\beta^k, \beta^{)k(}) \equiv u^k\{\phi_3^k(\beta), \lambda^k \phi_2^k(\beta) + (1-\lambda^k)\phi(\beta), 1/\phi(\beta)\}$$

where $\beta^{)k(} = (\beta^1, \beta^2, \ldots, \beta^{k-1}, \beta^{k+1}, \ldots, \beta^K)$, the vector β with β^k excluded.

As noted above, we are assuming that what happens under *noncooperation* is described by a Nash equilibrium with each country k choosing β^k, taking $\beta^{)k(}$ as given. Formally, then, the noncooperative solution is a vector $(\hat{\beta}^1, \hat{\beta}^2, \ldots, \hat{\beta}^K)$ such that, for each k, $\beta^k = \hat{\beta}^k$ maximizes $V^k(\beta^k, \hat{\beta}^{)k(})$. We can view *cooperation* as leading to the choice of any feasible vector β, in particular, one that maximizes $\Sigma w^k V^k(\beta^k, \beta^{)k(})$, where w^k is the weight given to the country k social welfare function and the summation is over the K countries.

We appraise the noncooperative solution under the assumptions that the world economy consists of identical countries and that the noncooperative solution for such a world is one with a common value of β^k, what is called a *symmetric* Nash equilibrium.[14] The assumption of identical countries simplifies the presentation and does not prejudice the results toward cooperation.[15] Very generally, if cooperation is desirable in a world of identical countries, then it is desirable in a world of dissimilar countries. We then have the following proposition:

PROPOSITION 4. *If countries are identical and Assumptions 1–4 hold, then monetary policies generally exist that imply a higher value of the common social welfare function u than is implied by any noncooperative solution with a common value of β^k for all k (any symmetric Nash equilibrium).*

The proof of this proposition in Appendix C shows that a small common departure of all the β^k from their common Nash equilibrium value will in general raise the value of the common function V, as defined in equation (30), from its value at the Nash equilibrium.

The proof shows, however, that there is no general presumption about whether the higher value of social welfare is achieved at lower or higher values of the β^k (with an easier or tighter monetary policy). It implies that if the social welfare function does not attach any weight to savers (to $\bar{r}^k$), then better cooperative outcomes are achieved at lower values of the β^k. If, alternatively, the welfare function does not attach any weight to the initial owners of currency (to p_1^k), then better cooperative outcomes are achieved at higher values of the β^k. These results also follow from an examination of the tradeoffs among triplets $(p_1^k, \bar{r}^k, 1/r)$ faced, on the one hand, by a country acting noncooperatively (taking other countries' policies as given) and those faced, on the other hand, by all countries acting jointly (and varying all the β^k in unison). As also shown in Appendix C, a given increase in $1/r$ (a benefit to borrowers) is achieved at the expense of a larger decrease in p_1^k (a cost to the initial old) when a country acts noncooperatively than when all countries act jointly. And a given increase in $1/r$ is achieved at the expense of a smaller decrease in $\bar{r}^k$ (a cost to savers) when a country acts noncooperatively than when all countries act jointly. Obviously, then, if the social welfare function attaches weight to the well-being of all three groups, then the better cooperative outcomes can occur at either lower or higher common values of the β^k. The following two examples illustrate these possibilities.

The examples are of a world economy with three identical countries. In each country, each generation consists of one saver endowed with 1 unit of the consumption good when young and nothing when old and one borrower endowed with nothing when young and 1.05 units of the consumption good when old. Each person has a utility function equal to the sum of the logarithms of first- and second-period consumption. These assumptions imply $S^k = 0.5$ and $D^k(r) = 1.05/2r$ for all k. We also assume that $\lambda^k = 0.1$ and $G^k = 0.005$ for all k. Finally, we let $u^k(p_1^k, \bar{r}^k, 1/r) = (0.01)\ \ln(p_1^k) + \alpha \ln(\bar{r}^k) + \ln(1/r)$, where α is a parameter.[16] If $\alpha = 1$, then the symmetric Nash equilibrium values are

$$(\beta^k, p_1^k, \bar{r}^k, 1/r) = (-0.267, 0.0341, 1.118, 1/1.139)$$

and, as can easily be verified, better outcomes are

[14] We do not prove existence of a symmetric Nash equilibrium. It is easy to make assumptions about V which guarantee such existence—namely, that for each possible common value for monetary policy chosen by other countries, country k has a unique best policy which is neither indefinitely easy (in the direction of -1) nor indefinitely tight (in the direction of ∞) and which depends in a continuous way on the policy chosen by other countries. However, it is hard to make appealing assumptions about the structure and the social welfare function u^k that imply these conditions.

