International pricing with costly consumer arbitrage[1]

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Abstract

Consumer arbitrage affects international pricing in several ways. If all consumers face the same arbitrage costs, a monopolist's profit increases with arbitrage costs, and world welfare declines with them (if output does not rise). If arbitrage costs differ across consumers, a monopolist may sell in a second country even if there is no local demand – it can use the second country to discriminate across consumers in the first country. Again, world welfare typically falls with arbitrage costs. When there is also local demand in the second country, world welfare may be increasing in arbitrage costs, even if output falls.

1   Introduction

Transport costs have declined rapidly over the years, and many borders are now much more open than before, while the borders of the European Com-

munity are subject to almost no customs interference anymore. This means that it has become less costly to arbitrage across markets so that the ability for firms to price discriminate on a geographical basis is severely curtailed. Competitive analysis would suggest that world surplus should rise as arbitrage costs fall. Monopoly analysis of third-degree price discrimination leads to the same result (as long as output does not fall).

Third-degree price discrimination involves selling in different markets delineated by exogenous characteristics at different prices. Pure third-degree discrimination may be rather rare, since in many cases some consumers can arbitrage between markets, although at some cost. Europeans buy their PCs in the United States for use at home, Americans used to buy German cars in Europe, etc. The boundaries of the markets are often blurred and firms account for the fact that consumers can cross from one market segment to another, but at some cost. Since consumers in the high-price market can then choose which market to buy in, there is self-selection among them. This introduces an element of second-degree price discrimination: the firm chooses prices that anticipate an endogenous split of consumers.

An analysis of pure second-degree price discrimination has some independent interest, and has not so far been applied in the context of international trade.[2] We show that a monopolist may wish to create a second market in another country in which there is no local demand for the product, in order to price discriminate across consumers in the first country. The analysis suggests that world welfare (and firm profit) rises as arbitrage costs fall.

We then integrate both second- and third-degree price discrimination

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[2]Indeed, there is almost no analysis of price discrimination with arbitrage costs in the literature. Lovell and Wertz (1981) consider discrimination in markets with leakage, but they do not explicitly model arbitrage.

within a common framework by analyzing monopoly pricing across countries when arbitrage is costly and there is local demand in the second country. A simple condition for profits to rise or fall with arbitrage costs depends on whether second- or third-degree discrimination is dominant. The analysis of pure second- and third-degree discrimination alone (described above) suggests that world welfare would fall with arbitrage costs. We find that, on the contrary, world welfare may increase if arbitrage is made more difficult.

The intuition for this result is as follows. The potential for third-degree price discrimination is curtailed when consumers in one country tend to buy in a second if the price there is low enough. However, the second country provides a channel for the monopolist to second-degree price discriminate among consumers from the first country if they can be sorted by arbitrage costs which are correlated with willingness to pay. In our model, higher arbitrage costs render second-degree price discrimination more effective. Hence profit rises with arbitrage costs if the third-degree effect dominates, and falls if the second-degree effect dominates. The welfare effects are also ambiguous. The most interesting case is when welfare improves in both countries. This can happen when higher arbitrage costs cause a large reduction in the number of consumers indulging in the (socially wasteful) operation of arbitrage. If the monopolist is based in the country from which consumers arbitrage, its profit may rise so much as to fully offset the decrease in consumer surplus there. Consumer surplus of residents in the other country also rises (the third-degree restraint having been relaxed), so that welfare rises in both countries. Hence the model provides an illustration of the proposition that higher non-tariff barriers to trade may enhance the welfare of both countries involved.

This welfare result goes in the same direction as the one in Malueg and

Schwartz (1994) who show that, if demand schedules in the various countries served by a monopolist are different enough, the possibility of parallel imports by unauthorized sellers may yield lower world welfare than third-degree price discrimination. The reason is that arbitrage forces the monopolist to set uniform prices across countries; prices may then be so high that the producer stops serving some countries, engendering a welfare loss if demands differ sufficiently.[3] In the Malueg-Schwartz model, there will be no parallel imports in equilibrium and the welfare gain from disallowing arbitrage stems from the fairly standard argument that more markets can be served under (third-degree) price discrimination. In our model which includes an additional layer of second-degree discrimination, the reason for the welfare gain is that fewer resources are wasted on costly arbitrage activities, though these do *not* necessarily disappear in equilibrium.

