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International Dependence and Contagion across Asset Classes: The Case of Poland*

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Abstract
We investigate the linkages between international financial markets and Poland, including stocks, bonds and foreign exchange. We work in a static copula framework, allow for asymmetry of tail behavior and use tail dependence as a measure of contagion. Even though we find the overall dependence to be strong, Polish assets are to a certain extent immune to contagion from global and emerging markets. Equities are prone to only mild contagion, foreign exchange and long-term bonds are even less affected, and short-term bonds appear insulated.

1. Introduction

Two important issues in finance are the degree to which assets are priced locally or globally and differences in dependence between asset prices during crises relative to normal times (Karolyi and Stulz, 2003). These issues have serious practical implications from both the policy and risk management perspectives. However, whereas there is a large body of work on stock market dependence and contagion among developed markets, there is relatively little research on dependence between developed and emerging markets, and less still on assets other than equities. In this paper we address these questions taking an emerging market perspective and investigate the dependence structure between financial assets in Poland and abroad, including stocks, bonds and foreign exchange, with particular interest in dependence in times of crisis. Poland is an emerging economy with the biggest financial market in terms of capitalization and turnover in Central and Eastern Europe, and the three asset classes there are arguably the most liquid in the region.

The simple measure of Pearson correlation is inadequate for investigating the dependence structure and contagion, as it is only appropriate to describe dependence in multivariate normal distribution and to some extent in other elliptical distributions (for details, see Embrechts *et al.*, 2002), whereas it is well documented that returns on asset prices exhibit fat tails and deviate strongly from normality (Gabaix *et al.*, 2003). A better alternative might be to use concordance measures, such as Spearman's rho or Kendall's tau, which can capture non-linear relations between any distributions. Still, rank correlations do not provide full information

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on the dependence structure, which in turn can be captured with copula functions—the method of choice for the present study.

A copula is a function that links together univariate (marginal) distribution functions to form a multivariate distribution function (Sklar, 1959). On one hand, each marginal distribution contains all the univariate information on a given variable. On the other hand, the multivariate distribution contains all the univariate *and* multivariate information. Therefore the function that links the two marginal distributions into the multivariate distribution contains complete information on the dependence between variables, including the behavior at the center of the multivariate distribution and in its tails. Copulas have become very popular in financial modeling (for a review, see Patton, 2009), as they make it possible to model separately each marginal distribution and the dependence structure and thus allow for far greater flexibility of the multivariate distribution than known multivariate extensions of univariate distributions. This feature is particularly important for our study, as it enables us to choose from a wide range of marginal distributions and dependence structures without necessarily making strong assumptions about the characteristics of the joint price process for any two assets. Another appealing aspect of the copula framework is that it allows us to model and test both the dependence in normal times and during extreme events simultaneously, without necessarily assuming that they are similar. In addition the notion of contagion arises naturally in the copula framework. Indeed, one of the most common definitions of contagion describes it as the probability of a crisis in one country (or asset) conditional on a crisis in another (Pericoli and Sbracia, 2003). This is tantamount to tail dependence, which is a copula property. As a result, the adopted definition of contagion does not require an ad hoc identification of crisis periods. In principle, the comovement in tails can be, *inter alia*, explained by common economic exposure (i.e. fundamentals) or herd behavior. We do not differentiate between these various mechanisms.

Following the *inference function for margins* (IFM) approach proposed by Joe and Hu (1996), which is standard in the copula literature, we first estimate the parameters of each of the univariate distributions and subsequently estimate the copula functions, given the margins. We limit ourselves to a static copula and therefore a static dependence structure. This is primarily because the goodness-of-fit tests are more developed for the static case compared to dynamic copulas and we find that this feature is crucial for correct inference concerning the dependence structure and contagion. Our approach can be thought of as trying to establish and test for a static dependence structure around which a dynamic one possibly evolves. We model univariate distributions using a wide range of ARMA-GARCH model specifications, apply a number of goodness-of-fit tests and choose the one that fits best in order to minimize the impact of misspecification of the margins on the estimated dependence structure. Then we analyze the dependence structure by testing a number of parametric copula families, using the method of Genest and Remillard (2009), which was proven to outperform other goodness-of-fit tests for copula functions.

We contribute to the literature on contagion in two ways. First, the vast majority of studies that use copula functions to gain insights into dependence among asset prices concentrates on relations between equity markets only (Jondeau and Rockinger, 2002; Aloui *et al.,* 2011; Christoffersen *et al.,* 2012). Less information is available on dependence between currencies (Patton, 2006; Benediktsdóttir and

Scotti, 2009; Dias and Embrechts, 2010) and still less on bonds (Garcia and Tsafack, 2011). Moreover, in contrast with research on equities, these studies focus almost exclusively on developed markets. Second, the dependence between Polish and foreign markets has been studied usually with factor models, multivariate GARCH or in a VAR framework (Scheicher, 2001; Serwa and Bohl, 2005; Li and Majerowska, 2008; Adam, 2013; Gjika and Horváth, 2013—among others). To our knowledge, this is the first study that employs a copula framework for the analysis of dependence and contagion between Polish and foreign assets, spanning not only equities, but also foreign exchange and bond prices.