[15] The assumption of identical countries in our model does have one special consequence. It implies that a symmetric cooperative optimum is achieved if each country both imposes capital controls which rule out any international borrowing and lending and chooses its monetary policy, its β, to maximize its social welfare function.

[16] Note that this particular u^k is actually a weighted sum of indirect utilities, since (aside from additive constants) the indirect utility function of the initial old person who holds the initial nominal debt of country k is $\ln(p_1^k)$, that of the country k saver in each generation is $\ln(\bar{r}^k)$, and that of any borrower is $\ln(1/r)$.

achieved at (slightly) lower values of the β^k. If $\alpha = 2$, then the symmetric Nash equilibrium values are

$$(\beta^k,\ p_1^k,\ \bar{r}^k,\ 1/r) = (0.558, 0.0638, 1.178, 1/1.217)$$

and better outcomes are achieved at higher values of the β^k.[17]

Perusal of Appendix C will verify that cooperation is desirable in this model because of the asymmetry discussed in the preview. One country's policy affects its borrowers and those of other countries in the same way, but affects its initial old and its savers in a quite different way than it affects the initial old and savers of other countries because each country's money is held only by its residents.[18] Since the asymmetry is more important the smaller is each country relative to the world economy, the desirability of cooperation in our model does not depend on countries being, in any sense, large relative to the world economy.

Conclusion

We have shown that in a particular model some form of coordination would improve the workings of a floating exchange rate system. We have identified crucial features of our model as separate country-by-country markets in currencies and an integrated world market in securities in which government borrowing affects interest rates. We have also suggested that our conclusion is likely to hold in most models that share these features. However, even if these features are accepted as approximating conditions in the actual world economy, our conclusion cannot be taken as recommending a particular policy.

Within the confines of our particular model, we have seen that no general conclusion emerges concerning whether better cooperative outcomes are achieved at tighter or easier monetary policies. How cooperation changes policies depends on the objective tradeoffs as well as on the weights each country assigns the utilities of different groups of people.

Since features within our model affect the form that desirable cooperation takes, we expect that departures from our model would also affect it. We can imagine reasonable departures in several directions:

- Different *economic environments*—for example, environments with more general asset demands, uncertainty, and nontraded goods.
- Different *policy choices*—for example, nonconstant sequences of policies and policies parameterized in different ways.
- Different *noncooperative equilibrium concepts*—for example, ones that do not treat the countries symmetrically and ones that have countries take into account the policy responses of other countries.
- Different *cooperative equilibrium concepts*—for example, ones that take account of enforcement problems arising from uncertainty.

We do not know how these departures would change the form that desirable cooperation would take.

Thus, although we have identified one set of general conditions about the world's markets in currencies and securities that imply some role for cooperation, our analysis does not imply a particular way of achieving desirable cooperation. It does suggest that there is a plausible rationale for the proposal that countries coordinate their macroeconomic policies under floating exchange rates. However, it leaves quite open what precise form that coordination should take.

[17] For the above form of $D^k(r)$, equation (20) is quadratic in r at a common value of β. This permits us to find an explicit form for the function V and for its partial derivative with respect to its first argument. For each example, the symmetric Nash equilibrium was found by numerically solving for the common value of the β^k for which that partial derivative is zero.

[18] The asymmetry and, we strongly suspect, the desirability of cooperation would not be present if all the initial old and all the savers held all the different currencies in the same proportion. The kind of asymmetry we find is implied as long as the currency of country k is held predominantly by its residents.