In Section 2, we set up the basic model. We then present two simple models of price discrimination in the presence of arbitrage costs. The first (Section 3) is more familiar, and deals with third-degree discrimination. Second-degree discrimination is addressed in Section 4. We then allow for both types together in Section 5, while Section 6 looks at a special case in order to establish the unexpected welfare result. Section 7 concludes the paper.

2 Preliminaries

Our research was motivated by the recent decision of the European Court to prevent a French firm from reimporting French cars from Belgium and selling them at a price below the standard price quoted in France. With

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[3]See also Schmalensee (1981) and Tirole (1983), chapter 3.

this example in mind, let there be two countries, C_F and C_B. A monopolist, based in C_F, sells its product in both countries. Its marginal production cost is zero, without further loss of generality. Let the prices at which it offers its product for sale be denoted by by p_F in C_F and p_B in C_B.

It is helpful to provide a disaggregate model of individual behaviour that can generate the demand side. This ensures that the demands can be given rigorous consumer-theoretic underpinnings, as well as motivating the example given in Section 6. The model uses the framework of Mussa and Rosen (1978). Each consumer buys one unit of the good if buying yields positive surplus. Let (θ, τ) denote a consumer type with willingness-to-pay $\theta \in [\underline{\theta}, \overline{\theta}]$ and arbitrage cost $\tau \in [\underline{\tau}, \overline{\tau}]$.

An individual's τ can be interpreted either as a direct cost of going to the other country (e.g. travelling to the U.S. to buy electrical goods or personal computers, which are typically much cheaper than in Europe).[4] It can also be the utility cost of buying the product designed for a different market (transformers may be needed for electrical appliances, and a car bought from an importer may not come with good after-sales service) and different people value such discrepancies differently.

Let the density of consumer types be given by $f(\theta, \tau)$. The conditional utility of a C_F-consumer of type (θ, τ) is:

$U_F = \theta - p_F$, if she buys in C_F,
$U_X = \theta - p_B - \tau$, if she buys in C_B,

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[4]Trips abroad need not be for the sole purpose of buying cheaper goods. For those who would travel anyway, τ is just the cost of bringing back the good (heavier bags and possible customs problems). For others, price differences may encourage more frequent trips and τ is then the difference between the cost of the trip and its intrinsic worth. All we need for the model is the idea that price differences can be exploited by some consumers. And the numbers can be substantial: tens of millions of tourists visit France or the U.S. each year.

$U_0 = 0$, if she does not buy.

Assuming that the support of f is the set S, the partition of C_F-consumers is given in Figure 1. Let:

$$F = \int_{p_F - p_B}^{\overline{\tau}} \int_{p_F}^{\overline{\theta}} f(\theta, \tau) d\theta d\tau$$

denote the measure of consumers for whom $U_F > 0$ and $U_F > U_X$; these are the C_F-consumers who buy in C_F. Similarly, let:

$$X = \int_{\underline{\tau}}^{p_F - p_B} \int_{p_B + \tau}^{\overline{\theta}} f(\theta, \tau) d\theta d\tau$$

denote the measure of consumers who cross, i.e. for whom $U_X > 0$ and $U_X > U_F$: these consumers buy from C_B.

[Insert Figure 1 here]

From the definitions of F and X (and from Figure 1), we note that:

$$\frac{\partial F}{\partial p_B} = \frac{\partial X}{\partial p_F} = \int_{p_F}^{\overline{\theta}} f(\theta, p_F - p_B) d\theta > 0, \tag{1}$$

and we henceforth assume that this holds.