The structure of the text is as follows: Section 2 presents a review of studies on contagion, with particular focus on studies including Poland and other CEE markets. In Section 3, we present an overview of the methodology and models used. Section 4 describes the data. The main results are presented in Sections 5 and 6, followed by our conclusions in Section 7. Some details concerning the methodology and results are provided in the *Appendix* available on the website of this journal.

2. Literature Review

While the notion of contagion has received considerable attention in both theoretical and empirical work, there is no universally accepted definition of contagion in the literature. From the empirical point of view, at least five major approaches can be identified (for a review of international studies of contagion, see Pericoli and Sbracia, 2003). We list them below, presenting the results from various studies of contagion between financial assets in Poland and abroad corresponding to each of the approaches. One of the possible definitions assumes that contagion occurs when cross-country comovement of asset prices cannot be explained by fundamentals. There are two major strands of models underpinning this approach. One is based on models of multiple equilibria where fundamentals alone cannot account for the shift from one equilibrium to another. The other one is based on models of incomplete information, where even mild differences in opinions or in the degree of uncertainty can produce significant changes in behavior. While this approach has a theoretical underpinning, its application necessitates identification of fundamental transmission channels. It is plausible that the analyzed fundamental transmission channels are just a subset of the ones in the real world, making any general statement about the fundamental or non-fundamental nature of comovements prone to criticism. Perhaps this is one of the reasons why all of the other and most common approaches define contagion through statistical properties of the time series. As a result, these other approaches do not differentiate between various transmission mechanisms and contagion could reflect common shocks, bilateral fundamental exposures or herding by investors, as long as these phenomena have similar implications for the statistical properties of the data. Within this group, the first approach defines contagion as a significant increase in the probability of a crisis in one asset (or country) conditional on a crisis in another. The second line of studies defines contagion as a volatility spillover from one asset (country) to another. The third definition identifies contagion as an increased probability of comovement between variables conditional on a crisis in one market. The last commonly encountered definition of contagion identifies it as a change in the transmission channel between markets following a crisis in a market.

By far the most popular line of research is that of volatility spillovers and changes in the transmission mechanism, with the latter often studied with “correlation breakdowns”. In line with other international studies, the vast majority of analyses investigating the contagion between Poland (or often more broadly the Central and Eastern European markets) and core, global markets are focused exclusively on the stock market. The foreign exchange and bond markets have received far less attention.

Scheicher (2001) was among the first studies on the subject. Based on VAR combined with multivariate GARCH, his study finds that between 1995 and 1997 returns of the three CEE-3 stock markets (Czech Republic, Hungary and Poland) were to some extent influenced by the global markets, but there is no evidence of volatility spillovers; the spillovers exist primarily inside the region, and not from Western financial markets. Serwa and Bohl (2005) apply the correlation breakdown method based on Corsetti *et al.* (2005) to study the changes in the dependence between CEE stock markets and their counterparts in Western Europe in 1997–2002. They discriminate between contagion, interdependence and breaks in the stock market relationship and do not find evidence of contagion to the CEE markets, only interdependence. Jokipii and Lucey (2007) study the contagion within the CEE-3 stock market banking indices in 1994–2002 and, based on a test for correlation breakdowns, they do find the Polish market to be relatively resilient, despite contagion between the Czech Republic and Hungary. Li and Majerowska (2008) study volatility spillovers from the United States and Germany to the Polish and Hungarian stock markets using a multivariate GARCH framework. In a sample covering daily returns between 1998 and 2005 they find very limited contagion from the core markets.

More recently, Syllignakis and Kouretas (2011) analyze changes of the transmission mechanism from global markets to seven CEE stock markets in 1997–2009 using a DCC multivariate GARCH on weekly returns. For the majority of countries, Poland included, the correlations with the US and Germany seem to be stable (around 0.5 in the case of Poland), with the exception of 2007–2009, when they suddenly spike (to almost 0.7 for Poland). They show that the macroeconomic fundamentals have substantial power in explaining these conditional correlations during the financial crisis. Hanousek and Kočenda (2011) use high-frequency, five-minute stock returns during 2004–2007 to study the volatility spillovers between the US and German markets and the CEE-3 markets in a GARCH framework. Controlling for macroeconomic announcements, they still find strong links across markets with mature ones strongly determining the behavior of the emerging economies. Horváth and Petrovski (2013) study the comovements of the CEE-3 and Southeastern European economies (Croatia, Macedonia and Serbia) with developed markets with a multivariate BEKK-GARCH estimated on daily returns over 2006–2011. They find the CEE-3 markets to be considerably more integrated with the global markets, with correlation remaining relatively stable at around 0.6, even during the recent crisis. Gjika and Horváth (2013) extend the data sample to cover daily data on CEE stock market returns from 2001 to 2011 and examine the volatility spillovers and changes in the transmission mechanism *vis-à-vis* the euro area (STOXX50) with an asymmetric DCC multivariate GARCH. The correlation increases in time, particularly after the CEE countries’ accession to the European Union, but the financial crisis does not seem to have altered the dependence pattern—the correlation for Poland rises from approximately 0.4 in 2002–2006 to 0.6 in 2007–2011. They also find some signs of asymmetry in volatility spillovers and conditional correlations.