Appendix A
Proofs of Propositions 1–3

PROPOSITION 1. *Under Assumptions* 1 *and* 2, *any stationary equilibrium has* $r > 1$ *and is a binding equilibrium.*

Proof. By (13) and $\beta^k > -1$, $h^k + b^k > 0$. But then, by (10) and Assumption 2, $r > 1$. To see that a nonbinding stationary equilibrium cannot exist, note that $R^k = r > 1$ and $h^k + b^k > 0$ imply that the right side of (9) is negative, which contradicts Assumption 1.

PROPOSITION 2. *Under Assumptions* 1–3, *a binding equilibrium with* $r < \tilde{r}$ *exists.*

Proof. We will show that equation (20) has a solution with $r \,\varepsilon\, (1, \tilde{r})$. That and Assumption 1 will imply immediately that the right side of (19) is positive and less than unity ($0 < R^k < r$) and that the right side of (21) is positive ($p_1^k > 0$).

Since $E(r)$ and $F(r; \beta, G)$ are continuous functions of r (for fixed β and G), to show that (20) has a solution with $r \,\varepsilon\, (1, \tilde{r})$ we need only show that $E(1) < F(1; \beta, G)$ and that $E(\tilde{r}) > F(\tilde{r}; \beta, G)$.

We have $E(1) = \Sigma[S^k - D^k(1)] - \Sigma\lambda^k S^k < -\Sigma\lambda^k S^k$, the inequality being a consequence of Assumption 2. We also have $F(1; \beta, G) = \Sigma(\lambda^k S^k - G^k)\beta^k > -\Sigma(\lambda^k S^k - G^k) \geq -\Sigma\lambda^k S^k$, both inequalities being consequences of Assumption 1. Thus, $E(1) < F(1; \beta, G)$.

Since $F(r; \beta, G)$ is increasing in β^k for each k, $F(\tilde{r}; \beta, G)$ is less than the limit of $F(\tilde{r}; \beta, G)$ as $\beta^k \to \infty$ for every k. This limit is $[\Sigma_k(\lambda^k S^k - G^k)]/(\tilde{r}-1)$. Therefore, Assumption 3 implies that $E(\tilde{r}) > F(\tilde{r}; \beta, G)$.

PROPOSITION 3. *Under Assumptions* 1–3 *and either Assumption* 4 *or the existence of an equilibrium with* $F(r; \beta, G) \geq 0$, *equation* (20) *has a unique solution with* $r < \tilde{r}$.

Proof. Since Proposition 3 is obviously true if $\beta^k = 0$ for all k, we proceed under the assumption that $\beta^k \neq 0$ for at least some k. Letting $f(r) \equiv F(r; \beta, G)$, we first establish that

$$\text{(A1)} \quad f'(r) < -(1/r)f(r).$$

Note that $f'(r) = -\Sigma x_k(r)y_k(r)$, where $x_k(r) = (\lambda^k S^k - G^k)\beta^k/[r(1+\beta^k) - \beta^k]$ and $y_k(r) = 1/[r - \beta^k/(1+\beta^k)]$. We also have that if $\beta^k > 0$, then $x_k(r) > 0$ and $y_k(r) > 1/r$, while if $\beta^k < 0$, then $x_k(r) < 0$ and $y_k(r) < 1/r$. Therefore, $-f'(r) = \Sigma x_k(r)y_k(r) > (1/r)\Sigma x_k(r) = (1/r)f(r)$. Thus, we have (A1).

Inequality (A1) says that $f(r)$ [which is identical to $F(r; \beta, G)$] is downward-sloping wherever F is not negative. Thus, if (20) has a solution where $f \geq 0$—say, at r_+—then it is the only solution. There cannot be a solution at $r > r_+$ because $E(r)$ is increasing and f can never get to a higher value than $f(r_+)$ without violating (A1). There cannot be a solution with $r < r_+$ because then f could never get to a value as great as $f(r_+)$ without violating (A1). [Note that we get an equilibrium where $F \geq 0$ if enough of the β^k are positive. Thus, if $\beta^k \geq 0$ for all k, then we have a unique solution to (20) without appeal to Assumption 4.]

When there is no solution with $F \geq 0$, we need Assumption 4. Since $E(1) < F(1; \beta, G) \equiv f(1)$, uniqueness is implied if $f'(r) < E'(r)$ at any solution.