3   Third-degree discrimination with arbitrage

The analysis discussed in this section is fairly standard. It serves as a benchmark for the later results. Suppose for the moment that the two countries are isolated in the sense that the monopolist, based in C_F, is free to set separate prices in each country, without worrying about arbitrage (in the context of

the model of the previous section, τ is prohibitive for all consumers). Suppose further that the profit functions (which, given our zero-cost assumption, are simply the total revenue functions) are strictly concave in price. Profit-maximizing prices can be characterized by:

$$F(\epsilon_F^F + 1) = B(\epsilon_B^B + 1) = 0, \tag{2}$$

where ϵ_j^i refers henceforth to the elasticity of demand i to price j (and we define own-price elasticities to be negative). Suppose that these solutions yield $p_F > p_B$. This unconstrained monopoly solution is still valid if $p_F - p_B$ is not larger than the arbitrage cost to C_F-consumers of getting the product from C_B.

Next, consider what happens when the cost of arbitrage (constant per unit and for now the same to all C_F-consumers) is less than the difference in the unconstrained prices. Denote this common cost by t (so $\tau = t$ for all consumers). Then, all the C_F-consumers would buy in C_B if the monopolist set the unconstrained prices. Realizing this, it would instead set prices under the constraint $p_F \leq p_B + t$. Given that revenue is strictly concave, the constraint binds, and the monopolist's problem:

$$max_{p_F, p_B} \Pi = p_F F + p_B B + \lambda(p_B + t - p_F),$$

where F and B are demands in C_F and C_B, yields the pricing rule:

$$F(\epsilon_F^F + 1) = -B(\epsilon_B^B + 1) = \lambda > 0. \tag{3}$$

At the corresponding prices, demand is inelastic in C_F (so p_F is below the unconstrained level), and is elastic in C_B (p_B is higher). Equivalently, marginal revenue (with respect to price) from C_F, $MR_F(p_F)$ is positive (since $\lambda > 0$) and equals minus the marginal revenue from C_B, $MR_B(p_B)$. The solution is illustrated in Figure 2.

[Insert Figure 2 here]

From the profit problem (and from the Figure), $d\Pi/dt = \lambda > 0$ so that raising the arbitrage cost raises profit by allowing a larger price spread and bringing the monopolist closer to the unconstrained optimum. Totally differentiating the equilibrium condition $MR_F(p_F) = -MR_B(p_F - t)$ shows that an increase of t raises p_F, decreases p_B and increases the price difference by Δt.[5]

A sufficient condition for world welfare (defined as the sum of consumer surplus in C_F and C_B plus profit) to fall as t rises is that output does not rise with t. The reason follows from the analysis of Schmalensee (1981): for a given amount of output to be allocated across two markets, total welfare is greater the more similar are the two prices.[6] Raising t raises the wedge between the prices and hence the difference between the willingness-to-pay of the marginal consumer in each market. If output does not rise, welfare then necessarily falls with t.[7] The results for third-degree price discrimination are now summarized:

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[5]Nahata et al. (1990) have shown that both prices can decrease or increase if the profit functions are not concave. See also Malueg (1992).

[6]This argument shows that welfare is higher without discrimination if output is not lower and both markets continue to be served. The argument was extended to non-constant marginal costs by Schwartz (1990).

[7]Output is independent of t if demand is linear in each market, so that welfare necessarily falls with t in this case. The outpu condition is sufficient, but not necessary, for a welfare decrease.

Proposition 1. If all consumers in C_F face the same arbitrage cost t, and under the conditions above, then increasing t raises profits, raises consumer surplus in C_B and reduces it in C_F. Total surplus falls if output does not rise.

If arbitrage costs of C_F-consumers differ, the monopolist might use these differences to discriminate across them. Even if there were no consumers in C_B, the monopolist might still want to sell goods there, targeting only C_F-consumers, in order to segment the C_F market. A similar situation arose recently when Canadian cigarette manufacturers exported a large fraction of their output to the United States, even though few US-citizens tend to buy Canadian cigarettes (since the taste is different) – the cigarettes were primarily sold to Canadians.[8] This type of situation is explored in the next Section.