There are only a handful of studies of contagion between other asset classes in CEE or Poland in particular, a feature which reflects the dominant position of stock markets in the studies of contagion in general. Where currencies are concerned, Bubák *et al.* (2011) use high-frequency, five-minute data over 2003–2009 to study the volatility spillovers between the CEE-3 currencies and the US dollar *vis-à-vis* the euro in a DCC multivariate GARCH. They find that before 2008 the volatilities of the Czech koruna and the Polish zloty are, *inter alia*, affected by the long-term volatility of the EUR/USD exchange rate, while the Hungarian forint is not. Claeys and Vašíček (2014) investigate the changes in the dependence between bond prices of the EU countries over 2000–2012 based on a factor VAR model. They identify contagion as a significant change in the coefficients of the model. They find contagion to be present for all countries, with non-domestic factors explaining approximately 60% of variance on average, but the contagion within Europe is particularly strong for the euro area countries (only 25% of variance is idiosyncratic), and it increased during the crisis. For the countries outside the euro area, the intra-European contagion is either primarily limited to the region, as in case of the CEE-3 countries (idiosyncratic factors account for 50–66% of variance), or hardly present at all. For Poland, the 65% of variance in bond spreads appears to be related to idiosyncratic factors, while half of the remaining spillover comes from the other CEE-3 countries. The importance of regional differences is underlined by Adam (2013) in a cross-section of sovereign CDS spreads, including CEE countries. He finds that intra-regional spillovers are significant, while contagion spills over from distressed countries and is largely liquidity-driven, with larger markets influencing smaller markets. Polish spreads are most closely linked with Hungarian spreads. Babecký *et al.* (2013) investigate the integration between many asset classes of the CEE-3 and global markets, including stocks, bonds, foreign exchange and money market, over 1995–2012 (the data coverage differs for individual assets). They measure the degree of comovement of particular markets and global markets using common regressions and panel models with linear effects. It appears that the sensitivity of equities and bonds is similar (approx. 0.4, with 0 interpreted as independence and 1 as full comovement), whereas the CEE-3 currencies appear to be insulated from global news.

The copula framework has been used increasingly in the literature to study contagion (e.g. Aloui *et al.*, 2011; Christoffersen *et al.*, 2012; Garcia and Tsafack, 2011), primarily due to its property as a complete characterization of dependence. Still, to our knowledge there are no studies on contagion based on copula functions for the CEE countries. Christiansen and Ranaldo (2009) study the linkages between stock markets of the new EU member states and old ones based on the frequency of simultaneous extreme returns on different markets (coexceedances). This is related to the notion of tail dependence in the copula framework, though it requires specification of the specific thresholds. For the new EU member states, they find significant linkages in extreme returns with stock markets in old EU countries, as well as asymmetry between positive and negative coexceedances.

3. The Method

This section presents, in an abbreviated form, the methodology used in the paper, which is described in detail in *Appendix 1*.

Any multivariate distribution can be decomposed into marginal distributions and a dependence function between them, which is called a copula. Under an additional assumption that marginal distributions are continuous, the copula function is uniquely determined, which proves particularly convenient for the estimation of parametric distributions. The joint log-likelihood is the sum of univariate log-likelihoods and the copula log-likelihood, which suggests an estimation procedure, called IFM. It consists of separate estimation of the parameters of marginal distributions and then copula parameters conditionally on marginal distributions' fixed parameters. This is a simplified method, compared to a computationally much more involved, though asymptotically efficient joint estimation of parameters for margins and copulas by maximum likelihood. In our application we limit ourselves to two-dimensional distributions, i.e. dependence between pairs of variables. We use Matlab R2011b, A. Patton's Copula Toolbox and J.P. LeSage's jplv7 toolbox.

In the first step, we parametrically specify the marginal distributions. We decided to model the data in the broad tradition of the ARMA-GARCH (including EGARCH and GJR variants) framework, which captures most of the stylized facts observed in financial data. The orders of lagged terms are limited to five in order to favor more parsimonious representations. We allow the error term in each of the models to follow either normal or *t*-Student distribution. In the post-estimation analysis, we carry out a number of goodness-of-fit tests and choose the right one with an information criterion. Finally, using the conditional cumulative distribution function of the selected model, we transform rates of return into a uniformly distributed U(0,1) variable, as required by the IFM method. It serves as an input for the second step of the IFM method.

For dependence modeling we chose a broad set of static, parametric functions, which are the most popular in the literature. They allow a wide range of dependence relations important for investigating the contagion effect, including asymmetry and varying degrees of tail dependence. The parameters of the copulas are obtained by maximizing the respective likelihood functions. We chose the best available goodness-of-fit test in the literature on copulas (Genest *et al*., 2009). It is based on the so-called "empirical copula" (a-theoretic information on the dependence structure). The idea is to compare the distance between the "empirical copula" and the estimated parametric copula.

The final question, if the above goodness-of-fit test admits more than one copula, concerns the choice of one particular function for further analysis. We chose the parametric copula with the shortest distance to the "empirical copula", but we also check the robustness of the results by averaging over all admitted models. Then we proceed to compute measures of contagion.