At any solution, we have the following string:

$$\text{(A2)} \quad \begin{aligned} f'(r) &< -(1/r)f(r) = -(1/r)E(r) \\ &= -(1/r)\,\Sigma(1-\lambda^k)S^k + (1/r)D(r) \\ &\leq -(1/r)\,\Sigma(1-\lambda^k)S^k - D'(r) < -D'(r) = E'(r). \end{aligned}$$

The first inequality is (A1); the second (an equality) uses the assumption that we are at a solution; the third (an equality) uses the definition of $E(r)$; the fourth uses Assumption 4 and the fact that $D(r) > 0$ at any $r < \tilde{r}$.

Appendix B
Expressions for the Effects of One Country's Policy Changes

Here we display expressions for the derivatives of key variables with respect to the real interest rate in a passive country and with respect to policy variables in the own-country.

Effects of a Change in the Real Interest Rate on Passive Country *k*

Derivative with respect to r of			From
(B1)	R^k:	$-(\chi^k\beta^k/\lambda^k S^k)$	(19)
(B2)	R^k/r:	$-[\chi^k(1+\beta^k)/\lambda^k S^k]$	(19)
(B3)	p_1^k:	$-\chi^k\beta^k(1+\beta^k)$	(21)
(B4)	C_1^k:	$D^{k'}(r) - \chi^k\beta^k(1+\beta^k)$	(20), (24)

Effects of a Change in Country 1's Monetary Policy

Derivative with respect to β^1 of			From
(B5)	r:	$\chi^1 r/a_0$*	(20)
(B6)	R^1:	$-(\chi^1 r/\lambda^1 S^1)[(\chi^1\beta^1/a_0) + (r-1)]$	(19), (20)
(B7)	R^1/r:	$-(\chi^1/\lambda^1 S^1)\{[\chi^1 r(1+\beta^1)/a_0] + (r-1)\}$	(19), (20)
(B8)	p_1^1:	$\chi^1 r\{1 - [\chi^1\beta^1(1+\beta^1)/a_0]\}$	(20), (21)
(B9)	$e_1^{1,k}$:	$(e_1^{1,k}/a_0)\left(\left\{\sum_{j=2}^K[\chi^j\beta^j(1+\beta^j) - D'(r)] \,/\, (1+\beta^1)[r(1+\beta^1) - \beta^1]\right\} + \chi^1\beta^1/[r(1+\beta^k) - \beta^k]\right)$	(21), (B3), (B8)
(B10)	C_1^1:	$\chi^1 r\left\{\sum_{k=2}^K[\chi^k\beta^k(1+\beta^k) - D^{k'}(r)] \div \sum_{k=1}^K[\chi^k\beta^k(1+\beta^k) - D^{k'}(r)]\right\}$	(20), (24)

*$a_0 \equiv \sum_{k=1}^K \chi^k\beta^k(1+\beta^k) - D'(r) > 0$ by Assumptions 1–4.

Effects of a Change in Country 1's Budget Policy With Monetary Accommodation

Derivative with respect to G^1 (with fixed β^1) of			From
(B11)	r:	$(-1/a_0)\{\beta^1/[r(1+\beta^1) - \beta^1]\}$	(20)
(B12)	R^1:	$a_1[-ra_0 + \chi^1(\beta^1)^2]$*	(19), (20)
(B13)	R^1/r:	$a_1[-a_0 + \chi^1\beta^1(1+\beta^1)]$	(19), (20)
(B14)	p_1^1:	$a_2[-ra_0 + \chi^1(\beta^1)^2]$*	(20), (21)
(B15)	$e_1^{1,k}$:	$-(e_1^{1,k}/a_0)\Big(\{\beta^1(1+\beta^k)/[r(1+\beta^1) - \beta^1][r(1+\beta^k) - \beta^k]\} + \Big\{\Big[\sum_{j=2}^{K}\chi^j\beta^j(1+\beta^j) - D^{j\prime}(r)\Big] / (\lambda^1 S^1 - G^1)\Big\}\Big)$	(21), (B3), (B14)
(B16)	C_1^1:	$-\{\beta^1/[r(1+\beta^1) - \beta^1]\}\Big\{\sum_{k=2}^{K}[\chi^k\beta^k(1+\beta^k) - D^{k\prime}(r)] \div \sum_{k=1}^{K}[\chi^k\beta^k(1+\beta^k) - D^{k\prime}(r)]\Big\}$	(20), (24)