4   Second-degree price discrimination

In order to isolate the effects of pure second-degree discrimination, suppose, for this section, that there is no domestic market for the monopolist's product in C_B, but that C_F-citizens differ as to their arbitrage costs. Let X denote the cross-border traffickers, and F denote the C_F-consumers buying in C_F. The demands from these consumers in C_F depend on p_B and p_F. Equivalently, we can write them as $X(p_B + t, p_F)$ and $F(p_F, p_B + t)$, with both demands decreasing in the first argument and increasing in the second one and here, t is that part of the arbitrage costs common to all consumers. This formulation

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[8]Canadian Prime Minister Jean Chretien told the House of Commons that illegal cigarettes account for 40% of Canada's \$9.3 billion tobacco market and up to two thirds in Quebec. See *The Washington Post,* Feb. 9, 1994. The high tax on tobacco in Canada likely accounts for much of the arbitrage, but the example does highlight that arbitrage can be an important factor.

enables us to analyze changes in arbitrage costs (so a one dollar increase raises the arbitrage cost by one dollar for all consumers). We also assume that (1) holds. We first show with a simple example that the monopolist may use C_B to discriminate across C_F-consumers.[9]

Within the context of the model sketched at the end of Section 2, suppose that there are only two consumer types. Type $\overline{\theta}$-consumers have prohibitive arbitrage costs $\overline{\tau}$, and $\underline{\theta}$-consumers have arbitrage costs $\underline{\tau}$, with $\overline{\theta} > \underline{\theta} > \underline{\tau}$.[10] Let there be $\overline{N}$ of the former type and $\underline{N}$ of the latter. The monopolist has two basic choices. It could sell only in C_F to everyone at price $p_F = \underline{\theta}$; or it could set $p_F = \overline{\theta}$, $p_B = \underline{\theta} - \underline{\tau}$ and let consumers self-select. Profits are respectively $\underline{\theta}(\overline{N} + \underline{N})$ and $\overline{\theta}\,\overline{N} + (\underline{\theta} - \underline{\tau})\underline{N}$. The condition for the monopolist to open the C_B market is $\overline{N}/\underline{N} \geq \underline{\tau}/(\overline{\theta} - \underline{\theta})$.[11] It shows that the monopolist is more likely to open a market in C_B the lower the transaction cost for the low willingness-to-pay consumers, the larger the difference in the willingness-to-pay parameters, and the larger the proportion of high willingness-to-pay types.

Leaving this example, to derive the elasticity formulae for monopoly pricing, assume that X and F are differentiable. The firm's profit function is:

$$\Pi = p_F F + p_B X. \tag{4}$$

At an interior maximum (and using (1)):

$$\frac{\partial \Pi}{\partial p_F} = p_F \frac{\partial F}{\partial p_F} + F + p_B \frac{\partial F}{\partial p_B} = 0 \tag{5}$$

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[9] "If there were no Belgium, it might be invented."

[10] It is sufficient that $\overline{\tau}$ exceed $\underline{\tau} + (\overline{\theta} - \underline{\theta})$ for arbitrage costs to be prohibitive for the $\overline{\theta}$-consumers. This would give them a negative utility, should they buy in C_B.

[11] If this condition does not hold, the monopolist sells to all consumers at the price $\underline{\theta}$.

and

$$\frac{\partial \Pi}{\partial p_B} = p_F \frac{\partial F}{\partial p_B} + p_B \frac{\partial X}{\partial p_B} + X = 0. \tag{6}$$

Using (5), one can see that the marginal revenue (w.r.t. price) from C_F-consumers buying in C_F is negative, suggesting that p_F is higher than if there were no arbitrage. The idea here is that the monopolist will concentrate on extracting more surplus from the top end of the C_F-consumer market in C_F, and will use C_B to get the lower end consumers.[12] Indeed, this is what happens in the model at the beginning of the section if $\overline{N}/\underline{N} < \underline{\tau}/(\overline{\theta} - \underline{\theta})$ (so the monopolist prefers to set the low price $\underline{\theta}$ and serve all, if consumers cannot be segmented through the device of the second market) and the condition for opening C_B is also met.