The definition of contagion employed in the present paper can be operationalized with the so-called asymptotic tail dependence coefficients (hereinafter referred to as "TDCs"). The coefficients describe the propensity of markets to crash or boom together, i.e. they measure the dependence between extreme outcomes of the variables. The upper (lower) TDC is a limiting probability of one variable exceeding (falling behind) a high-order (low-order) quantile, given that the other variable exceeds (falls behind) the same quantile. If the upper or lower TDC equals zero, the respective extreme values are independent; otherwise, we say that there is dependence between the extreme values of the variables considered. Importantly, for the copulas considered in this paper the TDCs are simple functions of copula parameters.

4. The Data on Polish and Global Financial Instruments

Our data set comprises foreign and Polish variables. Foreign variables include stock indices and sovereign bond yields in the United States (SP500, US2Y, US10Y) and Germany (DAX, DE2Y, DE10Y), the VIX index, the EUR/USD exchange rate and the euro area's banking equity index, as well as emerging market indices: sovereign bond spreads (EMBI), currency return (EMFX), equity (MSCI) and an index tracking the performance of carry trades (EMCARRY). For Poland, the variables include the main stock exchange index (WIG), banking sub-index (WIG BANKS), sovereign bond yields (PL2Y, PL10Y) and the EUR/PLN exchange rate. Detailed definitions of the variables and their transformations are presented in *Appendix 2*.

Copula modeling requires long samples to catch the phenomenon of tail events which are rare by definition. We prefer the longest possible sample, however our choice is constrained by the availability of the data. We use Bloomberg data, which are reliable for all the variables in the study only from 26 November 1999. Almost 15 years of daily data (up to 31 July 2014) produces 3,829 observations, a period long enough to cover a few business cycles with both calm and crisis periods in financial markets. Specifically, the sample we choose covers the financial crisis of 2008 characterized by extraordinary volatility of financial assets, which proved quite persistent, resulting in volatility clustering. We model this phenomenon using ARMA-GARCH models.

One problem we need to address is the non-synchronicity of the daily closing price data, which stems from the fact that US variables, specifically the S&P 500 equity index and the VIX option volatility index, are traded in a significantly different time zone than the European variables. This difference means that changes in, for example, US equity prices that occur after the Warsaw Stock Exchange has closed will not appear in Polish equities until the next trading day. As a result, information sets for the two markets differ, affecting price levels and potentially distorting the analysis of dependence. To a lesser extent, this may also affect variables in the same time zone in the case of markets closing at different times. There are a few ways of dealing with non-synchronous data known in the literature (for Central European countries' data analyzed together with US data; see, for example, Baumöhl and Výrost, 2010; Schotman and Zalewska, 2006). The most popular include using relatively long time spans and/or employing synchronized prices. The first method amounts to using weekly or monthly data instead of daily data, which decreases the non-overlapping share of two data sets with regard to daily data. However, moving to weekly prices does not solve the non-synchronicity problem, but only lessens its extent. The downside of using aggregate data is the loss of observations, which results in less efficient estimators. Moreover, averaging of extreme observations, which is achieved by changing the frequency from daily to weekly, is not desired when the focus is on contagion observed in extreme realizations. The second method amounts to using prices collected when all markets are simul-

Table 1 Descriptive Statistics

	Mean	Min	Max	St. dev.	Skew.	Kurtosis	ARCH(10)	Q(20)
EURPLN	8.75E-05	-0.038	0.053	0.007	0.413	7.667	366.8*	47.9*
PL2Y	2.52E-04	-0.630	0.691	0.107	0.251	12.197	345.3*	82.0*
PL10Y	7.24E-04	-0.759	0.981	0.086	0.663	20.637	358.8*	63.4*
WIG	6.96E-05	-0.085	0.061	0.014	-0.446	6.092	225.6*	30.0***
PLBANKS	2.51E-04	-0.087	0.087	0.017	-0.005	6.270	393.8*	38.7**
VIX	-1.41E-03	-0.285	0.351	0.063	0.400	4.729	153.8*	39.3**
EURUSD	8.85E-05	-0.026	0.039	0.006	0.191	5.341	155.1*	32.5**
EMCARRY	1.80E-04	-0.056	0.035	0.007	-1.278	11.844	218.7*	34.3**
EMFX	2.20E-04	-0.028	0.018	0.004	-0.883	11.269	602.0*	61.7*
DE2Y	-1.71E-03	-0.303	0.331	0.048	0.225	8.155	180.2*	55.5*
DE10Y	-1.71E-03	-0.192	0.228	0.046	0.200	4.432	197.1*	27.7
US2Y	-1.82E-03	-0.497	0.473	0.061	-0.100	9.430	387.5*	54.2*
US10Y	-1.54E-03	-0.473	0.266	0.067	-0.046	5.134	133.1*	22.6
EMBI	-4.35E-02	-57.137	97.830	9.293	1.080	16.936	745.9*	121.9*
SP500	-6.70E-05	-0.091	0.101	0.013	-0.506	10.394	524.6*	40.9*
DAX	1.35E-04	-0.073	0.107	0.016	0.010	6.740	411.1*	24.8
EUBANKS	-1.67E-04	-0.108	0.178	0.020	0.446	10.033	246.6*	49.8*
MSCI	1.60E-04	-0.100	0.101	0.014	-0.593	11.370	685.0*	125.7*

Notes: The table displays sample statistics for yield differences (bonds), spread differences (EMBI) and daily returns (other series) between 26 November 1999 and 31 July 2014, spanning 2246 observations for each series after excluding missing data points. ARCH(10) and Q(20) are the Lagrange multiplier test of no ARCH effects up to ten lags and the Ljung-Box statistics of no serial correlation up to 20 lags.