Effects of a Change in Country 1's Budget Policy Without Monetary Accommodation

Derivative with respect to G^1(with fixed H_2^1) of			From
(B17)	r:	$1/[a_0 - \chi^1\beta^1(1+\beta^1)]$	(26)
(B18)	R^1:	$(-1/\lambda^1 S^1)\big(\{(\lambda^1 S^1\psi + G^1)/[a_0 - \chi^1\beta^1(1+\beta^1)]\} + r\big)$	(25)
(B19)	R^1/r:	$-[(1+\psi)/r^2]/[a_0 - \chi^1\beta^1(1+\beta^1)]$	(25)
(B20)	p_1^1:	0	(27)
(B21)	$e_1^{1,k}$:	$(e_1^{1,k}/p_1^k)[\chi^k\beta^k(1+\beta^k)]/[a_0 - \chi^1\beta^1(1+\beta^1)]$	(21), (B3), (B20)
(B22)	C_1^1:	$\Big\{\sum_{k=2}^{K}[\chi^k\beta^k(1+\beta^k) - D^{k\prime}(r)]\Big\} \div \Big\{\sum_{k=1}^{K}[\chi^k\beta^k(1+\beta^k) - D^{k\prime}(r)] - D^{1\prime}(r)\Big\}$	(24), (26)

*$a_1 \equiv \{\lambda^1 S^1[r(1+\beta^1) - \beta^1]a_0\}^{-1} > 0.$

$a_2 \equiv [(1+\beta^1)\lambda^1 S^1]a_1 > 0.$

Appendix C
Proof of Proposition 4 and Derivation of the Model's Tradeoffs

Proof of Proposition 4

PROPOSITION 4. *If countries are identical and Assumptions* 1–4 *hold, then monetary policies generally exist that imply a higher value of the common social welfare function u than is implied by any noncooperative solution with a common value of β^k for all k (any symmetric Nash equilibrium).*

Proof. Let 1_{K-1} denote a $K-1$ element vector of 1's, and let $V_1(\cdot, \cdot)$ denote the partial derivative of V with respect to its first argument. We show that the derivative of $V[\beta, \beta(1_{K-1})]$ with respect to β is generally different from zero when it is evaluated at a symmetric Nash equilibrium, $\hat{\beta}$, which satisfies the first-order condition $V_1[\hat{\beta}, \hat{\beta}(1_{K-1})] = 0$.

Since $dV[\beta,\beta(1_{K-1})]/d\beta = V_1[\beta,\beta(1_{K-1})] + \{(K-1) \times V_{)(}[\beta, \beta(1_{K-1})]\}$, where $V_{)(}$ denotes the partial derivative of V with respect to any argument other than the first, and since $V_1[\hat{\beta}, \hat{\beta}(1_{K-1})] = 0$, our task is to derive an expression for $V_{)(}[\beta, \beta(1_{K-1})]$ and evaluate it at $\beta = \hat{\beta}$.

From (30),

$$\text{(C1)} \quad V_{)(}[\beta, \beta(1_{K-1})] = u_1(\partial\phi_3^k/\partial\beta^j) + u_2\lambda(\partial\phi_2^k/\partial\beta^j) + [(1-\lambda)u_2 - (u_3/r^2)](\partial\phi/\partial\beta^j)$$

$$\text{(C2)} \quad V_1[\beta, \beta(1_{K-1})] = u_1(\partial\phi_3^k/\partial\beta^k) + u_2\lambda(\partial\phi_2^k/\partial\beta^k) + [(1-\lambda)u_2 - (u_3/r^2)](\partial\phi/\partial\beta^k)$$

where u_i stands for the partial derivative of u with respect to its ith argument and where the partial derivatives of ϕ, ϕ_2^k, and ϕ_3^k—see equations (22), (28), and (29)—are computed as in Appendix B.