We are primarily interested in the effects of increasing t, the cost of arbitrage. From (4), using the Envelope Theorem and the fact that $\partial Z/\partial t = \partial Z/\partial p_B, Z = F, X$, we have:

$$\frac{d\Pi}{dt} = p_F \frac{\partial F}{\partial p_B} + p_B \frac{\partial X}{\partial p_B}. \tag{7}$$

The first term is positive and the second is negative. Using (6), we can write (7) as:

$$\frac{d\Pi}{dt} = -X. \tag{8}$$

The reason for which $d\Pi/dt$ is negative is clear: if it were positive, the monopolist's price would not be optimal, since it could raise profit by raising p_B. This would have exactly the same effect on demand shifts but would increase revenue from consumers in C_B.

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[12]The elasticity forms of (5) and (6) are $F(\epsilon_F^F + \epsilon_B^F + 1) = 0$ and $X(\epsilon_F^X + \epsilon_B^X + 1) = 0$, respectively. These equations show that revenue from each market segment is unchanged if both p_F and p_B are raised by one percent. Thus the total profit of the monopolist is unchanged. Despite the fact that the markets are interrelated, the elasticity equations exhibit a strong form of separability.

The change in consumer surplus is:

$$\frac{dCS}{dt} = -X(1 + \frac{dp_B}{dt}) - F\frac{dp_F}{dt}. \tag{9}$$

From (8) and (9), the change in total welfare, W, is:

$$\frac{dW}{dt} = -X(2 + \frac{dp_B}{dt}) - F\frac{dp_F}{dt}. \tag{10}$$

The analysis above gives us the following proposition:

Proposition 2. An increase in t decreases profits. Moreover, world welfare cannot increase if $dp_B/dt + 2 \geq 0$ and $dp_F/dt \geq 0$.

The conditions given in Proposition 2 are sufficient but not necessary for world welfare to fall with t. We could expect that the two price derivative conditions would usually hold. If transaction costs rise, we would expect p_B to fall to counteract the rise in t, but by less than the increase in t. Hence, we would expect $dp_B/dt > -1$ (a "no overshift" condition) and the first price condition would hold. The condition $dp_F/dt \geq 0$ is less obvious. We note that it holds (albeit weakly) in the example given at the beginning of this section, and that it also holds for the integrated model of Section 6. It is also true that if indeed $dp_B/dt > -1$, an increase in t will raise quantity demanded in C_F if no action is taken on p_F. One would then expect that p_F would be increased to bring back quantity demanded in C_F closer to where it was. We tentatively conclude that the "usual" case for this model is $dW/dt < 0$.

5 An integrated model

The pure second-degree model of the previous section is amended when there is a home market in C_B, by adding a term $p_B B(p_B)$ to the profit function.[13]

The new expression for the derivative of the profit function with respect to t is:

$$\frac{d\Pi}{dt} = -X - B(\epsilon_B^B + 1). \tag{11}$$

The sign of (11) can be positive or negative. The first term is negative, and is the second-degree effect (see equation (8)). The second term is the third-degree effect (see equation (3)). If (11) is negative, we say that second-degree price discrimination dominates; if positive, third-degree dominates. In the latter case, $\epsilon_B^B < -1$, so that if profit is to increase with transactions costs, then it must be the case that the price in C_B is higher than it would be if arbitrage costs were prohibitive (pure third-degree case). This analysis highlights the fundamental tension of arbitrage costs between second- and third-degree discrimination.

The welfare analysis for the general integrated model is too complex to be illuminating. For this reason, we provide a specific example in the next section. As background for this example, recall that world welfare falls with transaction costs for the pure third-degree price discrimination model with arbitrage (at least, as long as output does not rise); the same is true in the case of pure second-degree price discrimination (for reasonable equilibrium price derivatives). Nevertheless, even though all the qualifying conditions are met, the integrative model behaves quite differently from what would be expected from the behaviour of the component models.

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[13]The elasticity forms of the first-order conditions are then $F(\epsilon_F^F + \epsilon_B^F + 1) = 0$, which is the same expression as for the pure second-degree model, and $X(\epsilon_B^F + \epsilon_B^X + 1) + B(\epsilon_B^B + 1) = 0$, which differs from before by the addition of the second term that is MR_B.