*, ** and *** denote statistical significance at the 1%, 5% and 10% levels, respectively.

taneously open. Unfortunately, not all markets have common opening hours or such data are not readily available. A potential solution to the above challenges is to use respective opening and closing prices, so that the differences between the information sets of individual markets are minimized. This is the approach we take, calculating the returns as follows: for the Polish data we calculate returns (differences if appropriate) between two consecutive closing prices. For the US data (S&P 500 and VIX) we take the returns calculated on opening prices of two consecutive days. This allows us to significantly reduce the time mismatch. For example, for the pair S&P 500-WIG the mismatch is reduced from five hours (closing prices recorded at 10:00 PM and 5:00 PM CET respectively) to just 1.5 hours (S&P opening price at 3:30 PM CET). The resulting difference is effectively almost the same as for the weekly data and yet the dataset is substantially bigger (weekly data cover 40 hours, therefore the non-synchronicity stemming from the five-hour difference between US and European closing prices implies a 12% difference between *weekly* information sets; in our case, the 1.5-hour difference between the eight-hour trading sessions for our opening-closing *daily* data decreases the non-synchronicity from over 60% to just 18%).

Table 1 presents descriptive statistics of the dataset. All variables exhibit high kurtosis accompanied by frequently high absolute skewness. ARCH effects are present

Table 2 Spearman's ρ Coefficients

EURPLN	**VIX**	**EURUSD**	**EMCARRY**	**EMFX**
	0.220	0.011	-0.214	-0.341
PL2Y	**VIX**	**DE2Y**	**US2Y**	**EMBI**
	0.064	0.047	0.023	0.087
PL10Y	**VIX**	**DE10Y**	**US10Y**	**EMBI**
	0.111	0.111	0.063	0.115
WIG	**VIX**	**SP500**	**DAX**	**MSCI**
	-0.391	0.276	0.518	0.531
PLBANKS	**VIX**	**SP500**	**DAX**	**EUBANKS**
	-0.335	0.232	0.477	0.471

Note: The table reports Spearman's ρ calculated between Polish and foreign variables between 26 November 1999 and 31 July 2014.

in all of the variables and all but two show autocorrelation. These characteristics justify and motivate the use of ARMA-GARCH models for marginal distributions.

Table 2 reports unconditional dependence for pairs of variables based on Spearman's ρ. Transformation into ranks makes it a valid measure of monotone dependence without the stringent distributional assumptions that have been shown to be violated in *Table 1*. The variables are paired according to their class, with the exception of VIX, which is used for all asset classes, being a frequently used proxy for global factors relevant for all markets in similar studies, including emerging markets (Pan and Singleton, 2008). The general pattern is that Polish equities appear to be highly dependent on global factors that affect valuation on core markets and other emerging markets. This relationship appears considerably weaker for foreign exchange and weaker still in the case of bonds. However, it should be kept in mind that the concordance measure captures average dependence across the whole distribution and does not provide information about possibly different behavior in the tails or asymmetries.

An issue of the data which needs to be accommodated for when using copulas is the possibly negative average dependence between some variables. There are copula functions that do not allow negative dependence (i.e. Clayton, Gumbel), yet they may still be very good at capturing the dependence between transformed variables. For example, the Gumbel copula does exhibit upper tail dependence and no lower tail dependence. This is a plausible relationship between stocks in Poland and (inverted) VIX—there could be a higher propensity for WIG to fall when VIX increases significantly (bad news) than for WIG to increase when VIX falls (good news), consistent with the leverage effect for margins. Even though the asymmetric behavior could be perfectly reflected by the Gumbel copula, we would not see it, as this copula does not allow negative dependence. To allow this possibility, we estimate GARCH models on the original data and then transform the $\hat{u}_t$ series into $1-\hat{u}_t$ before using it as an input for copula estimation. This essentially reverses the rank of pseudo-observations and the dependence between variables from negative to positive without any other changes in its characteristics, so the interpretation of the results is straightforward. We use this approach for the four pairs that exhibit relatively high negative dependence (WIG-VIX, PLBANKS-VIX, EURPLN-EMCARRY and EURPLN-EMFX).