At a symmetric Nash equilibrium, the right side of (C2) is zero and $\partial\phi/\partial\beta^k = \partial\phi/\partial\beta^j$. These imply, by substitution from (C2) into (C1), that

$$\text{(C3)} \quad V_{)(}[\hat{\beta}, \hat{\beta}(1_{K-1})] = u_1[(\partial\phi_3^k/\partial\beta^j) - (\partial\phi_3^k/\partial\beta^k)] + u_2\lambda[(\partial\phi_2^k/\partial\beta^j) - (\partial\phi_2^k/\partial\beta^k)].$$

Since, by (28) and (29),

$$\text{(C4)} \quad (\partial\phi_3^k/\partial\beta^j) - (\partial\phi_3^k/\partial\beta^k) = -\lambda SR/[r(1+\beta) - \beta]$$

$$\text{(C5)} \quad (\partial\phi_2^k/\partial\beta^j) - (\partial\phi_2^k/\partial\beta^k) = R(r-1)/[r(1+\beta) - \beta]$$

we have

$$\text{(C6)} \quad V_{)(}[\hat{\beta}, \hat{\beta}(1_{K-1})] = -\{\lambda\hat{R}/[\hat{r}(1+\hat{\beta}) - \hat{\beta}]\} \times [u_1 S - u_2(\hat{r}-1)].$$

Although the terms in the second factor on the right side of (C6)—that involving u_1 and u_2—have opposite signs, they are generally not of equal magnitudes. Thus, $V_{)(}[\hat{\beta}, \hat{\beta}(1_{K-1})]$ is generally not zero.

Cooperative and Noncooperative Tradeoffs

Write (19) as $R^k = g(\beta^k, r)$ and (21) as $p_1^k = h(\beta^k, r)$. Also, let $dr/d\beta^k = \partial\phi(\beta)/\partial\beta^k$ and $dr/d\beta = \Sigma_k \partial\phi(\beta)/\partial\beta^k$. If these derivatives are evaluated at $\beta^k = \beta$ for all k, then $dr/d\beta = K dr/d\beta^k$, which is used below.

We begin by finding the tradeoffs between R^k and r. We have

$$\text{(C7)} \quad dR^k/d\beta^k = g_1 + g_2(dr/d\beta^k)$$

where g_i is the partial derivative of g with respect to its ith argument. Therefore,

$$\text{(C8)} \quad (dR^k/dr)_N = (dR^k/d\beta^k)/(dr/d\beta^k) = g_1/(dr/d\beta^k) + g_2$$

where N denotes *noncooperative* (holding $\beta^j = \hat{\beta}$ for $j \neq k$). Also,

$$\text{(C9)} \quad dR^k/d\beta = g_1 + g_2 dr/d\beta$$

and, therefore,

$$\text{(C10)} \quad (dR^k/dr)_C = (dR^k/d\beta)/(dr/d\beta) = g_1/(dr/d\beta) + g_2$$

where C denotes *cooperative* (varying all the β^k together). Therefore,

$$\text{(C11)} \quad (dR^k/dr)_C - (dR^k/dr)_N = -(K-1)g_1/(dr/d\beta) > 0$$

since $dr/d\beta^k > 0$ and, from (19), $g_1 = -(1-G/\lambda S)(r-1) \div [r(1+\beta^k) - \beta^k]^2 < 0$.

Then, since $\bar{r}^k = \lambda R^k + (1-\lambda)r$, we know immediately that $(d\bar{r}^k/dr)_C - (d\bar{r}^k/dr)_N$ is λ times the right side of (C11).

Finally, in exactly the same way as we got (C11), we get

$$\text{(C12)} \quad (dp_1^k/dr)_C - (dp_1^k/dr)_N = -(K-1)h_1/(dr/d\beta) < 0.$$

The inequality in (C12) is a consequence of $h_1 = (\lambda S - G)r \div [r(1+\beta^k) - \beta^k]^2 > 0$ [see (21)].

To get the corresponding tradeoffs between $\bar{r}^k$ and $1/r$ and that between p_1^k and $1/r$, simply multiply (C11) and (C12) by $-r^2$.

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