6 The integrated model with linear demands

We use the framework set out in Section 2, with the additional restriction that θ is uniformly distributed on $[0, 1]$ and $\tau = b\theta + t, b \in [0, 1), t > 0$.[14] Thus consumers differ over the total transaction cost: If, for example, the consumer characteristic θ is correlated with income, the assumption implies that rich C_F-consumers are less inclined to arbitrage; their valuation of, say, the time lost by buying in C_B is higher than that of poor consumers. Finally, let $1 - b - t > 0$, so that the C_F-consumers with highest θ get positive surplus from buying in C_B when $p_B = 0$.

Consumers in C_F partition themselves into three segments. These are derived by noting that the consumer indifferent between buying in C_F or in C_B is given by $\theta_{FF} = (p_F - p_B - t)/b$. The consumer indifferent between not buying and buying in C_B is $\theta_{FB} = (p_B + t)/(1 - b)$. The demand segments from consumers located in C_F are therefore $X(p_B + t, p_F) = \theta_{FF} - \theta_{FB}$ and $F(p_F, p_B + t) = 1 - \theta_{FF}$. Domestic demand by consumers in C_B is simply $B(p_B) = 1 - p_B/\alpha$, where $0 < \alpha < 1$, so that willingness to pay for C_B-consumers is less than that of C_F-consumers. Figure 3 illustrates the different demand configurations.[15]

[Insert Figure 3 here]

The profit maximizing first-order conditions yield:
$$p_F = \frac{(1 - b)(t + b) + \alpha(2 - b)}{2(1 - b + \alpha)}$$
and
$$p_B = \frac{\alpha(2 - 2b - t)}{2(1 - b + \alpha)}.$$

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[14]The model of Section 3 corresponds to $b = 0$.

[15]Note that $t + b > t/(1 - b)$, since we assumed earlier that $1 - b - t > 0$. Otherwise there would be no X-segment.

The appendix gives the proof of the following result:

Proposition 3. For the linear model, there is a set of parameters $\{t, b, \alpha\}$ such that:

(a) there exists an interior solution which maximizes profit;

(b) an increase in the minimum transaction cost t increases profits as well as world welfare even though world output decreases; moreover, $dp_F/dt > 0$ and $dp_B/dt + 2 > 0$.

Proof. See Appendix 1.[16]

For such parameter values, higher transaction costs to arbitrage cause p_F to rise and p_B to fall, thus increasing the distortion. Despite this, welfare rises. The reason for this surprising result is as follows. When t rises, the number of individuals purchasing abroad falls dramatically, more than offsetting the higher transaction costs for the remaining arbitrageurs, so aggregate transaction costs fall. For the parameter values used in the proof of Proposition 3, $\partial T/\partial t = -0.372$, where $T = \int_{\theta_{FB}}^{\theta_{FF}} (t + b\theta) d\theta$ is the total value of transaction costs. Thus, the direct social loss due to arbitrage falls when t rises and this reduction more than outweighs the extra distortion due to larger price differentials. Indeed, for all parameter values where welfare rose with t we found that total transaction costs fell.

The result is even more surprising since the intuition from Propositions

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[16]The proof is given for values $t = 0.05$, $b = 0.35$ and $\alpha = 0.20$. The parameter values for which the Proposition holds are not unique; we have computed ranges of parameters which give similar results. For example, there are values of b and t such that the Proposition is true for $\alpha \in [0.02, 0.40]$. When $\alpha = 0.2$ for example, t can take values in $[0.01, 0.095]$ for appropriate b. Finally, when $\alpha = 0.2, t = 0.05$ (the case chosen to illustrate Proposition 3), the result holds for $b \in [0.319, 0.627]$.

1 and 2 would suggest that if output falls while p_F rises, and p_B falls by less than Δt, then welfare should decline. Thus the full model is more than the sum of its parts. As Proposition 3 shows, the opposite happens. The model is also noteworthy in that it gives a case in which world welfare moves the same way as firm profits. What is good for Peugeot is good for France, and, here, for Belgium too.