Table 3 Results for the Marginal Distributions (Short View)

	Conditional Mean	Conditional Variance
EURPLN	ARMA(0,4)	EGARCH(5,5)
PL2Y	ARMA(1,1)	EGARCH(4,5)
PL10Y	ARMA(0,3)	EGARCH(5,5)
WIG	ARMA(3,5)	EGARCH(3,5)
PLBANKS	ARMA(1,5)	EGARCH(4,5)
VIX	ARMA(1,1)	EGARCH(5,5)
EURUSD	ARMA(3,2)	EGARCH(4,4)
EMCARRY	ARMA(1,1)	EGARCH(5,3)
EMFX	ARMA(5,4)	EGARCH(3,3)
DE2Y	ARMA(4,5)	EGARCH(4,4)
DE10Y	ARMA(1,0)	EGARCH(4,3)
US2Y	ARMA(1,2)	EGARCH(5,3)
US10Y	ARMA(0,3)	EGARCH(5,5)
EMBI	ARMA(0,2)	EGARCH(5,1)
SP500	ARMA(0,1)	EGARCH(5,5)
DAX	ARMA(2,2)	EGARCH(4,4)
EUBANKS	ARMA(0,2)	EGARCH(3,5)
MSCI	ARMA(2,1)	EGARCH(3,5)

Note: All models have *t*-Student error terms

5. Univariate GARCH Models—Empirical Results

Prior to choosing the models for marginals, we estimate a broad set of ARMA-GARCH models. The selection process is as follows: first, we use the standard goodness-of-fit tests described in Section 2 to discard the models with misspecifications. It turns out that we are able to find more than one model for each variable of interest that passes the Ljung-Box, Engle's ARCH and Berkowitz distribution tests at the 5% level of significance (for the lags in the respective tests, see *Table 1*). Second, to choose the right specification from the set of candidates we use the Akaike Information Criterion (AIC), selecting the model with the lowest AIC value.

The final results of the GARCH fitting procedure are reported in *Tables A–B* in *Appendix 3*. *Table 2* is an abridged version of those tables. No single model (autoregressive or moving average) dominates the conditional mean specifications, which are predominantly some versions of ARMA. However, a clear AR structure is confirmed for the DE10Y, while a moving average representation is chosen for six variables: EURPLN, PL10Y, US10Y, SP500, EMBI and EUBANKS. Interestingly, for all series the best fit is obtained by using the EGARCH model with *t*-Student error terms for the conditional variance equation. Therefore, the choice to include asymmetric models in the set of candidates proves to be right. The leverage terms are mostly significant, indicating that the response of volatility to shocks of positive and negative signs is different in the variables. There is no obvious tendency of a certain lag length structure chosen by the AIC; the best fit is achieved for rather high order representations with three or more lags out of five allowed[1] in the variance equation.

We also note that the sum of autoregressive and moving average parameters in the variance equation is often close to 1, which indicates that volatility exhibits a high level of persistence with large changes followed by other large changes and small changes followed by other small changes. Such higher-order models are often preferred when a long span of data is used, such as several years of daily data.

With the chosen model, we construct a transformed $U(0,1)$ variable according to the procedure described in *Appendix 1*, which is used for copula analysis.

6. Dependence and Contagion—Empirical Results

Table 4 presents copulas that pass the goodness-of-fit test and are chosen as the best description of the dependence between each analyzed pair. Table C in *Appendix 3* presents the results for all the copulas considered. *Table 4* shows that *t*-Student, Symmetrized Joe-Clayton (SJC) and Plackett copulas stand out as the most common dependence structures. Interestingly, the Gaussian copula, though not rejected by the goodness-of-fit test in some cases, is not chosen as the best in any pair. This suggests that the common assumption of a Gaussian dependence structure between financial variables is not necessarily the most appropriate and other copula specifications should be tested as well. We note that 40% of copulas chosen as the best do not allow for dependence in either the upper or lower tail, i.e. there is no tail dependence in their specifications. This is the first sign suggesting that tail dependence from global assets may not be an important feature of the Polish counterparts.

Four pairs of variables do not find any acceptable representation in the set of parametric copulas that we assess. These are the pairs which characterize dependence between Polish and German or EM equities. This does not imply independence, though, since formal goodness-of-fit tests of the independence copula were also carried out and the copula was rejected. The dependence between other Polish assets and their foreign counterparts is, on the contrary, well described by the copulas we consider. Importantly, after recalculating our results on different samples it appears that the problem concerns only the WIG-DAX dependence.

Another striking observation is that for a number of pairs the goodness-of-fit test admits many copulas, often with quite different properties. This is reported in *Table C* in *Appendix 3*. The most often allowed copula is *t*-Student, followed by SJC. It thus seems that with the data available, the goodness-of-fit test alone provides rather weak guidance in choosing the copula and the second criterion, the distance to the "empirical copula", needed to be introduced. Our conjecture is that if several copulas are admitted, these cases likely correspond to a weak dependence between the variables, with estimated copula parameters implying that the copula is *close* to independence.

In the next step, for each pair of variables we compute TDCs using the formulas in *Table A* in *Appendix 1* for the chosen copula. The respective coefficients are presented in *Table 5a*. Each cell of the table reports the lower and upper TDCs. Estimated TDCs equal to or higher than 0.05—the level which we consider economically significant—are bolded and underlined. It has to be acknowledged that

[1] Although it would be possible to allow for more lags, it would require significantly greater computational effort for this part of the study.