7 Conclusions

Firms may use third-degree price discrimination across countries, but large price differentials induce consumer arbitrage. This may be particularly important nowadays, due to the record numbers of tourists and business travellers. The economic analysis of firm pricing under (costly) arbitrage involves a combination of second- and third-degree price discrimination. That is, once consumers arbitrage, firms can use the difference in arbitrage costs to discriminate against consumers with high costs (and high willingness-to-pay). Since consumers "self-select," this is second-degree discrimination. Combining second- and third-degree price discrimination leads to the counterintuitive result that world welfare may increase as arbitrage is made more difficult

The model of this paper describes a producer who sells in two countries, and is able to price discriminate given that arbitrage is not perfect. Although a French consumer can buy a French car in Belgium, where it is less expensive than in France, he will have to undergo tedious and long (but quite inexpensive) formalities in order to register the car in France.[17] We show that it may

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[17]Several other situations can be described by the integrative model of second- and third-degree price discrimination. Many producers price discriminate but have to account for consumers who have some freedom to arbitrage for the better deal. Telephone calls are cheaper at evenings and week-ends, and some consumers postpone their calls to the

be in the interest of the French car producer, of France as a country, and of Belgium, to keep arbitrage from becoming too easy. And this is indeed one possible way to interpret the judgement, in the late 1980s, of the European Court of Justice, which ruled against Eurosystem, a French firm, that had started importing French cars from Belgium, and selling them in France at a discounted price.[18] The Court was caught between the general policy of abolishing all non-tariff barriers among EC countries, and protecting the French car industry. As the model shows, however, this move may have been welfare improving for both countries.

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evening (these are C_F-men who buy in C_B); but there are also consumers whose θ is high (the "rich" ones), and who call during the day (C_F-men who buy in C_F). Likewise, cheaper Apex air tickets are available for those customers who are ready to trade flexibility for rigid reservation dates and to spend the week-end away from home.

[18]The Court did not rule out Eurosystem importing cars from Belgium if they had been ordered by individual customers. The fact that the French manufacturer initiated the complaint suggests that thid-degree discrimination was dominant, in the sense of eq. (11). Note that in this context, τ reflects the welfare loss due to the lack (or the lower level) of after-sales service, as compared to that provided for domestically purchased cars.

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9   Appendix. Proof of Proposition 3

(a) Existence of equilibrium

Using the results of Section 6, for the parameter values $t = 0.05$, $b = 0.35$, $\alpha = 0.20$, we find:

$p_F = 0.3471, p_B = 0.1471,$
$\theta_{FF} = 0.4286, \theta_{FB} = 0.3032,$
$F(.) = 0.5714, X(.) = 0.1255, B(.) = 0.2645,$
$\Pi = 0.2557.$

The profit function is negative definite, so that this solution is a local maximum. To verify that it is a global maximum, we must also check that profit cannot be increased by choosing prices such that one or two of the market segments are not served, i.e. such that one or more of the demands $F(.), X(.)$ or $B(.)$ are zero. The various possibilities are illustrated in Figure 3.

(i) The monopolist could set p_B so high that nothing is sold in C_B. Hence, $X(.) = B(.) = 0$ and $p_F = 1/2$ with $\Pi = 1/4$; this is the monopoly profit for market C_F, which is less than the profit obtained above (0.2557).

(ii) $F(.) = B(.) = 0$. This is clearly dominated by (i), since the demand of C_F-consumers in C_F always exceeds the demand of C_F-consumers in C_B because of the positive transaction cost.

(iii) If $F(.) = 0$ and if $p_B < \alpha$, so that $X(.) > 0$ and $B(.) > 0$, profit can be increased by reducing p_F. The only effect (as long as p_F remains higher than $(p_B + t)/(1 - b)$), is to convert X-type buyers into F-type buyers. This is profitable because p_F exceeds p_B.