Table 4 Chosen Copulas

	VIX	EURUSD	EMCARRY(-1)	EMFX(-1)
EURPLN	Stud (0.07)	Stud (0.22)	Stud (0.15)	Fran (0.08)
	0.229 (0.021) 19.56 (9.59)	-0.003 (0.025) 4.50 (0.60)	-0.233 (0.021) 12.51 (3.54)	-2.249 (0.140)
	VIX	**DE2Y**	**US2Y**	**EMBI**
P 2Y	symJC (0.21)	Fran (0.5)	symJC (0.56)	Plac (0.79)
	5.91E-08 (5.89E-08) 0.005 (0.008)	0.336 (0.125)	1.90E-06 (4.06E-06) 1.91E-06 (4.52E-06)	1.341 (0.089)
	VIX	**DE10Y**	**US10Y**	**EMBI**
PL10Y	Plac (0.08)	Plac (0.05)	Plac (0.99)	symJC (0.2)
	1.400 (0.095)	1.474 (0.098)	1.225 (0.080)	0.001 (0.003) 0.05 (0.02)
	VIX(-1)	**SP500**	**DAX**	**MSCI**
WIG	Stud (0.19)	rGumb (0.5)	NA	NA
	-0.406 (0.017) 34.56 (24.90)	1.187 (0.026)		
	VIX(-1)	**SP500**	**DAX**	**MSCI**
PLBANKS	Stud (0.91)	symJC (0.79)	NA	NA
	-0.339 (0.019) 24.16 (13.00)	0.099 (0.024) 0.04 (0.02)		

Notes: The chosen copulas are the copulas with the lowest distance to the "empirical copula" among the admitted ones. The upper cell for a given pair of variables contains the name of the chosen copula and the *p*-value of the goodness-of-fit test (in parenthesis, H_0: copula is correct); the lower cell contains copula parameters and asymptotic standard errors (in parentheses). In the case of *t*-Student, the first parameter is the correlation coefficient and the second is the degree of freedom; in the case of Symmetrized Joe-Clayton (SJC), these correspond to the lower and the upper tail dependence coefficients, respectively. No copula was allowed for four pairs, which is reported as NA.

choosing one copula from the allowable set (defined by the goodness-of-fit test) and making inferences based solely on this particular copula risks ignoring potentially useful information contained in the other copulas that have passed the goodness-of-fit test but with a worse fit. Also, note that we have not formally tested whether the distances of the various copulas from the "empirical copula" are *significantly* different. It is well possible that the difference between the shortest distance and the second-shortest is actually statistically insignificant. As a cross-check, then, for each pair we compute also the average TDCs over copulas admitted by the goodness-of-fit test and present them in *Table 5b*. The results obtained by averaging are qualitatively very similar to the base results, though in some cases the tail dependence increased from null (or close to it) to a few percentage points, without ever reaching the 5% economic significance threshold, however. Three broad observations can be made from

Table 5a Estimated TDCs for the Chosen Copula

EURPLN	**VIX**	**EURUSD**	**EMCARRY(-1)**	**EMFX(-1)**
	0.002 / 0.002	**0.061** / **0.061**	4E-04 / 4E-04	0 / 0
PL2Y	**VIX**	**DE 2Y**	**US 2Y**	**EMBI**
	6E-08 / 0.005	0 / 0	2E-06 / 2E-06	0 / 0
PL10Y	**VIX**	**DE 10Y**	**US 10Y**	**EMBI**
	0 / 0	0 / 0	0 / 0	0.001 / **0.049**
WIG	**VIX(-1)**	**SP500**	**DAX**	**MSCI**
	7E-11 / 7E-11	**0.21** / 0	NA	NA
PLBANKS	**VIX(-1)**	**SP500**	**DAX**	**EUBANKS**
	2E-07 / 2E-07	**0.099** / 0.038	NA	NA

Notes: Each entry contains λ_L/λ_U. 0 denotes no (zero) tail dependence as implied by the chosen copula. No copula was allowed for four pairs, which is reported as NA. Values greater than or equal to 0.05 are in bold and underlined.

Table 5b Estimated TDCs Averaged over Admitted Copulas

EURPLN	**VIX**	**EURUSD**	**EMCARRY(-1)**	**EMFX(-1)**
	0.002 / 0.002	**0.062** / **0.062**	0.024 / 0.045	0.02 / 0.02
PL2Y	**VIX**	**DE 2Y**	**US 2Y**	**EMBI**
	0.009 / 0.019	0.002 / 0.004	0.001 / 0.002	0.01 / 0.01
PL10Y	**VIX**	**DE 10Y**	**US 10Y**	**EMBI**
	0.007 / 0.013	0.013 / 0.026	0.005 / 0.005	0.021 / 0.042
WIG	**VIX(-1)**	**SP500**	**DAX**	**MSCI**
	0.001 / 0.001	**0.105** / 0.039	NA	NA
PLBANKS	**VIX(-1)**	**SP500**	**DAX**	**EUBANKS**
	0.001 / 0.001	**0.062** / 0.013	NA	NA

Notes: Each entry contains λ_L/λ_U. No copula was allowed for four pairs, therefore TDCs are not computed. Values greater than or equal to 0.05 are in bold and underlined.

the TDC analysis. First, Polish assets in general do not seem to reveal a large susceptibility to contagion. Second, asset classes differ in this regard. Third, the responses to upturns and downturns in global markets are often asymmetric.