(iv) $X(.) = 0$, with $F(.) > 0$ and $B(.) > 0$. Profits necessarily increase by raising p_F to $(p_B+t)/(1-b)$, as long as this is less than $1/2$ (the unconstrained monopoly price in C_F). This condition holds in the present case since the maximum value of p_B consistent with $B(.) > 0$ is 0.20, and $t = 0.05, b = 0.35$. Again, such a move brings us to the boundary of the region where all demands are positive and the profit function is negative definite there.

(v) $B(.) = 0$, $F(.) > 0$, $X(.) > 0$. This corresponds to pure second-degree price discrimination, where the C_B market only helps the firm to discriminate among C_F-consumers. In this case, the profit of the firm is $\Pi = p_F F + p_B X$ and first-order conditions lead to $p_F = 1/2$ and $p_B = (1 - b - t)/2$, or $p_B = 0.3$ for the parameter values under consideration. These prices yield $X(.) = \theta_{FF} - \theta_{FB} = -10/91 < 0$, so that this solution is not interior. Hence, the optimizing solution for this region must lie on its boundary, and we have already shown that all other cases give lower profits.[19]

Therefore, the solution found is an equilibrium.[20]

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[19]The optimal profit is $\Pi = 0.2527$ (ignoring the constraints), which is lower than the profit of 0.2557 obtained for the case in which all three markets are served.

[20]Depending on the parameter values of α, t and b, the monopolist's choice of p_F and p_B may lead us to other regions of the partition in Figure 3. The monopolist will never choose prices such that $X = F = B = 0$ (since costs are zero and demand is positive), nor will it choose $X = F = 0$ with $B > 0$ (since $\alpha \leq 1$). By arguments (ii) and (iii) respectively, neither will it choose prices in regions $F = B = 0$, $X > 0$, or $F = 0$, $X > 0$, $B > 0$. If b or t is large enough with α small enough, the monopolist will choose $p_F = 1/2$ and will not sell in C_B, which is the region for which $X = B = 0$ and $F > 0$. Changing t has no effect in this region. If b and t are not too large, and α is small enough, the monopolist will choose to be in the region where $B = 0$, $X > 0$, $F > 0$. This is the case of pure second-degree discrimination treated in Proposition 2, and here both profit and welfare fall with t. If t or b is large, with α not too large, the monopolist's choice will be in the region where $X = 0$, $F > 0$, $B > 0$. This corresponds to standard third-degree discrimination and changes in t have no effect locally, except if $b = 0$ when the case described in Proposition 1 holds: then higher t increases profit and decreases welfare.

(b) Welfare results

We now determine the effects of increasing the transaction cost t on total demand, profit and welfare. From the expressions for $X(.)$ and $B(.)$ derived in Section 6, total demand is $Q = F(.) + X(.) + B(.) = 2 - \theta_{FB} - p_B/\alpha$, so:

$$\frac{dQ}{dt} = \frac{-\alpha - 1 - b}{2(1-b)(1-b+\alpha)}.$$

The effect on profit can be computed from (15). The changes in consumer surplus in C_F and C_B are respectively:

$$\frac{dCS_F}{dt} = -F\frac{dp_F}{dt} - X(1 + \frac{dp_B}{dt}) = -F\frac{1-b}{2(1-b+\alpha)} - X\frac{2-2b+\alpha}{2(1-b+\alpha)}$$

and

$$\frac{dCS_B}{dt} = -B\frac{dp_B}{dt} = -B\frac{-\alpha}{2(1-b+\alpha)}.$$

If the transaction cost rises, total output and consumer surplus in C_F always decrease, while consumer surplus in C_B always increases; the effect on the firm's profit is ambiguous. For the numerical example, we find:

(a) $\partial Q/\partial t = -0.7692$

(b) $\partial \Pi/\partial t = 0.3452$

(c) $\partial CS_F/\partial t = -0.3292$

(d) $\partial CS_B/\partial t = 0.0312$.

Summing (b) and (c) shows that total welfare (profits plus consumer surplus) rises in C_F. Thus, an increase in transaction (or arbitrage) costs decreases total output (which is expected), but it also increases profit and total welfare in both C_F and C_B.