Looking at particular asset classes, Polish equities appear most prone to contagion from the S&P 500, specifically in the lower tail (when contemporaneous crashes occur). This observation is valid both for the broad WIG index and for the banking sub-index (PLBANKS). TDCs for other pairs are rather economically insignificant (very close to zero) and these results remain largely unchanged if TDCs are averaged over admitted copulas. The above tail dependence coefficients are lower than the average probabilities found by Christoffersen *et al.* (2012) for a group of emerging and developed economies in the same period and the estimates reported by Aloui *et al.* (2011) among big emerging economies and the US stock market. These two studies differ with regard to asymmetry between the lower and upper tail dependence—Christoffersen *et al.* (2012) find the lower tail dependence to be considerably higher compared to the upper tail dependence, whereas Aloui *et al.* (2011) find no such asymmetry. For Polish stocks, both patterns emerge and, depending on the choice of foreign market, the Polish stock market exhibits symmetric or asymmetric tail dependence.

The Polish zloty appears more resilient to extreme changes in global markets than equities. The only meaningful dependence is found for the dependence on the EUR/USD exchange rate. If allowed for averaging (*Table 5b*), however, also the EMCARRY upper TDC becomes more pronounced. The above pattern may be interpreted with the notion of crash risk that is prevalent for emerging market currencies and which manifests itself in sudden depreciation of the Polish currency when carry trades unwind. Interestingly, although the overall dependence between the EUR/PLN and EUR/USD exchange rates is low (with Spearman's ρ of 1%), there is a 6% chance that they experience extreme changes together, in both the upper and lower tails. Low dependence in tranquil times should therefore not lead one to false complacency, as the strong dependence may reveal itself in times of stress. It also underscores the usefulness of copulas as a description of the full dependence structure—it may be that the correlation coefficients (both Pearson, which captures only average linear dependence, and Spearman, which captures average dependence) are close to zero, but the relationship is nevertheless far from comprising independence. Compared to contagion among the G10 currencies reported by Benediktsdóttir and Scotti (2009), the respective tail dependence between the EUR/PLN exchange rate and the VIX or the EUR/USD exchange rate are relatively low, though certainly not negligible. The literature provides mixed results concerning asymmetry—Patton (2006) and Benediktsdóttir and Scotti (2009) find it, whereas Dias and Embrechts (2010) report that the *t*-Student copula model provides the best fit. Our results suggest that both patterns are present for the zloty. Thus, similarly to equities, when considering potential dependence structure one should allow both for symmetric and asymmetric behavior in the tails.

Polish bonds differ from equities and foreign exchange in that they exhibit very limited contagion from foreign markets. Polish two-year yields do not seem to be affected by any of the external factors considered. Even if a copula admits some dependence, the computed TDC is close to zero. This may be due to the fact that yields on short-term bonds are generally determined mainly by expectations about future *local* interest rates. In the period under review, monetary policy in Poland, a not particularly open economy, operated under an inflation targeting framework and freely floating interest rates—a mix that probably contributed to interest rates primarily reflecting domestic conditions. For ten-year bonds, contagion is visible, though it is relatively weak compared to other asset classes and limited to Germany and other emerging markets (EMBI)—rapidly rising yields on German and emerging 10Y bonds translate to analogous bonds in Poland with a 3%–5% chance. This can be due to higher risk premia embedded in longer-term bonds, particularly credit risk, which has been found to comove with a global factor for a number of developed and emerging economies, as documented by Adam (2013) and Longstaff *et al.* (2011). Compared to the high degree of tail comovement between bonds on economically linked developed markets, found by Garcia and Tsafack (2011) to be usually far above 50%, the results suggest a much lower degree of contagion.

7. Conclusions

The analysis of contagion in the framework of the present paper leads to three broad conclusions. First, Polish assets seem to be, to a certain extent, immune to contagion from global and emerging markets alike. Second, asset classes differ with regard to vulnerability to contagion from foreign markets. Third, the reaction to upturns and downturns in global markets is often asymmetric. To be more specific, the highest degree of susceptibility to contagion occurs between Polish and US equities. The zloty's vulnerability to major shifts in the USD/EUR exchange rate is economically significant too, though to a smaller extent. As far as long-term bonds are concerned, contagion from global markets is relatively weak, while the lack of contagion in short-term yields may be interpreted as a result of Polish monetary policy independence. Perhaps surprisingly, extreme changes in the price of risk on global markets, as measured by the VIX index, do not increase the likelihood of extreme price changes in Polish assets. This underscores the differences in dependence between "average" and "extreme" returns.

With respect to economics, the results point to the potential benefits of international diversification. Though the tail dependence between Polish equities and global markets appears to be stronger than that of foreign exchange or bonds, it remains lower compared to contagion experienced by big emerging economies or between developed markets. The zloty does not seem to comove in tails with other emerging currencies and is independent from carry trades in G10 currencies; therefore, diversification benefits appear to be even higher, though not as high as in the case of Polish bonds.

From the methodological point of view, our results underscore the importance of a flexible modeling approach. The assumption of the Gaussian dependence structure between financial variables may not necessarily be appropriate and other copula specifications should be tested as well. In particular, in many pairs the optimal copulas exhibit potential tail behavior. Moreover, asymmetry in tails may be important—indeed, substantial differences in tail behavior were found for a couple of pairs. The coexistence of symmetric and asymmetric dependence structures as well as failure to model dependence for a couple of pairs suggest that a wider range of competing models, potentially including time-varying copulas, is advisable. We leave this for further research.

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